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Fixed-Time Adaptive Neural Tracking Control for a Class of Uncertain Nonlinear Pure-Feedback Systems

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ABSTRACT This paper presents fixed-time adaptive neural tracking control for a class of uncertain nonlinear pure-feedback systems. To overcome the design difficulty arising from the nonaffine structure of nonlinear pure-feedback systems, the mean value theorem is introduced to separate the nonaffine appearance of nonlinear pure-feedback systems. Radial basis function (RBF) neural networks are employed to approximate designed unknown functions $\hat{f}_i(Z_i)$. By combining RBFs and Lyapunov functions, a novel fixed-time controller is designed, and semiglobal uniform ultimate boundedness of all signals in the closed-loop control system is guaranteed in a fixed time. Sufficient conditions are given to ensure that the system has semiglobal fixed-time stability. The main purpose of this paper is to design a controller for an unknown nonlinear pure-feedback system so that the system output y can track the reference signal y_d. The simulation experiments indicate that the selection of sufficient design parameters makes the tracking error converge on a domain of the origin. Compared with the existing finite-time control and fixed-time control, the proposed fixed-time control scheme reduces the size of the tracking error.

INDEX TERMS Adaptive neural network, fixed-time control, nonlinear pure-feedback systems.

I. INTRODUCTION

Compared with general nonlinear systems such as lower-triangular systems or strict feedback nonlinear systems, the nonlinear pure-feedback system [1]–[3] is a more general system better reflecting actual situations. It has thus attracted considerable attention and been a focus of extensive research in recent years.

Neural networks have been widely applied in machine learning, image processing, nonlinear systems, and other fields since the first neural network model [4] based on single neuron construction was proposed in the 1940s. Among such work, the research and application of BP neural networks and RBF neural networks have significantly promoted the development of nonlinear systems. In 1995, Kanellakopoulos and other researchers [5], [6] proposed backstepping, which has been a powerful method in nonlinear system research. Backstepping adaptive control and neural network adaptive control have been rapidly developed and applied. Recent research on RBF neural network adaptive control has attracted considerable attention [7]–[15], and a method for analyzing the stability of neural network adaptive control based on the Lyapunov method reported elsewhere [16]–[18] has been proposed.

In the study of the nonlinear system tracking control, it has been found that both system stability and system transient performance should be considered. In recent years, finite-time stability has been a hot research topic in nonlinear systems, in which much progress has been made [19]–[24]. Although finite-time control ensures that the system converges within a finite time, the convergence time is generally related to the system's initial state. If the initial state deviates from the equilibrium point, the convergence time of the system will be much longer. To eliminate the dependence of the convergence time on the initial state, fixed-time control was proposed [25]–[28]. Polyakov et al. first proposed the problem of fixed-time control and defined fixed-time stability [27]. In recent years, fixed-time control has attracted considerable attention; for example, in the literature [29], fast fixed-time nonsingular terminal sliding mode control

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has been proposed for the chaos suppression problem in power systems. In addition, fixed-time adaptive neural network tracking control for a class of uncertain nonlinear systems has been suggested [30], along with fixed-time tracking control based on backstepping for strict feedback nonlinear systems [31]. In the literatures [25]–[31], nonstrict or strict feedback nonlinear systems have mostly been considered, which do not solve the problem of fixed-time control for more general nonlinear systems. Compared with these approaches, nonlinear pure-feedback systems are more general nonlinear systems. The purpose of this paper is to solve the problem of fixed-time control based on nonlinear pure-feedback systems and present sufficient conditions and design procedures that ensure semiglobal fixed-time stability.

The main contributions of this paper can be summarized as follows.

1) Introducing the concept of fixed-time control in nonlinear pure-feedback systems for the first time and extending the theory of fixed-time control to more general systems. Compared with strict or nonstrict feedback nonlinear systems reported elsewhere [25]–[31], the nonlinear pure-feedback system is a more general nonlinear system.
2) By introducing the RBF neural network and the fixed-time control theory, a fixed-time control algorithm for nonlinear pure-feedback systems is designed in this paper so that the RBF neural network can approximate the unknown functions and some functions that are difficult to calculate in the process of designing the fixed-time controller.
3) To design the virtual controllers α_i and the actual controller u, and present sufficient conditions and design procedures that ensure the semiglobal fixed-time stable.

The rest of this paper is arranged as follows. In section II, problem description and preliminaries are presented. In section III, which is aimed at resolving the problem of fixed-time tracking control, fixed-time adaptive neural tracking control for a class of uncertain nonlinear pure-feedback systems is proposed by adopting backstepping, RBF neural network, and Lyapunov function. In section IV, the stability of the closed-loop system and the semiglobal uniform ultimate boundedness of all signals in the closed-loop control system are proven. In section V, the correctness of the proposed control scheme is proven by simulation studies. The conclusion of this work is presented in section VI.

II. PROBLEM DESCRIPTION AND PRELIMINARIES

A. PROBLEM STATEMENT

Consider the following nonlinear pure-feedback system:

$$\begin{cases} \dot{x}_i(t) = f_i(\bar{x}_i(t), x_{i+1}(t)) \\ \dot{x}_n(t) = f_n(\bar{x}_n(t), u(t)) \\ y(t) = x_1(t) \end{cases} \tag{1}$$

where $i = 1, \ldots, n$, $\bar{x}_i = [x_1(t), \ldots, x_i(t)]^T \in R^i$ with $i = 1, \ldots, n$, $u(t) \in R$, and $y(t) \in R$ are system state variables, system input, and system output ,separately; $f_i(.)$ are unknown smooth nonaffine functions.

Using the mean value theorem [32], we have [33]

$$f_i(\bar{x}_i, x_{i+1}) = f_i(\bar{x}_i, x_{i0}) + h_{i\mu_i} \cdot (x_{i+1} - x_{i0}) \tag{2}$$

$$f_n(\bar{x}_n, u) = f_n(\bar{x}_n, x_{n0}) + h_{n\mu_n} \cdot (u - x_{n0}) \tag{3}$$

where $h_{i\mu_i} := h_i(\bar{x}_i, x_{\mu_i}) = \partial f_i(\bar{x}_i, x_{\mu_i})/.\partial x_{\mu_i}$ with $i = 1, 2, \ldots, n$, $x_{n+1} = u$, $x_{\mu_i} = \mu_i x_{i+1} + (1 - \mu_i)x_{i0}$, $0 < \mu_i < 1$, and x_{i0} are known at a given time t_0.

Then, system (1) can be rewritten as

$$\begin{cases} \dot{x}_i = f_i(\bar{x}_i, x_{i0}) + h_{i\mu_i} \cdot (x_{i+1} - x_{i0}) \\ \dot{x}_n = f_n(\bar{x}_n, x_{n0}) + h_{n\mu_n} \cdot (u - x_{n0}) \\ y = x_1 \end{cases} \tag{4}$$

It can be seen from system (4) that the mean value theorem separates $\bar{x}_i$ and x_{i+1}. It also separates system state variable $\bar{x}_n$ and system control input u for the controller whose design is presented in the next part of this paper.

The main purpose of this paper is to design fixed-time adaptive neural tracking control for a class of unknown nonlinear pure-feedback systems, so that the system output y can track the reference signal y_d, and all signals in the closed-loop system are uniform and ultimately bounded. For this purpose, the vector functions are defined as $\bar{y}_{d,i} = [y_d, y_d^{(1)}, \ldots, y_d^{(i)}]^T$, $i = 1, \ldots, n$, where $y_d^{(i)}$ is the ith derivative.

Assumption 1 [33]: Unknown smooth nonlinear functions $h_i(.)$ are bounded, and there are known positive constants, b and c, that satisfy $0 < b \le |h_i(.)| < c < \infty$, $\forall(\bar{x}_i, x_{i+1}) \in R^i \times R$. Without loss of generality, we assume $0 < b \le h_i(.)$, $i = 1, \ldots, n$.

Assumption 2 [33]: The reference signal vector functions $\bar{y}_{di}$ are known, continuous, and bounded, $\bar{y}_{di} \in \Omega_{di} \subset R^{i+1}$ with Ω_{di} being known compact sets, $i = 1, \ldots, n$.

B. FIXED-TIME

Definition 1 [30]: Consider the following nonlinear system:

$$\dot{x}(t) = f(x(t)) \tag{5}$$

where $x(t) \in R^n$ is a system variable, and $f(x(t))$ is a smooth nonlinear function. Assuming that system (5) satisfies stability under Lyapunov meaning, for any initial condition $x(0) \in \Omega$, the solution of system converges on Ω in a finite time T_s, that is, the finite convergence time $T_s \le T_{\max}$ is bounded, where $T_{\max}$ represents the upper bound of the convergence time.

Lemma 1 [34]: Consider the system (5). If there are design parameters $\phi_1 > 0$, $\phi_2 > 0$, $\alpha \in (1, +\infty)$, and $\beta \in (0, 1)$ to make

$$\dot{V}(x) \le -\phi_1 V^{\alpha}(x) - \phi_2 V^{\beta}(x) \tag{6}$$

where $V(x)$ is a continuous differentiable positive definite function, then system (5) is global fixed-time stable, and the

TABLE 1. Sufficient conditions and convergence time for finite-time and fixed-time stability.

Control algorithms	Sufficient condition	Convergence time
Finite-time	$\dot{V}(x) \le -\phi_2 V(x)^{\beta}$	$T \le T_{\max} := \frac{1}{\varphi_2(1-\beta)} V(x(0))^{1-\beta}$
Fixed-time	$\dot{V}(x) \le -\phi_1 V^{\alpha}(x) - \phi_2 V^{\beta}(x)$	$T \le T_{\max} := \frac{1}{\varphi_1(\alpha-1)} + \frac{1}{\varphi_2(1-\beta)}$

fixed convergence time satisfies

$$T \le T_{\max} := \frac{1}{\phi_1(\alpha-1)} + \frac{1}{\phi_2(1-\beta)}. \tag{7}$$

The advantage of the fixed-time control over the finite-time control is that the upper bound of the convergence time of fixed time has nothing to do with the initial conditions, only with the design parameters. Table 1 shows the convergence time of fixed-time control and finite-time control.

It can be seen from Table 1 that the convergence time of the finite-time control is related to the initial state $V(x(0))$, while the convergence time of the fixed-time control is only related to the design parameters.

Lemma 2 [30]: If there are some design parameters $\phi_1 > 0$, $\phi_2 > 0$, $\alpha \in (1, \infty)$, $\beta \in (0, 1)$,$\tau \in (0, \infty)$, and $\varpi \in (0, 1)$ such that

$$\dot{V}(x) \le -\phi_1 V^{\alpha}(x) - \phi_2 V^{\beta}(x) + \tau. \tag{8}$$

then the trajectory of this system (5) is practical fixed-time stable and the fixed time T can be estimated by

$$T \le T_{\max} := \frac{1}{\phi_1 \varpi(\alpha-1)} + \frac{1}{\phi_2 \varpi(1-\beta)}. \tag{9}$$

The residual set of the solution of system $\dot{x} = f(x)$ is given by

$$x \in \{V(x) \le \min\{(\frac{\tau}{(1-\varpi)\phi_1})^{\frac{1}{\alpha}}, (\frac{\tau}{(1-\varpi)\phi_2})^{\frac{1}{\beta}}\}\}. \tag{10}$$

Lemma 3 [35]: Let $x_1, x_2, \ldots, x_n \ge 0$. Then

$$\sum_{i=1}^{n} x_i^{\rho} \ge \left(\sum_{i=1}^{n} x_i\right)^{\rho}, \quad if\, 0 < \rho \le 1. \tag{11}$$

$$\sum_{i=1}^{n} x_i^{\rho} \ge n^{1-\rho}\left(\sum_{i=1}^{n} x_i\right)^{\rho}, \quad if\, 1 < \rho \le \infty. \tag{12}$$

Lemma 4 [36]: For $x \in R$ and any positive constant κ, satisfying

$$0 \le |x| < \kappa + \frac{x^2}{\sqrt{x^2+\kappa^2}}. \tag{13}$$

Lemma 5: For $y \ge x > 0$, $x, y \in R$ and any positive constant χ, then the following is satisfied

$$\frac{y}{\sqrt{y+\chi}} \ge \frac{x}{\sqrt{x+\chi}}. \tag{14}$$

Proof:

$$\begin{aligned} &\frac{y}{\sqrt{y+\chi}} - \frac{x}{\sqrt{x+\chi}} \\ &= \frac{y+\chi}{\sqrt{y+\chi}} - \frac{x+\chi}{\sqrt{x+\chi}} - \frac{\chi}{\sqrt{y+\chi}} + \frac{\chi}{\sqrt{x+\chi}} \\ &= (\sqrt{y+\chi} - \sqrt{x+\chi}) + (\frac{\chi}{\sqrt{x+\chi}} - \frac{\chi}{\sqrt{y+x}}) \\ &\ge 0. \end{aligned}$$

C. GAUSSIAN RADIAL BASIS NETWORKS

An RBF neural network [37], [38] is applied in this paper to approximate arbitrary continuous function. The mathematical expression of the RBF neural network is as follows:

$$\hat{\varphi} = W^T S(Z) \tag{15}$$

where $W = [w_1, w_2, \cdots, w_l]^T \in R^l$ represents weight vector, $l > 1$ is the node number, $Z \in \Omega_Z \subset R^q$ is input vector, q is input dimension, $S(Z) = [s_1(Z), s_2(Z), \ldots, s_l(Z)]^T \in R^l$ is basis vector function, $s_i(Z)$ is the output of the ith node, and the selection principle of $s_i(Z)$ is as described in the literature [39]. Generally, the selected basis functions $s_i(Z)$ are the following Gauss functions:

$$s_i(Z) = \exp(\frac{-(Z-\xi_i)^T(Z-\xi_i)}{r}), \quad i = 1, \ldots, l \tag{16}$$

where r is the width of the basis function, and $\xi_i = [\xi_{i1}, \xi_{i2}, \cdots, \xi_{iq}]^T$ is the center of the basis function.

Selecting sufficient node number l, the RBF neural network can approximate arbitrary continuous function $\varphi(Z)$ in compact set $\Omega_Z \in R^q$ with arbitrary accuracy ε.

$$\varphi(Z) = W^{*T} S(Z) + \delta(Z), \quad \forall Z \in \Omega_Z \in R^q \tag{17}$$

where $\delta(Z)$ is an approximation error and satisfies $|\delta(Z)| \le \varepsilon$, and W^* is a given ideal constant weight vector. For all $Z \in \Omega_Z$, W^* is the value of W that makes approximation error $\delta(Z)$ the smallest, whose definition is

$$W^* = \arg \min_{W \in R^l} \sup_{Z \in \Omega_Z} \{|\varphi(Z) - \hat{\varphi}(Z)|\}.$$

In this paper, let $\theta_i = \max\{\|W_i^*\|^2/b, i = 1, 2, \ldots, n\}$, $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$, where $\hat{\theta}_i$ is the estimation of unknown constant θ_i, $W_i^*$is the ideal weight vector of the RBF neural network in $i(1 \le i \le n)$step, b is the positive design parameter, b is related to Assumption 1, and $\|\cdot\|$ represents the norm.

Assumption 3: There are unknown constants Q_i, which make $|\tilde{\theta}_i| \le Q_i$, $i = 1, 2, \ldots, n$.

Lemma 6 [40]: Consider the Gaussian RBF networks (15) and (16). $\|S(Z)\|$ has an upper bound, such that

$$\|S(Z)\| \le \sum_{k=0}^{\infty} 3q(k+2)^{q-1} e^{-2p^2k^2/r^2} := s$$

where $p = 1/2\ (\min_{i \ne j} \|\xi_i - \xi_j\|)$.

III. DESIGN OF FIXED-TIME CONTROLLER

In this section, fixed-time adaptive neural tracking control for a class of unknown nonlinear pure-feedback systems is designed on the basis of backstepping. The coordinate transformation is as follows:

$$z_1 = x_1 - y_d, z_i = x_i - \alpha_{i-1}, \quad i = 2, \ldots, n \tag{18}$$

where α_i represents the virtual controller of the ith subsystem.

An RBF neural network is applied in this paper to approximate unknown functions $\hat{f}_i(Z_i)$,

$$\hat{f}_i(Z_i) = W_i^{*T} S_i(Z_i) + \delta_i(Z_i). \quad (19)$$

According to Young's inequality, Lemma 3, and Complete Square Formula, we have the following inequalities:

$$z_i\delta_i(Z_i) \leq bk_{i3}z_i^2 + \frac{\varepsilon_i^2}{4bk_{i3}} \quad (20)$$

$$z_iW_i^{*T}S_i(Z_i) \leq \frac{b\theta_i}{2\eta_i^2}S_i^T(Z_i)S_i(Z_i)z_i^2 + \frac{\eta_i^2}{2} \quad (21)$$

$$-bk_{i1}((\frac{1}{2}z_i^2)^\beta + (\frac{b\tilde{\theta}_i^2}{2\gamma})^\beta) \leq -bk_{i1}(\frac{1}{2}z_i^2 + \frac{b\tilde{\theta}_i^2}{2\gamma})^\beta$$

$$-bk_{i2}((\frac{1}{2}z_i^2)^2 + (\frac{b\tilde{\theta}_i^2}{2\gamma})^2) \quad (22)$$

$$\leq -bk_{i2}(\frac{1}{2}z_i^2 + \frac{b\tilde{\theta}_i^2}{2\gamma})^2 + \frac{k_{i2}b^2\tilde{\theta}_i^4}{4\gamma} + \frac{k_{i2}b^2z_i^4}{4\gamma} \quad (23)$$

where $Z_1 = [x_1, \hat{\theta}_i, y_d, \dot{y}_d] \in \Omega_{Z_1} \subset R^{3+1}$ and $Z_i = [x_1, x_2, \ldots, x_i, \hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_i, \bar{y}_{di}] \in \Omega_{Z_i} \subset R^{3i+1}$ with $2 \leq i \leq n$ are input vectors; η_i, k_{i1}, k_{i2}, k_{i3}, and γ are positive design parameters; $S_i(Z_i)$ are RBF basis function vectors; and $\delta_i(Z_i)$ are approximation errors and satisfy $|\delta(Z_i)| \leq \varepsilon_i$.

Step 1: According to $z_1 = x_1 - y_d$ and (18), we obtain

$$\dot{z}_1 = f_1(\bar{x}_1, x_{10}) + h_{1\mu_1}(x_2 - x_{10}) - \dot{y}_d. \quad (24)$$

Construct the following Lyapunov function as

$$V_1 = \frac{1}{2}z_1^2 + \frac{b\tilde{\theta}_1^2}{2\gamma}. \quad (25)$$

The time derivative of V_1 is

$$\dot{V}_1 = z_1f_1(\bar{x}_1, x_{10}) + z_1h_{1\mu_1}(x_2 - x_{10}) - z_1\dot{y}_d - \frac{b\tilde{\theta}_1\dot{\hat{\theta}}_1}{\gamma}. \quad (26)$$

Substituting $z_2 = x_2 - \alpha_1$ into (24) yields

$$\dot{V}_1 = z_1f_1(\bar{x}_1, x_{10}) + z_1h_{1\mu_1}z_2 + z_1h_{1\mu_1}\alpha_1 - z_1h_{1\mu_1}x_{10} - z_1\dot{y}_d - \frac{b\tilde{\theta}_1\dot{\hat{\theta}}_1}{\gamma}. \quad (27)$$

The virtual controller α_1 is designed as

$$\alpha_1 = -k_{11}(\frac{1}{2}z_1^2)^\beta/z_1 - k_{12}(\frac{1}{2}z_1^2)^2/z_1 - \frac{\hat{\theta}_1}{2\eta_1^2}S_1^T(Z_1)S_1(Z_1)z_1 - k_{13}z_1 - \frac{bk_{12}}{4\gamma}z_1^3 + x_{10} \quad (28)$$

where k_{11}, k_{12}, k_{13}, and η_1 are positive design parameters.

Remark 1: The virtual controllers α_i and the actual controllers u designed in [30] and [41]–[43] have similar power function z^{2q-1}, where the positive constant q meets $0 < q < 1$; when q is not selected properly, singularity will occur. For example, if $q = 1/3$, z^{2q-1} is meaningless at $z = 0$; meanwhile, if $q = 3/4$, z^{2q-1} is meaningless at the negative domain. To prevent the controller from being meaningless in the origin and negative field, we make the following restrictions on β:

$$\beta = \frac{q_1}{q_2} \quad (29)$$

where $\beta \in (0.5, 1)$, $q_1 \in (0, +\infty)$, $q_2 \in (0, +\infty)$, and q_2 is odd.

Remark 2: In Lemma 1, the stability of the system depends on the values of the power exponents α and β. In the field of fixed-time control and finite-time control, there are no rules for selecting the power exponents α and β, so in order to make the system stable, the values of α and β are generally selected by a cut-and-try method. The experimental results show that the stability of the system is sensitive to the values of α and β. Generally speaking, there are two power exponents α and β in the fixed-time control, while only one power exponent β exists in the finite-time control, so it is more convenient to select the power exponent β for the finite-time control. To solve this problem, this paper fixes the value of power exponent α and uses Complete Square Formula to make α equal to 2; so, we only need to consider the influence of the value of the power exponent β on the system.

Substituting α_1 into (27) yields

$$\dot{V}_1 = z_1\hat{f}_1(Z_1) + z_1h_{1\mu_1}z_2 - \frac{b\tilde{\theta}_1\dot{\hat{\theta}}_1}{\gamma} - bk_{11}(\frac{1}{2}z_1^2)^\beta - bk_{12}(\frac{1}{2}z_1^2)^2 - \frac{b\hat{\theta}_1}{2\eta_1^2}S_1^T(Z_1)S_1(Z_1)z_1^2 - bk_{13}z_1^2 - \frac{b^2k_{12}}{4\gamma}z_1^4 \quad (30)$$

where $\hat{f}_1(Z_1) = f_1(\bar{x}_1, x_{10}) - \dot{y}_d$. An RBF neural network (19) is applied to approximate $\hat{f}_1(Z_1)$ and introduce inequalities (20) and (21), then (30) can be written as

$$\dot{V}_1 = \frac{b\theta_1}{2\eta_1^2}S_1^T(Z_1)S_1(Z_1)z_1^2 + \frac{\eta_1^2}{2} + \frac{\varepsilon_1^2}{4bk_{13}} + z_1h_{1\mu_1}z_2 - \frac{b\tilde{\theta}_1\dot{\hat{\theta}}_1}{\gamma} - bk_{11}(\frac{1}{2}z_1^2)^\beta - bk_{12}(\frac{1}{2}z_1^2)^2 - \frac{b\hat{\theta}_1}{2\eta_1^2}S_1^T(Z_1)S_1(Z_1)z_1^2 - \frac{b^2k_{12}}{4\gamma}z_1^4. \quad (31)$$

Adding

$$-bk_{11}(\frac{b\tilde{\theta}_1^2}{2\gamma})^\beta - bk_{12}(\frac{b\tilde{\theta}_1^2}{2\gamma})^2 + bk_{11}(\frac{b\tilde{\theta}_1^2}{2\gamma})^\beta + bk_{12}(\frac{b\tilde{\theta}_1^2}{2\gamma})^2$$

to the right of (31) obtains

$$\dot{V}_1 = \frac{b\theta_1}{2\eta_1^2}S_1^T(Z_1)S_1(Z_1)z_1^2 + \frac{\eta_1^2}{2} + \frac{\varepsilon_1^2}{4bk_{13}} + z_1h_{1\mu_1}z_2 - \frac{b\tilde{\theta}_1\dot{\hat{\theta}}_1}{\gamma} - bk_{11}(\frac{1}{2}z_1^2)^\beta$$

$$-bk_{12}(\frac{1}{2}z_1^2)^2-\frac{b\hat{\theta}_1}{2\eta_1^2}S_1^T(Z_1)S_1(Z_1)z_1^2$$
$$-bk_{11}(\frac{b\tilde{\theta}_1^2}{2\gamma})^\beta-\frac{b^2k_{12}}{4\gamma}z_1^4-bk_{12}(\frac{b\tilde{\theta}_1^2}{2\gamma})^2$$
$$+bk_{11}(\frac{b\tilde{\theta}_1^2}{2\gamma})^\beta+bk_{12}(\frac{b\tilde{\theta}_1^2}{2\gamma})^2. \quad (32)$$

Substituting (22) and (23) into (32) and combining Assumption 3 yields

$$\dot{V}_1\le -bk_{11}(\frac{1}{2}z_1^2+\frac{b\tilde{\theta}_1^2}{2\gamma})^\beta-bk_{12}(\frac{1}{2}z_1^2+\frac{b\tilde{\theta}_1^2}{2\gamma})^2$$
$$+\frac{b\tilde{\theta}_1}{\gamma}(\frac{\gamma}{2\eta_1^2}S_1^T(Z_1)S_1(Z_1)z_1^2-\dot{\hat{\theta}}_1)$$
$$+z_1h_{1\mu_1}z_2+\sigma_1+\beta_1 \quad (33)$$

where $\sigma_1=\frac{\eta_1^2}{2}+\frac{\varepsilon_1^2}{4bk_{13}}$and $\beta_1=\frac{k_{12}b^2Q_1^4}{4\gamma}+bk_{11}(\frac{bQ_1^2}{2\gamma})^\beta+bk_{12}(\frac{bQ_1^2}{2\gamma})^2$.

The adaptive law is designed as

$$\dot{\hat{\theta}}_1=\frac{\gamma}{2\eta_1^2}S_1^T(Z_1)S_1(Z_1)z_1^2-\lambda\hat{\theta}_1 \quad (34)$$

where λ is a design positive parameter.

Substituting (34) into (33) yields

$$\dot{V}_1\le -bk_{11}(\frac{1}{2}z_1^2+\frac{b\tilde{\theta}_1^2}{2\gamma})^\beta-bk_{12}(\frac{1}{2}z_1^2+\frac{b\tilde{\theta}_1^2}{2\gamma})^2$$
$$+z_1h_{1\mu_1}z_2+C_1 \quad (35)$$

where $\frac{\lambda b\tilde{\theta}_1\hat{\theta}_1}{\gamma}\le\frac{\lambda b}{\gamma}(\frac{\theta_1^2}{2}-\frac{\tilde{\theta}_1^2}{2})\le\frac{\lambda b}{2\gamma}\theta_1^2$ and $C_1=\alpha_1+\beta_1+\frac{\lambda b}{2\gamma}\theta_1^2$.

Step 2: From $z_2=x_2-\alpha_1$, we obtain

$$\dot{z}_2=f_2(\bar{x}_2,x_{20})+h_{2\mu_2}(x_3-x_{20})-\dot{\alpha}_1$$

where

$$\dot{\alpha}_1=\frac{\partial\alpha_1}{\partial x_1}\dot{x}_1+\frac{\partial\alpha_1}{\partial y_d}\dot{y}_d+\frac{\partial\alpha_1}{\partial\hat{\theta}_1}\dot{\hat{\theta}}_1. \quad (36)$$

Construct the Lyapunov function as

$$V_2=V_1+\frac{1}{2}z_2^2+\frac{b\tilde{\theta}_2^2}{2\gamma}.$$

The derivative of V_2 is written as

$$\dot{V}_2\le -bk_{11}(\frac{1}{2}z_1^2+\frac{b\tilde{\theta}_1^2}{2\gamma})^\beta-bk_{12}(\frac{1}{2}z_1^2+\frac{b\tilde{\theta}_1^2}{2\gamma})^2$$
$$+z_2\hat{f}_2(Z_2)+z_2h_{2\mu_2}z_3+z_2h_{2\mu_2}\alpha_2$$
$$-z_2h_{2\mu_2}x_{20}-z_2\frac{\partial\alpha_1}{\partial\hat{\theta}_1}\dot{\hat{\theta}}_1-\frac{b\tilde{\theta}_2\dot{\hat{\theta}}_2}{\gamma}$$
$$+z_2M_1(Z_2)+C_1 \quad (37)$$

where

$$\hat{f}_2(Z_2)=f_2(\bar{x}_2,x_{20})-\frac{\partial\alpha_1}{\partial x_1}f_1(\bar{x}_1,x_{10})$$
$$-\frac{\partial\alpha_1}{\partial y_d}\dot{y}_d+z_1h_{1\mu_1}-M_1(Z_2). \quad (38)$$

Remark 3: $M_1(Z_2)$ is a smooth function, being used to overcome the design difficulty of $\dot{\hat{\theta}}_1\partial\alpha_1/\partial\hat{\theta}_1$.

Next, use RBF neural network (19) to approximate $\hat{f}_2(Z_2)$ and introduce inequalities (20) and (21). (37) can be written as

$$\dot{V}_2\le -bk_{11}(\frac{1}{2}z_1^2+\frac{b\tilde{\theta}_1^2}{2\gamma})^\beta-bk_{12}(\frac{1}{2}z_1^2+\frac{b\tilde{\theta}_1^2}{2\gamma})^2$$
$$+z_2h_{2\mu_2}z_3+\frac{b\theta_2}{2\eta_2^2}S_2^T(Z_2)S_2(Z_2)z_2^2+bk_{23}z_2^2$$
$$+\frac{\varepsilon_2^2}{4bk_{23}}-z_2\frac{\partial\alpha_1}{\partial\hat{\theta}_1}\dot{\hat{\theta}}_1+z_2h_{2\mu_2}\alpha_2+\frac{\eta_2^2}{2}$$
$$-z_2h_{2\mu_2}x_{20}-\frac{b\tilde{\theta}_2\dot{\hat{\theta}}_2}{\gamma}+z_2M_1(Z_2)+C_1. \quad (39)$$

The virtual controller α_2 is designed as

$$\alpha_2=-k_{21}(\frac{1}{2}z_2^2)^\beta/z_2-k_{22}(\frac{1}{2}z_2^2)^2/z_2$$
$$-\frac{\hat{\theta}_2}{2\eta_2^2}S_2^T(Z_2)S_2(Z_2)z_2-k_{23}z_2$$
$$-\frac{bk_{22}}{4\gamma}z_2^3+x_{20} \quad (40)$$

where k_{21}, k_{22}, k_{23}, and η_2 are positive design parameters.

The adaptive law is designed as

$$\dot{\hat{\theta}}_2=\frac{\gamma}{2\eta_2^2}S_2^T(Z_2)S_2(Z_2)z_2^2-\lambda\hat{\theta}_2. \quad (41)$$

Substituting (40) and (41) into (39) yields

$$\dot{V}_2\le -b\sum_{j=1}^{2}k_{j1}(\frac{1}{2}z_j^2+\frac{b\tilde{\theta}_j^2}{2\gamma})^\beta$$
$$-b\sum_{j=1}^{2}k_{j2}(\frac{1}{2}z_j^2+\frac{b\tilde{\theta}_j^2}{2\gamma})^2$$
$$+\sum_{j=1}^{2}C_j+z_2h_{2\mu_2}z_3$$
$$+z_2(M_1(Z_2)-\frac{\partial\alpha_1}{\partial\hat{\theta}_1}\dot{\hat{\theta}}_1) \quad (42)$$

where $\frac{\lambda b\tilde{\theta}_2\hat{\theta}_2}{\gamma}\le\frac{\lambda b}{\gamma}(\frac{\theta_2^2}{2}-\frac{\tilde{\theta}_2^2}{2})\le\frac{\lambda b}{2\gamma}\theta_2^2$ and $C_2=\sigma_2+\beta_2+\frac{\lambda b\theta_2^2}{2\gamma}$.

It can be seen from (42) that one of the difficulties is how to design the smooth function $M_1(Z_2)$, such that

$$M_1(Z_2)-\frac{\partial\alpha_1}{\partial\hat{\theta}_1}\dot{\hat{\theta}}_1\le 0.$$

Through Lemma 4, Lemma 5, Lemma 6, and (31), we obtain

$$-z_2\frac{\partial\alpha_1}{\partial\hat{\theta}_1}\dot{\hat{\theta}}_1\le -z_2\frac{\partial\alpha_1}{\partial\hat{\theta}_1}(\frac{\gamma}{2\eta_1^2}S_1^T(Z_1)S_1(Z_1)z_1^2-\lambda\hat{\theta}_1)$$
$$\le\frac{z_2^2(\frac{\partial\alpha_1}{\partial\hat{\theta}_1})^2(\frac{\gamma}{2\eta_1^2}S_1^T(Z_1)S_1(Z_1)z_1^2)^2}{\sqrt{z_2^2(\frac{\partial\alpha_1}{\partial\hat{\theta}_1})^2(\frac{\gamma}{2\eta_1^2}S_1^T(Z_1)S_1(Z_1)z_1^2)^2+\zeta_{2,1}^2}}$$

$$+z_2\frac{\partial\alpha_1}{\partial\hat{\theta}_1} \le \frac{z_2^2(\frac{\partial\alpha_1}{\partial\hat{\theta}_1})^2(\frac{\gamma}{2\eta_1^2}s^2z_1^2)^2}{\sqrt{z_2^2(\frac{\partial\alpha_1}{\partial\hat{\theta}_1})^2(\frac{\gamma}{2\eta_1^2}s^2z_1^2)^2+\zeta_{2,1}^2}} + z_2\frac{\partial\alpha_1}{\partial\hat{\theta}_1}\lambda\hat{\theta}_1. \tag{43}$$

Therefore, $M_1(Z_2)$ can be designed as

$$M_1(Z_2) = -\frac{z_2(\frac{\partial\alpha_1}{\partial\hat{\theta}_1})^2(\frac{\gamma}{2\eta_1^2}s^2z_1^2)^2}{\sqrt{z_2^2(\frac{\partial\alpha_1}{\partial\hat{\theta}_1})^2(\frac{\gamma}{2\eta_1^2}s^2z_1^2)^2+\zeta_{2,1}^2}} - \lambda\frac{\partial\alpha_1}{\partial\hat{\theta}_1}\hat{\theta}_1. \tag{44}$$

Substituting (44) into (42) yields

$$\begin{aligned}\dot{V}_2 \le& -b\sum_{j=1}^{2}k_{j1}(\frac{1}{2}z_j^2+\frac{b\tilde{\theta}_j^2}{2\gamma})^\beta \\ &-b\sum_{j=1}^{2}k_{j2}(\frac{1}{2}z_j^2+\frac{b\tilde{\theta}_j^2}{2\gamma})^2 \\ &+\sum_{j=1}^{2}C_j. + z_2h_{2\mu_2}z_3. \end{aligned} \tag{45}$$

Step k $(3 \le k \le n-1)$: From $z_k = x_k - \alpha_{k-1}$, we have

$$\dot{z}_k = f_k(\bar{x}_k, x_{k0}) + h_{k\mu_k}(x_{k+1}-x_{k0}) - \dot{\alpha}_{k-1}$$

where

$$\dot{\alpha}_{k-1} = \sum_{j=1}^{k-1}\frac{\partial\alpha_{k-1}}{\partial x_j}\dot{x}_j + \frac{\partial\alpha_{k-1}}{\partial\bar{y}_{d,k-1}}\dot{\bar{y}}_{d,k-1} + \sum_{j=1}^{k-1}\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}_j}\dot{\hat{\theta}}_j. \tag{46}$$

Constructing the Lyapunov function $V_k = V_k + z_k^2/2 + b\tilde{\theta}_k^2/2\gamma$, we have

$$\begin{aligned}\dot{V}_k \le& -b\sum_{j=1}^{k-1}k_{j1}(\frac{1}{2}z_j^2+\frac{b\tilde{\theta}_j^2}{2\gamma})^\beta \\ &-b\sum_{j=1}^{k-1}k_{j2}(\frac{1}{2}z_j^2+\frac{b\tilde{\theta}_j^2}{2\gamma})^2 \\ &+\sum_{j=1}^{k-1}C_j + z_k\hat{f}_k(Z_k) \\ &+z_kh_{k\mu_k}z_{k+1} + z_kh_{k\mu_k}\alpha_k \\ &-z_kh_{k\mu_k}x_{k0} - z_k\sum_{j=1}^{k-1}\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}_j}\dot{\hat{\theta}}_j \\ &-\frac{b\tilde{\theta}_k\dot{\hat{\theta}}_k}{\gamma} + z_kM_{k-1}(Z_k) \end{aligned} \tag{47}$$

where

$$\begin{aligned}\hat{f}_k(Z_k) =& f_k(\bar{x}_k, x_{k0}) - \sum_{j=1}^{k-1}\frac{\partial\alpha_{k-1}}{\partial x_j}f_j(\bar{x}_j, x_{j+1}) \\ &-\frac{\partial\alpha_{k-1}}{\partial\bar{y}_{d,k-1}}\dot{\bar{y}}_{d,k-1} + z_{k-1}h_{k-1/\mu_{k-1}} \\ &-M_{k-1}(Z_k). \end{aligned} \tag{48}$$

The RBF neural network (19) can be used to approximate $\hat{f}_k(Z_k)$ and introduce inequalities (20) and (21). Then (47) can be written as

$$\begin{aligned}\dot{V}_k \le& -b\sum_{j=1}^{k-1}k_{j1}(\frac{1}{2}z_j^2+\frac{b\tilde{\theta}_j^2}{2\gamma})^\beta \\ &-b\sum_{j=1}^{k-1}k_{j2}(\frac{1}{2}z_j^2+\frac{b\tilde{\theta}_j^2}{2\gamma})^2 \\ &+\sum_{j=1}^{k-1}C_j + \frac{b\theta_k}{2\eta_k^2}S_k^T(Z_k)S_k(Z_k)z_k^2 \\ &+\frac{\eta_k^2}{2} + bk_{k3}z_k^2 + \frac{\varepsilon_k^2}{4bk_{k3}} + z_kh_{k\mu_k}z_{k+1} \\ &+z_kh_{k\mu_k}\alpha_k - z_kh_{k\mu_{k2}}x_{k0} - z_k\sum_{j=1}^{k-1}\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}_j}\dot{\hat{\theta}}_j \\ &-\frac{b\tilde{\theta}_k\dot{\hat{\theta}}_k}{\gamma} + z_kM_{k-1}(Z_k). \end{aligned} \tag{49}$$

The virtual controller α_k is designed as

$$\begin{aligned}\alpha_k =& -k_{k1}(\frac{1}{2}z_k^2)^\beta/z_k - k_{k2}(\frac{1}{2}z_k^2)^2/z_k \\ &-\frac{\hat{\theta}_k}{2\eta_k^2}S_k^T(Z_k)S_k(Z_k)z_k - k_{k3}z_k \\ &-\frac{bk_{k2}}{4\gamma}z_k^3 + x_{k0} \end{aligned} \tag{50}$$

where k_{k1}, k_{k2}, k_{k3}, and η_k are positive design parameters.

The adaptive law is designed as

$$\dot{\hat{\theta}}_k = \frac{\gamma}{2\eta_k^2}S_k^T(Z_k)S_k(Z_k)z_k^2 - \lambda\hat{\theta}_k. \tag{51}$$

Substituting (50) and (51) into (49) yields

$$\begin{aligned}\dot{V}_k \le& -b\sum_{j=1}^{k}k_{j1}(\frac{1}{2}z_j^2+\frac{b\tilde{\theta}_j^2}{2\gamma})^\beta \\ &-b\sum_{j=1}^{k}k_{j2}(\frac{1}{2}z_j^2+\frac{b\tilde{\theta}_j^2}{2\gamma})^2 \\ &+\sum_{j=1}^{k}C_j + z_kh_{k\mu_k}z_{k+1} \\ &+z_k(M_{k-1}(Z_k) - \sum_{j=1}^{k-1}\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}_j}\dot{\hat{\theta}}_j) \end{aligned} \tag{52}$$

where

$$C_k = \sigma_k + \beta_k + \frac{\lambda b\theta_k^2}{2\gamma}.$$

Then, the smooth function $M_{k-1}(Z_k)$ is designed, such that

$$M_{k-1}(Z_k) - \sum_{j=1}^{k-1} \frac{\partial\alpha_{k-1}}{\partial\hat{\theta}_j}\dot{\hat{\theta}}_j \leq 0.$$

Through Lemma 4, Lemma 5, Lemma 6, and (51), we have

$$M_{k-1}(Z_k) = -\sum_{j=1}^{k-1} \frac{z_k(\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}_j})^2(\frac{\gamma}{2\eta_j^2}s^2z_j^2)^2}{\sqrt{z_k^2(\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}_j})^2(\frac{\gamma}{2\eta_j^2}s^2z_j^2)^2 + \varsigma_{k,j}^2}} - \lambda\sum_{j=1}^{k-1}\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}_j}\hat{\theta}_j. \tag{53}$$

Substituting (53) into (52) yields

$$\begin{aligned}\dot{V}_k \leq & -b\sum_{j=1}^{k} k_{j1}(\frac{1}{2}z_j^2 + \frac{b\tilde{\theta}_j^2}{2\gamma})^\beta \\ & - b\sum_{j=1}^{k} k_{j2}(\frac{1}{2}z_j^2 + \frac{b\tilde{\theta}_j^2}{2\gamma})^2 \\ & + \sum_{j=1}^{k} C_j + z_k h_{k\mu_k} z_{k+1}.\end{aligned} \tag{54}$$

Step n: From $z_n = x_n - \alpha_{n-1}$, we have

$$\dot{z}_n = f_n(\bar{x}_n, x_{n0}) + h_{n\mu_n}(u - x_{n0}) - \dot{\alpha}_{n-1}$$

where

$$\dot{\alpha}_{n-1} = \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}\dot{x}_j + \frac{\partial\alpha_{n-1}}{\partial\bar{y}_{d,n-1}}\dot{\bar{y}}_{d.n-1} + \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{\theta}_j}\dot{\hat{\theta}}_j. \tag{55}$$

Constructing the Lyapunov function $V_\text{n} = V_n + z_n^2/2 + b\tilde{\theta}_n^2/2\gamma$, we have

$$\begin{aligned}\dot{V}_n \leq & -b\sum_{j=1}^{n-1} k_{j1}(\frac{1}{2}z_j^2 + \frac{b\tilde{\theta}_j^2}{2\gamma})^\beta \\ & - b\sum_{j=1}^{n-1} k_{j2}(\frac{1}{2}z_j^2 + \frac{b\tilde{\theta}_j^2}{2\gamma})^2 \\ & + \sum_{j=1}^{n-1} C_j + z_n\hat{f}_n(Z_n) \\ & + z_n h_{n\mu_n} u - z_n h_{n\mu_n} x_{n0} \\ & - z_n\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{\theta}_j}\dot{\hat{\theta}}_j - \frac{b\tilde{\theta}_n\dot{\hat{\theta}}_n}{\gamma} \\ & + z_n M_{n-1}(Z_n)\end{aligned} \tag{56}$$

where

$$\begin{aligned}\hat{f}_n(Z_n) = & f_n(\bar{x}_n, x_{n0}) - \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}f_j(\bar{x}_j, x_{j+1}) \\ & - \frac{\partial\alpha_{n-1}}{\partial\bar{y}_{d,n-1}}\dot{\bar{y}}_{d,n-1} + z_{n-1}h_{n-1\mu_{n-1}} \\ & - M_{n-1}(Z_n).\end{aligned} \tag{57}$$

The RBF neural network (19) is used to approximate $\hat{f}_n(Z_n)$ and inequalities (20) and (21) are introduced. Then (56) can be written as

$$\begin{aligned}\dot{V}_n \leq & -b\sum_{j=1}^{n-1} k_{j1}(\frac{1}{2}z_j^2 + \frac{b\tilde{\theta}_j^2}{2\gamma})^\beta \\ & - b\sum_{j=1}^{n-1} k_{j2}(\frac{1}{2}z_j^2 + \frac{b\tilde{\theta}_j^2}{2\gamma})^2 \\ & + \sum_{j=1}^{n-1} C_j + \frac{b\theta_n}{2\eta_n^2}S_n^T(Z_n)S_n(Z_n)z_n^2 \\ & + \frac{\eta_n^2}{2} + bk_{n3}z_n^2 + \frac{\varepsilon_n^2}{4bk_{n3}} + z_n h_{n\mu_n} u \\ & - z_n h_{n\mu_n} x_{n0} - z_n\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{\theta}_j}\dot{\hat{\theta}}_j \\ & - \frac{b\tilde{\theta}_n\dot{\hat{\theta}}_n}{\gamma} + z_n M_{n-1}(Z_n).\end{aligned} \tag{58}$$

The actual controller u is designed as

$$\begin{aligned}u = & -k_{n1}(\frac{1}{2}z_n^2)^\beta/z_n - k_{n2}(\frac{1}{2}z_n^2)^2/z_n \\ & - \frac{\hat{\theta}_n}{2\eta_n^2}S_n^T(Z_n)S_n(Z_n)z_n \\ & - k_{n3}z_n - \frac{bk_{n2}}{4\gamma}z_n^3 + x_{n0}\end{aligned} \tag{59}$$

where k_{n1}, k_{n2}, k_{n3}, and η_n are positive design parameters.

The adaptive law is designed as

$$\dot{\hat{\theta}}_n = \frac{\gamma}{2\eta_n^2}S_n^T(Z_n)S_n(Z_n)z_n^2 - \lambda\hat{\theta}_n. \tag{60}$$

Substituting (59) and (60) into (58) yields

$$\begin{aligned}\dot{V}_n \leq & -b\sum_{j=1}^{n} k_{j1}(\frac{1}{2}z_j^2 + \frac{b\tilde{\theta}_j^2}{2\gamma})^\beta \\ & - b\sum_{j=1}^{n} k_{j2}(\frac{1}{2}z_j^2 + \frac{b\tilde{\theta}_j^2}{2\gamma})^2 \\ & + \sum_{j=1}^{n} C_j + z_n(M_{n-1}(Z_n) \\ & - \sum_{j=1}^{n-1}\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}_j}\dot{\hat{\theta}}_j)\end{aligned} \tag{61}$$

where

$$C_n = \sigma_n + \beta_n + \frac{\lambda b\theta_n^2}{2\gamma}.$$

The method of processing $M_{n-1}(Z_n)$ is the same as (43).

$$M_{n-1}(Z_n) = -\sum_{j=1}^{n-1} \frac{z_n(\frac{\partial\alpha_{n-1}}{\partial\hat{\theta}_j})^2(\frac{\gamma}{2\eta_j^2}s^2z_j^2)^2}{\sqrt{z_n^2(\frac{\partial\alpha_{n-1}}{\partial\hat{\theta}_j})^2(\frac{\gamma}{2\eta_j^2}s^2z_j^2)^2 + \varsigma_{n,j}^2}} - \lambda\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{\theta}_j}\hat{\theta}_j. \quad (62)$$

Substituting (62) into (61) yields

$$\dot{V}_n \le -b\sum_{j=1}^{n} k_{j1}(\frac{1}{2}z_j^2 + \frac{b\tilde{\theta}_j^2}{2\gamma})^\beta - b\sum_{j=1}^{n} k_{j2}(\frac{1}{2}z_j^2 + \frac{b\tilde{\theta}_j^2}{2\gamma})^2 + \tau \quad (63)$$

where $\sum_{j=1}^{n} C_j = \tau$.

Set $\phi_1 = \min(bk_{12}, bk_{22}, \ldots, bk_{n2})$, $\phi_2 = \min(bk_{11}, bk_{12}, \ldots, bk_{n1})$, and according to Lemma 3, we have

$$-b\sum_{j=1}^{n} k_{j1}(\frac{1}{2}z_j^2 + \frac{b\tilde{\theta}_j^2}{2\gamma})^\beta \le -\phi_2(\sum_{j=1}^{n}(\frac{1}{2}z_j^2 + \frac{b\tilde{\theta}_j^2}{2\lambda}))^\beta \quad (64)$$

$$-b\sum_{j=1}^{n} k_{j2}(\frac{1}{2}z_j^2 + \frac{b\tilde{\theta}_j^2}{2\gamma})^2 \le -\frac{\phi_1}{n}(\sum_{j=1}^{n}(\frac{1}{2}z_j^2 + \frac{b\tilde{\theta}_j^2}{2\gamma}))^2. \quad (65)$$

Substituting (64) and (65) into (63) yields

$$\begin{aligned}\dot{V}_n \le & -\phi_2(\sum_{j=1}^{n}(\frac{1}{2}z_j^2 + \frac{b\tilde{\theta}_j^2}{2\gamma}))^\beta \\ & -\frac{\phi_1}{n}(\sum_{j=1}^{n}(\frac{1}{2}z_j^2 + \frac{b\tilde{\theta}_j^2}{2\gamma}))^2 + \tau \\ = & -\phi_2V_n^\beta - \frac{\phi_1}{n}V_n^2 + \tau. \quad (66)\end{aligned}$$

At this point, the design of the controller is complete.

IV. STABILITY ANALYSIS

Theorem 1: For system (1), if the system satisfies Assumption 1-3 and adopts the virtual controller (50), the actual controller (59), and the adaptive law (51), the semiglobal uniform ultimate boundedness of all signals in the closed-loop system (1) will be guaranteed in a fixed time.

According to Lemma 2, proper parameters $k_{j1} > 0$, $k_{j2} > 0$, $k_{j3} > 0$, and $C_j > 0, j = 1, \ldots, n$, are designed so that (63) satisfies the following situation.

Case 1:

If $V_n > (\tau/(1-\phi)\phi_2)^\beta$, $\varpi \in (0,1)$, (63) can be written as

$$\dot{V}_n \le -\varpi\phi_2V_n^\beta - \frac{\phi_1}{n}V_n^2. \quad (67)$$

The solution of system(1) converges on the following compact set

$$x \in \{V(x) \le (\frac{\tau}{(1-\varpi)\phi_2})^{\frac{1}{\beta}}\}. \quad (68)$$

Fixed convergence time is

$$T \le T_{\max} := \frac{n}{\phi_1} + \frac{4}{\varpi\phi_2}. \quad (69)$$

Case 2:

If $V_n > (\tau n/(1-\varpi)\varphi_1)^{1/2}$, (63) can be written as

$$\dot{V}_n \le -\phi_2V_n^\beta - \varpi\frac{\phi_1}{n}V_n^2. \quad (70)$$

Then, the solution of system (1) converges on the following compact set

$$x \in \{V_n(x) \le (\frac{m}{(1-\varpi)\phi_1})^{\frac{1}{2}}\}. \quad (71)$$

Fixed convergence time is

$$T \le T_{\max} := \frac{n}{\varpi\phi_1} + \frac{4}{\phi_2}. \quad (72)$$

Combining case 1 and case 2, the system's solution converges on

$$x \in \{V_n(x) \le \min\{(\frac{\tau}{(1-\varpi)\phi_2})^{\frac{1}{\beta}}, (\frac{m}{(1-\varpi)\phi_1})^{\frac{1}{2}}\}\}. \quad (73)$$

Fixed convergence time is

$$T_s \le T_{\max} := \frac{n}{\varpi\phi_1} + \frac{4}{\varpi\phi_2}. \quad (74)$$

It can be seen from (67) and (70) that V_n is bounded, so z_j and $\tilde{\theta}_j$ are bounded. As $\hat{\theta}_j = \theta_j - \tilde{\theta}_j$, $\hat{\theta}_j$ are also bounded, $j = 1, \ldots, n$. As $z_1 = x_1 - y_d$, z_1 and y_d are bounded, x_1 is bounded. As α_1 is the function of z_1, y_d, $\dot{y}_d$, and $\hat{\theta}_1$, α_1 is bounded. As $z_2 = x_2 - \alpha_1$, x_2 is bounded. In the same way, we can deduce that α_{j-1} and $x_j, j = 1, .., n$ are bounded. Therefore, all signals in the closed-loop system are bounded.

Remark 4: The fixed-time control algorithm for the nonlinear pure-feedback system is different from previous nonlinear fixed-time control algorithms. The differences are as follows.

1) The unknown nonstrict nonlinear system proposed in [30] did not solve the nonaffine structure problem of system input $u(t)$. However, the fixed-time control algorithm proposed in this paper solves this problem.
2) The structure of some systems is too complex to use $f_i(x)$ directly to design controllers. The RBF neural network is used in this paper to approximate the unknown functions $f_i(\cdot)$, so that there is no need to know the information of $f_i(\bar{x}_i, x_{i+1})$. The avoids difficulties in the design of controllers resulting from the complex system structure.

3) To overcome the difficulties of designing $\sum_{j=1}^{k-1}(\partial\alpha_{k-1}/\partial\hat{\theta}_j)\dot{\hat{\theta}}_j$, $M_{k-1}(Z_k)$ are added in this paper. Designing $M_{k-1}(Z_k)$ makes

$$\sum_{j=1}^{n}\frac{\partial\alpha_{n-1}}{\partial\hat{\theta}_j}\dot{\hat{\theta}}_j - M_{n-1}(Z_n) \leq 0, \quad k = 2, \ldots, n.$$

V. SIMULATION RESULTS

Two simulation examples are studied in this section to verify the controller designed as described in the above paragraphs.

Example 1: Numerical example.

Consider the following nonlinear pure-feedback system:

$$\begin{cases} \dot{x}_1 = \frac{1-e^{-x_1}}{1+e^{-x_2}} + x_2^3 + x_2 e^{-1-x_1^2} \\ \dot{x}_2 = x_1^2 x_2^2 + 0.15(x_1^2 + x_2^2)u^3 + 0.1x_1^3 x_2^2 u \\ y = x_1 \end{cases} \tag{75}$$

where x_1 and x_2 are the state variables, u is the system input, and y is the system output. Choose the reference signal as $y_d = \sin(0.5t) + 0.5\sin(1.5t)$. The purpose of this example is to design the virtual controller, the actual controller, and the adaptive law so that the system output y tracks the reference signal y_d in a fixed time.

For system (75), the design is as follows:

$$\begin{aligned} \alpha_1 = & -k_{11}(\frac{1}{2}z_1^2)^{\beta}/z_1 - k_{12}(\frac{1}{2}z_1^2)^2/z_1 \\ & - \frac{\hat{\theta}_1}{2\eta_1^2}S_1^T(Z_1)S_1(Z_1)z_1 \\ & - k_{13}z_1 - \frac{bk_{12}}{4\gamma}z_1^3 + x_{10} \end{aligned} \tag{76}$$

$$\begin{aligned} u = & -k_{21}(\frac{1}{2}z_2^2)^{\beta}/z_2 - k_{22}(\frac{1}{2}z_2^2)^2/z_2 \\ & - \frac{\hat{\theta}_2}{2\eta_2^2}S_2^T(Z_2)S_2(Z_2)z_2 \\ & - k_{23}z_2 - \frac{bk_{22}}{4\gamma}z_2^3 + x_{20} \end{aligned} \tag{77}$$

$$\dot{\hat{\theta}}_1 = \frac{\gamma}{2\eta_1^2}S_1^T(Z_1)S_1(Z_1)z_1^2 - \lambda\hat{\theta}_1 \tag{78}$$

$$\dot{\hat{\theta}}_2 = \frac{\gamma}{2\eta_2^2}S_2^T(Z_2)S_2(Z_2)z_2^2 - \lambda\hat{\theta}_2 \tag{79}$$

where $z_1 = x_1 - y_d$, $z_2 = x_2 - \alpha_1$, $Z_1 = [x_1, \hat{\theta}_1, y_d, \dot{y}_d]$, and $Z_2 = [x_1, x_2, \hat{\theta}_1, \hat{\theta}_2, y_d, \dot{y}_d, \ddot{y}_d]$, choosing initial conditions as $[x_1(0), x_2(0)]^T = [0.3, 05]^T$ and $[\hat{\theta}_1(0), \hat{\theta}_2(0)]^T = [0, 0]^T$. Design parameters are chosen as follows: $\beta = 3/4$, $k_{11} = 5$,$k_{12} = 5$,$k_{13} = 5$,$k_{21} = 5$, $k_{22} = 5$, $k_{23} = 5$, $\gamma = 5$, $\eta_1 = 0.25$, $\eta_2 = 0.25$, $\lambda = 0.1$, $b = 2$, $x_{10} = 0.2$ and $x_{20} = 0.5$. We select a small number of Gauss function nodes. The width of the RBF neural network is set to 4. $W_1S_1(Z_1)$ includes seven nodes and $W_2S_2(Z_2)$ includes five nodes. Gauss function center is set as

$$\xi_1 = \begin{bmatrix} -2 & -1.5 & -1 & 0 & 1 & 1.5 & 2 \\ -2 & -1.5 & -1 & 0 & 1 & 1.5 & 2 \\ -2 & -1.5 & -1 & 0 & 1 & 1.5 & 2 \\ -2 & -1.5 & -1 & 0 & 1 & 1.5 & 2 \end{bmatrix}$$

$$\xi_2 = \begin{bmatrix} -2 & -1 & 0 & 1 & 2 \\ -2 & -1 & 0 & 1 & 2 \\ -2 & -1 & 0 & 1 & 2 \\ -3 & -2 & 0 & 2 & 3 \\ -3 & -2 & 0 & 2 & 3 \\ -3 & -2 & 0 & 2 & 3 \\ -3 & -2 & 0 & 2 & 3 \end{bmatrix}$$

Figure 1 displays the system output y and the reference signal y_d. It can be seen that the output y can effectively track the reference signal y_d. Figure 2 displays the system state variables x_1 and x_2. Figure 3 displays the system actual controller u. Figure 4 displays the system adaptive laws $\hat{\theta}_1$ and $\hat{\theta}_2$. Figure 5 displays the error between the system output y and the reference signal y_d.

It can be seen from Figures 1-5 that the system state variables x_1 and x_2 are bounded, the actual controller u is bounded, and the adaptive parameters $\hat{\theta}_1$ and $\hat{\theta}_2$ are bounded, so all signals in the closed-loop system (75) are bounded.

Example 2: Physical example.

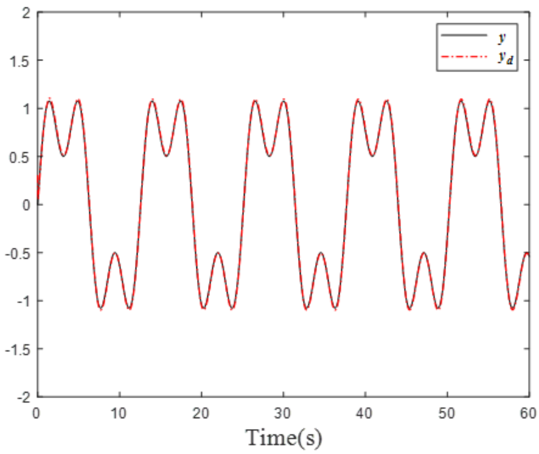

FIGURE 1. System output y and reference signal y_d.

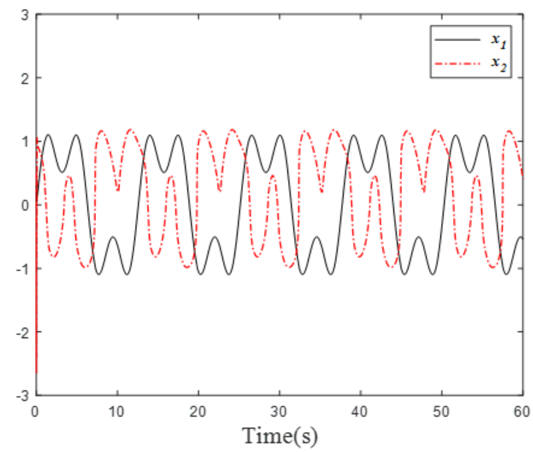

FIGURE 2. State variables x_1 and x_2.

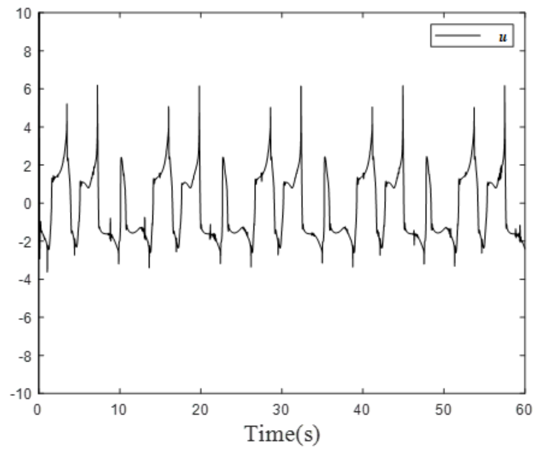

FIGURE 3. Actual control u.

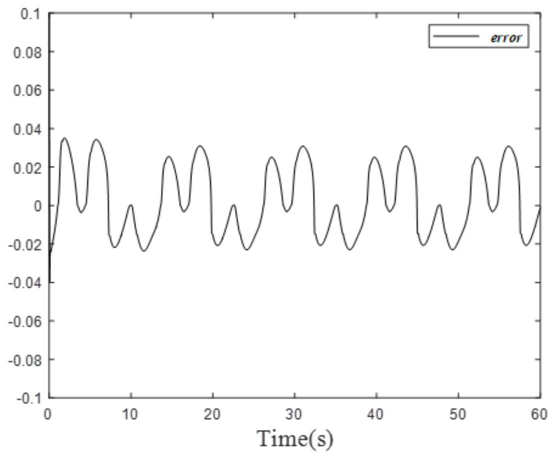

FIGURE 5. Tracking error $y - y_d$.

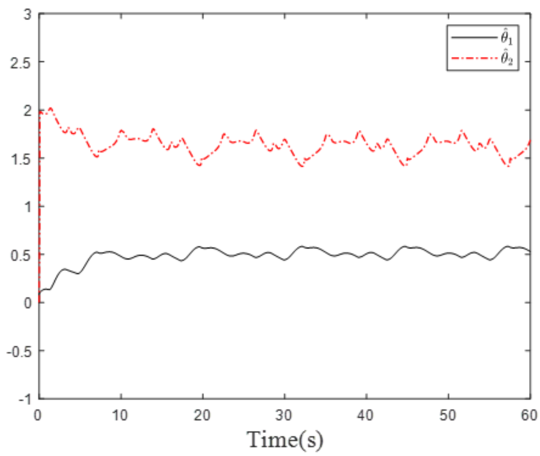

FIGURE 4. Adaptive parameters $\hat{\theta}_1$ and $\hat{\theta}_2$.

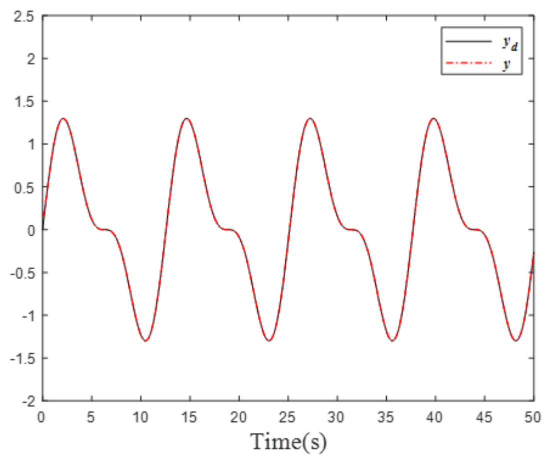

FIGURE 6. System output y and reference signal y_d.

Consider the following electromechanical system [41]:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = e_{21}x_2x_3^2 + e_{22}\sin x_1 + e_{23}x_2 - e_{23}x_2^2x_3^3 \\ \dot{x}_3 = e_{31}u + e_{32}x_2 + e_{33}x_3^2 - e_{33}x_2^2 \sin x_3 \\ y = x_1 \end{cases} \tag{80}$$

where $e_{21} = 1/M$, $e_{22} = -N/M$, $e_{23} = -B/M$, $e_{31} = 1/L$, $e_{32} = -K_B/L$, and $e_{33} = -R/L$. For the descriptions of M, N, B, L, K_B, and R, refer to [44]. The parameters are chosen as follows: $M = 0.0642$, $N = 1.1408$, $B = 0.0181$, $L = 0.025$, $K_B = 0.9$, and $R = 5.0$. Choose the reference signal as $y_d = \sin(0.5t) + 0.5\sin(t)$. The purpose of this example is to design a fixed-time controller to make the system output y track the reference signal y_d in a fixed time.

For system (80), the design is as follows:

$$\dot{\hat{\theta}}_2 = \frac{\gamma}{2\eta_2^2} S_2^T(Z_2)S_2(Z_2)z_2^2 - \lambda\hat{\theta}_2 \tag{81}$$

$$\dot{\hat{\theta}}_3 = \frac{\gamma}{2\eta_3^2} S_3^T(Z_3)S_3(Z_3)z_3^2 - \lambda\hat{\theta}_3 \tag{82}$$

$$\alpha_1 = -k_{11}\left(\frac{1}{2}z_1^2\right)^{\beta}/z_1 - k_{12}\left(\frac{1}{2}z_1^2\right)^{2}/z_1 - \frac{bk_{12}}{4\gamma}z_1^3 \tag{83}$$

$$\begin{aligned} \alpha_2 = & -k_{21}\left(\frac{1}{2}z_2^2\right)^{\beta}/z_2 - k_{22}\left(\frac{1}{2}z_2^2\right)^{2}/z_2 \\ & - \frac{\hat{\theta}_2}{2\eta_2^2}S_2^T(Z_2)S_2(Z_2)z_2 \\ & - k_{23}z_2 - \frac{bk_{22}}{4\gamma}z_2^3 + x_{20} \end{aligned} \tag{84}$$

$$\begin{aligned} u = & -k_{31}\left(\frac{1}{2}z_3^2\right)^{\beta}/z_3 - k_{32}\left(\frac{1}{2}z_3^2\right)^{2}/z_3 \\ & - \frac{\hat{\theta}_3}{2\eta_3^2}S_3^T(Z_3)S_3(Z_3)z_3 \\ & - k_{33}z_3 - \frac{bk_{32}}{4\gamma}z_3^3 + x_{30} \end{aligned} \tag{85}$$

where $z_i = x_i - \alpha_{i-1}$ with $i = 1, 2, 3$.

To show the effectiveness of our designed fixed-time controller, the controller is compared with previously designed the fixed-time controller [30] and the traditional finite-time controller [41].We apply the three controllers separately

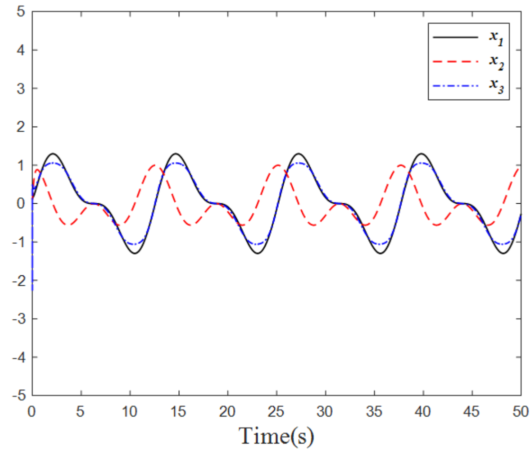

FIGURE 7. State variables x_1, x_2 and x_3.

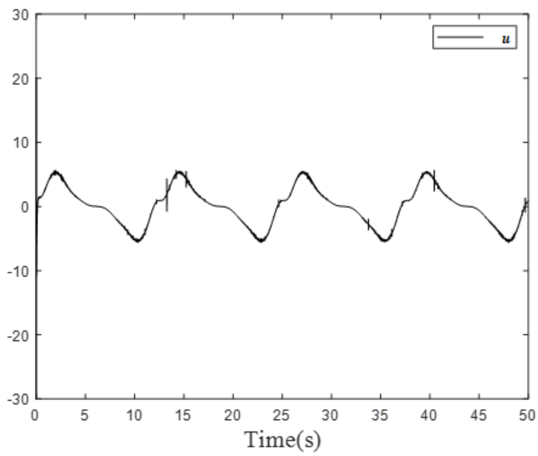

FIGURE 8. Actual control u.

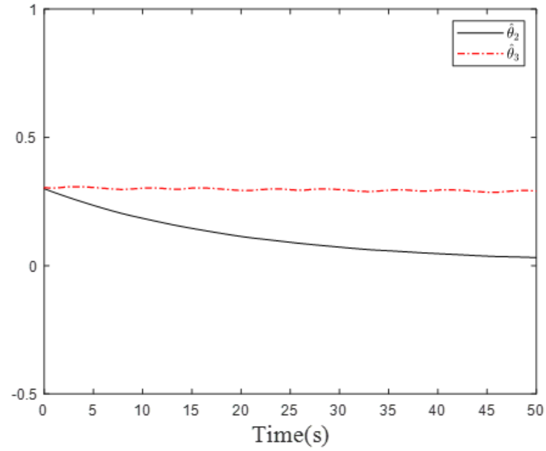

FIGURE 9. Adaptive parameters $\hat{\theta_2}$ and $\hat{\theta_3}$.

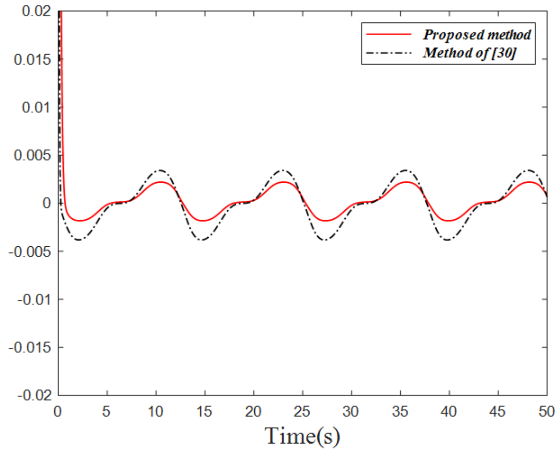

FIGURE 10. Tracking error $y - y_d$.

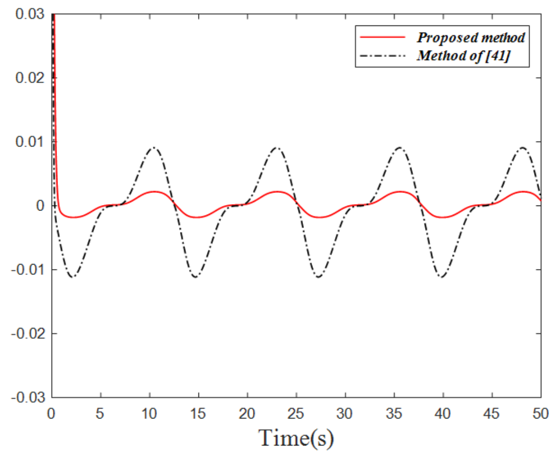

FIGURE 11. Tracking error $y - y_d$.

to an electromechanical system (80). For fair comparison, we choose the same design parameters as in [41], as follows: $k_{11} = k_{12} = 10$, $k_{21} = k_{22} = 10$, $k_{31} = k_{32} = 10$, $k_{23} = k_{33} = 60$, $x_{10} = x_{20} = x_{30} = 0.1$, and $b = 10$. The initial conditions are $[x_1(0), x_2(0), x_3(0)]^T = [0.15, 0.15, .15]^T$ and $[\hat{\theta}_2(0), \hat{\theta}_3(0)] = [0.3, 0.3]^T$. The design of RBF neural network is the same as in [41].

Remark 5: The design parameters of the controller designed in this paper increase $k_{12}, k_{22}, k_{23}, k_{32}$, and k_{33} compared with those in [41], and increase k_{23} and k_{33} compared with those in [30]. Therefore, when the design parameters of the controller are the same as in [30] and [41], the controller designed in this paper has two more design parameters k_{23} and k_{33}. Under the premise of system stability, the tracking error of the system is reduced continuously only by increasing the values of k_{23} and k_{33}. Therefore, the controller designed in this paper is more flexible.

Figure 6 displays the system output y and the reference signal y_d. It can be seen that the system output y can effectively track the reference signal y_d. Figure 7 displays the system state variables x_1, x_2, and x_3. Figure 8 displays the system actual controller u. Figure 9 displays the system adaptive parameters $\hat{\theta}_2$ and $\hat{\theta}_3$. Figure 10 displays the tracking error of the controller designed in this paper and the controller designed in [30]. Figure 11 displays the tracking error of the controller designed in this paper and the controller designed in [41].

As can be seen from Figures 6-11, the system state variables x_1, x_2, and x_3 are bounded, the actual controller u is bounded, the tracking error is bounded, and adaptive parameters $\hat{\theta}_2$ and $\hat{\theta}_3$ are bounded, so all signals in the closed-loop system (80) are bounded. It can be seen from Figures 10 and 11 that the proposed scheme has higher tracking performance with higher accuracy.

VI. CONCLUSION

In this paper, fixed-time control is applied to the nonlinear pure-feedback system, effectively solving difficulties in the design of a fixed-time controller arising from the nonaffine structure. The fixed-time controller designed in this paper enables the system output to track the reference signal in a fixed time, and the tracking error converges on a small domain of the origin in a fixed time. The final simulation further demonstrates the correctness of the design method used in this paper. In the next paper, we will delve into the problem of controller futility in the origin and negative fields and propose solutions.

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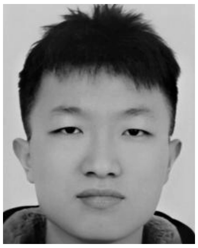

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