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Disruption Recovery for Urban Public Tram System: An Analysis of Replacement Service Selection

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ABSTRACT Disruption recovery is an imperative task for the tram system, which has caused widespread concerns in the public transport area. In this paper, how to provide replacement services for stranded passengers during short-term unplanned events has been investigated. A decision support tool for the tram company to dispatch appropriate replacement services by taxis or buses has been proposed. First, critical factors for disruption recovery decisions are identified as commuters' behaviors, recovery cost, and service level. Then, critical factors are model as replacement service selection strategies among bus, taxi, and bus-taxi hybrid. Finally, numerical analyses are conducted to shed light on key variables affecting the tram company's benefit function as well as replacement service decisions. The numerical analyses provide immediate managerial implications. The proposed method can be easily extended to real-time disruption recovery for urban public transit system by capturing passenger patterns and vehicle status through big data technologies.

INDEX TERMS Tram system, public transportation, disruption recovery service, collaboration.

I. INTRODUCTION

Tram system is a popular public transport in metropolises around the world [1]–[5]. Compared with private cars, tram system is superior in high-capacity, low-cost, and low-carbon emission, while is inferior in system robustness. It is highly vulnerable to the disruptive events such as power failure, explosion, terrorist rumor, vehicle breakdown, etc. [6]. Tram disruptions can not only cause passengers' travel inconvenience, but also results in complains or even lost faith to the system [7]. Therefore, how to respond quickly to unplanned events and maintain a smooth public transportation service is an imperative task for tram companies.

To mitigate disruption impacts, tram operators have attempted to employ recovery methods, in which buses are adopted frequently [8]. Recently, a new method emerged in major German cities started to working with local taxi providers to ensure replacement service, such as in Munich, Berlin and Frankfurt [9]. Taxis can arrive at the first time and provide timely emergency evacuation services. However, it takes a long time for the bus to reach the accident station, and interrupt the normal operation schedule of buses. When an unplanned short-term disruption happens, it is difficult to judge which vehicles should come to provide replacement service. These events inspire our interest to provide a systematic decision-making tool for the tram company to choose proper replacement vehicles for mitigating the impact of unplanned short-term disruptions.

The passengers' behavior during disruption is one of the most important elements in the decision-making process. If the replacement service is late or insufficient in capacity, passengers may abandon the system, leaving TOCs (Tram Operating Centers) with lots of complaints and goodwill loss. From the financial viewpoint, when taxis or buses are

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used to recover the transport service during disruptions, the tram company has to cover the entire recovery service cost, while the passengers have no obligation to pay. Consequently, the tram operators need to balance between the recovery cost and the service level, which is highly correlated to passengers' reaction to the replacement service. In terms of transport capacity, a sufficient number of taxis or buses need to be available to handle the on-board passenger volume in the event of disruptions. Moreover, the difficulty and the expenditure of having enough taxis or buses will differ significantly when the passenger volume, disruption duration, and service time vary.

This study aims to identify when the traditional replacement approach with buses should be replaced or supplemented with taxis. Our contributions contain three aspects: (1) to propose a decision support tool for tram operators to appropriately determine the recovery service provided by bus, taxi, or both; (2) to integrate the passengers behavior into decision model and then explore the balance between the recovery cost and service; (3) to derive theoretical results and use numerical analysis approach to generate managerial implications.

The remainder of the paper is organized as follows. Section 2 reviews related work on recovery strategies on disruption management in tram system. Section 3 describes the decision problem under consideration and explains the preliminaries such as assumptions and basic terms needed for mathematical modeling. Section 4 presents the mathematical models and their theoretical results and implications. A set of numerical examples and sensitivity analyses are conducted in Section 5 to demonstrate the managerial insights. Finally, a summary of findings from this study, future research extensions, and concluding remarks are given in Section 6.

II. LITERATURE REVIEW

The efforts made for disruption recovery can be classified into two categories: the operational level and the strategic level. Moreover, as a public service system, how to mitigate the effect of passengers is important for the tram system. In this section, the most related work is reviewed from three perspectives.

A. DISRUPTION RECOVERY FORM THE OPERATIONAL LEVEL

Generally, disruption recovery from the operational level can be classified into two categories: pro-active planning and optimization-based re-scheduling. Pro-active planning refers to design of a robust system to cope with disruptions [10]. For example, a stream of literature investigated the vulnerability of public transport system, which refers to assessing the reliability of the network structure [11]–[13]. The other methods in literature are ranging from structural improvement [14], flexibility timetable design [15], and mitigation process assessment [16]. The system re-scheduling for public transport systems aims to recovery the system quickly after disruption happens. Three most common questions have been examined in the literature: (a) ''timetable adjustment'' [17], which attempts to adjust train timetables in response to disruptions; (b)''rolling stock'' [18], which relies on available vehicle capacity to cope with unexpected events; and (c) ''crew re-scheduling'' [19], which tries to adjust workforce size and utilization rate to respond to disruptions.

B. DISRUPTION RECOVERY FROM THE STRATEGIC LEVEL

Management strategies for disruption recovery are investigated from the strategic level, in which collaboration is one of the most used strategies [20]. The potential collaborators for tram operating companies to provide disruption service recovery include bus and taxi companies. The bus collaborative mechanism, which is also called as bus bridging service, is widely adopted to respond to disruptions in urban transit rail systems with focus on how to set bus routes, how to design an efficient bus bridging network and how to allocate resources [8], [21]. The taxi collaborative recovery service is also investigated in previous literature [22]. Zeng *et al.* conducted a study to answer how a tram company should collaborate with a taxi company to provide efficient recovery service during disruption [6]. Yang and Chen investigated the pricing strategy between urban rail transit systems and ride-hailing platforms to collect vehicles for stranded passenger evacuation [23]. It has been pointed out that the key deficiencies of bus bridging service, namely slow response, deployment difficulty, and new uncertainties caused by buses [24]. Meanwhile, taxis are flexible and can response quickly. How to achieve complementary advantages of taxis and buses? What are the applicable scenarios of taxis, buses or both? These questions are still worthy of study.

C. RESEARCH ON PASSENGER BEHAVIOR FOR DISRUPTION MITIGATION

The last stream of literature related to our work is behavioral study in the context of disruption management. In recent years, there has been a wave of research on the passenger-oriented disruption management in public transport system. Some scholars have studied how to link the passenger flow under disruption with the mitigation strategies. For example, Cadarso *et al.* classified the passenger patterns under disruption into four categories, and the interaction between passenger flows and optimization rescheduling were formulated [25]. Veelenturf *et al.* took the dynamics of passenger behavior explicitly by assigning the dynamic passenger flows to the re-scheduled timetables [26]. There are also some research aimed at assessing the passenger inconvenience experienced during disruption. For example, Zhu and Goverde investigated passenger-oriented rescheduling problems and evaluated the contingency plan by aggregating each passenger inconvenience [27]. Durand *et al.* assess rescheduling strategies of the metro system considering the inconvenience of passengers, where societal passenger costs associated waiting times are used [28]. Yang *et al.* pointed out that the expected customer waiting time is generally considered as an important measure of service quality, since it can obviously affect passengers' decisions [29]. Existing literature provides a lot of reference ideas for systematic restoration of behavioral direction. What is the passenger reaction for the tram system under disruption? How to balance between service level and recovery cost for the tram company? These issues need to be explored in this study.

D. SUMMARY

The existing findings provide some fundamental elements for our research. In this article, we concentrate on how to appropriately determine the recovery service provided by bus, taxi, or both when the tram system are undergoing short-term disruptions. The existing findings provide some fundamental elements for our research, but unfortunately are not directly applicable to this problem. The challenge lies in the areas of quantifying the loss of the tram company and balancing between the replacement service level and the cost from the view of different passenger behaviors. How should the tram company make decisions as to whether to use taxis, buses or both? How to better utilize the advantages of both taxis and buses during disruption recovery? Our research aims to bridge the gaps in current literature by providing a set of models that take these factors into consideration to guide the involving parties to make efficient decisions. The findings of this study will provide a foundation for developing future long-term decision-support systems and procedures for public tram disruption management.

III. PROBLEM STATEMENT AND MODELING PRELIMINARIES

In this section, the research problem is discussed in subsection 3.A. Then the basic notations and assumptions as the preliminaries for modeling are introduced in subsection 3.B.

A. PROBLEM STATEMENT

The process of disruption recovery in a tram system is demonstrated in Figure 1. Consider the scenario when an unexpected interruption breaks down one or a few links between tram stations causing a temporary closure of the tram line. For example, in Figure 1, the link between station s1 and s2 is down and the trams within this area are blocked. Meanwhile, commuters along the closed tram line need to find alternative ways to arrive at their destination. Since the trams outside the blocked area can still move, the passengers could get to other transfer stations by trams. But commuters within the blocked area will either have to leave the system or wait for the TOC to arrange for a replacement service by taxis or buses.

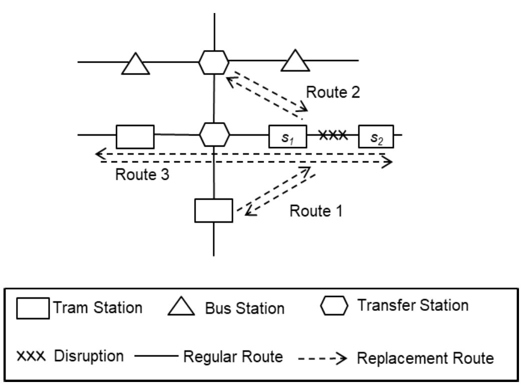

FIGURE 1. Schematic diagram of replacement service in a tram system.

The TOC has to decide on an alternative replacement route to transfer or to evacuate the passengers within the blocked area. A common response of TOC to such disruptions is to divert the passengers to the nearest tram station (i.e., Route 1 in Figure 1) or the nearest transfer station for buses or subways (i.e., Route 2 in Figure 1) depending on the situation. Sometimes, the TOC would provide the replacement service in parallel to the broken tram line (i.e., Route 3 in Figure 1). According to the statistics of the TOC in Munich, the portions of the three scenarios is 24%, 59% and 17%, respectively [9]. In this paper, we consider the first two cases only because the last case is usually dominated by bus service.

Once the recovery destination is selected, the TOC will consider three recovery service choices - bus, taxi, and both. If the tram company takes no action, all of the passengers would have to find alternatives on their own to reach their destinations and will likely require ticket refund, compensation, or even never use the tram again. We refer to this situation as complete passenger loss. However, if the tram company provides a replacement service, a portion of passengers will accept and wait for such a service. We consider this situation as partial loss. The difference between the two scenarios, called reduced loss, is adopted to measure the benefit of the tram company and support the TOC's decision-making during disruptions [6].

The biggest challenge for the taxi or bus company is to arrange the required number of vehicles by the desired time that should be shorter than the disruption duration. The potential vehicles can be either the idle ones in depots or those on duty and close to the disruption site. The efforts for arranging taxis or buses to provide recovery service include internal communications, locating vehicles, assigning drivers, making suficient capacity available and scheduling. In what follows, we first present our assumptions, basic terms, and notation list.

B. MODELING PRELIMINARIES: NOTATIONS AND ASSUMPTIONS

The decision variables are the fleet size N of taxis or buses, and hereinafter we use subscripts or superscripts t, b to represent the taxi replacement service and bus service, respectively. For example, the fleet size of taxis and buses are N_t and N_b, respectively. The main input parameters and notations are introduced first, for example, average arrival time of vehicles t_a, loss rate of passengers c_l, and round-trip service time t_s, based on which, the maximum service time tm and cost for arranging taxis or buses $C(N)$ can be calculated. A complete list of notations is given in Appendix A.

1) AVERAGE ARRIVAL TIME OF VEHICLES t_a

t_a denotes the average arrival time of vehicles, and can be estimated at each situation. For example, the arrival time of taxis t_a^t is negatively related to the number of vacant taxis available [30], while the average arrival time of buses t_a^b can be estimated by scheduling time plus the travel time of the available buses. Here, we assume that $t_a^t < t_a^b$, which implies that taxis are faster than buses.

2) TOTAL BLOCKED PASSENGER VOLUME P

Once an unplanned disruption happened, the TOC will first identify the blocked segment, which usually involves a couple of trams near the disruption place. Passengers outside the blocked area will transfer or shift to other transit line easily, while passengers within the blocked segment need the bus/taxi device to transfer them to the nearest transit station. In the modern advanced public transport system, the total blocked passenger volume could be estimated, and in this paper, we denote it as P.

3) PASSENGERS BEHAVIOR: LOSS RATE OF PASSENGERS l

By interviewing the managers of the tram systems in both Dalian and Munich, we have tried to understand the basic pattern of passengers' behaviors. Once the passengers are aware of disruption, they will decide to wait for the replacement service or to leave the system. Generally, the longer the waiting time, the more passengers will leave. Moreover, the decisions of passengers are intuitively dependent on two factors. The first one is the internal factor, which is the passenger's own business at the time of tram breakdown; for example, those on tight schedules will probably choose to leave right away to find other options. The other factor is the external environment for passengers to find alternatives, such as the convenience to find replacement vehicles, the distance to the nearest transfer station, as well as the extra fee that has to be paid by them to switch to an alternative. Clearly, the easier and cheaper it is to find an alternative, the more passengers will leave.

Let l be the departure rate of the disrupted passengers in unit time (#persons/unit time). Naturally, lower value of l captures the situations where the passengers react more patiently when the access to alternatives is harder, or during non-rush hour when people are not in a hurry. In contrast, the situations where disruptions occur during rush-hour, or the disruption site is close to a transfer station or business center, are more likely to be approximated with a higher value of l. Here, we assume the passengers' departure rate is constant. In reality, the passengers' leaving rate is likely to be increased as time passes by. For example, Yang et al. modeled the departure rate as an exponential function [30]. Although the formulation and numerical solutions using such a function are still available, the analyses based on them would be too complex to yield useful implications that we seek to provide.

4) PERCEIVED PASSENGER LOSS RATE c_l

This loss occurs when passengers leave to find alternative on their own instead of waiting for the replacement service, and can be quantitative or qualitative. For example, he/she may request ticket refund, which is quantitative, or complain to the people around, which may lead to reputation damage (goodwill loss), or loyalty loss. In our research, we use c_l to quantify the unit passenger's loss that the tram company could perceive. Naturally, a higher value of c_l means a higher service level the tram company would like to provide.

5) ROUND-TRIP SERVICE TIME t_s

The replacement route is chosen by the TOC, and can be the route from the disruption point to the nearest tram stop, or to a transfer station as shown in Figure 1. In our cases, each taxi or bus is assumed to serve multiple trips, because the replacement route is usually short in urban area. We denote the average round-trip service time of a vehicle as t_s, and clearly $t_s^t < t_s^b$.

6) RELATIONSHIP BETWEEN THE FLEET SIZE N AND THE MAXIMUM SERVICE TIME t_m

The arrival time of replacement vehicles t_a will break the whole process into two stages as shown in Figure2. Once the passengers are informed that there will be replacement service, they will wait from the beginning of disruption until taxis or buses arrive at time t_a. This is regarded as the first stage. During this stage, passenger volume will decrease with the loss rate l.

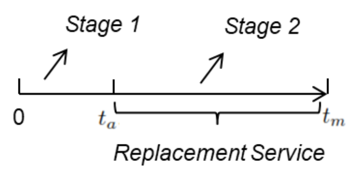

FIGURE 2. Process of disruption recovery service.

The second stage starts from the time when replacement vehicles arrives at t_a to the maximum service time when all waiting passengers are serviced at t_m. During this stage, some passengers will be transferred by taxis or buses, whereas others choose to leave as well because the taxis or buses cannot take all the passengers at once due to the capacity constraints. Here we assume that the leaving rate remains unchanged from stage 1 to stage 2. Then the following relationship can be obtained.

$$P - VN\left(t_m - t_a\right)/t_s - lt_m = 0. \tag{1}$$

Here, $(t_m - t_a)$ means the service time which is calculated by the end of service time t_m minus the arrival time of vehicles t_a. Then $(t_m - t_a)/t_s$ is the number of trips a vehicle could travel where t_s is the round-trip service time. When $(t_m - t_a)/t_s$ is multiplied by the capacity of a vehicle V, and the fleet size of vehicles N, it can yield the total served passenger volume $VN\left(t_m - t_a\right)/t_s$ $- lt_m$ is the passenger volume that discard the replacement service. In order to get a closed-form formulation, the integer constraint of the vehicle number is relaxed as a continuous one, which results in the following equation.

$$t_m = \frac{P - lt_a}{\mu + l} + t_a, \tag{2}$$

where $\mu = NV/t_s$.

7) COST FOR ARRANGING TAXIS OR BUSES $C(N)$

The taxi or bus company's expense on providing replacement service is correlated with the fleet size, and the total service time, which can be written as follows.

$$C\left(N\right) = c\left(t_m - t_a\right)N + kN^2. \tag{3}$$

The total cost in Eq. (3) is justified below. First, c is the fixed cost per unit time, and $c \geq 0$; for example, gasoline consumption and labor compensation fall into this category [29], [30]. The second part, kN^2, represents the taxi/bus company's variable operating cost that tends to have a non-linear relationship with the number of taxis/buses in service. The variable cost elements include communication cost, scheduling cost, goodwill loss, and opportunity cost for an empty taxi/bus traveling to the disruption site [29], [30]. It is also not difficult to see that the sum of these variable cost factors during disruptions are likely to rise rapidly as the fleet size increases which means that the marginal cost increases. Therefore, a quadratic function is used to capture this phenomenon.

IV. STAKEHOLDERS' DECISION PROCESS AND PROFIT FUNCTIONS

The decisions of the involving stakeholders will follow a sequential process shown in Figure3. When a disruption occurs, the TOC will estimate the passenger volume and disruption duration, determine the replacement service route, report the information to the taxi or bus company, and ask for replacement vehicles. The taxi/bus company will then determine the fleet size under the given information. During this process, passengers decide to wait or leave according to the waiting time spanning from the beginning of the replacement service to the time of final boarding. Finally, the tram company assesses the replacement service by balancing between the replacement cost and service rate. In what follows, the decision functions of each company under different service tools are introduced.

According to the sequential decision process, the decision functions under three replacement services, *taxi-only*, *bus-only*, and hybrid, are formulated in subsections 4.A, 4.B and 4.C. Moreover, two versions of the hybrid service, namely Passenger-Dividing strategy (called *P-strategy*) and Time-Dividing strategy (called *T-strategy*), are proposed to determine how taxis and buses work together.

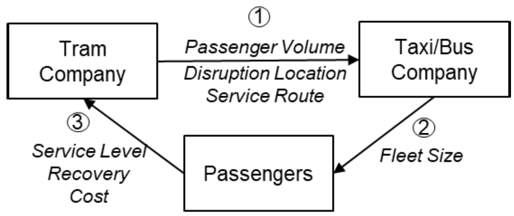

FIGURE 3. Sequential decision process of replacement service.

A. PROFIT FUNCTIONS UNDER TAXI-ONLY SERVICE

1) TAXI COMPANY'S PROFIT FUNCTION UNDER TAXI-ONLY SERVICE

We first obtain the taxi company's decision function under taxi-only service. According to the total cost function in Eq. (3), given a fleet size of taxis N_t, the taxi company's recovery service profit $\Pi_t\left(N_t\right)$ is calculated in Eq. (4). Note that the subscript or superscript t denotes the taxi-only service.

$$\Pi_t\left(N_t\right) = \left(p_t - c_t\right)\left(t_m^t - t_a^t\right)N_t - k_t N_t^2, \tag{4}$$

where p_t is the price of taxis per unit time (\$/unit time), c_t is the fixed cost of taxis per unit time (\$/unit time), k_t is scale factor of taxis' cost, and $\left(t_m^t - t_a^t\right)$ is the total taxi service time. Substituting Eq. (2) into Eq. (4), we can get

$$\Pi_t\left(N_t\right) = \frac{N_t\left(p_t - c_t\right)\left(P - lt_a^t\right)}{u_t + l} - k_t N_t^2, \tag{5}$$

where $u_t = \frac{N_t V_t}{t_s^t}$,$V_t$ is the capacity of a taxi, and t_s^t is the round-trip service time of taxis, $(P - lt_a^t)$ is the total passenger volume at taxis arrival.

Proposition 1: Let $\theta_t = V_t/t_s^t$. The taxi company's profit function in Eq. (5) is concave with respect to the fleet size, and the optimal fleet size can be obtained by the following expression (6), as shown at the bottom of the next page.

Proof: See Appendix B.

It can be pinpointed that the optimal N_t^* decreases with θ_t and k_t, because they are always in the denominator in Eq. (6). Consequently, N_t^* increases with the round-trip service time t_s^t, but deceases with capacity V_t and cost factor k_t. This can be explained straightforwardly that a smaller number of taxis will be needed if the vehicle could accommodate more passengers, or the cost is higher. The positive correlation between t_s^t and N_t^* implies that when the replacement service route is shorter, used during the non-rush hour, or the travel speed is faster, the fleet size will be smaller.

2) TRAM COMPANY'S BENEFIT FUNCTION UNDER TAXI-ONLY SERVICE

The tram company's decision-making function, which is referred to as reduced loss, is defined as the difference between the *complete loss* (i.e., loss of doing nothing during a disruption) and the *partial loss* (i.e., working with the taxi company to provide recovery service) [6]. The formulation of the tram company's reduced loss should consider both passenger loss and financial payment. Let $\Theta_t(\cdot)$ denote the tram company's reduced loss under taxi-only service, it can be calculated in Eq. (7).

$$\Theta_t\left(N_t\right) = c_l P - c_l t_m^t l - p_t N_t (t_m^t - t_a^t), \tag{7}$$

where c_l is the perceived loss of one passenger as defined in subsection 3.B. Then, $c_l P$ is the complete passenger loss when no replacement service is provided. Since $t_m^t l$ is the volume of leaving passengers when taxi replacement service is provided, $c_l t_m^t l$ is the partial passenger loss. $p_t N_t (t_m^t - t_a^t)$ is the payment to the taxi company, which is the financial loss from the tram company's standpoint. Therefore, Eq. (7) not only captures the behavior of passengers but also provides a compromising solution between recovery cost and service level.

Substituting Eq. (2) into Eq. (7), we can obtain the reduced loss under taxi-only service in Eq. (8), where the optimal taxi fleet size N_t^* is obtained according to Proposition 1.

$$\Theta_t\left(N_t^*\right) = \frac{N_t^*(P - l t_a^t)(\lambda_t - p_t)}{\mu_t^* + l}, \tag{8}$$

where $\lambda_t = c_l V_t / t_s^t$, and $\mu_t^* = N_t^* V_t / t_s^t$.

B. PROFIT FUNCTIONS UNDER BUS-ONLY SERVICE

1) BUS COMPANY'S PROFIT FUNCTION UNDER BUS-ONLY SERVICE

Similarly, the profit function of the bus company $\Pi_b\left(N_b\right)$ can be obtained in Eq (9).

$$\begin{aligned}\Pi_b\left(N_b\right) &= N_b\left(p_b - c_b\right)\left(t_m^b - t_a^b\right) - k_b N_b^2 \\ &= \frac{N_b\left(p_b - c_b\right)\left(P - l t_a^b\right)}{\mu_b + l} - k_b N_b^2,\end{aligned} \tag{9}$$

where $u_b = N_b V_b / t_s^b$, p_b and c_b denote the unit price and unit cost of buses (\$/unit time) respectively, t_a^b is the average arrival time of buses, $(t_m^b - t_a^b)$ is total service time, k_b is scale factor of buses, V_b is the capacity of a bus, and t_s^b is the round-trip service time of buses.

According to Eq. (9), the optimal bus fleet size N_b^* can be calculated as (10), as shown at the bottom of this page, where $\theta_b = \frac{V_b}{t_s^b}$.

It is obvious that the nature of Eqs. (9), (10) are similar to that of Eqs. (5), (6), and that the optimal bus fleet size N_b^* increases with the round-trip service time t_s^b, but deceases with the capacity V_b and the cost factor k_b.

2) TRAM COMPANY'S BENEFIT FUNCTION UNDER BUS-ONLY SERVICE

Let V_b, t_s^b, t_a^b, p_b denote the capacity, round-trip service time, average arrival time, and unit price of buses, respectively. The tram company's benefit function under the bus-only service, denoted as $\Theta_b(\cdot)$, can be formulated in Eq. (11).

$$\Theta_b\left(N_b^*\right) = \frac{N_b^*(P - l t_a^b)(\lambda_b - p_b)}{\mu_b^* + l} \tag{11}$$

where $\lambda_b = c_l V_b / t_s^b$, and $\mu_b^* = N_b^* V_b / t_s^b$.

Combining with the tram company's benefit function under taxi-only service in Eq. (8), we obtain Proposition 2 as follows.

Proposition 2: If the perceived unit passenger loss c_l satisfies the condition $c_l^t > p_t t_s^t / V_t$, the tram company will adopt the taxi-only service; if $c_l^b > p_b t_s^b / V_b$,,the tram company will adopt bus-only service.

Proof: Since $(P - l t_a^t)$ and $(P - l t_a^b)$ represent the served passenger volume, they are both positive. Therefore, setting Eq. (8) and Eq. (11) to be greater than zero will yield the solution given in Proposition 2.

Note that $p_t t_s^t$ is the payment for one taxi per trip, and when divided by capacity V_t, it yields the unit payment rate per served passenger by taxi. Similarly, $p_b t_s^b / V_b$ is the unit payment rate per served passenger by bus. Thus, the tram company's decision as whether to provide recovery service depends on the unit perceived passenger loss c_l and the unit payment rate $(p_t t_s^t / V_t, p_b t_s^b / V_b)$. It is interesting to find that the bottom line for the tram company to provide replacement service is determined by four parameters, namely perceived

$$\begin{aligned}N_t^* = \frac{2l}{3\theta_t} &+ \sqrt[3]{\frac{l^3}{27\theta_t^3} + \frac{(p_t - c_t)\left(P - l t_a^t\right)}{4k_t\theta_t^2} + \sqrt{\frac{(p_t - c_t)^2\left(P - l t_a^t\right)^2}{16k_t^2\theta_t^4} + \frac{l^3\left(p_t - c_t\right)\left(P - l t_a^t\right)}{54k_t\theta_t^5}}} \\ &+ \sqrt[3]{\frac{l^3}{27\theta_t^3} + \frac{(p_t - c_t)\left(P - l t_a^t\right)}{4k_t\theta_t^2} - \sqrt{\frac{(p_t - c_t)^2\left(P - l t_a^t\right)^2}{16k_t^2\theta_t^4} + \frac{l^3\left(p_t - c_t\right)\left(P - l t_a^t\right)}{54k_t\theta_t^5}}}.\end{aligned} \tag{6}$$

$$\begin{aligned}N_b^* = \frac{2l}{3\theta_b} &+ \sqrt[3]{\frac{l^3}{27\theta_b^3} + \frac{(p_b - c_b)\left(P - l t_a^b\right)}{4k_b\theta_b^2} + \sqrt{\frac{(p_b - c_b)^2\left(P - l t_a^b\right)^2}{16k_b^2\theta_b^4} + \frac{l^3\left(p_b - c_b\right)\left(P - l t_a^b\right)}{54k_b\theta_b^5}}} \\ &+ \sqrt[3]{\frac{l^3}{27\theta_b^3} + \frac{(p_b - c_b)\left(P - l t_a^b\right)}{4k_b\theta_b^2} - \sqrt{\frac{(p_b - c_b)^2\left(P - l t_a^b\right)^2}{16k_b^2\theta_b^4} + \frac{l^3\left(p_b - c_b\right)\left(P - l t_a^b\right)}{54k_b\theta_b^5}}},\end{aligned} \tag{10}$$

unit loss c_l, unit price p, round-trip service time t_s, and capacity of vehicle V.

In addition, let $\rho_1 = N_t^*(\mu_b^* + l)/N_b^*(\mu_t^* + l)$, $\rho_2 = (P - lt_a^t)/(P - lt_a^b)$, and $\rho_3 = (\lambda_t - p_t)/(\lambda_b - p_b)$, we obtain Proposition 3 as follows.

Proposition 3: If $\rho_1\rho_2\rho_3 > 1$, taxi-only service is better than bus-only service for the tram company. Otherwise, the bus-only service is better.

Proof: According to the tram company's benefit functions under taxi-only and bus-only services in Eqs. (8) and (11), and after some grouping, we can get

$$\frac{\Theta_t\left(N_t^*\right)}{\Theta_b\left(N_b^*\right)} = \frac{N_t^*(\mu_b^*+l)}{N_b^*(\mu_t^*+l)} \times \frac{(P-lt_a^t)}{P-lt_a^b} \times \frac{\lambda_t - p_t}{\lambda_b - p_b} = \rho_1\rho_2\rho_3.$$

If $\rho_1\rho_2\rho_3 > 1$, it will lead to the result.

C. PROFIT FUNCTIONS UNDER HYBRID APPROACH

Evidently, the taxi-based recovery service offers quick response but results in higher cost and limited capacity, while the bus-based replacement service offers larger capacity but slow responsiveness. Would the hybrid approach outperform the single-type-vehicle approach? We answer this question in this subsection through two hybrid approaches, namely *P-strategy*, and *T-strategy*. Both strategies determine how many passengers should be assigned to be transferred by taxis or buses, and how many taxis and buses should be ordered. The main difference between the two strategies is that taxis will stay in service after buses arrive under P-strategy, but taxis will quit the service after buses arrive under *T-Strategy*. Detailed process and mathematic models of the two strategies are illustrated in the following.

1) P-STRATEGY

In *P-Strategy*, the solution to assign passengers to either taxi or bus is by dividing the total passenger volume into two parts, which means a portion of passengers will be transferred by taxis while the rest of them will take buses. The optimal fleet size of taxis and buses will be determined accordingly. During the service process, taxis will not leave when buses arrive until the passengers assigned to taxis are all moved. This is done in such a way that taxis and buses will be called to take a fixed number of passengers separately. The implementation procedure of *P-Strategy* is summarized as follows.

a. Once an unplanned disruption is confirmed, both taxi and bus centers are notified, and will then start arranging the replacement service.

b. Taxis will arrive first to pick up a portion (denoted as β) of the affected passengers. The taxi company's profit in this situation can be calculated in Eq. (12). Note that the subscript P represents P-strategy.

$$\Pi_{t,P} = \frac{\beta N_{t,P}\left(p_t - c_t\right)\left(P - lt_a^t\right)}{\mu_t + \beta l} - k_t N_{t,P}^2, \tag{12}$$

The optimal taxi numbers, denoted as $N_{t,P}^*$ (note that $N_{t,P}^* \neq N_t^*$), can be determined by Eq. (13), as shown at the bottom of this page.

c. Buses will arrive at t_a^b after taxis to pick up the rest of the passengers, i.e., $(1-\beta)P$. During this process, taxis are still in service till the passengers assigned to taxis are all serviced. Similarly, the optimal bus number $N_{b,P}^*$ can be obtained by Eq. (14).

$$N_{b,P}^* = \frac{2l(1-\beta)}{3\theta_b} + \sqrt[3]{\varepsilon_1 + \varepsilon_2} + \sqrt[3]{\varepsilon_1 - \varepsilon_2}, \tag{14}$$

where,

$$\varepsilon_1 = \frac{(1-\beta)^3 l^3}{27\theta_b^3} + \frac{(1-\beta)\left(p_b - c_b\right)\left(P - lt_a^b\right)}{4k_b\theta_b^2},$$

$$\varepsilon_2 = \sqrt{\frac{(1-\beta)^2\left(p_b - c_b\right)^2\left(P - lt_a^t\right)^2}{16k_b^2\theta_b^4} + \frac{(1-\beta)^4 l^3\left(p_b - c_b\right)\left(P - lt_a^b\right)}{54k_b\theta_b^5}}.$$

Moreover, the bus company's profit can be calculated in Eq. (15).

$$\Pi_{b,P} = \frac{N_{b,P}(1-\beta)\left(p_b - c_b\right)\left(P - lt_a^b\right)}{\mu_b + l(1-\beta)} - k_b N_{b,P}^2. \tag{15}$$

Therefore, under such a procedure, the tram company's reduced loss, denoted as Θ_P, is the sum of taxi service part, denoted as $\Theta_{t,P}(N_{t,P}^*)$, and bus service part, denoted as $\Theta_{b,P}(N_{b,P}^*)$, as given in Eq. (16).

$$\begin{aligned}\Theta_P &= \Theta_{t,P}\left(N_{t,P}^*\right) + \Theta_{b,P}\left(N_{b,P}^*\right) \\ &= \frac{\beta N_{t,P}^*(P - lt_a^t)(\lambda_t - p_t)}{\mu_{t,P}^* + (1-\beta)l} \\ &\quad + \frac{N_{b,P}^*(1-\beta)(P - lt_a^b)(\lambda_b - p_b)}{\mu_{b,P}^* + (1-\beta)l},\end{aligned} \tag{16}$$

where $\mu_{t,P}^* = \frac{N_{t,P}^* V_t}{t_s^t}$, $\mu_{b,P}^* = \frac{N_{b,P}^* V_b}{t_s^b}$

$$\begin{aligned}N_{t,P}^* = \frac{2\beta l}{3\theta_t} &+ \sqrt[3]{\frac{\beta^3 l^3}{27\theta_t^3} + \frac{\beta\left(p_t - c_t\right)\left(P - lt_a^t\right)}{4k_t\theta_t^2} + \sqrt{\frac{\beta^2\left(p_t - c_t\right)^2\left(P - lt_a^t\right)^2}{16k_t^2\theta_t^4} + \frac{\beta^4 l^3\left(p_t - c_t\right)\left(P - lt_a^t\right)}{54k_t\theta_t^5}}} \\ &+ \sqrt[3]{\frac{\beta^3 l^3}{27\theta_t^3} + \frac{\beta\left(p_t - c_t\right)\left(P - lt_a^t\right)}{4k_t\theta_t^2} - \sqrt{\frac{\beta^2\left(p_t - c_t\right)^2\left(P - lt_a^t\right)^2}{16k_t^2\theta_t^4} + \frac{\beta^4 l^3\left(p_t - c_t\right)\left(P - lt_a^t\right)}{54k_t\theta_t^5}}}.\end{aligned} \tag{13}$$

Thus, the question herein needs to be answered is whether there exists β^*, such that the hybrid approach will outperform both single-type-vehicle methods, i.e., $\Theta_P(\beta^*) > \Theta_t(N_t^*)$ and $\Theta_P(\beta^*) > \Theta_b(N_b^*)$. Due to the complexity of the formulations, we can hardly derive any analytical result. Numerical examples will be conducted in section 5.3 to test the optimal β.

2) T-STRATEGY

The T-strategy means that when a disruption occurs, taxis will be used first. When buses arrive later, the rest passengers will be served by buses, and during this process, taxis will not be used. As shown in Figure 4, the process can be separated into three stages, namely no-service, taxi-service, and bus-service. In what follows, we derive the decision functions of the three companies. Note that we use "T" to denote parameters related to *T-strategy* of hybrid service.

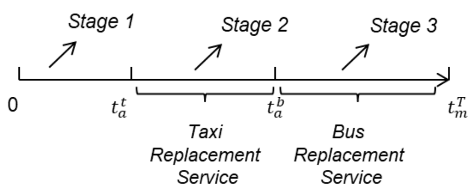

FIGURE 4. Process of hybrid service under T-strategy.

a. Taxis will arrive first to service from time of t_a^t until buses arrive at t_a^b, and $t_a^t < t_a^b$. Then, the taxi company's decision function can be written in Eq. (17).

$$\Pi_{t,T}\left(N_{t,T}\right) = (p_t - c_t)(t_a^b - t_a^t)N_{t,T} - k_t N_{t,T}^2. \quad (17)$$

Since the arrival time of buses t_a^b is a known constant, we can obtain the optimal taxi number $N_{t,T}^*$ in Eq. (18).

$$N_{t,T}^* = \frac{(p_t - c_t)(t_a^b - t_a^t)}{2k_t}. \quad (18)$$

b. After buses arrive at t_a^b, all taxis will quit the replacement service. Based on the conservative equation given in Eq. (19), the maximum service time t_m^T under the T-strategy can be obtained in Eq. (20).

$$P - (t_a^b - t_a^t)\mu_{t,T}^* - (t_m^T - t_a^b)\mu_{b,T}^* - lt_m^T = 0, \quad (19)$$

$$t_m^T = \frac{P - (t_a^b - t_a^t)\mu_{t,T}^* - lt_a^b}{\mu_{b,T}^* + l} + t_a^b. \quad (20)$$

where $\mu_{t,T}^* = N_{t,T}^* V_t / t_s^t$, $\mu_{b,T}^* = N_{b,T}^* V_b / t_s^b$.

c. Based on Eq. (20), the bus company's profit can be obtained in Eq. (21).

$$\Pi_{b,T}\left(N_{b,T}\right) = (p_b - c_b)(t_m^T - t_a^b)N_{b,T} - k_b N_{b,T}^2. \quad (21)$$

Thus, the optimal number of buses $N_{b,T}^*$ can be calculated in Eq. (22).

$$N_{b,T}^* = \frac{2l}{3\theta_b} + \sqrt[3]{\eta_1 + \eta_2} + \sqrt[3]{\eta_1 - \eta_2}, \quad (22)$$

in which η_1 and η_2, shown at the bottom of this page.

d. Then the tram company's reduced loss under T-strategy (denoted as Θ_T) is computed as the sum of taxi-service part (denoted as $\Theta_{t,T}$) and bus-service part (denoted as $\Theta_{b,T}$) in Eq. (23).

$$\begin{aligned}\Theta_T &= \Theta_{t,T}\left(N_{t,T}^*\right) + \Theta_{b,T}(N_{b,T}^*)\\ &= \left(t_a^b - t_a^t\right)(\lambda_t - p_t)N_{t,T}^*\\ &\quad + \frac{\left(P - (t_a^b - t_a^t)\mu_{t,T}^* - lt_a^b\right)(\lambda_b - p_b)N_{b,T}^*}{\mu_{b,T}^* + l} \quad (23)\end{aligned}$$

We will adopt the numerical analysis method to identify managerial implications in subsection 5.C.

V. NUMERICAL EXPERIMENTS AND MANAGERIAL IMPLICATIONS

A series of numerical experiments are carried out in this section to indicate how the input parameters affect the choice of recovery service type and to provide more managerial implications for public tram company to handle disruptions. A base example is first introduced in subsection 5.A. Based on the parameters defined in the base example, the taxi-only service and bus-only service are then compared in subsection 5.2. A set of experiments regarding the hybrid service under P-strategy and T-strategy are conducted in subsection 5.C.

A. BASE EXAMPLE SETTING AND RESULTS

The base values of the input parameters are given in Table 1. Most of the parameters reported are self-explanatory, and for the perceived value of passenger loss c_l, we multiply the average tram fare for one single trip (e.g. $2/trip) by a couple of times (e.g. 5) to estimate the value.

We use Matlab R2010a on a PC with 2.0 GHz Core-Duo and 8G memory to implement the numerical experiments. According to the base example given in Table 1, the values of optimal fleet size, the tram company's reduced loss using

$$\eta_1 = \frac{l^3}{27\theta_b^3} + \frac{(p_b - c_b)\left(P - (t_a^b - t_a^t)\mu_{t,T}^* - lt_a^b\right)}{4k_b\theta_b^2}$$

$$\eta_2 = \sqrt{\frac{(p_b - c_b)^2\left(P - (t_a^b - t_a^t)\mu_{t,T}^* - lt_a^b\right)^2}{16k_b^2\theta_b^4} + \frac{(p_b - c_b)\left(P - (t_a^b - t_a^t)\mu_{t,T}^* - lt_a^b\right)l^3}{54k_b\theta_b^5}}.$$

TABLE 1. **Parameters setting.**

Parameters	Value	Parameters	Value
P (#person)	200	k_t($)	0.5
c_l ($)	10	p_b($/min)	20
l (#person/min)	3	c_b($/min)	10
p_t($/min)	3	t_a^b (min)	20
c_t($/min)	1	V_b (#person)	60
t_a^t (min)	10	t_s^b (min)	10
V_t (#person)	4	k_b ($)	10
t_s^t(min)	5		

taxi-only and bus-only are summarized in Table 2. We can obtain the optimal fleet size according to Eqs. (6) and (10), i.e., $N_t^* = 6$, $N_b^* = 1$. Moreover, through Eqs. (5) and (8), we can calculate the taxi company's profit $\Pi_t(N_t^*) = \$242$, and the tram company's reduced loss $\Theta_t\left(N_t^*\right) = \$680$. Similarly, in bus-only recovery, the tram company's reduced loss is $\Theta_b\left(N_b^*\right) = \$670$, which is slightly lower than that of using taxi-only service. Consequently, the taxi-only service offers a higher level of reduced loss for the tram company and thus is a better alternative for recovery service during the entire disruption.

In Table 2, we further provide the results of other three indicators, namely *tram company's financial investment*, *service passenger volume*, and *payment per served passenger*, so as to gain a comprehensive understanding of the different recovery services. We can see that the bus-only service results in a higher expenditure ($400) than that of the taxi service ($390), although both vehicles can carry the same quantity of passengers (107 persons). We can find that the unit payment of taxi service is 3.64, which is lower than that of the bus

TABLE 2. **Results of the base example.**

Service type	Taxi-only	Bus-only
Optimal fleet size of taxis/buses, i.e., N^*	6	1
Maximum service time, i.e., t_m	31	31
Reduced loss of tram company, i.e., Θ	680	670
Profit of taxi/bus company, i.e., Π	242	169
Payment of the tram company, i.e., $(t_m - t_a)pN^*$	390	400
Served passenger volume, i.e., $(t_m - t_a)N^*V/t_s$	107	107
Payment per served passenger, i.e., pt_s/V	3.64	3.73

service (3.73). Therefore, Proposition 2 is verified since the unit payments of two services are both lower than the perceived passenger loss which equals to $c_l = 10$. Note that there is little difference between the results of taxi service and bus service, because we intend to use this as a base example for further sensitivity analysis to test the effect of critical parameters.

B. SENSITIVITY ANALYSIS FOR DECISIONS BETWEEN TAXI-ONLY AND BUS-ONLY SERVICE

In this section, the taxi-only service and bus-only service are compared, and the experiments are conducted to identify the key factors for decisions between the taxi-only service and bus-only service.

1) THE EFFECT OF PASSENGER VOLUME P AND PASSENGERS' LEAVING RATE *l*

The parameters related to passenger behavior, namely passenger volume P and passengers' leaving rate l are tested. Herein, P is set from 100 to 800 separated by 100 as an increment; l is set from 0 to 8 separated by every one unit. The experiments could capture the scenarios with different passenger volumes and when passengers' willingness to wait is decreased. The tram company's benefit under taxi-only and bus-only service, denoted Θ_t and Θ_b respectively, are compared and the results are shown in Figure 5.

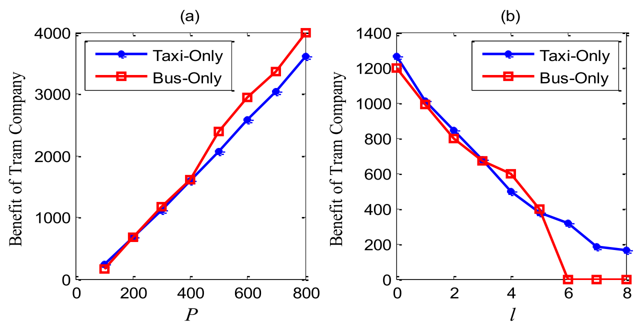

FIGURE 5. **Effect of *P* and *l* on the replacement service decisions. (a) Effect of *P*. (b) Effect of *l*.**

As shown in Figure 5 (a), when P increases, the bus-only service will be better, which could be explained that a larger passenger volume leads to economic scale for the bus-service with large capacity. Meanwhile, the results in Figure 5 (b) demonstrate that the effect of passengers' leaving rate is more complex. It implies that when passengers behave more patiently, i.e., $l \leq 3$, the taxi-only service is better. And with the increase in leaving rate, i.e., $3 < l \leq 5$, the bus-only service is better. This is mainly because taxis could arrive earlier to provide a faster service, and when passengers are more patient, taxis, of course, can take more passengers, which will consequently increase the benefit of the tram company. However, when $l > 5$, the taxi service shows its advantage again, because of the slow arrival time of buses and its large capacity. When buses arrive late, most of the passengers will leave when the leaving rate is higher, which may hurt the service level as well as the benefit of the tram company. We conclude that passengers' leaving rate has a high impact on the optimal decisions, and that both lower and higher leaving rates prefer the taxi-only service due to its quick response and flexibility.

2) THE EFFECT OF PERCEIVED PASSENGER LOSS C_L

It is interesting to see how the perceived passenger loss c_l, which can be regarded as an index for the service level, affects the decision. Here the value of c_l is set from 0 to 20 with an increment of 2. In order to get a comprehensive understanding of c_l, we choose six scenarios determined by the passenger volume, which are $P = 100, 150, 400, 500, 700, 900$. The tram company's reduced loss under taxi-only and bus-only service (Θ_t, Θ_b) are compared in Figure 6. The findings are anti-intuitive as follows. (1) When passenger volume is small, such as $P = 100$, the decision is not sensitive to cl, and taxi-only is always better, because when passenger volume is small, the flexibility and fast response of taxis can increase the service level. (2) When the passenger volume is increased, such as $P = 150, 400$, the taxi-only service is superior only if c_l is increased as well. For instance, only when $c_l > 6$ or $c_l > 12$ can the taxi-only service perform better for the cases of $P = 150$ and $P = 400$ respectively. (3) When the passenger volume reaches a higher level, such as $P \geq 500$, due to the limitation of capacity, the bus-only service is more favorable regardless of c_l. This practically means that the decision between taxi-only and bus-only service is not sensitive to cl especially when the passenger volume is higher or smaller, which could simply the decision process of

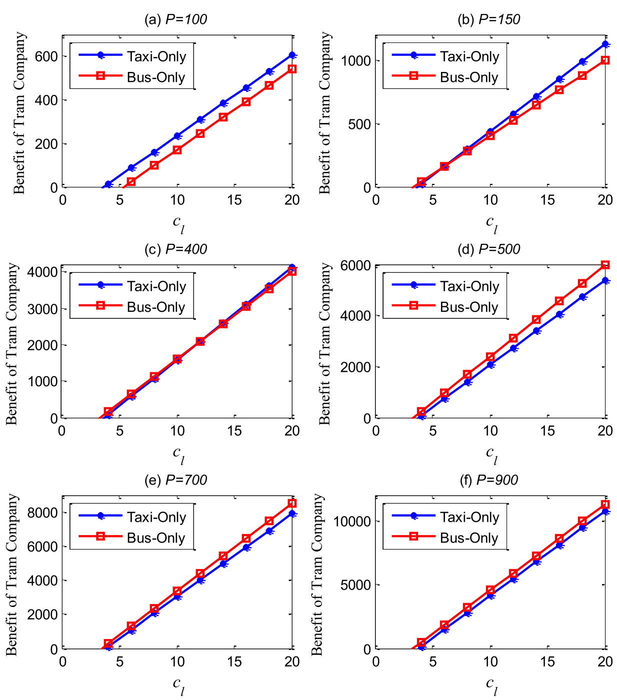

FIGURE 6. Effect of perceived passenger loss c_l on the replacement service decisions.

the TOCs to consider more on the passenger volume instead of perceived passenger loss.

3) THE EFFECT OF $(\lambda_t - p_t)/(\lambda_b - p_b)$

From Proposition 3 and associated computational experience, we have noticed that the ratio of $(\lambda_t - p_t)/(\lambda_b - p_b)$ has an important impact and deserves further investigation, where $\lambda_t = c_l V_t/t_s^t$, $\lambda_b = c_l V_b/t_s^b$. We set up two steps as follows. In the first step, we set the ratio $(\lambda_t - p_t)/(\lambda_b - p_b)$ as a fixed value $1/8$, and then let $\lambda_t - p_t$ and $\lambda_b - p_b$ increase simultaneously. The numerical results are shown in Figure 7, which indicate that at $(\lambda_t - p_t)/(\lambda_b - p_b) = 1/8$, the difference of the tram company's benefit under the two services is very small.

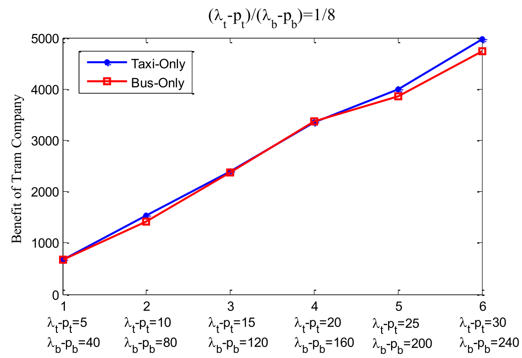

FIGURE 7. Effect of changing on $(\lambda_t - p_t)$ and $(\lambda_b - p_b)$.

In the second step, we decrease the ratio $(\lambda_t - p_t)/(\lambda_b - p_b)$ by reducing $\lambda_b - p_b$ gradually and fixing $\lambda_t - p_t$. The numerical results are presented in Figure 8. We can observe that the benefit of bus-only service, Θ_b, increase reversely with the ratio $(\lambda_t - p_t)/(\lambda_b - p_b)$. λ_b can be regarded as the perceived passenger loss per unit time; similarly, $\lambda_b - p_b$ can be interpreted as the profit per unit time for the bus service. Consequently, the decrease in the profit rate will bring the benefit level down.

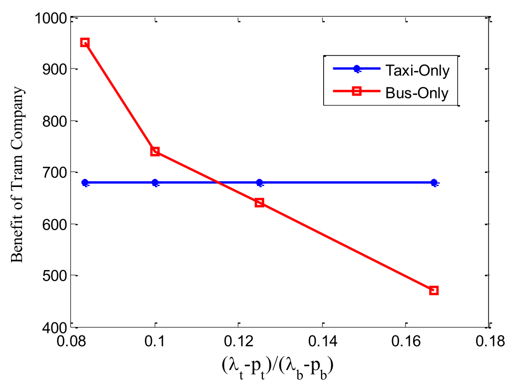

FIGURE 8. Effect of $(\lambda_t - p_t)/(\lambda_b - p_b)$ on the decision.

In general, the numerical results shown in Figure 7 and Figure 8 indicate that the ratio $(\lambda_t - p_t)/(\lambda_b - p_b)$ can highly affect the choice between the service types, in particular 1) If the ratio stays at a fixed value, the bus-only and taxi-only services are almost indifferent; 2) If the ratio goes down gradually, the bus-only service will gain popularity.

C. NUMERICAL EXPERIMENTS ON HYBRID SERVICE UNDER P-STRATEGY AND T-STRATEGY

Due to the complexity of the objective formulations, it is hard to derive any analytic results explicitly for the hybrid service. In this section we choose to conduct several numerical experiments to develop some managerial insights.

1) RESULTS OF HYBRID SERVICE UNDER P-STRATEGY

According to 4.3.1, the basic idea of P-Strategy is to find an optimal portion (β^*), of which the passengers will be served by taxi recovery service and the rest will use bus service. The optimal fleet size of taxis and buses will be determined accordingly. Here, the effect of hybrid service under P-strategy is conducted based on several scenarios with different passenger volume, namely $P = 100, 300, 500, 700, 900$. For each scenario, β is increased from 0 to 1 at 0.1 each time, and the results are shown in Figure 9. In each sub-figure, the tram company's benefit under the P-strategy (denoted as Θ_P) with respect to β is plotted, and the tram company's benefits under taxi-only service Θ_t and bus-only service Θ_b are used as the benchmark in each scenario. For example, Figure 9 (a) shows the benefit lines of bus-only and taxi-only, and demonstrates that when $P = 100$, the taxi-only is always the best service. In Figure 9 (b-e), the results show that by appropriately selecting the value of β, P-strategy will be superior to both taxi-only and bus-only services, for example, $\beta^* = 0.6$ when $P = 300$. Generally, Figure 9 (a-e) indicates that (1) when passenger volume is small, such as $P = 100$, P-strategy is the worst choice; (2) With the increase in P, the advantage of P-strategy is quite evident and for each scenario there exists one β^* that maximizes the benefit of the tram company, i.e., $\Theta_P(\beta^*) > \Theta_t(N_t^*)$ and $\Theta_P(\beta^*) > \Theta_b(N_b^*)$.

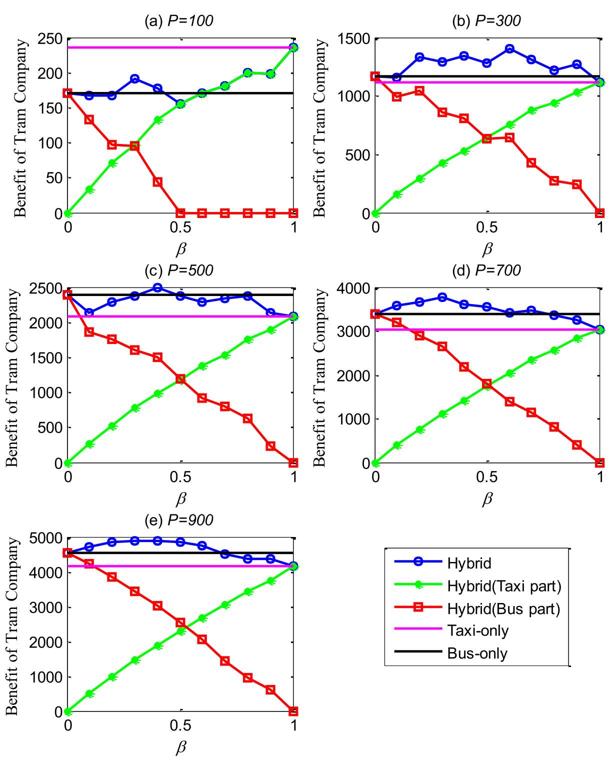

FIGURE 9. Results of hybrid service under P-strategy.

Furthermore, Figure 10 shows the changing of β^* with respect to P, where P increases from 100 to 900 at an interval of 100. We can observe that with the increase in P, β^* goes down gradually till reaches a stable value 0.3. Although there is a small fluctuation when $P = 200$, which caused by the integer constraint, the impact can be omitted.

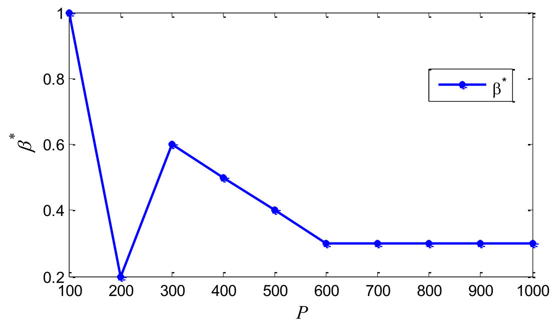

FIGURE 10. Changing trend of β^* with respect to P.

2) RESULTS OF HYBRID SERVICE UNDER T-STRATEGY AND COMPARISON OF NUMERICAL RESULTS

In T-strategy, taxis provide service before buses arrive, and after buses arrive, the taxis will exit the service process. Here, we choose 8 scenarios with different passenger volume changing from 100 to 1500 with an increment of 200. Here we show three more scenarios compared with subsection 5.C.1 for better understanding the differences between P-strategy and T-strategy. The results of the tram company's benefit under T-strategy as well as P-Strategy, Taxi-only and Bus-only services are presented in Figure 11. Based on the numerical results, we can compare the performance of different strategies.

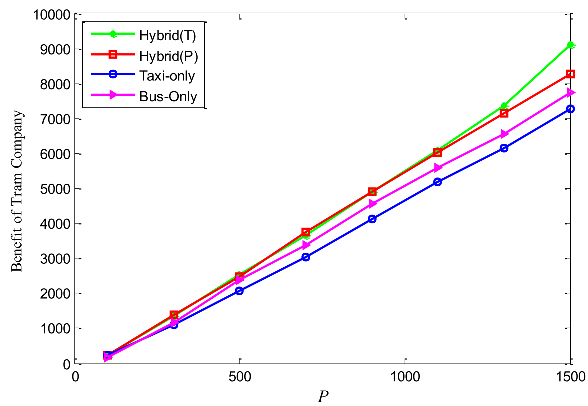

FIGURE 11. Comparisons amongst taxi-only, bus-only and hybrid services.

In summary: (1) it is still correct that when passenger volume is small ($P = 100$), the hybrid service will not be the best choice; (2) in the range of [300, 1000], P-Strategy performs the best; (3) when ($P \geq 1300$), the T-Strategy will be better than the P-Strategy, because T-Strategy calls for more buses when taxis will not be adopted after buses arrival; while the P-strategy will use some taxis which will reduce the needed number of buses and hence reduce the benefit of the tram company.

VI. CONCLUSION

This study has proposed a framework for public tram systems to select replacement services under short-term unplanned disruption. Firstly, this study has analyzed the replacement service options for the tram company to provide quick response and efficient service in the aftermath of unplanned disruptions in urban public tram systems. Secondly, this study has proposed a systematic decision tool for tram operating centers to answer two key questions: (1) whether to provide replacement service; and (2) which service type should be adopted among taxi-only, bus-only and hybrid? Finally, this work is a successful application of the collaborative partnership to the public transport systems. The passenger behavior is considered as the basis for the tram company to balance the recovery cost and replacement service level. It can provide guidelines for not only public tram systems, but other urban public transport network components such as light rail and subway systems as well.

The results obtained from numerical experiments suggest the following interesting and anti-intuitive managerial implications: (1) both scenarios with lowest or highest leaving rate of passengers favor the taxi-only service; (2) the effects of the passenger loss and average arrival time of vehicles behave quiet differently under different passenger volumes; (3) the ratio of profit rate per unit time between taxi-service and bus-service has large impact on the replacement selection decisions compared to vehicle price or passenger loss; (4) the optimal solutions of the proposed two hybrid strategies can be found by numerical searching method. Moreover, both hybrid strategies perform better under higher passenger volume scenario, and when the passenger volume is high enough, the T-strategy is the best choice which means taxis should quit the replacement service process after the bus arrival.

The promising directions of future study lie in three aspects. First, in the era of big data, how to capture the dynamic behavior pattern of passengers [31], as well as taxis and buses to achieve better coordinated rescue strategy is worthy of study in future. The second direction is to investigate the recovery route, which can be extended from one replacement route to multiple routes with the aim to not only increase the service level, but also allows for large scale bus bridging problem with different travel requirements. Last but not the least, how to build a reliable long-term collaboration relationship between the taxi, bus and tram companies to cope with uncertain disruptions is an important direction.

APPENDIXES

APPENDIX A

LIST OF NOTATIONS

l	Perceived loss of each lost passenger
c_l	Passengers' loss rate
P	Total blocked passenger volume
β	The portion of passenger assigned to taxis under P-strategy
N	Decision variable of fleet size
N_t	Fleet size of taxis under taxi-only service, a decision variable
N_b	Fleet size of buses under bus-only service, a decision variable
$N_{t,P}$	Fleet size of taxis in hybrid service under P-strategy
$N_{b,P}$	Fleet size of buses in hybrid service under P-strategy
$N_{t,T}$	Fleet size of taxis in hybrid service under T-strategy
$N_{b,T}$	Fleet size of buses in hybrid service under T-strategy
c	Average cost of replacement vehicles per unit time
c_t	Average cost of taxi service per unit time
c_b	Average cost of bus service per unit time
k	Cost factors related to fleet size N
k_t	Cost factors related to taxi's fleet size N_t
k_b	Cost factors related to bus's fleet size N_b
p	Variable payment rate per unit time
p_t	Variable payment rate per unit time for taxi service
p_b	Variable payment rate per unit time for bus service
V	Capacity parameter
V_t	Capacity of the taxi
V_b	Capacity of the bus
t_a	Average arrival time of the replacement vehicles
t_a^t	Average arrival time of taxis at the disruption site
t_a^b	Average arrival time of buses at the disruption site
t_s	The round-trip service time of replacement service
t_s^t	Round-trip service time of the taxi service
t_s^b	Round-trip service time of the bus service
t_m	Maximum service time
t_m^t	Maximum service time under the taxi-only service
t_m^b	Maximum service time under the bus-only service
t_m^P	Maximum service time under P-strategy
t_m^T	Maximum service time under T-strategy
C	Operational cost for replacement service
C_t	The taxi company's operational cost for replacement service
C_b	The bus company's operational cost for replacement service
Π	Profit function
Π_t	Taxi company's profit under the taxi-only service
Π_b	Bus company's profit under the bus-only service
$\Pi_{t,P}$	The taxi company's profit under P-strategy
$\Pi_{b,P}$	The bus company's profit under P-strategy
$\Pi_{t,T}$	The taxi company's profit under T-strategy
$\Pi_{b,T}$	The bus company's profit under T-strategy
Θ	The tram company's reduced loss

$$N_t^* = \frac{2l}{3\theta_t} + \sqrt[3]{\frac{l^3}{27\theta_t^3} + \frac{(p_t - c_t)\left(P - lt_a^t\right)}{4k_t\theta_t^2} + \sqrt{\frac{(p_t - c_t)^2\left(P - lt_a^t\right)^2}{16k_t^2\theta_t^4} + \frac{l^3(p_t - c_t)\left(P - lt_a^t\right)}{54k_t\theta_t^5}}} + \sqrt[3]{\frac{l^3}{27\theta_t^3} + \frac{(p_t - c_t)\left(P - lt_a^t\right)}{4k_t\theta_t^2} - \sqrt{\frac{(p_t - c_t)^2\left(P - lt_a^t\right)^2}{16k_t^2\theta_t^4} + \frac{l^3(p_t - c_t)\left(P - lt_a^t\right)}{54k_t\theta_t^5}}}$$

Θ_t The tram company's reduced loss under taxi-only
Θ_b The tram company's reduced loss under bus-only
Θ_P The tram company's reduced loss under P-strategy
Θ_t The tram company's reduced loss under T-strategy

APPENDIX B
PROOF OF PROPOSITION 1

According to the profit function of the taxi company as follows,

$$\Pi_t(N_t) = \frac{N_t(p_t - c_t)(P - lt_a^t)}{u_t + l} - k_t N_t^2,$$

we can the first-order and the second derivatives after relaxing the fleet size of taxis N_t as a real continuous variable,

$$\frac{\partial \Pi_t(N_t)}{\partial N_t} = \frac{P - lt_a^t}{u_t + l} - \frac{\left(P - lt_a^t\right)u_t}{(u_t + l)^2}(p_t - c_t) - 2k_t N_t,$$

$$\frac{\partial^2 \Pi_t(N_t)}{\partial N_t^2} = -\frac{\left(P - lt_a^t\right)V_t}{(u_t + l)^2 t_s^t}(1 - \frac{u_t}{u_t + l})(p_t - c_t) - 2k_t.$$

Since $p_t - c_t > 0$, $P - lt_a^t > 0$, $1 - u_t/(u_t + l) > 0$, and $k_t > 0$, we can get $\partial^2\Pi_t(N_t)/\partial N_t^2 < 0$. Then the taxi company's profit function $\Pi_t(N_t)$ is concave with respect to the fleet size N_t.

By setting the first-order derivative function $\partial\Pi_t(N_t)/\partial N_t$ to be equal to zero, the following equation can be obtained,

$$\frac{P - lt_a^t}{u_t + l} - \frac{\left(P - lt_a^t\right)u_t}{(u_t + l)^2}(p_t - c_t) - 2k_t N_t = 0.$$

According to the above equation, it has one real root can be written as the following form N_t^*, shown at the top of this page.

Therefore, the proof of Proposition 1 is completed.

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REFERENCES

[1] D. A. Hensher, "Why is light rail starting to dominate bus rapid transit yet again?" *Transp. Rev.*, vol. 36, no. 3, pp. 289–292, May 2016.
[2] D. Houston, A. Dang, J. Wu, Z. Chowdhury, and R. Edwards, "The cost of convenience; air pollution and noise on freeway and arterial light rail station platforms in Los Angeles," *Transp. Res. D, Transp. Environ.*, vol. 49, pp. 127–137, Dec. 2016.
[3] D. Pojani and D. Stead, "Policy design for sustainable urban transport in the global south," *Policy Des. Pract.*, vol. 1, no. 2, pp. 90–102, Apr. 2018.
[4] R. Buehler, J. Pucher, R. Gerike, and T. Götschi, "Reducing car dependence in the heart of Europe: Lessons from Germany, Austria, and Switzerland," *Transp. Rev.*, vol. 37, no. 1, pp. 4–28, Jan. 2017.
[5] China Association of Metros. (Mar. 30, 2019). *2018 Annual Statistics and Analysis Report of Urban Rail Transit*. [Online]. Available: http://www.camet.org.cn/index.php?m=content&c=index&a=show&catid=18&id=16219
[6] A. Z. Zeng, C. F. Durach, and Y. Fang, "Collaboration decisions on disruption recovery service in urban public tram systems," *Transp. Res. E, Logistics Transp. Rev.*, vol. 48, no. 3, pp. 578–590, May 2012.
[7] J. De Oña, R. De Oña, L. Eboli, C. Forciniti, and G. Mazzulla, "Transit passengers' behavioural intentions: The influence of service quality and customer satisfaction," *Transportmetrica A, Transp. Sci.*, vol. 12, no. 5, pp. 385–412, May 2016.
[8] J. G. Jin, K. M. Teo, and A. R. Odoni, "Optimizing bus bridging services in response to disruptions of urban transit rail networks," *Transp. Sci.*, vol. 50, no. 3, pp. 790–804, Aug. 2016.
[9] Y. Fang, C. Durach, A. Zeng, and J. Hou, "Coping with disruptions in public tram systems: Cases in Germany, China and the United States," in *Proc. 5th KES-Int. Conf. Intell. Decis. Technol.*, Sessimbra, Portugal, 2013, pp. 150–157.
[10] R. M. Lusby, J. Larsen, and S. Bull, "A survey on robustness in railway planning," *Eur. J. Oper. Res.*, vol. 266, no. 1, pp. 1–15, Apr. 2018.
[11] D. Sun and S. Guan, "Measuring vulnerability of urban metro network from line operation perspective," *Transp. Res. A, Policy Pract.*, vol. 94, pp. 348–359, Dec. 2016.
[12] X. Ding, S. Guan, D. J. Sun, and L. Jia, "Short turning pattern for relieving metro congestion during peak hours: The substance coherence of Shanghai, China," *Eur. Transp. Res. Rev.*, vol. 2018, no. 10, p. 28, Jun. 2018.
[13] G. Nian, F. Chen, Z. Li, Y. Zhu, and D. Sun, "Evaluating the alignment of new metro line considering network vulnerability with passenger ridership," *Transportmetrica A, Transp. Sci.*, vol. 15, no. 2, pp. 1402–1418, Nov. 2019.
[14] B. Pender, G. Currie, A. Delbosc, and Y. Wang, "Proactive recovery from rail disruptions through provision of track crossovers and bus bridging," *Transp. Res. Rec.*, vol. 2275, no. 1, pp. 68–76, Jan. 2012.
[15] M. Botte and L. D'Acierno, "Dispatching and rescheduling tasks and their interactions with travel demand and the energy domain: Models and algorithms," *Urban Rail Transit*, vol. 4, no. 4, pp. 163–197, Dec. 2018.
[16] M. Adnan, F. C. Pereira, C. L. Azevedo, K. Basak, K. Koh, H. Loganathan, H. P. Zhang, and M. Ben-Akiva, "Evaluating disruption management strategies in rail transit using SimMobility mid-term simulator: A study of Singapore MRT North-East line," in *Proc. 96th Annu. Meeting Transp. Res. Board*, Washington DC, USA, 2016, pp. 1–17.
[17] Y. Jiang and X. Zhou, "A connecting timetable rescheduling model for production and rail transportation with unexpected disruptions," *IEEE Access*, vol. 7, pp. 4284–4294, 2019.
[18] E. Van Der Hurk, L. Kroon, and G. Maróti, "Passenger advice and rolling stock rescheduling under uncertainty for disruption management," *Transp. Sci.*, vol. 52, no. 6, pp. 1391–1411, Dec. 2018.
[19] D.-Y. Lin and M.-R. Tsai, "Integrated crew scheduling and roster problem for trainmasters of passenger railway transportation," *IEEE Access*, vol. 7, pp. 27362–27375, 2019.
[20] D. Ivanov, A. Dolgui, and B. Sokolov, "Robust dynamic schedule coordination control in the supply chain," *Comput. Ind. Eng.*, vol. 94, pp. 18–31, Apr. 2016.

[21] A. De-Los-Santos, G. Laporte, J. A. Mesa, and F. Perea, "Evaluating passenger robustness in a rail transit network," *Transp. Res. C, Emerg. Technol.*, vol. 20, no. 1, pp. 34–46, Feb. 2012.

[22] Y. Westerlund and O. Cazemier, "The use of taxis for special and integrated public transport in sweden and The Netherlands," presented at the Int. Taxi Colloquium, Lisbon, Portugal, Sep. 2007.

[23] Z. Yang and X. Chen "Compensation decisions on disruption recovery service in urban rail transit," *Promet-Traffic Transp.*, vol. 31, no. 4, pp. 367–375, 2019.

[24] T. Darmanin, C. Lim, and H. Gan, "Public railway disruption recovery planning: A new recovery strategy for metro train Melbourne," in *Proc. 11th Asia–Pacific Ind. Eng. Manage. Syst. Conf.*, Melaka, Malaysia, 2010, pp. 1–6.

[25] L. Cadarso, Á. Marín, and G. Maróti, "Recovery of disruptions in rapid transit networks," *Transp. Res. E, Logistics Transp. Rev.*, vol. 53, pp. 15–33, Jul. 2013.

[26] L. P. Veelenturf, L. G. Kroon, and G. Maróti, "Passenger oriented railway disruption management by adapting timetables and rolling stock schedules," *Transp. Res. C, Emerg. Technol.*, vol. 80, pp. 133–147, Jul. 2017.

[27] Y. Zhu and R. M. Goverde, "Dynamic passenger assignment during disruptions in railway systems," in *Proc. 5th IEEE Int. Conf. Models Technol. Intell. Transp. Syst. (MT-ITS)*, Naples, Italy, Jun. 2017, pp. 146–151.

[28] A. Durand, N. Van Oort, and S. Hoogendoorn, "Assessing and improving operational strategies for the benefit of passengers in rail-bound urban transport systems," *Transp. Res. Rec.*, vol. 2672, no. 8, pp. 421–430, Dec. 2018.

[29] H. Yang, C. Fung, K. Wong, and S. Wong, "Nonlinear pricing of taxi services," *Transp. Res. A, Policy Pract.*, vol. 44, no. 5, pp. 337–348, Jun. 2010.

[30] H. Yang, M. Ye, W. H.-C. Tang, and S. C. Wong, "A multiperiod dynamic model of taxi services with endogenous service intensity," *Oper. Res.*, vol. 53, no. 3, pp. 501–515, Jun. 2005.

[31] X. Chu, J. Liu, D. Gong, and R. Wang, "Preserving location privacy in spatial crowdsourcing under quality control," *IEEE Access*, vol. 7, pp. 155851–155859, 2019.

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