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A Fuzzy Performance Evaluation Model for a Gearbox System Using Hidden Markov Model

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ABSTRACT In order to track and grasp the operation situations of the gearboxes, the vertical vibration signals of three different gear fault states, normal, worn and broken teeth, are collected via a gearbox vibration experiment. An online diagnosis and performance evaluation model with hidden Markov model (HMM) and fuzzy comprehensive evaluation is proposed. To address the limitation of maximum membership principle in the case of equal membership or the membership is very close to each other, a closeness evaluation strategy is proposed by defining the likelihood ratio of HMM as a similarity and selecting an combined membership function of the semi-trapezoidal and intermediate-ridge distribution. Results show that the online diagnosis has achieved a good performance with the similarity strategy. Compared with the evaluation strategy of the maximum membership principle, the proposed gearbox performance model with the closeness evaluation strategy is more accurately distinguished from the evaluation results of the broken teeth state and the worn state, especially for the case of the equal membership.

INDEX TERMS Closeness evaluation strategy, fault diagnosis, gearbox, hidden Markov model, performance evaluation.

I. INTRODUCTION

Due to a wide range of speed ratio, high reliability, and accurate transmission ratio, the gearbox is becoming one of important power transmission components, and has widely applied in heavy industry, light industry, construction and daily life [1]. However, the gearboxes are easy to fail due to frequently operated in harsh conditions such as heavy load or high speed, affecting the production efficiency and even endanger personal safety [2]. Therefore, it is necessary to track and grasp the running state of the gearbox timely and accurately, so as to ensure the safety and reliability of the gearbox.

Performance evaluation has been proven to be an effective technique in tracking and identifying the operational situation of the mechanical equipment in time[3]. So far, several artificial intelligent methods, such as logistic regression [4] support vector data description [5], [6], self-organizing mapping [7], neural network [8], Gaussian mixture model [9], have been widely employed in performance evaluation for mechanical equipment.

For bearing performance degradation evaluation, many efforts are made in improving the accuracy of the evaluation model. Xia *et al.* [10] evaluated the uncertainty of vibration by a grey prediction model. Ye *et al.* [11] predicted the vibration performance by chaos prediction model. Ren *et al.* [12] proposed a performance evaluation method by manifold space fuzzy k-master curve similarity analysis. Zhu *et al.* [13] proposed a performance degradation method by improved fuzzy entropy. Li *et al.* [14] applied a general mathematical morphology particle as a new indicator for bearing performance degradation evaluation. Liu *et al.* [15] proposed an evaluation method by ensemble empirical mode decomposition. Wang and Tsui [16] established a generalized dimensionless health indicator to evaluate bearing degradation. Rai and Upadhyay [17] pro-posed a performance degradation index based on self-organizing mapping to estimate the remaining service life.

For gearbox performance evaluation, however, there are a few existing works. Wang *et al.* [18] realized dynamic evaluation of gearbox by grey theory and bootstrap theory.

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Wang *et al.* [19] developed a predictive system to monitor gear performance degradation. Pan *et al.* [20] proposed a performance degradation assessment method based on complete ensemble empirical mode decomposition with adaptive noise and kernel principal component analysis. It is found that performance evaluation model based on artificial intelligent method suffers from the black-box problem, cumbersome data processing and modeling.

Hidden Markov model (HMM) has been gaining more attention on performance evaluation due to its model interpretation ability and high efficiency in processing the time-varying signals. Most notably, Li *et al.* [21] applied HMM in evaluating the reliability of generators. Jiang *et al.* [22] applied HMM in bearing performance evaluation. Zhang *et al.* [23] proposed a performance degradation index by orthogonal neighborhood retained projection and two-dimensional HMM. Wang *et al.* [24] identified the running state of the spot welding machining process by HMM. Liu *et al.* [25] proposed a bearing performance degradation evaluation method by orthogonal locally retained projection and continuous HMM. Zhou *et al.* [26] proposed a fault diagnosis by a shift-invariant dictionary learning and HMM. Li *et al.* [27] proposed an optimal Bayesian control policy by HMM. Mba *et al.* [28] proposed a state classification by stochastic resonance and HMM.

Due to the objective and clear evaluation results, fuzzy comprehensive evaluation method has been popular in performance evaluation [29]. Fang *et al.* [30] proposed a fuzzy comprehensive evaluation method based on supervisory control and data acquisition. Liu and Ma [31] proposed a fuzzy comprehensive evaluation method to determine the symmetry degree of three types of mechanical structure symmetries. Xu *et al.* [32] studied the application of fuzzy comprehensive evaluation method in tool cutting performance evaluation. In summary, we found that fuzzy comprehensive evaluation with the maximum membership principle is not available in the case of the equal membership.

For the above reasons, this paper focuses on the gearbox performance evaluation by a closeness evaluation strategy. To address the limitation of maximum membership principle in the case of the equal membership or the membership is very close to each other, we propose a closeness evaluation strategy by defining the likelihood ratio of HMM as the similarity and selecting a combined membership function of the semi-trapezoidal and intermediate-ridge distribution. Additionally, we perform a gearbox experiment to collect the vertical vibration signals of three different gear fault states for the purpose of verifying the proposed performance evaluation model based on a closeness evaluation strategy. For online diagnosis phase, affinity propagation (AP) clustering algorithm is employed to parameter initialization for HMM. By forward-backward, Viterbi, and Baum-Welch algorithms, we calculate the likelihood ratio of HMM, and define the likelihood ratio as a similarity for quantifying the degradation of the gearbox, so as to implement the online diagnosis.

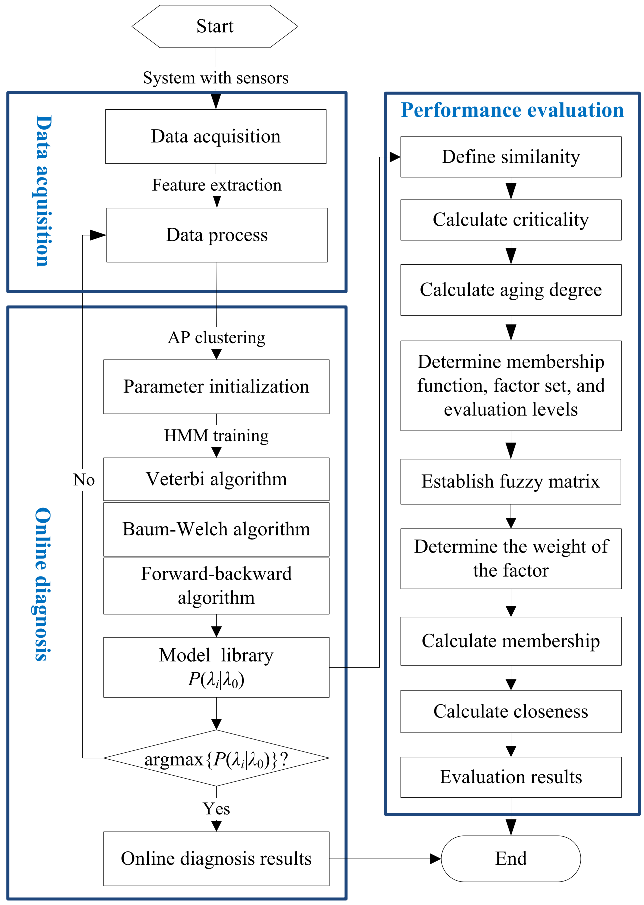


FIGURE 1. The procedure of the proposed gearbox performance evaluation model.

For performance evaluation phase, we present a fuzzy comprehensive evaluation model with a closeness evaluation strategy by converting similarity into closeness. By this closeness evaluation strategy, the limitation of maximum membership principle on the case of the equal membership is avoided. The structure of the paper is as follows: Section II describes the proposed fuzzy performance evaluation model; Section III introduces the online diagnosis method by HMM; Section IV discusses the online performance evaluation method by fuzzy comprehensive evaluation; Section V discusses the results of online diagnosis and performance evaluation for gearbox; Conclusions are given in Section VI.

II. PROPOSED FUZZ PERFORMANCE EVALUATION MODEL

To make clear the actual operating situation of gearboxes, we proposed a fuzzy performance evaluation model with HMM. The procedure of the proposed gearbox performance evaluation model is shown in Figure 1. There are three steps for developing gearbox state evaluation: (1) data acquisition and data processing; (2) online diagnosis; (3) online performance evaluation.

A. DATA ACQUISITION AND DATA PROCESSING

In this stage, many issues should be taken into account, such as sensor selection, sensor quantity, sensor mounting location, and reliability of measuring data. Owing to carry the dynamic information on machine state, the vibration signals are effective in gearbox fault diagnosis [33]. In order to ensure the reliability of measuring data, the acceleration sensor was calibrated before use, and the effect of sensor failure was eliminated. We collected the vertical vibration signals with normal, worn and fractured gears, and extracted the vibration frequency band energy of the gear as the observation value of the model, because the vibration energy of vertical vibration signal can reflect gearbox operating conditions.

Restricted to inherent characteristics of the sensors, such as limited precision and limited range, the raw signal must be processed by eliminating the outliner and zero-mean normalization before feature extraction. In addition, the external operating environment factors, such as noise, information redundancy, have an impact on gearbox diagnostics and performance evaluation results [34]. Therefore, it is necessary to mine and extract the effective feature parameter from the raw data. In this paper, the raw signals are divided into a plurality of short-term signals, and the vibration energy feature is extracted from each of the short-term signals to form an observation sequence of the HMM.

B. ONLINE DIAGNOSIS

In this section, three gear states with normal, worn, and broken teeth are taken into account, and the online diagnosis contains two phases. The first phase is training of HMM by a Baum-Welch algorithm, and the second phase is classification by a forward-backward algorithm. Three training models with normal, worn, and broken teeth, namely λ_0, λ_1, and λ_2, are established by the historical data collected from the above three predefined states in this paper, and the similarity between different fault states and normal states are described by the probability $p(\lambda_i|\lambda_0)$ of being in the fault state i for the normal state. Here, $\lambda_i (i \geq 1)$ denotes the training model in different fault states, and λ_0 is the training model in the normal state. Additionally, the correctness of the model can be verified by comparing the similarity between different fault states and normal states.

C. ONLINE EVALUATION

The online evaluation of the gearbox is implemented by fuzzy performance evaluation. By substituting the evaluation indicator converted from the similarity between different fault states and normal states into the established combined membership function, the online evaluation is implemented to evaluate the operating situations of the gearbox under different states. In addition, the differences in the evaluation results between the closeness and maximum membership evaluation strategies under different conditions are compared as well.

III. ONLINE DIAGNOSIS BY HMM

Due to the effects such as stress and shock, the amplitude of the vibration signal gradually increases with the performance degrading in the operation process over time. The fault occurs once the performance degrades exceeded a certain threshold. In this condition, we define a similarity to measure how similar the current measured value is to the normal or fault value. With such strategy, the similarity between the real-time vibration signal and the normal or specific fault vibration signal can be accurately quantified, so as to obtain the performance degradation or fault severity of the gearbox at any service rime. Base on the maximum membership principle, the smaller similarity between the current state and the normal state is, the worse the performance. That is, the system is more likely to occur fault. For the above reasons, we proposed to define the likelihood ratio of HMM as the similarity P, the likelihood ratio was obtained by forward-backward algorithm, and. the model λ was adjusted under the given observation and initial condition by the Baum-Welch algorithm.

Generally, HMM is defined as $\lambda = (N, M, \pi, A, B)$.

(1) N is the number of states. The individual states are defined as $S_t \in \{S_1, S_2, \cdots, S_N\}$.

(2) M is the number of observations. Suppose that the observations are $V_1, V_2, \cdots, V_M$, and the observations at time t is defined as o_t.

(3) $\pi = \{\pi_i, i = 1, 2, \cdots, N\}$ is the initial state distribution vector. $\pi_1 = 1, \pi_i = 0 (2 \leq i \leq N)$. Here, $0 \leq \pi_i \leq 1$, $\sum_{i=1}^{N} \pi_i = 1, 1 \leq i \leq N$.

(4) $A = \{a_{ij}\}$ is the state transition matrix, $A = a_{i,j} = p(S_j|S_i)$, $1 \leq i, j \leq n$ and $\sum_{i=1}^{N} a_{ij} = 1$.

(5) $B = \{b_i(k)\} (1 \leq i \leq M, 1 \leq k \leq N)$ is the emissions probability matrix. b_{ik} denotes the occur probability of the observation v_k at state i, i.e. $b_{ik} = P_r(o_t = v_k | q_t = s_i)$, here, $\sum_{i=1}^{N} b_{ik} = 1, 1 \leq i \leq N, 1 \leq k \leq N$.

The parameter λ of the probability $P(O|\lambda)$ can be obtained by using Baum-Welch algorithm. Let $\varsigma(i,j) = P(q_t = i, q_{t+1} = j | O, \lambda)$, and $\varsigma(i,j)$ can be obtained by using forward-backward algorithm.

$$\begin{aligned} \varsigma_t(i,j) &= \frac{P(q_t = S_i, q_{t+1} = S_j, O | \lambda)}{P(O | \lambda)} \\ &= \frac{\alpha_t(i) a_{i,j} b_j(o_{t+1}) \beta_{t+1}(j)}{P(O | \lambda)} \\ &= \frac{\alpha_t(i) a_{i,j} b_j(o_{t+1}) \beta_{t+1}(j)}{\sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_t(i) a_{i,j} b_j(o_{t+1}) \beta_{t+1}(j)} \end{aligned} \tag{1}$$

Under the condition that the initial parameter λ is known and the observed value O is measured, the probability of being in state S_i at time t is defined as follows:

$$\tau_t(i) = P(q_t = S_i|O, \lambda) = \frac{P(q_t = S_i, O|\lambda)}{P(O|\lambda)} = \frac{\alpha_t(i)\beta_t(i)}{\sum_{j=1}^{N} \alpha_t(j)\beta_t(j)} = \sum_{j=1}^{N} \varsigma_t(i,j) \quad (2)$$

The re-evaluation procedure of the parameter λ is as follows.

Step 1: Determine the initial N, M and the initial model $\lambda(\pi, N, M, A, B)$ according to the actual situation.

Step 2: Calculate the forward variable $\alpha_t(i)$ and the backward variable $\beta_{t+1}(j)$ according to the forward-backward algorithm; calculate $\varsigma(i,j)$ and $\tau_t(i)$ by Eq. (1) and Eq. (2).

Step 3: The parameter re-evaluation is developed by the following formulas.

$$\begin{cases} \bar{\pi}_i = \tau_1(i) \\ \bar{a}_{i,j} = \dfrac{\sum_{t=1}^{T-1} \varsigma(i,j)}{\sum_{t=1}^{T-1} \tau_t(i)} \\ \bar{b}_j(k) = \dfrac{\sum_{t=1}^{T} \tau_t(j)\,\delta(o_t, v_k)}{\sum_{t=1}^{T} \tau_t(j)} \end{cases} \quad (3)$$

where $\delta(o_t, v_k)$ is Dirac function.

$$\delta(o_t, v_k) = \begin{cases} 1, & o_t = v_k \\ 0, & o_t \neq v_k \end{cases} \quad (4)$$

Step 4: Calculate the probability $P(O|\lambda)$ by the re-evaluation parameters $\bar{\lambda} = (\bar{\pi}, \bar{A}, \bar{B})$. Repeat steps (2) and (3) until the absolute value of the difference between two successive probabilities is less than the given value.

Forward procedure is defined by

$$\alpha_t(i) = P(o_1, o_2, \cdots o_t, q_t = s_i|\lambda_0) \quad (5)$$

where $\alpha_t(i)$ is the joint probability of the observation value sequence $o_1, o_2, \ldots, o_t$ from the beginning of acquisition to the time t and the state S_i of the model at time t under the condition of obtaining the parameter λ_0.

$\alpha_t(i)$ can be defined as follows:

$$\alpha_1(i) = \pi_i b_i(o_1), \quad (1 \leq i \leq N) \quad (6)$$

$$\alpha_{t+1}(j) = \sum_{i=1}^{N} \alpha_t(i) a_{ij} b_j(o_{t+1}), \quad (1 \leq t \leq T-1; 1 \leq j \leq N) \quad (7)$$

$$P(\lambda|\lambda_0) = \sum_{i=1}^{N} \alpha_\gamma(i) \quad (8)$$

Backward procedure is similarly defined by

$$\beta_t(i) = P(o_{t+1}, o_{t+2}, \cdots, o_T|q_t = s_i, \lambda_0) \quad (9)$$

where $\beta_t(i)$ is the joint probability of partial observation value sequence $o_{t+1}, o_{t+2}, \ldots o_T$ from time t to final time model under the condition that the model parameters λ_0 is known and the model is in state S_i at time t.

TABLE 1. Degree of damage of different failure modes.

Components	Fault modes	Criticality
Gear	Tooth surface fatigue (pitting)	4.5
	Tooth surface wear (wear)	3.2
	Broken tooth	5.8
Bearing	Surface damage (pitting)	4.5
	Cage deformation	4.8
	Fracture	4.8
Shaft	Shaft crack	4.5
	Spindle break	5.8

$\beta_t(i)$ can be defined as follows:

$$\beta_t(i) = 1, \quad (1 \leq i \leq N) \quad (10)$$

$$\beta_t(i) = \sum_{j=1}^{N} a_{ij} b_j(o_{t+1}) \beta_{t+1}(j), \quad (t = T-1, T-2, \cdots, 1; 1 \leq i \leq N) \quad (11)$$

The likelihood ratio $P(\lambda|\lambda_i)$ of the observation model λ under a given model λ_i can be obtained by forward-backward algorithm, and the likelihood ratio between the current state and normal state is as follow:

$$P(\lambda|\lambda_0) = \sum_{i=1}^{N} \alpha_t(i)\beta_t(i) \quad (12)$$

IV. ONLINE PERFORMANCE EVALUATION BY FUZZY COMPREHENSIVE EVALUATION

For assessment of the gearbox degradation, we propose a performance evaluation model with fuzzy performance evaluation, and the procedures of the proposed fuzzy performance evaluation model are as follows.

Step 1: Define the similarity. We defined the forementioned likelihood ratio of HMM as a similarity for quantifying the degradation of the gearbox.

Step 2: Calculate the criticality. The criticality is quantified by risk matrix method, and the criticality is in range of [1], [10]. Table 1 shows the criticality for different gearbox fault modes. The criticality is calculated as.

$$E = \begin{cases} 3.2 & 10^{-5} < P \\ 3.2 - 0.13 \times \log_{10} \dfrac{P}{10^{-5}} & 10^{-15} < P \leq 10^{-5} \\ 4.5 - 0.13 \times \log_{10} \dfrac{P}{10^{-15}} & 10^{-20} < P \leq 10^{-15} \\ 5.8 - 0.13 \times \log_{10} \dfrac{P}{10^{-20}} & 10^{-30} < P \leq 10^{-20} \\ 10 & P \ll 10^{-30} \end{cases} \quad (13)$$

where E and P denotes the criticality and similarity, respectively.

Step 3: Calculate the aging degree. This paper introduces an aging degree to quantify the deterioration between the measured state and the normal state for gearbox, that is, the aging degree that in range of [0, 1] is used as an evaluation index, and the calculation formula is as follows:

$$g(x) = \begin{cases} 0 & x < e \\ \dfrac{x-e}{f-e} & e \le x \le f \\ 1 & x > f \end{cases} \tag{14}$$

where $g(x)$ is aging degree; ^{x} is the measured criticality; ^{e} and f is the lower and upper limit of the criticality respectively.

Since the lower and upper limit of the criticality is [1], [10], the aging degree can be calculated as follows:

$$g(x) = \begin{cases} 0 & x < 1 \\ \dfrac{x-1}{9} & 1 \le x \le 10 \\ 1 & x > 10 \end{cases} \tag{15}$$

Step 4: Determine performance evaluation level set and membership function. There is no consensus, however, on how to determine the performance evaluation level set. This paper divided the performance evaluation level set as four states: excellent, good, attention, and serious. Additionally, the membership function is the core of fuzzy performance evaluation, and the available membership functions include rectangular and semi-rectangular distributions, trapezoidal and semi-trapezoidal distributions, parabolic distributions, normal distributions, Ridge distribution, and Gaussian distributions.

Step 5: Calculate the membership. After step (4), the membership is quantified, and the membership matrix is $G = \{g_{mn}\}$. Here, g_{mn} denotes the membership of a given evaluation index corresponding to different evaluation levels. The matrix is derived from the monitoring data of the membership function and the performance evaluation level

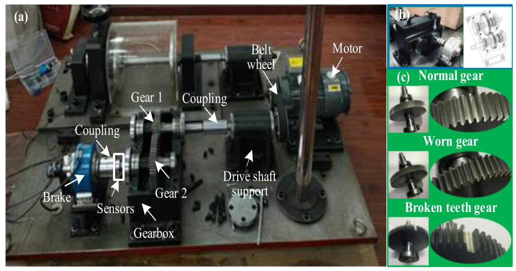
Step 6: Calculate the closeness. There are limitations in evaluation strategy with the maximum membership principle, especially for the case of the equal membership or the membership is very close to each other. In this condition, the elements are regarded as the equal effect on the system in the following situations [0.25, 0.25, 0.25, 0.25] or [0.4, 0.1, 0.4, 0.1] by the maximum membership principle. In practical, however, it is criticized as low-validity by the maximum membership principle when the membership values are so close. To address the problem, this paper proposed to use the closeness to make an evaluation. There are normal and abnormal evaluation results, and we judge the evaluation result as normal when $D_1 > D_2$, the opposite is judged as abnormal when $D_1 \le D_2$. Here, D_1 and D_2 denotes the closeness between the object to be evaluated and the normal state or the abnormal state respectively. The normal closeness D_1 and abnormal closeness D_2 are defined as follows:

$$\begin{cases} D_1 = D_{i1} + D_{i2} \\ D_2 = D_{i3} + D_{i4} \end{cases} \tag{16}$$

where D_{i1}, D_{i2}, D_{i3}, and D_{i4} is the closeness of the target evaluation object to excellent, good, attention, and serious state, respectively. D_{i1}, D_{i2}, D_{i3}, and D_{i4} can be calculated as follows:

$$\begin{cases} D_{ij} = \dfrac{D_m^-}{D_m^+ + D_m^-} \quad (j = 1, 2, 3, 4) \\ D_m^+ = \sqrt{\displaystyle\sum_{j=1}^{n} \left(v_{ij} - y_j^+\right)^2} \\ D_m^- = \sqrt{\displaystyle\sum_{j=1}^{n} \left(v_{ij} - y_j^-\right)^2} \end{cases} \tag{17}$$

where v_{ij} is the membership for each state; D_m^+ and D_m^- are the distances between the objects to be evaluated and their respective ideal state; y_j^+ and y_j^- is the positive and negative ideal values for the i th state respectively. The positive and negative ideal states Y^+ and Y^- is $Y^+ = [y_1^+, y_2^+, \cdots, y_n^+]$ and $Y^- = [y_1^-, y_2^-, \cdots, y_n^-]$ respectively.



(a). Experimental scenes. (b). Layout of vibration monitoring points. (c).Test gears.

FIGURE 2. Gearbox vibration experimental rig.

V. EXPERIMENTAL RESULTS AND DISCUSSION

A. THE GEARBOX VIBRATION EXPERIMENT

To verify the effectiveness of the proposed gearbox evaluation model, we performed a gearbox vibration experiment [35]. Figure 2 shows the gearbox vibration experimental rig, including a driven motor, a gearbox, a brake, a control device, a driven shaft support, and some corresponding electronic units. The experimental scenes are shown in Figure 2(a), layout of vertical vibration monitoring points are shown in Figure 2(b), and 3 testing gears are shown in Figure 2(c). It can be seen that from Figure 2, the working principle is as follows. The motor is employed to drive the gearbox, and a brake is to control the rotational speed. The vertical vibration signals were measured by the accelerometers installed on the shaft, and the collected signals were stored in the industrial personal computer by a data acquisition card. Three gearbox fault modes with normal, worn, and broken teeth were simulated, and 400 raw vibration signal were collected for each fault mode. As shown in Table 2, the signals of each state of the gear are divided into 40 groups, and an average value is extracted for every 10 data. Therefore, there are a total of 120 data samples. Where, the rotating speed is 1500r/min; the sampling frequency is 51.2 kHz; the sampling time is 1.37 min, and the data length is 2700.

TABLE 2. Description of the experimental data.

Fault modes	Condition	Raw vibration data	Samples
Normal	1500r/min	400	40
Worn	1500r/min	400	40
Broken teeth	1500r/min	400	40

B. EXPERIMENTAL RESULTS AND DISCUSSION

1) PARAMETER INITIALIZATION

As a matter of fact, parameter initialization for HMM is determined the initial value of the model $\lambda = (N, M, \pi, A, B)$. Due to the vibration data are continuous signals, however, the emission probability of HMM is calculated under discrete observations. Therefore, AP clustering algorithm is employed for scalar quantization. In this process, the continuous signals were divided into N-1 portions. AP clustering algorithm is employed to calculate the parameters N and M. The clustering number of AP clustering algorithm is taken as the value of the parameter N, and the clustering result is used as the parameter M. Due to the initial value of π and A have less influence on the clustering results of the model, we take a uniform value for the parameters π and A, i.e. $\pi = \left\{\frac{1}{N}\right\}_{N\times 1}$ and

$$A = \begin{bmatrix} 0 & \frac{1}{(N-1)} & \cdots & \frac{1}{(N-1)} \\ \frac{1}{(N-1)} & 0 & \cdots & \frac{1}{(N-1)} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{1}{(N-1)} & \frac{1}{(N-1)} & \cdots & 0 \end{bmatrix}.$$

The probability density function at the state S_j is as follows:

$$\begin{aligned} b_j(o_t) &= \sum_{l=1}^{M_j} c_{jl} \times b_{jl}(o_t) \\ &= \sum_{l=1}^{M_j} c_{jl} \times \frac{\exp\left(-0.5\left(o_t - u_{jl}\right)^T U_{jl}^{-1}\left(o_t - u_{jl}\right)\right)}{\sqrt{(2\pi)^d \times \left|U_{jl}\right|}} \end{aligned} \quad (18)$$

The following conditions shall be satisfied.

$$\int_{-\infty}^{+\infty} b_j(x)dx = 1, \quad 1 \le j \le N \quad (19)$$

Emissions probability matrix B is determined by the parameters $\{M, c, u, U, P\}$. Since the values of c, u and U have a great influence on the convergence results, it is not suitable to take random values, but the initial values of three parameters can be given according to the results of the AP clustering process.

(1) C_{jl} is the l th mixing coefficient in state s_j.

$$C_{jl} = \frac{\lambda}{\rho}, \quad c_{jl} \ge 0 \quad (20)$$

where λ and ρ is the number of observation vectors of x_{jl} and s_j respectively, and $\sum_{i=1}^{M} C_{jl} = 1$, $\quad 1 \le j \le N$.

(2) u_{jl} is the mean of the l th Gaussian distribution in state s_j.

$$u_{jl} = \ell \quad (21)$$

where ℓ is the uniform vector of the observation matrix x_{jl}.

(3) U_{jl} is the covariance matrix of the l th Gaussian distribution in state s_j.

$$U_{jl} = \theta \quad (22)$$

where θ is the covariance matrix of the observation vector x_{jl}; x_{jl} is the element in the observation matrix.

F is the time probability matrix, that is, the probability of the duration in a certain state. Especially,

$$F = \{P_i(d)\} = \left\{a_{ii}^{d-1}(1 - a_{ii})\right\}, \quad (1 \le d \le D) \quad (23)$$

where d is the dimension of the observation value; D represents the probability that E belongs to a certain state continuously in d frequency bands.

Since the initial value of F has little influence on the model, the value of F is taken uniformly.

$$F = \{1/D\}_{N \cdot D} \quad (24)$$

2) DETERMINATION OF MEMBERSHIP FUNCTION

The operation state of gearbox is divided into four states: excellent, good, attention and serious. The deterioration degree is used to evaluate the operation state of gearbox under different working conditions. Correspondingly, the membership were set to $r = [r_1, r_2, r_3, r_4]$, and it indicates the membership for each performance evaluation level. As the membership function of the combination of semi-trapezoid and middle ridge distribution has the advantages of simple shape and strong applicability, it is selected as the membership function of fuzzy comprehensive evaluation, and the distribution diagram is shown in Figure 3.

The membership function r_i can be determined by the aging degree. By Table 1, the membership function corresponding to different performance evaluation levels are determined as $g_1 = 0.24$, $g_2 = 0.39$, $g_3 = 0.53$, and $g_4 = 0.64$. The membership functions corresponding to each performance evaluation level are as follows:

$$r_1(g) = \begin{cases} 0 & g \le 0.24 \\ 1 & 0.24 < g \le 0.39 \\ 0 & g \ge 0.39 \end{cases} \quad (25)$$

$$r_2(g) = \begin{cases} 0 & g < 0.24 \\ 6.8g - 1.63 & 0.24 < g < 0.39 \\ 7.14g - 2.78 & 0.39 \le g < 0.53 \\ 0 & g \ge 0.53 \end{cases} \quad (26)$$

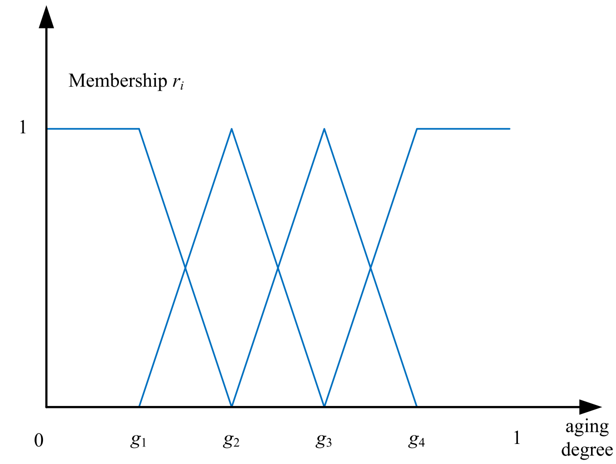


FIGURE 3. Membership function with the semi-trapezoidal and intermediate-ridge distribution.

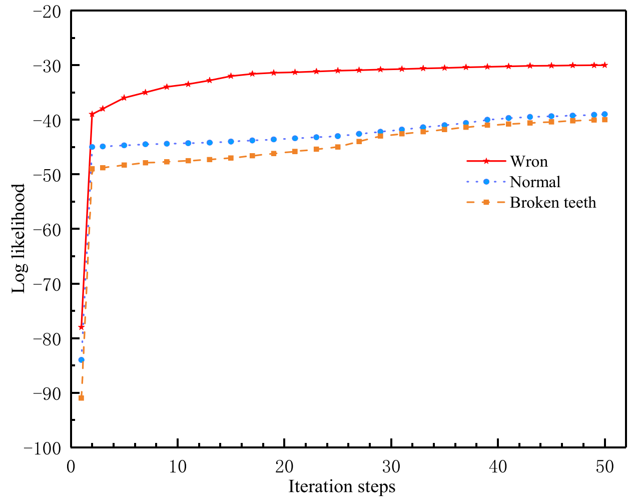


FIGURE 4. HMM training curve of three gear states.

$$r_3(g) = \begin{cases} 0 & g \leq 0.39 \\ 3.78 - 7.14g & 0.39 < g < 0.53 \\ 9.37g - 5 & 0.53 \leq g < 0.64 \\ 0 & g \geq 0.64 \end{cases} \tag{27}$$

$$r_4(g) = \begin{cases} 0 & g \leq 0.53 \\ 6 - 9.37g & 0.53 < g < 0.64 \\ 1 & g \geq 0.64 \end{cases} \tag{28}$$

C. ONLINE DIAGNOSIS

According to AP algorithm, we obtained the parameters $N = 4$, and $M = \{1, 3, 1\}$. Here, the attenuation coefficient is set to 0.5, and the maximum number of iteration steps is set to 40. The parameter initialization was implemented with the obtained AP clustering results, and the three training models λ_0, λ_1, and λ_2 are trained with a likelihood probability threshold of 0.1. Because the logarithmic probability can effectively prevent the likelihood probability from overflowing, the logarithmic probability of the three models is taken during model training, and the change curve of the logarithmic probability with the increase of iteration steps is shown in Figure 4.

By the above three models, the similarity $P(\lambda_1|\lambda_0)$ between the worn state and the normal state, and the similarity $P(\lambda_2|\lambda_0)$ between the broken teeth state and the normal state are determined, which are shown in Figure 5.

It can be seen from Figure 5 that the similarity of the worn state and the broken tooth state to the normal state are $6.86{*}10^{-17}$ and $5.73{*}10^{-25}$, respectively. It means the broken teeth state is a more serious fault than the worn state, due to the lower similarity with the normal state.

Similarly, the similarities between the state to be recognized and the normal, worn, and broken tooth are obtained by using the model λ_0, λ_1 and λ_2, which are shown in Figure 6.

It can be seen from Figure 6(a) that the similarities of the state to be recognized to the normal, the worn and the broken teeth state are $2.82{*}10^{-3}$, $5.4{*}10^{-17}$, and $3.44{*}10^{-25}$, respectively. The maximum similarity appears in the normal

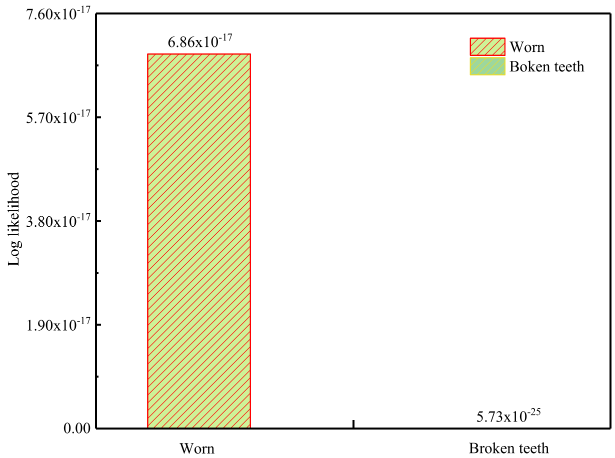


FIGURE 5. The similarity of the worn state and the broken teeth state to the normal state.

state, meaning that the state to be recognized is the normal state. From Figure 6(b), we see that the similarities of the state to be recognized to the normal, the worn and the broken teeth state are $6.86{*}10^{-17}$, $1.84{*}10^{-3}$, and $5.95{*}10^{-13}$, respectively. The maximum similarity appears in the worn state, meaning that the state to be recognized is worn state. It can be seen from Figure 6(c) that the similarities of the state to be recognized to the normal, the worn and the broken teeth state are $5.73{*}10^{-25}$, $8.17{*}10^{-13}$, and $7.33{*}10^{-3}$, respectively. The maximum similarity appears in broken teeth state, meaning that the state to be recognized is broken teeth state. Table 3 lists the similarity results of each state. In summary, it can be seen that the similarity results are consistent with the actual situations, proving that the validity of the established diagnosis model.

D. ONLINE PERFORMANCE EVALUATION

1) WORN STATE

According to Eq. (13), the similarity is converted as the criticality, and the criticality is A_1=4.5−0.13×$\log_{10}$

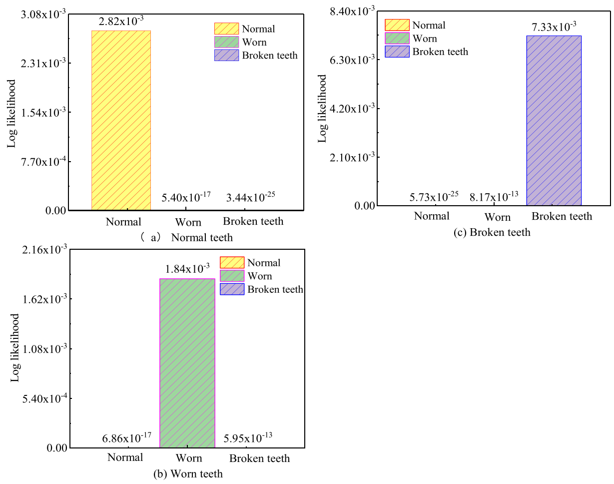


FIGURE 6. The similarity between the state to be recognized and the normal, worn, and broken teeth.

TABLE 3. The similarities of the state to be recognized to the normal, worn, and broken teeth.

Gear state / State to be recognized	Normal	Worn	Broken teeth
Normal	$2.82*10^{-3}$	$5.4*10^{-17}$	$3.44*10^{-25}$
Worn	$6.86*10^{-17}$	$1.84*10^{-3}$	$5.95*10^{-13}$
Broken teeth	$5.73*10^{-25}$	$8.17*10^{-13}$	$7.33*10^{-3}$

$\frac{p}{10^{-15}} = 4.65$. According to Eq. (15), the aging is obtained as $g_{worn} = 0.41$. Substituting the aging degree g_{worn} into each membership function, and the obtained membership matrix is $r_{worn} = [0, 0.15, 0.85, 0]$.

According to the principle of maximum membership degree, the membership degree of gearbox in the state of attention is the largest. According to the above calculation, the evaluation result can be attention. But in this paper, the comprehensive closeness degree is chosen to evaluate the operation state; the advantages of this method are not obvious here.

We set the gearbox in the ideal state of excellent, good, attention and serious as $\begin{cases} Y_1^+ = [1\ 0\ 0\ 0] \\ Y_2^+ = [0\ 1\ 0\ 0] \\ Y_3^+ = [0\ 0\ 1\ 0] \\ Y_4^+ = [0\ 0\ 0\ 1]. \end{cases}$

According to Eq. (17), the distances from the worn state to four ideal states $\lfloor D_{m1}^+, D_{m2}^+, D_{m3}^+, D_{m4}^+ \rfloor$ and the closeness can be obtained as shown in Table 4.

According to Eq. (16), we obtained the normal and abnormal closeness, with the values of $D_1 = 1.024$ and $D_2 = 1.362$, respectively. With such an evaluation strategy, it can be determined that the gearbox operating state is abnormal due to $D_1 < D_2$ Additionally, the evaluation result is attention as for $D_{i3} > D_{i4}$, i.e. the state to be evaluated is closer to attention. In this case, the evaluation result is same as the result by the maximum membership principle.

TABLE 4. Distance and closeness between the evaluated state (worn) and four ideal states.

Evaluation levels $j,(j\in[1,2,3,4])$	Distance D_{mj}	Closeness D_{ij}
Excellent	1.321	0.50
Good	1.202	0.52
Attention	0.212	0.86
Serious	1.321	0.50

TABLE 5. Distance and closeness between the evaluated state (broken teeth) and four ideal states.

Evaluation levels $j,(j\in[1,2,3,4])$	Distance D_{mj}	Closeness D_{ij}
Excellent	1.225	0.38
Good	1.225	0.38
Attention	0.665	0.53
Serious	0.750	0.82

2) BROKEN TEETH STATE

Similarly, according to Eq. (13), (15), and (25) to (28), we obtained the membership matrix of the broken teeth with the value of $r_{broken} = [0, 0, 0.53, 0.47]$. It is worth noting that if we only rely on the principle of maximum membership, it is difficult to make a reasonable judgment on the operation status of the gearbox for each membership under the condition of broken teeth, because the membership under attention and serious is not much different, and any single judgment of any membership level will discard the information of another membership level. This is also the reason why the comprehensive approach degree is adopted in this paper.

According to Eq. (17), the distances from the broken teeth state to four ideal states, and the closeness is obtained, which are shown in Table 5.

According to Eq. (16), we obtained the normal and abnormal closeness, with the values of D_1=0.76 and D_2=1.35, respectively. With such an evaluation strategy, it can be determined that the gearbox operating state is abnormal due to $D_1 < D_2$. Additionally, the evaluation result is serious as for $D_{i3} > D_{i4}$, i.e. the state to be evaluated is closer to serious.

Table 6 lists the evaluation results of the gearbox under different evaluation strategies. It can be seen from Table 6 that the evaluation strategy with the closeness can clearly distinguish the evaluation results of the broken tooth state and the wear state, avoiding the inapplicability of maximum membership principle in the case of the equal membership.

It is found that the broken teeth state is a more severe fault than the worn state base on the results in Figure 5, or comparing with the memberships of two fault modes. The evaluation strategy with the closeness is also clearly distinguished the evaluation results of the broken teeth state

TABLE 6. Comparison of evaluation results under different evaluation strategies.

	Maximum membership degree	Closeness
Worn	Attention	Attention
Broken teeth	Attention	Serious

from the worn state. However, the evaluation results with maximum membership principle are all attention state, indicating that the evaluation strategy with maximum membership principle brings some misleading to the actual operation and maintenance due to causing the above-mentioned judgment ambiguity by discarding information. Therefore, we can conclude that the proposed performance evaluation model has achieved good performance.

VI. CONCLUSION

This paper proposed a gearbox online diagnosis and performance evaluation approach, and performed a gearbox vibration experiment to verify the effectiveness of the proposed model at the PHM Laboratory in Jiangxi University of Science and Technology. The conclusions obtained are as follows:

1) HMM is applied to the evaluation of the operation state of the gearbox. After obtaining the observation value and initial parameters, Baum-Welch algorithm is used to adjust the parameters of the normal, worn and broken models. By defining the likelihood ratio of HMM as a similarity, the online diagnosis has achieved a good performance, indicating that the similarity strategy is effectively in gearbox fault diagnosis.
2) The forward-backward algorithm is used to quantify the similarity between states. By a closeness evaluation strategy, the evaluation results of the broken teeth state from the worn state is distinguished clearly. It is concluded that the closeness evaluation strategy is avoid the inapplicability of maximum membership principle in the case of the equal membership, and has achieved a good performance. Moreover, the comprehensive closeness evaluation strategy can identify different operation states more sensitively.

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