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New Results of Fuzzy Sampled-Data Control for Nonlinear Time-Delay Systems

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ABSTRACT In the manuscript, synthesis and stability analysis for fuzzy H_2/H_∞ sampled-data control of dynamical systems with delay behavior are studied by employing the input delay method. By using Lyapunov theory, a new H_2/H_∞ standard is derived and the fuzzy sampled-data controller is proposed to ensure H_∞ performance and H_2 performance simultaneously. Meanwhile, the control design is verified by two practical examples. Furthermore, experimental results also indicate that the H_2/H_∞ sampled-data control has a better performance.

INDEX TERMS Dynamical systems, delay behavior, fuzzy sampled-data control, $\mathrm{H}_2/\mathrm{H}_\infty$ control.

I. INTRODUCTION

Recently, fuzzy control method has always been a hot topic in the field of control and can be widely employed. Because of its logical reasoning abilities and superior approximation performance, T-S fuzzy model was adopted to capture the nonlinear dynamical system in [1]. Subsequently, fuzzy controllers were also designed to guarantee the system stability in [2]–[6]. Generally, in the field of control, the role of computer is to control the controlled plants as a digital controller. When the continuous-time measurement signal is processed by the digital computer, the measurement signal is sampled and quantized firstly. Next, the discrete-time signal is generated by a zero-order holder, and then transmitted back to the continuous-time control input signal.

By now, there have been a lot of reports on the synthesis and analysis of sampled-data control with fuzzy form [7]–[37]. Among these existing literatures, [7], [9], [12], [21], [23], [24], [28]–[31] had analyzed the stability of the system, [18], [19], [23], [32], [33] had carried on the stabilization, [8], [10], [13], [14], [16], [25], [27] had discussed the H_∞ control, [15], [21] had studied the H_2 GC control, [11], [17], [26] had discussed the tracking control, and [20], [22] had considered the filtering. Meanwhile, delay phenomena often occur in some engineering systems, such as manual control, aircraft stability, ship stability and nuclear reactor. Moreover, in practical systems, the phenomenon of time delay leads to oscillations and instability frequently. In [9], [10], [14], [16], [34], [35], some fuzzy sampled-data control schemes were designed for nonlinear dynamical systems with delay behavior. The control performance design was described clearly in [9]. In [10], the issue of reliable non-uniform H_∞ sampling fuzzy control was analyzed, in which the generalized model transformation and input delay method were employed. Meanwhile, H_∞ control was also investigated by utilizing Leibniz-Newton formula and Lyapunov-Krasovskii functional [14]. Furthermore, a distinctive delay-dependent stabilization standard was proposed in [16]. For T-S fuzzy systems with time delay and parametric uncertainties, a robust guaranteed cost sampled-data fuzzy control design method was proposed in [34]. In [35], the stabilization of a T-S fuzzy system with time delay was explored, in which the designed fuzzy controller of sampled-data contained both the current and delayed state information. In addition, if the H_2 performance and H_∞ performance can be optimized simultaneously, the control system will show a better performance. Nevertheless, fuzzy H_2/H_∞ sampled-data control issue of dynamical systems with delay behavior has not been explored yet.

H_2 performance focuses on the system state and the system input. H_∞ performance is mainly reflected in the gain between the system state and the external disturbance.

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However, H_2 control or H_∞ control can only achieve a single control performance. In the existing literatures, fuzzy sampled-data H_2 control and fuzzy sampled-data H_∞ control have been explored respectively. But, the dimension of LMIs is very large, which can bring about the conservatism of the results. Therefore, for some engineering systems, how to design a more convenient and easier to implement sampled-data controller with larger sampling interval is still an unsolved problem.

Based on the above researches, fuzzy H_2/H_∞ sampled-data control issue is proposed for dynamical systems with delay behavior. A new H_2/H_∞ criterion is constructed and formulated as the linear matrix inequalities (LMIs) form. The feasibility of sampled-data control scheme is tested by simulation results. Moreover, by considering both H_∞ performance and H_2 performance, the global optimization algorithm is achieved and a fuzzy H_2/H_∞ sampled-data control scheme is firstly investigated for dynamical systems with delay behavior. The presented control algorithm is less conservative, which reduces the dimension of the LMIs and shortens the implementation time.

The main innovation points of this paper are embodied in several aspects.

(I) The proposed control algorithm is less conservative, where the dimension of the LMIs is simplified, which adds the existence of the feedback gains, reduces the computational complexity and lowers the implementation time.

(II) The proposed method achieves a better performance, where fuzzy sampled-data controller has a larger sampling interval and shows a better attenuated level and faster state responses.

Notations: Throughout the manuscript, the notation $W > 0(< 0)$ means the positivity (negativity) W^T represent the transpose of a matrix W; and * describes the symmetric term of a symmetric matrix. It is assumed that the matrices are compatible.

II. PROBLEM FORMULATION

Analyze the T–S fuzzy model capturing the dynamical system with delay behavior:

Rule i : IF $\sigma_1(t)$ is N_{i1} and $\cdots$ and $\sigma_p(t)$ is N_{ip}, THEN

$$\dot{x}(t) = \hat{A}_i x(t) + \hat{A}_{id} x(t-d) + \hat{B}_i u(t) + \omega(t), \quad i = 1, \cdots, M \tag{1}$$

where the state, the input, and the disturbance are expressed by $x(t)$, $u(t)$, and $\omega(t)$, respectively; the matrices $\hat{A}_i$, $\hat{B}_i$, and $\hat{A}_{id}$ are time-invariant; and time delay d is a constant.

The global system is deduced as

$$\dot{x}(t)=\sum_{i=1}^{M} \xi_i(\sigma(t)) \left[\hat{A}_i x(t)+\hat{A}_{id} x(t-d)+\hat{B}_i u(t)+\omega(t)\right] \tag{2}$$

where $\xi_i(\sigma(t)) \geq 0$, $i = 1, 2, \ldots, M$, and $\sum_{i=1}^{M} \xi_i(\sigma(t)) = 1$.

The j th rule of fuzzy controller is as follows:

Rule j : IF $\sigma_1(t_k)$ is N_{j1} and $\cdots$ and $\sigma_p(t_k)$ is N_{jp},THEN

$$u(t) = G_j x(t_k), \quad t_k \leq t < t_{k+1}, \ j = 1, 2, \ldots, M$$

where G_j represents the gain, and $0 < t_{k+1} - t_k = h_k \leq h$.

Therefore, the global controller is shown as

$$u(t) = \sum_{j=1}^{M} \xi_j(\sigma(t_k)) G_j x(t_k). \tag{3}$$

Let $\tau(t) = t - t_k$, $t_k \leq t < t_{k+1}$. Obviously, the derivative $\dot{\tau}(t) = 1$ for $t \neq t_k$. By employing the input delay method, (3) is rewritten as

$$u(t) = \sum_{j=1}^{M} \xi_j(\sigma(t_k)) G_j x(t-\tau(t)) \tag{4}$$

By considering the system (2) and the controller (4), the closed-loop system is obtained as

$$\dot{x}(t) = \sum_{i=1}^{M}\sum_{j=1}^{M} \xi_i(\sigma(t))\xi_j(\sigma(t_k)) \Big[\hat{A}_i x(t) + \hat{A}_{id} x(t-d) + \hat{B}_i G_j x(t-\tau(t)) + \omega(t)\Big]. \tag{5}$$

where two delays $x(t-d)$ and $x(t-\tau(t))$ are independent.

Let us take into account the following H_∞ control performance

$$\int_0^\infty x^T(t) S_1 x(t) dt \leq \varepsilon^2 \int_0^\infty \omega^T(t)\omega(t) dt \tag{6}$$

where ε denotes a attenuation level and the matrix S_1 is positive-definite.

The H_2 control performance is as follows:

$$J = \int_0^\infty (x^T(t) S_2 x(t) + u^T(t) Q u(t))\, dt \tag{7}$$

where S_2 and Q are positive-definite matrices.

By analyzing a desired H_∞ disturbance rejection constraint in (6), the suboptimal H_2 control performance (7) is obtained. Our control objective aims to explore a fuzzy sampled-data controller to ensure the H_2/H_∞ performance for the closed-loop system (5).

III. FUZZY H2/H$_\infty$ SAMPLED-DATA CONTROL

Based on the LMIs, a fuzzy sampled-data H_2/H_∞ control criterion is proposed as follows.

Theorem 1: Considering the closed-loop system (5), for given matrices $S_1 > 0$, $S_2 > 0$, and $Q > 0$, scalars $\varepsilon > 0$, $h > 0$, and $\lambda > 0$, the H_2/H_∞ control performance in (6) and (7) is satisfied, simultaneously, if there exist matrices $\overline{Q_1} > 0$ and $\overline{Q_2} > 0$ satisfying the LMIs (8) and (9)

$$\mathbb{R}_{ij} = \begin{bmatrix} \mathbb{R}_{ij11} & \mathbb{R}_{ij12} & \mathbb{R}_{ij13} & \mathbb{R}_{ij14} & \mathbb{R}_{ij15} \\ * & \mathbb{R}_{ij22} & 0 & 0 & 0 \\ * & * & \mathbb{R}_{ij33} & \mathbb{R}_{ij34} & 0 \\ * & * & * & \mathbb{R}_{ij44} & \mathbb{R}_{ij45} \\ * & * & * & * & \mathbb{R}_{ij55} \end{bmatrix} < 0$$
$$i, j = 1, 2, \cdots, M \tag{8}$$

$$\mathbb{Z}_{ij} = \begin{bmatrix} \mathbb{Z}_{ij11} & \mathbb{Z}_{ij12} & \mathbb{Z}_{ij13} & 0 & \mathbb{Z}_{ij15} & \mathbb{Z}_{ij16} \\ * & \mathbb{Z}_{ij22} & 0 & 0 & 0 & 0 \\ * & * & \mathbb{Z}_{ij33} & \mathbb{Z}_{ij34} & \mathbb{Z}_{ij35} & 0 \\ * & * & * & \mathbb{Z}_{ij44} & 0 & 0 \\ * & * & * & * & \mathbb{Z}_{ij55} & \mathbb{Z}_{ij56} \\ * & * & * & * & * & \mathbb{Z}_{ij66} \end{bmatrix} < 0$$
$$i, j = 1, 2, \cdots, M \tag{9}$$

where

$$\begin{aligned}
\mathbb{R}_{ij11} &= \hat{A}_i\overline{W} + \overline{W}\hat{A}_i^T + \bar{Q}_1 - \overline{Q}_2 + \frac{1}{\varepsilon^2}I, \quad \mathbb{R}_{ij12} = \overline{W} \\
\mathbb{R}_{ij13} &= \hat{B}_i\bar{G}_j + \overline{Q}_2, \quad \mathbb{R}_{ij14} = \lambda\overline{W}\hat{A}_i^T, \ \mathbb{R}_{ij15} = \hat{A}_{id}\overline{W} \\
\mathbb{R}_{ij22} &= -\frac{1}{1+\lambda}S_1^{-1}, \quad \mathbb{R}_{ij33} = -\overline{Q}_2, \ \mathbb{R}_{ij34} = \lambda\overline{G}_j^T\hat{B}_i^T \\
\mathbb{R}_{ij44} &= -2\lambda\overline{W} + h^2\overline{Q}_2 + \frac{\lambda}{\varepsilon^2}I, \quad \mathbb{R}_{ij45} = \lambda\hat{A}_{id}\overline{W} \\
\mathbb{R}_{ij55} &= -\overline{Q}_1, \quad \mathbb{Z}_{ij11} = \hat{A}_i\bar{W} + \bar{W}\hat{A}_i^T + \overline{Q}_1 + \overline{Q}_2 \\
\mathbb{Z}_{ij11} &= \hat{A}_i\bar{W} + \bar{W}\hat{A}_i^T + \overline{Q}_1 + \overline{Q}_2, \quad \mathbb{Z}_{ij12} = \bar{W} \\
\mathbb{Z}_{ij13} &= \hat{B}_i\bar{G}_j + \overline{Q}_2, \quad \mathbb{Z}_{ij15} = \lambda\overline{W}\hat{A}_i^T, \ \mathbb{Z}_{ij16} = \hat{A}_{id}\overline{W} \\
\mathbb{Z}_{ij22} &= -S_2^{-1}, \quad \mathbb{Z}_{ij33} = -\bar{Q}_2, \ \mathbb{Z}_{ij34} = \overline{G}_j^T \\
\mathbb{Z}_{ij35} &= \lambda\overline{G}_j^T\hat{B}_i^T, \quad \mathbb{Z}_{ij44} = -Q^{-1}, \ \mathbb{Z}_{ij55} = -2\lambda\overline{W} + h^2\overline{Q_2} \\
\mathbb{Z}_{ij56} &= \lambda\hat{A}_{id}\overline{W}, \quad \mathbb{Z}_{ij66} = -\bar{Q}_1.
\end{aligned}$$

And, the control gains in sampled-data controller are $G_j = \bar{G}_j\overline{W}^{-1}, j = 1, 2, \cdots, M$.

Proof: Selecting the Lyapunov-Krasovskii functional as the candidate

$$V(x_t) = V_1(x) + V_2(x_t) + V_3(x_t) \tag{10}$$

where

$$\begin{aligned}
V_1(x) &= x^T(t)Wx(t) \\
V_2(x_t) &= V_2(x,t) = \int_{t-d}^{t} x^T(s)Q_1x(s)ds \\
V_3(x_t) &= V_3(x,t) = h\int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)Q_2\dot{x}(s)dsd\theta
\end{aligned}$$

with $W > 0$, $Q_1 > 0$, and $Q_2 > 0$.

The derivative of V is as follows:

$$\begin{aligned}
\dot{V}_1(x) &= \dot{x}^T(t)Wx(t) + x^T(t)W\dot{x}(t) \\
&= \sum_{i=1}^{M}\sum_{j=1}^{M}\xi_i(\sigma(t))\xi_j(\sigma(t_k))[x^T(t)\hat{A}_i^TWx(t) \\
&\quad + x^T(t-d)\hat{A}_{id}^TWx(t) \\
&\quad + x^T(t-\tau(t))G_j^T\hat{B}_i^TWx(t) + \omega^T(t)Wx(t) \\
&\quad + x^T(t)W\hat{A}_ix(t) + x^T(t)W\hat{A}_{id}x(t-d) \\
&\quad + x^T(t)W\hat{B}_iG_jx(t-\tau(t)) + x^T(t)W\omega(t)] \\
&= \sum_{i=1}^{M}\sum_{j=1}^{M}\xi_i(\sigma(t))\xi_j(\sigma(t_k))[x^T(t)\hat{A}_i^TWx(t) \\
&\quad + x^T(t-d)\hat{A}_{id}^TWx(t) \\
&\quad + x^T(t-\tau(t))G_j^T\hat{B}_i^TWx(t) + x^T(t)W\hat{A}_ix(t) \\
&\quad + x^T(t)W\hat{A}_{id}x(t-d) + x^T(t)W\hat{B}_iG_jx(t-\tau(t)) \\
&\quad + \frac{1}{\varepsilon^2}x^T(t)WWx(t) + \varepsilon^2\omega^T(t)\omega(t) \\
&\quad - (\frac{1}{\varepsilon}Wx(t) - \varepsilon\omega(t))^T(\frac{1}{\varepsilon}Wx(t) - \varepsilon\omega(t))] \\
&\le \sum_{i=1}^{M}\sum_{j=1}^{M}\xi_i(\sigma(t))\xi_j(\sigma(t_k))[x^T(t)\hat{A}_i^TWx(t) \\
&\quad + x^T(t-d)\hat{A}_{id}^TWx(t) \\
&\quad + x^T(t-\tau(t))G_j^T\hat{B}_i^TWx(t) + x^T(t)W\hat{A}_ix(t) \\
&\quad + x^T(t)W\hat{A}_{id}x(t-d) + x^T(t)W\hat{B}_iG_jx(t-\tau(t)) \\
&\quad + \frac{1}{\varepsilon^2}x^T(t)WWx(t) + \varepsilon^2\omega^T(t)\omega(t)].
\end{aligned} \tag{11}$$

$$\dot{V}_2(x_t) = x^T(t)Q_1x(t) - x^T(t-d)Q_1x(t-d). \tag{12}$$

By applying Lemma in [38], we can obtain

$$\begin{aligned}
&-h\int_{t-h}^{t}\dot{x}^T(s)Q_2\dot{x}(s)ds \\
&\le -\tau(t)\int_{t-\tau(t)}^{t}\dot{x}^T(s)Q_2\dot{x}(s)ds \\
&\le -\left(\int_{t-\tau(t)}^{t}\dot{x}(s)ds\right)^T Q_2\left(\int_{t-\tau(t)}^{t}\dot{x}(s)ds\right).
\end{aligned} \tag{13}$$

Leibniz-Newton formula is

$$\int_{t-h}^{t}\dot{x}(s)ds = x(t) - x(t-h). \tag{14}$$

By employing (13) and Leibniz-Newton formula, we get

$$\begin{aligned}
\dot{V}_3(x_t) &= h^2\dot{x}^T(t)Q_2\dot{x}(t) - h\int_{t-h}^{t}\dot{x}^T(s)Q_2\dot{x}(s)ds \\
&\le h^2\dot{x}^T(t)Q_2\dot{x}(t) - (x(t) \\
&\quad - x(t-\tau(t)))^TQ_2(x(t) - x(t-\tau(t))) \\
&= h^2\dot{x}^T(t)Q_2\dot{x}(t) - x^T(t)Q_2x(t) \\
&\quad + x^T(t-\tau(t))Q_2x(t) + x^T(t)Q_2x(t-\tau(t)) \\
&\quad - x^T(t-\tau(t))Q_2x(t-\tau(t)).
\end{aligned} \tag{15}$$

It should be clarified that for a given $\mu > 0$, we have

$$\begin{aligned}
0 &= -2\lambda\dot{x}^T(t)W\dot{x}(t) \\
&\quad + \lambda\dot{x}^T(t)W\left\{\sum_{i=1}^{M}\sum_{j=1}^{M}\xi_i(\sigma(t))\xi_j(\sigma(t_k))\left[\hat{A}_ix(t)\right.\right. \\
&\quad \left.\left. + \hat{A}_{id}x(t-d) + \hat{B}_iG_jx(t-\tau(t)) + \omega(t)\right]\right\} \\
&\quad + \lambda\{\sum_{i=1}^{M}\sum_{j=1}^{M}\xi_i(\sigma(t))\xi_j(\sigma(t_k))[\hat{A}_ix(t) + \hat{A}_{id}x(t-d) \\
&\quad + \hat{B}_iG_jx(t-\tau(t)) + \omega(t)]\}^TW\dot{x}(t) \\
&= -2\lambda\dot{x}^T(t)W\dot{x}(t) + \sum_{i=1}^{M}\sum_{j=1}^{M}\xi_i(\sigma(t))\xi_j(\sigma(t_k))
\end{aligned}$$

$$\begin{aligned}&\times \Big[\lambda\dot{x}^T(t)W\hat{A}_i x(t)+\lambda\dot{x}^T(t)W\hat{A}_{id}x(t-d)\\&+\ \lambda\dot{x}^T(t)W\hat{B}_iG_jx(t-\tau(t))\\&+\ \lambda\dot{x}^T(t)W\omega(t)+\lambda x^T(t)\hat{A}_i^TW\dot{x}(t)\\&+\lambda x^T(t-d)\hat{A}_{id}^TW\dot{x}(t)\\&+\lambda x^T(t-\tau(t))G_j^T\hat{B}_i^TW\dot{x}(t)+\lambda\omega^T(t)W\dot{x}(t)\Big]\\\le& -2\lambda\dot{x}^T(t)W\dot{x}(t)\\&+\sum_{i=1}^{M}\sum_{j=1}^{M}\xi_i(\sigma(t))\xi_j(\sigma(t_k))\Big[\lambda\dot{x}^T(t)W\hat{A}_ix(t)\\&+\lambda\dot{x}^T(t)W\hat{B}_iG_jx(t-\tau(t))+\lambda\dot{x}^T(t)W\hat{A}_{id}x(t-d)\\&+\lambda x^T(t)\hat{A}_i^TW\dot{x}(t)+\lambda x^T(t-d)\hat{A}_{id}^TW\dot{x}(t)\\&+\lambda x^T(t-\tau(t))G_j^T\hat{B}_i^TW\dot{x}(t)+\lambda\frac{1}{\varepsilon^2}\dot{x}^T(t)WW\dot{x}(t)\\&+\ \varepsilon^2\omega^T(t)\omega(t)\Big)\Big].\end{aligned}\tag{16}$$

By utilizing (11)-(13) and (15)-(16), we can acquire that

$$\dot{V}(x_t)\le\sum_{i=1}^{M}\sum_{j=1}^{M}\xi_i(\sigma(t))\xi_j(\sigma(t_k))\times\Big[\tilde{x}^T(t)\mathbb{C}'_{ij}\tilde{x}(t)+\bar{\varepsilon}^2\omega(t)^T\omega(t)\Big]\tag{17}$$

where

$$\tilde{x}(t)=[x^T(t)\quad x^T(t-\tau(t))\quad \dot{x}^T(t)\quad x^T(t-d)]^T$$
$$\bar{\varepsilon}=\sqrt{1+\lambda}\varepsilon$$
$$\mathbb{C}'_{ij}=\begin{bmatrix}\mathbb{C}'_{ij11}&\mathbb{C}'_{ij12}&\mathbb{C}'_{ij13}&\mathbb{C}'_{ij14}\\ *&\mathbb{C}'_{ij22}&\mathbb{C}'_{ij23}&0\\ *&*&\mathbb{C}'_{ij33}&\mathbb{C}'_{ij34}\\ *&*&*&\mathbb{C}'_{ij44}\end{bmatrix}.\tag{18}$$

with

$$\begin{aligned}\mathbb{C}'_{ij11}&=\hat{A}_i^TW+W\hat{A}_i+Q_1-Q_2+\frac{1}{\varepsilon^2}WW\\\mathbb{C}'_{ij12}&=W\hat{B}_iG_j+Q_2,\quad \mathbb{C}'_{ij13}=\lambda\hat{A}_i^TW,\ \mathbb{C}'_{ij14}=W\hat{A}_{id}\\\mathbb{C}'_{ij22}&=-Q_2,\quad \mathbb{C}'_{ij23}=\lambda G_j^T\hat{B}_i^TW\\\mathbb{C}'_{ij33}&=-2\lambda W+h^2Q_2+\frac{\lambda}{\varepsilon^2}WW,\quad \mathbb{C}'_{ij34}=\lambda W\hat{A}_{id}\\\mathbb{C}'_{ij44}&=-Q_1.\end{aligned}$$

Let $\tilde{S}=diag\big[(1+\lambda)S_1\ \ 0\ 0\ 0\big]$. Let $\mathbb{C}_{ij}=\mathbb{C}'_{ij}+\tilde{S}$, then

$$\mathbb{C}_{ij}=\begin{bmatrix}\mathbb{C}'_{ij11}+(1+\lambda)S_1&\mathbb{C}'_{ij12}&\mathbb{C}'_{ij13}&\mathbb{C}'_{ij14}\\ *&\mathbb{C}'_{ij22}&\mathbb{C}'_{ij23}&0\\ *&*&\mathbb{C}'_{ij33}&\mathbb{C}'_{ij34}\\ *&*&*&\mathbb{C}'_{ij44}\end{bmatrix}.\tag{19}$$

Left- and right-multiplying $\mathbb{C}_{ij}$ by $diag[\,W^{-1}\ W^{-1}\ W^{-1}\ W^{-1}\,]$ yields

$$\hat{\mathbb{C}}_{ij}=\begin{bmatrix}\mathbb{R}_{ij11}+(1+\lambda)\bar{S}_1&\mathbb{R}_{ij13}&\mathbb{R}_{ij14}&\mathbb{R}_{ij15}\\ *&\mathbb{R}_{ij33}&\mathbb{R}_{ij34}&0\\ *&*&\mathbb{R}_{ij44}&\mathbb{R}_{ij45}\\ *&*&*&\mathbb{R}_{ij55}\end{bmatrix}\tag{20}$$

where

$$\overline{W}=W^{-1},\quad \overline{Q_1}=W^{-1}Q_1W^{-1},\ \overline{Q_2}=W^{-1}Q_2W^{-1},$$
$$\bar{S}_1=W^{-1}S_1W^{-1},\quad \bar{G}_j=G_j\bar{W},\ j=1,2,\cdots,M.$$

By adopting the Schur complement in (8), we have $\hat{\mathbb{C}}_{ij}<0$. Therefore, $\mathbb{C}_{ij}<0$. Substituting $\mathbb{C}'_{ij}<-\tilde{S}$ into (17), one has

$$\dot{V}(x_t)\le-\tilde{x}^T(t)\tilde{S}\tilde{x}(t)+\bar{\varepsilon}^2\omega^T(t)\omega(t).\tag{21}$$

Owing to $\tilde{S}=diag\big[(1+\lambda)S_1\ 0\ 0\ 0\big]$, $\bar{\varepsilon}=\sqrt{1+\lambda}\varepsilon$, we get

$$\dot{V}(x_t)\le-(1+\lambda)x^T(t)S_1x(t)+(1+\lambda)\varepsilon^2\omega^T(t)\omega(t).\tag{22}$$

Integrating both sides of (22), there is

$$\int_0^\infty x^T(t)S_1x(t)dt\le\varepsilon^2\int_0^\infty\omega^T(t)\omega(t)dt.\tag{23}$$

Remark 1: Eq. (8) provides a new relaxed stability condition for the system (2). Unlike the existing works, in the proof, eq.(16) is introduced to consider the fuzzy relationship between $\dot{x}(t)$, $x(t)$, $x(t-\tau(t))$ and $x(t-d)$. By using of the LMIs, it is easy to determine the feedback gains.

Now, we continue to consider the H_2 control performance for the closed-loop system (5) in the absence of $\omega(t)$.

The derivative of V in the absence of $\omega(t)$ is as follows:

$$\begin{aligned}\dot{V}_1(x)&=\dot{x}^T(t)Wx(t)+x^T(t)W\dot{x}(t)\\&=\sum_{i=1}^{M}\sum_{j=1}^{M}\xi_i(\sigma(t))\xi_j(\sigma(t_k))[x^T(t)\hat{A}_i^TWx(t)\\&\quad+x^T(t-d)\hat{A}_{id}^TWx(t)+x^T(t-\tau(t))G_j^T\hat{B}_i^TWx(t)\\&\quad+x^T(t)W\hat{A}_ix(t)+x^T(t)W\hat{A}_{id}x(t-d)\\&\quad+x^T(t)W\hat{B}_iG_jx(t-\tau(t))].\end{aligned}\tag{24}$$

$$\dot{V}_2(x_t)=x^T(t)Q_1x(t)-x^T(t-d)Q_1x(t-d).\tag{25}$$

By using (15),

$$\begin{aligned}\dot{V}_3(x_t)\le\ & h^2\dot{x}^T(t)Q_2\dot{x}(t)-x^T(t)Q_2x(t)\\&+x^T(t-\tau(t))Q_2x(t)+x^T(t)Q_2x(t-\tau(t))\\&-x^T(t-\tau(t))Q_2x(t-\tau(t)).\end{aligned}\tag{26}$$

It is noted that for a given $\mu>0$, one has

$$\begin{aligned}0=&-2\lambda\dot{x}^T(t)W\dot{x}(t)\\&+\lambda\dot{x}^T(t)W\{\sum_{i=1}^{M}\sum_{j=1}^{M}\xi_i(\sigma(t))\xi_j(\sigma(t_k))[\hat{A}_ix(t)\\&+\hat{A}_{id}x(t-d)\\&+\hat{B}_iG_jx(t-\tau(t))]\}\\&+\lambda\{\sum_{i=1}^{M}\sum_{j=1}^{M}\xi_i(\sigma(t))\xi_j(\sigma(t_k))[\hat{A}_ix(t)+\hat{A}_{id}x(t-d)\\&+\hat{B}_iG_jx(t-\tau(t))]\}^TW\dot{x}(t)\\=&-2\lambda\dot{x}^T(t)W\dot{x}(t)\end{aligned}$$

$$
\begin{aligned}
&+\sum_{i=1}^{M}\sum_{j=1}^{M}\xi_i(\sigma(t))\xi_j(\sigma(t_k))[\lambda\dot{x}^T(t)W\hat{A}_ix(t)\\
&+\lambda\dot{x}^T(t)W\hat{A}_{id}x(t-d)+\lambda\dot{x}^T(t)W\hat{B}_iG_jx(t-\tau(t))\\
&+\lambda x^T(t)\hat{A}_i^TW\dot{x}(t)+\lambda x^T(t-d)\hat{A}_{id}^TW\dot{x}(t)\\
&+\lambda x^T(t-\tau(t))G_j^T\hat{B}_i^TW\dot{x}(t)]. \qquad (27)
\end{aligned}
$$

From (24)-(27), we can obtain that

$$
\begin{aligned}
&\dot{V}(x_t)+x^T(t)S_2x(t)+u^T(t)Qu(t)\\
&\quad\leq\sum_{i=1}^{M}\sum_{j=1}^{M}\xi_i(\sigma(t))\xi_j(\sigma(t_k))\tilde{x}^T(t)\Im_{ij}\tilde{x}(t) \qquad (28)
\end{aligned}
$$

where

$$
\tilde{x}(t)=[x^T(t)\ x^T(t-\tau(t))\ \dot{x}^T(t)\ x^T(t-d)]^T
$$

$$
\Im_{ij}=\begin{bmatrix}\Im_{ij11}&\Im_{ij12}&\Im_{ij13}&\Im_{ij14}\\ *&\Im_{ij22}&\Im_{ij23}&0\\ *&*&\Im_{ij33}&\Im_{ij34}\\ *&*&*&\Im_{ij44}\end{bmatrix} \qquad (29)
$$

with

$$
\begin{aligned}
\Im_{ij11}&=\hat{A}_iW+W\hat{A}_i^T+Q_1-Q_2+S_2,\\
\Im_{ij12}&=W\hat{B}_iG_j+Q_2\\
\Im_{ij13}&=\lambda\hat{A}_i^TW,\quad \Im_{ij14}=W\hat{A}_{id},\\
\Im_{ij22}&=-Q_2+G_j^TQG_j\\
\Im_{ij23}&=\lambda G_j^T\hat{B}_i^TW,\quad \Im_{ij33}=-2\lambda W+h^2Q_2,\\
\Im_{ij34}&=\lambda W\hat{A}_{id}\\
\Im_{ij44}&=-Q_1.
\end{aligned}
$$

Left- and right-multiplying $\Im_{ij}$ by $diag[\,W^{-1}\ \ W^{-1}\ \ W^{-1}\ \ W^{-1}\,]$ yields

$$
\hat{\Im}_{ij}=\begin{bmatrix}\mathbb{Z}_{ij11}+\overline{S_2}&\mathbb{Z}_{ij13}&\mathbb{Z}_{ij15}&\mathbb{Z}_{ij16}\\ *&\mathbb{Z}_{ij33}+\overline{G}_j^TQ\overline{G}_j&\mathbb{Z}_{ij35}&0\\ *&*&\mathbb{Z}_{ij55}&\mathbb{Z}_{ij56}\\ *&*&*&\mathbb{Z}_{ij66}\end{bmatrix} \qquad (30)
$$

where

$$
\begin{aligned}
&\overline{W}=W^{-1},\quad \overline{Q_1}=W^{-1}Q_1W^{-1},\ \overline{Q_2}=W^{-1}Q_2W^{-1},\\
&\overline{S_2}=W^{-1}S_2W^{-1},\quad \bar{G}_j=G_j\bar{W},\ j=1,\cdots,M.
\end{aligned}
$$

Applying the Schur complement to (9), there is $\hat{\Im}_{ij}<0$. And, we have $\Im_{ij}<0$ in (29). Thus, as for (28), we have

$$
\dot{V}(x_t)+x^T(t)S_2x(t)+u^T(t)Qu(t)<0 \qquad (31)
$$

which implies that $\dot{V}(x_t)<0$.

Integrating (31) from $t=0$ to $t=\infty$, there is

$$
\begin{aligned}
&V(x_t(\infty))-V(x_t(0))\\
&\quad+\int_0^{\infty}(x^T(t)S_2x(t)+u^T(t)Qu(t))dt<0. \qquad (32)
\end{aligned}
$$

Due to $V(x_t(\infty))=0$, one has

$$
J<V(x_t(0))=x^T(0)Wx(0). \qquad (33)
$$

The design of this paper is formulated as the following optimization problem.

Theorem 2: Considering the fuzzy closed-loop system (5), if the following problem

$$
\begin{aligned}
&\min_{\bar{W}}\ \ Trace(U)\\
&s.t.\ \bar{W}>0,\quad \overline{Q_1}>0,\ \overline{Q_2}>0,\ (8),\ (9),\ \text{and}\begin{bmatrix}U&I\\ *&\bar{W}\end{bmatrix}>0
\end{aligned} \qquad (34)
$$

has a solution $\overline{G_j}, j=1,\cdots,M$ and $\overline{W}$, then a whole optimal H_2/H_∞ control performance is achieved. The control gains are $G_j=\bar{G}_j\bar{W}^{-1}, j=1,2,\cdots,M$.

Remark 2: Fuzzy H_2/H_∞ sampled-data control is firstly discussed for time-delay systems. In practical engineering systems, the H_2/H_∞ sampled-data control is more appealing in achieving the desired control performance. And, the focus of H_∞ performance is mainly on the gain between the system state and the external disturbance. Meanwhile, both the system state and the system input are considered in H_2 performance. Furthermore, H_2/H_∞ control synthesizes the merits of H_∞ control and H_2 control.

Remark 3: The proposed control algorithm simplifies the dimension of the LMI, reduces the computational complexity, and is less conservative. At the same time, it increases the existence of the feedback gain and shortens the implementation time. The designed method has better performance, in which the fuzzy sampled-data controller has a larger sampling interval, a faster state response and a better attenuation level.

IV. SIMULATION EXAMPLES

The proposed design is demonstrated by two practical dynamical systems with delay. In addition, the superiority is obvious in the obtained simulation results.

Example 1: The truck-trailer system in [39] is

$$
\begin{aligned}
\dot{x}_1(t)&=-\tilde{a}\frac{v\bar{t}}{Lt_0}x_1(t)-(1-\tilde{a})\frac{v\bar{t}}{Lt_0}x_1(t-t_d)\\
&\quad+\frac{v\bar{t}}{lt_0}u(t)+w(t)\\
\dot{x}_2(t)&=\tilde{a}\frac{v\bar{t}}{Lt_0}x_1(t)+(1-\tilde{a})\frac{v\bar{t}}{Lt_0}x_1(t-t_d)\\
\dot{x}_3(t)&=\frac{v\bar{t}}{Lt_0}\sin(x_2(t)\\
&\quad+\tilde{a}(v\bar{t}/2L)x_1(t)+(1-\tilde{a})(v\bar{t}/2L)x_1(t-t_d)) \qquad (35)
\end{aligned}
$$

where $l=2.8, L=5.5, v=-1.0\,\tilde{a}=0.7, \bar{t}=2.0, t_0=0.5$, $w(t)$ is the external disturbance.

Let $x(t)=[x_1(t)\ x_2(t)x_3(t)]^T$. The truck-trailer system is represented as

$$
\dot{x}(t)=\sum_{i=1}^{2}\xi_i(\sigma(t))\left[\hat{A}_ix(t)+\hat{A}_{id}x(t-d)+\hat{B}_iu(t)+\omega(t)\right]
$$

TABLE 1. The dimensions of the LIMs.

Method	[22]	[10]	Theorem 1
Dimension	30	20	16

with

$$\hat{A}_1 = \begin{bmatrix} -\tilde{a}\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\ \tilde{a}\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\ \tilde{a}\dfrac{v^2\bar{t}^2}{2Lt_0} & \frac{v\bar{t}}{t_0} & 0 \end{bmatrix},$$

$$\hat{A}_{1d} = \begin{bmatrix} -(1-\tilde{a})\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-\tilde{a})\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-\tilde{a})\dfrac{v^2\bar{t}^2}{2Lt_0} & 0 & 0 \end{bmatrix}$$

$$\hat{A}_2 = \begin{bmatrix} -\tilde{a}\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\ \tilde{a}\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\ \tilde{a}\dfrac{dv^2\bar{t}^2}{2Lt_0} & \dfrac{dv\bar{t}}{t_0} & 0 \end{bmatrix},$$

$$\hat{A}_{2d} = \begin{bmatrix} -(1-\tilde{a})\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-\tilde{a})\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-\tilde{a})\dfrac{dv^2\bar{t}^2}{2Lt_0} & 0 & 0 \end{bmatrix}$$

$$\hat{B}_1 = \begin{bmatrix} \dfrac{v\bar{t}}{lt_0} \\ 0 \\ 0 \end{bmatrix}, \quad \hat{B}_2 = \begin{bmatrix} \dfrac{v\bar{t}}{lt_0} \\ 0 \\ 0 \end{bmatrix}, \quad d = 10t_0/\pi$$

$$\xi_1(\theta(t)) = \left(1 - \frac{1}{1+\exp(-3(\theta(t)-0.5\pi))}\right) \times \left(\frac{1}{1+\exp(-3(\theta(t)+0.5\pi))}\right)$$

$$\xi_2(\theta(t)) = 1 - \xi_1(\theta(t)).$$

By employing the methods of [22], [10], and Theorem 1, a lower dimension in the LMIs is obtained in this paper, see, Table 1. That is to say, the computation burden of this paper is smaller than that of the methods in [22] and [10].

Using the methods of [10] and Theorem 1, a larger sampling interval is obtained, see, Table 2.

TABLE 2. The maximum sampling interval with $\varepsilon = 1.0$.

Method	[10]	Theorem 1
$h_{\max}(t=0.5)$	0.374	0.432
$h_{\max}(t=1)$	0.315	0.378
$h_{\max}(t=2)$	0.251	0.312

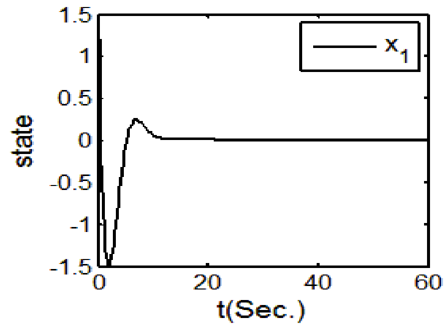

FIGURE 1. State response x_1.

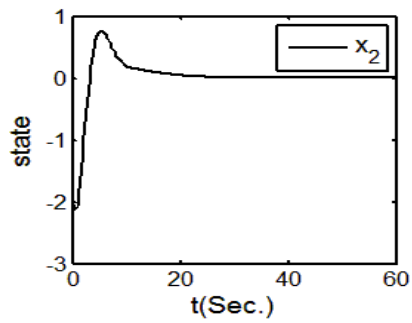

FIGURE 2. State response x_2.

When $\varepsilon = 1.0$, Theorem1 gives the maximum sampling interval $h = 0.432$. With $\lambda = 0.6$, $S_1 = diag\{1 \;\; 1 \;\; 0.1\} \times 10^{-8}$, $S_2 = diag\{1 \;\; 1 \; 0.1\} \times 10^{-8}$, $Q = 1 \times 10^{-5}$, fuzzy state feedback gains are

$$G_1 = [1.2749 \quad -0.6031 \quad 0.0122]$$
$$G_2 = [1.2749 \quad -0.6031 \quad 0.0122].$$

Experimental results are depicted in FIGUREs. 1-4.

The system stability is confirmed and the piecewise continuous behavior of the control input is also plotted.

When $h = 0.432$, we find that this paper shows faster state responses than those of the H_∞ method in [22].

Example 2: The stirred tank reactor system in [40] is

$$\dot{x}_1(t) = -\frac{1}{v}x_1(t) + D_\sigma(1-x_1(t))e^{\frac{x_2(t)}{1+x_2(t)/\gamma_0}} + (\frac{1}{v}-1)x_1(t-\tau)$$
$$\dot{x}_2(t) = \left(\frac{1}{v}+\beta\right)x_2(t) + HD_\sigma(1-x_1(t))e^{\frac{x_2(t)}{1+x_2(t)/\gamma_0}} + \left(\frac{1}{v}-1\right)x_2(t-\tau) + \beta u(t) + \beta w(t) \quad (36)$$

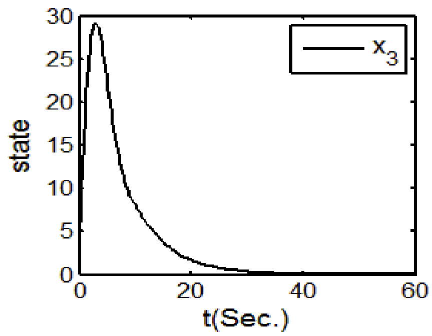

FIGURE 3. State response x_3.

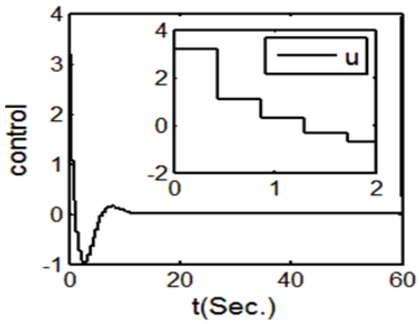

FIGURE 4. Control input u.

where $\gamma_0 = 20, H = 8, D_\sigma = 0.072, v = 0.8, \ \beta = 0.3$, $0 \le x_1(t) \le 1$, and $w(t)$ is the external disturbance.

Let $x(t) = [x_1(t), \ x_2(t)]^T$. The stirred tank reactor system is described as

$$\dot{x}(t) = \sum_{i=1}^{3} \xi_i(\sigma(t))[\hat{A}_i x(t) + \hat{A}_{id} x(t-\tau) + \hat{B}_i u(t) + \omega(t)]$$

with

$$\hat{A}_1 = \begin{bmatrix} -1.4274 & 0.0757 \\ -1.4189 & -0.9442 \end{bmatrix}, \quad \hat{A}_{1d} = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \end{bmatrix}$$

$$\hat{A}_2 = \begin{bmatrix} -2.0508 & 0.3958 \\ -6.4066 & 1.6268 \end{bmatrix}, \quad \hat{A}_{2d} = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \end{bmatrix}$$

$$\hat{A}_3 = \begin{bmatrix} -4.5279 & 0.3167 \\ -26.2228 & 0.9387 \end{bmatrix}, \quad \hat{A}_{3d} = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \end{bmatrix}$$

$$\hat{B}_1 = \begin{bmatrix} 0 \\ 0.3 \end{bmatrix}, \quad \hat{B}_2 = \begin{bmatrix} 0 \\ 0.3 \end{bmatrix}, \quad \hat{B}_3 = \begin{bmatrix} 0 \\ 0.3 \end{bmatrix}$$

$$\xi_1(x_2(t)) = \begin{cases} 1, & x_2 \le 0.8862 \\ 1 - \dfrac{x_2(t) - 0.8862}{2.7520 - 0.8862}, & 0.8862 < x_2 < 2.7520 \\ 0, & x_2 \ge 2.7520 \end{cases}$$

$$\xi_2(x_2(t)) = \begin{cases} 1 - \xi_1(x_2(t)), & x_2 \le 2.7520 \\ 1 - \xi_3(x_2(t)), & x_2 > 2.7520 \end{cases}$$

$$\xi_3(x_2(t)) = \begin{cases} 0, & x_2 \le 2.7520 \\ 1 - \dfrac{x_2(t) - 2.7520}{4.7052 - 2.7520}, & 2.7520 < x_2 < 4.7052 \\ 1, & x_2 \ge 4.7052 \end{cases}$$

TABLE 3. The dimensions of the LIMs.

Method	[22]	[10]	Theorem 1
Dimension	20	14	11

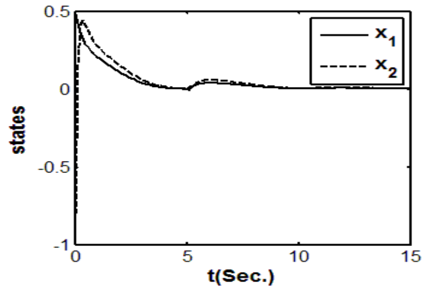

FIGURE 5. State responses.

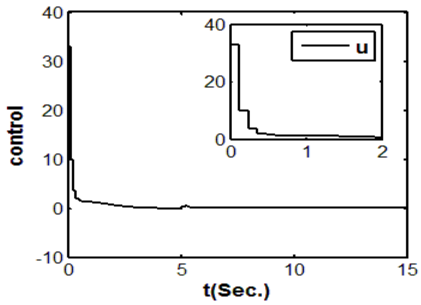

FIGURE 6. Control input.

By using the methods of [22], [10] and Theorem 1, a lower dimension in the LMIs is achieved, see, Table 3.

That is to say, the computation burden in this paper is smaller than that of the methods in [22] and [10].

When $h = 0.1$, there is $\varepsilon_{\min} = 0.0497$ under Theorem 1. The minimum disturbance attenuation is 0.0497, which is ineffective in the LMI conditions of H_∞ control [22]. This implies that the proposed fuzzy H_2/H_∞ sampled-data control method is superior to the existing H_∞ control method.

When $\varepsilon = 1.0$, Theorem 1 gives $h_{\max} = 0.222$. The maximum sampling interval is 0.222, which is infeasible in the LMI conditions of H_2 control [10]. This implies that the proposed fuzzy H_2 /H_∞ sampled-data control method is superior to the existing H_2 control method.

According to the design procedure, we get the optimal design parameters $\varepsilon = 0.0603, h = 0.12, \lambda = 0.2$. With $S_1 = diag\{9 \ \ 8\} \times 10^{-6}, S_2 = diag\{2 \ \ 6\} \times 10^{-4}, Q = 1 \times 10^{-6}$, Theorem 1 gives state feedback gains

$$G_1 = [30.2128 - 17.8191], \quad G_2 = [30.2128 - 17.8191]$$
$$G_3 = [30.2128 - 17.8191].$$

Simulation results under $\tau = 5$ are shown in FIGUREs. 5-6. The system stability is confirmed and the piecewise continuous behavior of the control input is also portrayed.

V. CONCLUSION

Fuzzy H_2/H_∞ sampled-data control for dynamical systems with delay behavior is addressed by using the input delay method. Based on Lyapunov theory, a fuzzy sampled-data controller is proposed, which can guarantee the H_∞ performance and H_2 performance concurrently. Moreover, a new H_2/H_∞ standard is derived. Both stability analysis and simulation results demonstrate the proposed design and the superiority have also been verified through comparative analysis. In the future, this method will be applied to uncertain dynamical systems and provide theoretical support for further research. Furthermore, we will pay attention to more developments on the sampled-data control, the adaptive neural control, the adaptive fuzzy tracking control, the event-triggered scheme and the stochastic control in the literatures.

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