Impact-Time-Control Guidance Law for Hypersonic Missiles in Terminal Phase

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ABSTRACT Recent researches on hypersonic vehicles adopted in military have gained a lot of interest because of its high flight speed and efficiency. However the guidance laws for hypersonic missiles against the anti-missile system to increase survivability and defense penetration ability are still need to study. One main countermeasure to deal with this issue is to implement simultaneous attack. Therefore in this paper an impact-time-control guidance law for hypersonic missiles to impact a stationary target at the same desired final time is presented. Time-to-go estimation for time-varying velocity is derived by using a method of approximate acceleration form to extend the estimation method for constant velocity. The impact-time-control guidance law is firstly given and applied in vertical plane based on proportional navigation guidance law to control the impact time in terminal phase. Then the conditions that limit the method to apply are discussed. To overcome this issue, the guidance law applied in lateral plane is derived. The simulations are implemented using each method in vertical and lateral planes respectively and show good results for hypersonic missiles to achieve simultaneous attack.

INDEX TERMS Hypersonic missile, impact-time-control, time-to-go estimation, time-varying velocity, simultaneous attack, vertical and lateral planes.

I. INTRODUCTION

Hypersonic vehicles (HSVs) are usually considered to have speeds at Mach 5 and above while cruise altitudes are usually 80,000 feet (25 km) and above [1]. With the development of the propulsion system [2]–[4], it allows HSVs to launch at a low speed or even stationary state to speed up to hypersonic speed. So hypersonic missiles (HSMs) are expected to launch from various platforms to destroy ships and ground targets in a foreseeable future. But on the other hand, because of the development of antimissile defense systems, such as interception missiles and close-in weapon systems (CIWS) which will intercept and destroy the incoming missiles from long and short ranges, it becomes more and more difficult for missiles to complete their missions. Now a common and practical countermeasure to increase the survivability of the missile weapon system is to launch several missiles and impact the target at a same final time. So developing impacttimecontrol guidance (ITCG) laws for hypersonic missiles to achieve simultaneous attack can be of great help to

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increase survivability and complete the missions. In order to control the impact time, time-to-go estimation is always used combining with certain guidance methods, such as proportional navigation guidance (PNG) law. The ITCG law was first suggested in [5] by Jeon for constant velocity missiles to impact stationary target. The guidance law could reduce the miss distance, minimum the control effort and control the impact time. The impact time error was defined as the error between the desired impact time and the impact time guided by PNG law and introduced into PNG law as a feedback loop combined with traditional optimal feedback loop. The linearized state equations were derived by small heading angle assumption and the navigation gain was constant. In [6] Jeon extended the previous work with nonlinear formulation and an arbitrary constant navigation gain. The target could be stationary or non-maneuvering moving. There are many other researches combining the time-to-go estimation with PNG law. In [7] Dhananjay obtained the time-to-go estimation by interpolation method. It could be used with large initial heading errors. The target was assumed to be stationary or have constant velocity and the missile velocity was constant. In [8] Cho derived a closed-form solution for time-to-go estimation

of pure proportional navigation guidance (PPNG) against stationary target without any linearization or approximation. The ITCG law was presented based on PPNG with a time-varying nonlinear navigation gain and guaranteed the asymptotic stability of the time-to-go error. In [9] Ghosh derived unified time-to-go algorithms as closed-form approximation functions of range, navigation gain and heading error based on recursive numerical computations. The algorithms could be applied in PNG laws with both negative and positive navigation gains in three-dimensional (3D) engagement scenarios against stationary targets and the targets whose velocities are lower or several times higher than the missiles.

Some ITCG laws are proposed based on nonlinear control theories, such as sliding-mode-based and Lyapunov-based guidance law. In [10], [11] Kumar and Cho respectively presented an ITCG law based on sliding mode control (SMC) for constant velocity missiles against stationary target or non-maneuvering moving target. In [10] the guidance law was derived using nonlinear engagement dynamics and could be applied even if the interceptor is launched with large heading angle error. In [11] Cho introduced a positive continuous nonlinear function of the missile's leading angle to avoid the singularity of guidance command and the guidance law could be applied regardless of the initial conditions and a wide range of the capture region can be guaranteed. In [12] Kim and [13] Saleem each presented an ITCG law based on Lyapunov theory for constant speed missiles against stationary target. In [12] Kim derived two-dimensional and three-dimensional ITGC laws using nonlinear kinematics. The singularity issue of the guidance law was analyzed. In [13] Saleem derived an exact closed-form impact time expression in terms of a beta function of the initial heading error and initial range.

Besides the two main categories of realizing ITCG aforementioned, there is another sort of methods using trajectory-shaping-guidance concept to derive the guidance law. In [14] Tekin presented an ITCG law based on polynomial look angle profile for constant speed missiles against stationary target. The guidance law applied the remaining engagement time instead of time-to-go estimation and the guidance gain was calculated by numerically solving an integral equation. In [15] Tekin extended the previous work by using adaptive guidance schemes by predicting the mean velocity to update the guidance gains in order to deal with the missiles time-varying velocity problem.

From the aforementioned articles it can be informed that the ITCG laws were mostly proposed for constant speed missiles. Even in [15] Tekin and [16] Zhou presented guidance laws considering the time-varying velocity problem, the speed and its varying range is relatively smaller compared with HSMs and this is one reason why the methods are no longer valid. In [17] Jiawei Wang presented an ITCG law for HSVs in terminal phase. The guidance law was implemented in the vertical plane and the time-to-go estimation was obtained by numerical integrating the PNG trajectory which might be too much computation for missile-borne computer.

The main effort of this article is to give accurate time-to-go estimation method for HSMs with time-varying velocity and to apply the method in impact-time-control guidance. The time-to-go estimation represents how much time it will take if the vehicle is guided by PNG law from the current moment to the moment it impacts the target and it is derived based on the traditional PNG law. Once the time-to-go estimation is obtained, it is compared with the desired time-to-go to obtain the time-to-go error. Then the time error is fed back into the guidance law to eliminate itself to complete the guidance loop and realize the time-to-go control. In general once the time-to-go error is eliminated to zero, the guidance law is equal to the traditional PNG law.

In this note an ITCG law combining PNG to achieve a desired impact time for HSMs is presented. The time-to-go estimation method for time-varying velocity is derived based on applying an assumption form of the acceleration. The performance comparison of the time-to-go estimations and ITCG laws between [5] and the method proposed in this article are presented in Section III. The guidance law is implemented in vertical and lateral planes respectively. The performances and differences are discussed.

This paper is organized as follows: Section II describes the basic assumptions and the engagement kinematics. Section III gives the time-to-go estimation mothed for time-varying velocity and two strategies to implement ITCG law in vertical and lateral planes respectively. The simultaneous attack simulation is presented in Section IV.

II. PROBLEM FORMULATION

A. BASIC ASSUMPTIONS

Before introducing the engagement kinematics, several basic assumptions are made as follows:

Assumption 1: The target is stationary.

Assumption 2: The missile is regarded as an ideal mass point model.

Assumption 3: The distances between each missile and the target are within a limit range, not too large from each other.

Note the assumptions above are very common. In Assumption 1, the target is assumed to be ship or ground stationary target. The velocity of the ship is much lower compared with the hypersonic missile and the terminal phase is relative short, so it can be regarded as stationary. In Assumption 2, it is assumed that there is no lag in guidance loop and actuator. The control system can completely meet the guidance commands. So only the guidance problem is concerned and the missile can be regarded as an ideal mass point model. In Assumption 3, the impact-time-control guidance has its ability limitation of adjusting the impact time, so the distances between each missile are limited.

B. ENGAGEMENT KINEMATICS

The engagement motion geometry is shown in Fig.1. *MT* is the line-of-sight (LOS), which is described by the azimuth angle ψ_A and the elevation angle θ_E.
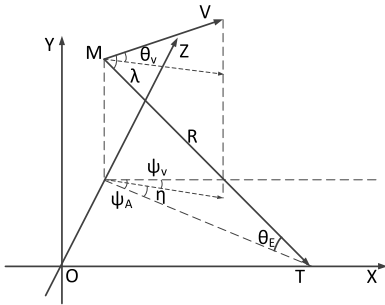
FIGURE 1. The engagement geometry.

The motion equations are given as follows:

$$\dot{x} = V \cos \theta_V \cos \psi_V$$
$$\dot{y} = V \sin \theta_V$$
$$\dot{z} = V \cos \theta_V \sin \psi_V \quad (1)$$

The LOS and LOS angle rate are given as follows:

$$\dot{R} = -V \cos \lambda$$
$$= V \sin \theta_V \sin \theta_E - V \cos \theta_V \cos (\psi_A - \psi_V) \cos \theta_E \quad (2)$$

$$\dot{\psi}_A = \frac{V \cos \theta_V \sin (\psi_A - \psi_V)}{R \cos \theta_E} \quad (3)$$

$$\dot{\theta}_E = \frac{V \cos \theta_V \cos (\psi_A - \psi_V) \sin \theta_E - V \sin \theta_V \cos \theta_E}{R} \quad (4)$$

The kinetic equations of the missile are given as follows:

$$\dot{V} = -\frac{D}{m} - g \sin \theta_V \quad (5)$$

$$\dot{\theta}_V = \frac{L \cos \gamma_V}{mV} - \frac{g \cos \theta_V}{V} \quad (6)$$

$$\dot{\psi}_V = \frac{L \sin \gamma_V}{mV \cos \theta_V} \quad (7)$$

where OYXZ is position coordinates. M is the missile. T is the target. R is the distance between the missile and the target. V is the velocity of the missile. θ_V is the fight path angle. ψ_V is the heading angle. γ_V is the bank angle. L and D are the aerodynamic lift and drag forces. m is the mass. λ is the look angle, the angle between the velocity and the LOS. η is the look angle in lateral plane. Anticlockwise angle is positive, otherwise is negative.

C. AERODYNAMIC MODEL
The aerodynamic model of the missile used in this article is the same in [18] The aerodynamic coefficients given in [18] are consisted of different parts such as the aerodynamic calculation part, the engine calculation part and the nozzle calculation part. The variables used to calculate the aerodynamic coefficients include Mach number Ma, angle of attack α, angle of sideslip β, deflection angles of left and right wings δ_{e1}, δ_{e2}, and deflection angle of vertical tail δ_r. Not all of the variables are used according to the specific situation in this article so $\beta = 0, \delta_{e1} = 0, \delta_{e2} = 0$ and $\delta_r = 0$ are

set while using the aerodynamic model, so the aerodynamic coefficients are given as:

$$C_L = C_{La} + C_{Le} \quad (8)$$
$$C_D = C_{Da} + C_{De} \quad (9)$$
$$C_{La} = C_{La} (Ma, \alpha)$$
$$= 0.1498 - 0.02751 Ma + 0.07235 \alpha$$
$$- 0.003368 \alpha Ma + 0.002343 Ma^2 + 0.001185 \alpha^2$$
$$C_{Le} = C_{Le} (Ma, \alpha)$$
$$= 0.7215 + 0.02635 \alpha + 0.1147 Ma$$
$$- 0.002795 \alpha Ma - 0.5782 Ma^{1/2}$$
$$C_{Da} = C_{Da} (Ma, \alpha)$$
$$= 0.05099 - 0.004863 Ma + 0.002967 \alpha$$
$$+ 0.001364 \alpha^2$$
$$C_{De} = C_{De} (Ma, \alpha)$$
$$= 0.002339 \alpha + 0.00012182 \alpha^2 - 0.00033126 \alpha Ma \quad (10)$$

The reference area is $S = 0.2986$ m^2, the mass of the missile is $m = 671.33$ kg. An exponential form of the air density ρ is given as [19]:

$$\rho = \rho_0 \exp \left(\frac{h_0 - h}{h_s} \right) \quad (11)$$

where h is the height, the detail is in [19].

III. IMPACT TIME CONTROL GUIDANCE LAW
A. ITCG LAW IMPLEMENTED IN VERTICAL PLANE
1) THE ENGAGEMENT GEOMETRY
If the terminal guidance law is implemented in vertical plane, the condition for the missile to enter the terminal phase should be as follows:

$$|\psi_A - \psi_V| \le \psi_T \quad (12)$$

where the angle ψ_T is a small angle close to zero. It means the missile can be regarded as moving in vertical plane. The engagement geometry is simplified as shown in Fig.2.

The LOS and LOS angle rate are given as follows:

$$\dot{R} = -V \cos \lambda \quad (13)$$
$$\dot{\theta}_E = \frac{V \sin \lambda}{R} \quad (14)$$

From the engagement geometry it can be derived that:

$$\lambda = \theta_E - \theta_V \quad (15)$$

2) THE TIME-TO-GO ESTIMATION METHOD
The traditional PNG law is given as:

$$\dot{\theta}_V = N \dot{\theta}_E \quad (16)$$

where N is the navigation gain of the PNG law.

The look angle rate is obtained by differentiating (15):

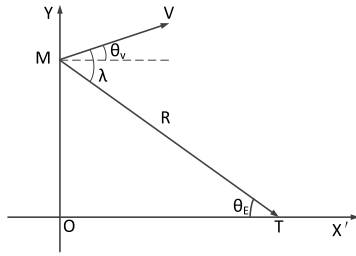$$\dot{\lambda} = \dot{\theta}_E - \dot{\theta}_V \quad (17)$$

FIGURE 2. The simplified engagement geometry in vertical plane.

The look angle rate can also be expressed by combining (14), (16) and (17):

$$\dot{\lambda} = \frac{(1-N) V \sin \lambda}{R} \tag{18}$$

The differential equation of the look angle λ and the distance R can be obtained by combining (13) with (18):

$$\frac{\dot{\lambda}}{\dot{R}} = \frac{\mathrm{d}\lambda}{\mathrm{d}R} = \frac{(N-1)\tan \lambda}{R} \tag{19}$$

By separating the variables the differential equation is obtained as:

$$\frac{(N-1)}{R}\mathrm{d}R = \frac{1}{\tan \lambda}\mathrm{d}\lambda \tag{20}$$

The integral equation is obtained by integrating (20):

$$\int_{R_0}^{0} \frac{(N-1)}{R}\mathrm{d}R = \int_{\lambda_0}^{0} \frac{1}{\tan \lambda}\mathrm{d}\lambda = \int_{\lambda_0}^{0} \frac{1}{\sin \lambda}\mathrm{d}\sin \lambda \tag{21}$$

where R_0 and λ_0 are initial value The solution of (21) is given as:

$$R = \frac{R_0}{|\sin \lambda_0|^{1/(N-1)}} |\sin \lambda|^{1/(N-1)} \tag{22}$$

The differential form of the distance R is obtained by transforming (13):

$$\dot{R} = \frac{\mathrm{d}R}{\mathrm{d}t} = -V \cos \lambda \tag{23}$$

Integrating (23) to obtain:

$$\int_{R_0}^{0} -\frac{1}{V \cos \lambda}\mathrm{d}R = \int_{0}^{t_f} \mathrm{d}t = t_f \tag{24}$$

where t_f is the final impact time. If the velocity V is considered as constant, t_f can be given as:

$$t_f = \frac{1}{V}\int_{0}^{R_0} \frac{1}{\cos \lambda}\mathrm{d}R = \frac{1}{V}\int_{0}^{R_0} \frac{1}{\sqrt{1-\sin^2 \lambda}}\mathrm{d}R \tag{25}$$

The final impact time t_f is obtained by using Taylor series expansion:

$$t_f = \frac{1}{V}\int_{0}^{R_0} \left(1 + \frac{1}{2}\sin^2 \lambda + \frac{3}{8}\sin^4 \lambda + \cdots \right)\mathrm{d}R \tag{26}$$

Taking (22) into (26) to obtain:

$$\begin{aligned} t_f &= \frac{1}{V}\int_{0}^{R_0} \left(1 + \frac{1}{2}\sin^2 \lambda + \frac{3}{8}\sin^4 \lambda + \cdots \right)\mathrm{d}R \\ &= \frac{R_0}{V}\left(1 + \frac{\sin^2 \lambda_0}{2(2(N-1)+1)} + \frac{3\sin^4 \lambda_0}{8(4(N-1)+1)} + \cdots \right) \end{aligned} \tag{27}$$

The solution is obtained by neglecting the higher order terms and taking $N = 3$ into (27):

$$t_f = \frac{R_0}{V}\left(1 + \frac{\sin^2 \lambda_0}{10}\right) \tag{28}$$

The time-to-go estimation is obtained by taking the value of any point in the flight to replace the initial value:

$$\hat{t}_{go} = \left(1 + \frac{\sin^2 \lambda}{10}\right)\frac{R}{V} \tag{29}$$

This is the time-to-go estimation given in [5], also the similar derivation procedure in some articles such as [20] for constant velocity situation. In the equation velocity V is constant. But the velocity changes significantly when considering hypersonic vehicle. So the constant velocity assumption is no longer valid. The problem is caused in (24) that the velocity V is assumed to be constant. In order to obtain a more accurate time-to-go estimation, a method to estimate the time-to-go with time-varying velocity is presented below. Inspired by [16] and [21], assuming that the drag force plays a main role during the flight and the acceleration has the following form:

$$\dot{V} = -\kappa V^2 \tag{30}$$

As in (5) if the gravity is neglected κ should be defined as:

$$\kappa = \frac{\rho S C_{D_0}}{2m} \tag{31}$$

where ρ is air density, S is the reference area, C_{D_0} is the zero lift drag coefficient which is assumed to be constant, m is the mass. In real flight condition, the acceleration can be obtained by accelerometer. So in the article, the real value of $\dot{V}$ is used instead of (31) to calculate κ:

$$\kappa = \frac{-D - mg \sin \theta_V}{mV^2} \tag{32}$$

The integral equation is obtained by transforming (30) into differential form:

$$\frac{1}{V^2}\mathrm{d}V = -\kappa \,\mathrm{d}t \tag{33}$$

The solution of (33) is given as:

$$V(t) = \frac{V_0}{1 + \kappa V_0 t} \tag{34}$$

A new integral equation is obtained by taking (34) into (23) to replace V:

$$\frac{\mathrm{d}R}{\mathrm{d}t} = -\frac{V_0}{1 + \kappa V_0 t}\cos \lambda \tag{35}$$

TABLE 1. Initial time-to-go estimation.

No.	t_{PNG}	$t_{[5]}$	$e_{t[5]}$	$e_{t[5]}\%$	t_{pro}	e_{tpro}	$e_{tpro}\%$
1	59.03	56.87	-2.16	-3.7%	60.52	1.49	2.5%
2	89.94	84.22	-5.72	-6.4%	92.37	2.43	2.7%
3	122.87	111.78	-11.09	-9.0%	126.44	3.57	2.9%
4	157.80	139.42	-18.38	-11.6%	162.71	4.91	3.1%
5	194.78	167.11	-27.67	-14.2%	201.28	6.50	3.3%

Annotations: t_{PNG}: total flight time guided by PNG law; $t_{[5]}$: time-to-go estimation using the method in [5]; $e_{t[5]}$: initial time-to-go estimation error using the method in [5]; $e_{t[5]}\%$: $e_{t[5]}/t_{PNG}*100\%$; t_{pro}: time-to-go estimation using the method proposed in this article; e_{tpro}: initial time-to-go estimation error using the method proposed in this article; $e_{tpro}\%$: $e_{tpro}/t_{PNG}*100\%$.

The differential equation is reconstructed by separating the variables:

$$\frac{1}{V_0 \cos \lambda} dR = -\frac{1}{1 + \kappa V_0 t} dt \qquad (36)$$

Integrating (36) to obtain:

$$\int_{R_0}^{0} -\frac{1}{V_0 \cos \lambda} dR = \int_{0}^{t_f} \frac{1}{1 + \kappa V_0 t} dt \qquad (37)$$

Noticing that the left side has the same form of (25) and the solution of (37) is given as:

$$\frac{R_0}{V_0} \left(1 + \frac{\sin^2 \lambda_0}{10} \right) = \frac{1}{\kappa V_0} \ln \left(1 + \kappa V_0 t_f \right) \qquad (38)$$

The final impact time t_f is obtained by solving (38):

$$t_f = \frac{e^{\kappa R_0 [1 + (\sin^2 \lambda_0)/10]} - 1}{\kappa V_0} \qquad (39)$$

Taking the value of any point in the flight to replace the initial value and the time-to-go estimation can be obtained as:

$$\hat{t}_{go} = \frac{e^{\kappa R [1 + (\sin^2 \lambda)/10]} - 1}{\kappa V} \qquad (40)$$

In order to verify whether the method presented in this article is valid, a simple simulation is conducted. The initial position of the missile is (0,20,0) km, velocity is 1800 m/s, $\theta_V = 0\deg$, $\psi_V = 0\deg$, the position of the target is (X,0,0) km, X = 100,150,200,250 and 300 respectively. The flight is guided by traditional PNG law with $N = 3$. To eliminate other interference factors, the air density is assumed to be $\rho = 0.2 kg/m^3$, the zero lift drag coefficient is assumed to be $C_{D_0} = 0.027$. The time-to-go estimation is given by the method in [5] and the method proposed in this article respectively. The results are shown in Figure.3, Figure.4 and Table.1.

In Figure.3 it informs that the time-to-go estimations obtained by [5] and proposed in this article both converge to zero while the missile approaches the target. But the initial time-to-go estimation error is much larger using [5] than proposed. Furthermore when the initial distance increases,
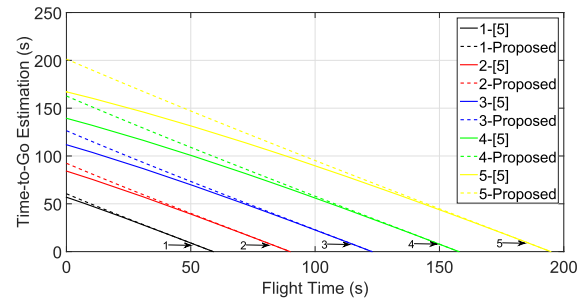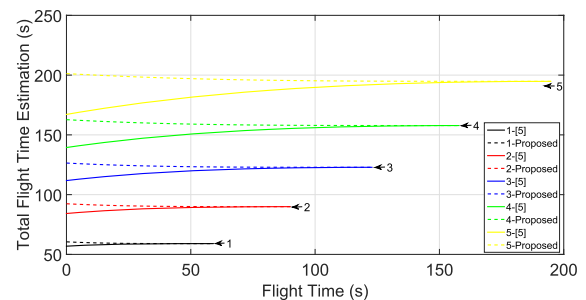

FIGURE 3. Flight time and time-to-go estimation.

FIGURE 4. Flight time and total flight time estimation.

TABLE 2. Time-to-go estimation with different air density.

No.	t_{PNG}	$t_{[5]}$	$e_{t[5]}$	$e_{t[5]}\%$	t_{pro}	e_{tpro}	$e_{tpro}\%$
1	122.87	111.78	-11.09	-9.0%	126.44	3.57	2.9%
2	138.36	111.78	-26.58	-19.2%	143.71	5.35	3.9%
3	156.03	111.78	-44.25	-28.4%	164.14	8.11	5.2%
4	175.99	111.78	-64.21	-36.5%	188.36	12.37	7.0%

Annotations: the same as Table.1.

the initial time-to-go estimation error increases much more using [5] than proposed. The results in Table.1 show the data in detail. The initial time-to-go estimation error of total flight time increases from -3.7% to -14.2% using [5] while it is 2.5% to 3.3% by using proposed method.

Aforementioned paragraph analyzed how the initial and flight time-to-go estimations vary with different downrange distances. In the following paragraph those estimations with different air density are investigated. The air density ρ is 0.2, 0.4, 0.6 and 0.8 kg/m^3 respectively. The position of the target is (200,0,0) km and the other conditions are unchanged. The results are shown in Figure.5, Figure.6 and Table.2.

In Figure.5 and Figure.6 it informs that the total flight time and the initial time-to-go estimation given by proposed method increase while the air density increases. But the estimation given by [5] keeps unchanged. This causes a large time-to-go estimation error at initial and during the flight. The results in Table.2 show the data in detail. The initial time-to-go estimation error of total flight time increases from -9% to -36.5% of total flight time using [5] while it is 2.9% to 7.0% by using proposed method.
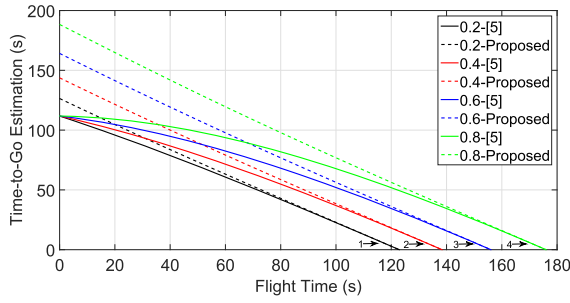
FIGURE 5. Flight time and time-to-go estimation with different air density.
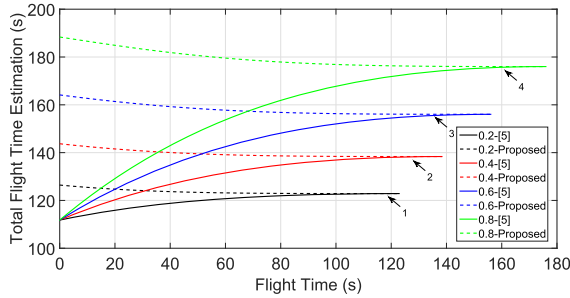

FIGURE 6. Flight time and total time estimation with different air density.

In Figure.4 and Figure.6 it informs that the total flight time estimation obtained by proposed method is a little larger than the real total flight time and eventually the error converge to zero. But the total flight time estimation obtained by [5] is much smaller than the real total flight time meanwhile the time error is negative. This negative time error may happen in conducting ITCG when a desired impact time is set and this will cause a problem that when time error becomes zero or negative, the law will lose the ability to control impact time and the ITCG law becomes invalid. This will be further discussed in the following section.

3) THE IMPACT-TIME-CONTROL GUIDANCE LAW

In Section 2) the time-to-go estimation for time-varying velocity is given. So the time error estimation is defined as follows:

$$\hat{e}_t = (t_d - t) - \hat{t}_{go} \quad (41)$$

where t_d is the desired impact time, t is flight time, $(t_d - t)$ means the desired time-to-go. Substituting (40) into (41) and taking the time derivative gives:

$$\begin{aligned}
\dot{\hat{e}}_t &= -\dot{\hat{t}}_{go} - 1 \\
&= -\frac{e^{\kappa R[1+(\sin^2 \lambda)/10]}}{\kappa V} \cdot \kappa \left(1 + \frac{\sin^2 \lambda}{10}\right) \cdot \dot{R} \\
&\quad - \frac{e^{\kappa R[1+(\sin^2 \lambda)/10]}}{\kappa V} \cdot \kappa R \left(\frac{2 \sin \lambda \cos \lambda}{10}\right) \cdot \dot{\lambda} \\
&\quad - \frac{e^{\kappa R[1+(\sin^2 \lambda)/10]} - 1}{\kappa} \cdot \left(\frac{-1}{V^2}\right) \cdot \dot{V} - 1 \quad (42)
\end{aligned}$$

Substituting (13)(18) (30) and $N = 3$ into (42) to obtain:

$$\begin{aligned}
\dot{\hat{e}}_t &= -\frac{e^{\kappa R[1+(\sin^2 \lambda)/10]}}{\kappa V} \cdot \kappa \left(1 + \frac{\sin^2 \lambda}{10}\right) \cdot (-V \cos \lambda) \\
&\quad - \frac{e^{\kappa R[1+(\sin^2 \lambda)/10]}}{\kappa V} \cdot \kappa R \left(\frac{2 \sin \lambda \cos \lambda}{10}\right) \cdot \left(-\frac{2V \sin \lambda}{R}\right) \\
&\quad - \frac{e^{\kappa R[1+(\sin^2 \lambda)/10]} - 1}{\kappa} \cdot \left(\frac{-1}{V^2}\right) \cdot \left(-\kappa V^2\right) - 1 \\
&= -e^{\kappa R[1+(\sin^2 \lambda)/10]} \left(-\cos \lambda - \frac{\sin^2 \lambda \cos \lambda}{2} + 1\right) \quad (43)
\end{aligned}$$

In (43) it is obvious that:

$$e^{\kappa R[1+(\sin^2 \lambda)/10]} > 0 \quad (44)$$

Then it is assumed that:

$$f(\lambda) = -\cos \lambda - \frac{\sin^2 \lambda \cos \lambda}{2} + 1 \quad (45)$$

Differentiating (45) to obtain:

$$f'(\lambda) = \frac{3}{2} \sin^3 \lambda \Rightarrow \begin{cases} f'(\lambda) > 0, & \lambda > 0 \\ f'(\lambda) = 0, & \lambda = 0 \\ f'(\lambda) < 0, & \lambda < 0 \end{cases} \quad (46)$$

So the minimum of $f(\lambda)$ is $f_{\min}(\lambda) = f(0) = 0$ and

$$f(\lambda) \geq f(0) = 0 \quad (47)$$

And according to (44) and (47) it can be inferred that:

$$\dot{\hat{e}}_t = -e^{\kappa R[1+(\sin^2 \lambda)/10]} \left(-\cos \lambda - \frac{\sin^2 \lambda \cos \lambda}{2} + 1\right) \leq 0 \quad (48)$$

According to (22), λ will not decrease to zero until R decreases to zero. So when $\lambda \neq 0$, $f(\lambda) > 0$. So it can be inferred that $\dot{\hat{e}}_t < 0$. In addition that $\hat{e}_{t0} > 0$, so $\hat{e}_t \to 0$ as time goes on.

The path angle rate command $\dot{\theta}_V$ is given as in [5]:

$$\dot{\theta}_V = N \dot{\theta}_E \left(\frac{3}{2} - \frac{1}{2} \sqrt{1 + \frac{240V^5}{(N V \dot{\theta}_E)^2 R^3} \hat{e}_t}\right) \quad (49)$$

This is the ITCG law given in [5], more detail is in [5]. According to the given guidance law (49), $\dot{\theta}_V \to N \dot{\theta}_E$ while $\hat{e}_t \to 0$. So eventually the time-to-go error $\hat{e}_t$ converges to zero and the ITCG law switches to traditional PNG law (16) As seen in (49), the guidance law work under the condition that $\hat{e}_t \geq 0$.

In the following part the simulations are conducted by using the ITCG laws with the time-to-go estimation method in [5]. traditional PNG law and proposed in this article respectively. The initial position of the target is (200,0,0) km. The other conditions are the same as previous. The navigation gain is $N = 3$. The desired impact time is set to be $t_d = 130$ s. The results are shown in Figure.7 to Figure.12.
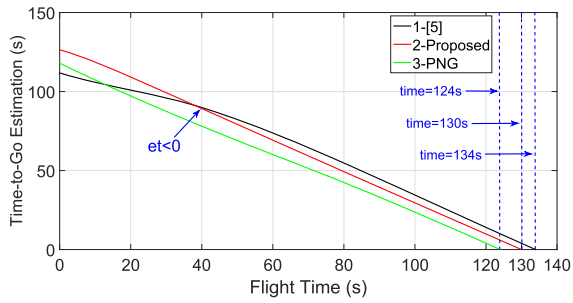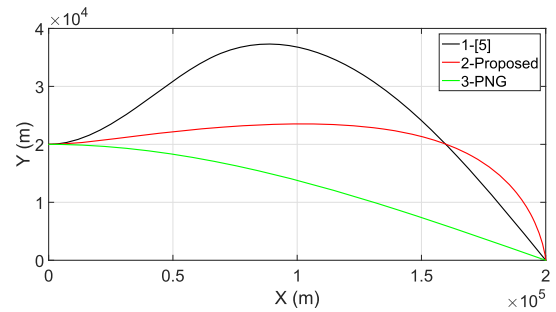
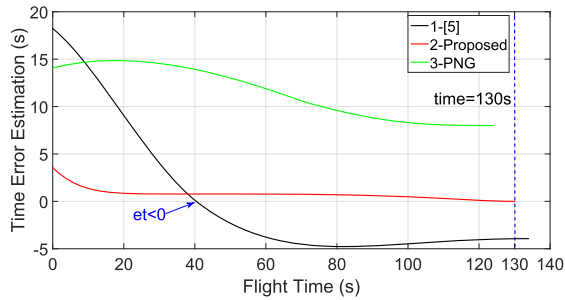**FIGURE 7.** Flight time and time-to-go estimation.

FIGURE 8. Flight time and time error estimation.

FIGURE 9. Flight time and look angle.

FIGURE 10. Flight time and path angle.

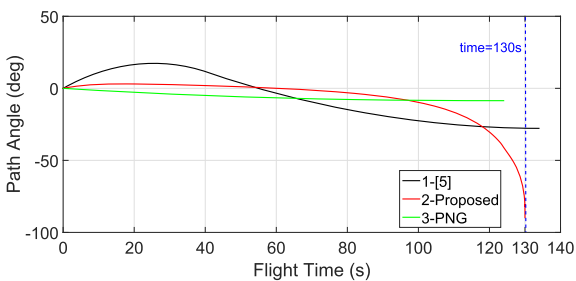**FIGURE 11.** Position X and Y.

FIGURE 12. Flight time and velocity.

In Figure.7 and Figure.8, it informs that the impact times of the missiles guided by traditional PNG law, the ITCG in [5] and proposed in this article are 124s, 134s and 130s respectively. The traditional PNG law doesn't have the ability to control impact time. The time-to-go estimation given by [5] reaches desired time-to-go and the time error reaches zero at about 40s and the guidance law alters to PNG law. This eventually results in a large impact time error. The reason

causes this problem is because there is a large time-to-go estimation error at initial and during the flight. This drives guidance law to increase the path angle and look angle to eliminate the time error it estimates and because of this it causes unnecessary increase in path angle, look angle and height as shown in Figure.9 to Figure.11. On the other hand, the proposed method not only has a good performance in time-to-go estimation to meet the desired impact time constraint but also results in a promising impact angle. A further research could be based on this to study about impact time and angle control guidance law.

So far an ITCG law using time-to-go estimation for time-varying velocity based on PNG law implemented in vertical plane is presented. It has a promising performance as shown previous. But there are still same disadvantages may limit its implement:

(1)The conditions for entering the terminal phase.

As discussed previously, to apply the guidance law in vertical plane, the heading angle ψ_V and the azimuth angle ψ_A must satisfy the constraint that $|\psi_A - \psi_V|$ is close to zero. This may be too strict in real situation The missile always has a larger seeker's field-of-view than the constraint. This means the missile has to adjust the heading angle first to satisfy the constraint once the target is in the seeker's field-of-view and this may sacrifice a part of the performance of the missile.

(2)The real air density varies a lot with height

The time-to-go estimation equation is derived based on assumption that the acceleration has the form of $\dot{V} = -\kappa V^2$ where the value of the variable κ is much relevant with the air density. While the guidance law is applied in vertical plane, the guidance law will enforce the path angle and look angle

to change to eliminate the estimated time error and this could cause a large rang varying in height and result in a large variation in air density. This will influence the accuracy of the proposed method. But this can be avoided by choosing a suitable desired impact time t_d and navigation gain N to make the front trajectory of the terminal phase flat and straight to reduce the change in height. This can reduce the air density influence and result in a large impact angle.

Although it may have some limits to apply the guidance law in vertical plane, it is still a promising method. In order to improve the performance and make it easier to apply, in the following section the ITCG law implemented in lateral plane is derived. If the ITCG law is implemented in lateral plane, the constraint of the heading angle and azimuth angle to enter the terminal phase will be free. Meanwhile it is no longer required to change the path angle to control impact time and this will reduce the air density influence on the accuracy of the proposed method.

B. ITCG LAW IMPLEMENTED IN LATERAL PLANE
1) THE ENGAGEMENT GEOMETRY
The LOS and LOS angle rate are given in (2) and shown in Figure.1. From the engagement geometry the relationship between the heading angle ψ_V and the azimuth angle ψ_A can simply be given as:

$$\eta = \psi_A - \psi_V \tag{50}$$

2) THE TIME-TO-GO ESTIMATION AND ITCG LAW
In lateral plane, the LOS and velocity is given as:

$$R_{xz} = R \cos \theta_E \tag{51}$$
$$V_{xz} = V \cos \theta_V \tag{52}$$

From previous work, as given in (40), the time-to-go estimation in lateral plane is obtained as:

$$\hat{t}_{go} = \frac{e^{\kappa R_{xz}[1+(\sin^2 \eta)/10]} - 1}{\kappa V_{xz}} \tag{53}$$

In order to apply the upper equation, the key point is to obtain κ accurately. As in (30), it is assumed that:

$$\dot{V}_{xz} = -\kappa V_{xz}^2 \tag{54}$$

Differentiating (52) to obtain:

$$\dot{V}_{xz} = \dot{V} \cos \theta_V - V \dot{\theta}_V \sin \theta_V \tag{55}$$

So κ_L is given as:

$$\kappa_L = \frac{V \dot{\theta}_V \sin \theta_V - \dot{V} \cos \theta_V}{V^2 \cos^2 \theta_V} \tag{56}$$

In the equation, κ_L represents variable κ in lateral plane. The desired impact time t_d, the flight time t, the time-to-go estimation $\hat{t}_{go}$ and the impact time error estimation $\hat{e}_t$ are defined the same as in (41).
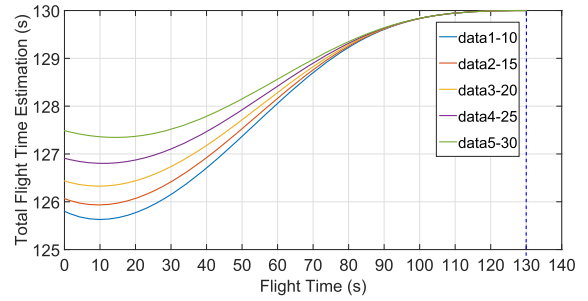
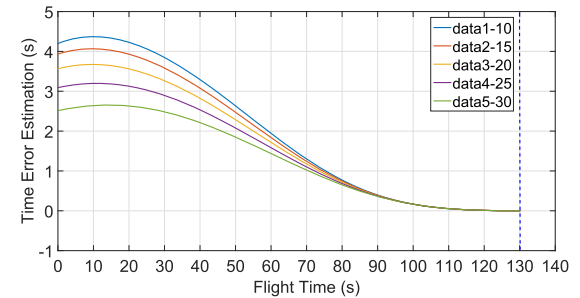**FIGURE 13.** Flight time and total flight time estimation.

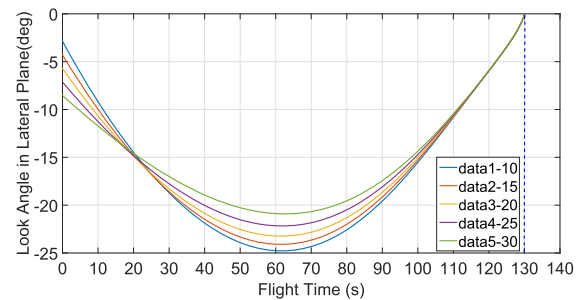**FIGURE 14.** Flight time and time error estimation.

FIGURE 15. Flight time and look angle in lateral plane.

In lateral plane, the ITCG law is given as:

$$\dot{\psi}_V = N \dot{\psi}_A \left(\frac{3}{2} - \frac{1}{2} \sqrt{1 + \frac{240 V_{xz}^5}{(N V_{xz} \dot{\psi}_A)^2 R_{xz}^3} \hat{e}_t} \right) \tag{57}$$

In vertical plane, the PNG law is given as:

$$\dot{\theta}_V = N \dot{\theta}_E \tag{58}$$

Simulations are conducted to verify whether the method presented aforementioned is valid. The initial position of the missile is (0,20,Z) km, Z =10,15,20,25 and 30 respectively and the other conditions are the same as previous. The results are shown in Figure.13 to Figure.17 and Table.3.

In Table.3 it informs that the initial time-to-go estimations obtained by the proposed method are quiet close to the impact time guided by PNG. The estimation errors are about 2.2%. In Figure.13 and Figure.14 it shows that the total flight time approaches the desired impact time and the time error estimation approaches zero while the flight time increases.

TABLE 3. Initial time-to-go estimation.

No.	t_{PNG}	t_{pro}	e_{tpro}	$e_{tpro}\%$
1	123.08	125.80	2.72	2.21%
2	123.33	126.07	2.74	2.22%
3	123.69	126.44	2.75	2.22%
4	124.15	126.91	2.76	2.22%
5	124.71	127.49	2.78	2.23%

Annotations: the same as Table.1.

TABLE 4. The initial state of the missiles.

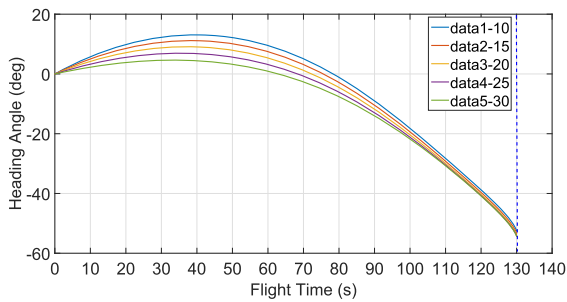| Missile number	Position / km	Velocity / m/s
1	(0,20,0)	1800
2	(-10,25,0)	2000
3	(10,15,0)	1900
4	(0,20,20)	1800
5	(0,20,-30)	1800

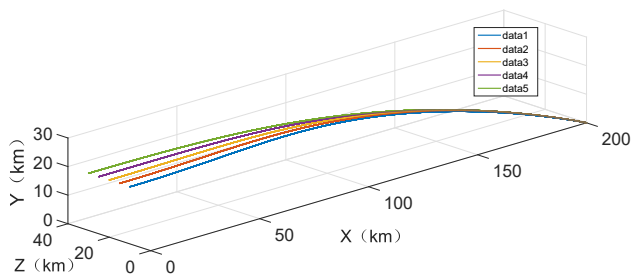**FIGURE 16.** Flight time and heading angle.

FIGURE 17. Flight trajectory in 3D view.

In Figure.15 to Figure.17 it shows that the look angle varies in a reasonable range and finally approaches zero. The guidance law adjusts the impact time by altering the heading and look angle. It enforces the velocity direction to point to the target eventually.

The results show the ITCG law is valid to apply in lateral plane. This frees the vertical plane to apply other guidance laws to realize other objectives, such as impact angle control guidance. This can be further studied.

IV. SIMULATION

The simulation scenario is assumed that several missiles are at the beginning of their terminal phases against the same stationary target Each missile has different position and velocity. The position of the target is (200,0,0) km. The positions of the missiles are shown in Table.4.

The initial heading and path angles are assumed to be $\psi_V = 0$ deg, $\theta_V = 0$ deg. The air density is the same as previous. The impact time of each missile guided by traditional PNG law and the initial time-to-go estimations are shown in Table.5. The desired impact time is set to be 130s. Missile 1, 2 and 3 are guided by the ITCG law applied in
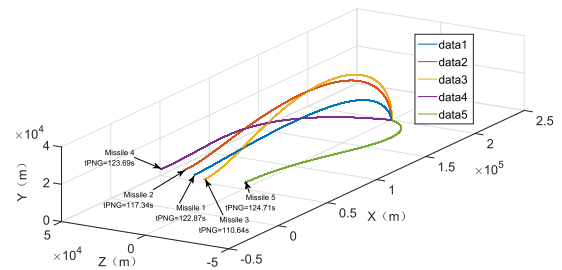

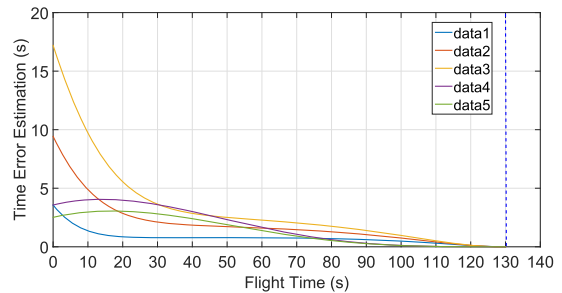**FIGURE 18.** The trajectories in 3D view.

FIGURE 19. Flight time and the time error estimations.
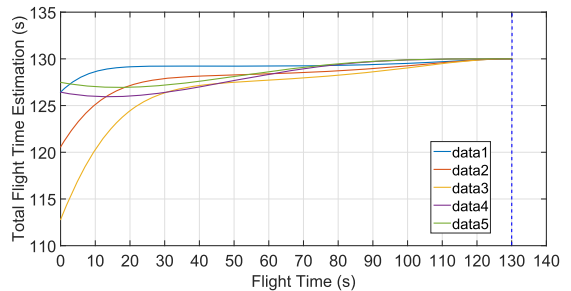

FIGURE 20. Flight time and total flight time estimations.

vertical plane. Missile 4 and 5 are guided by the ITCG law applied in lateral plane. The ITGC law is used to enforce all the missiles to impact the target at the same final time to realize simultaneous attack. The simulation results are shown in Figure.18 to Figure.20.

From Figure.18 to Figure.20 it informs that the missiles started from different positions and impacted the target at the same final flight time. The trajectories in Figure.18 showed the missile impacted the target from different directions at the same time. The simulation results showed the proposed ITCG laws in this article applied in vertical and lateral planes

TABLE 5. PNG impact time and initial time-to-go estimations.

Missile number	t_{PNG} / s	Initial time-to-go estimation/s
1	122.87	126.44
2	117.34	120.16
3	110.64	112.80
4	123.69	126.44
5	124.71	127.49

TABLE 6. Cep and impact time.

Missile number	Impact point (X,Y,Z)/ m	Miss distance/m	Impact time/s
1	(200000.0,0,0)	0.0	130.0
2	(200000.0,0,0)	0.0	130.0
3	(200000.0,0,0)	0.0	130.0
4	(199997.9,0.446,2.12)	3.02	130.0
5	(199998.4,0.343,-1.65)	2.32	130.0

are valid in controlling the impact time of different missiles accurately to realize simultaneous attack and improve the survival and damage ability of the missiles

V. CONCLUSION
The ITCG laws used to guide missiles to impact a stationary target at the same desired time are given in this article The time-to-go estimation for time-varying velocity is derived based on combining the estimation of constant velocity situation with an approximate form of acceleration. Then the ITCG laws are designed in vertical and lateral plane respectively. Several conditions which may influence the performance of the ITCG law are anaylzed as well. The ITCG law applied in vertical plane not only showed its ability to control the impact time but also showed a promising result in impact angle. The law applied in lateral plane freed the vertical channel to apply other guidance method. All these gain much interest in further research. The simulation results showed the ITCG law given can enforce the missiles of different initial conditions at the beginning of the terminal phase to impact the target at the same desired final time accurataly.

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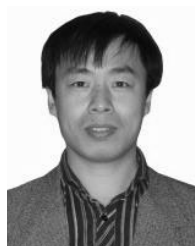
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