Received January 8, 2020, accepted January 30, 2020, date of publication February 11, 2020, date of current version February 20, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.2973180

Estimation of Rice Factor Ratio for Doubly Selective Fading Channels

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This work was supported in part by the National Natural Science Foundation of China under Grant 61671394 and Grant 61971362, in part by the National Key Research and Development Program of China under Grant 2018YFC0809200, in part by the Science and Technology Program of Shenzhen, China, under Grant JSGG20170414090428464, and in part by the European Union's Horizon 2020 Research and Innovation Program under Grant 654462 (STEMM-CCS).

ABSTRACT In wireless communication systems, Rice factor ratio (RFR) defined as $K/(1+K)$ is a key parameter not only to evaluate the quality of communication channel since it can reveal the severity of the small-scale fading, but also to be employed as *a priori* information for estimation of other parameters such as frequency. Consequently, its estimation is important for a variety of wireless application scenarios. In this paper, we propose an estimation algorithm on the RFR for the received signals that are disturbed by the Rician doubly selective fading channels and additive noise. During the estimation periods, we initially utilize the known signals to multiply the received signals. Second-order and fourth-order statistics are then employed to further deal with the processed signals mentioned above, which disposes of influence of some unnecessary parameters, e.g., indistinguishable multipaths, maximum Doppler shift, Doppler shift, and noise variance. Finally, a useful expression on the RFR estimation is derived for the Rician frequency selective fast fading channels by flexibly mathematical calculation. Furthermore, the presented method only uses the maximum estimation values of the second-order and fourth-order statistics defined in this paper, which can reduce the computational complexity. Importantly, the investigated scheme is robust to the signal-to-noise ratio over 0 dB and frequency offset (maximum Doppler shift and Doppler shift), and shows a slight improvement on the estimation performance with an increase of the aided data length. The performance and benefits of the proposed approach are verified and evaluated through computer simulations.

INDEX TERMS Rice factor ratio, Rician doubly selective fading channels, second-order and fourth-order statistics.

I. INTRODUCTION

In the past several years, to satisfy the essential requirements of the traffic growth, the latency reduction and the energy efficiency in the wireless communications, a variety of advanced technologies have been introduced in the existing literature [1]–[5]. Further, many wireless communication environments for short-term fading can be described as a statistic model characterized by Rician distribution, which results in received signals with different magnitudes, angles of arrival and phases that comprise one specular or light-of-sight (LoS) signal and many random reflected, diffracted, or scattered versions of the transmitted signal. As the key term for the Rician fading channels, Rice factor ratio (RFR) is defined as $K/(1+K)$, where K represents Rice factor, which is useful for the evaluation of communication link quality and provides *a priori* knowledge on the estimation of some parameters. Therefore, like the Rice factor, its estimation is of practical importance in all kinds of wireless scenarios, e.g., adaptive modulation,

The associate editor coordinating the review of this manuscript and approving it for publication was Kezhi Wang.

channel characterization, localization application, optimal power loading of transmit diversity systems, mobile station velocity estimation, link budget as well as frequency estimation [6]–[10].

The Rice factor estimation is a sustained research subject; thus many methods have been addressed in the past few decades. The existing literature is mainly catalogued into the envelope-based works and the phase-based ones. In the envelope-based literature, Greenwood and Hanzo [11] estimated the Rice factor by comparing probability density function (PDF) of the received envelope data to the hypothesis Rician envelope PDFs using a suitable goodness-of-fit test. In [12], the maximum likelihood estimation (MLE) method was employed to the Rice factor, but it needs a root search procedure. To reduce the computation of the MLE scheme, in [13]–[15], the moment-based approaches were presented to estimate the Rice factor. Naimi and Azemi [16] suggested two generalized moment-based estimators for the Rice factor when shadowing was considered.

The additional phase information provided by the in-phase and quadrature components of the complex baseband samples, which the aforementioned methods fail to utilize, was exploited to estimate the Rice factor. The tractable MLE schemes of the polar signals were presented in [17] and [18]. The estimator for the Rice factor was addressed in [19], which was based on the statistics of the channel phase derivative, i.e., the first moment and zero-crossing rate of the received signal instantaneous frequency. An estimator only used phase was suggested in [20], where Doppler shift of the dominant component was known and the dominant source was not perpendicular to the mobile trajectory.

All the Rice factor estimators mentioned above assume that the channel is time-varying and narrowband. However, with requirements for high data rate and development of wireless communications, lots of current wireless communication standards and systems are time-varying and wideband. In order to estimate the Rice factor for wideband channels, Sasaoka *et al.* [21] proposed an estimator for multiple input multiple output channels, where the Rice factor is computed as the ratio of the power in the first delay bin to the power in the other delay bins. References [22] and [23] employed the moment approach to calculate the Rice factor on each delay bin or carrier frequency. However, each bin does not contain all multipath components in the channel [24]. A classical moment based Rice factor estimator was described in [24] for narrowband temporal selective fading from the single-snapshot wideband measurements.

Among the above mentioned Rice factor and RFR estimators except for [7], [9] and [10], they only considered the channel coefficients, and therefore are unable to apply for modulated signals with additive noise [7]. To deal with this problem, Chen and Beaulieu [25] considered data-aided and non-data-aided (NDA) methods and utilized autocorrelation of the received signal (namely, the second-order fading statistics) to estimate the Rice factor when *a priori* knowledge of the normalized maximum Doppler shift was assumed. Nevertheless, the NDA estimator cannot be applied to M-ary phase shift keying (MPSK) modulated signals [7]. The fourth-order cross-moments based Rice factor estimator was investigated in [7] for the MPSK modulated signals. The suggested approaches in [9] and [10] presented the estimation of the RFR for the purpose of frequency estimation. Moreover, these estimators presented in [7], [9], [10] and [25] only consider the frequency flat fading channels. Except for previous estimators published in the literature, to the best of the authors' knowledge, the estimation of the RFR has not been investigated for Rician doubly selective fading channels which are always encountered in wireless communication systems, especially for high rate wireless communications.

Based on the aforementioned analyses, we propose an estimation algorithm on the RFR for the Rician doubly selective fading channels. In the estimation stage, we first multiply the known signals and the received signals together. Then, we employ second-order and fourth-order statistics to cope with the processed signals mentioned above, where some unnecessary parameters, such as indistinguishable multipaths, frequency offset (maximum Doppler shift and Doppler shift) and noise variance, are cancelled. We finally derive a useful expression of the RFR estimation, where the Rician frequency selective fast fading channels and additive noise are encountered. The main contributions of this paper are summarized as follows.

1) The proposed method can be applied to such scenarios where the transmitted signals are through the Rician doubly selective fading channels and disturbed by the additive noise.
2) The proposed method does not need *a priori* knowledge of the frequency offset (the maximum Doppler shift and the Doppler shift) and extra parameter estimation in the procedure of the RFR estimation, which cannot only reduce the algorithm complexity, but also avoid error propagation.
3) The proposed method only employs the maximum estimation values of the second-order and fourth-order statistics defined in this paper, which further reduces the computational complexity.
4) The proposed method is robust to signal-to-noise ratio (SNR) over 0 dB, the maximum Doppler shift and the Doppler shift, and its performance exhibits a slightly improved change with the increase of the known data length aided for estimation, which enhances its applications.

The remainder of this paper is organized as follows. Section II explains the background with respect to the received signal model such as the Rician frequency selective fast fading channels, and states the problem to be solved. Section III proposes the estimation algorithm on the RFR for the wireless fading channels mentioned above in detail. The investigated method in this work is verified and evaluated by the computer simulations in Section IV, and the conclusion part in Section V summarizes the results and discusses possible applications of the proposed approach.

II. PRELIMINARY AND FORMULATION

In the wireless communication systems equipped with a single antenna where the transmitted signals experience the Rician fading, the received baseband signals can be denoted as

$$r(t) = s(t) \otimes h(t, \tau) + n(t), \tag{1}$$

where $s(t)$ is the transmitted signal, $h(t, \tau)$ is the Rician doubly selective fading channel coefficient, $n(t)$ is the additive white Gaussian noise (AWGN), and $\otimes$ represents the convolution operator.

For the Rician doubly selective fading channel coefficient [26], [27], $h(t, \tau)$, its complex baseband representation can be written as

$$h(t, \tau) = \sqrt{\frac{K}{1+K}} h_{\text{LoS}}(t)\delta(\tau - \tau_0) + \sqrt{\frac{1}{1+K}} \sum_{\ell=0}^{L-1} P_\ell h_{\text{NLoS},\ell}(t)\delta(\tau - \tau_\ell), \tag{2}$$

where P_ℓ is the magnitude of the ℓth path, τ_ℓ is the propagation delay at the ℓth path, L is the number of paths, $\delta(\cdot)$ is the Kronecker delta function, and $h_{\text{LoS}}(t)$ and $h_{\text{NLoS},\cdot}(t)$ are the LoS and non-LoS (NLoS) components, respectively. It should be noted that, according to the definition of the Rice factor [26], [27], $E\{|h_{\text{LoS}}(t)|^2\} = E\{|\sum_{\ell=0}^{L-1} P_l h_{\text{NLoS},\ell}(t)|^2\}$, where $E\{\cdot\}$ stands for the statistical average operator. It should also be mentioned that $K/(1+K)$, which is to be estimated, will approach 1 as K increases; on the contrary, it will be close or equal to 0 when K decreases, so one of its functions is as an indicator similar to the Rice factor for the evaluation on the quality of the wireless communication links.

The existing Rician channel models in the literature (see [7], [18], [20] and the references therein) assume: the LoS fading component, $h_{\text{LoS}}(t) = \exp(j2\pi f_d t\cos\theta_0 + j\phi_0)$, where f_d, θ_0, and ϕ_0 are the maximum Doppler shift, angle of arrival, and initial phase, respectively; the NLoS fading component, $h_{\text{NLoS},\cdot}(t)$, is non-frequency selectivity with Rayleigh distribution and unit power, whose mean and autocorrelation function are zero and zero-order Bessel function, respectively, and whose real and imaginary parts are independent Gaussian random variables that can be modeled by various schemes addressed in [28].

Based on the analyses and discussions mentioned above, we have known that the initial path is the first arrival one. Thus, without loss of generality, $\tau_0 = 0$ is assumed. Utilizing this assumption and substituting (2) into (1), we can discretize and rewrite it as

$$r(k) = \left(\sqrt{\frac{K}{1+K}} h_{\text{LoS}}(k) + \sqrt{\frac{1}{1+K}} P_0 h_{\text{NLoS},0}(k)\right) s(k) + \sqrt{\frac{1}{1+K}} \sum_{\ell=1}^{L-1} P_\ell h_{\text{NLoS},\ell}(k) s(k - \iota_\ell) + n(k). \tag{3}$$

From the above preliminary and formulation, it shows that some parameters such as the maximum Doppler shift, the angle of arrival, the initial phase, the magnitude of the multipaths, the number of multipaths, the transmitted signal, and AWGN should be cancelled to estimate the RFR, when the Rician frequency selective fast fading channels and additive noise are encountered. Furthermore, as we all know, error propagation will exist if extra parameters need to be estimated in the procedure of the estimation on the RFR. In order to suppress the redundant parameters and variables mentioned above and solve these issues encountered, we propose a novel algorithm for the RFR estimation in the succeeding section.

III. PROPOSED ESTIMATION ALGORITHM ON RICE FACTOR RATIO FOR DOUBLY SELECTIVE FADING CHANNELS

In this section, we firstly derive a useful expression on the estimation of the RFR. In the procedure of derivation, we initially multiply the received signals and the known signals together. Then, we define the second-order and fourth-order statistics of the processed signals mentioned above. Employing these two statistics, the expression on the estimation of the RFR is proposed for the doubly selective fading channels as follows. Finally, we conclude this algorithm based on the theoretical derivation and estimation of the RFR.

A. DERIVED EXPRESSION ON THE ESTIMATION OF THE RICE FACTOR RATIO

From (3), we can clearly see that it is difficult to directly extract the RFR, because of the unnecessary parameters, the transmitted signals, and AWGN. Consequently, we first multiply the known signals and the received signals together by employing data-aided method. That is to say, using the property of the known signals, i.e., $s(k)s^*(k) = 1$, $k = 1, \cdots, N$, we can obtain

$$\begin{aligned}\check{r}(k) &= r(k) \cdot s^*(k) \\ &= \sqrt{\frac{K}{1+K}} h_{\text{LoS}}(k) + \sqrt{\frac{1}{1+K}} P_0 h_{\text{NLoS},0}(k) \\ &\quad + \sqrt{\frac{1}{1+K}} \sum_{\ell=1}^{L-1} P_\ell h_{\text{NLoS},\ell}(k) s(k - \iota_\ell) s^*(k) + w(k),\end{aligned} \tag{4}$$

where $w(\cdot)$ also maintains the same statistical property with the $n(\cdot)$.

To extract the RFR from (4), the second-order and fourth-order statistics are defined as

$$R_{\check{r}\check{r}}(m) \triangleq E\left\{\check{r}^*(k) \cdot \check{r}(k+m)\right\}, \tag{5}$$

$$R_{\check{r}\check{r}\check{r}\check{r}}(l, m, q) \triangleq E\left\{\check{r}^*(k) \cdot \check{r}(k+l) \cdot \check{r}(k+m) \cdot \check{r}^*(k+q)\right\}. \tag{6}$$

In the following, we provide two propositions to describe the second-order and fourth-order statistics of $\check{r}$, which will be used to derive the RFR via some mathematical operations.

Proposition 1: By substituting (4) into (5) and doing some mathematical operations, the second-order statistic of the $\check{r}$ is given by

$$R_{\breve{r}\breve{r}}(m) = \frac{K}{1+K}\exp\left(j2\pi\breve{f}_d m\right) + \frac{1}{1+K}J_0\left(2\pi f_d m\right) \cdot \left(P_0^2 + \sum_{\ell=1}^{L-1} P_\ell^2\delta(m)\right) + \sigma^2\delta(m), \quad (7)$$

where $\breve{f}_d = f_d\cos\theta_0$ is the Doppler shift, $J_0(\cdot)$ is the zero-order Bessel function, the mean of the additive noise $w(\cdot)$ is assumed as 0 for sake of simplicity, and σ^2 is the variance of the $w(\cdot)$.

Proof: See the Appendix A for details. ■

Proposition 2: By substituting (4) into (6) and doing some other mathematical operations, the fourth-order statistic of the $\breve{r}$ is expressed as

$$\begin{aligned}
&R_{\breve{r}\breve{r}\breve{r}\breve{r}}(l,m,q)\\
&= \left(\frac{K}{1+K}\right)^2 \exp\left(j2\pi\breve{f}_d(l+m-q)\right)\\
&\quad + \frac{K}{(1+K)^2}\exp\left(j2\pi\breve{f}_d l\right)J_0\left(2\pi f_d(m-q)\right)\Big(P_0^2\\
&\quad + \sum_{\ell=1}^{L-1}P_\ell^2\delta(m-q)\Big) + \frac{K}{1+K}\exp\left(j2\pi\breve{f}_d l\right)\sigma^2\delta(m-q)\\
&\quad + \frac{K}{(1+K)^2}\exp\left(j2\pi\breve{f}_d m\right)J_0\left(2\pi f_d(l-q)\right)\Big(P_0^2\\
&\quad + \sum_{\ell=1}^{L-1}P_\ell^2\delta(l-q)\Big) + \frac{K}{1+K}\exp\left(j2\pi\breve{f}_d m\right)\sigma^2\delta(l-q)\\
&\quad + \frac{K}{(1+K)^2}\exp\left(j2\pi\breve{f}_d(l-q)\right)J_0(2\pi f_d m)\Big(P_0^2\\
&\quad + \sum_{\ell=1}^{L-1}P_\ell^2\delta(m)\Big) + \frac{K}{1+K}\exp\left(j2\pi\breve{f}_d(l-q)\right)\sigma^2\delta(m)\\
&\quad + \frac{K}{(1+K)^2}\exp\left(j2\pi\breve{f}_d(m-q)\right)J_0(2\pi f_d l)\Big(P_0^2\\
&\quad + \sum_{\ell=1}^{L-1}P_\ell^2\delta(l)\Big) + \frac{K}{1+K}\exp\left(j2\pi\breve{f}_d(m-q)\right)\sigma^2\delta(l)\\
&\quad + \frac{1}{(1+K)^2}\\
&\quad \cdot\Big(J_0(2\pi f_d l)J_0\left(2\pi f_d(m-q)\right)\Big(P_0^2 + \sum_{\ell=1}^{L-1}P_\ell^2\delta(l)\Big)\\
&\quad \cdot\Big(P_0^2 + \sum_{\ell=1}^{L-1}P_\ell^2\delta(m-q)\Big) + J_0(2\pi f_d m)J_0\left(2\pi f_d(l-q)\right)\\
&\quad \cdot\Big(P_0^2 + \sum_{\ell=1}^{L-1}P_\ell^2\delta(m)\Big)\Big(P_0^2 + \sum_{\ell=1}^{L-1}P_\ell^2\delta(l-q)\Big)\Big)\\
&\quad + \frac{1}{1+K}J_0\left(2\pi f_d l\right)\sigma^2\delta(m-q)\Big(P_0^2 + \sum_{\ell=1}^{L-1}P_\ell^2\delta(l)\Big)\\
&\quad + \frac{1}{1+K}J_0\left(2\pi f_d m\right)\sigma^2\delta(l-q)\Big(P_0^2 + \sum_{\ell=1}^{L-1}P_\ell^2\delta(m)\Big)\\
&\quad + \frac{1}{1+K}J_0\Big(2\pi f_d(l-q)\Big)\sigma^2\delta(m)\Big(P_0^2 + \sum_{\ell=1}^{L-1}P_\ell^2\delta(l-q)\Big)\\
&\quad + \frac{1}{1+K}J_0\Big(2\pi f_d(m-q)\Big)\sigma^2\delta(l)\Big(P_0^2 + \sum_{\ell=1}^{L-1}P_\ell^2\delta(m-q)\Big)\\
&\quad + \sigma^4\Big(\delta(l)\delta(m-q) + \delta(m)\delta(l-q)\Big). \quad (8)
\end{aligned}$$

Proof: See the Appendix B for the derivation. ■

Obviously, the same terms are included in (7) and (8), and accordingly we can do some simple algebra operations on these two equations to derive the RFR. Through setting $l = m = q = 0$ in (8), we obtain

$$\begin{aligned}
&R_{\breve{r}\breve{r}\breve{r}\breve{r}}(0,0,0)\\
&= \left(\frac{K}{1+K}\right)^2 + \frac{4K}{(1+K)^2}\sum_{\ell=0}^{L-1}P_\ell^2 + \frac{4K}{1+K}\sigma^2\\
&\quad + \frac{4}{1+K}\sigma^2\sum_{\ell=0}^{L-1}P_\ell^2 + \frac{2}{(1+K)^2}\left(\sum_{\ell=0}^{L-1}P_\ell^2\right)^2 + 2\sigma^4\\
&= -\left(\frac{K}{1+K}\right)^2 + 2\left(\frac{K}{1+K} + \frac{1}{1+K}\sum_{\ell=0}^{L-1}P_\ell^2 + \sigma^2\right)^2. \quad (9)
\end{aligned}$$

We set $m = 0$ in (7) and have

$$R_{\breve{r}\breve{r}}(0) = \frac{K}{1+K} + \frac{1}{1+K}\sum_{\ell=0}^{L-1}P_\ell^2 + \sigma^2. \quad (10)$$

Substituting (10) into (9), we obtain

$$R_{\breve{r}\breve{r}\breve{r}\breve{r}}(0,0,0) = -\left(\frac{K}{1+K}\right)^2 + 2R_{\breve{r}\breve{r}}^2(0). \quad (11)$$

According to the property of the Rice factor, i.e., $K \geq 0$, and with a simple algebra operation, the RFR can be derived as

$$\frac{K}{1+K} = \sqrt{2R_{\breve{r}\breve{r}}^2(0) - R_{\breve{r}\breve{r}\breve{r}\breve{r}}(0,0,0)}. \quad (12)$$

In (12), if we utilize $\hat{R}_{\breve{r}\breve{r}}(0) = \hat{R}_{\breve{r}\breve{r}}(m)|_{m=0}$ and $\hat{R}_{\breve{r}\breve{r}\breve{r}\breve{r}}(0,0,0) = \hat{R}_{\breve{r}\breve{r}\breve{r}\breve{r}}(l,m,q)|_{l=m=q=0}$, where $\hat{R}_{\breve{r}\breve{r}}(m) = \frac{1}{N-m}\sum_{k=1}^{N-m}\breve{r}^*(k)\breve{r}(k+m)$ and $\hat{R}_{\breve{r}\breve{r}\breve{r}\breve{r}}(l,m,q) = \frac{1}{N-\max\{l,m,q\}}\sum_{k=1}^{N-\max\{l,m,q\}}\breve{r}^*(k)\breve{r}(k+l)\breve{r}(k+m)\breve{r}^*(k+q)$ respectively denote the estimations of $R_{\breve{r}\breve{r}}(m)$ and $R_{\breve{r}\breve{r}\breve{r}\breve{r}}(l,m,q)$, the estimator of the RFR can be expressed as

$$\widehat{\left(\frac{K}{1+K}\right)} = \sqrt{2\hat{R}_{\breve{r}\breve{r}}^2(0) - \hat{R}_{\breve{r}\breve{r}\breve{r}\breve{r}}(0,0,0)}. \quad (13)$$

B. CONCLUDED ALGORITHM ON THE ESTIMATION OF THE RICE FACTOR RATIO

Based on the analyses, discussions, and mathematical derivations mentioned above, we can clearly see that the estimation algorithm on the RFR proposed in this paper does not need to estimate the extra parameters such as the indistinguishable multipaths and noise variance, and to employ *a priori* knowledge of the frequency offset (the maximum Doppler shift and the Doppler shift), which also reduces the algorithm complexity and avoids error propagation. Except for

Algorithm 1 Estimation of Rice Factor Ratio for Doubly Selective Fading Channels

Input: $r(k)$

Output: $\widehat{\left(\frac{K}{1+K}\right)}$

1: Get $\breve{r}(k) = r(k) \cdot s^*(k)$

2: Calculate $\hat{R}_{\breve{r}\breve{r}}(0)$ which is the estimation of the (5) when $m = 0$

3: Calculate $\hat{R}_{\breve{r}\breve{r}\breve{r}\breve{r}}(0, 0, 0)$ that is the estimation of the (6) when $l = m = q = 0$

4: **return** $\widehat{\left(\frac{K}{1+K}\right)} = \sqrt{2\hat{R}^2_{\breve{r}\breve{r}}(0) - \hat{R}_{\breve{r}\breve{r}\breve{r}\breve{r}}(0, 0, 0)}$

the above merits, it can be noticed from the equation of the estimation on the RFR that only maximum estimation values of the second-order and fourth-order statistics defined in this paper are utilized, which further reduces the computational complexity so that the addressed scheme is more suitable for such application scenarios requiring low computational loads. Before proceeding to further discussion on performance evaluation, we conclude the proposed algorithm on the RFR estimation for the Rician doubly selective fading channels in Algorithm 1.

This algorithm clearly describes the mechanism of the estimation on the RFR. Generally speaking, except for the simple mathematical form of the estimated component, we further need to understand the performance of the proposed method. Thus, we will provide a detailed performance evaluation on the suggested algorithm for several conditions of interest through the computer simulation as follows.

IV. PERFORMANCE EVALUATION

Performance verification of the proposed RFR estimation algorithm is carried out in this section. Such scenarios are considered for the simulations and performance analyses, where the transmitted signals over the Rician doubly selective fading channels and additive noise are received. The Zadoff-Chu sequence is employed for known data, and carrier frequency is set as 2 GHz. The Rician non-frequency selective fast fading channel coefficient and Rayleigh frequency flat fast fading channel coefficients are generated by [28]. We have chosen $L = 3$ and 4 multipaths scenarios for performance comparisons, respectively. For convenience, we denote these two scenarios as S1 and S2, and the magnitudes of useful components for the S1 and S2 are as follows, S1: $P_0 = 0.55$, $P_1 = 0.72$, and $P_2 = 0.42$; and S2: $P_0 = 0.65$, $P_1 = 0.52$, $P_2 = 0.45$, and $P_3 = 0.32$. All simulation results are obtained by averaging over 10,000 Monte-Carlo trails and the RFR is assumed as a constant value in the estimation procedure.

A. BIAS OF THE ESTIMATED RICE FACTOR RATIO VERSUS K

In this subsection, the bias of the estimated RFR against K from 1 to 40, i.e., $K/(1 + K) \in [1/2, 40/41]$ is shown in Fig. 1 for the S1 and S2, where $N = 1024$, the mobile

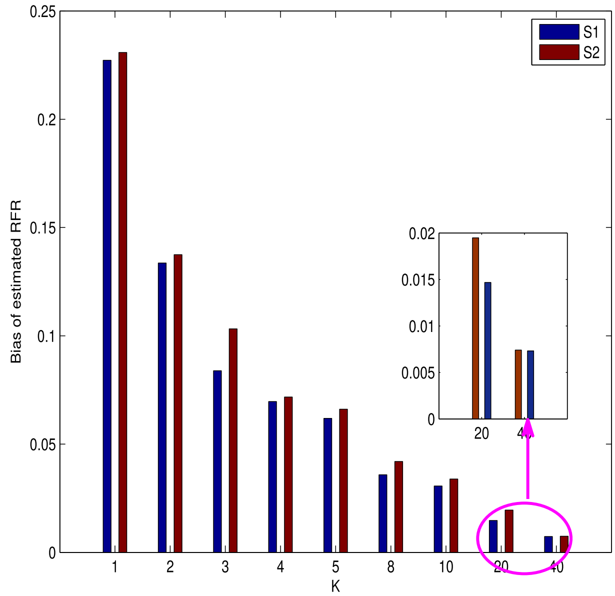

FIGURE 1. Comparisons of the bias on the estimated RFR versus the K for different wireless scenarios.

travels at a speed of 200 km/h, angle of arrival $\theta_0 = \pi/4$, and SNR is set as 20 dB. From this simulation experiment, it can be seen that the bias becomes smaller and smaller with the increments of the K. Furthermore, it is also shown that the bias of the estimated RFR for the S1 is less than that for the S2. As predicted, it verifies the effect of the scenarios with rich multipaths on the estimation performance of the RFR.

B. STD OF THE ESTIMATED RICE FACTOR RATIO VERSUS SNR

In the following subsections, the standard deviation (STD) of the estimated RFR is chosen as the performance index to further evaluate our proposed algorithm. The STD of the estimated RFR versus SNR is illustrated in Fig. 2, where most of parameters are the same as the ones considered in the previous section except for the K and SNR. In this section, the Rice factor is set as 10, 20 and 40, i.e., its corresponding the RFR is 10/11, 20/21 and 40/41, respectively, and the SNR varies from 2 to 20 dB. As seen from the Fig. 2, the STD of the estimation on the RFR is approximately from 0.22 to 0.26 for $K = 10$, from 0.16 to 0.19 for $K = 20$ and from 0.12 to 0.15 for $K = 40$ at the different wireless scenarios. Furthermore, we clearly see that the STD of the RFR estimation is somewhat unchanged when the SNR is higher than 0 dB in all scenarios considered in this paper. Further speaking, the investigated algorithm is robust at the SNR of over 0 dB for various scenarios we consider.

C. STD OF THE ESTIMATED RICE FACTOR RATIO VERSUS FREQUENCY OFFSET (MAXIMUM DOPPLER SHIFT AND DOPPLER SHIFT)

In this subsection, the influence of the maximum Doppler shift and the Doppler shift on the estimation performance of the proposed scheme is separately investigated

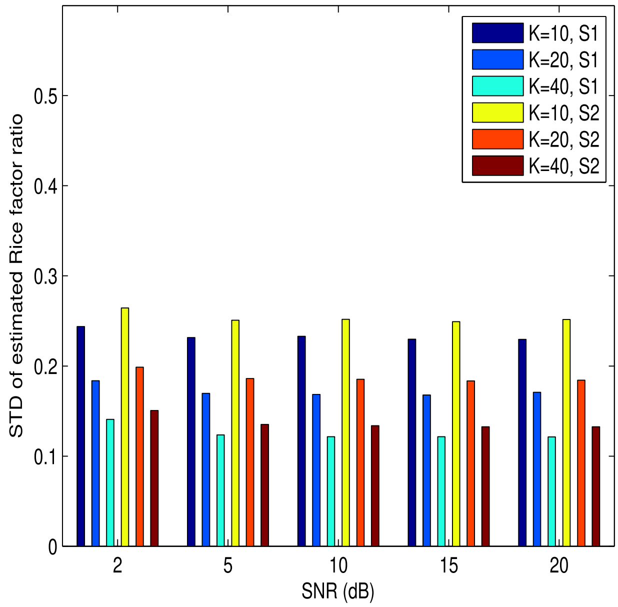

FIGURE 2. Comparisons of the STD on the estimated RFR versus the SNR for different wireless scenarios at $K = 10$, $K = 20$ and $K = 40$, respectively.

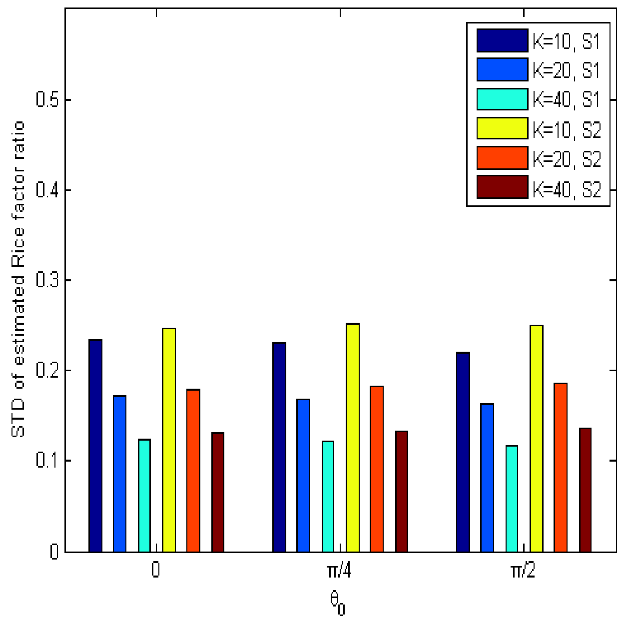

FIGURE 4. Comparisons of the STD on the estimated RFR versus the angle of arrival for different wireless scenarios at $K = 10$, $K = 20$ and $K = 40$, respectively.

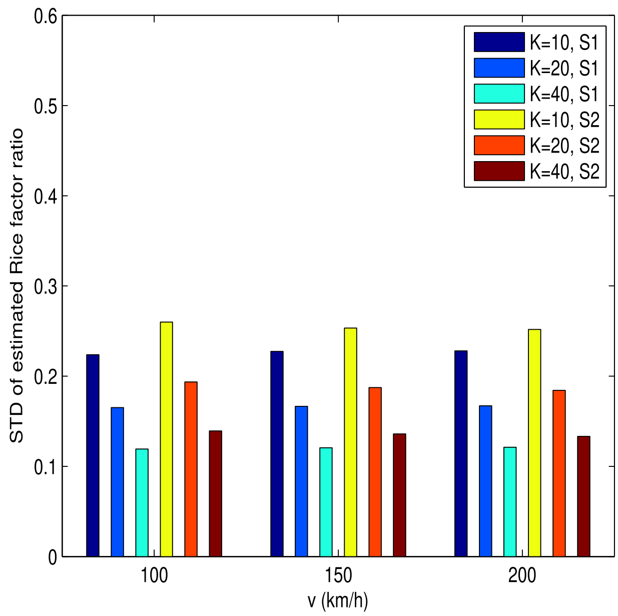

FIGURE 3. Comparisons of the STD on the estimated RFR versus the mobile speed for different wireless scenarios at $K = 10$, $K = 20$ and $K = 40$, respectively.

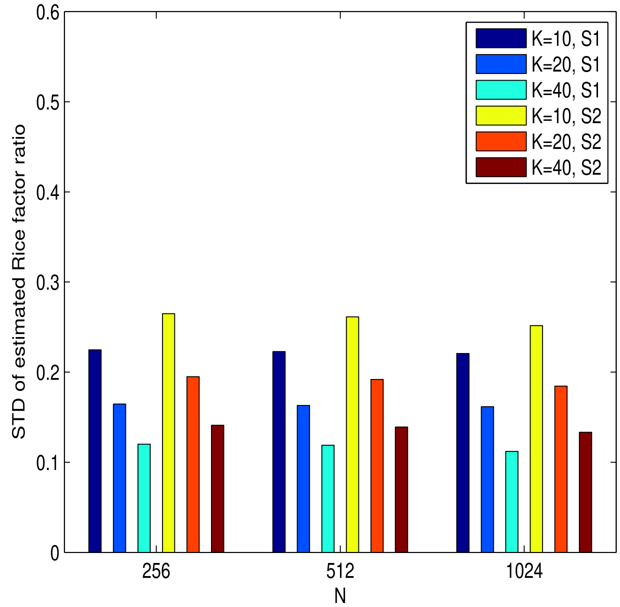

FIGURE 5. Comparisons of the STD on the estimated RFR versus the N for different wireless scenarios at $K = 10$, $K = 20$ and $K = 40$, respectively.

in Fig. 3 and Fig. 4. Some parameters such as N, SNR and K are set as the ones in the first and second scenario, respectively. In the first simulation, we employ the relation between the mobile speed and the maximum Doppler shift, i.e., $f_d = f_c v/c$, where f_c is the carrier frequency, c is the light speed and v is the mobile speed, and assume that the angle of arrival $\theta_0 = \pi/4$ and the mobile travels at different speeds from 100 km/h to 200 km/h. Namely, in Fig. 3, the effect of the maximum Doppler shift on the performance of the estimated RFR is evaluated. It can be seen from this figure that the estimated STD of the RFR is almost invariant when the speeds vary from the range mentioned above. In another simulation, we assume that the mobile speed is set as 200 km/h, and the angle of arrival θ_0 varies from 0 to $\pi/2$. That is to say, in this simulation, we investigate the influence of the Doppler shift on the estimation performance. It can be noticed from the Fig. 4 that the STD of the estimated RFR is almost unchanged for various angles of arrival. From the above results, we can conclude that the estimated RFR is robust to the maximum Doppler shift and the Doppler shift, which can enhance application scenarios.

D. STD OF THE ESTIMATED RICE FACTOR RATIO VERSUS N

In this subsection, we evaluate the effect of the length of the known data N on the estimation performance in Fig. 5. Except for N from 256 to 1024, most of the parameters are the ones adopted in the above subsections, i.e., the SNR, the mobile speed and the angle of arrival are set as 20 dB, 200 km/h, and $\pi/4$, respectively. The RFR is set as 10/11, 20/21 and 40/41, viz., $K = 10, 20$ and 40. It can be seen from simulation results that the STD of the estimated RFR shows a little change when $N \geq 256$, and slightly reduces as the N increases. Thus, as we expected, the proposed method is more suitable for such applications with the low computational complexity.

V. CONCLUSION

In this paper, the corresponding background of the parameter that is to be estimated and its formulation are introduced, and an estimation algorithm on the RFR is proposed for doubly selective fading channels. The addressed method first processes the received signals by employing the property of the known data, which is to be further handled by utilizing two propositions with the second-order and fourth-order statistics. Based on the above analyses and mathematical operations, a useful expression on the estimated RFR is presented for the scenarios mentioned in the preliminary. The investigated approach does not need extra parameter estimation and *a priori* information on the maximum Doppler shift or the Doppler shift, which not only avoids error propagation, but also reduces the algorithm complexity. Meanwhile, the suggested method only utilizes the maximum estimation values of the second-order and fourth-order statistics, which can further reduce the computational complexity. Furthermore, the proposed scheme is robust with respect to the SNR over 0 dB, the maximum Doppler shift and the Doppler shift, and has a small change on the STD with the increasing length of the known data, which is proven to be the potential applications where detailed scenarios were not considered and the proposed scheme is more suitable for the scenarios with the low computational complexity. Finally, these performance evaluations are carried out by computer simulation experiments, and their simulation results demonstrate the superior benefits of the presented algorithm.

APPENDIXES

APPENDIX A
PROOF OF THE PROPOSITION 1

This appendix presents the proof on the second-order statistic of the $\check{r}$. During the computation of the (7), $E\{w(k)\} \equiv 0$, $E\{w^*(k)\} \equiv 0$, $E\{w^*(k)w(k+m)\} \equiv \sigma^2\delta(m)$, $E\{h_{\mathrm{NLoS},\cdot}(k)\} \equiv 0$, $E\{h^*_{\mathrm{NLoS},\cdot}(k)\} \equiv 0$, $E\{h^*_{\mathrm{NLoS},\ell}(k)h_{\mathrm{NLoS},\ell'}(k+m)\} \equiv J_0(2\pi f_d m)\,\delta(\ell-\ell')$, and $E\{s^*(k-\iota_\ell)s(k)s(k+m-\iota_\ell)s^*(k+m)\} \equiv \delta(m)$ are utilized.

Based on the above identities, we obtain

$$
\begin{aligned}
&R_{\check{r}\check{r}}(m)\\
&= E\{\check{r}^*(k)\cdot\check{r}(k+m)\}\\
&= E\Bigg\{\Bigg(\sqrt{\frac{K}{1+K}}\exp(-j2\pi\check{f}_d k - j\phi_0)\\
&\quad+\sqrt{\frac{1}{1+K}}P_0 h^*_{\mathrm{NLoS},0}(k)\\
&\quad+\sqrt{\frac{1}{1+K}}\sum_{\ell=1}^{L-1}P_\ell h^*_{\mathrm{NLoS},\ell}(k)s^*(k-\iota_\ell)s(k)+w^*(k)\Bigg)\\
&\quad\cdot\Bigg(\sqrt{\frac{K}{1+K}}\exp\left(j2\pi\check{f}_d(k+m)+j\phi_0\right)\\
&\quad+\sqrt{\frac{1}{1+K}}P_0 h_{\mathrm{NLoS},0}(k+m)\\
&\quad+\sqrt{\frac{1}{1+K}}\sum_{\ell=1}^{L-1}P_\ell h_{\mathrm{NLoS},\ell}(k+m)s(k+m-\iota_\ell)s^*(k+m)\\
&\quad+w(k+m)\Bigg)\Bigg\}\\
&=\frac{K}{1+K}\exp\left(j2\pi\check{f}_d m\right)+\frac{1}{1+K}J_0(2\pi f_d m)\\
&\quad\cdot\left(P_0^2+\sum_{\ell=1}^{L-1}P_\ell^2\delta(m)\right)+\sigma^2\delta(m). \qquad (14)
\end{aligned}
$$

APPENDIX B
PROOF OF THE PROPOSITION 2

In this appendix, we will derive the fourth-order statistic of the $\check{r}$. Besides the identities employed in (7), in the process of calculating (8), $E\{(h_{\mathrm{NLoS},\cdot}(k+l)h_{\mathrm{NLoS},\cdot}(k+m))^*\} \equiv 0$, $E\{h_{\mathrm{NLoS},\cdot}(k+m)h_{\mathrm{NLoS},\cdot}(k+l)\} \equiv 0$, $E\{h^*_{\mathrm{NLoS},\cdot}(k)h_{\mathrm{NLoS},\cdot}(k+m)h_{\mathrm{NLoS},\cdot}(k+l)\} \equiv 0$, $E\{h_{\mathrm{NLoS},\cdot}(k+l)h_{\mathrm{NLoS},\cdot}(k+m)h^*_{\mathrm{NLoS},\cdot}(k+q)\} \equiv 0$, $E\{h^*_{\mathrm{NLoS},\cdot}(k)h_{\mathrm{NLoS},\cdot}(k+m)h^*_{\mathrm{NLoS},\cdot}(k+q)\} \equiv 0$, $E\{h^*_{\mathrm{NLoS},\cdot}(k)h_{\mathrm{NLoS},\cdot}(k+l)h^*_{\mathrm{NLoS},\cdot}(k+q)\} \equiv 0$, $E\{h^*_{\mathrm{NLoS},\ell}(k)h_{\mathrm{NLoS},\ell'}(k+l)h_{\mathrm{NLoS},\ell''}(k+m)h^*_{\mathrm{NLoS},\ell'''}(k+q)\} \equiv E\{h^*_{\mathrm{NLoS},\ell}(k)h_{\mathrm{NLoS},\ell'}(k+l)\}E\{h_{\mathrm{NLoS},\ell''}(k+m)h^*_{\mathrm{NLoS},\ell'''}(k+q)\} + E\{h^*_{\mathrm{NLoS},\ell}(k)h_{\mathrm{NLoS},\ell''}(k+m)\}E\{h_{\mathrm{NLoS},\ell'}(k+l)h^*_{\mathrm{NLoS},\ell'''}(k+q)\}$, $E\{(w(k+m)w(k+l))^*\} \equiv 0$, $E\{w(k+m)w(k+l)\} \equiv 0$, $E\{w^*(k)w(k+m)w(k+l)\} \equiv 0$, $E\{w(k+l)w(k+m)w^*(k+q)\} \equiv 0$, $E\{w^*(k)w(k+m)w^*(k+q)\} \equiv 0$, $E\{w^*(k)w(k+l)w^*(k+q)\} \equiv 0$, $E\{w^*(k)w(k+l)w(k+m)w^*(k+q)\} \equiv E\{w^*(k)w(k+l)\}E\{w(k+m)w^*(k+q)\} + E\{w^*(k)w(k+m)\}E\{w(k+l)w^*(k+q)\}$, $E\{s(k+m-\iota_\ell)s^*(k+m)s^*(k+q-\iota_\ell)s(k+q)\} \equiv \delta(m-q)$, and $E\{s(k+l-\iota_\ell)s^*(k+l)s^*(k+q-\iota_\ell)s(k+q)\} \equiv \delta(l-q)$ are employed.

Substituting (4) into (6), we obtain

$$
\begin{aligned}
&R_{\check{r}\check{r}\check{r}\check{r}}(l,m,q)\\
&=E\{\check{r}^*(k)\cdot\check{r}(k+l)\cdot\check{r}(k+m)\cdot\check{r}^*(k+q)\}\\
&=E\Bigg\{\Bigg(\underbrace{\sqrt{\frac{K}{1+K}}\exp(-j2\pi\check{f}_d k-j\phi_0)}_{A1}\\
&\quad\underbrace{+\sqrt{\frac{1}{1+K}}\left(P_0h^*_{\mathrm{NLoS},0}(k)+\sum_{\ell=1}^{L-1}P_\ell h^*_{\mathrm{NLoS},\ell}(k)s^*(k-\iota_\ell)s(k)\right)}_{A2}
\end{aligned}
$$

$$+\underbrace{w^*(k)}_{A3}\Big)\Big(\underbrace{\sqrt{\frac{K}{1+K}}\exp\left(j2\pi\breve{f}_d(k+l)+j\phi_0\right)}_{B1}$$

$$+\underbrace{\sqrt{\frac{1}{1+K}}P_0h_{\mathrm{NLoS},0}(k+l)}_{B2}$$

$$+\underbrace{\sqrt{\frac{1}{1+K}}\sum_{\ell=1}^{L-1}P_\ell h_{\mathrm{NLoS},\ell}(k+l)s(k+l-\iota_\ell)s^*(k+l)}_{B2}$$

$$+\underbrace{w(n+l)}_{B3}\Big)\Big(\underbrace{\sqrt{\frac{K}{1+K}}\exp\left(j2\pi\breve{f}_d(k+m)+j\phi_0\right)}_{C1}$$

$$+\underbrace{\sqrt{\frac{1}{1+K}}P_0h_{\mathrm{NLoS},0}(k+m)}_{C2}$$

$$+\underbrace{\sqrt{\frac{1}{1+K}}\sum_{\ell=1}^{L-1}P_\ell h_{\mathrm{NLoS},\ell}(k+m)s(k+m-\iota_\ell)s^*(k+m)}_{C2}$$

$$+\underbrace{w(n+m)}_{C3}\Big)\Big(\underbrace{\sqrt{\frac{K}{1+K}}\exp\left(-j2\pi\breve{f}_d(k+q)-j\phi_0\right)}_{D1}$$

$$+\underbrace{\sqrt{\frac{1}{1+K}}P_0h^*_{\mathrm{NLoS},0}(k+q)}_{D2}$$

$$+\underbrace{\sqrt{\frac{1}{1+K}}\sum_{\ell=1}^{L-1}P_\ell h^*_{\mathrm{NLoS},\ell}(k+q)s^*(k+q-\iota_\ell)s(k+q)}_{D2}$$

$$+\underbrace{w^*(n+q)}_{D3}\Big)\Big\}. \tag{15}$$

By employing the above identities, (15) is composed by

$$E\{A1\cdot B1\cdot C1\cdot D1\} = \left(\frac{K}{1+K}\right)^2\exp\left(j2\pi\breve{f}_d(l+m-q)\right), \tag{16}$$

$$\begin{aligned}E\{A1\cdot B1\cdot C1\cdot D2\} &= E\{A1\cdot B1\cdot C1\cdot D3\}\\ &= E\{A1\cdot B1\cdot C2\cdot D1\}\\ &= E\{A1\cdot B1\cdot C2\cdot D3\}\\ &= E\{A1\cdot B1\cdot C3\cdot D1\}\\ &= E\{A1\cdot B1\cdot C3\cdot D2\}\\ &= 0,\end{aligned} \tag{17}$$

$$\begin{aligned}E\{A1\cdot B1\cdot C2\cdot D2\} &= \frac{K}{(1+K)^2}\exp\left(j2\pi\breve{f}_d l\right)\\ &\cdot J_0\left(2\pi f_d(m-q)\right)\left(P_0^2+\sum_{\ell=1}^{L-1}P_\ell^2\delta(m-q)\right),\end{aligned} \tag{18}$$

$$E\{A1\cdot B1\cdot C3\cdot D3\} = \frac{K}{1+K}\exp\left(j2\pi\breve{f}_d l\right)\sigma^2\delta(m-q), \tag{19}$$

$$\begin{aligned}E\{A1\cdot B2\cdot C1\cdot D1\} &= E\{A1\cdot B2\cdot C1\cdot D3\}\\ &= E\{A1\cdot B2\cdot C2\cdot D1\}\\ &= E\{A1\cdot B2\cdot C2\cdot D2\}\\ &= E\{A1\cdot B2\cdot C2\cdot D3\}\\ &= E\{A1\cdot B2\cdot C3\cdot D1\}\\ &= E\{A1\cdot B2\cdot C3\cdot D2\}\\ &= E\{A1\cdot B2\cdot C3\cdot D3\}\\ &= 0,\end{aligned} \tag{20}$$

$$\begin{aligned}E\{A1\cdot B2\cdot C1\cdot D2\} &= \frac{K}{(1+K)^2}\exp\left(j2\pi\breve{f}_d m\right)\\ &\cdot J_0\left(2\pi f_d(l-q)\right)\left(P_0^2+\sum_{\ell=1}^{L-1}P_\ell^2\delta(l-q)\right),\end{aligned} \tag{21}$$

$$\begin{aligned}E\{A1\cdot B3\cdot C1\cdot D1\} &= E\{A1\cdot B3\cdot C1\cdot D2\}\\ &= E\{A1\cdot B3\cdot C2\cdot D1\}\\ &= E\{A1\cdot B3\cdot C2\cdot D2\}\\ &= E\{A1\cdot B3\cdot C2\cdot D3\}\\ &= E\{A1\cdot B3\cdot C3\cdot D1\}\\ &= E\{A1\cdot B3\cdot C3\cdot D2\}\\ &= E\{A1\cdot B3\cdot C3\cdot D3\}\\ &= 0,\end{aligned} \tag{22}$$

$$E\{A1\cdot B3\cdot C1\cdot D3\} = \frac{K}{1+K}\exp\left(j2\pi\breve{f}_d m\right)\sigma^2\delta(l-q), \tag{23}$$

$$\begin{aligned}E\{A2\cdot B1\cdot C1\cdot D1\} &= E\{A2\cdot B1\cdot C1\cdot D2\}\\ &= E\{A2\cdot B1\cdot C1\cdot D3\}\\ &= E\{A2\cdot B1\cdot C2\cdot D2\}\\ &= E\{A2\cdot B1\cdot C2\cdot D3\}\\ &= E\{A2\cdot B1\cdot C3\cdot D1\}\\ &= E\{A2\cdot B1\cdot C3\cdot D2\}\\ &= E\{A2\cdot B1\cdot C3\cdot D3\}\\ &= 0,\end{aligned} \tag{24}$$

$$\begin{aligned}E\{A2\cdot B1\cdot C2\cdot D1\} &= \frac{K}{(1+K)^2}\exp\left(j2\pi\breve{f}_d(l-q)\right)\\ &\cdot J_0(2\pi f_d m)\left(P_0^2+\sum_{\ell=1}^{L-1}P_\ell^2\delta(m)\right),\end{aligned} \tag{25}$$

$$\begin{aligned}E\{A2\cdot B2\cdot C1\cdot D1\} &= \frac{K}{(1+K)^2}\exp\left(j2\pi\breve{f}_d(m-q)\right)\\ &\cdot J_0(2\pi f_d l)\left(P_0^2+\sum_{\ell=1}^{L-1}P_\ell^2\delta(l)\right),\end{aligned} \tag{26}$$

$$
\begin{aligned}
&E\{A2 \cdot B2 \cdot C1 \cdot D2\} \\
&\quad = E\{A2 \cdot B2 \cdot C1 \cdot D3\} \\
&\quad = E\{A2 \cdot B2 \cdot C2 \cdot D1\} \\
&\quad = E\{A2 \cdot B2 \cdot C2 \cdot D3\} \\
&\quad = E\{A2 \cdot B2 \cdot C3 \cdot D1\} \\
&\quad = E\{A2 \cdot B2 \cdot C3 \cdot D2\} \\
&\quad = 0, \qquad (27)
\end{aligned}
$$

$$
\begin{aligned}
&E\{A2 \cdot B2 \cdot C2 \cdot D2\} \\
&\quad = \frac{1}{(1+K)^2}\Big(J_0(2\pi f_d l)J_0\Big(2\pi f_d(m-q)\Big) \\
&\qquad \cdot \Big(P_0^2 + \sum_{\ell=1}^{L-1} P_\ell^2 \delta(l)\Big)\Big(P_0^2 + \sum_{\ell=1}^{L-1} P_\ell^2 \delta(m-q)\Big) \\
&\qquad + J_0(2\pi f_d m)J_0\Big(2\pi f_d(l-q)\Big) \\
&\qquad \cdot \Big(P_0^2 + \sum_{\ell=1}^{L-1} P_\ell^2 \delta(m)\Big)\Big(P_0^2 + \sum_{\ell=1}^{L-1} P_\ell^2 \delta(l-q)\Big)\Big), \qquad (28)
\end{aligned}
$$

$$
\begin{aligned}
&E\{A2 \cdot B2 \cdot C3 \cdot D3\} \\
&\quad = \frac{1}{1+K} J_0\left(2\pi f_d l\right) \sigma^2 \delta(m-q) \\
&\qquad \cdot \left(P_0^2 + \sum_{\ell=1}^{L-1} P_\ell^2 \delta(l)\right), \qquad (29)
\end{aligned}
$$

$$
\begin{aligned}
&E\{A2 \cdot B3 \cdot C1 \cdot D1\} \\
&\quad = E\{A2 \cdot B3 \cdot C1 \cdot D2\} \\
&\quad = E\{A2 \cdot B3 \cdot C1 \cdot D3\} \\
&\quad = E\{A2 \cdot B3 \cdot C2 \cdot D1\} \\
&\quad = E\{A2 \cdot B3 \cdot C2 \cdot D2\} \\
&\quad = E\{A2 \cdot B3 \cdot C3 \cdot D1\} \\
&\quad = E\{A2 \cdot B3 \cdot C3 \cdot D2\} \\
&\quad = E\{A2 \cdot B3 \cdot C3 \cdot D3\} \\
&\quad = 0, \qquad (30)
\end{aligned}
$$

$$
\begin{aligned}
&E\{A2 \cdot B3 \cdot C2 \cdot D3\} \\
&\quad = \frac{1}{1+K} J_0\left(2\pi f_d m\right) \sigma^2 \delta(l-q) \\
&\qquad \cdot \left(P_0^2 + \sum_{\ell=1}^{L-1} P_\ell^2 \delta(m)\right), \qquad (31)
\end{aligned}
$$

$$
\begin{aligned}
&E\{A3 \cdot B1 \cdot C1 \cdot D1\} \\
&\quad = E\{A3 \cdot B1 \cdot C1 \cdot D2\} \\
&\quad = E\{A3 \cdot B1 \cdot C1 \cdot D3\} \\
&\quad = E\{A3 \cdot B1 \cdot C2 \cdot D1\} \\
&\quad = E\{A3 \cdot B1 \cdot C2 \cdot D2\} \\
&\quad = E\{A3 \cdot B1 \cdot C2 \cdot D3\} \\
&\quad = E\{A3 \cdot B1 \cdot C3 \cdot D2\} \\
&\quad = E\{A3 \cdot B1 \cdot C3 \cdot D3\} \\
&\quad = 0, \qquad (32)
\end{aligned}
$$

$$
\begin{aligned}
&E\{A3 \cdot B1 \cdot C3 \cdot D1\} \\
&\quad = \frac{K}{1+K} \exp\Big(j2\pi \breve{f}_d(l-q)\Big)\sigma^2\delta(m), \qquad (33)
\end{aligned}
$$

$$
\begin{aligned}
&E\{A3 \cdot B2 \cdot C1 \cdot D1\} \\
&\quad = E\{A3 \cdot B2 \cdot C1 \cdot D2\} \\
&\quad = E\{A3 \cdot B2 \cdot C1 \cdot D3\} \\
&\quad = E\{A3 \cdot B2 \cdot C2 \cdot D1\} \\
&\quad = E\{A3 \cdot B2 \cdot C2 \cdot D2\} \\
&\quad = E\{A3 \cdot B2 \cdot C2 \cdot D3\} \\
&\quad = E\{A3 \cdot B2 \cdot C3 \cdot D1\} \\
&\quad = E\{A3 \cdot B2 \cdot C3 \cdot D3\} \\
&\quad = 0, \qquad (34)
\end{aligned}
$$

$$
\begin{aligned}
&E\{A3 \cdot B2 \cdot C3 \cdot D2\} \\
&\quad = \frac{1}{1+K} J_0\Big(2\pi f_d(l-q)\Big)\sigma^2\delta(m) \\
&\qquad \cdot \left(P_0^2 + \sum_{\ell=1}^{L-1} P_\ell^2 \delta(l-q)\right), \qquad (35)
\end{aligned}
$$

$$
\begin{aligned}
&E\{A3 \cdot B3 \cdot C1 \cdot D1\} \\
&\quad = \frac{K}{1+K} \exp\Big(j2\pi \breve{f}_d(m-q)\Big)\sigma^2\delta(l), \qquad (36)
\end{aligned}
$$

$$
\begin{aligned}
&E\{A3 \cdot B3 \cdot C1 \cdot D2\} \\
&\quad = E\{A3 \cdot B3 \cdot C1 \cdot D3\} \\
&\quad = E\{A3 \cdot B3 \cdot C2 \cdot D1\} \\
&\quad = E\{A3 \cdot B3 \cdot C2 \cdot D3\} \\
&\quad = E\{A3 \cdot B3 \cdot C3 \cdot D1\} \\
&\quad = E\{A3 \cdot B3 \cdot C3 \cdot D2\} \\
&\quad = 0, \qquad (37)
\end{aligned}
$$

$$
\begin{aligned}
&E\{A3 \cdot B3 \cdot C2 \cdot D2\} \\
&\quad = \frac{1}{1+K} J_0\Big(2\pi f_d(m-q)\Big)\sigma^2\delta(l) \\
&\qquad \cdot \left(P_0^2 + \sum_{\ell=1}^{L-1} P_\ell^2 \delta(m-q)\right), \qquad (38)
\end{aligned}
$$

$$
\begin{aligned}
&E\{A3 \cdot B3 \cdot C3 \cdot D3\} \\
&\quad = \sigma^4\Big(\delta(l)\delta(m-q) + \delta(m)\delta(l-q)\Big). \qquad (39)
\end{aligned}
$$

Based on the above detailed derivations, we conclude the proposition 2.

ACKNOWLEDGMENT

The authors would like to thank the anonymous referees for their valuable comments and helpful suggestions to improve the quality and readability of the early version of the paper.

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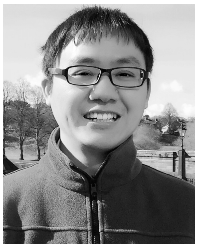

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