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An Improved Grey Prediction Evolution Algorithm Based on Topological Opposition-Based Learning

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ABSTRACT The grey prediction evolution algorithm based on the even grey model (GPEAe) proposed by Z.B.Hu et al. in 2019 is a competitively stochastic real-parameter optimization algorithm with characters of simple code, less parameters and strong exploration capability. To improve the algorithmic overall performance, a topological opposition-based learning strategy (TOBL) is first developed to enhance its exploitation capability in this paper. The TOBL determines offsprings by calculating the Manhattan distances between the current best individual and all the vertices of the hypercube inspired by the opposition-based learning strategy. An improved grey prediction evolutionary algorithm based on the TOBL (TOGPEAe) is then proposed. The performance of the TOGPEAe is tested on CEC2005, CEC2014 benchmark functions and a test suite composed of six engineering design problems. The experimental results of the TOGPEAe are very competitive compared with those of the original GPEAe and other state-of-the-art algorithms.

INDEX TERMS Engineering design problems, grey prediction evolution algorithm, topological opposition-based learning.

I. INTRODUCTION

The optimization methods can be divided into traditional optimization methods and meta-heuristic optimization methods [1]. In recent years, meta-heuristic algorithms have attracted more and more attention because of its simple structure, easy implementation, independent of gradient information, avoiding local optimum, and wide application in engineering problems [2], [3]. According to the difference of inspired objects, the meta-heuristic algorithms can be divided into four categories: (I) based on nature evolution phenomena: genetic algorithm (GA) [4], differential evolution (DE) [5]–[7], covariance matrix adaptation evolution strategy (CMAES) [8], and backtracking search optimization algorithm (BSA) [9], [10]; (II) inspired by biological social activities (mainly are swarm intelligence algorithms [11]): particle swarm optimization (PSO) [12], cuckoo search (CS) [13], artificial bee colony algorithm (ABC) [14] and ant colony optimization algorithm (ACO) [15]; (III) based on physical phenomena: simulated annealing algorithm (SA) [16], gravitational search algorithm (GSA) [17], ray optimization algorithm (RO) [18], small-world optimization algorithm (SWOA) [19] and curved space optimization (CSO) [20]; (IV) inspired by mathematical models: estimation of distribution algorithm (EDA) [21], grey prediction evolution algorithm (GPEAe) [22].

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Unlike other meta-heuristic algorithms, the GPEAe [22] proposed by Z.B.Hu et al. in 2019 is inspired by the even grey model of the grey theory [23]–[25]. The novel evolutionary algorithm inspired by a mathematical model treats population series as time series, and then uses the even grey model to predict its offsprings. It has the characteristics of simple code, less parameters and strong exploration capability, and has been successfully applied to the environmental economic dispatch (EED) problem. In order to improve its overall performance by enhancing exploitation capability, an improved grey prediction evolution algorithm based on topological opposition-based learning (TOGPEAe) is proposed in this paper.

The core innovation of the TOGPEAe comes from a topological opposition-based learning strategy (TOBL). The TOBL is proposed on basis of the opposition-based learning strategy (OBL) which was proposed by Tizhoosh in 2005 [26]. The OBL is one of the most successful strategies to enhance algorithmic exploitation capability. It has been successfully applied to various optimization algorithms [27]–[33]. Moreover, many scholars have studied [34]–[36] and proposed many improved OBL strategies [37]–[44]. However, almost all improved OBL strategies have to compute at least one additional fitness (function) value. This will increase computational overhead.

Unlike the original and improved OBL strategies, the proposed TOBL determines candidate solutions by calculating the Manhattan distances between the best individual and all the vertices of the hypercube inspired by the OBL. Compared with the calculation of fitness values for other improved OBL strategies, the Manhattan distances has much less computational overhead.

The main contributions of this paper are as followings.

- ***Proposed an improved grey prediction evolution algorithm (TOGPEAe)***: The TOGPEAe is developed by adding the TOBL in front of the selection operator of the original GPEAe. The TOBL enhances the algorithmic exploitation capability by guiding the individuals to learn from the current best individual. The TOGPEAe achieves remarkable results in CEC2005 and CEC2014 benchmark function sets, and is successfully applied to solve engineering design problems.
- ***Proposed a novel learning strategy (TOBL)***: Topological opposition-based learning strategy (TOBL) is an improved OBL strategy. The strategy possesses the characteristics of lower computational overhead, strong exploitation capability and larger candidate solution space.

The remainder of this paper is organized as follows. Section 2 introduces the original GPEAe. As the main contribution of this paper, a detailed explanation of the TOBL and the TOGPEAe is presented in Section 3. In Section 4, the TOGPEAe is evaluated on CEC2005, CEC2014 benchmark functions and six engineering design problems. Finally, the concluding remarks and future work are summarized in Section 5.

II. BASIC GREY PREDICTION EVOLUTION ALGORITHM

Like other meta-heuristics algorithms, the beginning of the GPEAe is the process of initializing population. The GPEAe then uses its peculiar reproduction operator to generate trial population, rather than mutation and crossover operators. Finally, greedy selection is used to obtain the most potential individuals into the next generation.

A. INITIALISATION

In the initialization of the GPEAe, it generates $3N$ D-dimension individuals in feasible region. The individuals are expressed as $\vec{x}_i^g = (x_{i,1}^g, x_{i,2}^g, \cdots, x_{i,D}^g)$, where $i = 1, 2, \cdots, N$, $g = 0, 1, 2, \cdots, g_{max}$ is the current generation and g_{max} is the maximum number of generation.

Like most meta-heuristics, the GPEAe uses the following formula to randomly generate the jth dimension of the ith individual within the feasible region:

$$x_{i,j}^g = Low_j + rand \cdot (Up_j - Low_j) \tag{1}$$

Here $g = 0, 1, 2$, $rand$ represents a random number of the uniform distribution from 0 to 1, Low_j and Up_j are the lower and upper boundary of j^{th} dimension of i^{th} individual, respectively.

Noting. The GPEAe must initialize three generation populations. Each generation has N individuals, from top to bottom are $X^2(g = 2)$, $X^1(g = 1)$, $X^0(g = 0)$, respectively.

B. REPRODUCTION OPERATOR

Let X^{g-2}, X^{g-1}, X^g $(g \geq 2)$ be a successive series of three population. Individuals $\vec{x}_{r1}$, $\vec{x}_{r2}$, $\vec{x}_{r3}$ are randomly selected from X^{g-2}, X^{g-1}, X^g respectively, they are used as an individual series. Let $\vec{u}_i^g = (u_{i,1}^g, u_{i,2}^g, \cdots, u_{i,j}^g, \cdots u_{i,D}^g)$ be the trial vector in the trial population U^g of the population X^g. Let $d_{12} = |x_{r1,j} - x_{r2,j}|$, $d_{13} = |x_{r1,j} - x_{r3,j}|$, $d_{23} = |x_{r2,j} - x_{r3,j}|$, and $Maxd_r = max\{d_{12}, d_{23}, d_{13}\}$, and $Mind_r = min\{d_{12}, d_{23}, d_{13}\}$. Then the reproduction operator is formulated as follows:

$$u_{i,j}^g = \begin{cases} (1 - e^{\alpha}) \cdot (x_{r1,j} - \frac{\beta}{\alpha}) \cdot e^{-3\alpha}, & \text{if} \quad Maxd_r \geq \delta, \\ \frac{4x_{r3,j} + x_{r2,j} - 2x_{r1,j}}{3}, & \text{elseif} \quad Mind_r < \delta \\ x_{r3,j} + \omega \cdot Maxd_r, & \text{otherwise.} \end{cases} \tag{2}$$

Here the δ is a parameter for controlling forecast, it belongs to $[0.001, 0.1]$. α is the grey development coefficient, β is the grey control parameter, and ω is a disturbance coefficient, respectively. We obtain them by the following equation:

$$\begin{cases} \alpha = \frac{2(x_{r2,j} - x_{r3,j})}{x_{r2,j} - x_{r3,j}}. \\ \beta = \frac{2((x_{r2,j})^2 + x_{r1,j} \cdot x_{r2,j} - x_{r1,j} \cdot x_{r3,j})}{x_{r2,j} + x_{r3,j}}, \\ \omega = rand(-1, 1) \cdot \left(0.01 - \frac{3.99(I - M)}{M}\right) \end{cases} \tag{3}$$

Here $rand(-1, 1)$ is a random number with uniform distribution from -1 to 1. M is the maximum number of iteration and I is the current iteration number.

C. SELECTION

In this state, we select individuals with better fitness value from trial population U^g and target population X^g according to a greedy selection mechanism, which are used to update

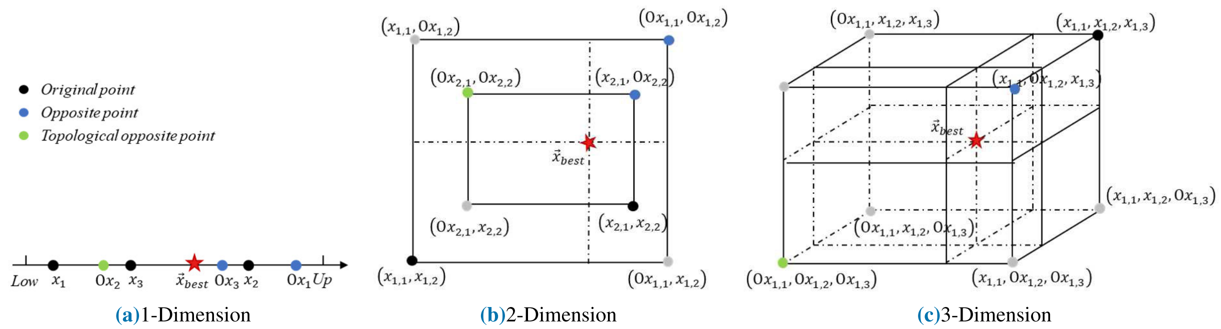

FIGURE 1. **Original point and its potential topological opposite points of the TOBL.**

the population X^g. For the minimum problems, if the solution value $\vec{x}_i^g$ is smaller than $\vec{u}_i^g$, then $\vec{x}_i^g$ is retained, whereas $\vec{x}_i^g$ is replaced by $\vec{u}_i^g$. This process is shown as following:

$$\vec{x}_i^{g+1} = \begin{cases} \vec{u}_i^g, & \text{if} \quad f(\vec{u}_i^g) < f(\vec{x}_i^g), \\ \vec{x}_i^g, & \text{otherwise.} \end{cases} \tag{4}$$

III. IMPROVED GREY PREDICTION EVOLUTION ALGORITHM BASED ON TOPOLOGICAL OPPOSITION-BASED LEARNING

The main contents of this section are as follows:

- Proposing a topological opposition-based learning operator (TOBL)
- Proposing an improved algorithm based on the TOBL, namely grey prediction evolutionary algorithm based on the TOBL (TOGPEAe)

A. TOPOLOGICAL OPPOSITION-BASED LEARNING STRATEGY (TOBL)

The OBL has only one complete opposite point, in which each dimension is opposite to the original value. In addition, the OBL keeps good individuals into the next generation by comparing fitness values of target individuals and corresponding opposition individuals. Inspired by the OBL, a topological opposition-based learning (TOBL) strategy is proposed.

Definition 1 (Topological opposite point): Let $\vec{x}_i = (x_{i,1}, x_{i,2}, \cdots, x_{i,j}, \cdots, x_{i,D})$ be a point in D-dimensional search space, its topological opposite point $T\vec{x}_i = (Tx_{i,1}, Tx_{i,2}, \cdots, Tx_{i,j}, \cdots, Tx_{i,D})$ can be defined as follows:

$$Tx_{i,j} = \begin{cases} Ox_{i,j}, & \text{if} \quad \left|x_{best,j} - x_{i,j}\right| > \left|x_{best,j} - Ox_{i,j}\right|, \\ x_{i,j}, & \text{otherwise.} \end{cases} \tag{5}$$

Here, $Ox_{i,j}$ is the j^{th} dimension of the opposite point $O\vec{x}_i$, and $x_{best,j}$ is the j^{th} dimension of the current best individual $\vec{x}_{best}$.

$$Ox_{ij} = Low_j + Up_j - x_{ij} \quad j = 1, 2, \cdots, D \tag{6}$$

In fact, the $T\vec{x}_i$ is the point with the smallest Manhattan distance between the current best individual and all the vertices of the hypercube inspired by the opposition-based learning strategy. For example, take the 3-Dimension case of Fig.1, the black dot $(x_{1,1}, x_{1,2}, x_{1,3})$ is an original point, the rest seven vertices of the cube are a set of alternative points for the topological opposite point. Each vertex (vector) of those has at least one dimension which is changed according to the opposite formula (6). When the best current individual locates the position marked by a red pentacle in the figure, the blue point $(x_{1,1}, Ox_{1,2}, x_{1,3})$ has the smallest Manhattan distance from the best current individual. So the point $(x_{1,1}, Ox_{1,2}, x_{1,3})$ is the topological point of the original point. The other two subgraphs (1-Dimension and 2-Dimension) are similar.

Comparing with the original OBL strategy and some improved OBL strategies, the TOBL strategy has the following two advantages.

- **More alternative points.** The TOBL has 2^D potential opposite points for each original point, while OBL has just only one. In fact, each vertex of the hypercube is a potential TOBL point.
- **Less computational overhead.** The least Manhattan distance for the TOBL is implemented by the formula (5). There is no computation for fitness function, but which is inevitable for the OBL or improved OBL strategies.

B. FLOW AND PSEUDO CODE OF IMPROVED ALGORITHM

As mentioned above, the basic GPEAe has the strong capability of global search. The proposed TOGPEAe uses the TOBL to enhance its local search capability, and therefore achieves the relative balance between exploration and exploitation capability.

As with the basic GPEAe, after three population initialization (formula (1)), the TOGPEAe realizes the function-optimized process by looping the reproduction (formula (2)) operator and the selection operator (formula (4)) for updating the population. What makes TOGPEAe different is that it adds the above TOBL before the selection operator on every individual of the current population. The flow chart of the TOGPEAe is shown in the Fig.2. and pseudo code is shown in Alg.1.

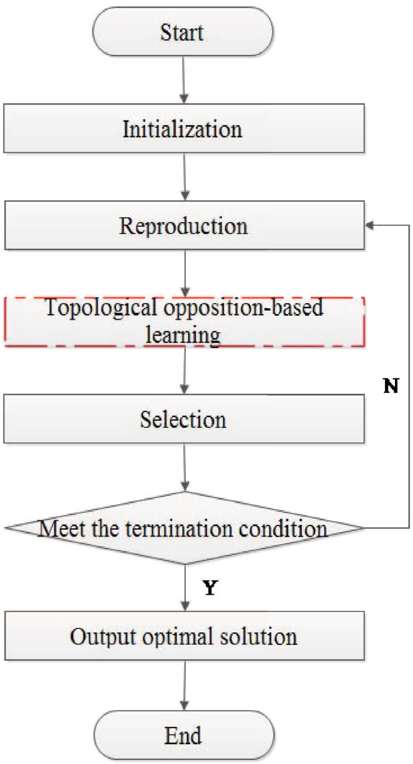

FIGURE 2. Flow chart of the TOGPEAe.

Algorithm 1 The pseudo code for the TOGPEAe

Input: N, D, M, δ, Low, Up
Output: Optimal solution $f(x)$
Initialization
Initialize X^2, X^1, X^0 according to the formula (1);
for $g = 3 : M$ **do**
  Reproduction
  for $i = 1 : N, j = 1 : D$ **do**
    Three individuals $\vec{x}_{r1}, \vec{x}_{r1}, \vec{x}_{r1}$ are randomly selected from X^{g-2}, X^{g-1} and X^g, respectively ;
    Let $d_{12} = |x_{r1,j} - x_{r2,j}|$, $d_{13} = |x_{r1,j} - x_{r3,j}|$, $d_{23} = |x_{r2,j} - x_{r3,j}|$, and
    $Maxd_r = max\{d_{12}, d_{23}, d_{13}\}$, and
    $Mind_r = min\{d_{12}, d_{23}, d_{13}\}$;
    if $Maxd_r \geq \delta$ **then**
      $u_{i,j}^g = (1 - e^{\alpha}) \cdot (x_{r1,j} - \frac{\beta}{\alpha}) \cdot e^{-3\alpha}$
    else if $Mind_r < \delta$ **then**
      $u_{i,j}^g = \frac{4x_{r3,j}+x_{r2,j}-2x_{r1,j}}{3}$;
    else
      $u_{i,j}^g = x_{r3,j} + \omega \cdot Maxd_r$;
    end
    $u_{i,j}^g = x_{r3,j} + \omega \cdot Md_r$;
  end
  Boundary processing method
  Topological opposition-based Learning
  for $i = 1 : N, j = 1 : D$ **do**
    $Ou_{i,j}^g = Low_j + Up_j - u_{i,j}^g$;
    if $\left|x_{best,j}^g - u_{i,j}^g\right| > \left|x_{best,j}^g - Ou_{i,j}^g\right|$ **then**
      $Tu_{i,j}^g = Ou_{i,j}^g$;
    else
      $Tu_{i,j}^g = u_{i,j}^g$;
    end
  end
  Selection
  for $i = 1 : N$ **do**
    if $fitness(T\vec{u}_i^g) < fitness\ (\vec{x}_i^g)$ **then**
      $\vec{x}_i^{g+1} = T\vec{u}_i^g$;
    else
      $\vec{x}_i^{g+1} = \vec{x}_i^g$;
    end
  end
end
Note. M is the maximum number.

IV. NUMERICAL EXPERIMENTS

In order to verify the performance of the proposed TOGPEAe, two numerical experiments have been performed in this section. The first experiment is investigated based on CEC2005 [45] (including 10-dimension and 30-dimension) and CEC2014 (30-dimension) [46] benchmark test functions. The details of CEC2005 and CEC2014 are shown in Tab.1 and Tab.2, respectively. The second experiment is carried out in six engineering design problems. All experiments are executed in MATLAB R2012a with an Intel(R) Core(TM) i5-4590 CPU @ 3.30GHz with 4 GB RAM.

A. EXPERIMENTS FOR BENCHMARK FUNCTIONS

To prove the effectiveness of the TOGPEAe, in this experiment, LBSA, BSA, SaDE [47], CLPSO [48], PSOFIPS [49], TLBO, ETLBO [50], PSOFDR [51], OBSA [52], GPEAe and some other state-of-the-art algorithms are compared with the TOGPEAe on CEC2005 and CEC2014 benchmark functions. To ensure fairness, each comparison algorithm independently runs 30 times. The size of population (N) is set to 50, and the dimension (D) is 10 and 30 respectively. In addition, the termination condition depends on the maximum number of iterations (M), which is set to $100 * D$.

1) RESULTS IN CEC2005 ON 10-DIMENSIONAL (10D) DATA

The CEC2005 test functions include unimodal functions, multimodal functions and hybrid composition functions. They are used to evaluate the different performance of the algorithm. From Tab.1, among the CEC2005 test functions, F1-F5 are unimodal functions, F6-F14 are multimodal functions and F15-F25 are hybrid composition functions.

In this part, the statistical results are summarized in Tab.3, including the best value (Best), mean value (Mean), and standard deviation (Std) obtained using the TOGPEAe, LBSA, BSA, CLPSO, PSOFIPS, TLBO, ETLBO, PSOFDR, OBSA, and GPEAe algorithms. With respect to the Best

TABLE 1. CEC2005 benchmark functions.

Function type	Function	Function name	Properties	Dimision	Range	Global optimum
Unimodel function	F1	Shifted Sphere function	UN	10/30	[-100,100]	-450
	F2	Shifted Schwefels Problem 1.2	UN	10/30	[-100,100]	-450
	F3	Shifted Rotated High Conditioned Elliptic Function	UN	10/30	[-100,100]	-450
	F4	Shifted Schwefels Problem 1.2 with Noise in Fitness	UN	10/30	[-100,100]	-450
	F5	Schwefels Problem 2.6 with Global Optimum on Bounds	UN	10/30	[-100,100]	-310
Multimodal function	F6	Shifted Rosenbrocks Function	MN	10/30	[-100,100]	390
	F7	Shifted Rotated Griewanks Function without Bounds	MN	10/30	[-600,600]	-180
	F8	Shifted Rotated Ackleys Function with Global Optimum on Bounds	MN	10/30	[-32,32]	-140
	F9	Shifted Rastrigins Function	MS	10/30	[-5,5]	-330
	F10	Shifted Rotated Rastrigins Function	MN	10/30	[-5,5]	-330
	F11	Shifted Rotated Weierstrass Function	MN	10/30	[-0.5,0.5]	-330
	F12	Schwefels Problem 2.13	MN	10/30	[-pi,pi]	-460
	F13	Expanded Extended Griewanks plus Rosenbrocks Function(F8,F2)	MN	10/30	[-3,1]	-130
	F14	Shifted Rotated Expanded Scaffers F6	MN	10/30	[-100,100]	-300
Hybrid composition function	F15	Hybrid Composition Function	MS	10/30	[-5,5]	120
	F16	Rotated Hybrid Composition Function	MN	10/30	[-5,5]	120
	F17	Rotated Hybrid Composition Function with Noise in Fitness	MN	10/30	[-5,5]	120
	F18	Rotated Hybrid Composition Function	MN	10/30	[-5,5]	10
	F19	Rotated Hybrid Composition Function with a Narrow Basin for the Global Optimum	MN	10/30	[-5,5]	10
	F20	Rotated Hybrid Composition Function with the Global Optimum on the Bounds	MN	10/30	[-5,5]	10
	F21	Rotated Hybrid Composition Function	MN	10/30	[-5,5]	360
	F22	Rotated Hybrid Composition Function with High Condition Number Matrix	MN	10/30	[-5,5]	360
	F23	Non-Continuous Rotated Hybrid Composition Function	MN	10/30	[-5,5]	360
	F24	Rotated Hybrid Composition Function	MN	10/30	[-5,5]	260
	F25	Rotated Hybrid Composition Function without Bounds	MN	10/30	[-5,5]	260

value, the TOGPEAe ranks first for functions F1, F2, F4-F6, F11, F18-F20, and F24 and ranks last only one function F14. So, it should be noted that the TOGPEAe performs significantly better than its competitors on unimodal functions and hybrid composition functions. The GPEAe is superior to other nine algorithms in functions F3 and F8. LBSA performs best on functions F1, F2, F4, F5, F9 and F15, F18-F20, F22-F24. Comparing the Mean value and the Std value of the ten algorithms, although the performance of the TOGPEAe is not the best among all functions, it outperforms some competitors. As can be observed from Tab.3, the average rank of the TOGPEAe (2.88) in terms of the Best is only worse than that of LBSA (2.76), and it is better than those of other eight algorithms. That is to say, the TOGPEAe is competitive in the solution accuracy of the algorithm, but the robustness of the algorithm is poor.

2) RESULTS IN CEC2005 ON 30-DIMENSIONAL (30D) DATA

This part mainly discusses the experimental results in CEC2005 on 30D. The Best, Mean, and Std of the error values over 30 runs for all comparison algorithms are reported in Tab.4. Moreover, the ranking and the average ranking of all algorithms for 25 functions are also given in Tab.4, and the ranking is based on the Best error value. It can be observed from the table that the TOGPEAe ranks first on functions F3, F8, F10, F21, F23, and F24 when only the Best value are considered. The GPEAe outperforms the other nine comparison algorithms on functions F5, F11 and F12. LBSA is better than other nine algorithms on functions F6, F9, F13, F16 and F17. The average ranking in terms of the Best obtained by the TOGPEAe is 3.04, which is the smallest among all comparison algorithms. From these results, we can see that the TOGPEAe is competitive with the other nine algorithms in terms of algorithmic solution accuracy.

TABLE 2. CEC2014 benchmark functions.

Function type	Function	Function name	Dimision	Range	Global optimum
Unimodal Functions	F1	Rotated High Conditioned Elliptic Function	30	[-100,100]	100
	F2	Rotated Bent Cigar Function	30	[-100,100]	200
	F3	Rotated Discus Function	30	[-100,100]	300
Simple Multimodal Functions	F4	Shifted and Rotated Rosenbrock's Function	30	[-100,100]	400
	F5	Shifted and Rotated Ackley's Function	30	[-100,100]	500
	F6	Shifted and Rotated Weierstrass Function	30	[-100,100]	600
	F7	Shifted and Rotated Griewank's Function	30	[-100,100]	700
	F8	Shifted Rastrigin's Function	30	[-100,100]	800
	F9	Shifted and Rotated Rastrigin's Function	30	[-100,100]	900
	F10	Shifted Schwefel's Function	30	[-100,100]	1000
	F11	Shifted and Rotated Schwefel's Function	30	[-100,100]	1100
	F12	Shifted and Rotated Katsuura Function	30	[-100,100]	1200
	F13	Shifted and Rotated HappyCat Function	30	[-100,100]	1300
	F14	Shifted and Rotated HGBat Function	30	[-100,100]	1400
	F15	Shifted and Rotated Expanded Griewank's plus Rosenbrock's Function	30	[-100,100]	1500
	F16	Shifted and Rotated Expanded Scaffer's F6 Function	30	[-100,100]	1600
Hybrid Function 1	F17	Hybrid Function 1 (N=3)	30	[-100,100]	1700
	F18	Hybrid Function 2 (N=3)	30	[-100,100]	1800
	F19	Hybrid Function 3 (N=4)	30	[-100,100]	1900
	F20	Hybrid Function 4 (N=4)	30	[-100,100]	2000
	F21	Hybrid Function 5 (N=5)	30	[-100,100]	2100
	F22	Hybrid Function 6 (N=5)	30	[-100,100]	2200
Composition Functions	F23	Composition Function 1 (N=5)	30	[-100,100]	2300
	F24	Composition Function 2 (N=3)	30	[-100,100]	2400
	F25	Composition Function 3 (N=3)	30	[-100,100]	2500
	F26	Composition Function 4 (N=5)	30	[-100,100]	2600
	F27	Composition Function 5 (N=5)	30	[-100,100]	2700
	F28	Composition Function 6 (N=5)	30	[-100,100]	2800
	F29	Composition Function 7 (N=3)	30	[-100,100]	2900
	F30	Composition Function 8 (N=3)	30	[-100,100]	3000

3) RESULTS IN CEC2014 ON 30-DIMENSIONAL (30D) DATA

The above experiments mainly discuss the experimental results of CEC2005. This part of the experiment focuses mainly on CEC2014 and the details of the 30 functions in CEC2014 are shown in Tab.2. From the table, it can be observed that the functions of CEC2014 are divided into four types according to different properties. F1-F3 are unimodal functions, F4-F16 are multimodal functions, F17-F22 are hybrid functions, and F23-F30 are composition functions. Different types of test functions are helpful to verify the different performance of an algorithm. Specifically speaking, unimodal functions are effective in evaluating the exploitation capability of an algorithm, while multimodal functions are useful to verify the exploration capability of an algorithm.

Same as above, Tab.5 and Tab.6 report the statistical results in terms of the Best, Mean, and Std of the error value obtained using the TOGPEAe, PSOFIPS, SaDE, CLPSO, CBSA, CLBSA, TLBO, DGSTLBO and GPEAe. Moreover, the tables also report the ranked results of the nine algorithms on the Best error value of each function. The results for the Best obtained by the TOGPEAe on functions F11, F12, F14, F17 are better than those of other eight competitors. When considering the Mean value and the Std value, although the results are less than satisfactory, there is still some competitiveness in some functions. In addition, the average ranking of the Best value for the 30 functions shows that the TOGPEAe ranks better than other seven comparison algorithms, and only worse than SaDE. From these results, we can see that the TOGPEAe is competitive with the other eight algorithms in terms of algorithmic solution accuracy but not good algorithmic robustness.

B. THE SIGN TEST

The sign test [53] is a common method to determine whether there is significant difference between two algorithms. In this paper, the Best value is used as the target of the sign test. The signs "$+$", "$\approx$", and "$-$" represent that the TOGPEAe performs better, almost the same, and worse than other comparison algorithms, respectively, and " Total" represents total number of test functions. The results are reported in Tab.7 and thirteen pairs of algorithms are compared. For LBSA, the TOGPEAe performs better than it on eighteen

TABLE 3. Comparative results of ten algorithms for 10D problems of CEC2005.

F	AL	LBSA	BSA	CLPSO	PSOFIPS	TLBO	ETLBO	PSOFDR	OBSA	GPEAe	TOGPEAe
	Best	**-450.000**	**-450.000**	**-450.000**	**-450.000**	**-450.000**	**-450.000**	**-450.000**	**-450.000**	**-450.000**	**-450.000**
F1	Mean	-450.000	-450.000	-450.000	-450.000	-450.000	-450.000	-439.674	-450.000	-450.000	-450.000
	Std	0.000	0.000	0.000	0.000	0.000	0.000	32.653	0.000	0.000	0.000
	Rank	1	1	1	1	1	1	1	1	1	1
	Best	**-450.000**	**-450.000**	-449.906	**-450.000**	**-450.000**	**-450.000**	**-450.000**	-449.791	**-450.000**	**-450.000**
F2	Mean	-450.000	-449.997	-449.676	-450.000	-450.000	-450.000	-450.000	-446.527	-450.000	-450.000
	Std	0.000	0.005	0.177	0.000	0.000	0.000	0.000	3.336	0.000	0.000
	Rank	1	1	2	1	1	1	1	3	1	1
	Best	251.786	3316.003	333184.289	48691.653	22503.152	23119.940	6576.186	35974.764	**-383.061**	-213.343
F3	Mean	5652.945	21518.124	695430.984	114431.841	70188.530	132295.917	143065.100	427940.360	19403.699	21742.204
	Std	7686.985	18993.641	263113.140	54898.334	47419.217	98983.396	156627.508	471417.610	31289.775	25519.252
	Rank	3	4	10	9	6	7	5	8	1	2
	Best	**-450.000**	-449.994	-449.361	**-450.000**	**-450.000**	**-450.000**	**-450.000**	-448.989	**-450.000**	**-450.000**
F4	Mean	-450.000	-449.800	-445.693	-450.000	-450.000	-450.000	-438.829	-430.129	-435.064	-438.139
	Std	0.000	0.306	2.683	0.000	0.000	0.000	23.550	19.385	45.869	35.980
	Rank	1	2	3	1	1	1	1	4	1	1
	Best	**-310.000**	**-310.000**	**-310.000**	-289.340	**-310.000**	**-310.000**	**-310.000**	-304.344	**-310.000**	**-310.000**
F5	Mean	-310.000	-310.000	-310.000	-233.009	-310.000	-310.000	-310.000	-228.735	-309.983	-309.997
	Std	0.000	0.000	0.000	33.727	0.000	0.000	0.000	64.078	0.060	0.004
	Rank	1	1	1	3	1	1	1	2	1	1
	Best	390.001	390.091	390.348	393.841	**390.000**	**390.000**	390.002	**390.000**	**390.000**	**390.000**
F6	Mean	390.548	392.105	391.599	394.658	392.592	392.167	399.415	404.196	392.210	392.074
	Std	1.112	2.741	1.610	0.405	3.041	2.313	18.471	11.599	2.117	2.542
	Rank	2	4	5	6	1	1	3	1	1	1
	Best	1087.046	1087.046	1087.046	**-179.949**	1087.046	1087.046	1087.266	-179.933	1087.046	1087.046
F7	Mean	1087.046	1087.082	1087.208	-179.810	1087.046	1087.046	1087.288	-179.816	1087.046	1087.046
	Std	0.000	0.076	0.045	0.098	0.000	0.000	0.197	0.078	0.000	0.000
	Rank	3	3	3	1	3	3	4	2	3	3
	Best	-119.728	-119.812	-119.792	-119.851	-119.831	-119.628	-119.800	-119.722	**-119.995**	-119.994
F8	Mean	-119.643	-119.681	-119.624	-119.640	-119.628	-119.648	-119.714	-119.607	-119.763	-119.773
	Std	0.053	0.085	0.081	0.090	0.087	0.066	0.075	0.072	0.172	0.186
	Rank	8	5	7	3	4	10	6	9	1	2
	Best	**-330.000**	**-330.000**	**-330.000**	**-330.000**	-326.020	-327.015	-329.005	**-330.000**	-325.025	-326.020
F9	Mean	-330.000	-330.000	-330.000	-329.333	-322.342	-322.737	-327.314	-329.343	-318.624	-319.496
	Std	0.000	0.000	0.000	0.723	2.928	2.696	1.487	0.844	5.024	5.480
	Rank	1	1	1	1	4	3	2	1	5	4
	Best	-325.724	-324.562	-322.465	-324.188	-325.025	-327.015	**-328.010**	-323.468	-323.035	-327.015
F10	Mean	-323.493	-320.479	-316.698	-317.409	-319.885	-321.513	-319.259	-310.290	-314.178	-315.658
	Std	1.987	3.542	3.125	5.428	4.132	3.593	6.612	9.254	7.778	7.234
	Rank	3	5	9	6	4	2	1	7	8	2
	Best	93.737	92.408	94.425	92.922	91.059	92.619	90.702	92.209	**90.003**	**90.003**
F11	Mean	95.188	94.889	95.319	93.633	94.002	94.632	91.962	95.001	92.503	91.649
	Std	0.746	1.196	0.572	0.536	1.652	0.997	1.023	1.271	2.096	1.334
	Rank	8	5	9	7	3	6	2	4	1	1
	Best	-459.985	-455.477	-431.128	-455.719	**-460.000**	**-460.000**	-459.496	-458.383	-459.991	-459.090
F12	Mean	-444.398	-438.290	-363.666	-422.531	1445.385	794.693	-203.253	-197.091	4011.390	8067.781
	Std	16.545	16.021	65.389	47.468	5027.194	2860.501	507.838	645.215	11368.329	13947.801
	Rank	3	8	9	7	1	1	4	6	2	5
	Best	-129.830	**-129.932**	-129.769	-129.109	-129.447	-129.384	-129.745	-129.663	-129.539	-129.496
F13	Mean	-129.666	-129.697	-129.564	-128.810	-129.196	-129.044	-129.352	-129.353	-128.795	-128.684
	Std	0.073	0.118	0.118	0.146	0.165	0.256	0.271	0.151	0.689	0.847
	Rank	2	1	3	10	8	9	4	5	6	7
	Best	-297.288	-297.108	-296.948	-297.642	-297.566	-297.701	**-298.570**	-297.095	-297.140	-296.805
F14	Mean	-296.902	-296.694	-296.591	-297.170	-297.215	-297.081	-297.241	-296.517	-296.279	-296.273
	Std	0.207	0.205	0.151	0.306	0.249	0.459	0.685	0.253	0.276	0.276
	Rank	5	7	9	3	4	2	1	8	6	10
	Best	**120.000**	**120.000**	120.002	120.460	195.411	197.129	176.891	120.003	320.000	319.102
F15	Mean	120.030	124.232	140.582	209.829	450.644	401.382	387.275	170.949	558.026	506.381
	Std	0.089	12.918	28.900	99.415	141.887	156.564	203.967	99.357	160.052	160.438
	Rank	1	1	2	4	6	7	5	3	9	8
	Best	218.507	**209.614**	223.542	221.355	219.015	217.668	222.865	221.217	224.130	217.663
F16	Mean	223.754	231.537	257.700	232.105	228.617	230.096	254.799	245.791	265.506	261.474
	Std	3.450	10.701	14.508	10.777	9.831	9.972	29.896	10.049	24.022	30.899
	Rank	4	1	9	7	5	3	8	6	10	2
	Best	227.764	237.089	252.680	**216.798**	218.722	222.809	219.166	230.774	231.386	232.587
F17	Mean	241.073	249.588	269.953	251.164	246.701	228.562	247.905	272.287	273.108	261.463
	Std	11.036	10.448	16.875	19.976	19.773	5.540	21.479	29.610	25.500	30.899
	Rank	5	9	10	1	2	4	3	6	7	8
	Best	**310.000**	**310.000**	559.128	**310.000**	**310.000**	393.632	700.558	**310.000**	**310.000**	**310.000**
F18	Mean	603.702	393.335	702.404	580.649	709.009	715.153	864.345	831.072	409.531	410.630
	Std	253.504	155.192	86.522	277.920	237.366	240.232	96.241	157.059	98.317	115.838
	Rank	1	1	3	1	1	2	4	1	1	1
	Best	**310.000**	**310.000**	656.476	**310.000**	417.269	366.021	698.725	310.009	**310.000**	**310.000**
F19	Mean	572.111	404.878	752.056	442.020	756.348	716.366	864.003	711.459	376.461	388.051
	Std	278.437	162.270	46.800	214.463	201.804	227.717	131.102	241.647	85.562	94.651
	Rank	1	1	5	1	4	3	6	2	1	1
	Best	**310.000**	**310.000**	566.636	**310.000**	**310.000**	486.451	693.240	314.640	**310.000**	**310.000**
F20	Mean	442.654	410.210	666.374	748.635	791.997	896.631	681.122	595.677	385.139	401.137
	Std	211.159	177.868	75.428	251.178	186.662	164.780	117.945	102.402	89.969	93.409
	Rank	1	1	4	1	1	3	5	2	1	1
	Best	660.019	**585.919**	790.540	660.000	660.000	660.000	660.000	660.000	660.000	660.000
F21	Mean	910.002	832.592	846.708	830.619	1110.454	931.239	1212.545	1002.404	1224.552	1208.020
	Std	171.591	86.672	28.555	222.507	367.780	360.290	400.425	269.495	259.316	286.449
	Rank	3	1	4	2	2	2	2	2	2	2
	Best	**660.000**	**660.000**	1131.022	1125.586	1116.253	1110.458	1109.829	**660.000**	**660.000**	1092.625
F22	Mean	1074.270	1079.861	1141.724	1131.082	1150.209	1135.155	1177.217	961.467	1124.834	1155.915
	Std	145.718	148.046	7.787	3.327	29.625	24.736	79.315	231.505	95.883	65.594
	Rank	1	1	7	6	5	4	3	1	1	2
	Best	**785.173**	914.095	**785.173**	919.468	919.468	919.468	919.468	919.468	919.468	919.468
F23	Mean	829.609	918.677	906.039	1069.522	1178.115	1237.380	1431.577	1275.131	1252.007	1341.320
	Std	56.624	1.797	42.468	179.455	232.518	257.118	315.479	146.070	283.554	259.152
	Rank	1	2	1	3	3	3	3	3	3	3
	Best	**460.000**	**460.000**	460.211	460.001	**460.000**	**460.000**	760.000	**460.000**	**460.000**	**460.000**
F24	Mean	460.000	460.000	696.247	588.505	460.000	460.000	1126.804	500.000	470.000	500.000
	Std	0.000	0.000	252.248	86.471	0.000	0.000	239.476	103.723	54.772	103.724
	Rank	1	1	3	2	1	1	4	1	1	1
	Best	1075.267	1074.953	1077.376	**460.000**	1074.200	1071.563	1071.868	**460.000**	640.447	635.241
F25	Mean	1079.394	1080.375	1079.655	612.785	1078.221	1076.067	1339.389	660.000	652.761	651.749
	Std	2.258	2.634	2.151	80.691	2.460	20650.000	217.394	164.002	7.371	9.267
	Rank	8	7	9	1	6	4	5	1	3	2
Average	rank	**2.76**	2.96	5.16	3.52	3.12	3.36	3.36	3.56	3.08	2.88

TABLE 4. Comparative results of ten algorithms for 30D problems of CEC2005.

F	AL	LBSA	BSA	CLPSO	PSOFIPS	TLBO	ETLBO	PSOFDR	OBSA	GPEAe	TOGPEAe
	Best	**0.000E+00**	1.173E-16	3.244E-10	7.480E-07	2.640E-26	1.379E-27	**0.000E+00**	5.684E-14	9.965E-16	4.390E-16
F1	Mean	0.000E+00	3.995E-16	4.839E-10	1.057E-06	5.836E-26	3.138E-27	9.330E+02	6.253E-14	2.400E-03	4.323E-14
	Std	0.000E+00	2.267E-16	1.409E-10	3.274E-07	2.769E-26	1.661E-27	1.632E+03	1.734E-14	9.000E-03	2.063E-13
	Rank	1	4	8	9	3	2	1	7	6	5
	Best	1.714E-02	2.450E+03	5.676E+03	3.818E+02	3.527E-05	**2.493E-06**	8.972E-03	4.243E+02	7.090E+00	6.109E-01
F2	Mean	4.995E-02	2.644E+03	6.355E+03	4.560E+02	7.480E-05	6.632E-05	9.374E+01	2.005E+03	1.094E+02	2.836E+01
	Std	4.852E-02	2.038E+02	7.050E+02	6.497E+01	3.424E-05	7.342E-05	8.638E+01	1.270E+03	8.214E+01	2.994E+01
	Rank	4	9	10	7	2	1	3	8	6	5
	Best	7.127E+05	2.784E+06	4.296E+07	1.138E+07	6.838E+05	6.838E+05	1.084E+06	1.313E+06	2.453E+05	**2.382E+05**
F3	Mean	9.240E+05	3.069E+06	4.814E+07	1.265E+07	1.344E+06	1.331E+06	1.757E+06	7.732E+06	1.085E+10	8.466E+05
	Std	3.486E+05	2.791E+05	5.129E+06	1.170E+06	6.881E+05	6.881E+05	6.131E+05	3.513E+06	6.797E+05	4.270E+05
	Rank	4	7	9	8	3	3	5	6	2	1
	Best	2.659E+02	8.510E+03	1.505E+04	2.016E+03	1.736E+02	4.067E+02	5.912E+02	8.304E+03	1.145E+04	5.487E+03
F4	Mean	7.097E+02	9.404E+03	1.613E+04	3.019E+03	2.128E+03	8.611E+02	7.308E+02	1.759E+04	1.993E+04	1.073E+04
	Std	4.298E+02	7.848E+02	1.800E+03	8.707E+02	2.915E+03	7.453E+02	2.258E+02	5.206E+03	5.708E+03	4.126E+03
	Rank	2	8	10	5	1	3	4	7	9	6
	Best	2.501E+03	3.130E+03	3.523E+03	3.009E+03	4.197E+03	3.678E+03	5.077E+03	3.310E+03	**1.880E+03**	2.594E+03
F5	Mean	3.029E+03	3.334E+03	4.315E+03	3.104E+03	4.546E+03	4.161E+03	6.156E+03	5.079E+03	3.230E+03	3.619E+03
	Std	7.939E+02	1.767E+02	9.405E+02	1.055E+02	3.140E+02	4.601E+02	1.750E+03	9.557E+02	8.465E+02	6.076E+02
	Rank	2	5	7	4	9	8	10	6	1	3
	Best	**3.069E+00**	4.494E+01	1.529E+01	2.714E+01	2.109E+01	2.207E+01	2.290E+02	2.495E+01	5.794E+01	4.261E+01
F6	Mean	2.958E+01	7.803E+01	3.974E+01	2.891E+01	2.297E+01	3.895E+01	1.638E+07	9.279E+01	1.443E+03	1.323E+03
	Std	4.229E+01	3.027E+01	2.123E+01	2.546E+00	2.780E+00	3.169E+01	2.646E+07	2.766E+01	3.176E+03	2.265E+03
	Rank	1	8	2	6	3	4	10	5	9	7
	Best	4.696E+03	4.696E+03	4.696E+03	2.859E-01	4.696E+03	4.696E+03	4.696E+03	**1.760E-01**	4.696E+03	4.696E+03
F7	Mean	4.696E+03	4.696E+03	4.696E+03	3.846E-01	4.696E+03	4.696E+03	4.918E+03	5.078E-01	4.696E+03	4.696E+03
	Std	1.575E-12	1.720E-08	0.000E+00	9.646E-02	1.575E-12	9.905E-13	3.844E+02	1.931E-01	5.380E-02	7.100E-03
	Rank	3	3	3	2	3	3	3	1	3	3
	Best	2.091E+01	2.093E+01	2.085E+01	2.092E+01	2.097E+01	2.084E+01	2.089E+01	2.082E+02	2.013E+01	**2.012E+01**
F8	Mean	2.096E+01	2.298E+01	2.093E+01	2.093E+01	2.098E+01	2.089E+01	2.092E+01	2.095E+01	2.070E+01	2.083E+01
	Std	5.689E-02	4.595E-02	8.812E-02	1.807E-02	1.689E-02	6.181E-02	2.802E-02	5.909E-02	3.684E-01	3.083E-01
	Rank	6	8	4	7	9	3	5	10	2	1
	Best	**0.000E+00**	1.134E+00	2.719E-04	4.420E+01	8.258E+01	4.179E+01	4.754E+01	1.294E-01	5.671E+01	2.189E+01
F9	Mean	1.184E-15	2.227E+00	5.082E-04	5.904E+01	9.618E+01	6.567E+01	5.164E+01	4.961E+00	8.388E+01	4.815E+01
	Std	2.051E-15	1.295E+00	2.078E-04	1.441E+01	2.185E+01	3.011E+01	5.605E+00	4.199E+00	2.043E+01	2.125E+01
	Rank	1	4	2	7	10	6	8	3	9	5
	Best	7.131E+01	6.963E+01	1.608E+02	1.788E+02	5.771E+01	8.743E+01	9.451E+01	8.846E+01	6.666E+01	**2.686E+01**
F10	Mean	7.969E+01	8.281E+01	1.647E+02	1.822E+02	9.228E+01	9.913E+01	1.141E+02	1.575E+02	1.268E+02	6.888E+01
	Std	7.575E+00	1.541E+01	5.872E+00	5.261E+00	4.150E+01	1.259E+01	1.892E+01	5.376E+01	2.555E+01	3.715E+01
	Rank	5	4	9	10	2	6	8	7	3	1
	Best	2.661E+01	2.843E+01	2.685E+01	2.625E+01	2.930E+01	3.526E+01	1.551E+01	1.913E+01	**5.866E+00**	8.506E+00
F11	Mean	2.833E+01	3.091E+01	2.798E+01	2.879E+01	3.467E+01	3.714E+01	1.803E+01	2.586E+01	2.606E+01	2.871E+01
	Std	1.680E+00	2.377E+00	1.205E+00	2.252E+00	4.680E+00	1.629E+00	2.803E+00	2.798E+00	1.410E+01	1.351E+01
	Rank	6	8	7	5	9	10	3	4	1	2
	Best	1.698E+03	1.447E+04	4.022E+04	1.962E+04	6.257E+03	3.727E+03	3.259E+03	3.634E+03	1.559E+03	4.311E+03
F12	Mean	2.976E+03	2.801E+04	4.411E+04	2.555E+04	9.523E+03	1.550E+04	2.882E+04	1.536E+04	1.282E+05	9.979E+00
	Std	1.281E+03	1.173E+04	3.443E+03	9.809E+03	3.881E+03	1.420E+04	4.351E+04	9.177E+03	2.832E+05	1.826E+05
	Rank	2	8	10	9	7	5	3	4	1	6
	Best	**1.771E+00**	2.408E+00	3.162E+00	1.347E+01	3.912E+00	2.682E+00	2.293E+00	2.314E+00	2.737E+00	2.063E+00
F13	Mean	1.883E+00	2.524E+00	3.409E+00	1.358E+01	4.248E+00	4.063E+00	2.870E+00	3.382E+00	6.534E+00	5.587E+00
	Std	1.203E-01	1.192E-01	2.156E-01	1.025E-01	5.438E-01	1.202E+00	7.919E-01	4.748E-01	4.297E+00	3.926E+00
	Rank	1	5	8	10	9	6	3	4	7	2
	Best	1.276E+01	1.266E+01	1.298E+01	1.266E+01	1.278E+01	1.303E+01	**1.112E+01**	1.229E+01	1.270E+01	1.210E+01
F14	Mean	1.286E+01	1.309E+01	1.312E+01	1.280E+01	1.289E+01	1.309E+01	1.185E+01	1.291E+01	1.270E+01	1.210E+01
	Std	1.284E-01	3.739E-01	1.258E-01	1.223E-01	1.513E-01	7.225E-02	7.922E-01	2.685E-01	3.063E-01	4.167E-01
	Rank	6	4	8	4	7	9	1	3	5	2
	Best	3.000E+02	6.421E+01	9.610E+01	3.007E+02	3.977E+02	4.268E+02	4.831E+02	2.000E+02	5.000E+02	2.000E+02
F15	Mean	4.015E+02	7.111E+01	1.214E+02	3.544E+02	4.713E+02	4.746E+02	5.258E+02	2.367E+02	5.000E+02	2.000E+02
	Std	1.013E+02	1.159E+01	2.545E+01	6.430E+01	9.623E+01	4.142E+01	5.548E+01	4.901E+01	2.051E+00	1.041E+02
	Rank	4	1	2	5	6	7	8	3	9	3
	Best	**7.797E+01**	1.240E+02	1.775E+02	2.087E+02	1.078E+02	1.912E+02	1.245E+02	1.075E+02	1.592E+02	8.071E+01
F16	Mean	8.450E+01	1.484E+02	2.018E+02	2.507E+02	2.400E+02	2.652E+02	2.413E+02	1.892E+02	1.592E+02	8.071E+01
	Std	5.658E+00	3.611E+01	2.174E+01	5.874E+01	2.269E+02	1.175E+02	1.107E+02	9.878E+01	1.421E+02	1.607E+02
	Rank	1	5	8	10	4	9	6	3	7	2
	Best	**1.198E+02**	1.964E+02	2.574E+02	2.388E+02	1.399E+02	1.323E+02	1.267E+02	1.840E+02	1.949E+02	1.253E+02
F17	Mean	1.511E+02	2.117E+02	3.001E+02	2.812E+02	2.639E+02	2.404E+02	1.932E+02	3.262E+02	1.949E+02	1.253E+02
	Std	4.484E+01	1.541E+01	4.652E+01	6.842E+01	1.708E+02	1.690E+02	1.022E+02	1.391E+02	1.681E+02	1.985E+02
	Rank	1	8	10	9	5	4	3	6	7	2
	Best	9.108E+02	9.119E+02	9.090E+02	8.310E+02	9.105E+02	8.000E+02	9.187E+02	**2.000E+02**	8.000E+02	9.084E+02
F18	Mean	9.186E+02	9.153E+02	9.098E+02	8.323E+02	9.148E+02	8.771E+02	9.388E+02	9.148E+02	9.298E+02	9.183E+02
	Std	6.806E+00	3.761E+00	7.681E-01	1.686E+00	4.459E+00	6.679E+01	2.396E+01	2.253E+01	9.298E+02	9.183E+02
	Rank	7	8	5	3	6	2	9	1	2	4
	Best	9.136E+02	9.147E+02	9.096E+02	8.318E+02	9.201E+02	9.903E+02	9.172E+02	**2.000E+02**	8.000E+02	9.073E+02
F19	Mean	9.137E+02	9.161E+02	9.100E+02	8.323E+02	9.594E+02	9.289E+02	9.449E+02	8.632E+02	8.000E+02	9.073E+02
	Std	1.897E-01	2.038E+00	4.197E-01	4.396E-01	5.067E+01	2.414E+01	2.678E+01	6.018E+01	3.823E+01	7.831E+00
	Rank	6	7	5	3	9	10	8	1	2	4
	Best	**8.000E+02**	9.128E+02	9.078E+02	8.310E+02	9.141E+02	9.136E+02	9.327E+02	9.098E+02	**8.000E+02**	9.096E+02
F20	Mean	8.779E+02	9.149E+02	9.088E+02	8.321E+02	9.475E+02	9.216E+02	9.501E+02	9.020E+06	8.000E+02	9.096E+02
	Std	6.752E+01	2.841E+00	9.501E-01	9.107E-01	3.759E+01	8.330E+00	2.351E+01	9.503E+00	4.974E+01	6.463E+00
	Rank	1	6	3	2	8	7	9	5	1	4
	Best	**5.000E+02**	**5.000E+02**	5.427E+02	**5.000E+02**	**5.000E+02**	**5.000E+02**	8.549E+02	**5.000E+02**	**5.000E+02**	**5.000E+02**
F21	Mean	5.000E+02	5.000E+02	6.508E+02	5.000E+02	9.551E+02	5.000E+02	9.728E+02	5.973E+02	5.000E+02	5.000E+02
	Std	1.969E-13	1.842E-13	1.376E+02	3.922E-06	3.941E+02	8.247E-13	1.750E+02	2.291E+02	2.764E+02	8.150E-13
	Rank	1	1	2	1	1	1	3	1	1	1
	Best	9.266E+02	9.617E+02	8.984E+02	**5.287E+02**	8.831E+02	8.747E+02	8.648E+02	9.733E+02	9.038E+02	8.660E+02
F22	Mean	9.372E+02	9.709E+02	9.189E+02	5.290E+02	9.220E+02	9.904E+02	8.860E+02	1.039E+03	9.038E+02	8.660E+02
	Std	1.765E+01	8.997E+00	2.153E+01	2.608E-01	3.630E+01	3.054E+01	2.439E+01	4.147E+01	4.965E+01	3.693E+01
	Rank	8	9	6	1	5	4	2	10	7	3
	Best	**5.342E+02**	**5.342E+02**	6.255E+02	**5.342E+02**	8.094E+02	1.184E+03	8.767E+02	**5.342E+02**	**5.342E+02**	**5.342E+02**
F23	Mean	5.342E+02	5.342E+02	6.655E+02	5.342E+02	1.052E+03	1.226E+03	1.030E+03	7.727E+02	5.342E+02	5.342E+02
	Std	4.661E-04	1.987E-05	5.669E+01	2.583E-04	2.102E+02	3.965E+01	1.329E+02	2.544E+02	4.974E+01	6.463E+00
	Rank	1	1	2	1	3	5	4	1	1	1
	Best	**2.000E+02**	**2.000E+02**	9.471E+02	2.176E+02	**2.000E+02**	**2.000E+02**	9.399E+02	**2.000E+02**	**2.000E+02**	**2.000E+02**
F24	Mean	2.000E+02	2.000E+02	9.489E+02	2.183E+02	2.000E+02	2.000E+02	9.579E+02	2.676E+02	2.000E+02	2.000E+02
	Std	0.000E+00	1.897E-12	1.994E+00	9.720E-01	4.702E-11	3.212E-12	2.510E+01	2.527E+02	3.104E+02	1.310E+02
	Rank	1	1	4	2	1	1	3	1	1	1
	Best	9.922E+02	1.007E+03	1.000E+03	2.000E+02	9.817E+02	1.009E+03	1.036E+03	**2.000E+02**	2.150E+02	2.149E+02
F25	Mean	9.938E+02	1.016E+03	1.008E+03	2.070E+02	1.006E+03	1.052E+03	1.064E+03	3.418E+02	2.150E+02	2.149E+02
	Std	1.593E+00	8.040E+00	1.268E+01	1.213E+01	2.774E+01	4.316E+01	2.582E+01	3.618E+02	2.656E+02	5.477E+01
	Rank	5	7	6	1	4	8	9	1	3	2
Average	rank	3.2	5.56	6.0	5.24	5.16	5.08	5.24	4.32	4.2	**3.04**

TABLE 5. Comparative results of ten algorithms for 30D problems of CEC2014.

F	AL	PSOFIPS	SaDE	CLPSO	CBSA	CLBSA	TLBO	DGSTLBO	GPEAe	TOGPEAe
	Best	5.60E+06	1.23E+05	2.14E+07	2.17E+07	5.69E+07	**5.77E+04**	3.15E+06	6.15E+05	1.31E+06
F1	Mean	1.02E+07	4.41E+05	2.92E+07	4.12E+07	1.61E+08	8.28E+05	1.04E+07	6.89E+06	6.54E+06
	Std	2.88E+06	3.04E+05	4.33E+06	1.19E+07	6.60E+07	7.89E+05	8.61E+06	4.29E+06	3.49E+06
	Rank	6	2	7	8	9	1	5	3	4
	Best	3.63E+03	**0.00E+00**	2.76E+02	1.75E+08	1.21E+10	2.82E+01	1.48E+03	6.36E+05	5.42E+06
F2	Mean	1.13E+04	2.84E-15	1.26E+03	3.20E+08	2.43E+10	1.44E+02	4.59E+06	8.43E+07	1.79E+07
	Std	5.27E+03	8.99E-15	7.40E+02	6.61E+07	7.40E+09	1.72E+02	1.11E+07	1.05E+08	2.26E+07
	Rank	5	1	3	8	9	2	4	6	7
	Best	1.32E+03	**0.00E+00**	5.76E+01	9.14E+02	1.36E+04	2.90E+02	**6.71E-01**	2.66E+02	6.44E+02
F3	Mean	6.94E+03	4.30E-12	7.67E+02	2.37E+03	3.09E+04	2.39E+03	1.44E+01	5.37E+03	5.47E+03
	Std	5.03E+03	1.25E-11	1.20E+03	1.05E+03	9.52E+03	1.77E+03	1.68E+01	4.72E+03	4.07E+03
	Rank	8	1	3	7	9	5	2	4	6
	Best	**2.53E+01**	6.73E+01	9.56E+01	1.13E+02	6.39E+02	6.74E+01	8.79E+01	7.39E+01	7.88E+01
F4	Mean	2.67E+01	8.29E+01	1.16E+02	1.56E+02	2.46E+03	9.70E+01	1.46E+02	1.65E+02	1.44E+02
	Std	6.37E-01	2.70E+01	1.33E+01	2.13E+01	1.56E+03	3.36E+01	3.78E+01	5.18E+01	4.37E+01
	Rank	1	2	7	8	9	3	6	4	5
	Best	2.09E+01	2.05E+01	2.04E+01	2.51E+01	1.58E+02	2.09E+01	2.09E+01	**2.00E+01**	**2.00E+01**
F5	Mean	2.10E+01	2.06E+01	2.05E+01	4.14E+01	7.61E+02	2.10E+01	2.10E+01	2.03E+01	2.05E+01
	Std	5.71E-02	5.89E-02	4.58E-02	9.01E+00	3.65E+02	5.23E-02	4.34E-02	3.01E-01	3.56E-01
	Rank	4	3	2	5	6	4	4	1	1
	Best	**2.75E+00**	**0.00E+00**	1.60E+01	7.26E+01	3.05E+02	1.17E+01	1.23E+01	1.65E+01	1.69E+01
F6	Mean	6.19E+00	2.61E-01	1.70E+01	9.93E+01	7.24E+02	1.54E+01	1.67E+01	2.39E+01	2.16E+01
	Std	2.22E+00	4.92E-01	6.10E-01	1.09E+01	2.63E+02	2.14E+00	3.45E+00	3.96E+00	3.46E+00
	Rank	2	1	5	8	9	3	4	6	7
	Best	7.08E-05	**0.00E+00**	1.37E-03	1.91E+00	9.95E+01	7.84E-12	1.20E-01	1.03E+00	1.05E+00
F7	Mean	2.56E-03	0.00E+00	3.92E-03	4.00E+00	2.41E+02	6.74E-02	1.01E+00	1.66E+00	1.40E+00
	Std	6.75E-03	0.00E+00	1.61E-03	6.84E-01	7.28E+01	8.69E-02	1.50E+00	5.53E-01	3.12E-01
	Rank	3	1	4	8	9	2	5	6	7
	Best	4.32E+01	**0.00E+00**	2.65E-04	1.36E+01	1.49E+02	5.07E+01	3.48E+01	3.09E+01	4.18E+01
F8	Mean	6.60E+01	1.99E-01	5.86E-01	1.86E+01	1.90E+02	7.37E+01	7.67E+01	7.68E+01	5.89E+01
	Std	1.22E+01	4.20E-01	6.20E-01	1.86E+00	1.89E+01	2.11E+01	2.45E+01	1.76E+01	1.80E+01
	Rank	7	1	2	3	9	8	5	4	6
	Best	1.21E+02	4.47E+01	5.52E+01	7.30E+01	1.49E+02	5.87E+01	6.41E+01	**4.38E+01**	5.87E+01
F9	Mean	1.53E+02	7.70E+01	7.79E+01	9.56E+01	1.98E+02	7.68E+01	9.84E+01	8.49E+01	7.22E+01
	Std	1.61E+01	1.68E+01	1.49E+01	9.56E+00	2.47E+01	1.08E+01	3.08E+01	1.71E+01	2.17E+01
	Rank	7	2	3	6	8	4	5	1	4
	Best	1.35E+03	**3.39E-02**	1.83E+01	7.75E+01	3.97E+03	4.64E+02	1.59E+03	1.78E+03	1.92E+03
F10	Mean	1.87E+03	2.56E+00	2.50E+01	1.35E+02	4.78E+03	1.72E+03	2.39E+03	3.79E+03	3.79E+03
	Std	4.44E+02	4.09E+00	5.21E+00	2.35E+01	4.01E+02	7.41E+02	4.71E+02	1.37E+03	1.91E+03
	Rank	5	1	2	3	9	4	6	7	8
	Best	5.23E+03	3.18E+03	2.98E+03	2.22E+03	4.08E+03	6.01E+03	2.42E+03	2.43E+03	**2.11E+03**
F11	Mean	5.76E+03	4.11E+03	3.24E+03	2.80E+03	5.08E+03	6.71E+03	3.39E+03	4.81E+03	4.53E+03
	Std	3.43E+02	4.80E+02	2.18E+02	2.58E+02	5.45E+02	3.92E+02	5.45E+02	1.38E+03	1.45E+03
	Rank	8	6	5	2	7	9	3	4	1
	Best	1.97E+00	7.98E-01	4.12E-01	2.44E-01	6.09E-01	2.13E+00	2.48E+00	7.13E-02	**5.34E-02**
F12	Mean	2.62E+00	1.05E+00	5.42E-01	4.72E-01	2.19E+00	2.64E+00	2.75E+00	7.44E-01	7.74E-01
	Std	2.82E-01	1.24E-01	8.53E-02	6.66E-02	6.22E-01	2.47E-01	2.62E-01	1.06E+00	1.10E+00
	Rank	7	6	4	3	5	8	9	2	1
	Best	2.89E-01	**2.62E-01**	2.77E-01	2.63E-01	2.74E+00	3.50E-01	3.09E-01	3.10E-01	3.26E-01
F13	Mean	3.46E-01	3.09E-01	4.18E-01	3.89E-01	3.88E+00	4.88E-01	4.71E-01	5.26E-01	4.93E-01
	Std	3.30E-02	3.58E-02	7.14E-02	5.27E-02	5.51E-01	1.14E-01	1.13E-01	1.23E-01	1.15E-01
	Rank	4	1	3	2	9	8	5	6	7
	Best	2.54E-01	2.25E-01	2.97E-01	2.20E-01	3.51E+01	2.11E-01	2.26E-01	1.58E-01	**1.42E-01**
F14	Mean	3.07E-01	2.81E-01	3.44E-01	2.83E-01	9.26E+01	2.88E-01	2.88E-01	2.62E-01	2.63E-01
	Std	3.63E-02	2.64E-02	3.38E-02	3.87E-02	2.41E+01	4.70E-02	4.92E-02	5.14E-02	5.02E-02
	Rank	7	5	8	4	9	3	6	2	1
	Best	1.41E+01	**7.32E+00**	9.09E+00	1.55E+01	5.91E+02	1.03E+01	1.54E+01	1.16E+01	1.12E+01
F15	Mean	1.58E+01	9.71E+00	1.09E+01	1.99E+01	8.76E+03	1.80E+01	3.75E+01	2.76E+01	2.43E+01
	Std	9.26E-01	1.28E+00	9.63E-01	2.08E+00	1.12E+04	5.93E+00	2.19E+01	1.03E+01	7.54E+00
	Rank	6	1	2	8	9	3	7	5	4
	Best	1.13E+01	1.13E+01	1.07E+01	**9.78E+00**	1.04E+01	1.14E+01	1.01E+01	1.05E+01	1.00E+01
F16	Mean	1.18E+01	1.15E+01	1.11E+01	1.05E+01	1.17E+01	1.20E+01	1.11E+01	1.23E+01	1.23E+01
	Std	2.50E-01	2.07E-01	2.31E-01	3.26E-01	5.87E-01	4.35E-01	6.62E-01	6.67E-01	6.54E-01
	Rank	7	7	6	1	4	8	3	5	2
	Best	2.44E+05	1.73E+03	1.07E+06	9.41E+05	3.61E+04	3.29E+04	3.54E+04	1.08E+04	**1.34E+03**
F17	Mean	3.76E+05	1.33E+04	2.07E+06	3.46E+06	4.28E+05	1.91E+05	1.67E+05	5.51E+04	7.12E+04
	Std	1.12E+05	6.50E+03	6.62E+05	1.54E+06	4.64E+05	1.73E+05	2.13E+05	6.14E+04	5.59E+04
	Rank	7	2	9	8	6	4	5	3	1

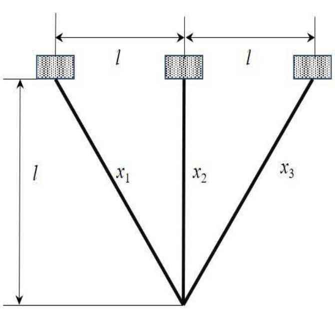

FIGURE 3. Three-bar truss design problem.

functions, almost the same on thirteen functions, and worse than on nineteen functions. Considering SaDE, the performance of the TOGPEAe is a little unsatisfactory since the TOGPEAe only performs better than it on seven functions. However, the results in Tab.7 show that the TOGPEAe performs much better than eleven comparison algorithms (including BSA, CLPSO, PSOFIPS, TLBO, ETLBO, PSOFDR, OBSA, CBSA, CLBSA, DGTLBO and GPEAe). From what has been discussed above, the TOGPEAe is very competitive in solving benchmark functions problems.

C. EXPERIMENTS FOR CONSTRAINED ENGINEERING DESIGN PROBLEMS

The second experiment is tested on six constrained engineering design problems, including the three-bar truss design problem, pressure vessel design problem, tension/compression spring design problem, welded beam design problem, speed reducer design problem and gear train problem. The formulation of these engineering design problems are given in Appendix.A. These problems have been

TABLE 6. Continued Tab.5: Comparative results of ten algorithms for 30D problems of CEC2014.

F	AL	PSOFIPS	SaDE	CLPSO	CBSA	CLBSA	TLBO	DGSTLBO	GPEAe	TOGPEAe
	Best	4.78E+02	**3.09E+01**	3.44E+02	1.54E+06	2.80E+02	1.35E+02	6.17E+01	9.35E+01	7.22E+01
F18	Mean	1.59E+03	6.89E+01	4.92E+02	4.54E+06	2.46E+04	2.93E+03	8.71E+02	2.99E+03	3.07E+03
	Std	8.50E+02	3.43E+01	9.30E+01	1.89E+06	1.06E+05	2.35E+03	1.02E+03	2.77E+03	3.05E+03
	Rank	8	1	7	9	6	5	2	4	3
	Best	1.01E+01	**4.74E+00**	8.20E+00	1.33E+01	2.24E+01	5.77E+00	9.65E+00	6.75E+00	6.41E+00
F19	Mean	1.19E+01	5.55E+00	1.01E+01	2.04E+01	8.25E+01	2.12E+01	2.71E+01	1.61E+01	1.60E+01
	Std	8.29E-01	6.59E-01	1.02E+00	5.81E+00	3.96E+01	2.55E+01	2.86E+01	1.61E+01	1.80E+01
	Rank	7	1	5	8	9	2	6	4	3
	Best	2.36E+03	**1.91E+01**	2.48E+03	1.05E+03	3.45E+02	6.68E+02	2.03E+02	2.51E+02	**1.42E+02**
F20	Mean	5.81E+03	4.31E+01	5.95E+03	6.95E+03	7.38E+03	1.54E+03	4.28E+02	2.16E+03	3.31E+03
	Std	2.71E+03	2.96E+01	3.35E+03	3.35E+03	7.07E+03	9.69E+02	1.77E+02	2.59E+03	3.67E+03
	Rank	8	1	9	7	5	6	3	4	2
	Best	8.47E+04	**2.21E+02**	1.25E+05	1.26E+05	2.10E+03	1.89E+04	4.09E+03	1.65E+03	**1.48E+03**
F21	Mean	1.49E+05	2.49E+03	3.39E+05	6.64E+05	4.18E+04	9.72E+04	2.20E+04	1.57E+04	1.73E+04
	Std	6.40E+04	2.80E+03	1.22E+05	2.63E+05	5.34E+04	9.01E+04	2.22E+04	1.18E+04	1.79E+04
	Rank	7	1	8	9	4	6	5	3	2
	Best	1.67E+02	**2.51E+02**	1.59E+02	1.84E+02	1.61E+02	1.64E+02	1.49E+02	4.26E+02	3.03E+02
F22	Mean	2.25E+02	1.36E+02	2.70E+02	4.19E+02	3.98E+02	2.81E+02	3.14E+02	6.55E+02	6.87E+02
	Std	7.27E+01	7.92E+01	5.98E+01	1.04E+02	1.63E+02	1.08E+02	1.41E+02	1.83E+02	1.65E+02
	Rank	6	1	3	7	4	5	2	9	8
	Best	3.14E+02	3.15E+02	3.15E+02	**2.00E+02**	3.34E+02	3.15E+02	3.15E+02	3.15E+02	3.16E+02
F23	Mean	3.14E+02	3.15E+02	3.15E+02	2.26E+02	3.87E+02	3.15E+02	3.15E+02	3.17E+02	3.17E+02
	Std	1.57E-04	0.00E+00	2.29E-01	4.77E+01	2.07E+01	1.23E-11	4.43E-01	1.35E+00	1.93E+00
	Rank	2	3	3	1	5	3	3	3	4
	Best	2.23E+02	2.24E+02	2.25E+02	**2.00E+02**	2.60E+02	**2.00E+02**	**2.00E+02**	2.30E+02	2.26E+02
F24	Mean	2.24E+02	2.25E+02	2.27E+02	2.00E+02	2.79E+02	2.00E+02	2.00E+02	2.44E+02	2.48E+02
	Std	5.46E-01	5.21E-01	1.01E+00	5.57E-04	9.88E+00	2.20E-03	9.68E-04	8.52E+00	8.19E+00
	Rank	2	3	4	1	7	1	1	6	5
	Best	2.04E+02	2.03E+02	2.08E+02	**2.00E+02**	2.16E+02	**2.00E+02**	**2.00E+02**	2.01E+02	2.01E+02
F25	Mean	2.07E+02	2.06E+02	2.10E+02	2.00E+02	2.24E+02	2.01E+02	2.02E+02	2.11E+02	2.09E+02
	Std	2.45E+00	2.49E+00	1.62E+00	8.04E-09	4.39E+00	2.39E+00	3.62E+00	8.67E+00	7.21E+00
	Rank	4	3	5	1	6	1	1	2	2
	Best	**1.00E+02**	**1.00E+02**	**1.00E+02**	**1.00E+02**	1.01E+02	**1.00E+02**	**1.00E+02**	**1.00E+02**	**1.00E+02**
F26	Mean	1.70E+02	1.00E+02	1.00E+02	1.00E+02	1.43E+02	1.10E+02	1.10E+02	1.01E+02	1.04E+02
	Std	4.83E+01	7.09E-02	3.00E-02	4.88E-02	4.93E+01	3.15E+01	3.15E+01	1.24E-01	1.83E+01
	Rank	1	1	1	1	2	1	1	1	1
	Best	3.76E+02	3.00E+02	4.17E+02	**2.00E+02**	4.44E+02	4.02E+02	4.06E+02	7.47E+02	4.04E+02
F27	Mean	4.34E+02	3.62E+02	4.38E+02	3.62E+02	8.61E+02	5.47E+02	7.94E+02	9.23E+02	9.26E+02
	Std	3.50E+01	5.04E+01	1.33E+01	9.59E+01	2.83E+02	1.66E+02	2.15E+02	1.63E+02	8.92E+01
	Rank	3	2	7	1	8	4	6	9	5
	Best	3.89E+02	8.06E+02	8.64E+02	**2.00E+02**	1.99E+03	9.20E+02	9.96E+02	1.12E+03	1.25E+03
F28	Mean	3.98E+02	8.51E+02	9.21E+02	5.74E+02	2.66E+03	1.24E+03	1.43E+03	1.66E+03	1.58E+03
	Std	1.16E+01	2.76E+01	3.47E+01	4.65E+02	4.48E+02	3.51E+02	4.37E+02	3.06E+02	2.22E+02
	Rank	2	3	4	1	9	5	6	7	8
	Best	**2.12E+02**	6.69E+02	7.11E+03	1.12E+04	1.45E+03	1.25E+03	9.91E+02	1.43E+03	1.24E+03
F29	Mean	2.14E+02	9.67E+02	3.11E+04	3.37E+04	7.77E+06	3.31E+06	3.08E+06	6.28E+05	2.09E+03
	Std	1.02E+00	1.47E+02	1.52E+04	1.09E+04	1.58E+07	5.39E+06	4.99E+06	2.39E+06	5.15E+02
	Rank	1	2	8	9	7	5	3	6	4
	Best	**5.98E+02**	5.90E+02	5.62E+03	7.35E+03	3.87E+03	1.62E+03	3.37E+03	4.09E+03	3.25E+03
F30	Mean	7.00E+02	1.15E+03	7.94E+03	1.38E+04	7.38E+04	3.35E+03	6.47E+03	1.13E+04	1.09E+04
	Std	8.54E+01	5.64E+02	2.72E+03	3.52E+03	4.24E+04	1.35E+03	3.43E+03	1.38E+04	6.80E+03
	Rank	2	1	8	9	6	3	5	7	4
Average	rank	4.9	**2.2**	4.9	5.2	7.1	4.2	4.27	4.47	4.1

TABLE 7. Comparisons between TOGPEAe and other algorithms in Sign Tests.

Method	$-$	$\approx$	$+$	Total
LBSA	19	13	18	50
BSA	11	12	27	50
CLPSO	22	5	53	80
PSOFIPS	27	11	42	80
TLBO	24	17	39	80
ETLBO	11	13	24	50
PSOFDR	13	7	30	50
OBSA	15	11	24	50
CBSA	10	1	19	30
CLBSA	1	0	29	30
DGTLBO	14	1	15	30
SaDE	22	1	7	30
GPEAe	25	21	34	80

used to test the performance of different algorithms. In addition, this paper adopts Deb's heuristic constraint handling method from numerous types of constraint handling strategies to handle the constraint. Each problem independently runs 30 times, and the best function value (Best), the mean function value (Mean), the worst function value (Worst), the standard function deviation (Std), and the function evaluation times (FEs) are reported. The following provides the parameter settings for different engineering design problems. Here, 'N' is the population size, 'T' represents the maximum number of iterations and 'D' is the dimension of the problem.

- Three-bar truss problem: $N = 20,\ T = 500,\ D = 2$.
- Pressure vessel problem: $N = 20,\ T = 2000,\ D = 4$.
- Tension/compression spring problem: $N = 20,\ T = 1000,\ D = 3$.
- Welded beam problem: $N = 20,\ T = 2000,\ D = 4$.
- Speed reducer problem: $N = 20,\ T = 1000,\ D = 7$.
- Gear train problem: $N = 20,\ T = 500,\ D = 4$.

1) THREE-BAR TRUSS DESIGN PROBLEM

The three-bar truss design problem is to minimize the volume of the structural optimization problem. Fig.3 shows the different components of this problem and the formulation is shown in Appendix A.1. This problem is solved by differential evolution with dynamic stochastic selection (DEDS) [54], hybrid evolution algorithm (HEAA) [55], hybrid particle swarm optimization with differential evolution (PSO-DE) [56], differential evolution with level comparison (DELC) [57], mine blast algorithm (MBA) [3] and crow search algorithm (CSA) [58]. The statistical results obtained by all algorithms are reported in Tab.8 and Tab.9.

TABLE 8. Comparison of best solutions for the three-bar truss design problem.

Method	DEDS	HEAA	PSO-DE	DELC	MBA	CSA	TOGPEAe
X_1	0.788675	0.788680	0.788675	0.788675	0.788675	0.788675	0.788697
X_2	0.408248	0.408234	0.408248	0.408248	0.408560	0.408248	0.408185
$g_1(X)$	1.77E-08	NA	-5.29E-11	NA	-5.29E-11	-3.23E-12	1.00E-06
$g_2(X)$	-1.464102	NA	-1.463748	NA	-1.463748	-1.464102	-1.4642
$g_3(X)$	-0.535898	NA	-0.536252	NA	-0.536252	-0.535898	-0.5358
$f(X)$	263.895843	263.895843	263.895843	263.895843	263.895852	263.895843	**263.895712**

TABLE 9. Comparison of statistical results for the three-bar truss design problem.

Method	Worst	Mean	Best	Std	FEs
DEDS	263.895849	263.895843	263.895843	9.7E-07	15000
HEAA	263.896099	263.895865	263.895843	4.9E-05	15000
PSO-DE	263.895843	263.895843	263.895843	4.5E-10	17600
DELC	263.895843	263.895843	263.895843	4.3E-14	10000
MBA	263.915983	263.897996	263.895852	3.93E-03	13280
CSA	263.895845	263.895843	263.895843	2.64E-07	13720
TOGPEAe	263.903833	263.896840	**263.895712**	1.5E-03	**9980**

TABLE 10. Comparison of best solutions for the pressure vessel design problem.

Method	GA-DT	MDE	CPSO	HPSO	DELC	ABC	BSA-SAε	BSA	TOGPEAe
X_1	0.812500	0.812500	0.812500	0.812500	0.812500	0.812500	0.812500	0.812500	0.812500
X_2	0.437500	0.437500	0.437500	0.437500	0.437500	0.437500	0.437500	0.437500	0.437500
X_3	42.097400	42.098400	42.091300	42.098400	42.098400	42.098400	42.098400	42.098400	42.098500
X_4	176.654000	176.636000	176.746500	176.636600	176.636600	176.636600	176.636600	176.636600	176.636600
$g_1(X)$	-3.01E-03	0	-1.37E-06	NA	NA	0	-9.5E-10	-3.69E-08	1.00E-06
$g_2(X)$	-3.58E-02	-0.035881	-3.59E-04	NA	NA	-0.358810	-3.59E-2	-0.358810	-0.035900
$g_3(X)$	-24.759300	-0.000000	-118.768700	NA	NA	-0.000226	-1.2E-4	-0.095446	1.003E-06
$g_4(X)$	-63.346000	-63.363900	-63.253500	NA	NA	-63.363000	-63.363000	-63.284200	-63.364000
$f(X)$	6059.946300	**6059.701700**	6061.077700	6059.714300	6059.714300	6059.714300	6059.714300	6059.715000	6059.708015

The Tab.8 compares the best solution obtained by the TOGPEAe with those obtained by other algorithms. From the table, the TOGPEAe obtains the best solution at $x = (0.788697, 0.408185)$ in the current literature, and the corresponding function value of the best solution equals to 263.895712. On the other hand, Tab.9 reports the Best value, the Mean value, the Worst value, the Std value and the FEs obtained by all comparison algorithms on this problem. It can be observed from the table that the FEs value obtained by the TOGPEAe (equals 9980) is the smallest among all comparison algorithms. This indicates that the TOGPEAe not only finds out the current best solution to this problem, but also holds the smallest FEs among all algorithms. That is, the TOGPEAe has a faster convergence rate than other algorithms to this optimization problem.

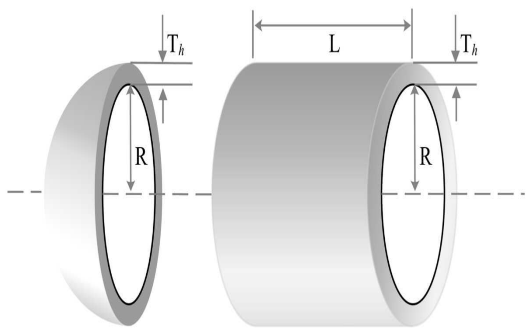

FIGURE 4. Pressure vessel design problem.

2) PRESSURE VESSEL DESIGN PROBLEM

Fig.4 shows the different components of the pressure vessel design problem. The pressure vessel design problem, which was firstly proposed by Kannan and Kramer [59], can be seen as a nonlinear objective function consisting of three linear inequality constraints and one nonlinear inequality constraint. There are four design variables in this problem, including the thickness of the shell (x_1), the thickness of the head (x_2), the inner radius (x_3) and the length of the cylindrical section (x_4), where x_1 and x_2 are the integer multiples of 0.0625. The formulation of this problem is shown in Appendix A.2.

The TOGPEAe is used to solve this problem and compared with other eight famous algorithms, such as genetic algorithm based on dominance tournament selection (GA-DT) [60], modified differential evolution (MDE) [61], coevolutionary particle swarm optimization (CPSO) [62], hybrid particle swarm optimization (HPSO) [63], DELC, artificial bee colony algorithm (ABC) [64], BSA-SAε [65]

TABLE 11. Comparison of best solutions for the pressure vessel design problem.

Method	Worst	Mean	Best	Std	FEs
GA-DT	6469.322000	6177.253300	6059.964300	130.929700	80000
MDE	6059.701700	6059.701700	**6059.701700**	1.0E-12	24000
CPSO	6363.804100	6147.133200	6061.077700	86.45	30000
HPSO	6288.677000	6099.932300	6059.714300	86.20	81000
DELC	6059.714300	6059.714300	6059.714300	2.1E-11	30000
ABC	NA	6245.308100	6059.714700	2.05E+02	30000
BSA-SAε	6116.780400	6074.368200	6059.7143 00	1.71E+01	80000
BSA	6771.596900	6221.286100	6059.715000	2.03E+02	60000
TOGPEAe	6771.592860	6127.051094	6059.708015	1.32E+02	**20620**

TABLE 12. Comparison of best solutions for the tension compression spring design problem.

Method	X_1	X_2	X_3	$g_1(X)$	$g_2(X)$	$g_3(X)$	$g_4(X)$	$f(X)$
GA-DT	0.051989	0.363965	10.890522	-1.30E-05	-2.10E-05	-4.061338	-0.722698	0.012681
MDE	0.051688	0.356692	11.290483	-0.000000	-0.000000	-4.053734	-0.727090	**0.012665**
CPSO	0.051728	0.357644	11.244543	-8.45E-04	-1.26E-05	-4.051300	-0.727090	0.012675
HPSO	0.051706	0.357126	11.265083	NA	NA	NA	NA	**0.012665**
DEDS	0.051689	0.356718	11.288965	NA	NA	NA	NA	**0.012665**
HEAA	0.051690	0.356729	11.288294	NA	NA	NA	NA	**0.012665**
DELC	0.051689	0.356718	11.288966	NA	NA	NA	NA	**0.012665**
ABC	0.051749	0.358179	11.203763	-0.000000	-0.000000	-4.0566663	-0.726713	**0.012665**
MBA	0.051656	0.35594	11.344665	0	0	-4.052248	-0.728268	**0.012665**
SSOC	0.051689	0.356718	11.288965	NA	NA	NA	NA	**0.012665**
BSA-SAε	0.051989	0.356727	11.288425	-7.70E-09	-3.30E-09	-4.054	-0.728	**0.012665**
BSA	0.516940	0.356845	11.281488	-1.05E-07	-1.77E-08	-4.054037	-0.727640	**0.012665**
TOGPEAe	0.051640	0.355540	11.358313	9.89E-07	9.97E-07	-4.051500	-0.728500	**0.012665**

TABLE 13. Comparison of statistical results for the tension compression spring design problem.

Method	Worst	Mean	Best	Std	FEs
GA-DT	0.012973	0.012742	0.012681	5.90E-05	80000
MDE	0.012674	0.012666	0.012665	2.0E-06	24000
CPSO	0.012924	0.012730	0.012675	5.20E-05	23000
HPSO	0.012719	0.012707	0.012665	1.58E-05	81000
DEDS	0.012738	0.012669	0.012665	1.25E-05	24000
HEAA	0.012665	0.012665	0.012665	1.4E-09	24000
DELC	0.012666	0.012665	0.012665	1.3E-07	20000
ABC	NA	0.012709	0.012665	0.012813	30000
MBA	0.012900	0.012713	0.012665	6.30E-05	**7650**
SSOC	0.012868	0.012765	0.012665	9.29E-05	25000
BSA-SAε	0.012666	0.012665	0.012665	1.62E-07	80000
BSA	0.012669	0.012666	0.012665	7.24E-07	43220
TOGPEAe	0.013005	0.012719	0.012665	7.55E-05	20000

and backtracking search optimization algorithm (BSA). The statistical results of all comparison algorithms are listed in Tab.10 and Tab.11. As shown in Tab.10, MDE obtains the best solution at $x = (0.8125, 0.4375, 42.0984, 176.6360)$ with the objective function value $f(x) = 6059.7071$. The TOGPEAe is better than other seven algorithms (GA-DT,CPSO, HPSO, DELC, ABC, BSA-SAε and BSA), and is only worse than MDE. In addition, in Tab.11, the Mean value and the Std value are worse than some comparison algorithms, but the FEs of the TOGPEAe (equals 20620) is lower than other eight algorithms. That is to say, the TOGPEAe determines a better best solution by using the smallest computational overhead. All in all, the above results show that the TOGPEAe has a certain competitiveness on this optimization problem.

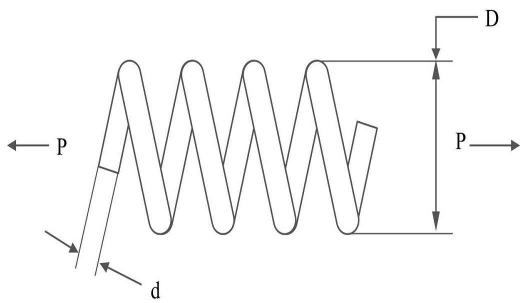

FIGURE 5. Tension/compression spring design problem.

3) TENSION/COMPRESSION SPRING DESIGN PROBLEM

The tension/compression spring design problem consists of three continuous variables (wire diameter (x_1), coil diameter (x_2) and the number of active coil (x_3)) and four nonlinear inequality constrains. The schematic diagram of the structure of the tesion/compression spring design problem is shown in Fig.5. The formulation of this problem is shown in Appendix A.3. For this problem, the results of the TOGPEAe is compared with GA-DT, MDE, CPSO, HPSO, DEDS, HEAA, DELC, ABC, MBA, social spider optimization (SSOC) [66], BSA-SAε and BSA. The statistical results are reported in Tab.12 and Tab.13.

As shown in Tab.12, except for GA-DT and CPSO, other eleven algorithms obtain the best solution which is equal to $f(x) = 0.012665$ (including the TOGPEAe, MDE, HPSO, DEDS, HEAA, DELC, ABC, MBA, SSOC, BSA-SAε and BSA). Moreover, Tab.13 shows that the Std of the TOGPEAe

TABLE 14. Comparison of best solutions for the welded beam design problem.

Method	GA-DT	MDE	CPSO	HPSO	ABC	MBA	BSA-SAε	BSA	BSAISA	TOGPEAe
$X_1(h)$	0.205986	0.205730	0.202369	0.205730	0.205730	0.205729	0.205730	0.205730	0.205730	0.205730
$X_2(l)$	3.471328	3.470489	3.544214	3.470489	3.470489	3.470493	3.470489	3.470489	3.470489	3.470467
$X_3(t)$	9.020224	9.036624	9.048210	9.036624	9.036624	9.036626	9.036624	9.036624	9.036624	9.036624
$X_4(b)$	0.206480	0.205730	0.205723	0.205730	0.205730	0.205729	0.205730	0.205730	0.205730	0.205730
$g_1(X)$	-0.074092	-0.000335	-12.839980	NA	0.000000	-0.001614	-1.55E-10	-5.32E-07	0	1.00E-06
$g_2(X)$	-0.266227	-0.000753	-1.247467	NA	-0.000002	-0.016911	-4.30E-09	-9.02E-06	0	1.00E-06
$g_3(X)$	-4.95E-04	-0.000000	-1.49E-03	NA	0.000000	-2.40E-07	-1.55E-15	-7.86E-12	-5.55E-17	1.00E-06
$g_4(X)$	-3.430044	-3.432984	-3.429347	NA	-3.432984	-3.432982	-3.4330	-3.432984	-3.43294	-3.433000
$g_5(X)$	-0.080986	-0.080730	-0.079381	NA	-0.080730	-0.080729	-8.07E-02	-0.080730	-0.080730	-0.080700
$g_6(X)$	-0.235514	-0.235540	-0.235536	NA	-0.235540	-0.235540	-0.2355	-0.235540	-0.235540	-0.235500
$g_7(X)$	-58.666440	-0.000882	-11.681355	NA	0.000000	-0.001464	-1.85E-10	-1.13E-07	-5.46E-12	1.00E-06
$f(X)$	1.728226	1.724852	1.728024	1.724852	1.724852	1.724853	1.724852	1.724852	1.724852	**1.724851**

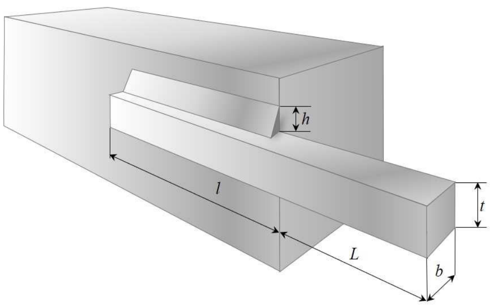

FIGURE 6. Welded beam design problem.

TABLE 15. Comparison of statistical results for the welded beam design problem.

Method	Worst	Mean	Best	Std	FEs
GA-DT	1.993408	1.792654	1.728226	7.47E-02	80000
MDE	1.724854	1.724853	1.724852	NA	**24000**
CPSO	1.782143	1.748831	1.728024	1.29E-02	**24000**
HPSO	1.814295	1749040	1.724852	4.00E-02	81000
ABC	NA	1.741913	1.724852	3.10E-02	30000
MBA	1.724853	1.724853	1.724853	6.94E-19	47340
BSA-SAε	1.724852	1.724852	1.724852	8.11E-10	80000
BSA	1.724854	1.724852	1.724852	2.35E-07	45480
BSAISA	1.724854	1.724852	1.724852	2.96E-07	29000
TOGPEAe	1.731209	1.725033	**1.724851**	9.31E-04	36180

is worse than some other algorithms, but the FEs of the TOGPEAe is second only to MBA with 7650 FEs. Based on the above results, it can be concluded that the TOGPEAe is suitable for this problem.

4) WELDED BEAM DESIGN PROBLEM

The aim of the welded beam design problem is to minimize the manufacturing cost of welded beam. As shown in Fig.6 and Appendix A.4. There are four design variables and seven constrains, two of which are linear inequality constraints and five of which are non-linear inequality constraints. The TOGPEAe is used to solve the problem and the comparison algorithm is as follows: GA-DT, MDE, CPSO, HPSO, ABC, MBA, BSA-SAε, BSA and BSAISA. The comparison results are shown in Tab.14 and Tab.15.

As shown in Tab.14, the best solution obtained by the TOGPEAe at $x = (0.205730, 3.470467, 9.036624, 0.20730)$ with the objective function value $f(x) = 1.724850$ is the smallest among all comparison algorithms. From Tab.15, the smallest FEs is 24000 obtained by MDE and CPSO, while the FEs of the TOGPEAe on this problem is 36180. This indicates that the TOGPEAe can obtain a better solution by sacrificing a little of computational overhead. Combining Tab.14 and Tab.15, we can see that the TOGPEAe is very competitive in all comparison algorithms.

5) SPEED REDUCER DESIGN PROBLEM

Fig.7 shows the sketch of the speed reducer design problem. The speed reducer design problem can be described as a single objective optimization problem with eleven constraints and seven continuous design variables. The variables are represented by $x_1, x_2, x_3, x_4, x_5, x_6, x_7$, where x_3 is an integer variable. From x_1 to x_7, they represent the face width, the module of teeth, the number of teeth on pinon, the face width, the length of the first shaft between bearing, the length of the second shaft between bearing, the diameter of first shaft, and the diameter of the first shaft, respectively. The formulation of this problem is shown in A.5.

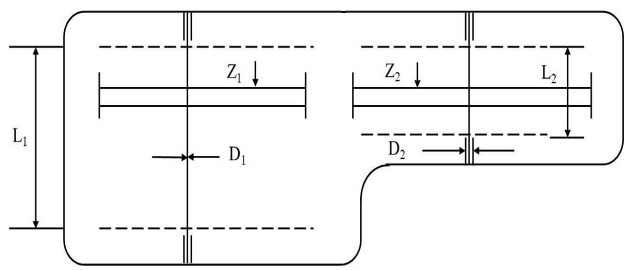

FIGURE 7. Speed reducer design problem.

Tab.16 compares the best solutions of the TOGPEAe, MDE, DEDS, DELC, HEAA, PSO-DE, MBA, BSA and BSAISA. The best solution obtained by the TOGPEAe is $x = (3.500000, 0.700000, 17.000000, 7.300002, 7.715310, 3.350214, 5.286653)$, and the corresponding function value is $f(x) = 2994.468269$. From the table, the TOGPEAe is better than all other algorithms. As shown in Tab.17, the FEs of the TOGPEAe, which equal 19,680, ranks third among all

TABLE 16. Comparison of best solutions for the speed reducer design problem.

Method	MDE	DEDS	DELC	HEAA	PSO-DE	MBA	BSA	BSAISA	TOGPEAe
X_1	3.500010	3.500000	3.500000	3.500000	3.500000	3.500000	3.500000	3.500000	3.500000
X_2	0.700000	0.700000	0.700000	0.700000	0.700000	0.700000	0.700000	0.700000	0.700000
X_3	17.000000	17.000000	17.000000	17.000013	17.000000	17.000000	17.000000	17.000000	17.000000
X_4	7.300156	7.300000	7.300000	7.300428	7.300000	7.300033	7.300000	7.300000	7.300002
X_5	7.800027	7.715320	7.715320	7.715377	7.800000	7.715772	7.715320	7.715320	7.715310
X_6	3.350221	3.350215	3.350215	3.350231	3.350215	3.350218	3.350215	3.350215	3.350214
X_7	5.286685	5.286654	5.286654	5.286664	5.286683	5.286654	5.286654	5.286654	5.286653
$f(X)$	2996.356689	2994.471066	2994.471066	2994.488107	2996.348167	2994.482453	2994.471066	2994.471066	**2994.468269**

TABLE 17. Comparison of statistical results for the speed reducer design problem.

Method	Worst	Mean	Best	Std	FEs
MDE	2996.390137	2996.367220	2996.356689	8.2E-03	24000
DEDS	2994.471066	2994.471066	2994.471066	3.58E-12	30000
DELC	2994.752311	2994.613368	2994.471066	7.0E-02	40000
HEAA	2994.471066	2994.471066	2994.488107	1.9E-12	30000
PSO-DE	2996.348204	2996.348174	2996.348167	6.4E-06	54350
MBA	2999.652444	2996.769019	2994.482453	1.56	**6300**
BSA	2994.471066	2994.471066	2994.471066	9.87E-11	25640
BSAISA	2994.471095	2994.471067	2994.471066	5.40E-06	15860
TOGPEAe	2994.471205	2994.469132	**2994.468269**	6.6E-04	19680

TABLE 18. Comparison of best solutions for the gear train design problem.

Method	ABC	MBA	CSA	TOGPEAe
X_1	49	43	49	43
X_2	16	16	19	16
X_3	19	19	16	19
X_4	43	49	43	49
$f(X)$	**2.7E-12**	**2.7E-12**	**2.7E-12**	**2.7E-12**

TABLE 19. Comparison of statistical results for the gear train design problem.

Method	Worst	Mean	Best	Std	FEs
ABC	NA	3.6E-10	**2.7E-12**	5.52E-10	**60**
MBA	2.1E-08	2.5E-09	**2.7E-12**	3.94E-09	1120
CSA	3.2E-08	2.06E-09	**2.7E-12**	5.1E-09	100000
TOGPEAe	2.4E-09	6.3E-10	**2.7E-12**	8.6E-10	1400

algorithms except for MBA (6300) and BSAISA (15860). We can conclude that the TOGPEAe can obtain a better solution than MBA and BSAISA by sacrificing a small amount of computational overhead on this problem.

6) GEAR TRAIN DESIGN PROBLEM

The gear train design problem aims to minimize the gear ratio of the gear train and Fig.8 is a schematic diagram of this problem. The gear train design problem has no function constraints, and only four design variables (x_1, x_2, x_3, x_4). Each variable is an integer from 12 to 60 and means the number of teeth on corresponding gear. The formulation of this problem is shown in A.6.

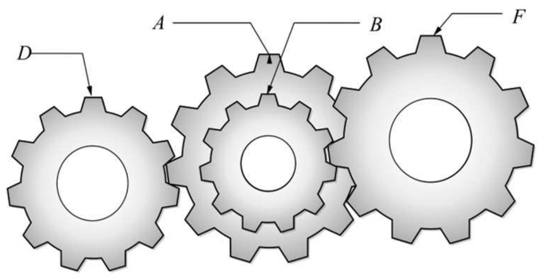

FIGURE 8. Gear train design problem.

Four state-of-the-art algorithms are used to solve this problem, including ABC, MBA, CSA and the TOGPEAe. The comparison results are reported in Tab.18 and Tab.19. Tab.18 compares the best solutions for the gear train design problem. It can be observed from the table that all algorithms reach the same optimal value and the function value is $f(x) = 2.7E - 12$. On the other hand, from Tab.19, the EFs value of the TOGPEAe (equals to 1400) is only better than CSA, and is worse than other two algorithms (ABC, MBA). Therefore, the TOGPEAe is competitive on the gear train design problem.

D. NUMERICAL ANALYSIS AND DISCUSSION

In order to further analyze the comprehensive performance of the TOGPEAe, this section conducts an in-depth analysis of the TOGPEAe from the following two aspects: 1)For benchmark functions experiments, we list the functions that the TOGPEAe ranks first in CEC2005 and CEC2014, and analyze the proposed algorithm based on the properties of these functions. 2)For constrained engineering design problems, we list the optimal results and the minimum FEs based

TABLE 20. Comparison for the best solution and FEs.

Engineering design problem	Well-known smallest Best	Best for TOGPEAe	Well-known smallest FEs(Algorithm)	FEs for TOGPEAe (Rank)
Three-bar truss	263.895843	**263.895712**	10000 (DELC)	**9980** (1)
Pressure vessel	6059.714300	**6059.708015**	24000 (MDE)	**20620** (1)
Tension/Compression spring	**0.012665**	**0.012665**	7650 (MBA)	20000 (2)
Welded beam	1.724852	**1.724851**	20000 (DELC)	36180 (6)
Speed reducer	2994.471066	**2994.468269**	6300 (MBA)	19680 (3)
Gear train	**2.7E-12**	**2.7E-12**	60 (ABC)	1400 (3)

on the table of the above 6 engineering problems and compare them with that of the proposed TOGPEAe. The statistical results are shown in Tab.20.

1) COMPREHENSIVE DISCUSSIONS BASED ON THE BENCHMARK FUNCTIONS EXPERIMENTS

The effectiveness of the TOGPEAe is tested on CEC2005 and CEC2014. CEC2005 and CEC2014 contain different types of functions, such as unimodal functions, multimodal functions and hybrid, composition functions. According to the results of Tab.3 the TOGPEAe performs best in functions F1, F2, F4, F5, F6, F11, F18-F20, F24, where F1-F6 are unimodal functions and the remaining five functions are multimodal functions. It indicates that when solving 10-dimensional functions, the TOGPEAe not only performs well in dealing with unimodal functions, but also is competitive in solving multimodal functions. As shown in Tab.4, the TOGPEAe ranks first among F3, F8, F10, F21, F23, and F24. In the 30-dimensional function test, where only F3 is unimodal function. It shows that the TOGPEAe is more suitable for dealing with multimodal functions. Whether in 10-D or 30-D of CEC2005, the average rank of the TOGPEAe is very competitive in terms of the 'Best' value. In the experiment of CEC2014 (showed in Tab.5 and Tab.6), the TOGPEAe ranks first for functions F5, F11, F12, F14, F17, F20, F21, F26. The average rank of the TOGPEAe is 4.1, it is only worse than SaDE (2.2). In summary, the TOGPEAe is more effective in obtaining better quality solutions.

2) COMPREHENSIVE DISCUSSIONS BASED ON THE ENGINEERING DESIGN PROBLEMS

Several following conclusions can be drawn about the TOGPEAe according to Tab.20.

Accuracy: The Best value in the above tables are the reflection of the accuracy for each algorithm in solving engineering optimization problems. The smaller the Best value, the better the solution accuracy. As shown in Tab.20, the Best value obtained by the TOGPEAe is the smallest on all six engineering design problems. Therefore, the solution accuracy of the TOGPEAe is obviously superior to other comparison algorithms.

Convergence speed: The FEs value reflects the computational overhead when achieving the current optimal function value. The smaller the FEs value is, the faster the convergence rate. From Tab.20, the FEs value of the TOGPEAe ranks first in terms of the three-bar truss design problem and pressure vessel design problem. In the other four engineering optimization problems, the FEs value of the TOGPEAe is not the smallest, but the TOGPEAe obtains better feasible solutions than other algorithms by sacrificing computational overhead. That is to say, the TOGPEAe is also very competitive compared with other algorithms when considering the FEs value.

Robustness: The Mean value and the Std value of all the previous tables are the reflection of the robustness for each algorithm in solving engineering optimization problems. The smaller the value is, the higher the robustness. Compared with other algorithms, the Mean value and the Std value of the TOGPEAe are somewhat unsatisfactory. All in all, the robustness of the TOGPEAe is not competitive enough.

According to the above analyses, two characteristics of the TOGPEAe can be observed. On the one hand, the TOGPEAe is very competitive improved algorithm in solving global optimization problem. On the other hand, although the TOGPEAe's robustness needs to be improved, the overall performance of the TOGPEAe is excellent.

V. CONCLUSION

The GPEAe as a new and competitive evolutionary algorithm with simple code, few parameters and strong exploration capability, its overall performance can still be further improved. In this paper, a new strategy called topological opposition-based learning (TOBL) is first developed. It is then planted in front the selection operator of the basic GPEAe to form a improved algorithm: grey prediction evolution algorithm based on topological opposition-based learning (TOGPEAe). The TOBL determines offsprings by calculating the Manhattan distances between the current best individual and all the vertices of the hypercube inspired on the original OBL strategy. It guides individuals of the TOGPEAe to learn from the best individual of the current generation to enhance the local exploitation capability without increasing the computation overhead.

In order to demonstrate the performance of the TOGPEAe, we tested the TOGPEAe on CEC2005, CEC2014 benchmark functions and a test suite composed of six engineering design problems, and compared the experimental results of the TOGPEAe with many state-of-the-art algorithms. The numerical results on both the benchmark functions and engineering design problems indicate that the proposed TOGPEAe is effective and promising for global optimization. Although the TOGPEAe obtains a good performance in our numerical experiments, we can still find that the standard deviation(Std) of the TOGPEAe is somewhat unsatisfactory on some complex benchmark functions. Our future works are to further research on the robustness of the TOGPEAe under the premise of ensuring the convergence and high precision.

VI. ENGINEERING DESIGN PROBLEMS

A.1. Three-bar truss design problem

$$\begin{aligned}
&\min f(x) = (2\sqrt{2}x_1 + x_2) \times l \\
&\text{subject to}: \ g_1(x) = \frac{\sqrt{2}x_1 + x_2}{\sqrt{2}x_1^2 + 2x_1x_2}\mathrm{P} - \sigma \le 0 \\
&\qquad g_2(x) = \frac{x_2}{\sqrt{2}x_1^2 + 2x_1x_2}\mathrm{P} - \sigma \le 0 \\
&\qquad g_3(x) = \frac{1}{\sqrt{2}x_2 + x_1}\mathrm{P} - \sigma \le 0 \\
&\qquad 0 \le x_i \le 1, \quad i = 1, 2 \\
&\qquad l = 100\,\mathrm{cm}, \quad \mathrm{P} = 2\,\mathrm{kN/cm^2}, \ \sigma = 2\,\mathrm{kN/cm^2}
\end{aligned}$$

A.2. Pressure vessel design problem

$$\begin{aligned}
&\min f(x) = 0.6224x_1x_3x_4 + 1.7781x_2x_3^2 + 3.1661x_1^2x_4 \\
&\qquad + 19.84x_1^2x_3 \\
&\text{subject to}: \ g_1(x) = -x_1 + 0.0193x_3 \le 0 \\
&\qquad g_2(x) = -x_2 + 0.00954x_3 \le 0 \\
&\qquad g_3(x) = -\pi x_3^2x_4 - (4/3)\pi x_3^3 + 1296000 \le 0 \\
&\qquad g_4(x) = x_4 - 240 \le 0 \\
&\qquad 0 \le x_i \le 100, \quad i = 1, 2 \\
&\qquad 10 \le x_i \le 200, \quad i = 3, 4
\end{aligned}$$

A.3. Tension/compression spring design problem

$$\begin{aligned}
&\min f(x) = (x_3 + 2)x_2x_1^2 \\
&\text{subject to}: \ g_1(x) = -x_2^3x_3/(71785x_1^4) + 1 \le 0 \\
&\qquad g_2(x) = (4x_2^2 - x_1x_2)/(12566(x_2x_1^3 - x_1^4)) \\
&\qquad\qquad + 1/(5108x_1^2) - 1 \le 0 \\
&\qquad g_3(x) = -140.45x_1/(x_2^2x_3) + 1 \le 0 \\
&\qquad g_4(x) = (x_1 + x_2)/1.5 - 1 \le 0 \\
&\qquad 0.05 \le x_1 \le 2.00 \\
&\qquad 0.25 \le x_2 \le 1.30 \\
&\qquad 2.00 \le x_3 \le 15.00
\end{aligned}$$

A.4. Welded beam design problem

$$\begin{aligned}
&\min f(x) = 1.10471x_1^2x_2 + 0.04811x_3x_4(14 + x_2) \\
&\text{subject to}: g_1(x) = \tau(x) - \tau_{max} \le 0 \\
&\qquad g_2(x) = \sigma(x) - \sigma_{max} \le 0 \\
&\qquad g_3(x) = x_1 - x_4 \le 0 \\
&\qquad g_4(x) = 0.10471x_1^2 + 0.04811x_3x_4(14 + x_2) \\
&\qquad\qquad - 5 \le 0 \\
&\qquad g_5(x) = 0.125 - x_1 \le 0 \\
&\qquad g_6(x) = \delta(x) - \delta_{max} \le 0 \\
&\qquad g_7(x) = P - P_c(x) \le 0 \\
&\qquad 0.1 \le x_i \le 2, \quad i = 1, 4 \\
&\qquad 0.1 \le x_i \le 10, \quad i = 2, 3
\end{aligned}$$

where

$$\begin{aligned}
&\tau(x) = \sqrt{(\tau')^2 + 2\tau'\tau''\frac{x_2}{2R} + (\tau'')^2}, \ \tau' = \frac{P}{\sqrt{2}x_1x_2}, \ \tau'' = \frac{MR}{J} \\
&M = P(L + \frac{x_2}{2}), \quad R = \sqrt{\frac{x_2^2}{4} + (\frac{x_1 + x_3}{2})^2}, \\
&J = 2\{\sqrt{2}x_1x_2[\frac{x_2^2}{12} + (\frac{x_1 + x_3}{2})^2]\} \\
&\sigma(x) = \frac{6PL}{x_4x_3^2}, \delta(x) = \frac{4PL^3}{Ex_3^3x_4}, \\
&P_c(x) = \frac{4.013E\sqrt{(x_3^2x_4^6/36)}}{L^2} \times (1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}) \\
&P = 6000\,\mathrm{lb}, \quad L = 14\mathrm{in}, \quad E = 30 \times 10^6\,\mathrm{psi}, \\
&G = 12 \times 10^6\,\mathrm{psi} \\
&\tau_{max} = 13600\,\mathrm{psi}, \quad \sigma_{max} = 30000\,\mathrm{psi}, \quad \delta_{max} = 0.25\,\mathrm{in}
\end{aligned}$$

A.5. Speed reducer design problem

$$\begin{aligned}
&\min f(x) = 0.7854x_1x_2^2(3.3333x_3^2 + 14.9334x_3 - 43.0934) \\
&\qquad - 1.508x_1(x_6^2 + x_7^2) + 7.4777(x_6^3 + x_7^3) \\
&\qquad + 0.7854(x_4x_6^2 + x_5x_7^2) \\
&\text{subject to}: g_1(x) = \frac{27}{x_1x_2^2x_3} - 1 \le 0 \\
&\qquad g_2(x) = \frac{397.5}{x_1x_2^2x_3^2} - 1 \le 0 \\
&\qquad g_3(x) = \frac{1.93x_4^3}{x_2x_6^4x_3} - 1 \le 0 \\
&\qquad g_4(x) = \frac{1.93x_5^3}{x_2x_7^4x_3} - 1 \le 0 \\
&\qquad g_5(x) = \frac{[(745x_4/(x_2x_3))^2 + 16.9 \times 10^6]^{1/2}}{110x_6^3} \\
&\qquad\qquad -1 \le 0 \\
&\qquad g_6(x) = \frac{[(745x_5/(x_2x_3))^2 + 157.5 \times 10^6]^{1/2}}{85x_7^3} \\
&\qquad\qquad -1 \le 0 \\
&\qquad g_7(x) = \frac{x_2x_3}{40} - 1 \le 0 \\
&\qquad g_8(x) = \frac{5x_3}{x_1} - 1 \le 0
\end{aligned}$$

$$g_9(x) = \frac{x_1}{12x_2} - 1 \leq 0$$
$$g_{10}(x) = \frac{1.5x_6 + 1.9}{x_4} - 1 \leq 0$$
$$g_{11}(x) = \frac{1.1x_7 + 1.9}{x_5} - 1 \leq 0$$

where

$$2.6 \leq x_1 \leq 3.6,$$
$$0.7 \leq x_2 \leq 0.8,$$
$$17 \leq x_3 \leq 28,$$
$$7.3 \leq x_4, x_5 \leq 8.3,$$
$$2.9 \leq x_6 \leq 3.9,$$
$$5.0 \leq x_7 \leq 5.5$$

A.6. Gear train design problem

$$\min f(x) = \left(\frac{1}{6.391} - \frac{x_3 x_2}{x_1 x_4}\right)^2$$
$$\text{subject to} : 12 \leq x_1, x_2, x_3, x_4 \leq 60$$

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