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Equilibrium Stability of Asymmetric Evolutionary Games of Multi-Agent Systems With Multiple Groups in Open Electricity Market

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ABSTRACT The asymptotically stable equilibrium points of asymmetric evolutionary games for multiple groups with multiple game strategies are obtained. The evolution of four groups and more multiple groups in open electricity market are described by the replicator dynamics. Multiple game strategies of the power generation enterprises in open electricity market are studied. Besides, the corresponding asymptotic stability conditions are given by the Lyapunov stability criterion.

INDEX TERMS Asymptotically stable, evolutionary game theory, multiple game strategies, open electricity market.

I. INTRODUCTION

Game theory has been applied into numerous fields, such as economics [1], energy management [2], and communication [3]. In a real game system, the players may be bounded by rationality, uncertainty, complexity, and opportunism [4]. Thus, the asymmetric games of multi-agent systems have been studied [5]. The players in a game system with conventional game theory should be intelligent at every time in the iteration game process to obtain the suitable strategies for their interests. Nevertheless, the player could not be intelligent at every time.

The players in a group could be stay in the game system with a long time; All these game groups in the game system could be balanced with long term. Therefore, the equilibrium state of the game groups will be more important than the game process. The equilibrium points of a grouped game with three groups and two strategies have been calculated in open electricity market [6]. However, the strategies of the game groups in open electricity market are more than two strategies in a real open electricity market. Hence, the equilibrium points of a grouped game with multiple groups and multiple strategies should be calculated.

With the developments of technology (e.g. storage and demand side response), electricity consumers in open electricity market are evolved into energy prosumers [7], [8], which are combined energy consumers and energy producers. The emergence of prosumers has changed the structure of the open electricity market. Furthermore, the opening degree of the open electricity power market is continuously improving [9], [10]. The reasonable market mechanism can play a positive role in the development of the competitive power market [11], which is conducive to guiding market participants to make correct decisions and maintaining the stability of market operation [12]. A huge challenge that is how to find the equilibrium state to balance the interests of all parties in an open electricity market under the new situation. At present, the two-group game and the three-group game in the open electricity market have been studied by scholars [6]. However, the equilibrium stability of asymmetric evolutionary games with multi-group with multi-strategy game has not been studied. Therefore, the equilibrium points of multiple groups with multiple strategies in the open electricity market are calculated. Replicator dynamics is applied to solve the evolution dynamics of quadripartite groups in the open

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electricity market, i.e., power generation enterprise groups (PGEG), power grid groups (PGG), prosumers, and power user groups (PUG). Then, the number of groups for evolutionary game is extended to n. Furthermore, the equilibrium points and the asymptotic stability conditions of the evolutionary game are solved.

The rest of the paper is organized as follows: Section II introduces the relevant knowledge about evolutionary game theory. In Section III, the evolutionary game model of four groups in open power market is established. In Section IV, the evolutionary equilibrium points and the asymptotic stability conditions of four-dimensional game under replicator dynamics are solved. Section V calculates the evolutionary equilibrium points and their asymptotic stability conditions of n-group game. In Section VI, the evolutionary game model of three power generation enterprises is established. Section VII briefly concludes this paper.

II. EVOLUTIONARY GAME THEORY

A. ELEMENTS OF EVOLUTIONARY GAME

An evolutionary game model is a strategic interaction that includes the following elements [13], [14].

1) Populations: individuals with similar characteristics with a group (Population). Each group has its own set of strategies.
2) Payoff function: the income corresponding to some kind of action is related to the chosen strategy by the participant and the proportion distribution of the current different strategies.
3) Dynamics: the dynamics reflect the learning and imitation process of the game participants. Under the influence of evolutionary dynamics, lower payment strategies will be replaced by higher payment strategies over time.
4) Equilibrium: the equilibrium is the convergence and stability of the evolution.

B. REPLICATOR DYNAMICS

Replicator dynamics was first proposed by *Peter D. Taylor* and *Leo B. Jonker* (1978) [15] and named by *Peter Schuster* and *Karl Sigmund* (1983) [16].

Let $\Delta = \{(x_1, x_2, \ldots, x_m) | \sum_{i=1}^{m} x_i = 1, x_i \geq 0 \, for \; i = 1, 2, \ldots, m\}$ denote the strategy of a group in the evolutionary game, i.e., the strategy set of this population (group) is composed of m strategies, and the ratio of selecting the i^{th} strategy in the group is x_i. Strategy Δ is a pure strategy if and only if $\exists x_i = 1 (i = 1, 2, \ldots, m)$, i.e., all individuals in this group choose the same strategy in the game.

Therefore, replicator dynamics of a group can be represented as follows

$$\frac{dx_i}{dt} = x_i \cdot (E_i - E_{av}), \quad i = 1, 2, \ldots, m. \tag{1}$$

where E_i denote expected revenue corresponding to the i^{th} pure strategy; $E_{av} = \sum_{i=1}^{m} x_i \cdot E_i$ represent average expected revenue of the group. When the revenue of a strategy is higher than the average revenue, the number of individuals selected in the strategy policy will increase in the group [17]. Over time, low-expectation revenue strategy will be replaced by high-expectation revenue strategy.

According to (1), the final equilibrium state of a group is related to the distribution of the payment parameters in the game.

C. EVOLUTIONARILY STABLE STRATEGY

Inspired by the idea of biological evolution, *J. Maynard Smith* and *G. R. Price* introduced the idea of evolution in biological theory into game theory, and presented the concept of evolutionarily stable strategy (ESS) in evolutionary game theory [18], [19]. Ever since ESS was proposed, evolutionary game theory has developed rapidly. In a two-group symmetric game, strategy $x^* \in \Delta$ is an ESS if and only if

1) Nash equilibrium condition: $E(x, x^*) \leq E(x^*, x^*)$;
2) Stability condition: $E(x^*, x) > E(x^*, x^*) \, if \; E(x, x^*) = E(x^*, x^*)$.

where $x \in \Delta$ and $x \neq x^*$, $E(x^*, x)$ represents the benefits derived from the selection of strategy x^* and strategy x by two groups, respectively. In the asymmetric game, the definition of ESS is similar to that of the symmetric game [20], [21].

In symmetric two-group games, a strategy is an ESS if and only if this strategy is an asymptotically stable [22]. Studying the asymptotic stability of the equilibrium point would be more appropriate than ESS, in some cases.

III. FOUR-DIMENSIONAL GAME IN TYPICAL SCENARIO OF OPEN ELECTRICITY MARKET

A. GROUPS IN GAME SYSTEMS

In the typical scenario of an open electricity market, the game groups are assumed to be set as follows. (1)

1) Power generation enterprise groups (PGEG): the large power plant groups, e.g. fossil-fuel power stations, hydroelectric power stations, nuclear power plants, wind farms, photovoltaic power plants, etc.
2) Power grid groups (PGG): the groups for transporting electrical energy.
3) Prosumers: the economically motivated entities that consume, produce, store energy, and transport electricity. Prosumers maximize their benefits by optimizing their economic decisions regarding energy utilization.
4) Power user groups (PUG): a collection of individuals or units that consume electrical energy (excluding prosumers).

B. TRADING STRATEGY

The power flow of electric energy during the trading process is shown in FIGURE 1. The arrow direction in FIGURE 1

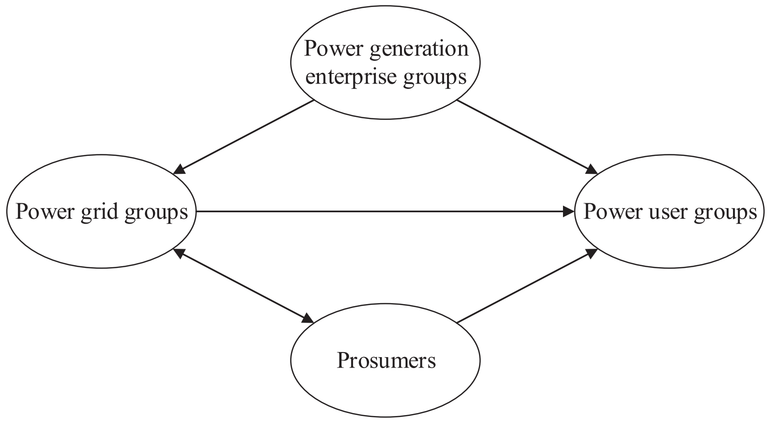


FIGURE 1. Power flow diagram for power trading rules.

indicates the possible power flow of electric energy during the transaction.

Suppose that the executable strategies of the PGEG, the PGG, prosumers and PUG are $\{S_{a1}, S_{a2}\}$, $\{S_{b1}, S_{b2}\}$, $\{S_{c1}, S_{c2}\}$, $\{S_{d1}, S_{d2}\}$, respectively.

1) Strategies for PGEG: Strategy S_{a1} means that PGEG sell electricity to PGG, the probability of choosing this strategy is x $(0 \le x \le 1)$. Strategy S_{a2} means that PGEG directly sell electricity to PUG in direct power supply mode, the probability of choosing this strategy is $(1 - x)$.
2) Strategies for PGG: Strategy S_{b1} means that PGG purchase electricity from PGEG and prosumers, sell electricity to PUG, the probability of choosing this strategy is y $(0 \le y \le 1)$. Strategy S_{b2} means that PGG purchase electricity from PGEG, sell electricity to PUG, while the probability of choosing this strategy is $(1 - y)$.
3) Strategies for prosumers: Strategy S_{c1} means that prosumers sell electricity to PGG and PUG, the probability of choosing this strategy is z $(0 \le z \le 1)$. Strategy S_{c2} means that prosumers purchase electricity from PGG, sell electricity to PUG, the probability of choosing this strategy is $(1 - z)$.
4) Strategies for PUG: Strategy S_{d1} means that PUG purchase a large amount of electricity from PGEG, and purchase a small amount of electricity from PGG and prosumers; the probability of choosing this strategy is w $(0 \le w \le 1)$. Strategy S_{d2} means that PUG only purchasing electricity from PGG and prosumers; the probability of choosing this strategy is $(1 - w)$.

C. PAYOFF DISTRIBUTION

Set $\{S_{ai}, S_{bi}, S_{ci}, S_{di}\}$ $(i \in \{1, 2\})$ is the strategy set, when the four groups (i.e., PGEG, PGG, prosumers, and PUG) choose strategy S_{ai}, S_{bi}, S_{ci}, and S_{di}, respectively. And the payoff distribution of these four groups is given in TABLE 1.

TABLE 1. Payoff distribution of strategy set for four groups.

Strategy set	S_{d1}	S_{d2}
S_{a1}, S_{b1}, S_{c1}	(a_1, b_1, c_1, d_1)	(a_2, b_2, c_2, d_2)
S_{a1}, S_{b1}, S_{c2}	(a_3, b_3, c_3, d_3)	(a_4, b_4, c_4, d_4)
S_{a1}, S_{b2}, S_{c1}	(a_5, b_5, c_5, d_5)	(a_6, b_6, c_6, d_6)
S_{a1}, S_{b2}, S_{c2}	(a_7, b_7, c_7, d_7)	(a_8, b_8, c_8, d_8)
S_{a2}, S_{b1}, S_{c1}	(a_9, b_9, c_9, d_9)	$(a_{10}, b_{10}, c_{10}, d_{10})$
S_{a2}, S_{b1}, S_{c2}	$(a_{11}, b_{11}, c_{11}, d_{11})$	$(a_{12}, b_{12}, c_{12}, d_{12})$
S_{a2}, S_{b2}, S_{c1}	$(a_{13}, b_{13}, c_{13}, d_{13})$	$(a_{14}, b_{14}, c_{14}, d_{14})$
S_{a2}, S_{b2}, S_{c2}	$(a_{15}, b_{15}, c_{15}, d_{15})$	$(a_{16}, b_{16}, c_{16}, d_{16})$

IV. SOLUTION OF FOUR-DIMENSIONAL GAME OF OPEN ELECTRICITY MARKET

A. SOLUTION STEPS

All the strategies in a population satisfy $\sum_{i=1}^{m} \frac{dx_i}{dt} = 0$. Each group has only two strategies that assumed to be in Section III-B. Hence, replicator dynamics of each group can be expressed by a differential equation, and four differential equations are included in the game.

Let the value of each differential equation 0, the equilibrium points of the game can be found, which mean that the state of the system does not change without disturbance. The asymptotic stability of the equilibrium points can be judged by Lyapunov indirect method. Let J be the Jacobian matrix of replicator dynamics. For each equilibrium point, if all the real parts of the eigenvalues of the matrix J are less than 0; this equilibrium point is an asymptotically stable equilibrium point (ASEP); and the system is in an asymptotically stable state. If the matrix J has at least one eigenvalue on the imaginary axis; the system is in a critical stable state and its stability needs to be determined by other methods. Otherwise, this equilibrium is an asymptotically unstable equilibrium point (AUEP); and the system is in an asymptotically unstable state.

The steps for solving the ASEP of replicator dynamics can be summarized as follows.

1) Establish replicator dynamics based on the payoff distribution and solving the equilibrium points;
2) Establish the Jacobian matrix of replicator dynamics;
3) If the real part of the eigenvalue of the Jacobian matrix corresponding to the equilibrium point is negative value, the equilibrium point is an ASEP.

B. REPLICATOR DYNAMICS OF FOUR-DIMENSIONAL GAME

The replicator dynamics of the four-group (dimensional) game can be shown as

$$\begin{cases} \dfrac{dx}{dt} = x\tilde{x}D_1 \\ \dfrac{dy}{dt} = y\tilde{y}D_2 \\ \dfrac{dz}{dt} = z\tilde{z}D_3 \\ \dfrac{dw}{dt} = w\tilde{w}D_4, \end{cases} \tag{2}$$

$$\begin{cases} D_1 = (a_1 - a_9)yzw + (a_2 - a_{10})yz\tilde{w} \\ \quad + (a_3 - a_{11})y\tilde{z}w + (a_4 - a_{12})y\tilde{z}\tilde{w} \\ \quad + (a_5 - a_{13})\tilde{y}zw + (a_6 - a_{14})\tilde{y}z\tilde{w} \\ \quad + (a_7 - a_{15})\tilde{y}\tilde{z}w + (a_8 - a_{16})\tilde{y}\tilde{z}\tilde{w} \\ D_2 = (b_1 - b_5)xzw + (b_2 - b_6)xz\tilde{w} \\ \quad + (b_3 - b_7)x\tilde{z}w + (b_4 - b_8)x\tilde{z}\tilde{w} \\ \quad + (b_9 - b_{13})\tilde{x}zw + (b_{10} - b_{14})\tilde{x}z\tilde{w} \\ \quad + (b_{11} - b_{15})\tilde{x}\tilde{z}w + (b_{12} - b_{16})\tilde{x}\tilde{z}\tilde{w} \\ D_3 = (c_1 - c_3)xyw + (c_2 - c_4)xy\tilde{w} \\ \quad + (c_5 - c_7)x\tilde{y}w + (c_6 - c_8)x\tilde{y}\tilde{w} \\ \quad + (c_9 - c_{11})\tilde{x}yw + (c_{10} - c_{12})\tilde{x}y\tilde{w} \\ \quad + (c_{13} - c_{15})\tilde{x}\tilde{y}w + (c_{14} - c_{16})\tilde{x}\tilde{y}\tilde{w} \\ D_4 = (d_1 - d_2)xyz + (d_3 - d_4)xy\tilde{z} \\ \quad + (d_5 - d_6)x\tilde{y}z + (d_7 - d_8)x\tilde{y}\tilde{z} \\ \quad + (d_9 - d_{10})\tilde{x}yz + (d_{11} - d_{12})\tilde{x}y\tilde{z} \\ \quad + (d_{13} - d_{14})\tilde{x}\tilde{y}z + (d_{15} - d_{16})\tilde{x}\tilde{y}\tilde{z}, \end{cases} \tag{3}$$

$$\begin{cases} \tilde{x} = 1 - x \\ \tilde{y} = 1 - y \\ \tilde{z} = 1 - z \\ \tilde{w} = 1 - w. \end{cases} \tag{4}$$

C. SOLUTION OF THE EQUATIONS

There are 3^4 or 81 types of solutions in (2), i.e., the game consists of 81 types equilibrium points. With the aim of solving (2), a total of two situations are discussed as follows.

1) Situation 1 (pure strategy): $\forall a \in \{x, y, z, w\} \Rightarrow a \in \{0, 1\}$, this situation has 2^4 or 16 types of solutions;
2) Situation 2 (mixed strategy): $\exists a \in \{x, y, z, w\} \Rightarrow a \notin \{0, 1\}$, this situation has $(3^4 - 2^4)$ or 65 types of solutions.

In the situation 1, according to the steps described in Section IV-A, the equilibrium points and their asymptotic stability conditions (TABLE 2) can be obtained. For each equilibrium point, the condition of the asymptotically stable equilibrium is composed of four inequalities. For instance, if the state of the system is at the equilibrium point (0, 0, 0, 1) and satisfies the condition ($a_7 < a_{15}, b_{11} < b_{15}, c_{13} < c_{15}, d_{16} < d_{15}$), the system is in an asymptotically stable state. At one point, the parameters of the system changed to make the $d_{15} < d_{16}$, (0, 0, 0, 1) is not the ASEP of the system. In fact, if the parameter distribution of the system at this time satisfies ($a_7 < a_{15}, b_{11} < b_{15}, c_{13} < c_{15}, d_{16} > d_{15}$), the equilibrium state of the asymptotic stability of the system can be changed from (0, 0, 0, 1) to (0, 0, 0, 0), or from (0, 0, 0, 1) to (0, 0, 1, 0), etc. The system may end up with 12 final evolutionary states. Adding another 11 inequality conditions can be inferred, i.e., a total of 15 corresponding inequality conditions so that the ASEP of the system can eventually evolve from (0, 0, 0, 1) to (0, 0, 0, 0).

In the situation 2, any of the equilibrium point of mixed strategies in a two-strategy multi-group game is a critical stable equilibrium point or an AUEP which will be proven in Section V-B.

TABLE 2. Equilibrium point and its asymptotic stability condition.

Equilibrium point (x, y, z, w)	Asymptotic stability condition
$(0,0,0,0)$	$a_8 < a_{16}, b_{12} < b_{16}, c_{14} < c_{16}, d_{15} < d_{16}$
$(0,0,0,1)$	$a_7 < a_{15}, b_{11} < b_{15}, c_{13} < c_{15}, d_{16} < d_{15}$
$(0,0,1,0)$	$a_6 < a_{14}, b_{10} < b_{14}, c_{16} < c_{14}, d_{13} < d_{14}$
$(0,0,1,1)$	$a_5 < a_{13}, b_9 < b_{13}, c_{15} < c_{13}, d_{14} < d_{13}$
$(0,1,0,0)$	$a_4 < a_{12}, b_{16} < b_{12}, c_{10} < c_{12}, d_{11} < d_{12}$
$(0,1,0,1)$	$a_3 < a_{11}, b_{15} < b_{11}, c_9 < c_{11}, d_{12} < d_{11}$
$(0,1,1,0)$	$a_2 < a_{10}, b_{14} < b_{10}, c_{12} < c_{10}, d_9 < d_{10}$
$(0,1,1,1)$	$a_1 < a_9, b_{13} < b_9, c_{11} < c_9, d_{10} < d_9$
$(1,0,0,0)$	$a_{16} < a_8, b_4 < b_8, c_6 < c_8, d_7 < d_8$
$(1,0,0,1)$	$a_{15} < a_7, b_3 < b_7, c_5 < c_7, d_8 < d_7$
$(1,0,1,0)$	$a_{14} < a_6, b_2 < b_6, c_8 < c_6, d_5 < d_6$
$(1,0,1,1)$	$a_{13} < a_5, b_1 < b_5, c_7 < c_5, d_6 < d_5$
$(1,1,0,0)$	$a_{12} < a_4, b_8 < b_4, c_2 < c_4, d_3 < d_4$
$(1,1,0,1)$	$a_{11} < a_3, b_7 < b_3, c_1 < c_3, d_4 < d_3$
$(1,1,1,0)$	$a_{10} < a_2, b_6 < b_2, c_4 < c_2, d_1 < d_2$
$(1,1,1,1)$	$a_9 < a_1, b_5 < b_1, c_3 < c_1, d_2 < d_1$

V. THE N-DIMENSIONAL GAME IN THE TYPICAL SCENARIO OF ELECTRICITY MARKET

Suppose that all the n ($n \geq 3$, $n \in Z$) groups have two pure strategies, i.e., $\{S_{i0}, S_{i1}\}$ ($i = 1, 2, \ldots, n$). For the i^{th} group, its strategy S_{i0}, S_{i1} chosen at a probability of p_i, $(1 - p_i)$, respectively.

A. REPLICATOR DYNAMICS SYSTEM OF THE N-GROUP GAME AND ITS JACOBIAN MATRIX

For strategy set $\{S_{1,m_1}, S_{2,m_2}, \ldots, S_{n,m_n}\}$ ($m_i \in \{0, 1\}, i = 1, 2, \ldots, n$). The payoff of strategy S_{i,m_i} can be expressed as

$$a_{i,M} \quad (M = \sum_{k=1}^{n} m_k \times 2^{n-k} + 1). \tag{5}$$

The replicator dynamics of the n-group game can be shown as

$$\begin{cases} \frac{dp_1}{dt} = p_1(1-p_1)D_1 \\ \frac{dp_2}{dt} = p_2(1-p_2)D_2 \\ \quad\vdots \\ \frac{dp_j}{dt} = p_j(1-p_j)D_j \\ \quad\vdots \\ \frac{dp_n}{dt} = p_n(1-p_n)D_n, \end{cases} \tag{6}$$

$$D_i = \sum [(a_{i,j} - a_{i,j+2^{n-i}}) \prod_{k=1,k\neq i}^{n} p_k{}^{1-x_k}(1-p_k)^{x_k}]$$

$$\begin{cases} j = \sum_{k=1}^{n} x_k \times 2^{n-k} + 1; \\ x_k \in \{0, 1\}; \\ x_i = 0; \\ i = 1, 2, \ldots, n. \end{cases} \tag{7}$$

The Jacobian matrix of replicator dynamics is shown as

$$J_n = \begin{bmatrix} (1-2p_1)D_1 & p_1(1-p_1)\frac{\partial D_1}{\partial p_2} & \ldots & p_1(1-p_1)\frac{\partial D_n}{\partial p_K} \\ p_2(1-p_2)\frac{\partial D_2}{\partial p_1} & (1-2p_2)D_2 & \ldots & p_2(1-p_2)\frac{\partial D_2}{\partial p_n} \\ \vdots & \vdots & \ddots & \vdots \\ p_n(1-p_n)\frac{\partial D_n}{\partial p_1} & p_n(1-p_n)\frac{\partial D_n}{\partial p_2} & \ldots & (1-2p_K)D_n \end{bmatrix}. \tag{8}$$

B. EQUILIBRIUM POINTS OF THE N-GROUP GAME

The equilibrium points of this system can be obtained from the (9).

$$\begin{cases} p_1(1-p_1)D_1 = 0 \\ p_2(1-p_2)D_2 = 0 \\ \quad\vdots \\ p_j(1-p_j)D_j = 0 \\ \quad\vdots \\ p_n(1-p_n)D_n = 0 \end{cases} \tag{9}$$

Equation (9) contain 3^n types of solutions (3^n types of the equilibrium points). Similar to the approach in Section IV-C, a total of two situations are discussed as follows. (1)

1) Situation 1 (pure strategy): $\forall p_i \in \{0, 1\}, i \in \{1, 2, \ldots, n\}$, this situation has 2^n types of solutions.
2) Situation 2 (mixed strategy): $\exists p_i \notin \{0, 1\}, i \in \{1, 2, \ldots, n\}$, this situation has $(3^n - 2^n)$ types of solutions.

In the situation 2, at least one of the values in p_i ($i = 1, 2, \ldots, n$) is not 0 or 1. Suppose $\{p_{l,1}, p_{l,2}, \ldots, p_{l,n_1}\} \subset \{p_1, p_2, \ldots, p_n\}$ and $\forall p_{l,i}$ ($i = 1, 2, \ldots, n_1$) $\notin \{0, 1\}$, according to Eq. (9), $D_{l,i} = 0$. Then, assume $\{p_{z,1}, p_{z,2}, \ldots, p_{z,n_1}\} \subset \{p_1, p_2, \ldots, p_n\}$ and $\forall p_{z,i}$ ($i = 1, 2, \ldots, n - n_1$) $\in \{0, 1\}$. $p_{l,1}, p_{l,2}, \ldots, p_{l,n_1}, p_{z,1}, p_{z,2}, \ldots, p_{z,n-n_1}$ is a new arrangement of $p_1, p_2, \ldots, p_n$.

The characteristic polynomial of (8) is (10).

$$|\lambda E - J_n| = \prod_{j=1}^{n-n_1} (\lambda - (-1)^{p_{z,j}} D_{z,j}) \left|\lambda E - J_{n_1}\right| = 0 \tag{10}$$

$$J_{n_1} = \begin{bmatrix} 0 & h_{l,1}\frac{\partial D_{l,1}}{\partial p_{l,2}} & \ldots & h_{l,1}\frac{\partial D_{l,1}}{\partial p_{l,n_1}} \\ h_{l,2}\frac{\partial D_{l,2}}{\partial p_{l,1}} & 0 & \ldots & h_{l,2}\frac{\partial D_{l,2}}{\partial p_{l,n_1}} \\ \vdots & \vdots & \ddots & \vdots \\ h_{l,n_1}\frac{\partial D_{l,n_1}}{\partial p_{l,1}} & h_{l,n_1}\frac{\partial D_{l,n_1}}{\partial p_{l,2}} & \ldots & 0 \end{bmatrix} \tag{11}$$

$$h_{l,j} = p_{l,j}(1-p_{l,j}), \quad j = 1, 2, \ldots n_1 \tag{12}$$

where $\left|\lambda E - J_{n_1}\right| = 0$ is a unitary n_1-degree polynomial equation with n_1 complex roots. According to (11), $\sum_{j=1}^{n_1} \lambda_j = tr(J_{n_1}) = 0$, $\lambda_1, \lambda_2, \ldots, \lambda_{n_1}$ are the eigenvalues of J_{n_1}. Then, $\exists Re(\lambda_t) \geq 0$ ($t = 1, 2, \ldots, n_1$). And all of the eigenvalues of J_{n_1} are also the eigenvalues of J_n. Therefore, J_n has one real non-negative eigenvalue at least. In the situation 2, any the equilibrium point of the system is a critical stable equilibrium point or an AUEP.

In the situation 1, when $\forall p_i \in \{0, 1\}$, $i \in \{1, 2, \ldots, n\}$, i.e., the components of the equilibrium point all take values of 0 or 1, the Jacobian matrix can be expressed as

$$J_n' = diag\{(-1)^{p_1}D_1, (-1)^{p_2}D_2, \ldots, (-1)^{p_n}D_n\}. \tag{13}$$

$$\left|\lambda E - J'_n\right| = 0 \tag{14}$$

The characteristic polynomial of J_n' is (14). For the equilibrium points in the situation 1, each polynomial in (7) is the sum of 2^{n-1} parts, and the value of $2^{n-1} - 1$ parts is 0. All eigenvalues of J_n' are real numbers, and the conditions that the values of the eigenvalues are negative can be expressed as

$$(-1)^{p_i}(a_{i,t+1} - a_{i,t+2^{n-i}+1}) < 0$$

$$\begin{cases} i = 1, 2, \ldots, n \\ t = \sum_{k=1,k\neq i}^{n} (1-p_k) \times 2^{n-k}. \end{cases} \tag{15}$$

In a n-group game, if each group has only two pure strategies, the asymptotically stable equilibrium points are $\{p_1, p_2, \ldots, p_n\}$ ($p_i \in \{0, 1\}$, $i = 1, 2, \ldots, n$), when the conditions in (15) are satisfied. Similar to the discussion in Section IV-C, when a certain equilibrium point satisfies n inequality conditions in (15), this equilibrium point is an ASEP. However, when the distribution of the payoff parameters of the system is changed, the equilibrium state of the system may evolve into another equilibrium state or unstable state. Therefore, the government policy to monitor the electricity market can change the distribution of the payment parameters and the ASEP of n groups in the electricity market.

Consequently, each participant is effectively integrated into the open competitive electricity market, so that the open electricity market can develop healthily and sustainably.

VI. THREE-GROUP AND MULTI-STRATEGY GAME IN THE TYPICAL SCENARIO OF OPEN ELECTRICITY MARKET

In this section, the number of strategies for each group is expanded to sixty. Set $\{S_{ai}, S_{bj}, S_{ck}\}$ $(i, j, k = 1, 2, 3, \ldots, 60)$ is the strategy set, when the three groups choose strategy S_{ai}, S_{bj}, S_{ck}, respectively.

The payoffs corresponding to the strategy set $\{S_{ai}, S_{bj}, S_{ck}\}$ are $\{a_H, b_H, c_H\}$ $(H = 3600i + 60j + k - 3660)$, respectively. Then, evolutionary game theory is used to simulate the process of market clearing on the power generation enterprises (PGEs).

A. PURE STRATEGY EQUILIBRIUM POINTS OF MULTI-STRATEGY EVOLUTIONARY GAME

The replicator dynamics of the three-group and multi-strategy game can be shown as

$$\begin{cases} \dfrac{dx_i}{dt} = x_i(E_{ai} - \bar{E}_a) \\ \dfrac{dy_i}{dt} = y_i(E_{bi} - \bar{E}_b) \qquad i = 1, 2, 3, \ldots, 60 \\ \dfrac{dz_i}{dt} = z_i(E_{ci} - \bar{E}_c), \end{cases} \tag{16}$$

$$\begin{cases} \bar{E}_a = E_{a60} + \sum_{i=1}^{59} x_i(E_{ai} - E_{a60}) \\ \bar{E}_b = E_{b60} + \sum_{i=1}^{59} y_i(E_{bi} - E_{b60}) \\ \bar{E}_c = E_{c60} + \sum_{i=1}^{59} z_i(E_{ci} - E_{c60}). \end{cases} \tag{17}$$

where x_i, y_i, z_i $(i = 1, 2, 3, \ldots, 59)$ represent the probability of the $1th$-group, the $2th$-group, and the $3th$-group choose strategy S_{ai}, S_{bi}, S_{ci}, respectively. The probabilities of choosing strategy S_{a60}, S_{b60}, S_{c60} for the three groups are $1 - \sum_{i=1}^{59} x_i$, $1 - \sum_{i=1}^{59} y_i$, and $1 - \sum_{i=1}^{59} z_i$, respectively. Benefits E_{ai}, E_{bi}, and E_{ci} denote the expected benefits of the $1th$-group, the $2th$-group and the $3th$-group, when the $1th$-group, the $2th$-group and the $3th$-group choose S_{ai}, S_{bi} and S_{ci}, respectively. Benefits $\bar{E}_a$, $\bar{E}_b$, $\bar{E}_c$ mean the average expected benefits of the three groups. The equilibrium points of pure strategy $\left\{x_i = 1, y_j = 1, z_k = 1\right\}$ $(i, j, k = 1, 2, \ldots, 60)$ can be obtained from the (16) and the (17).

The Jacobian matrix of the (16) is shown as the (18), as shown at the bottom of the next page. The order of the Jacobian matrix J_3 is 117, and the matrix J_3 consists of nine sub-blocks of the 59-order matrix. The non-diagonal sub-blocks of the matrix J_3 are all zero matrices.

The asymptotic stability conditions of the equilibrium point $\{x_i = 1, y_j = 1, z_k = 1\}$ $(i, j, k = 1, 2, \ldots, 60)$ can be obtained from (18) and expressed in (22). For each equilibrium point, each group has 59 inequality constraints, i.e., 177 constraints which can be represented by the max function to ensure its asymptotic stability, and the number of constraints corresponds to the order of the Jacobian matrix.

$$\begin{cases} \max\{a_{3600i_1+60j+k-3660}\} < a_H, \\ (i_1 = 1, 2, \ldots, 59;\ i = 60). \\ \max\{a_{3600i_1+60j+k-3660}\} < a_{212340+60j+k} < a_H, \\ (i_1 = 1, 2, \ldots, 59;\ i_1 \neq i;\ i < 60). \\ \max\{b_{3600i+60j_1+k-3660}\} < b_H, \\ (j_1 = 1, 2, \ldots, 59;\ j = 60). \\ \max\{b_{3600i+60j_1+k-3660}\} < b_{3600i+k-60} < b_H, \\ (j_1 = 1, 2, \ldots, 59;\ j_1 \neq j;\ j < 60). \\ \max\{c_{3600i+60j+k_1-3660}\} < c_H, \\ (k_1 = 1, 2, \ldots, 59;\ k = 60). \\ \max\{c_{3600i+60j+k_1-3660}\} < c_{3600i+60j-3600} < c_H, \\ (k_1 = 1, 2, \ldots, 59;\ k_1 \neq k;\ k < 60). \end{cases} \tag{22}$$

where $H = 3600i + 60j + k - 3660$.

B. EVOLUTIONARY GAME OF THE POWER GENERATION SIDE MARKET CLEARING

Suppose that the three PGEs are the PGE_1, the PGE_2, the PGE_3 corresponding to the $1th$-group, the $2th$-group and the $3th$-group in Section VI-A, respectively.

The bidding price strategies of each power generation enterprise are Price_i $(i, j = 1, 2, 3, 4;\ \text{Price}_i < \text{Price}_j \Leftrightarrow i < j)$. And the bidding power strategies of each power generation enterprise are Power_i $(i, j = 1, 2, \ldots, 14, 15;\ \text{Power}_i < \text{Power}_i \Leftrightarrow i < j)$. In the game, the bidding strategy of each power generation enterprise is composed of the bidding price and the bidding power. Hence, its strategy set has 4×15 or 60 elements. This paper assumes that each generator enterprise has only one generator unit. The power generation cost $C(P)$ of the power generation enterprises can be fitted by quadratic function approximate.

$$C(P) = \begin{cases} a \cdot P^2 + b \cdot P + c, & P \in [P_{\min}, P_{\max}] \\ 0, & P \notin [P_{\min}, P_{\max}] \end{cases} \tag{23}$$

where $P_{\max}$, $P_{\min}$, and P are the upper and lower limits of power generation capacity and the bidding power of the power generation companies, respectively; and a, b, c are constants. The benefits of the power generation enterprises after bidding can be expressed as $B_{clear} \cdot P - C(P)$; B_{clear} represents the uniform clearing electricity price.

Bidding mechanism:

1) Bidding stage: the bidding strategies of the PGE_1, the PGE_2, the PGE_3 are $\{B_1, P_1\}$, $\{B_2, P_2\}$, $\{B_3, P_3\}$, respectively; price B_i $(i = 1, 2, 3)$ is the bidding price of the PGE_i and P_i denotes its bidding power;

2) Price settlement stage: the PGEs trade electricity in the order of the bidding prices from low value to high value, until the power supply to meet the load demand. The bidding price of the last power generation enterprise to satisfy the load demand is called the marginal electricity price, i.e., the uniform clearing electricity price. When the bidding price of a power generation enterprises is higher than the unified clearing electricity price, the enterprise failed to bid. All the successful

$$J_3 = \begin{bmatrix} J_a & 0 & 0 \\ 0 & J_b & 0 \\ 0 & 0 & J_c \end{bmatrix} \tag{18}$$

$$J_a = \begin{cases} \begin{bmatrix} E_{a1}-E_{ai} & 0 & \cdots & 0 & \cdots & 0 \\ 0 & E_{a2}-E_{ai} & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ E_{a60}-E_{a1} & E_{a60}-E_{a2} & \cdots & E_{a60}-E_{ai} & \cdots & E_{a60}-E_{a59} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & E_{a59}-E_{ai} \end{bmatrix}, \\ i = 1, 2, \ldots, 59 \\ \\ diag(E_{a1} - E_{a60}, E_{a2} - E_{a60}, \ldots, E_{a59} - E_{a60}), \\ i = 60. \end{cases} \tag{19}$$

$$J_b = \begin{cases} \begin{bmatrix} E_{b1}-E_{bj} & 0 & \cdots & 0 & \cdots & 0 \\ 0 & E_{b2}-E_{bj} & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ E_{b60}-E_{b1} & E_{b60}-E_{b2} & \cdots & E_{b60}-E_{bj} & \cdots & E_{b60}-E_{b59} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & E_{b59}-E_{bj} \end{bmatrix}, \\ j = 1, 2, \ldots, 59 \\ \\ diag(E_{b1} - E_{b60}, E_{b2} - E_{b60}, \ldots, E_{b59} - E_{b60}), \\ j = 60. \end{cases} \tag{20}$$

$$J_c = \begin{cases} \begin{bmatrix} E_{c1}-E_{ck} & 0 & \cdots & 0 & \cdots & 0 \\ 0 & E_{c2}-E_{ck} & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ E_{c60}-E_{c1} & E_{c60}-E_{c2} & \cdots & E_{c60}-E_{ck} & \cdots & E_{c60}-E_{c59} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & E_{c59}-E_{ck} \end{bmatrix}, \\ k = 1, 2, \ldots, 59 \\ \\ diag(E_{c1} - E_{c60}, E_{c2} - E_{c60}, \ldots, E_{c19} - E_{c60}), \\ k = 60. \end{cases} \tag{21}$$

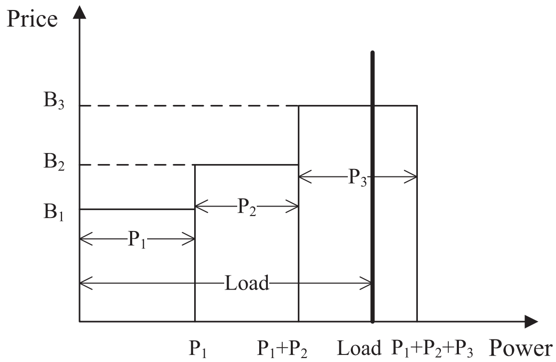


FIGURE 2. Schematic diagram of bidding for the power generation enterprises.

bidding power generation enterprises sell electricity at the unified clearing price.

3) Power settlement stage: the electricity sales of the power generation enterprises whose bidding price is lower than the clearing price are their bidding power. When the total bidding power exceeds the load demand, the transaction amount of electricity for each power generation enterprise whose bidding price is equal to the clearing price is $\tilde{P}_{ei}$. Power P_{ei} is the corresponding bidding power for this power generation enterprise; *Load* is load demand; and P_s is the cumulative transaction electricity of the power generation enterprises whose bidding price is lower than the clearing price. Considering the upper and lower limit constraints of power, this paper assumes that when the transaction electricity of a power generation enterprise is higher than the upper limit of power, its adjusted trading electricity is its upper limit of power. On the contrary, when the transaction electricity is below the lower limit of its capacity, the adjusted trading electricity of the power generation enterprise is zero. Under the condition of satisfying the constraint of upper and lower limits of power, each successful power generation enterprise adds electricity according to the proportion of its bidding power value to meet the adjusted power deficit. When the adjusted power failed to meet the load demand, all the power generation enterprises failed to bid.

$$\tilde{P}_{ei} = \frac{P_{ei}}{\sum P_{ei}} \cdot (Load - P_s) \tag{24}$$

The schematic diagram of bidding for the power generation enterprises is shown in FIGURE 2. Suppose that $B_1 < B_2 < B_3$ and $P_1 + P_2 < Load < P_1 + P_2 + P_3$, the unified clearing electricity price is B_3. When the output power of the PGE_3 meets its upper and lower bound constraints, the amount of electricity traded by the PGE_1, the PGE_2, the PGE_3 is P_1, P_2 and $Load - P_1 - P_2$, respectively. The power generation cost coefficients and power ranges of the three power generation enterprises are shown in TABLE 3. And the price strategies available for each power generator are 60 \$/MW·h, 70 \$/MW·h, 80 \$/MW·h, 90 \$/MW·h, respectively. The power strategies available for each power generator are $\frac{P_{\max}-P_{\min}}{14}(i-1)+P_{\min}$ $(i = \{1, 2, \ldots, 15\})$.

TABLE 3. Power generation enterprise parameters.

Enterprise	a [\$/MW^2·h]	b [\$/MW·h]	c [\$/h]	$P_{\min}$ [MW]	$P_{\max}$ [MW]
PGE_1	0.002	16.5	700	20	130
PGE_2	0.00211	16.5	680	20	130
PGE_3	0.00398	19.7	450	25	162

TABLE 4. Bidding strategy corresponding to game equilibrium point.

Load	Bidding strategy	Transaction power	Clearing pricing
100 MW·h	{35.71,60}, {35.71,60}, {103.28,60}	(20.44, 20.44, 59.12)	60 \$/MW·h
300 MW·h	{130,60}, {130,60}, {162,60}	(92.42, 92.42, 115.16)	60 \$/MW·h

When the *Load* is 100 MW·h and 300 MW·h, the optimal bidding strategy corresponding to the equilibrium point of the game of the three PGEs is shown in TABLE 4. The transaction power corresponds to the trading power obtained by the three power generation enterprises in the current situation. Payoff distribution is the main factor affecting the distribution of equilibrium point and its asymptotic stability. Under different trading mechanisms, the payoff distribution of the power generation enterprises is different, resulting in different distribution of the asymptotically stable equilibrium points. In complex games, the number of the asymptotically stable equilibrium points may be small or does not exist. The market regulator makes a reasonable transaction mechanism. For example, trading mechanisms that consider demand-side response or government regulation, so that the payoff distribution and the equilibrium points distribution of the game are evolving towards the desired direction.

VII. CONCLUSION

This paper mainly discusses the asymptotic stability of multi-group asymmetric evolutionary game in an open electricity market. The two-strategy four-group game and the two-strategy n-group game's replicator dynamics system are solved, respectively. And the asymptotically stable equilibrium points and their asymptotic stability equilibrium conditions are obtained. The bidding behavior of three power generation enterprises is simulated based on evolutionary game theory. Under the appropriate trading mechanism, the game of three power generation enterprises can achieve an asymptotically stable equilibrium state. In addition, the distribution of the asymptotically stable equilibrium points can be changed by changing the distribution of payoff parameters in the system. Decision makers are responsible for market design and interventions needed to achieve the goals, which makes the evolution of power system more reasonable.

However, the discussion on the equilibrium stability of the evolutionary game theory model established in the typical scenario of the open electricity market is not very strict. In the future, the game of the multi-agent continuous behavior in the open electricity market could be studied. Furthermore, the constraints of complex networks could be considered to make the model more consistent with actual power systems. At the same time, another equilibrium state, which makes the stable equilibrium point of the game easier to be solved and analyzed, will be considered.

REFERENCES

[1] A. Antoci and F. Sabatini, "Online networks, social interaction and segregation: An evolutionary approach," *J. Evol. Econ.*, vol. 28, no. 4, pp. 859–883, Sep. 2018.

[2] L. Han, T. Morstyn, and M. McCulloch, "Incentivizing prosumer coalitions with energy management using cooperative game theory," *IEEE Trans. Power Syst.*, vol. 34, no. 1, pp. 303–313, Jan. 2019.

[3] X. Wang, H. Zhang, Y. Tian, C. Zhu, and V. C. M. Leung, "Optimal distributed interference mitigation for small cell networks with non-orthogonal multiple access: A locally cooperative game," *IEEE Access*, vol. 6, pp. 63107–63119, 2018.

[4] D. Liu, W. Lv, H. Li, and J. Tang, "Bargaining model of labor disputes considering social mediation and bounded rationality," *Eur. J. Oper. Res.*, vol. 262, no. 3, pp. 1064–1071, Nov. 2017.

[5] M. Otte, M. Kuhlman, and D. Sofge, "Competitive target search with multi-agent teams: Symmetric and asymmetric communication constraints," *Auton. Robot.*, vol. 42, no. 6, pp. 1207–1230, Aug. 2018.

[6] L. Cheng and T. Yu, "Nash equilibrium-based asymptotic stability analysis of multi-group asymmetric evolutionary games in typical scenario of electricity market," *IEEE Access*, vol. 6, pp. 32064–32086, 2018.

[7] S. Grijalva and M. U. Tariq, "Prosumer-based smart grid architecture enables a flat, sustainable electricity industry," in *Proc. ISGT*, Jan. 2011, pp. 1–6.

[8] I. G. Ciuciu, R. Meersman, and T. Dillon, "Social network of smart-metered homes and SMEs for grid-based renewable energy exchange," in *Proc. 6th IEEE Int. Conf. Digit. Ecosyst. Technol. (DEST)*, Jun. 2012, pp. 1–6.

[9] D. Divenyi, B. Polgari, A. Sleisz, P. Sores, and D. Raisz, "Algorithm design for European electricity market clearing with joint allocation of energy and control reserves," *Int. J. Electr. Power Energy Syst.*, vol. 111, pp. 269–285, Oct. 2019.

[10] P. V. Gomes and J. T. Saraiva, "State-of-the-art of transmission expansion planning: A survey from restructuring to renewable and distributed electricity markets," *Int. J. Electr. Power Energy Syst.*, vol. 111, pp. 411–424, Oct. 2019.

[11] D. Streimikiene and I. Siksnelyte, "Sustainability assessment of electricity market models in selected developed world countries," *Renew. Sustain. Energy Rev.*, vol. 57, pp. 72–82, 2016.

[12] Y. Jiang, J. Hou, Z. Lin, F. Wen, J. Li, C. He, C. Ji, Z. Lin, Y. Ding, and L. Yang, "Optimal bidding strategy for a power producer under monthly pre-listing balancing mechanism in actual sequential energy dual-market in China," *IEEE Access*, vol. 7, pp. 70986–70998, 2019.

[13] D. Friedman, "On economic applications of evolutionary game theory," *J. Evol. Econ.*, vol. 8, no. 1, pp. 15–43, Mar. 1998.

[14] J. M. Smith, "Evolutionary game theory," *Phys. D, Nonlinear Phenomena*, vol. 22, nos. 1–3, pp. 43–49, 1997.

[15] P. D. Taylor and L. B. Jonker, "Evolutionary stable strategies and game dynamics," *Math. Biosci.*, vol. 40, no. 1, pp. 145–156, 1978.

[16] P. Schuster and K. Sigmund, "Replicator dynamics," *J. Theor. Biol.*, vol. 100, no. 3, pp. 533–538, 1983.

[17] M. Plank, "Some qualitative differences between the replicator dynamics of two player and N player games," *Nonlinear Anal., Theory, Methods Appl.*, vol. 30, no. 3, pp. 1411–1417, 1997.

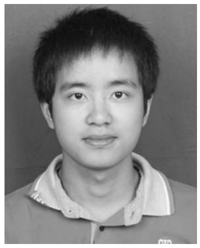
[18] J. M. Smith and G. R. Price, "The logic of animal conflict," *Nature*, vol. 246, no. 5427, pp. 15–18, 1973.

[19] J. M. Smith, "The theory of games and the evolution of animal conflicts," *J. Theor. Biol.*, vol. 47, no. 1, pp. 209–221, 1974.

[20] R. Cressman, "Frequency-dependent stability for two-species interactions," *Theor. Population Biol.*, vol. 49, no. 2, pp. 189–210, 1996.

[21] Q. Q. He, T. J. Feng, Y. Tao, B. Zhang, and T. Ji, "Asymmetric evolutionary game dynamics based on individuals' own volition," *J. Theor. Biol.*, vol. 454, pp. 118–125, 2018.

[22] M. Bukowski and J. Miekisz, "Evolutionary and asymptotic stability in symmetric multi-player games," *Int. J. Game Theory*, vol. 33, no. 1, pp. 41–54, Dec. 2004.



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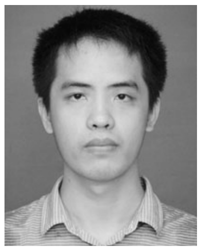

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