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Stability Analysis of Neutral-Type Cohen-Grossberg Neural Networks With Multiple Time-Varying Delays

LI WAN[1] AND QINGHUA ZHOU[2]
[1]School of Mathematics and Computer Science, Wuhan Textile University, Wuhan 430073, China
[2]School of Mathematics and Physics, Qingdao University of Science and Technology, Qingdao 266061, China

Corresponding author: Qinghua Zhou (zqhmath@126.com)

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ABSTRACT This paper deals with the problem for stability of neutral-type Cohen-Grossberg neural networks involving delay parameters. In the neutral-type neural networks, the states of the neurons involve multiple time-varying delays and time derivative of states of neurons include discrete time delays. We note that the neutral-type neural network cannot be expressed in the vector-matrix form due to multiple time-varying delays and discrete neutral delays, which leads to linear matrix inequality approach can not be employed to obtain stability conditions of this type of Cohen-Grossberg neural networks. Therefore, it is difficult for stability analysis of this type of Cohen-Grossberg neural networks to find suitable Lyapunov-Krasovskii functional and effective method. This paper constructs an appropriate Lyapunov-Krasovskii functional and employs M-matrix property to derive new sufficient conditions ensuring the global asymptotic stability of the equilibrium point of the neutral-type Cohen-Grossberg neural networks with multiple time-varying delays in the states and discrete delays in the time derivative of the states. The obtained stability conditions are easy to validate by testing basic matrix property. A constructive example is presented to indicate applicability of the obtained stability criteria. Compared with the existed references, the networks we studied are more general and the derived results develop and generalize the known results.

INDEX TERMS Neutral-type Cohen-Grossberg neural networks, multiple delays, global asymptotic stability, Lyapunov-Krasovskii functional.

I. INTRODUCTION

A class of neural network was named Cohen-Grossberg neural network in 1983 and it can describe some models from population biology and neurobiology, for example, the well-known Hopfield neural networks [1]. Since then, Cohen-Grossberg neural networks have been attracted considerable attention due to their successful applications in signal processing, pattern recognition, optimization, associative memories and finite-time consensus. These applications are based largely on the global stability theory or finite-time stability theory [2]–[5]. The early stability analysis of neural network did not consider the effect of time delay on the networks. In fact, time delays always exist in the signal transmission among the neurons due to the finite speeds of transmission and/or the traffic congestion. Time delays have many types such as constant delays, time-varying delays, distributed delays, leakage delays, and proportional delays so on [6]–[8]. Various delays are one of important factors that lead to instability or oscillation of the neural network. In addition, time delay may also appear in the derivatives of states. Neutral-type neural networks containing the information of past state derivatives have been proved to be useful systems in some specific application areas, for example, population ecology, propagation and diffusion models [9]–[11]. Recently, the stability analysis of neutral-type neural networks has been received considerable attention [12]–[23]. It is noted that neutral-type neural networks studied in [12]–[23] can be written in the matrix-vector forms and the stability criteria are presented in the linear matrix inequality forms.

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Compared with [12]–[23], the neutral-type Cohen-Grossberg neural networks studied in this paper cannot be expressed in the vector-matrix form due to multiple time-varying delays and discrete neutral delays, which leads to linear matrix inequality approach can not be employed to obtain stability conditions of this type of Cohen-Grossberg neural networks. Therefore, it is difficult for stability analysis of this type of Cohen-Grossberg neural networks to find suitable Lyapunov-Krasovskii functional and effective method. On the other hand, to our best knowledge, there are few published results about global asymptotic stability for neutral-type Cohen-Grossberg neural networks with multiple time-varying delays in states and discrete neutral delays in time derivative of states. These facts have been the main motivations of the current paper to focus on the global asymptotic stability of this type Cohen-Grossberg neural networks. This paper constructs an appropriate Lyapunov-Krasovskii functional and employs the property of M-matrix to derive new sufficient conditions ensuring the global asymptotic stability of the equilibrium point of this type Cohen-Grossberg neural network. Compared with [23]–[28], the neutral-type Cohen-Grossberg neural networks studied in this paper are more general, the proposed Lyapunov-Krasovskii functional is novel and the obtained results provide novel stability conditions. In particular, the obtained results generalize the known results in [28].

II. PRELIMINARIES

Consider the following neutral-type Cohen-Grossberg neural networks with multiple time-varying delays:

$$\dot{x}_i(t) = d_i(x_i(t))\Big\{ - c_i(x_i(t)) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} f_j(x_j(t-\tau_{ij}(t))) + u_i \Big\} + \sum_{j=1}^{n} e_{ij}\dot{x}_j(t-\xi_j), \quad (1)$$

where constants $u_i, e_{ij}, \xi_j, a_{ij}$ and b_{ij} denote external input, coefficients of the time derivative of the delayed state, the neutral delays and the strengths of the neuron interconnections, respectively. The time delays $\tau_{ij}(t)$ satisfies

$$0 \le \tau_{ij}(t) \le \tau,\ \dot{\tau}_{ij}(t) \le \bar{\tau} < 1, \quad t \ge 0.$$

Amplification function $d_i(\cdot)$, behaved function $c_i(\cdot)$ and activation function $f_j(\cdot)$ are continuous and there exist some positive constants $\underline{c}_i, \underline{d}_i, \overline{d}_i$ and $l_i, i = 1, \cdots, n$, such that for all $x, y \in R, x \neq y$,

$$0 < \underline{c}_i \le \frac{c_i(x) - c_i(y)}{x-y}, \quad 0 < \underline{d}_i \le d_i(x) \le \bar{d}_i, \quad |f_i(x) - f_i(y)| \le l_i|x-y|.$$

The initial conditions are $x_i(t) = \varphi_i(t)$ and $\dot{x}_i(t) = \phi_i(t) \in C([-\max\{\tau,\xi\},0],R)$, where $\xi = \max_{1\le j\le n}\{\xi_j\}$, $C([-\max\{\tau,\xi\},0],R)$ is the set of all continuous functions from $[-\max\{\tau,\xi\},0]$ to R.

Remark 1: Compared with [24] and [27], we do not require $\frac{c_i(x)-c_i(y)}{x-y}$ has upper bound. It implies that our conditions are less conservative.

By using the well-known Brouwers fixed-point theorem, it can be proved that system (1) has at least one equilibrium point $(x_1^*, \cdots, x_n^*)^T$. Let $y_i(t) = x_i(t) - x_i^*, i = 1, \cdots, n$, then system (1) turns to

$$\dot{y}_i(t) = \widetilde{d}_i(y_i(t))\Big\{ - \widetilde{c}_i(y_i(t)) + \sum_{j=1}^{n} a_{ij}\widetilde{f}_j(y_j(t)) + \sum_{j=1}^{n} b_{ij}\widetilde{f}_j(y_j(t-\tau_{ij}(t)))\Big\} + \sum_{j=1}^{n} e_{ij}\dot{y}_j(t-\xi_j), \quad (2)$$

where for $i, j = 1, \cdots, n$,

$$\begin{aligned} \widetilde{d}_i(y_i(t)) &= d_i(y_i(t)+x_i^*), \widetilde{c}_i(y_i(t)) \\ &= c_i(y_i(t)+x_i^*) - c_i(x_i^*), \\ \widetilde{f}_j(y_j(t)) &= f_j(y_j(t)+x_j^*) - f_j(x_j^*), \\ \widetilde{f}_j(y_j(t-\tau_{ij}(t))) &= f_j(y_j(t-\tau_{ij}(t))+x_j^*) - f_j(x_j^*). \end{aligned}$$

It is obvious that if $y(t) = 0$ of system (2) is global asymptotical stability, then the equilibrium point of system (1) is global asymptotical stability. It is easy to obtain for $i = 1, 2, \cdots, n$,

$$0 < \underline{d}_i \le \widetilde{d}_i(y_i(t)) \le \bar{d}_i,\ \underline{c}_i y_i^2(t) \le \widetilde{c}_i(y_i(t))y_i(t), \quad |\widetilde{f}_i(y_i(t))| \le l_i|y_i(t)|. \quad (3)$$

III. MAIN RESULTS

In this section, we will establish some sufficient conditions to ensure global asymptotic stability of the origin of system (2) by constructing a suitable Lyapunov-Krasovskii functional and using the property of M-matrix.

Define the matrix $W = (W_{ij})_{n\times n}$ with

$$\begin{aligned} W_{ii} &= \underline{c}_i\underline{d}_i - \overline{d}_i l_i(|a_{ii}| + \frac{|b_{ii}|}{1-\bar{\tau}}), \\ W_{ij} &= -\overline{d}_j l_i(|a_{ji}| + \frac{|b_{ji}|}{1-\bar{\tau}}), \quad i \neq j \end{aligned}$$

and the constants

$$\begin{aligned} \alpha_i &= p_i\underline{c}_i\underline{d}_i - \sum_{j=1}^{n} p_j\overline{d}_j l_i(|a_{ji}| + \frac{|b_{ji}|}{1-\bar{\tau}}), \\ \beta_i &= p_i\gamma - \sum_{j=1}^{n} p_j|e_{ji}|, \quad i = 1, 2, \cdots, n, \end{aligned}$$

in which $0 < \gamma < 1$. From [29], we know that if the real part of every eigenvalue of W is positive, then W is a nonsingular M-matrix and there exist positive numbers $p_1, p_2, \cdots, p_n$ such that $\alpha_i > 0, \forall i$. Therefore, we may state the following result.

Theorem 1: Suppose that W is a nonsingular M-matrix, that is, there exist positive numbers $p_1, p_2, \cdots, p_n$ such that $\alpha_i > 0, i = 1, 2, \cdots, n$. Moreover, for the same $p_1, p_2, \cdots, p_n, \beta_i > 0, i = 1, 2, \cdots, n$. Then $y(t) = 0$ of system (2) is globally asymptotically stable.

Proof: Constructing the following Lyapunov-Krasovskii functional

$$\begin{aligned} V(y(t)) &= \sum_{i=1}^{n} p_i[1-\gamma sgn(y_i(t))sgn(\dot{y}_i(t))]sgn(y_i(t))y_i(t) \\ &\quad + \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n} p_j\gamma \int_{t-\xi_j}^{t} |\dot{y}_j(s)|ds \\ &\quad + \frac{1}{1-\bar{\tau}}\sum_{i=1}^{n}\sum_{j=1}^{n} p_i\overline{d}_i|b_{ij}|l_j \int_{t-\tau_{ij}(t)}^{t} |y_j(s)|ds \\ &\quad + \eta\sum_{i=1}^{n}\sum_{j=1}^{n}\int_{t-\tau_{ij}(t)}^{t} |y_j(s)|ds, \end{aligned} \tag{4}$$

in which η is positive number whose value is to be identified later.

Computing $\dot{V}(y(t))$ along the trajectories of system (2), we derive

$$\begin{aligned} &\dot{V}(y(t)) \\ &= \sum_{i=1}^{n} p_i[1-\gamma sgn(y_i(t))sgn(\dot{y}_i(t))]sgn(y_i(t))\dot{y}_i(t) \\ &\quad + \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n} p_j\gamma|\dot{y}_j(t)| - \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n} p_j\gamma|\dot{y}_j(t-\xi_j)| \\ &\quad + \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{p_i\overline{d}_i|b_{ij}|l_j}{1-\bar{\tau}}(|y_j(t)| - (1-\dot{\tau}_{ij}(t))|y_j(t-\tau_{ij}(t))|) \\ &\quad + \eta\sum_{i=1}^{n}\sum_{j=1}^{n}[|y_j(t)| - (1-\dot{\tau}_{ij}(t))|y_j(t-\tau_{ij}(t))|] \\ &\le \sum_{i=1}^{n} p_i[1-\gamma sgn(y_i(t))sgn(\dot{y}_i(t))]sgn(y_i(t))\dot{y}_i(t) \\ &\quad + \sum_{i=1}^{n} p_i\gamma sgn(\dot{y}_i(t))\dot{y}_i(t) - \sum_{i=1}^{n} p_i\gamma|\dot{y}_i(t-\xi_i)| \\ &\quad + \sum_{i=1}^{n}\sum_{j=1}^{n} p_i\overline{d}_i|b_{ij}|l_j(\frac{|y_j(t)|}{1-\bar{\tau}} - |y_j(t-\tau_{ij}(t))|)] \\ &\quad + \eta\sum_{i=1}^{n}\sum_{j=1}^{n}[|y_j(t)| - (1-\bar{\tau})|y_j(t-\tau_{ij}(t))|]]. \end{aligned} \tag{5}$$

For $y_i(t) \neq 0$, it follows from (3) that

$$\begin{aligned} &p_i[1-\gamma sgn(y_i(t))sgn(\dot{y}_i(t))]sgn(y_i(t))\dot{y}_i(t) \\ &\quad + p_i\gamma sgn(\dot{y}_i(t))\dot{y}_i(t) \\ &= p_i sgn(y_i(t))\dot{y}_i(t) \\ &= -p_i sgn(y_i(t))\widetilde{d}_i(y_i(t))\widetilde{c}_i(y_i(t)) \\ &\quad + p_i sgn(y_i(t))\widetilde{d}_i(y_i(t))\sum_{j=1}^{n} a_{ij}\widetilde{f}_j(y_j(t)) \\ &\quad + p_i sgn(y_i(t))\widetilde{d}_i(y_i(t))\sum_{j=1}^{n} b_{ij}\widetilde{f}_j(y_j(t-\tau_{ij}(t))) \\ &\quad + p_i sgn(y_i(t))\sum_{j=1}^{n} e_{ij}\dot{y}_j(t-\xi_j) \\ &\le -p_i sgn(y_i(t))\widetilde{d}_i(y_i(t))\frac{\widetilde{c}_i(y_i(t))y_i(t)}{y_i(t)} \\ &\quad + p_i\overline{d}_i\sum_{j=1}^{n}|a_{ij}||\widetilde{f}_j(y_j(t))| \\ &\quad + p_i\overline{d}_i\sum_{j=1}^{n}|b_{ij}||\widetilde{f}_j(y_j(t-\tau_{ij}(t)))| \\ &\quad + p_i\sum_{j=1}^{n}|e_{ij}||\dot{y}_j(t-\xi_j)| \\ &\le -p_i\underline{c}_i\underline{d}_i|y_i(t)| + p_i\overline{d}_i\sum_{j=1}^{n}|a_{ij}|l_j|y_j(t)| \\ &\quad + p_i\overline{d}_i\sum_{j=1}^{n}|b_{ij}|l_j|y_j(t-\tau_{ij}(t))| + p_i\sum_{j=1}^{n}|e_{ij}||\dot{y}_j(t-\xi_j)|. \end{aligned}$$

For $y_i(t) = 0$, it follows from (3) and $0 < \gamma < 1$ that

$$\begin{aligned} &p_i[1-\gamma sgn(y_i(t))sgn(\dot{y}_i(t))]sgn(y_i(t))\dot{y}_i(t) \\ &\quad + p_i\gamma sgn(\dot{y}_i(t))\dot{y}_i(t) \\ &= p_i\gamma sgn(\dot{y}_i(t))\dot{y}_i(t) \\ &= -p_i\gamma sgn(\dot{y}_i(t))\widetilde{d}_i(y_i(t))\widetilde{c}_i(y_i(t)) \\ &\quad + p_i\gamma sgn(\dot{y}_i(t))\widetilde{d}_i(y_i(t))\sum_{j=1}^{n} a_{ij}\widetilde{f}_j(y_j(t)) \\ &\quad + p_i\gamma sgn(\dot{y}_i(t))\widetilde{d}_i(y_i(t))\sum_{j=1}^{n} b_{ij}\widetilde{f}_j(y_j(t-\tau_{ij}(t))) \\ &\quad + p_i\gamma sgn(\dot{y}_i(t))\sum_{j=1}^{n} e_{ij}\dot{y}_j(t-\xi_j) \\ &\le -p_i\gamma sgn(\dot{y}_i(t))\widetilde{d}_i(y_i(t))\widetilde{c}_i(y_i(t)) \\ &\quad + p_i\overline{d}_i\sum_{j=1}^{n}|a_{ij}||\widetilde{f}_j(y_j(t))| \\ &\quad + p_i\overline{d}_i\sum_{j=1}^{n}|b_{ij}||\widetilde{f}_j(y_j(t-\tau_{ij}(t)))| \\ &\quad + p_i\sum_{j=1}^{n}|e_{ij}||\dot{y}_j(t-\xi_j)| \\ &\le -p_i\underline{c}_i\underline{d}_i|y_i(t)| + p_i\overline{d}_i\sum_{j=1}^{n}|a_{ij}|l_j|y_j(t)| \\ &\quad + p_i\overline{d}_i\sum_{j=1}^{n}|b_{ij}|l_j|y_j(t-\tau_{ij}(t))| + p_i\sum_{j=1}^{n}|e_{ij}||\dot{y}_j(t-\xi_j)|, \end{aligned}$$

where $p_i\gamma sgn(\dot{y}_i(t))\widetilde{d}_i(y_i(t))\widetilde{c}_i(y_i(t)) = p_i\underline{c}_i\underline{d}_i|y_i(t)| = 0$ when $y_i(t) = 0$.

Therefore, for any $y_i(t) \in R$, one derives

$$\begin{aligned} p_i[1-\gamma sgn(y_i(t))sgn(\dot{y}_i(t))]sgn(y_i(t))\dot{y}_i(t) \\ +p_i\gamma sgn(\dot{y}_i(t))\dot{y}_i(t) \\ \le -p_i\underline{c}_i\underline{d}_i|y_i(t)| + p_i\overline{d}_i\sum_{j=1}^{n}|a_{ij}|l_j|y_j(t)| \\ +p_i\overline{d}_i\sum_{j=1}^{n}|b_{ij}|l_j|y_j(t-\tau_{ij}(t))| \\ +p_i\sum_{j=1}^{n}|e_{ij}||\dot{y}_j(t-\xi_j)|. \end{aligned} \tag{6}$$

From (5) and (6), it follows

$$\begin{aligned} \dot{V}(y(t)) \le & -\sum_{i=1}^{n}p_i\underline{c}_i\underline{d}_i|y_i(t)| \\ & +\sum_{i=1}^{n}\sum_{j=1}^{n}p_i\overline{d}_i|a_{ij}|l_j|y_j(t)| \\ & +\sum_{i=1}^{n}\sum_{j=1}^{n}p_i|e_{ij}||\dot{y}_j(t-\xi_j)| \\ & -\sum_{i=1}^{n}p_i\gamma|\dot{y}_i(t-\xi_i)| \\ & +\sum_{i=1}^{n}\sum_{j=1}^{n}p_i\overline{d}_i|b_{ij}|l_j\frac{|y_j(t)|}{1-\overline{\tau}} \\ & +\eta\sum_{i=1}^{n}\sum_{j=1}^{n}[|y_j(t)|-(1-\overline{\tau})|y_j(t-\tau_{ij}(t))|] \\ \le & -\sum_{i=1}^{n}[\alpha_i-n\eta]|y_i(t)| - \sum_{i=1}^{n}\beta_i|\dot{y}_i(t-\xi_i)| \\ & -\eta(1-\overline{\tau})\sum_{i=1}^{n}\sum_{j=1}^{n}|y_j(t-\tau_{ij}(t))|. \end{aligned} \tag{7}$$

From (7), we obtain

$$\begin{aligned} \dot{V}(y(t)) &\le -\sum_{i=1}^{n}[\alpha_i-n\eta]|y_i(t)| \\ &\le -[\min_{1\le i\le n}\{\alpha_i\}-n\eta]\sum_{i=1}^{n}|y_i(t)|. \end{aligned}$$

Therefore, we can choose $\eta < \frac{\min_{1\le i\le n}\{\alpha_i\}}{n}$ such that $\dot{V}(y(t)) < 0$ for $y(t) \neq 0$. If $y(t) = 0$ and $(\dot{y}_1(t-\xi_1), \cdots, \dot{y}_n(t-\xi_n))^T \neq 0$, then it follows from (7) that

$$\dot{V}(y(t)) \le -\min_{1\le i\le n}\{\beta_i\}\sum_{i=1}^{n}|\dot{y}_i(t-\xi_i)| < 0.$$

If $y(t) = (\dot{y}_1(t-\xi_1), \cdots, \dot{y}_n(t-\xi_n))^T = 0$ and $(y_j(t-\tau_{ij}))_{n\times n} \neq 0$, then it follows from (7) that

$$\dot{V}(y(t)) \le -\eta(1-\overline{\tau})\sum_{i=1}^{n}\sum_{j=1}^{n}|y_j(t-\tau_{ij}(t))| < 0.$$

If $y(t) = (\dot{y}_1(t-\xi_1), \cdots, \dot{y}_n(t-\xi_n))^T = 0$ and $(y_j(t-\tau_{ij}))_{n\times n} = 0$, then $\dot{V}(y(t)) = 0$. Thus, $\dot{V}(y(t)) < 0$ except for the origin, which shows $y(t) = 0$ is asymptotic stability.

In addition, it follows from (4) that

$$\begin{aligned} V(y(t)) &\ge \sum_{i=1}^{n}p_i[1-\gamma sgn(y_i(t))sgn(\dot{y}_i(t))]|y_i(t)| \\ &\ge \min_{1\le i\le n}\{p_i\}(1-\gamma)\sum_{i=1}^{n}|y_i(t)| \\ &= \min_{1\le i\le n}\{p_i\}(1-\gamma)\|y(t)\|_1, \end{aligned}$$

which implies that $V(y(t)) \to \infty$ as $\|y(t)\| \to \infty$, that is, $V(y(t))$ is radially unbounded. Thus, $y(t) = 0$ is global asymptotic stability.

Remark 2: We note that Theorem 1 generalizes the result in [28]. That is, Theorem 1 in [28] can be taken as a corollary of our result. If $\overline{\tau} = 0$, then the stability conditions of Theorem 1 are delay-independent.

In order to use Theorem 1 conveniently, we usually choose that $p_1 = \cdots = p_n = 1$.

Corollary 1: Let γ be a positive constant such that $0 < \gamma < 1$ and

$$\tilde{\beta}_i := \gamma - \sum_{j=1}^{n}|e_{ji}| > 0, \quad \forall i.$$

Suppose that

$$\tilde{\alpha}_i := \underline{c}_i\underline{d}_i - \sum_{j=1}^{n}\overline{d}_jl_i(|a_{ji}| + \frac{|b_{ji}|}{1-\overline{\tau}}) > 0, \quad \forall i.$$

Then $y(t) = 0$ of system (2) is global asymptotical stability.

Clearly, system (1) includes the following neutral-type Cohen-Grossberg neural networks studied in [24], [26]

$$\begin{aligned} \dot{x}_i(t) &- \sum_{j=1}^{n}e_{ij}\dot{x}_j(t-\xi_j) \\ &= d_i(x_i(t))\Big\{-c_i(x_i(t)) + \sum_{j=1}^{n}a_{ij}f_j(x_j(t)) \\ &\quad + \sum_{j=1}^{n}b_{ij}f_j(x_j(t-\tau_j)) + u_i\Big\}, \end{aligned} \tag{8}$$

the following neutral-type Hopfield neural networks considered in [23], [25]

$$\begin{aligned} \dot{x}_i(t) &- \sum_{j=1}^{n}e_{ij}\dot{x}_j(t-\xi_j) \\ &= -c_ix_i(t) + \sum_{j=1}^{n}a_{ij}f_j(x_j(t)) + \sum_{j=1}^{n}b_{ij}f_j(x_j(t-\tau_j)) + u_i \end{aligned} \tag{9}$$

and investigated in [28]

$$\dot{x}_i(t) - \sum_{j=1}^{n} e_{ij}\dot{x}_j(t-\xi_j) = -c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} f_j(x_j(t-\tau_{ij})) + u_i. \quad (10)$$

Corollary 2: Under the conditions of Theorem 1, the equilibrium point of system (8) is globally asymptotically stable.

Corollary 3: Suppose that there exist positive numbers $p_1, p_2, \cdots, p_n, \gamma$ such that $0 < \gamma < 1$,

$$\alpha_i = p_i c_i - \sum_{j=1}^{n} p_j l_i(|a_{ji}| + |b_{ji}|) > 0,$$
$$\beta_i = p_i\gamma - \sum_{j=1}^{n} p_j |e_{ji}| > 0, \quad i = 1, 2, \cdots, n.$$

Then the equilibrium point of system (9) and (10) is globally asymptotically stable.

Example 1: Consider system (1) with the following system matrices and the network functions:

$$A = \begin{pmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & -1 \end{pmatrix},$$
$$B = \begin{pmatrix} 0.5 & 0.5 & -0.5 & 0.5 \\ -0.5 & -0.5 & 0.5 & -0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 \\ -0.5 & -0.5 & -0.5 & 0.5 \end{pmatrix},$$

$d_1(x) = 1.5 + 0.5sinx, d_2(x) = 1.5 - 0.5cosx$, $d_3(x) = 1.5 - 0.5sinx, d_4(x) = 1.5 + 0.5cosx$, $c_i(x) = 9x, f_i(x) = 0.5\ tanh(x), \tau_{ij}(t) = 0.5sint$, $i = j; \tau_{ij}(t) = 0.5cost, i \neq j; i, j = 1, 2, 3, 4.$

Therefore, we calculate that $l_i = \bar{\tau} = 0.5, \underline{d}_i = 1$, $\overline{d}_i = 2, \underline{c}_i = 9, i = 1, 2, 3, 4$, and

$$W = \begin{pmatrix} 7 & -2 & -2 & -2 \\ -2 & 7 & -2 & -2 \\ -2 & -2 & 7 & -2 \\ -2 & -2 & -2 & 7 \end{pmatrix}.$$

It is obvious that W is a nonsingular M-matrix and we can choose $p_i = 1$ such that $\tilde{\alpha}_i > 0, i = 1, \cdots, 4$. Therefore, if W in this example is a nonsingular M-matrix and $\|E\|_1 < \gamma$, then the stability conditions of Corollary 1 hold.

Now, we choose $c_1(x) = c_2(x) = 9x, c_3(x) = c_4(x) = 8x$, then we calculate

$$W = \begin{pmatrix} 7 & -2 & -2 & -2 \\ -2 & 7 & -2 & -2 \\ -2 & -2 & 6 & -2 \\ -2 & -2 & -2 & 6 \end{pmatrix}.$$

From [28] and [29], we know that all eigenvalues of the matrix W are positive and W is a nonsingular M-matrix. Therefore, from Example 1 in [28], we know that there exist positive numbers p_1, p_2, p_3 and p_4 such that all α_i and β_i in Theorem 1 are positive. Thus, the stability conditions of Theorem 1 hold if W in this case is a nonsingular M-matrix and $||E||_1 < 1$.

IV. CONCLUSION

This paper has discussed global asymptotic stability of the equilibrium point of neutral-type Cohen-Grossberg neural networks involving delay parameters. Since the neural networks contain multiple time-varying delays in the states and discrete delays in the time derivative of the states, the neural networks cannot be expressed in the vector-matrix forms, which leads to leads to linear matrix inequality approach can not be employed to obtain stability conditions of this type of Cohen-Grossberg neural networks. Therefore, for stability analysis of this type of neural system, it is difficult to find suitable Lyapunov-Krasovskii functional and effective method. By constructing an appropriate Lyapunov-Krasovskii functional and using the property of M-matrix, new sufficient conditions have been derived to guarantee the global asymptotic stability of the equilibrium point of neutral-type Cohen-Grossberg neural networks with multiple time-varying delays and with multiple time-varying delays and discrete neutral delays. The obtained stability conditions are easy to validate by testing basic matrix property. Compared with the existed references, the networks we studied are more general and the derived results develop and generalize the known results. In the future work, we will study exponential stability for neutral-type Cohen-Grossberg neural networks with multiple time-varying delays (or other delays) and investigate stability problem of neutral-type complex-valued Cohen-Grossberg neural networks.

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LI WAN received the Ph.D. degree from Nanjing University, Nanjing, China. He was a Postdoctoral Fellow with the Department of Mathematics, Huazhong University of Science and Technology, Wuhan, China. Since August 2006, he has been with the School of Mathematics and Computer Science, Wuhan Textile University, Wuhan. He is the author or coauthor of more than 30 journal articles. His research interests include nonlinear dynamic systems, neural networks, and control theory.

QINGHUA ZHOU received the Ph.D. degree from Nanjing University, Nanjing, China. From August 2007 to August 2019, she was with the Department of Mathematics, Zhaoqing University, Zhaoqing, China. Since September 2019, she has been with the School of Mathematics and Physics, Qingdao University of Science and Technology, Qingdao, China. She is the author or coauthor of more than 25 journal articles. Her research interests include nonlinear dynamic systems and neural networks.