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Formation Control of High-Order Swarm Systems With Time-Varying Delays and Switching Interconnections

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ABSTRACT The paper investigates the formation control problem for high-order linear swarm systems with limited communications such as time-varying delays and switching interconnections. Firstly, the problem description is given including the dynamics of high-order swarm systems, formation protocol and the definitions of formation maintenance and tracking. Secondly, four formation conditions are derived: the former three are related to formation function, reference trajectory, auxiliary functions and network topology; the fourth one is equivalent to the stability of N-1 time-delay systems. In order to get lower conservative criteria, the Free-weighting Matrices(FWM) approach is employed to analyze the stabilization problem. Finally, the allowance upper bound of delays is calculated through solving the feasible linear matrix inequalities (LMIs). Numerical examples and simulation results are given to demonstrate the effectiveness and benefit on reducing conservativeness of the proposed method.

INDEX TERMS Formation tracking, formation maintenance, high-order swarm systems, time-varying delays, switching interconnections.

I. INTRODUCTION

Formation control of swarm systems is widely applied in surveillance and reconnaissance [1]–[3], target searching and localization [4]–[7], telecommunication relay [8], space exploration and source seeking [9], [10]. Main approaches for formation problem include leader-follower [11], behavior approach [12] and virtual structure [13]. However, these methods still have some shortcomings such as lower robustness, complex modeling or mass communication. Recently, the distributed formation control strategy based on consensus algorithm has been widely discussed [14], [15]. It is shown that the traditional approach aforementioned can be regarded as the special case of the consensus-based method [14]. Moreover, some drawbacks of the traditional approaches can be overcome.

Formation maintenance is one of the interesting topics when referring to the formation control problem. The consensus-based strategy is adopted to investigate the formation maintenance of first-order systems (e.g. wheeled vehicles [16], [17], MAS [18]) or second-order systems (e.g. MAS [14], multi-UAV systems [19], [20]) in the early studies. The high-order systems with undirected topology, which consists by a series of second order models, is discussed [21]. However, the eigenvalues of Laplacian matrix may be complex number when the network topology is directed. In such a case, it is difficult to analyze the formation feasibility. In [22], the formation control is investigated under the directed topology using the state and output feedback method. Fully distributed time-varying formation control problem was discussed for high-order linear MASs with directed network by developing an adaptive output-feedback approach [23].

In practical applications, the network limitations are taking into consideration like communication delays or switching topologies. The literature points out that the delays will affect the formation convergence time of the second-order swarm systems [24]. The influence of delay on formation and convergence speed can be improved by introducing diverse self-delay. Similarity, the formation control with both position delay and velocity delay is investigated [25]. The conclusion can be drawn that the increase of communication delay

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does not make the performance worse definitely. Further, the formation feasibility such as delay-independent or dependent condition is derived using the Nyquist criterion [26] or Lyapunov appraoches [27]–[31]. The latter method seems better when dealing with the time-varying delays and switching topologies [32]–[35]. The formation feasibility depends on the constructed Lyapunov functional in this cases.

Another application is the formation tracking or enclosing the moving target. The analysis and design is investigated with one leader [36], [37] or multiple leaders [15], [38]–[41]. These results aimed at the ideal communication conditions. The formation tracking problem with time-varying delays is presented, and the protocol is derived based on the full state information [42]. The formation tracking with switching topologies is discussed when solving the moving-target-enclosing problem [43]. The formation and consensus tracking with switching topologies is discussed [44]. The necessary and sufficient condition of the formation tracking with switching topologies is given [45]. As far as we know, few literature focus on the formation tracking with both delays and switching topologies.

This paper mainly focusses on the formation control problem of high-order swarm systems with time-varying delays and switching topologies. Different from the existing results, the main contributions of this paper are threefold. First, the unified framework is presented to solve the formation maintenance and tracking problem with limited communication conditions. Four formation conditions are derived. Second, both time-varying delays and switching topologies are considered. The similar limitations are considered when the formation maintenance is discussed [32], [34]. However, there are some challenges on auxiliary function design and stability analysis in terms of formation tracking. The existing results only study the case with network delays [41] or switching topologies [42]–[45]. Third, the proposed method in this paper has lower conservativeness. The formation condition is analyzed using the stability analysis of time-delay systems [32], [34], [42]. The derived sufficient condition has larger conservativeness due to the constructed Lyapunov or Lyapunov-Kravoskii functional. The FWM approach is adopted to analyze the stabilization problem in this paper, and the LMIs criteria with lower conservativeness are given. Furthermore, the upper bound of time-varying delays and the formation control gains can be obtained through justifying the feasible of LMIs.

The rest of this paper is organized as follows. In Section II, basic concepts and useful results on graph theory are introduced. In Section III, the problem descriptions are presented, and the necessary and sufficient conditions are also given. Further, the stability issues are discussed, and the algorithm procedure is derived. Numerical simulations and discussions are shown in Section IV. Finally, some concluding remarks are stated in Section V.

Throughout this paper, for any complex vector x, real matrix X and $\lambda \in \mathbb{C}$, let $\hat{x} = [\mathrm{Re}(x)^T, \mathrm{Im}(x)^T]^T$, $\Lambda_X = diag\{X, X\}$ and $\Phi_\lambda = \begin{bmatrix} \mathrm{Re}(\lambda)I & -\mathrm{Im}(\lambda)I \\ \mathrm{Im}(\lambda)I & \mathrm{Re}(\lambda)I \end{bmatrix}$, where I is an identity matrix and $\mathbf{0}$ is zero matrix with appropriate dimensions.

II. GRAPH THEORY AND RELATED LEMMAS

A. GRAPH THEORY

A graph $G = (V, \varepsilon, W)$ consists of vertices set $V = \{v_1, v_2, \ldots, v_N\}$, edges set $\varepsilon = \left\{(v_i, v_j) : v_i, v_j \in V\right\}$ and adjacency matrix $W = [w_{ij}] \in \mathbb{R}^{N\times N}$. $e_{ij} = (v_i, v_j)$ denotes a directed edge from v_i to v_j. The positive element w_{ij} describes the weight of the edge e_{ji}, and $w_{ij} > 0$ means node v_i can receive information from node v_j, that is $e_{ji} \in \varepsilon$. If there exists $e_{ij} \in \varepsilon$ for any $e_{ji} \in \varepsilon$, the graph G is defined as undirected graph. Otherwise, it is a digraph. For $i \in \{1, 2, \ldots, N\}$, there is $w_{ii} = 0$. The in-degree of the node v_i is defined as $\deg_{in}(v_i) = \sum_{j=1}^{N} w_{ij}$. Let $D = diag\left\{\deg_{in}(v_1), \deg_{in}(v_2), \ldots, \deg_{in}(v_N)\right\}$ denote the diagonal matrix with the in-degree of each node along the diagonal. The Laplacian matrix of the digraph G is defined as $L = D - W$.

B. RELATED LEMMAS

Lemma 1 [46]: There exists at least one zero eigenvalue for the Laplacian matrix, and $\mathbf{1}$ is the associated eigenvector. That is $L\mathbf{1} = 0$. If the digraph G has a spanning tree, then 0 is a simple eigenvalue of L, and all the other eigenvalues have positive real parts.

Lemma 2 (Schur Complement) [47]: For a given symmetric matrix $S = S^T = \begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix}$, where $S_{11} \in \mathbb{R}^{r\times r}$, the following three conditions are equivalent: (1) $S < 0$; (2) $S_{11} < 0$, $S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$; (3) $S_{22} < 0$, $S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

Lemma 3 [48]: If there exists a symmetric matrix X such that $\begin{bmatrix} P_1 + X & Q_1 \\ * & R_1 \end{bmatrix} > 0$ and $\begin{bmatrix} P_2 - X & Q_2 \\ * & R_2 \end{bmatrix} > 0$ hold at the same time, then it's necessary and sufficient condition is $\begin{bmatrix} P_1 + X & Q_1 & Q_2 \\ * & R_1 & 0 \\ * & * & R_2 \end{bmatrix} > 0$.

III. MAIN RESULTS

A. PROBLEM FORMULATION

Consider a high-order linear swarm systems

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), \quad i = 1, 2, \ldots, N \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$ is the state variable and $u_i(t) \in \mathbb{R}^m$ is the control protocol. Assume that B is a full column rank, and the communication relationship between agents can be described as a digraph G with a spanning tree structure.

The distributed control protocol with no leader is given as follows

$$u_i^{FT}(t) = u_{i1}^{FT}(t) + u_{i2}^{FT}(t) + u_{i3}^{FT}(t),$$
$$u_{i1}^{FT}(t) = K_1[x_i(t) - h_i(t) - r(t)],$$

$$u_{i2}^{FT}(t) = v_i(t) + s_i(t),$$
$$u_{i3}^{FT}(t) = K_2 \sum_{j\in\mathcal{N}_i} w_{ij} \{[x_j(t-d_t) - h_j(t-d_t) - r(t-d_t)] - [x_i(t-d_t) - h_i(t-d_t) - r(t-d_t)]\}. \quad (2)$$

in which K_1 and K_2 are constant gain matrices with appropriate dimension. $v_i(t) \in \mathbb{R}^m$ and $s_i(t) \in \mathbb{R}^m$ are the external auxiliary functions that depends on the formation $h(t)$ and the reference trajectory $r(t)$ respectively. In the following discussions, we assume that $h(t)$ and $r(t)$ are uniform continuously differentiable. d_t denotes the time-varying delays between agents. The neighbors set of agent i is described as $\mathcal{N}_i$. The switching topologies set is defined as $\{G_1, G_2, \ldots, G_\kappa\}$, $\kappa \geq 1$. When $t \in [-d_t, 0]$, $x_i(t) = \varsigma_i(t)$. Here, $\varsigma_i(t)$ is a bounded vector-valued function.

Definition 1 (Formation Tracking): considering time-varying delays and switching topologies, if there exist the protocol Eq.(2) such that

$$\lim_{t\to\infty}(x_i(t) - h_i(t) - r(t)) = 0, \quad i = 1, 2, \ldots, N \quad (3)$$

then the swarm systems Eq.(1) realize the desired time-varying formation $h_i(t), i = 1, 2, \ldots, N$ and track the reference trajectory $r(t)$.

If the reference trajectory $r(t) \equiv 0$, then the formation tracking problem is reduced to the maintenance problem. The distributed control protocol without $r(t)$ and $s_i(t)$ is

$$u_i^F(t) = u_{i1}^F(t) + u_{i2}^F(t) + u_{i3}^F(t),$$
$$u_{i1}^F(t) = K_1[x_i(t) - h_i(t)],$$
$$u_{i2}^F(t) = v_i(t),$$
$$u_{i3}^F(t) = K_2 \sum_{j\in\mathcal{N}_i} w_{ij} \{[x_j(t-d_t) - h_j(t-d_t)] - [x_i(t-d_t) - h_i(t-d_t)]\}. \quad (4)$$

Definition 2 (Formation Maintenance): considering time-varying delays and switching topologies, if there exist the protocol Eq.(4) such that

$$\lim_{t\to\infty}(x_i(t) - h_i(t)) = 0, \quad i = 1, 2, \ldots, N \quad (5)$$

then the swarm systems Eq.(1) realize the desired time-varying formation $h_i(t), i = 1, 2, \ldots, N$.

B. NECESSARY AND SUFFICIENT CONDITION

According to the Definition 1, Theorem 1 gives the necessary and sufficient conditions when the swarm systems realize the formation tracking.

Theorem 1: the swarm systems Eq.(1) applied protocol Eq.(2) achieve the formation $h_i(t), i = 1, 2, \ldots, N$ and track the trajectory $r(t)$ if and only if the following four conditions hold:

i)

$$\lim_{t\to\infty}\{(I_N \otimes \tilde{B}_2 A)h(t) - (I_N \otimes \tilde{B}_2)\dot{h}(t) + (I_N \otimes \tilde{B}_2 A)\tilde{r}(t) - (I_N \otimes \tilde{B}_2)\dot{\tilde{r}}(t)\} = 0 \quad (6)$$

ii)

$$\lim_{t\to\infty}\{(I_N \otimes \tilde{B}_1 A)h(t) - (I_N \otimes \tilde{B}_1)\dot{h}(t) + (I_N \otimes I)v(t) + (I_N \otimes I)s(t) + (I_N \otimes \tilde{B}_1 A)\tilde{r}(t) - (I_N \otimes \tilde{B}_1)\dot{\tilde{r}}(t)\} = 0 \quad (7)$$

iii) system (8) is asymptotically stable.

$$\dot{\xi}_1(t) = (A + BK_1)\xi_1(t) \quad (8)$$

iv) for $i = 2, 3, \ldots, N$, system (9) is asymptotically stable.

$$\dot{\xi}_i(t) = (A + BK_1)\xi_i(t) - \lambda_{\sigma(t),i}BK_2\xi_i(t-d_t) \quad (9)$$

where $\tilde{B} = [\tilde{B}_1^T, \tilde{B}_2^T]^T$ is a non-singular matrix satisfying $\tilde{B}_1 B = I$ and $\tilde{B}_2 B = \mathbf{0}$. The vector $\tilde{r}(t)$ is defined as $\underbrace{[r^T(t), \ldots, r^T(t)]^T}_{N}$, and the state variable $\xi(t)$ is the same dimension as $x(t)$. $\lambda_{\sigma(t),i}(i = 2, 3, ...N)$ are the non-zero eigenvalues of Laplacian matrix $L_{\sigma(t)}$, and the piecewise constant $\sigma(t) : [0, +\infty) \to \{1, 2, \ldots, \kappa\}$ denotes the switching signal among the digraph set $\{G_1, G_2, \ldots, G_\kappa\}$, $\kappa \geq 1$.

Proof: Substitution Eq.(2) into Eq.(1) gives the closed-loop equation when $t > 0$.

$$\dot{x}(t) = [I_N \otimes (A+BK_1)]x(t) - (L_{\sigma(t)} \otimes BK_2)x(t-d_t) + \Delta(t),$$
$$\Delta(t) = (I_N \otimes B)v(t) + (I_N \otimes B)s(t) - (I_N \otimes BK_1)h(t) - (I_N \otimes BK_1)\tilde{r}(t) + (L_{\sigma(t)} \otimes BK_2)h(t-d_t), \quad (10)$$

where $x(t) = \varsigma(t)$ when $t \in [-d_t, 0]$. $x(t)$, $v(t)$, $s(t)$ and $\varsigma(t)$ are vectors defined as follows

$$x(t) = [x_1^T(t), x_2^T(t), \ldots, x_N^T(t)]^T,$$
$$v(t) = [v_1^T(t), v_2^T(t), \ldots, v_N^T(t)]^T,$$
$$s(t) = [s_1^T(t), s_2^T(t), \ldots, s_N^T(t)]^T,$$
$$\varsigma(t) = [\varsigma_1^T(t), \varsigma_2^T(t), \cdots, \varsigma_N^T(t)]^T.$$

Let

$$\tilde{x}_i(t) = x_i(t) - h_i(t) - r(t), \quad i = 1, 2, \ldots, N,$$

then

$$\tilde{x}(t) = [\tilde{x}_1^T(t), \tilde{x}_2^T(t), \cdots, \tilde{x}_N^T(t)]^T$$

Rewriting Eq.(10) as

$$\dot{\tilde{x}}(t) = [I_N \otimes (A+BK_1)]\tilde{x}(t) - (L_{\sigma(t)} \otimes BK_2)\tilde{x}(t-d_t) + \tilde{\Delta}(t),$$
$$\tilde{\Delta}(t) = (I_N \otimes A)h(t) + (I_N \otimes A)\tilde{r}(t) + (I_N \otimes B)v(t) + (I_N \otimes B)s(t) - (I_N \otimes I_n)\dot{h}(t) - (I_N \otimes I_n)\dot{\tilde{r}}(t). \quad (11)$$

Here, $\tilde{x}(t) = \varsigma(t) - h(t) - r(t)$ when $t \in [-d_t, 0]$.

According to the Definition 1, the swarm systems Eq.(1) achieve the formation $h(t)$ and track the trajectory $r(t)$ when two conditions are satisfied:

$$\lim_{t\to\infty}\tilde{\Delta}(t) = 0 \quad (12)$$

and the close-loop system

$$\dot{\tilde{x}}(t)=[I_N\otimes(A+BK_1)]\tilde{x}(t)-(L_{\sigma(t)}\otimes BK_2)\tilde{x}(t-d_t) \quad (13)$$

is asymptotically stable for all $\sigma(t)$.

Multiplied by $I\otimes\tilde{B}$ at both sides of Eq.(12), it turns out that Eq.(12) is equivalent to Eq.(6) and Eq.(7).

For another, there exists a nonsingular matrix $E_{\sigma(t)}$ satisfying $E^{-1}_{\sigma(t)}L_{\sigma(t)}E_{\sigma(t)} = J_{\sigma(t)}$ according to Laplacian matrix property. That means $J_{\sigma(t)}$ is the Jordan expression of $L_{\sigma(t)}$.

Define

$$\xi=(E^{-1}_{\sigma(t)}\otimes I)\tilde{x}(t),$$

then Eq.(13) can be rewritten as

$$\dot{\xi}(t)=[I_N\otimes(A+BK_1)]\xi(t)-(J_{\sigma(t)}\otimes BK_2)\xi(t-d_t) \quad (14)$$

Obviously, closed-loop system Eq.(14) is stable if and only if the systems Eq.(8) and Eq.(9) are stable simultaneously by Lemma 1. Since the above process are equivalent transformations, four conditions described by Eq.(6), Eq.(7), Eq.(8) and Eq.(9) are necessary and sufficient conditions of the formation tracking problem. □

Similarly, the necessary and sufficient conditions for formation maintenance problem can be derived as follows.

Corollary 1: the swarm systems Eq.(1) applied protocol Eq.(4) achieve the formation $h_i(t)$, $i=1,2,\ldots,N$ if and only if the following four conditions hold:

i)

$$\lim_{t\to\infty}(I_N\otimes\tilde{B}_2A)h(t)-(I_N\otimes\tilde{B}_2)\dot{h}(t)=0 \quad (15)$$

ii)

$$\lim_{t\to\infty}\{(I_N\otimes\tilde{B}_1A)h(t)-(I_N\otimes\tilde{B}_1)\dot{h}(t)+(I_N\otimes I)v(t)\}=0 \quad (16)$$

iii) and iv) are same as the ones in Theorem 1.

Remark 1: Based on the above discussions, the condition i and ii in Theorem 1 or Corollary 1 can be satisfied through selecting suitable $h(t)$, $r(t)$, $v(t)$ and $s(t)$. If $A+BK_1$ is Hurwitz matrix, then system Eq.(8) is stable. This condition can be satisfied through eigenvalues configuration. That means suitable K_1 can be determined easily. In the following, the asymptotical stability of time-delay systems Eq.(9) will be discussed for $i=2,3,\ldots N$. Moreover, the allowance upper bound of delay can be calculated by numerical tools.

C. STABILITY OF TIME-DELAY SYSTEMS

Assume that time-varying delay $0\le d(t)\le d_{up}$ and its derivative $\dot{d}(t)\le d_{var}$, $0<d_{var}<1$.

Theorem 2: $\forall i=2,3,\ldots,N$, if there exist the real matrices $\mathcal{P}=\mathcal{P}^T>0$, $\mathcal{Q}=\mathcal{Q}^T>0$, $\mathcal{R}=\mathcal{R}^T>0$, $\mathcal{V}$, constant number a and non-zero constant number b such that the following inequality holds:

$$\begin{bmatrix} \omega_{11} & \omega_{12} & \omega_{13} & 0 & \mathcal{P} \\ * & \omega_{22} & \omega_{23} & d_{up}\mathcal{R} & 0 \\ * & * & -d_{up}\mathcal{R} & 0 & 0 \\ * & * & * & -d_{up}\mathcal{R} & 0 \\ * & * & * & * & -\Lambda_1 \end{bmatrix}<0 \quad (17)$$

where

$$\begin{aligned}
\omega_{11} &= \Lambda_2P+P\Lambda_2^T+ab^{-1}\Phi_{\lambda_{\sigma(t),i}}\Lambda_3\Lambda_4 \\
&\quad +ab^{-1}\Lambda_4^T\Lambda_3^T\Phi^T_{\lambda_{\sigma(t),i}}-a^2b^{-2}(1-d_{var})\Lambda_1, \\
\omega_{12} &= -b^{-1}\Phi_{\lambda_{\sigma(t),i}}\Lambda_3\Lambda_4+\mathcal{P}+ab^{-1}\Lambda_1+ab^{-2}(1-d_{var})\Lambda_1, \\
\omega_{13} &= d_{up}\left(\mathcal{P}\Lambda_2^T+ab^{-1}\Lambda_4^T\Lambda_3^T\Phi^T_{\lambda_{\sigma(t),i}}\right), \\
\omega_{22} &= -2b^{-1}\Lambda_1-b^{-2}(1-d_{var})\Lambda_1, \\
\omega_{23} &= -d_{up}b^{-1}\Lambda_4^T\Lambda_3^T\Phi^T_{\lambda_{\sigma(t),i}}, \\
\Lambda_1 &= diag\{\mathcal{Q},\mathcal{Q}\}, \\
\Lambda_2 &= diag\{A+BK_1,A+BK_1\}, \\
\Lambda_3 &= diag\{B,B\}, \\
\Lambda_4 &= diag\{\mathcal{V},\mathcal{V}\}.
\end{aligned}$$

then system Eq.(9) is asymptotically stable with the controller gain $K_2=\mathcal{V}\mathcal{Q}^{-1}$.

Proof: For the directed switching topologies, the eigenvalues of the Laplacian matrices have both real part and imaginary part. For $i=2,3,\ldots,N$, system Eq.(9) is equivalent to the following system.

$$\dot{\xi}_i(t)=\Lambda_2\xi_i(t)-\Phi_{\lambda_{\sigma(t),i}}\Lambda_3\Lambda_5\xi_i(t-d_t) \quad (18)$$

where $\Lambda_5=diag\{K_2,K_2\}$ and the matrix is defined as follows

$$\Phi_{\lambda_{\sigma(t),i}}=\begin{bmatrix} \mathrm{Re}(\lambda_{\sigma(t),i})I & -\mathrm{Im}(\lambda_{\sigma(t),i})I \\ \mathrm{Im}(\lambda_{\sigma(t),i})I & \mathrm{Re}(\lambda_{\sigma(t),i})I \end{bmatrix}.$$

By Newton Leibniz formula, we have

$$\xi_i(t)(t)-\xi_i(t-d_t)-\int_{t-d_t}^{t}\dot{\xi}_i(s)=0 \quad (19)$$

For any free-weighting matrices M_1 and M_2, the following equation holds.

$$2[\xi_i^T(t)M_1^T+\xi_i^T(t-d_t)M_2^T] \times[\xi_i(t)-\xi_i(t-d_t)-\int_{t-d_t}^{t}\dot{\xi}_i(s)ds]=0 \quad (20)$$

Similarly, for any matrices $X_{m_kn_k}(m_k=1,2;m_k\le n_k\le 2)$ with proper dimension, one has

$$\begin{bmatrix} \xi_i(t) \\ \xi_i(t-d_t) \end{bmatrix}^T\begin{bmatrix} \Theta_{12} & \Theta_{12} \\ * & \Theta_{22} \end{bmatrix}\begin{bmatrix} \xi_i(t) \\ \xi_i(t-d_t) \end{bmatrix}=0 \quad (21)$$

where $\Theta_{m_kn_k}=d_{up}(X_{m_kn_k}-X_{m_kn_k})$, $m_k=1,2$, $m_k\le n_k\le 2$.

Choose a common Lyapunov-Krasovskii functional as follows

$$V(t,\xi_i)=\xi_i^T(t)P\xi_i(t)+\int_{t-d_t}^{t}\xi_i^T(s)Q\xi_i(s)ds +\int_{-d_{up}}^{0}\int_{t+\theta}^{t}\dot{\xi}_i^T(s)R\dot{\xi}_i(s)dsd\theta \quad (22)$$

Here, $P = P^T > 0$, $Q = Q^T > 0$, $R = R^T > 0$ are the real matrices with proper dimensions.

Taking the derivative of $V(t, \xi_i)$ gives

$$\begin{aligned}\dot{V}(t,\xi_i) &= 2\xi_i^T(t)P\dot{\xi}_i(t) + \xi_i^T(t)Q\xi_i(t)\\ &\quad -(1-\dot{d}(t))\xi_i^T(t-d_t)Q\xi_i(t-d_t)\\ &\quad +d_{up}\dot{\xi}_i^T(t)R\dot{\xi}_i(t) - \int_{t-d_{up}}^{t}\dot{\xi}_i^T(s)R\dot{\xi}_i(s)ds \end{aligned} \tag{23}$$

From Eq.(20), Eq.(21) and Eq.(23), one can obtain

$$\begin{aligned}\dot{V}(t,\xi_i) &\le 2\xi_i^T(t)P\dot{\xi}_i(t) + d_{up}\dot{\xi}_i^T(t)R\dot{\xi}_i(t) + \xi_i^T(t)Q\xi_i(t)\\ &\quad -(1-d_{var})\xi_i^T(t-d_t)Q\xi_i(t-d_t)\\ &\quad -\int_{t-d_t}^{t}\dot{\xi}_i^T(s)R\dot{\xi}_i(s)ds\\ &\quad +2[\xi_i^T(t)M_1^T + \xi_i^T(t-d_t)M_2^T]\\ &\quad \times[\xi_i(t) - \xi_i(t-d_t) - \int_{t-d_t}^{t}\dot{\xi}_i(s)ds]\\ &\quad +\begin{bmatrix}\xi_i(t)\\ \xi_i(t-d_t)\end{bmatrix}^T\begin{bmatrix}\Theta_{12} & \Theta_{12}\\ * & \Theta_{22}\end{bmatrix}\begin{bmatrix}\xi_i(t)\\ \xi_i(t-d_t)\end{bmatrix}\\ &= \eta_{i1}^T(t)(H_i + d_{up}\Gamma_i^T R\Gamma_i)\eta_{i1}(t)\\ &\quad -\int_{t-d_t}^{t}\eta_{i2}^T(t,s)\Psi\eta_{i2}(t,s)ds \end{aligned} \tag{24}$$

in which

$$\Gamma_i = \begin{bmatrix}\Lambda_2 & -\Phi_{\lambda_{\sigma(t),i}}\Lambda_3\Lambda_5\end{bmatrix},\quad H_i = \begin{bmatrix}H_{11} & H_{12}^i\\ * & H_{22}\end{bmatrix},$$

$$\begin{aligned}H_{11} &= P\Lambda_2 + \Lambda_2^T P + Q + M_1^T + M_1 + d_{up}X_{11},\\ H_{12}^i &= -P\Phi_{\lambda_{\sigma(t),i}}\Lambda_3\Lambda_5 - M_1^T + M_2 + d_{up}X_{12},\\ H_{22} &= -(1-d_{var})Q - M_2^T - M_2 + d_{up}X_{22},\\ \eta_{i1}(t) &= [\xi_i^T(t)\ \xi_i^T(t-d_t)]^T,\\ \eta_{i2}(t,s) &= [\xi_i^T(t)\quad \xi_i^T(t-d_t)\quad \dot{\xi}_i^T(s)]^T,\\ \Psi &= \begin{bmatrix}X_{11} & X_{12} & M_1^T\\ * & X_{22} & M_2^T\\ * & * & R\end{bmatrix}.\end{aligned}$$

The conclusion can be drawn that system Eq.(18) is asymptotically stable if $\dot{V}(t, \xi_i) < 0$, which is equivalent to

$$H_i + d_{up}\Gamma_i^T R\Gamma_i < 0,\quad \Psi \ge 0 \tag{25}$$

By Lemma 2, one can obtain

$$\begin{aligned}& H_i + d_{up}\Gamma_i^T R\Gamma_i < 0\\ \Leftrightarrow & \begin{bmatrix}H_i & d_{up}\Gamma_i^T\\ * & -d_{up}R^{-1}\end{bmatrix} < 0\\ \Leftrightarrow & \begin{bmatrix}-H_i - d_{up}X & -d_{up}\Gamma_i^T\\ * & d_{up}R^{-1}\end{bmatrix} > 0\end{aligned} \tag{26}$$

where

$$X = \begin{bmatrix}X_{11} & X_{12}\\ * & X_{22}\end{bmatrix},\quad \mathcal{H}_i = \begin{bmatrix}\mathcal{H}_{11} & \mathcal{H}_{12}^i\\ * & \mathcal{H}_{22}\end{bmatrix},$$

$$\begin{aligned}\mathcal{H}_{11} &= P\Lambda_2 + \Lambda_2^T P + Q + M_1^T + M_1,\\ \mathcal{H}_{22} &= -(1-d_{var})Q - M_2^T - M_2,\\ \mathcal{H}_{12}^i &= -P\Phi_{\lambda_{\sigma(t),i}}\Lambda_3\Lambda_5 - M_1^T + M_2.\end{aligned}$$

On the other hand,

$$\Psi > 0 \Leftrightarrow \begin{bmatrix}d_{up}X_{11} & d_{up}X_{12} & d_{up}M_1^T\\ * & d_{up}X_{22} & d_{up}M_2^T\\ * & * & d_{up}R\end{bmatrix} > 0.$$

Obviously, Eq.(25) is equivalent to the following two inequalities hold simultaneously.

$$\begin{aligned}&\begin{bmatrix}-H_i - d_{up}X & -d_{up}\Gamma_i^T\\ * & d_{up}R^{-1}\end{bmatrix} > 0\\ &\begin{bmatrix}d_{up}X_{11} & d_{up}X_{12} & d_{up}M_1^T\\ * & d_{up}X_{22} & d_{up}M_2^T\\ * & * & d_{up}R\end{bmatrix} > 0\end{aligned} \tag{27}$$

It can be rearranged by Lemma 3,

$$\Xi = \begin{bmatrix}\mathcal{H}_{11} & \mathcal{H}_{12}^i & d_{up}\Lambda_2 & d_{up}M_1^T\\ * & \mathcal{H}_{22} & -d_{up}\Phi_{\lambda_{\sigma(t),i}}\Lambda_3\Lambda_5 & d_{up}M_2^T\\ * & * & -d_{up}R^{-1} & 0\\ * & * & * & -d_{up}R\end{bmatrix} < 0 \tag{28}$$

Define

$$W = \begin{bmatrix}P & 0\\ M_1 & M_2\end{bmatrix},\quad \mathcal{A}_i = \begin{bmatrix}\Lambda_2 & -\Phi_{\lambda_{\sigma(t),i}}\Lambda_3\Lambda_5\\ I & -I\end{bmatrix},$$

and $M_1 = aP$, $M_2 = bQ$, in which $b \neq 0$. The reversible matrix of W is

$$W^{-1} = \begin{bmatrix}P^{-1} & 0\\ -ab^{-1}Q^{-1} & b^{-1}Q^{-1}\end{bmatrix} \tag{29}$$

Moreover, let

$$\Omega = \begin{bmatrix}W^{-1} & 0 & 0\\ 0 & I & 0\\ 0 & 0 & R^{-1}\end{bmatrix}.$$

Taking the congruent transformation $\Omega^T\Xi\Omega$ yields

$$\begin{bmatrix}\mathcal{H}_{11}^i & d_{up}W^{-T}\Gamma_i^T & d_{up}\Upsilon^T\\ * & -d_{up}R^{-1} & 0\\ * & * & -d_{up}R^{-1}\end{bmatrix} < 0 \tag{30}$$

where $\mathcal{H}_{11}^i = \mathcal{A}_iW^{-1} + W^{-T}\mathcal{A}_i^T + W^{-T}\mathrm{diag}\{Q, -(1-d_{var})Q\}W^{-1}$ and $\Upsilon = \begin{bmatrix}0 & R^{-1}\end{bmatrix}$. Let $\mathcal{P} = P^{-1}$, $\mathcal{R} = R^{-1}$, $\Lambda_1 = Q^{-1}$ and $\mathcal{V} = K_2\mathcal{Q}$, one can obtain

$$\begin{bmatrix}\omega_{11} + \mathcal{P}\Lambda_{\mathcal{Q}}^{-1}\mathcal{P} & \omega_{12} & \omega_{13} & 0\\ * & \omega_{22} & \omega_{23} & d_{up}\mathcal{R}\\ * & * & -d_{up}\mathcal{R} & 0\\ * & * & * & -d_{up}\mathcal{R}\end{bmatrix} < 0 \tag{31}$$

where ω_{11}, ω_{12}, ω_{13}, ω_{22} and ω_{23} are defined in Eq.(17). By Lemma 3, Eq.(31) is equivalent to Eq.(17). In a conclusion, if there exist the real matrices $\mathcal{P} = \mathcal{P}^T > 0$, $\mathcal{Q} = \mathcal{Q}^T > 0$, $\mathcal{R} = \mathcal{R}^T > 0$, $\mathcal{V}$, constant number a and non-zero constant number b such that Eq.(17) holds, then the derivative of Lyapunov-Krasovskii functional $V(t, \xi_i)$ is negative definite. This implies that system Eq.(9) ($i = 2, 3, \ldots, N$) is asymptotically stable with the controller gain $K_2 = \mathcal{V}\mathcal{Q}^{-1}$. □

Remark 2: With the allowance delay upper bound d_{up}, the controller gain K_2 can be calculated through solving the LMIs feasibility described by Eq.(17). In fact, for the matrix $\Phi_{\lambda_{\sigma(t),i}}, i = 2, 3, \ldots, N$ in Theorem 2, there is no need to calculate all eigenvalues. In terms of the digraphs, only four cases $\tilde{\lambda}_1 = \max(\lambda^i_{Re}) + j\max(\lambda^i_{Im})$, $\tilde{\lambda}_2 = \max(\lambda^i_{Re}) - j\max(\lambda^i_{Im})$, $\tilde{\lambda}_3 = \min(\lambda^i_{Re}) + j\max(\lambda^i_{Im})$ and $\tilde{\lambda}_4 = \min(\lambda^i_{Re}) - j\max(\lambda^i_{Im})$ are discussed. Here, λ^i_{Re} and λ^i_{Im} are the real and imaginary part of $\lambda_{\sigma(t),i}, i = 2, 3, \ldots, N$ respectively. This approach reduces the amount of calculation, especially when the number of the agents in the swarm systems is large [49].

D. SOLVING ALGORITHM OF FORMATION PROTOCOL

The algorithm procedure of formation protocol is given in the followings.

Algorithm 1:

1) Given the specified formation $h_i(t), i = 1, 2, \ldots, N$ and reference trajectory $r(t)$ of the swarm systems. $h_i(t), i = 1, 2, \ldots, N$, $r(t)$, A and B should satisfy the constraint condition Eq.(6) when $r(t) \neq 0$ or Eq.(15) when $r(t) = 0$.

2) Find the gain K_1 by the eigenvalues configuration of $A + BK_1$, which assure that Eq.(8) is stable.

3) If $r(t) \neq 0$, compute the auxiliary function $v(t)$ and $s(t)$ by Eq.(7). If $r(t) = 0$, the function $v(t)$ is derived by Eq.(16).

4) The allowable upper bound d_{up} of delays can be obtained by Theorem 2 through solving feasible LMIs. Meanwhile, the controller gain K_2 can be computed when the specified delay and its upper bound are given.

5) Obtain the formation protocol $u_i^{FT}(t)$ described by Eq.(2) or $u_i^F(t)$ described by Eq.(4).

IV. NUMERICAL SIMULATIONS AND DISCUSSIONS

The proposed formation control methods will be evaluated by applying it to a high-order swarm system with delays and switching topologies. The effectiveness and advantages in conservativeness are shown by means of numerical simulations.

Consider a three-order swarm systems with eight agents, which are described by Eq.(1) with $A = \begin{bmatrix} -2 & 0 & 1 \\ 0 & 0 & 1 \\ -2 & 4 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Each agent follows the time-varying formation

$$\begin{aligned} h_i(t) &= [h_{i1}(t), h_{i2}(t), h_{i3}(t)]^T, \\ h_{i1}(t) &= 4\cos(t + (i-1)\pi/4), \\ h_{i2}(t) &= 2\sin(t + (i-1)\pi/4), \\ h_{i3}(t) &= 2\sin(t + (i-1)\pi/4), \quad i = 1, 2, \ldots, 8. \end{aligned}$$

Time-varying switching communication topologies with 0.3s intervals are shown in Fig.1.

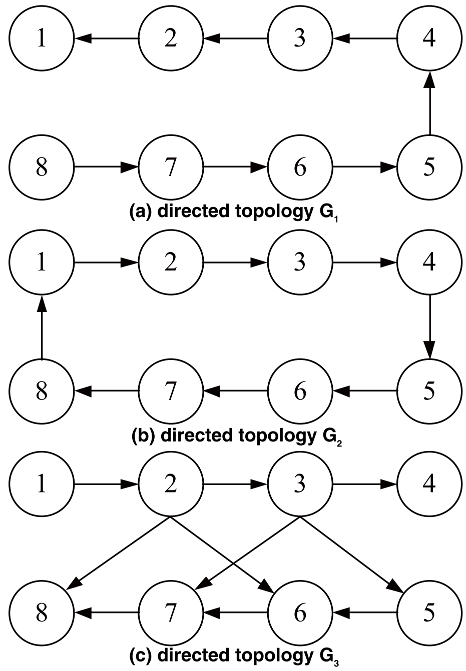

FIGURE 1. Switching interactions for high-order swarm systems.

A. TIME-VARYING FORMATION TRACKING

The eight agents are required to preserve the time-varying circle formation and keep rotating around the formation center

$$h_i(t) = [5\sin\beta, 5\cos\beta, 5\sin\beta]^T, \quad i = 1, 2, \ldots, 8,$$

where $\beta = 0.25t + 0.25(i-1)\pi$. The formation center follows the predefined reference trajectory $r(t)$, which is described by the following piecewise expressions.

$$r(t) = \begin{cases} [t, t, 0.5t]^T, 0 \leq t < 20 \\ [t, t, 10]^T, 20 \leq t < 40 \\ [2t-40, -1.5t+100, 10]^T, 40 \leq t < 50 \\ [2t-40, -1.5t+100, -0.25t+22.5]^T, \\ \qquad 50 \leq t < 70. \end{cases}$$

It is noted that $h_i(t), i = 1, 2, \ldots, 8$ and $r(t)$ should satisfy the condition described by Eq.(6).

By applying Algorithm 1 and Theorem 2, K_1, $v(t)$, $s(t)$ and K_2 can be well designed respectively. Choose $K_1 = \begin{bmatrix} 1.5 & 0 & 0.5 \\ 0 & -0.5 & -2.5 \\ 0.5 & -2.5 & -5.5 \end{bmatrix}$ by assigning the eigenvalues of $A + BK_1$ at $-0.5 + 2.1213i$, $-0.5 + 2.1213i$ and -0.5.

By Eq.(7), one can obtain the auxiliary functions

$$v_i(t) = \begin{bmatrix} 1.25cos\beta + 5sin\beta \\ -1.25sin\beta - 5sin\beta \\ 1.25cos\beta - 15sin\beta - 5\cos\beta \end{bmatrix}, \quad i = 1, 2, \ldots, 8,$$

and

$$s(t)=\begin{cases}[1, -2t+1, -12t+2]^T, & 0 \le t < 20\\ [2t-39, -39, -2t-200]^T, & 20 \le t < 40\\ [4t-118, -41.5, 10t-680]^T, & 40 \le t < 50\\ [5.5t-193, 1.5t-116, 17.5t-1056.5]^T, & \\ \qquad 50 \le t < 70. & \end{cases}$$

By Theorem 2, the upper bound of the delays is 0.65 through justifying the feasible of LMI, and the corresponding controller gain is

$$K_2 = \begin{bmatrix} 0.0838 & -0.0004 & 0.0218 \\ -0.0004 & 0.0838 & -0.0218 \\ -0.0218 & 0.0218 & 0.0842 \end{bmatrix}.$$

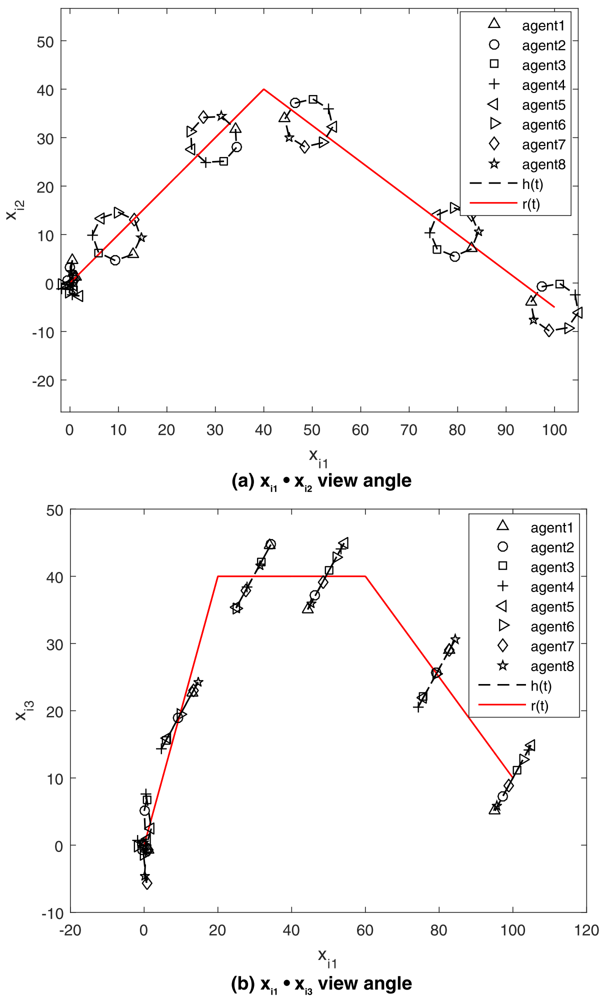

FIGURE 2. The three-dimension state evolution snapshots of the swarm systems under delays $d(t) = 0.33 + 0.32\sin(t)$ and switching topologies G_1, G_2, G_3 with 0.3s interval time.

Fig.2 shows the position evolutions of the eight agents in three-dimensional space, which are marked by different icons. After a period of time, the eight agents moved to the circumference of a circle uniformly and kept rotating around the center. Meanwhile, the formation center followed the

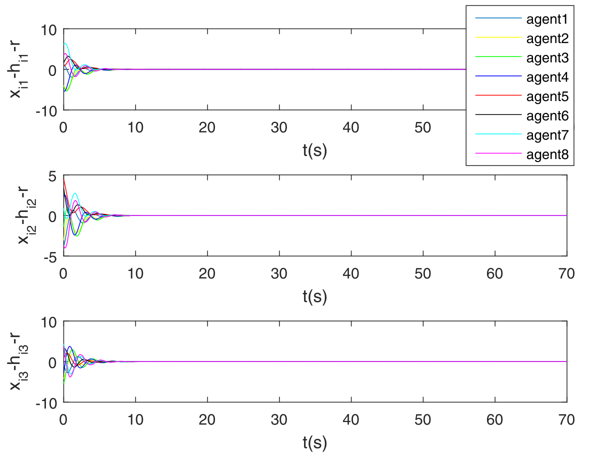

FIGURE 3. The errors evolution of the swarm systems under delays $d(t) = 0.33 + 0.32\sin(t)$ and the switching topologies G_1, G_2, G_3 with 0.3s interval time.

time-varying reference $r(t)$. The consensus errors between state variable $x(t)$, formation $h(t)$ and reference $r(t)$ are given in Fig.3.

From the curves, the errors trend to zero when the swarm systems realize formation tracking under the maximum delay $d_{up} = 0.65$ and switching topologies G_1, G_2, G_3 with 0.3s interval time. Further, the convergence time of three-dimensional variables are almost the same with different initial values from several simulation experiments. The conclusion can be made that the swarm systems achieve formation tracking under specified communication delay and switching topologies.

B. CONSERVATIVENESS COMPARISONS

To compare the conservativeness, the results when solving the formation maintenance problem will be investigated. The upper bounds of communication delays are calculated for fixed and switching topologies using Matlab LMI toolbox.

TABLE 1. The upper bounds comparisons with different methods.

	switching topologies G_1,G_2,G_3	fixed topology G_1	fixed topology G_2	fixed topology G_3
[27]	-	0.66	0.41	0.58
[49]	0.41	0.66	0.41	0.58
[50]	-	0.91	0.72	0.85
Corollary 1	0.65	1.14	0.65	0.74

As shown in Table 1, the allowance upper bounds are bigger than the existing results [27], [49], [50]. The FWM and the optimized parameter a and b improve the conservativeness. On the other side, the proposed method of this paper is not related to the dimension of the swarm systems. Only four eigenvalues are needed to calculate instead of all topologies. Therefore, the calculations is smaller. From this point of view, the proposed method has better adaptability when the agent number is huge.

V. CONCLUSION

A novel formation control method for high-order linear swarm systems with time-varying delays and switching topologies was discussed. Mainly, the following contributions were concluded in this paper:

1) The necessary and sufficient conditions of the formation tracking and maintenance problem were derived with same solution framework.

2) Both time-varying delays and switching topologies were taking into consideration. Proper formation protocol can be synthesized through eigenvalues configuration, selecting suitable auxiliary functionals and solving the feasible solution of LMIs.

3) Free-weight matrices method was employed to justify the negative definite of the Lyapunov-Krasovskii functional derivative. This assured lower conservativeness and smaller calculations.

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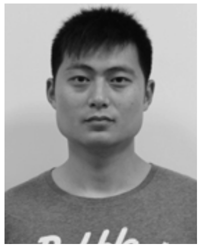

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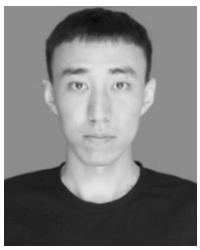

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