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Robust Parameter Estimation for a Class of Nonlinear System With EM Algorithm

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ABSTRACT This paper is concerned with the robust parameter estimation for linear parameter varying (LPV) finite impulse response (FIR) model. The practical process data are typically polluted by outliers and conventional parameter estimation methods may fail to derive an unbiased estimate. In order to deal with outliers, the Laplace noise model is adopted and the robust system model for the described parameter estimation problem is established. The robust parameter estimation for LPV FIR model is formulated and solved in the EM algorithm scheme and the equations to estimate all the unknown parameters are derived. The efficacy of the proposed method is verified through a numerical simulation and a chemical unit.

INDEX TERMS Expectation-maximization algorithm, Laplace distribution, LPV FIR model, outliers, robust parameter estimation.

I. INTRODUCTION

The modern industrial processes typically perform complex production tasks and exhibit complicated process dynamics, resulting in a nonlinear system [1]–[5]. The nonlinear system identification has been a challenging theoretical and engineering problem over decades and many approaches have been developed. Among the developed nonlinear system identification approaches, the linear parameter varying (LPV) have become one of the most attractive approach due to its close connection with control analysis and synthesis of LPV system [6].

The LPV model has a parametrized linear structure with time-varying model parameters expressed as linear combinations of certain basis function with respect to scheduling variables. The identification of LPV time-delay system with parameter-varying time-delay and constant input time-delay was respectively considered in [7] and the unknown model parameters and time-delays were estimated. In [8], the identification of LPV system based on incomplete data set was handled and two algorithms to estimate the multiple-model LPV finite impulse response (FIR) and the multiple-model LPV output error (OE) model were developed. In [9], the LPV system identification with noisy scheduling variable data was handled. The scheduling dynamic was described by using a state space model and unknown parameters in LPV model and scheduling dynamic model were estimated. The above mentioned works are focused on the local LPV identification approach and the LPV model is approximated by combinations of multiple local models. Another attractive way is to estimate the parameter polynomial coefficients of LPV model directly. In [10], the LPV prediction error method was developed and the unknown parameters in the LPV Box-Jenkins model were estimated though the numerical optimization of the one-step ahead prediction error function. The refined instrumental variable method was proposed in [11] to identify the LPV Box-Jenkins model.

The industrial process data are commonly polluted by outliers which should be carefully dealed with in order to obtain consistent parameter estimates [12]. Conventional method is to detect the outliers in data and omit them, so that the problem is transformed into performing parameter estimation with missing data [13]. However, the selection of the criterion to detect the outliers is difficult especially for complex dynamic processes. The robust statistical models, such as the contaminated Gaussian model [14], Laplace model, and t-distribution model, are typically employed to deal with outliers in identification.

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In this paper, the robust identification of LPV system is considered and the LPV FIR model is adopted. The outlier is commonly encountered in system identification and it can be caused by malfunction of sensors, system fault, data recording or reading error, external system disturbance and so on. The outliers have posed significant difficulty on LPV FIR model identification and traditional LPV identification methods are not able to provide an unbiased parameter estimates. To handle outliers in LPV FIR model parameter estimation, the robust model is constructed with the Laplace distribution. Thanks to the mathematical decompostion of the Laplace distribution, the robust identification algorithm is developed. Due to the difficulties of optimizing the log likelihood function directly, the considered robust parameter estimation is formulated in EM algorithm scheme. Hence the main contributions of current work are:

1) The output abnormality is considered in the identification of LPV FIR system and a robust identification strategy is introduced.
2) The decomposition of the Laplace distribution is applied in the identification process and the unbaised parameter estimates are obtained.
3) The robust parameter estimation is realized with the EM algorithm and the maximum likelihood estimates (MLEs) are obtained.

The outline of the rest paper is: The problem description is given in Section 2 and the robust system model is established with Laplace distribution; The EM algorithm is firstly introduced in Section 3 and the mathematical derivations of the robust parameter estimation algorithm for LPV FIR model are given; The simulation verification of the proposed method is presented in Section 4 and the final section concludes the work.

II. PROBLEM DESCRIPTION

Consider a nonlinear system described by the following discrete linear parameter varying (LPV) finite impulse response (FIR) model:

$$y_t = G(z_t, q^{-1})u_t + \varepsilon_t, \tag{1}$$

where z_t, y_t, and u_t are measurable scheduling variable, output variable, and input variable, respectively; ε_t is the zero-mean measurement white noise; the $G(z_t, q^{-1})$ is the transfer function between u_t and y_t with FIR model structure

$$G(z_t, q^{-1}) = c_1(z_t)q^{-1} + c_2(z_t)q^{-2} + \cdots + c_n(z_t)q^{-n}, \tag{2}$$

and the model parameters $\{c_j(z_t)\}_{j=1,\cdots,n}$ are functions of time-varying scheduling variable z_t

$$c_j(z_t) = c_{j,0} + \sum_{m=1}^{M} c_{j,m}\chi_m(z_t), \quad j = 1, \cdots, n, \tag{3}$$

in which $\{\chi_j(z_t)\}_{j=1,\cdots,M}$ are the meromorphic functions of z_t and n and M are the orders of the function polynomial and the order of the LPV FIR model, respectively.

Conventionally, the Gaussian noise assumption is made and maximum likelihood (ML) method is employed to estimate the model parameters. However, the Gaussian noise model is sensitive to outliers which makes the derived parameter estimates biased. In order to deal with outliers, the Laplace noise model is developed

$$\begin{aligned}\varepsilon_t &\sim Laplace(\varepsilon_t|0, \gamma)\\ &= \frac{1}{2}\sqrt{\frac{2}{\gamma}} \exp\left\{-\sqrt{\frac{2}{\gamma}}|\varepsilon_t|\right\}.\end{aligned} \tag{4}$$

Based on the model (1), the output y_t follows

$$\begin{aligned}y_t &\sim Laplace(y_t|G(z_t, q^{-1})u_t, \gamma)\\ &= \frac{1}{2}\sqrt{\frac{2}{\gamma}} \exp\left\{-\sqrt{\frac{2}{\gamma}}|y_t - G(z_t, q^{-1})u_t|\right\}.\end{aligned} \tag{5}$$

According to the property of Laplace distribution, the above distribution can be written as

$$\begin{aligned}y_t &\sim Laplace(y_t|G(z_t, q^{-1})u_t, \gamma)\\ &= \int \mathcal{N}(y_t|G(z_t, q^{-1})u_t, \omega_t)p(\omega_t|\gamma)d\omega_t\end{aligned} \tag{6}$$

where

$$\begin{aligned}&\mathcal{N}(y_t|G(z_t, q^{-1})u_t, \omega_t)\\ &\quad = \frac{1}{\sqrt{2\pi\omega_t}} \exp\left(-\frac{(y_t - G(z_t, q^{-1})u_t)^2}{2\omega_t}\right),\end{aligned} \tag{7}$$

and

$$p(\omega_t|\gamma) = \frac{1}{\gamma} \exp\left(-\frac{\omega_t}{\gamma}\right). \tag{8}$$

Denote the scheduling variable data $\{z_t\}_{t=1,\cdots,L}$, the input data $\{u_t\}_{t=1,\cdots,L}$, and the output data $\{y_t\}_{t=1,\cdots,L}$ in training data set as $z_{1:L}$, $u_{1:L}$, and $y_{1:L}$, respectively. The robust parameter estimation for LPV FIR model is to estimate the unknown model parameters θ and scale parameter γ based on the training data set with output measurements polluted by the outliers.

III. MATHEMATICAL FORMULATION OF THE ROBUST PARAMETER ESTIMATION PROBLEMS IN EM ALGORITHM SCHEME

Intuitively, the ML method provides an alternative to solve the described robust parameter estimation problems. The log likelihood of training set data is

$$\begin{aligned}&\log(y_{1:L}, z_{1:L}, u_{1:L}|\theta, \gamma)\\ &= \sum_{t=1}^{L} \log\left(\frac{1}{2}\sqrt{\frac{2}{\gamma}} \exp\left\{-\sqrt{\frac{2}{\gamma}}|y_t - G(z_t, q^{-1})u_t|\right\}\right)\\ &= L\log\left(\frac{1}{2}\sqrt{\frac{2}{\gamma}}\right) - \sqrt{\frac{2}{\gamma}}\sum_{t=1}^{L}|y_t - G(z_t, q^{-1})u_t|,\end{aligned} \tag{9}$$

which is nonlinear with respects to model parameters θ and scale parameter γ. The direct optimization of this log likelihood function can be very difficult through numerical optimization algorithms. The robust parameter estimation problems can be simplified and solved in the EM algorithm scheme.

A. THE BRIEF INTRODUCTION TO EM ALGORITHM

Through introducing latent variables and/or missing variables, the EM algorithm is able to simplify the ML estimation problem and performs the optimization of log likelihood function by alternating expectation (E) step and maximization (M) step. Denote the observed data set and missing data set as $\mathcal{Y}$ and $\mathcal{Z}$, respectively. The complete data set is then $\{\mathcal{Z}, \mathcal{Y}\}$. The moments of functions with respects to latent variables and/or missing variables are calculated in E-step and the conditional expectation of complete data log likelihood function is maximized with respects to unknown parameters in M-step to derive the parameter estimates. The overflow of the EM algorithm is described as [15]

1) Parameter initialization and set iteration number $s = 1$.
2) E-step: Calculate the following Q-function based on current parameter estimates Ω^s

$$Q(\Omega|\Omega^s) = E_{\mathcal{Z}|\mathcal{Y},\Omega^s}\{\log p(\mathcal{Z}, \mathcal{Y}|\Omega)\}. \tag{10}$$

3) M-step: Maximize the Q-function to derive the new parameter estimates

$$\Omega^{s+1} = \arg\max_{\Omega} Q(\Omega|\Omega^s). \tag{11}$$

4) If convergence of the algorithm is met, stop; Else, set $s = s + 1$ and return to E-step.

B. MATHEMATICAL DERIVATIONS IN EM ALGORITHM SCHEME

Introducing the latent variable ω_t and denoting the data $\{\omega_t\}_{t=1,\cdots,L}$ as $\omega_{1:L}$. The missing data set is constructed as $\mathcal{Z} = \{\omega_{1:L}\}$ and the observed data set is built as $\mathcal{Y} = \{y_{1:L}, u_{1:L}, z_{1:L}\}$. The unknown model parameter vector is $\Omega = \{\theta, \gamma\}$.

1) E-STEP

The log likelihood of data $\{\mathcal{Z}, \mathcal{Y}\}$ is

$$\begin{aligned}\log p(\mathcal{Z}, \mathcal{Y}|\Omega) &= \log p(y_{1:L}, u_{1:L}, z_{1:L}, \omega_{1:L}|\Omega)\\ &= \log p(y_{1:L}|u_{1:L}, z_{1:L}, \omega_{1:L}, \Omega)\\ &\quad + \log p(\omega_{1:L}|u_{1:L}, z_{1:L}, \Omega)\\ &\quad + \log p(u_{1:L}, z_{1:L}|\Omega).\end{aligned} \tag{12}$$

The term $\log p(y_{1:L}|u_{1:L}, z_{1:L}, \omega_{1:L}, \Omega)$ is further expanded into

$$\begin{aligned}&\log p(y_{1:L}|u_{1:L}, z_{1:L}, \omega_{1:L}, \Omega)\\ &= \sum_{t=1}^{L} \log p(y_t|y_{1:t-1}, u_{1:L}, z_{1:L}, \omega_{1:L}, \Omega)\\ &= \sum_{t=1}^{L} \log p(y_t|z_t, \omega_t, u_{1:t-1}, \Omega).\end{aligned} \tag{13}$$

The derivation of above formula is based on the fact that the output y_t is only related with z_t, ω_t, input data $u_{1:t-1}$, and parameter Ω.

The term $\log p(\omega_{1:L}|u_{1:L}, z_{1:L}, \Omega)$ is further written as

$$\begin{aligned}&\log p(\omega_{1:L}|u_{1:L}, z_{1:L}, \Omega)\\ &= \sum_{t=1}^{L} \log p(\omega_t|\omega_{1:t-1}, u_{1:L}, z_{1:L}, \Omega)\\ &= \sum_{t=1}^{L} \log p(\omega_t|\gamma),\end{aligned} \tag{14}$$

according to the fact that the latent variable ω_t depends only on the parameter γ.

Since the $u_{1:L}$ and $z_{1:L}$ are recorded data, the term $\log p(u_{1:L}, z_{1:L}|\Omega)$ is a constant, denoted as C_1. Then the system log-likelihood function can be rewritten as

$$\begin{aligned}\log p(\mathcal{Z}, \mathcal{Y}|\Omega) = \sum_{t=1}^{L} \log p(y_t|z_t, \omega_t, u_{1:t-1}, \Omega)\\ + \sum_{t=1}^{L} \log p(\omega_t|\gamma) + C_1.\end{aligned} \tag{15}$$

As exhibited in Eq. (10), the Q-function is derived as

$$\begin{aligned}Q(\Omega|\Omega^s) &= E_{\mathcal{Z}|\mathcal{Y},\Omega^s}\{\log p(\mathcal{Z}, \mathcal{Y}|\Omega)\}\\ &= E_{\omega_{1:L}|\mathcal{Y},\Omega^s}\Big\{\sum_{t=1}^{L} \log p(y_t|z_t, \omega_t, u_{1:t-1}, \Omega)\\ &\quad + \sum_{t=1}^{L} \log p(\omega_t|\gamma) + C_1\Big\}\end{aligned} \tag{16}$$

Based on formulas (7) and (8), the above formula is rewritten as

$$\begin{aligned}Q(\Omega|\Omega^s) &= E_{\omega_{1:L}|\mathcal{Y},\Omega^s}\Big\{-\frac{1}{2}\log 2\pi - \frac{1}{2}\sum_{t=1}^{L}\log\omega_t\\ &\quad - \sum_{t=1}^{L}\frac{(y_t - G(z_t, q^{-1})u_t)^2}{2\omega_t} - L\log\gamma\\ &\quad - \frac{1}{\gamma}\sum_{t=1}^{L}\omega_t + C_1\Big\}\\ &= -\frac{1}{2}\log 2\pi - \frac{1}{2}\sum_{t=1}^{L}\int p(\omega_t|\mathcal{Y}, \Omega^s)\log\omega_t d\omega_t\\ &\quad - \sum_{t=1}^{L}\frac{(y_t - G(z_t, q^{-1})u_t)^2}{2}\int p(\omega_t|\mathcal{Y}, \Omega^s)\\ &\quad \times \frac{1}{\omega_t} d\omega_t - L\log\gamma - \frac{1}{\gamma}\sum_{t=1}^{L}\int p(\omega_t|\mathcal{Y}, \Omega^s)\\ &\quad \times \omega_t d\omega_t + C_1.\end{aligned} \tag{17}$$

The posterior distribution $p(\omega_t|\mathcal{Y}, \Omega^s)$ is

$$\begin{aligned} p(\omega_t|\mathcal{Y}, \Omega^s) &= p(\omega_t|y_{1:L}, u_{1:L}, z_{1:L}, \Omega^s) \\ &= \frac{p(y_t|u_{1:t-1}, z_t, \omega_t, \Omega^s)p(\omega_t|\Omega^s)}{\int p(y_t|u_{1:t-1}, z_t, \omega_t, \Omega^s)p(\omega_t|\Omega^s)d\omega_t} \\ &= \frac{\mathcal{N}(y_t|G^s(z_t, q^{-1})u_t, \omega_t)p(\omega_t|\gamma^s)}{Laplace(y_t|G^s(z_t, q^{-1})u_t, \gamma^s)} \\ &= GIG\left(\omega_t|\frac{1}{2}, G^s(z_t, q^{-1})u_t, \sqrt{\frac{1}{\gamma^s}}\right). \quad (18) \end{aligned}$$

Moreover, the integrals in Eq. (17) are calculated as

$$\int p(\omega_t|\mathcal{Y}, \Omega^s)\frac{1}{\omega_t}d\omega_t = \sqrt{\frac{2}{\gamma^s(y_t - G^s(z_t, q^{-1})u_t)^2}} \triangleq \left\langle\frac{1}{\omega_t}\right\rangle. \quad (19)$$

and

$$\int p(\omega_t|\mathcal{Y}, \Omega^s)\omega_t d\omega_t = \sqrt{\frac{\gamma^s(y_t - G^s(z_t, q^{-1})u_t)^2}{2}} \times \frac{\mathcal{B}_{3/2}\left(\sqrt{\frac{2(y_t - G^s(z_t, q^{-1})u_t)^2}{\gamma^s}}\right)}{\mathcal{B}_{1/2}\left(\sqrt{\frac{2(y_t - G^s(z_t, q^{-1})u_t)^2}{\gamma^s}}\right)} \triangleq \langle\omega_t\rangle, \quad (20)$$

where $\mathcal{B}_\alpha(\delta)$ is the second-kind modified Bessel function with order α evaluated at δ.

The Q-function can be finally written as

$$Q(\Omega^s|\Omega) = J(\theta) + J(\gamma) + C_2 \quad (21)$$

where

$$\begin{aligned} J(\theta) &= -\sum_{t=1}^{L}\frac{(y_t - G^s(z_t, q^{-1})u_t)^2}{2}\left\langle\frac{1}{\omega_t}\right\rangle, \\ J(\gamma) &= -L\log\gamma - \frac{1}{\gamma}\sum_{t=1}^{L}\langle\omega_t\rangle, \quad (22) \end{aligned}$$

and the terms that not related with unknown parameters

$$C_2 = -\frac{L}{2}\log 2\pi - \frac{1}{2}\sum_{t=1}^{L}\langle\log\omega_t\rangle + C_1. \quad (23)$$

2) M-STEP

So as to calculate the parameter γ, the derivative is taken over $J(\gamma)$ with respect to γ and equating it to zero

$$-L\frac{1}{\gamma} + \frac{1}{\gamma^2}\sum_{t=1}^{L}\langle\omega_t\rangle = 0.$$

It is easy to derive that

$$\gamma = \frac{1}{L}\sum_{t=1}^{L}\langle\omega_t\rangle. \quad (24)$$

Algorithm 1 Robust Parameter Estimation Algorithm for LPV FIR Model

Input: The training data set $C_{obs} = \{y_{1:L}, u_{1:L}, z_{1:L}\}$
Output: The optimal estimates $\Omega^* = \{\theta^*, \gamma^*\}$

1: Perform parameter initialization Ω^1 and let $s = 1$;
2: **repeat**
3: Calculate the expectations $\langle 1/\omega_t\rangle$ and $\langle\omega_t\rangle$ according to formulas (19) and (20), respectively;
4: Update the parameter estimates θ and γ according to formulas (28) and (24), respectively;
5: Set $s = s + 1$;
6: **until** convergence ($\left\|\frac{\theta^{s+1} - \theta^s}{\theta^s}\right\| < 10^{-3}$);
7: Once the optimal estimates are obtained, the real output data estimates can be calcualted by simulating the estimated model $\hat{y}_t = \phi_t^T\theta^*$.

Based on formulas (1) to (3), the LPV FIR model can be rewritten into the linear regression form

$$y_t = \phi_t^T\theta + \varepsilon_t, \quad (25)$$

where

$$\begin{aligned} \phi_t = [&u_{t-1}\ \chi_1(z_t)u_{t-1}\ \cdots\ \chi_M(z_t)u_{t-1}\ u_{t-2} \\ &\chi_1(z_t)u_{t-2}\cdots\chi_M(z_t)u_{t-2}\ \cdots\ u_{t-n}\ \cdots\chi_M(z_t)u_{t-n}]^T, \end{aligned}$$

and

$$\theta = [c_{1,0}\ c_{1,1}\cdots c_{1,M}\ c_{2,0}\ c_{2,1}\ \cdots\ c_{2,M}\cdots c_{n,0}\cdots\ c_{n,M}]^T. \quad (26)$$

The cost function $J(\theta)$ is then transformed into

$$J(\theta) = -\sum_{t=1}^{L}\frac{(y_t - \phi_t^T\theta)^2}{2}\left\langle\frac{1}{\omega_t}\right\rangle. \quad (27)$$

To calculate the parameter θ, the derivative is taken over $J(\theta)$ with respect to θ and setting it to zero

$$-\frac{1}{2}\sum_{t=1}^{L}\left\langle\frac{1}{\omega_t}\right\rangle(2\phi_t\phi_t^T\theta - 2\phi_t y_t) = 0.$$

The formula to estimate the θ is then derived as

$$\theta = \left(\sum_{t=1}^{L}\left\langle\frac{1}{\omega_t}\right\rangle\phi_t\phi_t^T\right)^{-1}\sum_{t=1}^{L}\left\langle\frac{1}{\omega_t}\right\rangle\phi_t y_t. \quad (28)$$

The proposed robust parameter estimation algorithm for LPV FIR model is summarized in Algorithm 1.

IV. VERIFICATION

A. NUMERICAL VERIFICATION

Consider a nonlinear system described by the following LPV FIR model

$$y_t = c_1(z_t)u_{t-1} + c_2(z_t)u_{t-2} + c_3(z_t)u_{t-3} + \varepsilon_t, \quad (29)$$

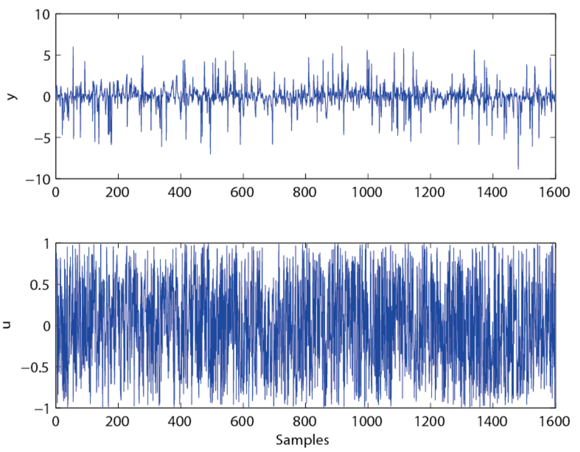


FIGURE 1. The generated input and output data.

where

$$c_1(z_t) = 1.5328 - 1.3438z_t + 0.1562z_t^2$$
$$c_2(z_t) = 1.3813 - 1.8750z_t + 0.6250z_t^2$$
$$c_3(z_t) = 1.1969 - 2.0625z_t + 0.9375z_t^2. \quad (30)$$

The input signal is chosen as the zero-mean Gaussian noise $u_t = 1 + 2\mathcal{N}(0, 1)$ and the scheduling variable z_t varies as a periodic signal $z_t = 0.5 + 0.5sin(0.15\pi t)$. The uniformly distributed interferences are set as the outliers. 10% outliers are added to the output data and the outliers are generated following the uniform distribution in $[-6, 6]$. By using the software Matlab, the input and output data for parameter estimation are shown in Fig. 1.

The proposed Algorithm 1 is applied to estimate unknown parameters of the LPV FIR model based on the noisy data with outliers. The parameter estimation accuracy (PEA) $(1 - ||\theta - \theta^*||_2/||\theta||_2) * 100\%$ is used to evaluate the performance of the proposed algorithm and the value is 97.615%. It can be seen from this result that the estimated parameters converge to the true parameters.

In order to illustrate the advantages of the proposed algorithm with respect to the existing LPV identification methods, the prediction error method (PEM) for LPV model in [10] and recursive least squares (RLS) method for LPV model in [16], one comparison is performed. The latter two methods are also applied to the data set in Fig. 1 and the estimated parameters of these three methods are given in Table 1. The parameter estimation accuracy of these three methods are 97.6150%, 61.2853%, and 50.2946%, respectively. As shown in the results, the performance of conventional PEM-LPV method and RLS-LPV method degrade greatly; The proposed method suppresses the negative effects of outliers and provides satisfactory parameter estimates.

The Monte Carlo (MC) simulation is then utilized to throughoutly verify the performance of the proposed algorithm. The MC simulations, each with 100 different noise sequences, are conducted under the signal-to-noise ratio (SNR), 5dB, 10dB, 15dB, and 20dB, and outlier ratios, 5%, 10%, and 20%. The mean and standard deviation of estimated parameter in each MC simulation are calculated and part of the results are shown in Fig. 2. In this figure, the red $*$ and red bar denote the mean value and standard derivation of parameter estimates, respectively. Suppose $\{\hat{\theta}_i\}_{i=1,2,...,100}$ represent 100 parameter estimates obtained in each Monte Caro simulation, then the resulted mean and standard deviation are computed as

TABLE 1. The estimated parameters of the proposed method, the PEM method, and the RLS method.

True	Proposed	PEM-LPV	RLS-LPV
1.5328	1.5405	1.5340	1.8288
-1.3438	-1.3745	-1.3419	-2.8998
0.1562	0.1833	0.1808	0.7545
1.3813	1.3812	1.2713	1.4780
-1.8750	-1.9310	-1.3530	-1.8266
0.6250	0.6901	0.1412	0.9734
1.1969	1.1990	1.1523	1.6944
-2.0625	-2.0796	-3.0037	-2.9268
0.9375	0.9340	1.9743	1.2485
PEA	97.6150%	61.2853%	50.2946%

TABLE 2. The bias and variance norms of estimated parameters in MC simulations.

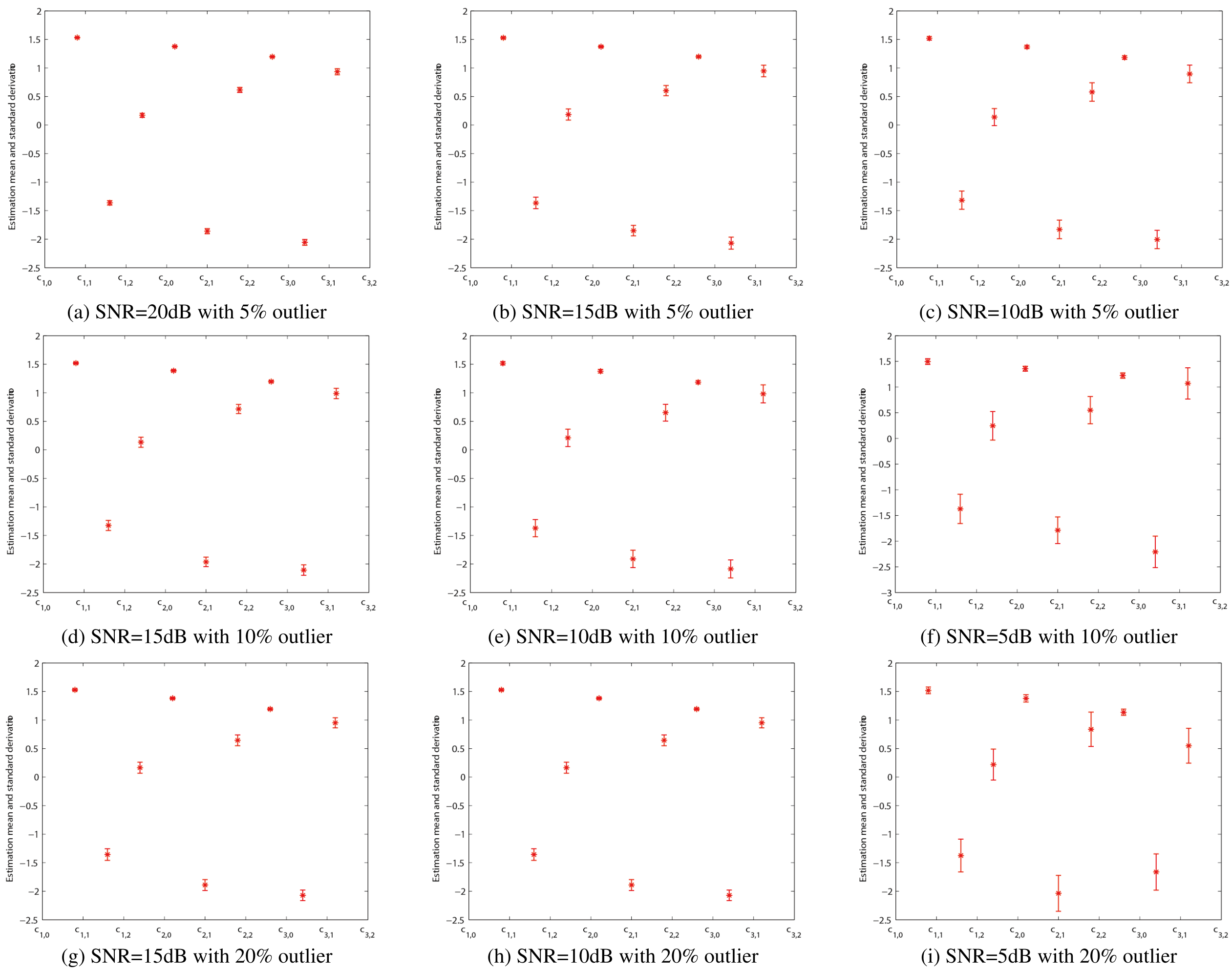
	10%		20%	
	BN	VN	BN	VN
20dB	0.0374	0.0059	0.0242	0.0082
15dB	0.1463	0.0187	0.0343	0.0223
10dB	0.0913	0.0574	0.2602	0.0668
5dB	0.2526	0.1994	0.6251	0.2217

$$\theta_{mean} = \frac{\sum_{i=1}^{100} \hat{\theta}_i}{100},$$
$$\theta_{std} = \sqrt{\frac{\sum_{i=1}^{100} (\hat{\theta}_i - \theta_{mean})^2}{100}}. \quad (31)$$

The bias norm (BN) $||\theta - E(\theta^*)||_2$ and variance norm (VN) $||E(\theta^* - E(\theta^*))||_2$ are two performance indexes used to evaluate the accuracy of estimated parameters in MC simulations, where the notation $\|\cdot\|_2$ represent the second norm of a certain vector. The calculated performance indexes are given in Table 2. It can be seen from this Table, the bias and variance norms are close to zero, which indicates that the proposed parameter estimation algorithm can not only handle the noise but also deal with the outliers and estimated parameters converge to the true parameter.

B. THE CHEMICAL PROCESS EXPERIMENTAL VERIFICATION

In chemical and biological engineering, a fundamental reactor is the continuous flow stirred tank reactor (CSTR). The feed materials are continuously pumped into the reactor and perfectly mixed in this column. The chemical reaction takes place and the products are transfered to the subsequent production unit. The temperature of the reactor is controlled by manipulating the flow rate of the coolant. The first-principal



(a) SNR=20dB with 5% outlier (b) SNR=15dB with 5% outlier (c) SNR=10dB with 5% outlier
(d) SNR=15dB with 10% outlier (e) SNR=10dB with 10% outlier (f) SNR=5dB with 10% outlier
(g) SNR=15dB with 20% outlier (h) SNR=10dB with 20% outlier (i) SNR=5dB with 20% outlier

FIGURE 2. **The mean and standard deviation of parameter estimates from MC simulations under different SNRs and outlier ratios.**

model of the CSTR is given as [10]

$$\begin{aligned}\frac{dC_A(t)}{dt} &= \frac{q(t)}{V}(C_{A0}(t) - C_A(t)) - k_0 C_A(t)\exp(\frac{-E}{RT(t)})\\ \frac{dT(t)}{dt} &= -\frac{(\Delta H)k_0 C_A(t)}{\rho C_p}\exp\left(\frac{-E}{RT(t)}\right) + \frac{q(t)}{V}(T_0(t)\\ &\quad - T(t)) + \frac{\rho_c C_{pc}}{\rho C_p V}q_c(t)\left\{1 - exp\left(\frac{-hA}{q_c(t)\rho C_p}\right)\right\}\\ &\quad \times (T_{c_0}(t) - T(t)),\end{aligned} \tag{32}$$

where the definitions and steady values of process variables are presented in [10].

The concentration of reactant A, $C_A(t)$, and the flow rate of feed material, $q(t)$, and the flow rate of coolant, are chosen as output variable, input variable, and scheduling variable, respectively. The scheduling variable signal is set to $2sin(0.02\pi t) + 100$ and the input signal is set to a random binary sequence. The practical process data of CSTR are recorded and shown in Fig. 3. The output noise and 10% outliers are added to the output data. The proposed method is used to identify the LPV FIR model with orders $n = 10$ and $M = 2$ for the CSTR. The simulated output $\hat{y}$ of the identified model is compared with the recorded real output data y and the result is shown in Fig. 4. The performance

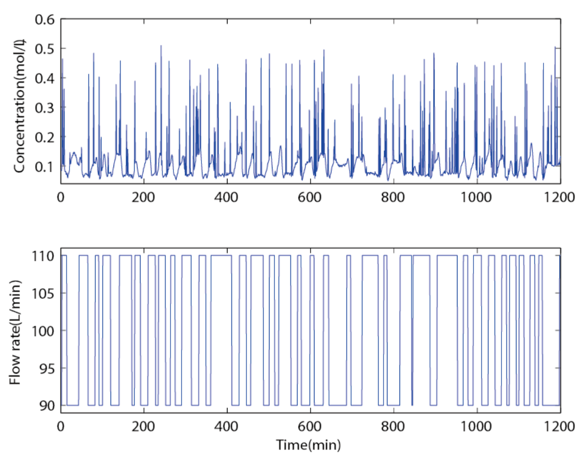


FIGURE 3. **The input and output data of the CSTR system used for identification.**

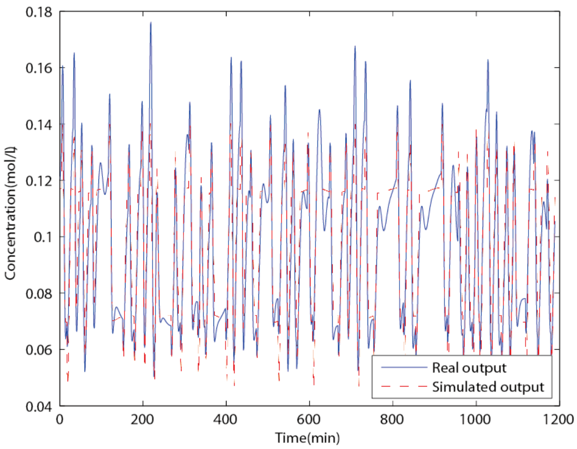


FIGURE 4. The comparison of the real output and model simulated output for identification data set.

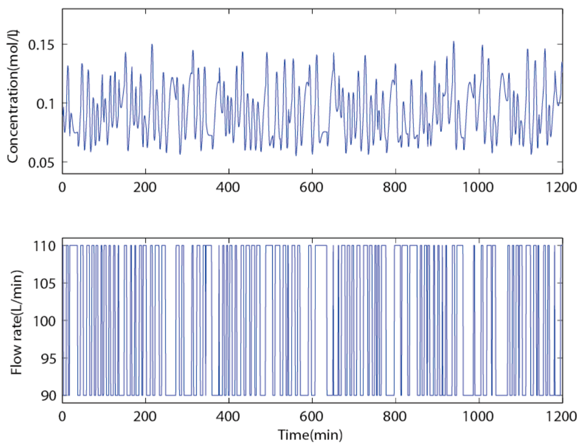


FIGURE 5. The input and output data of the CSTR system used for model validation.

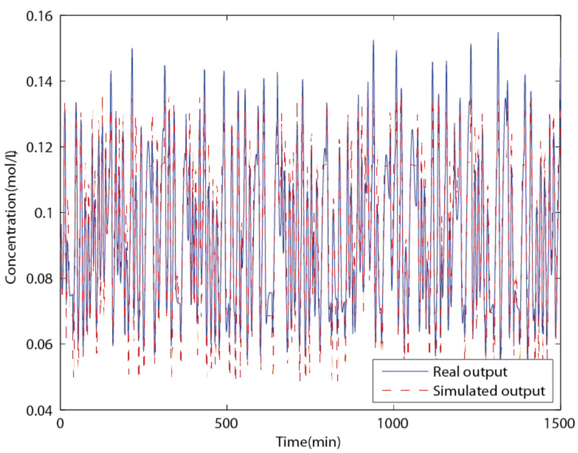


FIGURE 6. The comparison of the real output and model simulated output for validation data set.

index to evaluate the fitting accuracy, defined as $(1 - var(y - \hat{y})/var(y))$, is calculated and the result is 85.69%. The generalization ability of the identified model is verified through a validation data set shown in Fig. 5. The validation result is presented in Fig. 6 and the fitting accuracy between the simulated model output and real CSTR output is 86.94%. The results show that the proposed method can diminish the influence of noise and outliers imposed on model identification and provides a satisfactory process model for the CSTR.

V. CONCLUSION

This paper considers the parameter estimation for LPV FIR model with output data polluted by outliers. The Laplace noise model is adopted to deal with the outliers and the probability model to describe the LPV FIR process is established. In order to avoid solving the complex log likelihood function optimization problem directly, the considered parameter estimation problem is formulated in the EM algorithm framework. The iterative equations to estimate the unknown model parameters and scale parameter are derived and the outliers in data are dealed with adaptively. But as the identification data quality tends to be worse, the performance of the proposed algorithm degrades. Hence more robust identification strategy should be explored in the further studies.

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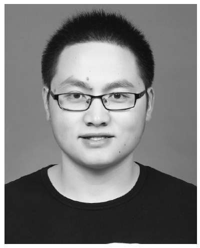

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