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On Scheduling Policies With Heavy-Tailed Dynamics in Wireless Queueing Systems

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ABSTRACT This paper takes a system view and studies a wireless queueing system where heavy-tailness may occur both at the traffic arrival and in the form of the multi-user interference. With the rapid development of AI technologies, this heavy-tailed traffic model has become more prevalent in the current network system, such as the file or data size used in the deep learning algorithm. We first re-visit the standard asymmetric queueing system with a mix of heavy-tailed and light-tailed traffic, but under a new variable-rate service model that not only better models the dynamics of the wireless medium but also includes the previous models as special cases. We then focus on the scheduling problem when heavy-tailed interference disrupts the serving link. The performance of queueing policies is investigated during an ON/OFF renewal channel process with heavy-tailed OFF periods, and the expected queue length and the throughput characteristic is studied under the priority as well as max-weight scheduling policies. The results show that the expected queue length of the heavy queue cannot be maintained as finite even under the most favorable priority policy. On the other hand, a priority policy can guarantee the finiteness of an expected queue length for the light queue, but the system is not throughput optimal any longer. It is further shown that no benefit can be provided by the max-weight scheduling policy to the light queue for the queue length behavior in a steady-state, though the system is always throughput optimal.

INDEX TERMS Heavy-tailed interference, queueing analysis, scheduling, artificial intelligence.

I. INTRODUCTION

Scheduling policy is an important problem in a networking area, and we have seen lots of literature for variant systems, including wireless [1], ad-hoc [2], and multi-hop networks [3]. In this field, simple traffic models were considered in early research, such as Poisson or Markov-modulated processes, as well as simple service models. However, the network traffic nowadays has begun to show strong correlations and statistical similarity [4]. As these simple traffic models are not enough [5], *heavy-tailed* distributions have been adopted to model such traffic in networking area. In these packetized networks, the file sizes vary quite significantly, and data traffic is shown to be more bursty. In these packetized networks, the file sizes vary quite significantly, and data traffic is shown to be more bursty. For example, with the rapid development of AI technoglogies [6]–[9], the data size used in the deep learning algorithm might be heavy-tailed [10].

A majority of the existing scheduling policies analyses, when heavy-tailed traffic exists, focuses on *asymmetric* queueing models, which shows how the system behavior is influenced by heavy-tailed *traffic*. In these models, there is generally a mix of heavy-tailed and light-tailed traffic [11]–[13], which can describe the heterogeneous traffic types in the network. Regarding the performance metric, two criteria are often considered: *expected queue length* and

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throughput optimality. Throughput is a widely used performance metric, since it shows the best performance region that a scheduling policy can achieve. Second, since the delay is proportional to the queue length, a finite expected queue length in the steady-state is preferred whenever possible.

The authors [13] investigate a single-server system with two parallel wireline queues, where one queue has heavy-tailed traffic (''heavy queue'') while the other has light-tailed traffic (''light queue''). They show that no scheduling policy can guarantee the heavy queue's queue length asymptotic, and the light queue may act as poorly as the heavy queue if an inappropriate policy is selected, including the famous max-weight policy [14]. Based on this work, [15] obtains the queue length distribution in a steady-state, and characterizes its tail coefficient for both the max-weight-α and log-max-weight policies, which prefers to provide more service to the light queue compared to the max-weight policy. The authors in [14] prove that max-weight is throughput optimal. The authors in [16] study the delay stability of the max-weight policies for more complex networks. They propose a novel approach that combines the fluid model with either renewal theory (for instability results) or stochastic Lyapunov theory (for stability results) in [17]. Recently, the authors [18] have extended their work to multi-hop networks, and show that the back-pressure-α policy can stabilize the light flows delay while the original back-pressure policy cannot, which is similar to the result for the max-weight policies in the single-hop case.

In the wireless queueing problems, *channel dynamics* have to be considered. The authors in [19] introduce an ON/OFF channel between the server and the queue, and study both the max-weight-α and log-max-weight policies. They characterize the tail behavior of the queue length in steady-state under heavy-tailed arrival processes. The same ON/OFF channel model is also considered in [20], where they study the impact of heavy-tailed traffic on the response time. Both inter-queue and intra-queue scheduling choices are considered to optimize the delay, and the analysis shows that the response time tail of the light queue is sensitive to the intra-queue policy choice.

All of the aforementioned works have laid the theoretical foundation for asymmetric queueing models with both heavy-tailed and light-tailed traffic. However, they may not be enough when we consider a *wireless* queueing system. This is because some unique characteristics for a wireless system have not been fully considered in previous studies. First, the literature only considers channels that are either always on [13], [15], [16], or can be modeled by some light-tailed ON/OFF channels, e.g., the time-varying ON/OFF channels with independent and identically distributed (i.i.d.) Bernoulli processes [19]. Such channel modeling assumes that the channel only experiences short time correlations, which might not be true in reality. A better capture for the wireless dynamics, such as large-scale and small-scale *channel fading* [21] needs to be considered.

Second, the heavy-tailed distributions are only considered in the *traffic* model. However, for better understanding the influence of heavy-tailed distributions in a wireless system, we should not only consider the traffic arrival, but also the potential heavy-tailed *interference* from other users. This is because of the distinctive property of wireless communications, that is, the *broadcasting* nature [21]. As a result, if the data transmission for one user shows some form of heavy-tailed behavior [22], it will result in a a *heavy-tailed* interference to other users in the system when the interference is strong.

In this paper, we perform a systematic study on the impact of heavy-tailed dynamics to the scheduling policies in a wireless queueing system. First we introduce a general *variable-rate service model* that can be used to study the standard asymmetric queueing system with a mix of heavy-tailed and light-tailed distributions for either traffic arrival or multi-user interference. This new model incorporates the effect of channel fading and multi-user interference, and thus better captures the dynamics of the wireless medium. We first analyze the model with heavy-tailed traffic arrivals and general time-varying service rate, and study the expected queue-length behavior in a steady-state of priority, max-weight, and max-service-rate scheduling policies. We then focus on the scheduling problem in the presence of heavy-tailed interference, which can be modeled by the proposed variable-rate service model with a heavy-tailed OFF period to one queue (''heavy queue'') and light-tailed for the other (''light queue'').[1] It is shown that the queue length for the heavy queue is unstable even under the most favorable priority-for-H policy. Thus, no scheduling policies can guarantee the heavy queue's queue-length stability. Regarding the light queue, we show that the queue-length distribution is light-tailed under the priority-for-L policy, but throughput optimal does not exist any more. For the max-weight scheduling policy, a threshold-base performance for the light queue is discovered. If the arrival rate for the light queue is below some threshold, any scheduling policy can stabilize the light queue. On the contrary, when the arrival rate to the light queue is greater than the threshold, the expected queue length in a steady-state is proven to be infinite under the max-weight scheduling policy. Hence, no benefit can be provided by the max-weight scheduling to the light queue in terms of the steady-state queue-length performance, although the system is throughput optimal.

To the best of the authors' knowledge, no result has been obtained for a wireless network that contains both heavy-tailed and light-tailed *interference*. From a methodological point of view, the main technical difficulty for this model comes from the correlation of interference across time slots, which makes the queue-length behavior no longer a Markov process. As we will see, the technicality renders the standard tools, such as Lyapunov functions,

[1]Only priority and max-weight policies are studied for the interference model, as the max-service-rate policy is not applicable.

difficult to use. We thus resort to the creation of *artificial queues* that can provide *delay bounds* to the original model, and use an asymptotic argument to derive the steady-state results.

The rest of the paper is organized as follows. Section II shows the system model and some preliminaries. The impact of mixing heavy-tailed and light-tailed traffic arrival is studied with a general variable-rate service model in Section III. We then analyze the performance of priority and max-weight scheduling policies with heavy-tailed multi-user interference in Section IV. We conclude the paper in Section V, with a discussion for future directions.

II. SYSTEM MODEL AND PRELIMINARIES

A. DEFINITIONS

We first show some relevant definitions, which will be extensively used in the remainder of this paper.

Definition 1 (Heavy-Tailness): A nonnegative random variable X is called heavy-tailed if $\mathbb{E}[X^2] = \infty$. It is light-tailed otherwise.

We note that there are different definitions of heavy-tailness in the literature. For example, light-tailed is sometimes defined if the random variable is of exponential type, and heavy-tailed otherwise. Our definition is the same as [16], [23].

Definition 2 (Tailed Coefficient): The tailed coefficient C of a nonnegative random variable X is defined as $C = \min\{C \in \mathbb{R}^+ | \mathbb{E}[X^C] = \infty\}$.

Definition 3 (Rate Stability): A queue $Q(t)$ is stable if

$$\lim_{t\to\infty} \frac{Q(t)}{t} = 0 \text{ with probability 1.}$$

Based on the queueing theorem, a queue is stable if and only if the mean service rate is greater than the mean arrival rate.

Definition 4 (Capacity Region): The capacity region of a queueing system with K queues is the rate region where the mean rate tuple $(\lambda_1, \cdots, \lambda_K)$ is stably supportable, i.e., the queues can be stable for some scheduling policy. Here λ_i denotes the mean arrival rate for queue i, $i \in \{1, \cdots, K\}$.

Definition 5 (Throughput Optimality): We call a scheduling policy throughput optimal if the queues can be stabilized when the mean rate tuple is within the capacity region.

Definition 6 (Stable-State Queue Length): A queue $Q(t)$ is stable and its stable-state queue length is defined as

$$q = \lim_{t\to\infty} \mathbb{E}[Q(t)].$$

B. THE WIRELESS QUEUEING SYSTEM MODEL

In this section, we introduce the general service-rate queueing model. We consider a time-slotted wireless system with a single server and two parallel queues H and L, as illustrated in Figure 1. The server can serve at most one queue at a time and the service rate is a normalized 1 packet per time slot.[2] Both queues are assumed to have infinite buffer. We denote the queue lengths as $Q_H(t)$ and $Q_L(t)$, and their arrival processes $A_H(t)$ and $A_L(t)$, respectively, where $A_i(t)$ is the number of packets arriving at queue i in time slot t, $i \in \{H, L\}$. We further use q_H and q_L to denote the steady-state queue length when they exist. We assume that the arrival process $A_i(t)$ is stationary with mean λ_i and tail coefficient C_i, $i \in \{H, L\}$. All arrivals are assumed to be i.i.d. over time and of each queue, and occur at the end of each time slot. We will refer to queue H and L as the "heavy queue" and "light queue", respectively, although the heavy-tailed and light-tailed distributions may refer to either traffic arrival or channel OFF period, depending on the context.

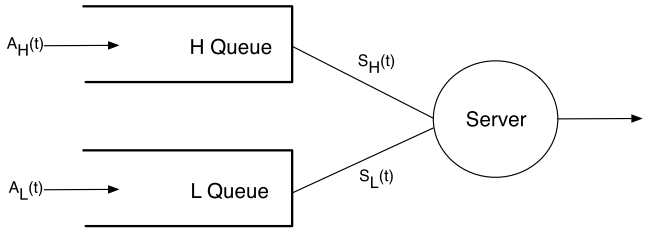

FIGURE 1. The general asymmetric parallel queueing model with traffic arrivals of two queues and time-varying wireless channels.

Both queues are connected to the server via wireless time-varying channels, which fluctuate over time according to some random processes. In practice, this fluctuation may be due to channel noise or other interference in the wireless medium. Specifically, we denote the wireless channel between queue i and server at time t as $G_i(t)$, which is stationary with mean g_i, $i \in \{H, L\}$. Correspondingly, the available service rate, which is determined by a combination of the service rate and the channel state, is a time-varying random process and denoted as $S_i(t)$ for $i \in \{H, L\}$. We assume that $S_i(t)$ is stationary, has mean s_i, and has support $\mathcal{R}_i$, $i \in \{H, L\}$. The variable service rate processes $S_H(t)$ and $S_L(t)$ are independent of the arrival processes and of each other. We assume that the channel conditions, or equivalently the service rates, are known by the queues before the scheduling decision. Note that the channel states may not be i.i.d. over time, which is particularly suitable when the interference is persistent. This is a case that will be considered in Section IV.

III. HEAVY-TAILED TRAFFIC ARRIVALS

We first analyze the general variable-rate service model of Section II with a mix of heavy-tailed and light-tailed *traffic arrival* distributions. To that end, we further assume that the arrival processes to the L (H) queue, $A_L(t)$ ($A_H(t)$), is light-tailed (heavy-tailed) according to Definition 1. When the scheduler chooses to serve queue i, the service rate is a random process $S_i(t)$, which is i.i.d. over time and across queues. We assume that both $S_H(t)$ and $S_L(t)$ are light-tailed,

[2]The service rate can be a random process in our model. Since we also consider the random channel variations, such service rate randomness can be absorbed into the channel variation, while keeping a normalized packet rate.

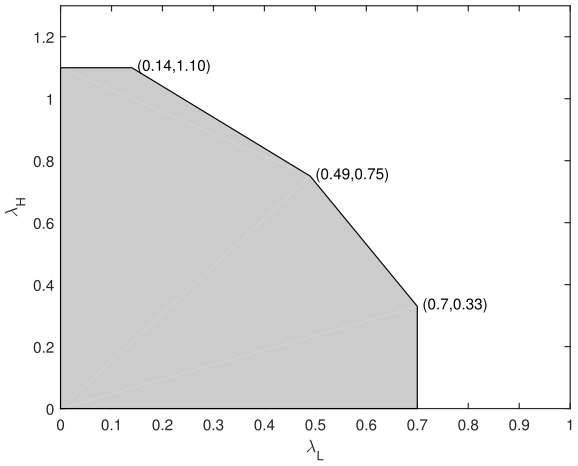

FIGURE 2. The capacity region Λ of the heavy-tailed traffic arrival model with $p_0 = 0.3, p_1 = 0.7$ and $r_0 = 0.2, r_1 = 0.5, r_2 = 0.3$.

i.i.d. over time, and distributed according to some discrete distributions with $P\{S_L(\cdot) = k\} = p_k$, $k \in \mathcal{R}_L$, and $P\{S_H(.) = j\} = r_j$, $j \in \mathcal{R}_H$. Obviously, $s_L = \sum_{k\in\mathcal{R}_L} p_k k$, $s_H = \sum_{j\in\mathcal{R}_H} p_j j$.

For this model, the capacity region, according to Definition 4, can be easily derived and is given by

$$\Lambda = \bigcup_{a_{kj}\in\mathcal{A}} \{(\lambda_H, \lambda_L)\} | 0 < \lambda_L < \sum_{k\in\mathcal{R}_L}\sum_{j\in\mathcal{R}_H} a_{kj}p_k r_j k,$$
$$0 < \lambda_H < \sum_{k\in\mathcal{R}_L}\sum_{j\in\mathcal{R}_H} (1 - a_{kj})p_k r_j j\}, \quad (1)$$

where

$$\mathcal{A} \triangleq \{a_{kj} : a_{k0}=1, a_{0j}=0, 0 \le a_{kj} \le 1, \forall k \in \mathcal{R}_L, j \in \mathcal{R}_H\}. \quad (2)$$

The boundary of the capacity region consists of a piecewise linear curve. Let (Λ_L, Λ_H) denote a boundary point of the capacity region, then we have that $0 < \Lambda_L < s_L$. Moreover, for a given Λ_L, we can determine the corresponding Λ_H by solving the following linear programming problem:

$$\begin{aligned} \underset{\{a_{kj}\}}{\text{maximize}} \quad & \sum_{k\in\mathcal{R}_L}\sum_{j\in\mathcal{R}_H}(1 - a_{kj})p_k r_j j \\ \text{subject to} \quad & \sum_{k\in\mathcal{R}_L}\sum_{j\in\mathcal{R}_H} a_{kj}p_k r_j k = \Lambda_L \\ & a_{k0} = 1, \quad \forall k \in \mathcal{R}_L \\ & a_{0j} = 0, \quad \forall j \in \mathcal{R}_H \\ & 0 \le a_{kj} \le 1, \quad \forall k \in \mathcal{R}_L,\ j \in \mathcal{R}_H. \end{aligned} \quad (3)$$

A pictorial illustration of the capacity region is shown as the gray area in Fig. 2.

A. PRIORITY FOR H

In this policy, the scheduler selects queue H as long as $S_H(t)Q_H(t) > 0$, and serves queue L in other scenarios. Note that the policy does not need to know either the actual queue length or the service rate, only that they are nonzero. Again, this represents the best-case scenario for the heavy queue, and the steady-state queue length analysis will shed light into the behavior of the heavy queue. We have the following result.

Proposition 1: Under the priority-for-H scheduling policy, the heavy queue is stable and $q_H = \infty$.

Proof: It is equivalent to proving

$$\lim_{t\to\infty} \mathbb{E}[Q_H(t)] = \infty.$$

The main idea behind the proof is to consider the renewal intervals that commence at the beginning of each busy period of queue H. Define the renewal reward process $R(t) = Q_H(t)$. We have

$$\lim_{t\to\infty} \mathbb{E}[R(t)] = \frac{\mathbb{E}[R]}{\mathbb{E}[T]},$$

from the key renewal theorem, where $\mathbb{E}[R]$ denotes the expected reward accumulated over a renewal interval, and $\mathbb{E}[T] < \infty$ is the mean renewal interval. It is therefore enough to show that

$$\mathbb{E}[\sum_{t=0}^{T} Q_H(t)] = \infty. \quad (4)$$

To prove Eqn. (4), let us condition on the busy period commencing at time 0 with a burst of size b to queue H. After this instant, assuming for the sake of a lower bound that there are no further bursts arriving at queue H, then with high probability, queue H drains at rate s_H. So the reward is at least $\mathcal{O}(b)$ for $\mathcal{O}(b)$ time slots. Thus, for some constant K,

$$\mathbb{E}[\sum_{t=0}^{T} Q_H(t)] \ge \mathbb{E}[Kb \cdot b] = \mathbb{E}[Kb^2] = \infty,$$

where the last expectation is infinite because the initial burst size has tail coefficient C_H that is smaller than 2. □

B. PRIORITY FOR L

The priority-for-L policy is similarly defined as Section III-A. We have the following main result.

Proposition 2: Under the priority-for-L scheduling policy, the following statements hold:

1) *If $\lambda_H > (1 - \lambda_L/s_L + p_0\lambda_L/s_L)s_H$, the heavy queue is unstable, and no steady-state exists.*
2) *If $\lambda_H < (1 - \lambda_L/s_L + p_0\lambda_L/s_L)s_H$, the heavy queue is stable, and $q_H = \infty$.*
3) *The light queue is stable and $q_L < \infty$.*

Proof: For the light queue, we can prove $q_L < \infty$ by considering the Lyapunov function

$$V_L(Q_L(t)) = \frac{1}{2}Q_L^2(t).$$

Expanding the term $\mathbb{E}[V_L(Q_L(t+1))|Q_L(t)]$ at $Q_L(t)$, we have:

$$\mathbb{E}[V_L(Q_L(t+1))|Q_L(t)]$$
$$= \mathbb{E}[\frac{1}{2}(Q_L(t) + A_L(t) - \min\{Q_L(t), S_L(t)\})^2|Q_L(t)]$$

$$
\begin{aligned}
&= V_L(Q_L(t)) + \mathbb{E}[A_L(t) - S_L(t) \\
&\quad + \max\{0, S_L(t) - Q_L(t)\}|Q_L(t)]Q_L(t) \\
&\quad + \mathbb{E}[(A_L(t) - \min\{Q_L(t), S_L(t)\})^2|Q_L(t)] \\
&\le V_L(Q_L(t)) + (\lambda_L - s_L)Q_L(t) \\
&\quad + \mathbb{E}[\max\{0, S_L(t) - Q_L(t)\}|Q_L(t)]Q_L(t) \\
&\quad + \mathbb{E}[A_L^2(t) + S_L^2(t)|Q_L(t)] \\
&\le V_L(Q_L(t)) + (\lambda_L - s_L)Q_L(t) + \mathbb{E}[S_L^2(t)] \\
&\quad + \mathbb{E}[A_L^2(t) + S_L^2(t)].
\end{aligned}
$$

Since $A_L(t)$ and $S_L(t)$ are both light-tailed, we have $\mathbb{E}[A_L^2(t)] < \infty$ and $\mathbb{E}[S_L^2(t)] < \infty$. Hence, there exists a positive constant b such that

$$\mathbb{E}[V_L(Q_L(t+1))|Q_L(t)] \le V_L(Q_L(t)) + (\lambda_L - s_L)Q_L(t) + b. \tag{5}$$

Therefore, using the Lyapunov theory, we can add up inequalities (5) over all $t \in \{0, 1, \ldots, T-1\}$ and divide by T, and then we have:

$$q_L \le \frac{b}{s_L - \lambda_L} < \infty.$$

For the heavy queue, it can be served only when $S_L(t) = 0$ or when $S_L(t) > 0$ but $Q_L(t) = 0$. Since the light queue has a finite expected queue length, according to Little's Theorem, we have that

$$P\{Q_L(t) = 0\} = 1 - \frac{\lambda_L}{s_L}.$$

Therefore, the average service rate of H is

$$p_0 s_H + (1 - p_0)(1 - \frac{\lambda_L}{s_L})s_H = (1 - \lambda_L/s_L + p_0\lambda_L/s_L)s_H$$

almost surely, which is equivalent to $\sum_{k\in\mathcal{R}_L, j\in\mathcal{R}_H}(1 - a_{kj})p_k r_j j$ when $a_{0j} = 0$, $a_{k0} = 1$, $a_{kj} = \frac{\lambda_L}{s_L}$.

However, when $\Lambda_L = \lambda_L$, we have

$$\Lambda_H = \max_{a_{kj}\in\mathcal{A}}\{\sum_{k\in\mathcal{R}_L}\sum_{j\in\mathcal{R}_H}(1 - a_{kj})p_k r_k j| \sum_{k\in\mathcal{R}_L}\sum_{j\in\mathcal{R}_H} a_{kj}p_k r_j k = \lambda_L\},$$

from the capacity region (1). By choosing $a_{0j} = 0$, $a_{i0} = 1$ and $a_{ij} = \frac{\lambda_L - s_L r_0}{(1-r_0)s_L}$, we know $\sum_{k\in\mathcal{R}_L}\sum_{j\in\mathcal{R}_H} a_{kj}p_k r_j k = \lambda_L$ is satisfied. Also, since $\frac{\lambda_L - s_L r_0}{(1-r_0)s_L} < \frac{\lambda_L}{s_L}$, we have

$$
\begin{aligned}
\Lambda_H &\ge \sum_{k\in\mathcal{R}_L}\sum_{j\in\mathcal{R}_H}(1 - \frac{\lambda_L - s_L r_0}{(1-r_0)s_L})p_k r_j j \\
&> \sum_{k\in\mathcal{R}_L}\sum_{j\in\mathcal{R}_H}(1 - \frac{\lambda_L}{s_L})p_k r_j j \\
&\ge (1 - \lambda_L/s_L + p_0\lambda_L/s_L)s_H,
\end{aligned}
$$

which means that there must exist a rate pair $\{\lambda_L, \lambda_H\}$ in the capacity region that satisfies $\lambda_H > (1 - \lambda_L/s_L + p_0\lambda_L/s_L)s_H$. In this case, the heavy queue is unstable, and no steady-state exists. As a result, the priority-for-L policy is not throughput optimal.

If $\lambda_H < (1 - \lambda_L/s_L + p_0\lambda_L/s_L)s_H$, then q_H is stable. However, the expected queue length of queue H is infinite even under the priority-for-H policy, so it will remain infinite under the priority-for-L policy. □

Proposition 3: Under any nonidling scheduling policy, if $\lambda_L < s_L r_0$, then $q_L < \infty$.

Proof: Suppose queue L is only served when event $\{S_H(t) = 0\}$ occurs. Since $S_H(\cdot)$ and $S_L(\cdot)$ are both i.i.d. and $S_L(\cdot)$ is light-tailed, we can derive that the service rate of queue L (denoted as S_{Le}) in every time slot is light-tailed with mean $s_L r_0$. Precisely, $P\{S_{Le} = 0\} = 1 - r_0 + r_0 p_0$, $P\{S_{Le} = k\} = r_0 p_k$ for $k \in \mathcal{R}_L$. Therefore, as in Proposition 2, if $\lambda_L < s_L r_0$, we can similarly prove that $\mathbb{E}[q_L] < \infty$. Moreover, the service opportunity is always offered to queue L when $S_H(t) = 0$, no matter what policy it is. Therefore, if $\lambda_L < s_L r_0$, $\mathbb{E}[q_L] < \infty$ holds under any nonidling scheduling policy. □

C. MAX WEIGHT

The max-weight policy, unlike the priority policies, makes use of the queue length information at each time slot. Hence, we assume that the scheduler knows $Q_H(t)$ and $Q_L(t)$ in addition to the service rate information $S_H(t)$ and $S_L(t)$. More precisely, the scheduler compares

$$S_H(t)Q_H(t) \gtrless S_L(t)Q_L(t)$$

and chooses to serve the queue that wins the competition.

For the max-weight policy, the following result states that the steady-state queue length will be infinite for large λ_L.

Proposition 4: Under max-weight scheduling with a bounded service rate $S_L(.) \le S_{Lmax}$ for queue L, if $\lambda_L > s_L r_0$, then $\mathbb{E}[Q_L] = \infty$.

Proof: Conditioning on the busy period commencing with a burst of size b at the heavy queue, $Q_H(t) > S_{Lmax}Q_L(t)$ is almost surely satisfied. Therefore, the heavy queue will drain at a rate of at most s_H with high probability, and the light queue will build up at a rate of $\lambda_L - s_L r_0$ with high probability. In order to get more service than $s_L r_0$, queue L will at least build up to $\mathcal{O}(b)$ level. After that, the two queues drain together, with most of the slots being used to serve queue H. Since queue H drains at most by $\mathcal{O}(b)$, queue L stays at $\mathcal{O}(b)$ level for at least $\mathcal{O}(b)$ time slots. Therefore, we have

$$\mathbb{E}[\sum_{i=0}^{T} Q_L(i)] \ge \mathbb{E}[Kb \cdot b] = \mathbb{E}[Kb^2] = \infty,$$

where T is the renewal interval of queue L's busy period. Similar to the proof of Proposition 1, we have that $\mathbb{E}[Q_L] = \infty$. □

D. MAX SERVICE RATE

The max-service-rate policy is similar to the *multi-user diversity* scheme [24], [25] and is often used in wireless communication systems. It chooses to serve the nonempty queue

that has the highest instantaneous service rate. Specifically, the scheduler compares $S_H(t)\cdot\mathbb{1}_{\{Q_H(t)>0\}} \gtrless S_L(t)\cdot\mathbb{1}_{\{Q_L(t)>0\}}$ and serves the queue that wins the competition.

We characterize the behavior of both queues in Proposition 5. Interestingly, the stability results for both queues exhibit some threshold effects.

Proposition 5: Under the max-service-rate policy,

1) *queue H is heavy-tailed with tail coefficient $C_H - 1$ if $\lambda_H < \mu_H$, and unstable otherwise;*
2) *queue L is light-tailed if $\lambda_L < \mu_L$, and unstable otherwise.*

Proof: For queue H, the average service rate is

$$\begin{aligned}\mu_H &= \mathbb{E}[S_H|S_H \geq S_L] + P\{Q_L(t)=0\}\mathbb{E}[S_H|S_H < S_L] \\ &= \sum_{k\in\mathcal{R}_L}\sum_{j\in\mathcal{R}_H}(1-a_{kj})p_k r_j j,\end{aligned}$$

where $a_{0j} = 0, a_{k0} = 1, a_{kj} = P\{Q_H(t) = 0\}\mathbb{1}_{k\leq j} + P\{Q_L(t) > 0\}\mathbb{1}_{k>j}$. Therefore, similar as in the proof of Proposition 1, we know if $\lambda_H < \mu_H$, queue H is stable and has a infinite expected queue length. If $\lambda_H > \mu_H$, queue H is unstable and no steady-state exists.

For queue L, the average service rate is

$$\begin{aligned}\mu_L &= \mathbb{E}[S_L|S_H < S_L] + P\{Q_H(t)=0\}\mathbb{E}[S_H|S_H \geq S_L] \\ &= \sum_{k\in\mathcal{R}_L}\sum_{j\in\mathcal{R}_H}a_{kj}p_k r_j k,\end{aligned}$$

where $a_{0j} = 0, a_{k0} = 1, a_{kj} = P\{Q_H(t) = 0\}\mathbb{1}_{k\leq j} + P\{Q_L(t) > 0\}\mathbb{1}_{k>j}$. Therefore, similar to the proof of Proposition 2, we can derive that if $\lambda_L < \mu_L$, queue L has a finite expected queue length. However, if $\lambda_L > \mu_L$, queue L is unstable and no steady-state exists.

Obviously, $\{\mu_L, \mu_H\}$ is in the capacity region. Hence, if $\Lambda_L = \mu_L$, then $\Lambda_H \geq \mu_H$. Therefore, if $\Lambda_L < \mu_L$, then $\Lambda_H > \mu_H$. As a result, the max-service-rate policy is not throughput optimal. □

Note that this is a general service-rate queueing model, which not only better models the wireless channel dynamics but also includes the previous ON/OFF models as special cases. For the scheduling policies, similar properties hold for both models.

IV. HEAVY-TAILED MULTI-USER INTERFERENCE

A. A MOTIVATIONAL EXAMPLE

A heterogeneous wireless cellular network is considered, where femtocells are placed overlaid with macrocells. For femtocells, such deployment allows them to share the spectrum with the macrocell and thus the spectrum utilization can be improved. A typical scenario is depicted in Figure 3, where both the macrocell and femtocell have their respective users. There is a femto user FU1, which is in the interference range of the macrocell, while another femto user FU2 is not. Hence, if heavy-tailed traffic is transmitted on the downlink in the macrocell, such as the log-normal type of distributions for call durations as reported in [26], FU1 will experience heavy-tailed interference. While FU2 does not suffer from such interference, but may be influenced by some out-of-cell interference, which is light-tailed.

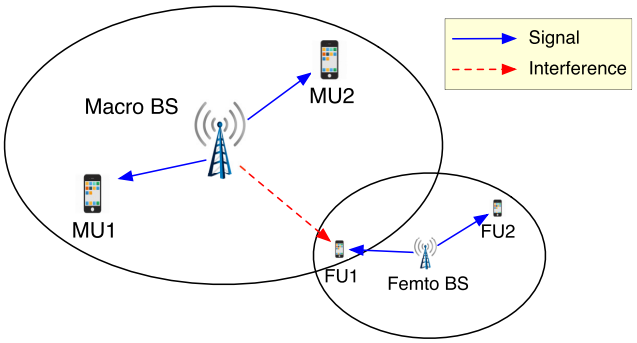

FIGURE 3. A motivational example of mix of both heavy-tailed and light-tailed interference in a heterogeneous cellular network.

TABLE 1. Summary of parameters.

Parameters	Expected Value	Variance
A_H	λ_H	ϕ_H
A_L	λ_L	ϕ_L
O_H	a_1	b_1
U_H	a_2	$\infty(b_2)$
O_L	a_3	b_3
U_L	a_4	b_4

Denoting the packet processing capability of FUn as $V_n(t)$ at time t, the femtocell needs to decide how to send each FU's packets for them to process, with knowledge of $V_n(t)$, and/or $Q_n(t)$, $n \in \{H, L\}$. As we can see, this problem can be analyzed using the general variable service rate model.

B. ASSUMPTIONS AND PRELIMINARY ANALYSIS

We consider the variable service rate model described in Section II-B, but only highlight the new assumptions for the heavy-tailed interference problem in this section. We consider both arrival processes A_H and A_L to be light-tailed, as the heavy-tailness is reflected in the channel process instead of the arrival.[3] The G_n is modeled as an ON/OFF renewal process $\{O_n, U_n\}$, with alternating independent ON period O_n and OFF period U_n, for $n \in \{H, L\}$. The process O_H, U_L, O_L are light-tailed. However, U_H is heavy-tailed because of the heavy-tailed interference. We provide a summary of these parameters in Table 1.

Similar to [13], [19], when the service rate is 1 packet per time slot, the capacity region for this model can be characterized as:

$$\begin{aligned}\{(\lambda_H,\lambda_L)|\lambda_H < \frac{a_1}{a_1+a_2} \triangleq p_H, \lambda_L < \frac{a_3}{a_3+a_4} \triangleq p_L, \\ \lambda_H + \lambda_L < p_H + p_L - p_H p_L\}. \quad (6)\end{aligned}$$

From (6), we can see that the capacity region is pentagonal, where an example is shown in Fig. 4. In our analysis, we only

[3] A more general approach would be to combine heavy-tailed traffic arrival with heavy-tailed interference. However, such approach would result in various combinations of the mixture of heavy-tailed and light-tailed arrival and interference, which is beyond the scope of this paper.

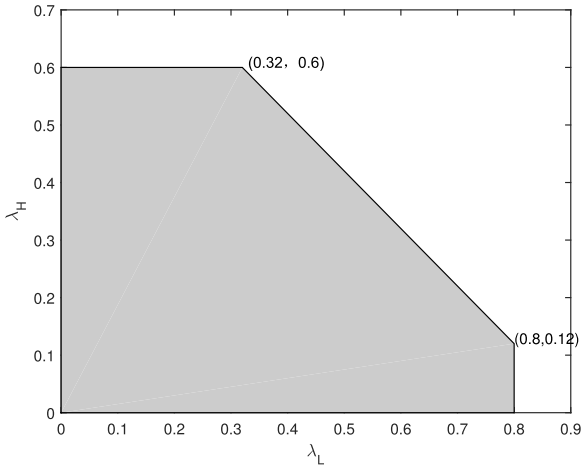

FIGURE 4. The capacity region Λ of the heavy-tailed interference model with $\lambda_L = 0.8$, $\lambda_H = 0.6$.

focus on the case in which the rate pair is in the capacity region.

In this section, we investigate the scheduling performance of the heavy-tailed interference system under different policies. We focus on both priority (for queue H and L) and max-weight policies in the analysis. The max-service-rate policy becomes trivial because of the ON/OFF nature of the channel and the constant service rate. Also, it is clear that a scheduling policy is only useful in determining which queue to serve when both G_H and G_L are ON. As only one channel is ON and the other is OFF, the queue with the ON channel should be selected. Furthermore, no queue can be served when both channels are OFF.

We should emphasize that our model is different from the existing literature. The main difference is that the heavy-tailed distribution applies to the OFF period of queue H's channel, instead of the traffic arrival. As a result, some key differences will be brought about by the *heavy-tailed channel dynamics* even if we apply the same scheduling policy in the new model. Methodologically, *Lyapunov functions* are often adopted to deal with this type of problems. However, this method requires the queue length $Q_n(t)$ to be a Markov process. Note that the one-step queue length evolution can be written as

$$Q_n(t+1) = Q_n(t) + A_n(t) - S_n(t)G_n(t)\mathbb{1}_{\{Q_n(t)>0, n \text{ is scheduled}\}} \tag{7}$$

where $G_n(t)$ denotes whether channel n is ON. In the previous models [13], [15]–[20], the channel is either always ON or the ON/OFF processes are i.i.d. Bernoulli processes. As a result, (7) satisfies the Markovian requirement. However, in the heavy-tailed interference model, channel $\{G_n(t)\}$ is an ON/OFF renewal process, making $Q_n(t)$ no longer a Markov process.

C. PRIORITY FOR L POLICY

We consider the priority-for-L policy, where queue L is served as long as it is nonempty and channel G_L is ON.

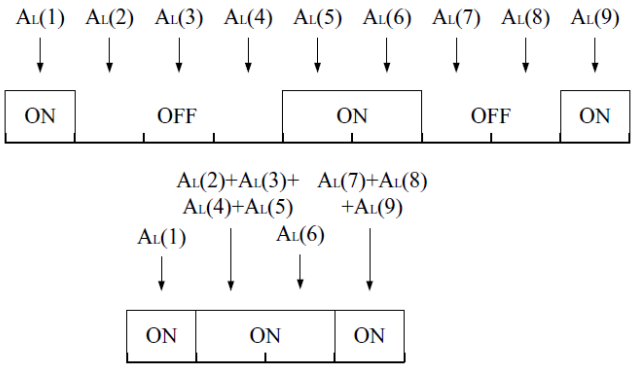

FIGURE 5. An example of the *artificial queue* L_1.

The result for the expected queue length and throughput optimality is shown as follows.

Proposition 6: Under the priority-for-L policy, the expected queue length of queue L in steady-state is finite. However, when $\lambda_H > p_H(1-\lambda_L)$, the system is not throughput optimal.

Proof: We will first prove that queue L is queue-length stable. For each OFF period U_L of queue L, the cumulative arrival is $A_{U_L} = \sum_{i=1}^{N} A_L(i)$. The expected value of A_{U_L} is derived by:

$$\begin{aligned}
\mathbb{E}[A_{U_L}] &= \mathbb{E}[\mathbb{E}[A_{U_L}|U_L = N]] \\
&= \mathbb{E}[\mathbb{E}[\sum_{i=1}^{N} A_L(i)|U_L = N]] \\
&= \mathbb{E}[N\lambda_L] \\
&= a_4\lambda_L.
\end{aligned}$$

The variance of A_{U_L} can be derived as:

$$\begin{aligned}
\text{Var}[A_{U_L}] &= \mathbb{E}[A_{U_L}^2] - (\mathbb{E}[A_{U_L}])^2 \\
&= \mathbb{E}[\mathbb{E}[A_{U_L}^2|U_L = N]] - (\mathbb{E}[A_{U_L}])^2 \\
&= \mathbb{E}[\mathbb{E}[(\sum_{i=1}^{N} A_L(i))^2|U_L = N]] - a_4^2\lambda_L^2 \\
&= \mathbb{E}[N(\phi_L + \lambda_L^2) + N(N-1)\lambda_L^2] - a_4^2\lambda_L^2 \\
&= \mathbb{E}[N\phi_L + N^2\lambda_L^2] - a_4^2\lambda_L^2 \\
&= a_4\phi_L + (a_4^2 + b_4)\lambda_L^2 - a_4^2\lambda_L^2 \\
&= a_4\phi_L + \lambda_L^2 b_4.
\end{aligned}$$

Since no packet is allowed to be transmitted when the channel is OFF, the queue L behavior will be the same as a new *artificial queue* L_1, where we combine the time line only with the ON periods of queue L and remove all the OFF periods. The number of arrivals in the first slot of an ON period O_L is assumed to be the sum of the arrivals of the current time slot and all arrivals in the previous OFF period. We show an example of the *artificial queue* L_1 in Figure 5. In order to prove queue L is stable, it is enough to show that arrival rate for queue L_1 is less than 1. The arrivals of queue L_1 can be treated as the sum of two flows, A_L for each slot

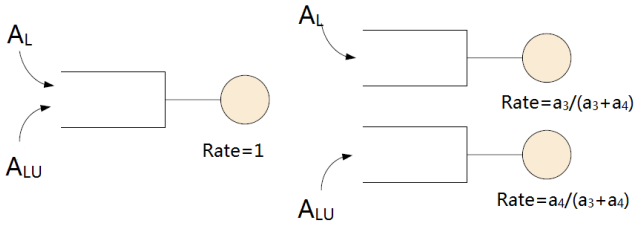

FIGURE 6. An illustration of the *artificial queue* L_2.

and A_{U_L} at the first time slot for each ON period. Denote $A_{LU}(t) = A_{U_L}(t) \cdot \mathbb{1}_{\{\text{t is the first time slot of an } O_L \text{ period}\}}$, then the arrival process for queue L_1 can be formulated as

$$A_{L_1}(t) = A_L(t) + A_{LU}(t).$$

Further, we notice that all the ON periods form a renewal process, where the process A_{LU} is as a renewal reward process associated. From renewal theory [27], the average arrival rate for A_{LU} is provided by following:

$$\lim_{T\to\infty}[\frac{1}{T}\int A_{U_L} \cdot \mathbb{1}_{\text{t is the first time slot of an } O_L \text{ period}} dt] = \frac{\mathbb{E}[A_{LU}]}{\mathbb{E}[O_L]} = \frac{a_4\lambda_L}{a_3}.$$

Therefore, we obtain the total arrival rate for queue L_1, given by $\lambda_L + a_4\lambda_L/a_3 = \lambda_L(a_3+a_4)/a_3 < 1$, where the last holds because $\lambda_L < \frac{a_3}{a_3+a_4}$. Hence, we conclude that queue L_1 is stable, which means that queue L is stable.

Next, we will show that the expected queue length of queue L in steady-state is finite. A second *artificial queue* system L_2 with two subqueues L_{21} and L_{22} is considered, as depicted in Figure 6. The arrival for queue L_{21} is A_L and the server's service rate is $\frac{a_3}{a_3+a_4}$. The other queue L_{22} has the arrival A_{LU} at the service rate $\frac{a_4}{a_3+a_4}$. We know that the queue length of L_1 will be smaller than the sum of the queue length of L_{21} and L_{22}, since some service will be wasted if either queue L_{21} or L_{22} is empty. Similarly, we consider another *artificial queue* system L_{23} with the arrival process $A_{U_L}(t)$ and the service rate $\frac{a_4}{a_3+a_4}O_L$ packets per slot. The queue length performance of queue L_{22} in an O_L period should be identical as that of queue L_{23} in any time slot. Therefore, we know that the expected queue length of queue L_{22} and L_{23} should be the same. The expected queueing delay of the queue L_{21} and L_{23}, which are discrete-time *Geo/G/1* queues, is provided by [28]:

$$\mathbb{E}[D] = \frac{\lambda^2 b_2 - \lambda\rho + \lambda_2 b}{2\lambda(1-\rho)}, \tag{8}$$

where λ and λ_2 are the first and second moment of the arrival in each slot, b and b_2 are the first and second moment of the service rate for each arrival, and ρ is the traffic intensity. We can see that the expected queue lengths are finite under both queue L_{21} and L_{23}, according to Eqn. (8) and Little's Law. Hence, we conclude that the expected queue length of queue L is finite.

Finally, we will prove that the system is no longer throughput optimal when $\lambda_H > p_H(1-\lambda_L)$. For queue H, it can

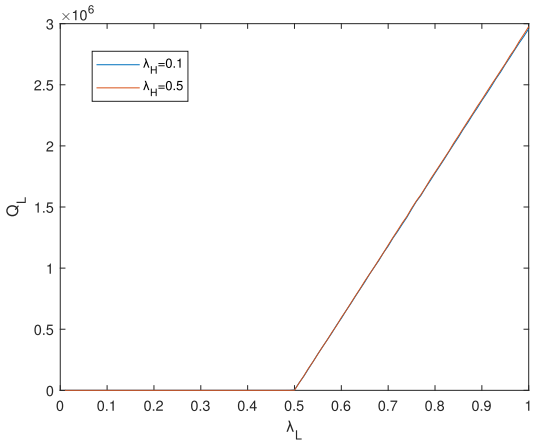

FIGURE 7. Under the priority-for-L, the queue length Q_L versus the arrival rate λ_L for $\lambda_H = 0.1$ and $\lambda_H = 0.5$, respectively.

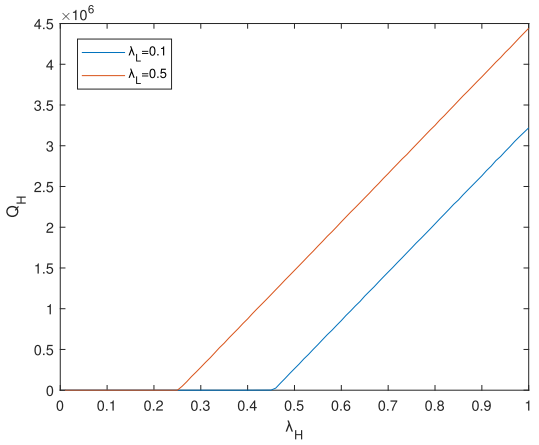

FIGURE 8. Under the priority-for-L, the queue length Q_H versus the arrival rate λ_H for $\lambda_L = 0.1$ and $\lambda_L = 0.5$, respectively.

only get service under two conditions:1) when G_H is ON and G_L is OFF; or 2) when both G_H and G_L are ON and queue L is empty. Since queue L is queue length stable under the priority-for-L policy, by applying Little's law, we know that the fraction of time that queue L is busy is equal to $\frac{\lambda_L}{p_L}$. As a result, the average service rate of queue H is

$$\mu_H = p_H(1-p_L) + p_H p_L(1-\frac{\lambda_L}{p_L}) = p_H(1-\lambda_L).$$

If $\lambda_H > p_H(1-\lambda_L)$, the average service rate is smaller than the average arrival rate for queue H. Thus queue H will not be stable even if the mean rate (λ_H, λ_L) is in the capacity region. Therefore, we conclude that this scheduling policy is not throughput optimal. □

Here we show some simulation results in Fig. 7 and 8. We randomly generate the traffic arrival A_L, A_H, channel ON period O_L, O_H and channel OFF period U_L following the light-tailed distribution and channel OFF period U_H following the heavy-tailed Pareto distribution. Assume that the expectations are satisfied $a_1 = a_2 = a_3 = a_4 = 3$. Figure 7 shows the queue length change in Q_L with λ_L under

the priority-for-L for $\lambda_H = 0.1$ and $\lambda_H = 0.5$, respectively. When $\lambda_L < 0.5$, the queue length of queue L is finite on both condition. Once $\lambda_L > 0.5$, Q_L becomes infinite because $\lambda_L > p_H = a_3/(a_3+a_4) = 3/(3+3) = 0.5$. Besides, we plot the queue length change in Q_H with λ_H under the priority-for-L for $\lambda_L = 0.1$ and $\lambda_L = 0.5$, respectively, in Figure 8. When $\lambda_L = 0.1$, the queue length of queue H is finite on condition of $\lambda_H < p_H(1-\lambda_L) = 0.5 * (1-0.1) = 0.45$. In case of $\lambda_H > 0.45$, the arrival rate is greater than the average service rate and Q_H becomes unstable. The condition of $\lambda_L = 0.5$ is similar to $\lambda_L = 0.1$ elaborated above.

D. PRIORITY FOR H POLICY

For the priority-for-H policy, the server will serve queue H whenever it is nonempty and channel G_H is ON. Proposition 7 shows a negative result for the expected queue length under this policy.

Proposition 7: For the priority-for-H scheduling policy, the expected queue length of queue H in steady-state is infinite.

Proof: Similar to the previous deduction, for each OFF period U_H of queue H, the cumulative arrival during this period is given by $A_{U_H} = \sum_{i=1}^{N} A_H(i)$. We can obtain the mean and variance of A_{U_H} as:

$$\mathbb{E}[A_{U_H}] = a_2\lambda_H,$$

and

$$\mathrm{Var}[A_{U_H}] = a_2\phi_H + \lambda_H^2 b_2 = \infty,$$

respectively. As no packet is allowed to be transmitted during the OFF period, the queue behavior of queue H will be the identical as a new *artificial queue* H_1, where the time consists of the ON periods of queue H, and remove all the OFF periods. The number of arrivals in the first slot of an ON period O_H is equal to the sum of all arrivals in the previous OFF period and the arrivals of the current time slot. We denote $A_{HU}(t) = A_{U_H}(t) \cdot \mathbb{1}_{\{\text{t is the first time slot of an } O_H \text{ period}\}}$. We have the arrival process of the *artificial queue* H_1 as

$$A_{H_1}(t) = A_H(t) + A_{HU}(t).$$

Similarly we consider two *artificial queue*: H_2 with the arrival process A_{HU} and the service rate 1 packet per time slot; and H_3 with the arrival process A_{U_H} and the service rate O_H packets per time slot. The queue length performance of queue H_2 in an O_H period should be identical as the queue length performance of queue H_3 in any time slot. Therefore, the expected queue length of queue H_2 should be the same as H_3. Since $\mathrm{Var}[A_{U_H}]$ is infinite, the expected queue length of queue H_3 is infinite. As a result, the expected queue length of queue H_2 is also infinite according to Eqn. (8). It is clear that the expected queue length of queue H_1 is greater than the expected queue length of queue H_2, which leads to the conclusion. □

In the system considered above, the priority-for-H policy is the best one that prefers the heavy queue. We thus claim that the expected queue length of queue H is always infinite under any scheduling policy.

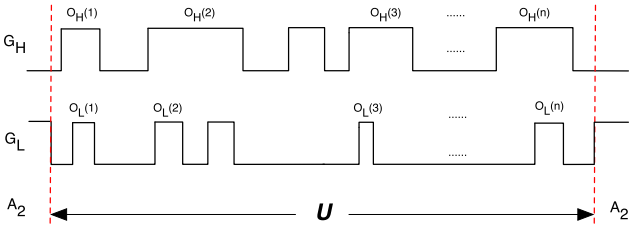

FIGURE 9. A schematic illustration of U.

E. MAX-WEIGHT POLICY

For the max-weight scheduling policy, the server compares the length of $Q_H(t)$ and $Q_L(t)$ when both channels are ON, and serves the longer queue. This subsection is to analyze the queue length performance under the max-weight scheduling policy. Because of a technical reason, we assume that the process O_H is always less than some constant O_{H_m}.

Proposition 8: For the max-weight scheduling policy, the expected queue length of queue L in steady-state is:

1) *finite, if* $\lambda_L < (1-p_H)p_L$;
2) *infinite, if* $\lambda_L > (1-p_H)p_L$.

Proof:

There are four subcases of the channel conditions as follows:

1) $P_1 \triangleq (1-p_H)(1-p_L)$, if both G_H and G_L are OFF, denoted as case A_1;
2) $P_2 \triangleq (1-p_H)p_L$, if G_H is OFF and G_L is ON, denoted as case A_2;
3) $P_3 \triangleq p_H(1-p_L)$, if G_H is ON and G_L is OFF, denoted as case A_3; and
4) $P_4 \triangleq p_H p_L$, if both G_H and G_L are ON, denoted as case A_4.

First, we will show that the expected queue length of queue L is finite only if $\lambda_L < (1-p_H)p_L$. This is done by proving that the expected queue length of queue L in steady-state is finite when queue L get served at A_2, that is, the conclusion holds no matter what scheduling policy is used.

Suppose queue L can only be served at A_2. Define U as the time duration between two successive A_2's. As queue L cannot be served during U, we know that O_L occurs only in the O_H period, rather than in the U_H period. An illustration of U is shown in Figure 9.

Clearly, for each O_L that occurs during U, we have

$$O_L(i) < O_{H_m}.$$

Since each $O_L(i)$ is i.i.d., we have

$$\mathbb{P}[O_L(i) \le O_{H_m}] = \mathbb{P}[O_L \le O_{H_m}] \triangleq p_0.$$

We use N to denote the number of time slots that O_L occurs during U, then we can obtain that

$$\mathbb{P}[N = n] \le \mathbb{P}[O_L(1) \le O_{H_m}, \cdots, O_L(n) \le O_{H_m}]$$

$$= \prod_{i=1}^{n} \mathbb{P}[O_L(i) \le O_{H_m}]$$
$$= p_0^n.$$

Thus we have

$$\mathbb{E}[N] \le \sum_{n=0}^{\infty} n p_0^n = \frac{p_0}{(1-p_0)^2},$$
$$\mathbb{E}[N^2] \le \sum_{n=0}^{\infty} n^2 p_0^n = \frac{p_0(p_0+1)}{(1-p_0)^3}.$$

We can show that

$$0 < U \le \sum_{j=1}^{N} O_L(j) + \sum_{k=1}^{N+1} U_L(k).$$

Hence, the mean and variance of U satisfy

$$\begin{aligned} m &\triangleq \mathbb{E}[U] \\ &\le \mathbb{E}[\sum_{k=1}^{N} O_L(k) + \sum_{k=1}^{N+1} U_L(k) | N = n] \\ &= \mathbb{E}[a_3 \cdot n + a_4 \cdot (n+1)] \\ &= a4 + (a_3 + a_4)\mathbb{E}[N] \\ &\le a4 + (a_3 + a_4) \cdot \frac{p_0}{(1-p_0)^2} \\ &< \infty, \end{aligned}$$

and

$$\begin{aligned} \sigma^2 &\triangleq \mathrm{Var}[U] \\ &= \mathbb{E}[U^2] - \mathbb{E}[U]^2 \\ &\le \mathbb{E}[(\sum_{k=1}^{N} O_L(k) + \sum_{k=1}^{N+1} U_L(k))^2 | N = n] - m^2 \\ &= \mathbb{E}[n(b_3 + a_3^2) + n(n-1)a_3^2 + (n+1)(b_4 + a_4^2) \\ &\quad + (n+1)na_4^2 + 2n(n+1)a_3a_4] - m^2 \\ &= \mathbb{E}[n^2(a_4^2 + a_3^2 + 2a_3a_4) + n(b_3 + b_4 + 2a_4^2 \\ &\quad + 2a_3a_4) + a_4^2 + b_4] - m^2 \\ &= (a_4^2 + a_3^2 + 2a_3a_4)\mathbb{E}[N^2] + (b_3 + b_4 + 2a_4^2 \\ &\quad + 2a_3a_4)\mathbb{E}[N] + a_4^2 + b_4 - m^2 \\ &\le (a_4^2 + a_3^2 + 2a_3a_4)\frac{p_0(p_0+1)}{(1-p_0)^3} + (b_3 + b_4 + 2a_4^2 \\ &\quad + 2a_3a_4)\frac{p_0}{(1-p_0)^2} + a_4^2 + b_4 - m^2 \\ &< \infty. \end{aligned}$$

In each U, we have the cumulative arrival is $A_U = \sum_{i=1}^{U} A_L(i)$. Similar to the proof of Proposition 6, we can obtain the mean and variance of A_U as

$$\mathbb{E}[A_U] = m\lambda_L,$$
$$\mathrm{Var}[A_U] = m\phi_L + \lambda_L^2 \sigma^2.$$

Because packets can only be transmitted during A_2, the queue behavior of queue L will be identical to a new *artificial queue* L_3, where the time line only consists of the A_2 periods without all other periods. Denote $A_{U2}(t) = A_U(t) \cdot \mathbb{1}_{\{\text{t is the first time slot of an } A_2 \text{ period}\}}$. The arrival process of queue L_3 can be written as

$$A_{L_3}(t) = A_L(t) + A_{U2}(t).$$

Also we notice that all packets arriving at queue L arrive at queue L_3, with the only exception that they arrive in shorter periods. Let T_2 denote as the time period of A_2, and T_0 as the time period between the first slot of A_2 and the last slot before the next A_2. Then we have

$$\sum_{i=1}^{T_2} A_{L_3}(i) = \sum_{i=1}^{T_0} A_L(i).$$

Since $P_2 = \mathbb{E}[T_2/T_0]$, we have

$$\mathbb{E}[A_{L_3}] = \mathbb{E}[A_L]/P_2 = \frac{\lambda_L}{P_2} < 1$$

This proves that queue A_{L_3} is stable. We also have

$$\mathbb{E}[A_{U2}] = \mathbb{E}[A_{L_3}] - \mathbb{E}[A_L] = \frac{\lambda_L}{P2} - \lambda_L < 1 - P_2.$$

Next, we will prove that the expected queue length of queue L_3 is finite. We consider a second *artificial queue* system L_4 with two subqueues L_{41} and L_{42}. The arrival process of queue L_{41} is A_L and the server's service rate is P_2. The other queue L_{42} serves the arrival A_{U2} and the rate is $1 - P_2$. We also consider another *artificial queue* L_{43} with the arrival process A_U and the service rate A_2 packets per slot. The queue length performance of queue L_{42} in an A_2 period and the queue length of queue L_{43} in any time slot should be the same. Hence, we know that the expected queue length of queue L_{42} and L_{43} should be identical.

The expected queueing delays of queue L_{41} and L_{43} are both finite according to Eqn. (8). Combined with Little's Law, we conclude that the expected queue lengths of queue L_{41} and L_{43} are finite. Note that the actual queue length of queue L_3 will be smaller than the summation of queue L_{41} and L_{42}, since some service will be wasted if either queue L_{41} or L_{42} is empty. Hence, the expected queue length of queue L_3 is finite, and as a result the expected queue length of queue L is finite.

Then we will study the case $\lambda_L > (1 - p_H)p_L$. In this subcase, queue L has to contend for another $\lambda_L - P_2$ fraction of time from sub-case 4 in order that the average service rate is greater than the average arrival rate. Now we consider a packet arrives at queue L at time slot τ, the queueing delay of this packet is denoted as $W_L(\tau)$. We notice that $W_L(\tau)$ consists of four parts, T_1, T_2, T_3, T_4, denoting the number of slots for the four sub-cases. We consider the condition $Q_H(\tau) > Q_L(\tau)$. The scenario with best delay performance occurs when no packet arrives at queue H after time slot τ. The scheduling policy keeps selecting queue H until both queue lengths become equal, and then queue L is scheduled

when it is available. We denote T as the least time such that $Q_L(\tau+T) > Q_H(\tau+T)$. Then, we have

$$(W_L(\tau)-T)\mathbb{1}_{\{t\in A_2\cup A_4\}} \geq Q_L(\tau), \quad \forall\, t \in (\tau+T,\cdots,\tau+W_L(\tau)).$$

By taking expectation on both sides conditioned on the event $\Gamma \triangleq \{Q_H(\tau) > Q_L(\tau)\}$, we have

$$\mathbb{E}[(W_L(\tau)-T)|\Gamma]P(t\in A_2\cup A_4) \geq \mathbb{E}[Q_L(\tau)|\Gamma],$$

which means that

$$\mathbb{E}[(W_L(\tau)-T)|\Gamma] \geq \frac{1}{P_2+P_4}\mathbb{E}[Q_L(\tau)|\Gamma]. \tag{9}$$

Notice that T must satisfy the following:

$$Q_L(\tau)+\sum_{k=0}^{T}A_L(\tau+k)-T_2 \geq Q_H(\tau)-T_3-T_4.$$

Again taking the conditional expectation on both sides, and together with the ergodic property of T_2, T_3, T_4, we have

$$\mathbb{E}[\sum_{k=0}^{T}A_L(\tau+k)+(P_3+P_4-P_2)T|\Gamma] \geq \mathbb{E}[Q_H(\tau)-Q_L(\tau)|\Gamma].$$

Because T is a stopping time, according to the Wald's equation, we have

$$\mathbb{E}[T|\Gamma] \geq \frac{1}{\lambda_L+P_3+P_4-P_2}\mathbb{E}[Q_H(\tau)-Q_L(\tau)|\Gamma]. \tag{10}$$

Substituting (10) into (9), we have

$$\begin{aligned}
&\mathbb{E}[W_L(\tau)|\Gamma] \\
&\geq \frac{1}{\lambda_L+P_3+P_4-P_2}\mathbb{E}[Q_H(\tau)-Q_L(\tau)|\Gamma] \\
&\quad+\frac{1}{P_2+P_4}\mathbb{E}[Q_L(\tau)|\Gamma] \\
&= \frac{1}{\lambda_L+P_3+P_4-P_2}\mathbb{E}[Q_H(\tau)|\Gamma]+(\frac{1}{P_2+P_4} \\
&\quad-\frac{1}{\lambda_L+P_3+P_4-P_2})\mathbb{E}[Q_L(\tau)|\Gamma] \\
&\geq \frac{1}{\lambda_L+P_3+P_4-P_2}\mathbb{E}[Q_H(\tau)|\Gamma]+(\frac{1}{P_2+P_4} \\
&\quad-\frac{1}{P_3+P_4})\mathbb{E}[Q_L(\tau)|\Gamma].
\end{aligned}$$

If

$$\frac{1}{P_2+P_4}-\frac{1}{P_3+P_4} > 0,$$

we have

$$\mathbb{E}[W_L(\tau)|\Gamma] > \frac{1}{P_3+P_4}\mathbb{E}[Q_H(\tau)|\Gamma] = \infty, \tag{11}$$

which concludes that the expected queue length of queue L is infinite based on Little's Law.

Otherwise,

$$(\frac{1}{P_3+P_4}-\frac{1}{P_2+P_4})\mathbb{E}[Q_L(\tau)|\Gamma]+\mathbb{E}[W_L(\tau)|\Gamma] \geq \frac{1}{P_3+P_4}\mathbb{E}[Q_H(\tau)|\Gamma]. \tag{12}$$

Again from Little's Law, we have

$$\mathbb{E}[W_L(\tau)] = \frac{1}{\lambda_L}\mathbb{E}[Q_L(\tau)].$$

Now, if $\mathbb{E}[Q_L(\tau)]$ is infinite, we obtain the conclusion directly. Otherwise, $\mathbb{E}[Q_L(\tau)|\Gamma]$ is finite if $\mathbb{E}[Q_L(\tau)]$ is finite. We denote V as an upper bound for $\mathbb{E}[Q_L(\tau)|\Gamma]$. We can rewrite (12) as

$$\begin{aligned}
&\mathbb{E}[W_L(\tau)|\Gamma] \\
&\geq \frac{1}{P_3+P_4}\mathbb{E}[Q_H(\tau)|\Gamma]-(\frac{1}{P_3+P_4}-\frac{1}{P_2+P_4})V \\
&\geq \mathbb{E}[Q_H(\tau)|\Gamma]-(\frac{1}{P_3+P_4}-\frac{1}{P_2+P_4})V.
\end{aligned} \tag{13}$$

Next, we consider the event $\Gamma^c = \{Q_H(\tau) \leq Q_L(\tau)\}$. It follows that

$$\begin{aligned}
&\mathbb{E}[W_L(\tau)|\Gamma^c] \geq \mathbb{E}[Q_H(\tau)|\Gamma^c] \\
&\geq \mathbb{E}[Q_H(\tau)|\Gamma^c]-(\frac{1}{P_3+P_4}-\frac{1}{P_2+P_4})V.
\end{aligned} \tag{14}$$

Combining (13) and (14), we have

$$\mathbb{E}[W_L(\tau)] \geq \mathbb{E}[Q_H(\tau)]-(\frac{1}{P_3+P_4}-\frac{1}{P_2+P_4})V. \tag{15}$$

From Proposition 6, we have $\mathbb{E}[Q_H(\tau)]$ is infinite. Therefore, the expected queue length of queue L is infinite by Little's Law and (15), when $\lambda_L > (1-p_H)p_L$. This concludes the proof. □

V. CONCLUSION

The scheduling policies are studied in a wireless system with both heavy-tailed and light-tailed dynamics, which may come from either traffic arrivals or multi-user interference. For heavy-tailed traffic arrivals, we studied a more general variable-rate service model, in which channel dynamics (as opposed to simple ON/OFF) are incorporated. We then focused on the more challenging heavy-tailed multi-user interference problem, and found that the heavy queue has an unstable queue length asymptotic, which cannot be guaranteed by any scheduling policies. Regarding the light queue, we proved that the priority-for-L policy can stabilize its queue length, though not throughput optimal any more. Max-weight scheduling is throughput optimal but cannot provide the queue length stability for the light queue. Furthermore, there exists a threshold effect for the light queue. When the traffic arrival rate for the light queue is less than the threshold, its queue length will be light-tailed under any scheduling policy. Neither the priority nor max-weight policy provides a satisfying queueing performance for the considered model with heavy-tailed interference. Some prior study [19] has

shown that revised max-weight policies, including log-max-wight and max-weight-α, can guarantee better performance for heavy-tailed traffic arrivals. We will investigate these scheduling policies under heavy-tailed traffic interference in the future work.

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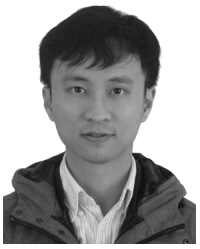

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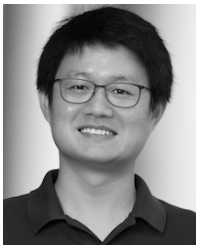

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