Integrating Multivariate Statistical Analysis Into Six Sigma DMAIC Projects: A Case Study on AISI 52100 Hardened Steel Turning

ROGÉRIO SANTANA PERUCHI[1], **PAULO ROTELA JUNIOR**[1], **TARCÍSIO G. BRITO**[2], **ANDERSON P. PAIVA**[2], **PEDRO P. BALESTRASSI**[2], **AND LAVÍNIA M. MENDES ARAÚJO**[1]

[1]Universidade Federal da Paraíba, João Pessoa 58051-900, Brazil
[2]Universidade Federal de Itajubá, Itajubá 37500-903, Brazil

Corresponding author: Paulo Rotela Junior (paulorotela@ct.ufpb.br)

ABSTRACT DMAIC (define, measure, analyze, improve and control) is one of the most utilized methods for guiding practitioners in the decision-making process of quality improvement projects. Industrial processes commonly deal with multiple critical-to-quality (CTQ) characteristics. When these characteristics are correlated, multivariate statistical techniques should be applied. This paper aims to propose a domain-specific Six Sigma method, the MDMAIC (multivariate DMAIC). The new stepwise procedure helps practitioners not only to reduce problem dimension but also to take account of the correlation structure among CTQs during the decision-making process. Principal component analysis has been applied for assessing the measurement system, analyzing process stability and capability, as well as modeling and optimizing multivariate manufacturing processes. A hardened steel turning case has been presented for proposal validation. The result analysis has shown that the MDMAIC was very successful in leading the practitioner during the steps and phases of the quality improvement project. The multivariate capability index of the enhanced process emphasized the substantial economic improvement.

INDEX TERMS Six sigma, dmaic, quality improvement, principal component analysis, multiobjective optimization.

I. INTRODUCTION

Continuous improvement has been implemented to several firms as a quality management strategy due to its capacity of providing higher competitive advantages [1]–[3]. Currently, Six Sigma has been adopted as a refined continuous improvement philosophy to improve organizational efficiency and customer satisfaction by decreasing operating costs and increasing profits [4]–[6]. It is defined by Linderman et al. [7] as an organized and systematic methodology for not only improving a strategic process but also developing new products and services. By this methodology, the significant reductions in defect rates are often achieved using statistical and scientific methods.

Initially used as a method to reduce variation, DMAIC (define, measure, analyze, improve and control) has been

The associate editor coordinating the review of this manuscript and approving it for publication was Seyedali Mirjalili.

implemented in practice as a generic approach for problem solving [5], [8], [9]. This method is, as any other problem solving approach, subjected to power/generality trade-off, which has first resulted in the evolution towards a more generality and later into a large number of domain-specific adaptations. De Mast & Lokkerbol [10] have concluded that DMAIC method is applicable to a wide range of well-structured and semi-structured problems. It serves as routine to organize problems, in order to turn them into well-structured problems.

Several researches have applied the DMAIC method as structured procedure to solving manufacturing problems with multiple CTQs. Some manufacturing applications are summarized as follows: automotive [11]–[14], casting [15], direct selling [16], extrusion [17], iron ore [18], printed circuit boards [19], [20], textile [21], [22], touch panel [23], white goods [24], services [1], [25], [26] and education [27]. In addition to the aforementioned papers, the book ''World
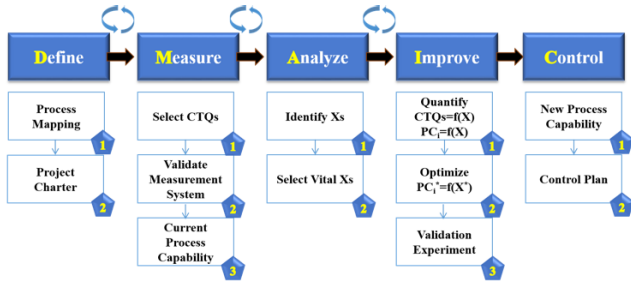
FIGURE 1. MDMAIC's phases and steps.

class application of Six Sigma: real world examples of success'', by Antony et al. [28], brings other interesting manufacturing applications. A worth mentioning paper is Chang et al. [29], which describes the application of a Six Sigma project, using DMAIC, for integrating statistical process control (SPC) to engineering processes control. In the analyze phase, the authors have used multivariate control charts, Hotelling T2, to evaluate six quality characteristics of a curing process of high-pressure hose products. However, a Six Sigma project, in fact, cannot be restricted to SPC techniques.

Taking into account that industrial processes commonly deal with multiple critical-to-quality (CTQ) characteristics [14], few researches have been conducted using multivariate approaches and DMAIC procedure to solving manufacturing problems. In such complex systems, the correlation among CTQs cannot be neglected due to its influence on the optimization results [30]. This effect destabilizes the mathematical models producing errors in the regression coefficients. As a result, estimated models are unable to represent the objective or constraint functions properly [31]–[34].

This research aims to propose a domain-specific DMAIC, the MDMAIC (Multivariate: Define, Measure, Analyze, Improve, Control), to solving manufacturing problems with multiple correlated CTQs. Principal component analysis (PCA) has been utilized for integrating a multivariate approach to the Six Sigma method.

II. MULTIVARIATE SIX SIGMA METHOD
The generic Six Sigma's stepwise strategy proposed by De Koning & De Mast [8] has been modified to conceive a method for dealing with multivariate processes. Fig. 1 presents the proposed multivariate method, the MDMAIC. In the following subsections are shown PCA and the main multivariate statistical techniques, based on PCA, in MDMAIC method.

A. PRINCIPAL COMPONENT ANALYSIS
PCA has been extensively used to summarize the common patterns of variation among variables [35], [36]. Algebraically, PCA is a linear combination ℓ of q random variables $\mathbf{CTQ}_1, \mathbf{CTQ}_2, \ldots, \mathbf{CTQ}_q$. Geometrically, these combinations determine a new coordinate system when rotating the original system [37], [38]. The coordinates of the axes

now have the variables $\mathbf{CTQ}_1, \mathbf{CTQ}_2, \ldots, \mathbf{CTQ}_q$ and represent the direction of the maximum. The principal components are uncorrelated and depend only on the variance-covariance matrix Σ, or the correlation matrix $\mathbf{R}$, of original variables. PCA development does not require the assumption of multivariate normality. PCA provides pairs of eigenvalues-eigenvectors $(\lambda_1, \mathbf{e_1}), (\lambda_2, \mathbf{e_2}), \ldots, (\lambda_q, \mathbf{e_q})$, where $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_q \geq 0$ are eigenvalues for obtaining percentage of explanation for each principal component and $\mathbf{e}_i$ are eigenvectors for estimating the component scores using (1).

$$\mathbf{PC}_i = \mathbf{e}_i^T \mathbf{CTQ} = \mathbf{e}_{1i}\mathbf{CTQ}_1 + \ldots + \mathbf{e}_{qi}\mathbf{CTQ}_q \quad i = 1, 2, \ldots, q \tag{1}$$

B. DEFINE PHASE
Initially, the relevant process should be mapped in order to provide the same level of knowledge for the project's team. SIPOC (suppliers-input-process-output-customers) is a simple and useful tool for identifying suppliers, inputs, the high-level process flow, outputs, and customers. Moreover, the project charter should be created, stating the problem, objectives, goals, scope, schedule, team, and potential financial benefits of the project [28].

C. MEASURE PHASE
1) SELECT CTQS AND VALIDATE MEASUREMENT SYSTEM
After selecting the critical-to-quality characteristics (CTQ), the measurement system should be validated. ANOVA (analysis of variance) method for GR&R studies can be applied only to univariate data. In dealing with multiple correlated CTQs, multivariate methods are more suitable for estimating the evaluation indices of these measurement systems [36], [39], [40]. A multivariate GR&R model using q quality characteristics, p parts, o operators, and r replicates can be written as (2) [36], [40]:

$$\mathbf{PC}_l = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk} \begin{cases} i = 1, & 2, \ldots, p \\ j = 1, & 2, \ldots, o \\ k = 1, & 2, \ldots, r \\ l = 1, & 2, \ldots, q \end{cases} \tag{2}$$

where μ is a constant and $\alpha_i, \beta_j, (\alpha\beta)_{ij}, \varepsilon_{ijk}$ are independent normal random variables with zero mean and variance, $\sigma_\alpha^2, \sigma_\beta^2, \sigma_{\alpha\beta}^2$, and σ_ε^2, for part-to-part (process), operator, part* operator interaction, and error term, respectively. Similarly to a univariate model, these variance components can be translated into GR&R notation as (3):

$$\sigma_P^2 = \sigma_\alpha^2, \quad \sigma_{repeatability}^2 = \sigma_\varepsilon^2, \quad \sigma_{reproducibility}^2 = \sigma_\beta^2 + \sigma_{\alpha\beta}^2$$
$$\sigma_{MS}^2 = \sigma_{repeatability}^2 + \sigma_{reproducibility}^2, \quad \sigma_T^2 = \sigma_P^2 + \sigma_{MS}^2 \tag{3}$$

After variance components have been calculated for a multivariate GR&R study, $R\&R_{PCi}$ and ndc_{PCi} indices for v (with $v \leq q$) principal components can be estimated by using (4) and (5). Then, multivariate evaluation indices can

be agglutinated as (6) and (7):

$$\%R\&R_{PC_i} = \left(\frac{\sigma_{MS;PC_i}}{\sigma_{T;PC_i}} \right) 100\% \quad \forall i = 1, 2, \ldots, v \quad (4)$$

$$ndc_{PC_i} = \sqrt{\frac{2\sigma_{P;PC_i}^2}{\sigma_{MS;PC_i}^2}} = \sqrt{2} \frac{\sigma_P}{\sigma_{MS}} \quad \forall i = 1, 2, \ldots, v \tag{5}$$

$$\%R\&R_m = \left[\prod_{i=1}^{v} \%R\&R_{PC_i} \right]^{1/v} \tag{6}$$

$$ndc_m = \left[\prod_{i=1}^{v} ndc_{PC_i} \right]^{1/v} \tag{7}$$

2) CURRENT PROCESS CAPABILITY

Process capability indices have been widely used to determine supplier's ability to deliver quality products [41]. Nevertheless, it is only recommended that the capability be evaluated when the process is under statistical control. Control charts are statistical tools commonly applied to assess process stability.

According to Montgomery [42], Hotelling T^2 is the most familiar procedure for monitoring and controlling the mean vector of the process. Using $i = 2, 3 \ldots, n$ sample size, $j = 1, 2, \ldots, q$ quality characteristics and $k = 1, 2, \ldots, m$ subgroups, the test statistic T^2 is given by:

$$T^2 = n \left(\overline{\mathbf{CTQ}} - \overline{\overline{\mathbf{CTQ}}} \right)^T \mathbf{S}^{-1} \left(\overline{\mathbf{CTQ}} - \overline{\overline{\mathbf{CTQ}}} \right) \tag{8}$$

where, $\overline{\mathbf{CTQ}}$ is the mean of CTQ_{ijk} values, $\overline{\overline{\mathbf{CTQ}}}$ is the mean vector of $\overline{\mathbf{CTQ}}_j$ (mean of CTQ_{jk} values), and $\mathbf{S}$ is the sample covariance matrix.

For retrospective analyses (phase 1), control limits for the T^2 control chart can be estimated as follows:

$$\begin{aligned} UCL &= \frac{q(m-1)(n-1)}{mn-m-q+1} F_{\alpha, q, mn-m-q+1} \\ CL &= \frac{q(m-1)(n-1)}{mn-m-q+1} F_{q, mn-m-q+1}^{-1}(0.5) \\ LCL &= 0 \end{aligned} \tag{9}$$

where F represents the F distribution.

Process variability can be monitored by the sample generalized variance, $|\mathbf{S}|$. This statistic, which is the determinant of the sample covariance matrix, can be used as a measure of multivariate dispersion [42]. Control limits for $|\mathbf{S}|$ would be:

$$\begin{aligned} UCL &= \frac{|\mathbf{S}|}{b_1} \left(b_1 + 3b_2^{1/2} \right) \\ CL &= |\mathbf{S}| \\ LCL &= \frac{|\mathbf{S}|}{b_1} \left(b_1 - 3b_2^{1/2} \right) \end{aligned} \tag{10}$$

where

$$b_1 = \frac{1}{(n-1)^q} \prod_{j}^{q} (n-j) \tag{11}$$

and

$$b_2 = \frac{1}{(n-1)^{2q}} \prod_{j=1}^{q} (n-j) \left[\prod_{r=1}^{q} (n-r+2) - \prod_{r=1}^{q} (n-r) \right] \tag{12}$$

In (10), if the calculated LCL is less than zero, the lower control limit is assumed to be zero.

Taking consideration of in-control systems, process capability indices (PCIs) provide numerical measures of whether or not a manufacturing process is capable to meet a predetermined level of production specification [41], [43]. C_p, C_{pk}, C_{pm} and C_{pmk} are the most used PCIs and are calculated as such:

$$C_p = \frac{USL - LSL}{6\sigma} \tag{13}$$

$$C_{pk} = \min \left\{ \frac{USL - \overline{CTQ}}{3\sigma}, \frac{\overline{CTQ} - LSL}{3\sigma} \right\} = \frac{d - \left| \overline{CTQ} - M \right|}{3\sigma} \tag{14}$$

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + \left(\overline{CTQ} - T \right)^2}} \tag{15}$$

$$\begin{aligned} C_{pmk} &= \min \left\{ \frac{USL - \overline{CTQ}}{3\sqrt{\sigma^2 + \left(\overline{CTQ} - T \right)^2}}, \frac{\overline{CTQ} - LSL}{3\sqrt{\sigma^2 + \left(\overline{CTQ} - T \right)^2}} \right\} \\ &= \frac{d - \left| \overline{CTQ} - M \right|}{3\sqrt{\sigma^2 + \left(\overline{CTQ} - T \right)^2}} \end{aligned} \tag{16}$$

where USL and LSL are the upper and lower specification limits respectively, T is the target value, $\overline{CTQ}$ is the process mean, σ is the process standard deviation, $M = (USL + LSL)/2$ is the mid-point of the specification interval and $d = (USL - LSL)/2$ is the half length of the specification interval.

For the multivariate case, (1) is used to determine the specification limits of the ith principal component [42]:

$$LSL_{PC_i} = \mathbf{e}_i^T \mathbf{LSL} \quad USL_{PCi} = \mathbf{e}_i^T \mathbf{USL} \quad T_{PC_i} = \mathbf{e}_i^T \mathbf{T} \tag{17}$$

where $\mathbf{LSL}$, $\mathbf{USL}$ and $\mathbf{T}$ must be standardized specification limits if correlation matrix is used.

Wang & Chen [44] proposed the MPCIs (multivariate process capability indices) MC_p, MC_{pk}, MC_{pm} and MC_{pmk} as follows:

$$MC_{pk} = \left(\prod_{i=1}^{\xi} C_{pk;PC_i} \right)^{1/v} \tag{18}$$

where

$$\begin{aligned} C_{pk;PC_i} &= \min \left\{ \frac{USL_{PC_i} - \overline{PC_i}}{3\sigma_{PC_i}}, \frac{\overline{PC_i} - LSL_{PC_i}}{3\sigma_{PC_i}} \right\} \\ &= \frac{d - \left| \overline{PC_i} - M \right|}{3\sigma_{PC_i}} \end{aligned} \tag{19}$$

TABLE 1. Analysis of variance table for crossed study with two factors.

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	F-Values
Factor A	$a \square 1$	— —	$\dfrac{SS_A}{DF_A}$	$\dfrac{MS_A}{MS_{Error}}$
Factor B	$b \square 1$	— —	$\dfrac{SS_B}{DF_B}$	$\dfrac{MS_B}{MS_{Error}}$
Interaction AB	$(a \square 1)(b \square 1)$	$SS_{Total} \square SS_{Error} \square SS_A \square SS_B$	$\dfrac{SS_{AB}}{DF_{AB}}$	$\dfrac{MS_{AB}}{MS_{Error}}$
Error	$n \square p$	—	$\dfrac{SS_{Error}}{DF_{Error}}$	
Total	$n \square 1$	—		

is the univariate measure of capability for the ith principal component, $\sigma_{PC_i} = \sqrt{\lambda_i}$ and v denotes the number of principal components used to assess the capability. Similarly, they defined MC_p, MC_{pm} and MC_{pmk} by replacing $C_{pk;PCi}$ with $C_{p;PCi}$, $C_{pm;PCi}$, $C_{pmk;PCi}$, respectively, for $i = 1, 2, \ldots, v$.

D. ANALYZE PHASE

In this phase, key process variables that cause defects should be identified. Design of experiments (DOE) along with hypothesis testing, analysis of variance (ANOVA) and Pareto chart are effective statistical tools for process modeling and optimization [45]. In order to determine which factors are statistically significant, the analysis of variance in Table 1 can be estimated.

Where a is the number of levels in factor A; b is the number of levels in factor B; n is the number of observations; p is the number of factors; $\overline{PC_{i..}}$ is the mean of the ith level of factor A; $\overline{PC_{...}}$ is the overall mean of principal components; $\overline{PC_{.j.}}$ is the mean of the jth level of factor B; $\overline{PC_{ijk}}$ is the principal component at the ith level of factor A, j^{th} level of factor B, and k^{th} replicate; and $\overline{PC_{ij.}}$ is the mean of the ith level of factor A and j^{th} level of factor B.

After estimating each component in the ANOVA table, each factor and interactions are evaluated by p-values taking 0.05 as significance level. After the screening stage, designs that are more comprehensive should be implemented in order to build a mathematical model and then finding the factor settings that produce optimal process performance [45].

E. IMPROVE PHASE
1) QUANTIFY RELATIONSHIP BETWEEN XS, CTQS AND PCS
After "the vital few" controllable factors have already been identified, response surface methods are useful designs for process optimization. If there is curvature in the experimental region, the approximating function, such as the second-order

model in (18), is usually employed [35].

$$ctq = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum_{i<j} \sum \beta_{ij} x_i x_j + \varepsilon \quad (20)$$

where β is the polynomial coefficient, x is controlled factors, k is the number of factors and ε is the random error term.

The ordinary least squares (OLS) method is utilized to estimate β coefficients by using:

$$\hat{\beta} = \left(\mathbf{X}^T \mathbf{X} \right)^{-1} \mathbf{X}^T \mathbf{CTQ} \quad (21)$$

where $\mathbf{X}$ is the matrix of independent variables and $\mathbf{CTQ}$ is the dependent variable. Curvature is assessed by the analysis of center points in the experimental design. Using (1), v principal components can also be fitted by a second-order model according to Eqs. (20) and (21).

2) OPTIMIZE PROCESS THROUGH PCS
According to Montgomery & Woodall [46], one of the main features of Six Sigma is the focus on variability reduction around the process' target. This information can be translated into MOOP as MSE (mean square error) functions for simultaneous optimization of mean and variance. The multivariate version of mean square error functions (MMSE), based on PCA, can be written as such [38], [47]:

$$\text{Minimize: } MMSE_v = \left[\prod_{i=1}^{v} (MMSE_i \, | \lambda_i \geq 1) \right]^{1/v}$$

$$= \left\{ \prod_{i=1}^{v} \left[(PC_i - T_{PC_i})^2 + \lambda_i \, | \lambda_i \geq 1 \right] \right\}^{1/v}$$

$$\text{Subject to: } X^T X \leq \rho^2 \quad (22)$$

where

$$PC_i = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum_{i<j} \sum \beta_{ij} x_i x_j \quad (23)$$

TABLE 2. Multivariate GR&R dataset.

	k=1		k=2		k=3	
i	R_a	R_y	R_a	R_y	R_a	R_y
1	0.26	1.54	0.26	1.71	0.28	1.64
2	0.53	3.31	0.50	3.48	0.55	2.99
3	0.50	3.55	0.49	3.06	0.47	3.29
4	0.30	2.30	0.27	2.18	0.31	1.78
5	0.55	3.12	0.55	3.08	0.54	2.82
6	0.58	3.30	0.58	3.13	0.55	3.07
7	0.20	1.42	0.18	1.36	0.17	1.29
8	1.35	9.30	1.47	10.47	1.45	10.31
9	1.06	5.45	1.06	5.08	1.10	4.94
10	0.53	3.31	0.57	3.30	0.56	2.99

and T_{PCi} is the target value of the ith principal component calculated by Eq. (17). Optimum values can be obtained by finding the stationary point of the multivariate fitted surface. The objective is to obtain $\mathbf{X}^*$ that can minimize not only the distance of expected mean (PC_i) from the target (T_{PC_i}) but also the process variability (λ_i). The objective function is subjected only to the experimental region of interest defined by $\mathbf{X}^T\mathbf{X} \leq \rho^2$. For a central composite design, a logical choice is the experimental axial distance [35], [38]. To solve this constrained nonlinear optimization system, GRG (generalized reduced gradient) is one of the most robust optimization algorithms [47], [48]. Validation experiments are required to verify whether the optimal solution is feasible.

F. CONTROL PHASE

Before implementing ongoing measures and actions to sustain improvement, process capability analysis must be conducted in order to check the optimized process capability. Finally, a phase 2 control chart study can be used for monitoring the mean vector of future production. The control limits are as follows [42]:

$$UCL = \frac{q(m+1)(n-1)}{mn-m-q+1}F_{\alpha,q,mn-m-q+1}$$

$$CL = \frac{q(m+1)(n-1)}{mn-m-q+1}F^{-1}_{q,mn-m-q+1}(0.5)$$

$$LCL = 0 \qquad (24)$$

III. NUMERICAL EXAMPLE

A. DEFINE PHASE

Hardened steel turning is a precision machining process highly productive and cost effective [49]. In this dry turning tests of the AISI 52100 steel (1.03% C; 0.23% Si; 0.35% Mn; 1.40% Cr; 0.04% Mo; 0.11% Ni; 0.001% S; 0.01%), a CNC lathe, with maximum rotational speed of 4000 rpm and power of 5.5 kW, was operated. Wiper mixed ceramic inserts ($Al_2O_3 + TiC$, ISO code CNGA 120408 S01525WH), coated with a very thin layer of titanium nitride (TiN, Sandvik-Coromant GC 6050), was utilized. The workpieces, made up with dimensions of Ø 49 mm × 50 mm, were previously quenched and tempered. After the heat treatment, hardness was measured between 49 and 52 HRC, up to a depth of 3 mm below the surface. A tool holder with negative geometry, ISO code DCLNL 1616H12 and entering angle
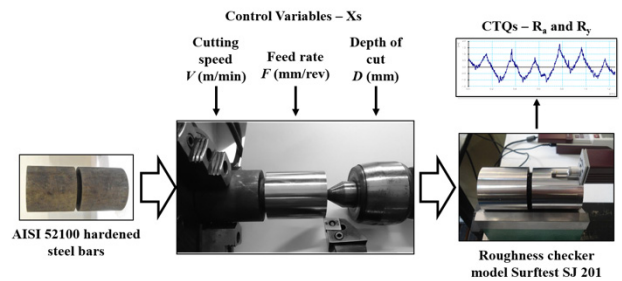

FIGURE 2. Overview of AISI 52100 hardened steel turning.

$\chi_r = 95°$, has been adopted. Fig. 2 illustrates the AISI 52100 hardened steel turning.

B. MEASURE PHASE

1) SELECT CTQS AND VALIDATE MEASUREMENT SYSTEM

Roughness parameters such as R_a (arithmetic average) are widely used in most manufacturing processes for assessing the quality of surface finishing of a work piece [39]. However, R_a alone is incapable of describing a surface completely. Hence, R_y (maximum roughness), which provides information about the deterioration of the vertical surface part, has also been adopted as a critical-to-quality characteristic.

To validate the measurement system, the multivariate GR&R study used $p = 10$ parts, $o = 1$ operator, and $r = 3$ replicates (Table 2). A portable roughness checker, set to a cut-off length of 0.25 mm, was utilized (Fig. 2).

Table 3 shows that PCA was applied to R_a and R_y roughness parameters using the correlation matrix. PC_1 explained 97.91% of total variation from original CTQs and was the only principal component scores evaluated. These scores were adjusted by using analysis of variance, according to the model in (2). Table 4 presents the square root of variance component for this multivariate GR&R study. (4) - (7) were used to calculate the measurement system' evaluation indices. $\%R\&R_m = 7.74\%$ and $ndc_m = 18$ suggest that the measurement system is deemed acceptable (guidelines for acceptable measurement system: $\%R\&R_m < 10\%$ and $ndc_m > 5$ [29]).

2) CURRENT PROCESS CAPABILITY

Before assessing process capability, control charts should be used to verify process stability. Using (8)-(12), Fig. 3 shows Hotelling T^2 and $|\mathbf{S}|$ control charts for checking mean and

TABLE 3. Principal component analyses based on correlation matrices.

	GR&R		Baseline		DOE		Validation	
	PC_1	PC_2	PC_1	PC_2	PC_1	PC_2	PC_1	PC_2
Eigenvalues	1.958	0.042	1.921	0.079	1.966	0.034	1.912	0.088
Proportion	97.91%	2.09%	96.04%	3.97%	98.29%	1.72%	95.61%	4.40%
Eigenvectors	PC_1	PC_2						
R_a	0.707	0.707						
R_v	0.707	-0.707						

TABLE 4. Multivariate GR&R results.

Source	σ_i	%
$\%R\&R_m$	0.112	7.74
Part-to-part	1.446	99.70
Total variation	1.450	100.00
ndc_m	18	

FIGURE 3. Hotelling T2 and |S| control charts for baseline study.

TABLE 5. Estimating the multivariate process capability index.

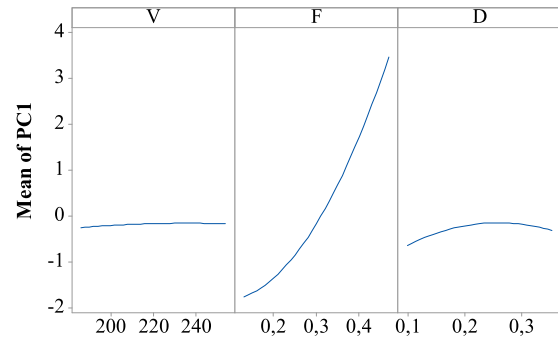|	Baseline		Optimized	
	R_a	R_v	R_a	R_v
$\overline{CTQ}$	0,681	3,108	0,184	1,531
σ	0,038	0,270	0,032	0,234
USL	0,800	3,300	0,800	3,300
$\left(USL - \overline{CTQ}\right)/\sigma$	3,147	0,711	19,143	7,550
	PC_1		PC_1	
USL	2,727		18.872	
σ	1.386		1.383	
MC_{pk}	0.66		4.55	
MC_{pk} (target)	> 1.33			

FIGURE 4. Main effects plot for PC1.

covariance stabilities, respectively. As can be seen from these charts, the multivariate process is under statistical control.

Turning now to the process capability analysis, Table 5 summarizes some descriptive statistics and specification limits. Table 3 shows that PC_1 explains 96.04% of total variation from the original CTQs, so that PC_1 was the only one principal component considered. For this particular case, there is only upper specification limits; thereby, MC_{pk} has been the only one multivariate process capability index estimated. Specification limit for PC_1 was calculated by using (17). After that, (18) and (19) were used for obtaining the multivariate process capability index. $MC_{pk} = 0.66$ determines that about 2% of defects are expected in this multivariate process. The goal is to increase this MPCI in order to obtain at least 1.33.

C. ANALYZE PHASE
In this phase, influencing factors and causes that affect CTQs' behavior are identified and the most significant ones are selected. Table 6 presents the control variables with their respective levels for building a central composite design. It was adopted 8 corner points, 6 axial points, 5 center points and $\rho = 1.682$ in this response surface design. The sequential set of experimental runs was conducted and stored in Table 7. Table 3 provides the PCA results with 98.29% of total variation accounting for the first principal component.

Before fitting a response surface model for PC_1, analysis of variance was assessed in order to identify the adequacy of a full quadratic model. As can be seen from Table 8, there were several non-significant terms included in the full quadratic model. Additionally, Fig. 4 illustrates how significant each factor was. It is essential to highlight that the most variation in PC_1 is due to the feed rate factor. In order to find the best fit for this turning process, several models were analyzed, taking account of lack-of-fit test, Anderson-Darling normality test for residuals and adjusted coefficient of determination (R^2_{Adj}). Non-significant impact on PC_1 was provided by the cutting speed factor. Therefore, this factor was removed from the final reduced response surface model, as seen in Table 8.

TABLE 6. Control factors and their respective levels.

Factor	Symbol	Levels				
Cutting speed (m/min)	V	186,36	200,00	220,00	240,00	253,64
Feed rate (mm/rev)	F	0,13	0,20	0,30	0,40	0,47
Depth of cut (mm)	D	0,10	0,15	0,23	0,30	0,35

TABLE 7. Central composite design.

Control factors			CTQs	
V	F	D	R_a	R_y
200	0,20	0,15	0,26	1,41
240	0,20	0,15	0,21	1,49
200	0,40	0,15	1,07	4,06
240	0,40	0,15	0,96	4,20
200	0,20	0,30	0,26	1,75
240	0,20	0,30	0,32	2,24
200	0,40	0,30	1,11	4,70
240	0,40	0,30	1,08	4,36
186	0,30	0,23	0,65	2,81
254	0,30	0,23	0,76	2,86
220	0,13	0,23	0,16	1,25
220	0,47	0,23	1,56	7,45
220	0,30	0,10	0,66	2,58
220	0,30	0,35	0,59	2,67
220	0,30	0,23	0,62	2,69
220	0,30	0,23	0,65	3,15
220	0,30	0,23	0,60	2,59
220	0,30	0,23	0,68	2,72
220	0,30	0,23	0,70	2,85

D. IMPROVE PHASE

1) QUANTIFY RELATIONSHIP BETWEEN XS, CTQS AND PCS

Response surface models for R_a, R_t and PC_1 were built by using (18) and (19). The reduced models described in (25) - (27) can be illustrated through contour and surface plots. As shown in Fig. 5, low level of feed rate minimizes the scores of PC_1.

$$R_a = 0.656 + 0.405F + 0.011D + 0.058F^2 - 0.026D^2 \quad (25)$$

$$R_y = 2.676 + 01.527F + 0.149D + 0.511F^2 \quad (26)$$

$$PC_1 = -0.179 + 1.573F + 0.095D + 0.355F^2 - 0.105D^2 \quad (27)$$

2) OPTIMIZE PROCESS THROUGH PCS

Owing to the fact that PC_1 is positively related to R_a and R_y (see eigenvectors in Table 3), minimizing PC_1 is the same as minimizing both R_a and R_y. Therefore, the original constrained multi-objective optimization problem can be simplified by a constrained single objective problem, using (22) and (23), as follows:

$$\text{Minimize: } MMSE_1 = \left(PC_1 - T_{PC_1}\right)^2 + \lambda_1$$
$$= [PC_1 - (-1.864)]^2 + 1.966$$
$$\text{Subject to: } \mathbf{X}^T\mathbf{X} \leq 1.682^2 \quad (28)$$

In this particular case, the target for each CTQ was obtained from $T_{CTQ} = \underset{X \in \Omega}{Min} [CTQ_i(\mathbf{X})]$ where Ω denotes the experimental region of interest defined by $\mathbf{X}^T\mathbf{X} \leq 1.682^2$.
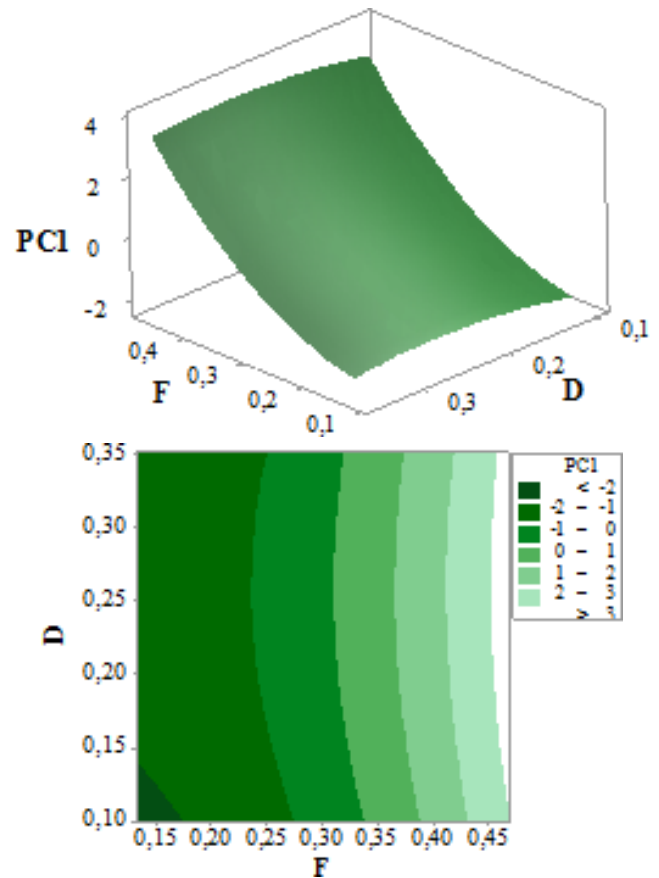

FIGURE 5. Contour and surface plots for PC1.

Applying the GRG algorithm into Eq. (29), the optimal setting using coded units was (-1,480; -0,799) for this multivariate process. This solution ($F = 0,152$ mm/rev and $D = 0,165$ mm – uncoded units), which attends all the constraints, must be validated by a pilot test in order to compare the optimized process capability to the baseline.

E. CONTROL PHASE

Hotelling T^2 and $|\mathbf{S}|$ control charts were applied, by using (8)-(12), to verify mean and covariance stabilities, respectively. As can be seen from Fig. 6, the multivariate process is in-control. Assessing now process capability, Table 5 summarizes some descriptive statistics and specification limits for the validation test. Table 3 determines that PC_1 explains 95.61% of total variation from the original roughness parameters, thus PC_1 was the only one principal component taken into account. Specification limit for PC_1 was calculated by

TABLE 8. Analysis of variance for PC1.

Source	Full quadratic model	Reduced model
Model	0,000[a]	0,000
Linear	0.000	0,000
V	0,779	-[b]
F	0.000	0,000
D	0,284	0,196
Square	0,009	0,000
V*V	0,852	-
F*F	0,002	0,000
D*D	0,228	0,150
2-Way Interaction	0,860	-
V*F	0,461	-
V*D	0,707	-
F*D	0,961	-
Lack-of-Fit test	0,051	0,123
Normality test for residuals	0,928	0,166
R^2_{Adj}	95,20%	96,61%

[a] p-value at 5% level of significance
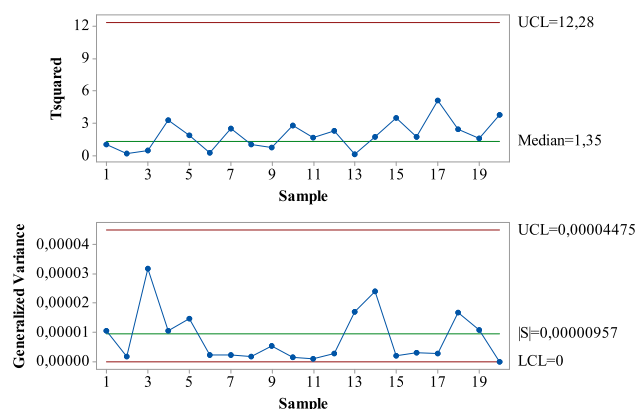[b] term excluded from the model

FIGURE 6. Hotelling T2 and |S| control charts for validation study.

using (17) and MC_{pk} by using (18) and (19). According to the index $MC_{pk} = 4.55$, the probability of producing defective parts was extremely reduced at levels lower than 0.00%. Further, phase 2 control charts, using Eq. (21) as control limit, could be applied for sustaining improvements.

Basically, the computational complexity of the proposed method is increased by having to conduct PCA before performing the other MSA, SPC, DOE and MOOP studies. On the other hand, the reduction in dimensionality imposed by PCA results in reduced computational efforts when conducting the other studies of MSA, SPC, DOE and MOOP. Thus, the number of principal components is usually smaller than the original set of correlated variables.

Finally, this numerical example showed how to define, measure, analyze, improve and control processes with multiple CTQs using Six Sigma/DMAIC and PCA. When evaluating systems with multiple CTQs, univariate methods often generate inconclusive results for validating measurement systems, determining process capability and defining the optimum operating condition. Multivariate methods reduce the size of the problem and provide conclusive results when conducting MSA, SPC, DOE, and MOOP studies.

IV. CONCLUSIONS

Complex industrial processes generally deal with multiple correlated critical-to-quality characteristics. Thus, multivariate statistical techniques are required to measure, analyze, improve and control such applications. Literature presents several papers applying multivariate approaches to either MSA, SPC, DOE or MOOP problems. Nevertheless, combining these approaches with a well-structured procedure is demanded to adequately solving problems of multivariate manufacturing processes. The domain-specific MDMAIC method was proposed to integrate contemporary multivariate techniques into the decision-making process of Six Sigma projects. The numerical example has shown how a multivariate process can be properly assessed and optimized. Additionally, the following conclusions are addressed:

- PCA effectively reduced problem dimension while applying the multivariate version of MSA, SPC, DOE and MOOP techniques;
- According to the evaluation indices %$R\&R_m$ and ndc_m, the measurement system that assesses roughness parameters was validated by using the multivariate GR&R study;
- Multivariate process capability indices were calculated in order to determine the economic losses before and after process improvement;
- The multivariate manufacturing process was enhanced by adopting the successful RSM-PCA approaches coupled with MSE functions for simultaneously optimizing mean and variance of multiple correlated CTQs. The MC_{pk} has been increased from 0.66 to 4.55;

Finally, the numerical example of the hardened steel turning process has shown that the MDMAIC method was considered efficient and effective in leading the practitioner to the problem solution.

REFERENCES

[1] M. V. Sunder and N. R. Kunnath, "Six Sigma to reduce claims processing errors in a healthcare payer firm," *Prod. Planing Control*, to be published, doi: 10.1080/09537287.2019.1652857.

[2] M. Butler, M. Szwejczewski, and M. Sweeney, "A model of continuous improvement programme management," *Prod. Planning Control*, vol. 29, no. 5, pp. 386–402, Apr. 2018.

[3] T. J. Douglas and W. Q. Judge, "Total quality management implementation and competitive advantage: The role of structural control and exploration," *Acad. Manage. J.*, vol. 44, no. 1, pp. 158–169, Feb. 2001.

[4] V. Sunder M., L. S. Ganesh, and R. R. Marathe, "A morphological analysis of research literature on Lean Six Sigma for services," *Int. J. Oper. Prod. Manage.*, vol. 38, no. 1, pp. 149–182, Jan. 2018.

[5] R. McAdam and B. Lafferty, "A multilevel case study critique of six sigma: Statistical control or strategic change?" *Int. J. Oper. Prod. Manage.*, vol. 24, no. 5, pp. 530–549, May 2004.

[6] A. Boon Sin, S. Zailani, M. Iranmanesh, and T. Ramayah, "Structural equation modelling on knowledge creation in Six Sigma DMAIC project and its impact on organizational performance," *Int. J. Prod. Econ.*, vol. 168, pp. 105–117, Oct. 2015.

[7] K. Linderman, R. G. Schroeder, S. Zaheer, and A. S. Choo, "Six Sigma: A goal-theoretic perspective," *J. Oper. Manag.*, vol. 21, no. 2, pp. 193–203, Mar. 2003.

[8] H. De Koning and J. De Mast, "A rational reconstruction of Six-Sigma's breakthrough cookbook," *Int. J. Qual. Rel. Manage.*, vol. 23, no. 7, pp. 766–787, 2006.

[9] S. S. Chakravorty, "Six Sigma programs: An implementation model," *Int. J. Prod. Econ.*, vol. 119, no. 1, pp. 1–16, May 2009.

[10] J. de Mast and J. Lokkerbol, "An analysis of the Six Sigma DMAIC method from the perspective of problem solving," *Int. J. Prod. Econ.*, vol. 139, no. 2, pp. 604–614, 2012.

[11] F. G. Amitrano, C. C. A. Estorilio, L. de Oliveira Franzosi Bessa, and K. Hatakeyama, "Six Sigma application in small enterprise," *Concurrent Eng.*, vol. 24, no. 1, pp. 69–82, Mar. 2016.

[12] B. Bilgen and M. Şen, "Project selection through fuzzy analytic hierarchy process and a case study on Six Sigma implementation in an automotive industry," *Prod. Planning Control*, vol. 23, no. 1, pp. 2–25, Jan. 2012.

[13] E. V. Gijo and J. Scaria, "Process improvement through Six Sigma with Beta correction: A case study of manufacturing company," *Int. J. Adv. Manuf. Technol.*, vol. 71, nos. 1–4, pp. 717–730, Mar. 2014.

[14] M. Tanco, E. Viles, L. Ilzarbe, and M. J. Alvarez, "Implementation of design of experiments projects in industry," *Appl. Stochastic Models Bus. Ind.*, vol. 25, no. 4, pp. 478–505, Jul. 2009.

[15] A. Kumaravadivel and U. Natarajan, "Application of Six-Sigma DMAIC methodology to sand-casting process with response surface methodology," *Int. J. Adv. Manuf. Technol.*, vol. 69, nos. 5–8, pp. 1403–1420, Nov. 2013.

[16] C. Wei, G. Sheen, C. Tai, and K. Lee, "Using Six Sigma to improve replenishment process in a direct selling company," *Supply Chain Manage.*, vol. 15, no. 1, pp. 3–9, Jan. 2010.

[17] H. Ketan and M. Nassir, "Aluminium hot extrusion process capability improvement using Six Sigma," *Adv. Prod. Eng. Manage.*, vol. 11, no. 1, pp. 59–69, Mar. 2016.

[18] J. A. Garza-Reyes, M. Al-Balushi, J. Antony, and V. Kumar, "A Lean Six Sigma framework for the reduction of ship loading commercial time in the iron ore pelletising industry," *Prod. Planning Control*, vol. 27, no. 13, pp. 1092–1111, 2016.

[19] K. Kaija, V. Pekkanen, M. Mäntysalo, S. Koskinen, J. Niittynen, E. Halonen, and P. Mansikkamäki, "Inkjetting dielectric layer for electronic applications," *Microelectron. Eng.*, vol. 87, no. 10, pp. 1984–1991, Oct. 2010.

[20] K. L. Lee and C. C. Wei, "Reducing mold changing time by implementing Lean Six Sigma," *Qual. Rel. Eng. Int.*, vol. 26, no. 4, pp. 387–395, 2010.

[21] A. R. Mukhopadhyay and S. Ray, "Reduction of yarn packing defects using Six Sigma methods: A case study," *Qual. Eng.*, vol. 18, no. 2, pp. 189–206, Jul. 2006.

[22] J. J. Cardiel Ortega, R. Baeza Serrato, and R. A. Lizárraga Morales, "Development of a system dynamics model based on Six Sigma methodology," *Ingeniería e Investigación*, vol. 37, no. 1, p. 80, Apr. 2017.

[23] J. J. Lynch and M. N. Chen, "A Six Sigma Approach to Touch Panel Quality Improvement," *Prod. Planing Control*, vol. 20, no. 5, pp. 445–454, 2009.

[24] E. Atmaca and S. S. Girenes, "Lean Six Sigma methodology and application," *Qual. Quantity*, vol. 47, no. 4, pp. 2107–2127, 2013.

[25] M. V. Sunder and J. Antony, "Six-Sigma for improving top-box CSAT for a banking call centre," *Prod. Planning Control*, vol. 8, no. 2, pp. 159–178, 2015.

[26] M. V. Sunder, S. Mahalingam, and M. S. N. Krishna, "Improving patients' satisfaction in a mobile hospital using Lean Six Sigma–a design-thinking intervention," *Prod. Planning Control*, to be published, doi: 10.1080/09537287.2019.1654628.

[27] V. Sunder M. and S. Mahalingam, "An empirical investigation of implementing Lean Six Sigma in higher education institutions," *Int. J. Qual. Rel. Manage.*, vol. 35, no. 10, pp. 2157–2180, Nov. 2018.

[28] J. Antony, A. Kumar, and R. Banuelas, *World Class Applications of Six Sigma*. Evanston, IL, USA: Routledge, 2006.

[29] S. I. Chang, T.-R. Tsai, D. K. J. Lin, S.-H. Chou, and Y.-S. Lin, "Statistical process control for monitoring nonlinear profiles: A Six Sigma project on curing process," *Qual. Eng.*, vol. 24, no. 2, pp. 251–263, Apr. 2012.

[30] L. G. D. Lopes, T. G. Brito, A. P. Paiva, R. S. Peruchi, and P. P. Balestrassi, "Robust parameter optimization based on multivariate normal boundary intersection," *Comput. Ind. Eng.*, vol. 93, pp. 55–66, Mar. 2016.

[31] G. E. P. Box, W. G. Hunter, J. F. Macgregor, and J. Erjavec, "Some problems associated with the analysis of multiresponse data," *Technometrics*, vol. 15, no. 1, pp. 33–51, Feb. 1973.

[32] C.-H. Chiao and M. Hamada, "Analyzing experiments with correlated multiple responses," *J. Qual. Technol.*, vol. 33, no. 4, pp. 451–465, Oct. 2001.

[33] J. H. F. Gomes, A. P. Paiva, S. C. Costa, P. P. Balestrassi, and E. J. Paiva, "Weighted multivariate mean square error for processes optimization: A case study on flux-cored arc welding for stainless steel claddings," *Eur. J. Oper. Res.*, vol. 226, no. 3, pp. 522–535, May 2013.

[34] A. I. Khuri and M. Conlon, "Simultaneous optimization of multiple responses represented by polynomial regression functions," *Technometrics*, vol. 23, no. 4, pp. 363–375, Nov. 1981.

[35] A. P. Paiva, J. R. Ferreira, and P. P. Balestrassi, "A multivariate hybrid approach applied to AISI 52100 hardened steel turning optimization," *J. Mater. Process. Technol.*, vol. 189, nos. 1–3, pp. 26–35, Jul. 2007.

[36] R. S. Peruchi, P. P. Balestrassi, A. P. de Paiva, J. R. Ferreira, and M. de Santana Carmelossi, "A new multivariate gage R&R method for correlated characteristics," *Int. J. Prod. Econ.*, vol. 144, no. 1, pp. 301–315, 2013.

[37] R. A. Johnson and D. W. Wichern, *Applied Multivariate Statistical Analysis*. Englewood Cliffs, NJ, USA: Prentice-hall, 2007.

[38] A. P. Paiva, E. J. Paiva, J. R. Ferreira, P. P. Balestrassi, and S. C. Costa, "A multivariate mean square error optimization of AISI 52100 hardened steel turning," *Int. J. Adv. Manuf. Technol.*, vol. 43, nos. 7–8, pp. 631–643, Aug. 2009.

[39] R. S. Peruchi, A. P. Paiva, P. P. Balestrassi, J. R. Ferreira, and R. Sawhney, "Weighted approach for multivariate analysis of variance in measurement system analysis," *Precis. Eng.*, vol. 38, no. 3, pp. 651–658, Jul. 2014.

[40] F.-K. Wang and T.-W. Chien, "Process-oriented basis representation for a multivariate gauge study," *Comput. Ind. Eng.*, vol. 58, no. 1, pp. 143–150, Feb. 2010.

[41] C.-W. Wu, W. L. Pearn, and S. Kotz, "An overview of theory and practice on process capability indices for quality assurance," *Int. J. Prod. Econ.*, vol. 117, no. 2, pp. 338–359, Feb. 2009.

[42] W. M. Bowen, "Introduction to statistical quality control," *J. Qual. Technol.*, vol. 29, no. 3, pp. 357–359, Jul. 1997.

[43] M. Scagliarini, "Multivariate process capability using principal component analysis in the presence of measurement errors," *AStA Adv. Stat. Anal.*, vol. 95, no. 2, pp. 113–128, Jun. 2011.

[44] F. K. Wang and J. C. Chen, "Capability index using principal components analysis," *Qual. Eng.*, vol. 11, no. 1, pp. 21–27, Sep. 1998.

[45] D. Montgomery, *Design and Analysis of Experiments*. New York, NY, USA: Wiley 2013.

[46] D. C. Montgomery and W. H. Woodall, "An overview of six sigma," *Int. Stat. Rev.*, vol. 76, no. 3, pp. 329–346, 2008.

[47] A. P. de Paiva, J. H. F. Gomes, R. S. Peruchi, R. C. Leme, and P. P. Balestrassi, "A multivariate robust parameter optimization approach based on Principal Component Analysis with combined arrays," *Comput. Ind. Eng.*, vol. 74, pp. 186–198, Aug. 2014.

[48] T. G. Brito, A. P. Paiva, J. R. Ferreira, J. H. F. Gomes, and P. P. Balestrassi, "A normal boundary intersection approach to multiresponse robust optimization of the surface roughness in end milling process with combined arrays," *Precis. Eng.*, vol. 38, no. 3, pp. 628–638, Jul. 2014.

[49] L. C. S. Rocha, A. P. de Paiva, P. Rotela Junior, P. P. Balestrassi, P. H. da Silva Campos, and J. P. Davim, "Robust weighting applied to optimization of AISI H13 hardened-steel turning process with ceramic wiper tool: A diversity-based approach," *Precis. Eng.*, vol. 50, pp. 235–247, Oct. 2017.

ROGÉRIO SANTANA PERUCHI received the degree in mechanical production engineering in 2009, the master's degree in production engineering from the Federal University of Itajubá, in 2011, and the Ph.D. degree in production engineering from the Federal University of Itajubá, in 2014, with a sandwich period at The University of Tennessee at Knoxville, in 2013. He held a postdoctoral position in production engineering from the Federal University of Itajubá, in 2015. He served as an Associate Professor with the Faculty of Engineering, Federal University of Goiás—Regional Catalão, in 2016. He is currently a Level 2 Researcher with the National Council for Scientific and Technological Development. He is also an Associate Professor with the Department of Production Engineering, Technology Center, Federal University of Paraíba. Has experience in process engineering in the automotive sector. He has worked with scientific research in the areas of six sigma, quality management, statistical quality control, statistical process modeling, and multiobjective optimization.

PAULO ROTELA JUNIOR received the B.Sc. degree in production engineering and computer information systems from Centro Universitário de Itajubá (FEPI), the M.B.A. degree, and the M.Sc. and D.Sc. degrees in production engineering from the Federal University of Itajubá (UNIFEI), in 2010, 2012, and 2015, respectively. He was a Postdoctoral Researcher with UNIFEI, from 2016 to 2018. He is currently an Assistant Professor with the Department of Production Engineering, Federal University of Paraíba (UFPB). He is also a Lead Researcher with the Applied Quantitative Methods Study Group, UFPB. He is also a Coordinator of the Laboratory of Applied Quantitative Methods (LabMeQA), UFPB. He has expertise in modeling and optimization for portfolio analysis. His research interests include economic engineering and investment analysis in renewable energy, accounting, finance and operations research.

TARCÍSIO G. BRITO received the degree in mechanical engineering, specialization in environmental engineering, the master's degree in mechanical engineering, and the Ph.D. degree in production engineering from the Federal University of Itajubá, in 2015. He is currently an Adjunct Professor C1 with the Federal University of Itajubá, Brazil. Has experience in mechanical engineering and production, acting on the following subjects: manufacturing processes, technical drawing I and II, machine elements, machine dynamics, and quality control. He works in the area of design and analysis of experiments and operational research.

ANDERSON P. PAIVA received the degree in mechanical engineering, the master's degree in production engineering, and the Ph.D. degree in mechanical engineering from UNIFEI. He is currently an Associate Professor III with the Federal University of Itajubá (UNIFEI / IEPG). He works in the area of Design and experiment analysis, multivariate statistics and optimization methods. His main line of research is manufacturing process optimization.

PEDRO P. BALESTRASSI received the degree in electrical engineering from the Federal University of Espírito Santo, in 1988, the master's degree in electrical engineering from the Federal University of Itajubá, in 1992, the Ph.D. degree in production engineering from the Federal University of Santa Catarina, in 2000, and the sandwich Ph.D. degree in industrial engineering from Texas A&M University, USA. From 2005 to 2006, he was a Visiting Professor in operational research with The University of Texas at Austin, USA. From 2010 to 2011, he was a Visiting Professor in industrial engineering with The University of Tennessee at Knoxville. He was a Visiting Professor with the Polytechnic University of Catalonia (BarcelonaTECH), in 2017. He is currently a Titular Professor with the Institute of Production Engineering and Management (IEPG), Federal University of Itajubá (UNIFEI). He is also a CNPq Research Productivity Fellow (Level 1D). He works mainly in the areas of quality engineering, statistics, experiment design, forecasting, and artificial neural networks.

LAVÍNIA M. MENDES ARAÚJO is currently pursuing the degree in production engineering with the Federal University of Paraíba, Campus I, João Pessoa. She currently holds a sandwich degree in France through the BRAFITEC Program with Grenoble INP's School of Industrial Engineering. She was a member of the Study Group on Applied Quantitative Methods, UFPB. She did an internship in her area of training at Hospital Nossa Senhora das Neves and at Paraibana Gas Company (PBGás). She was a University Extension Fellow with the Solidary Enterprises Incubator (INCUBES/UFPB). Accounting Technician by IFPB, Campus João Pessoa.

● ● ●