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A Hybrid Genetic Algorithm Based on Information Entropy and Game Theory

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ABSTRACT To overcome the disadvantages of traditional genetic algorithms, which easily fall to local optima, this paper proposes a hybrid genetic algorithm based on information entropy and game theory. First, a calculation of the species diversity of the initial population is conducted according to the information entropy by combining parallel genetic algorithms, including using the standard genetic algorithm (SGA), partial genetic algorithm (PGA) and syncretic hybrid genetic algorithm based on both SGA and PGA for evolutionary operations. Furthermore, with parallel nodes, complete-information game operations are implemented to achieve an optimum for the entire population based on the values of both the information entropy and the fitness of each subgroup population. Additionally, the Rosenbrock, Rastrigin and Schaffer functions are introduced to analyse the performance of different algorithms. The results show that compared with traditional genetic algorithms, the proposed algorithm performs better, with higher optimization ability, solution accuracy, and stability and a superior convergence rate.

INDEX TERMS Genetic algorithm, partheno-genetic algorithm, information entropy, game theory, parallel genetic.

I. INTRODUCTION

A. GENETIC ALGORITHM

A Genetic Algorithm refers to a computational model that simulates the natural selection of Darwin's biological evolution and the biological evolution process of a genetic mechanism; specifically, this approach involves searching for the optimal solution by simulating the natural evolution process. Genetic algorithms are typically based on finding a potential solution set for a given population. After the original population is generated, according to the evolutionary principle of survival of the fittest, each subsequent generation evolves to produce a better approximate solution considering the fitness level and size of the problem domain for individual selection in each generation. Then, the processes of crossover and mutation occur with genetic operators. Finally, a population that represents a new solution set is produced. Generally, this process results in an epigenetic population and is similar to natural population evolution methods but with better performance related to environmental adaptability. In addition, by decoding the optimal individual from the previous generation, the approximate optimal solution of the problem can be obtained.

The genetic algorithm process can be approximately expressed as follows.

As an intelligent algorithm for solving NP problems, genetic algorithms have been extensively studied.

Davis edited and published the book "Handbook of Genetic Algorithms", which covers many application examples of genetic algorithms in the engineering, technology and scientific computing fields.

In "Genetic Algorithms for Search, Optimization and Machine Learning", Goldberg proposed combining Pareto theory and genetic algorithms in economics to solve optimization problems.

Bernabe introduced the cell genetic algorithm with real-number coding to solve continuous optimization problems, and the results were better than those for other algorithms.

Nagham proposed a new structured population approach for genetic algorithms based on the customs, behaviours and patterns of the human community.

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Tu Chengyuan et al. proposed overcoming the problem of "premature convergence" by constructing new genetic operators, such as restoration, reconstruction and self-intersection operators, in the so-called partheno-genetic algorithm.

Generally, the above algorithms are relatively classic genetic algorithms; they have efficient search capabilities, but there are still inherent drawbacks to these algorithms. To find a better solution to the NP problem, many scholars have studied hybrid genetic algorithms and improved genetic algorithms.

Li Jia proposed a new genetic algorithm, namely, a hybrid genetic algorithm that integrated the tabu search algorithm into a genetic algorithm.

Zhang Tao et al. also proposed a new hybrid genetic algorithm in which a genetic algorithm and the 3-OPT algorithm are combined. This approach utilizes the efficient global search ability of the genetic algorithm and the local search ability of the 3-OPT algorithm.

Chen Xiangzhou et al. embedded a reversal operator in the genetic algorithm and improved the convergence speed of the algorithm in the late stage of operation.

Zhang et al. introduced the idea of cloning in genetic algorithms. Dai Xiaoming et al. introduced the concept of parallel evolution in genetic algorithms. Fang Xia et al. applied an immunity algorithm with a genetic algorithm to solve the VRP problem.

The hybrid algorithm effectively balances the diversity and convergence properties of the algorithms and embodies the advantages of each algorithm. However, the diversity of most hybrid algorithms is relatively low, and the information exchange between populations is not considered. With the advent of interdisciplinary studies, scholars have introduced information entropy and game theory into genetic algorithms to further optimize these algorithms.

Xue Feng et al. used information entropy to generate the initial population, which increased the diversity of the initial population and provided the basis for subsequent processing.

Chen Xiaofeng used the concept of information entropy to improve and fuse quantum evolution algorithms and immune genetic algorithms and proposed a quantum immune genetic algorithm based on information entropy.

Wei Qinfang et al. proposed a genetic algorithm for wireless sensor network intrusion detection based on information entropy.

Yang Mei and others borrowed relevant ideas from game theory for strategy optimization and proposed a hybrid optimization mechanism for multi-subgroup and multi-strategy methods; this approach improved the search accuracy and convergence speed, but it requires the selection of subpopulations.

The optimization ability of the genetic algorithm is reflected by the diversity of the population and the convergence speed of the algorithm. The traditional genetic algorithm easily falls to local optima, and this issue is largely related to the diversity of individuals in the population. Therefore, it is necessary to maintain the diversity of the population in each generation. Additionally, the genetic algorithm involves genetic and selection operations; thus, whether the new population is retained and how it is retained will affect the final optimization result and the speed of optimization. Therefore, the concept of information entropy is introduced in this paper. Sexual optimization and game theory optimization are applied for new populations generated by genetic operations. Based on a review of relevant studies, scholars have begun to study the combination of information entropy and genetic algorithms and the combination of game theory and genetic algorithms to optimize algorithms. However, applications involving information entropy, game theory and genetic algorithms for optimization have not been reported in China.

B. INFORMATION ENTROPY

Entropy was introduced in the 1860s by the German physicist Clausius as a concept of thermodynamics. This term was originally used to describe the amount of energy conversion and the direction of transformation. In 1854, Clausius defined the state function entropy of a reversible thermodynamic system. The mathematical expression is as follows.

$$ds = \frac{dQ}{dT} \tag{1}$$

A change in entropy is equal to the ratio of heat absorption to the temperature from state X0 to state X in a reversible element-based process.

Information entropy is an important concept in the field of information theory. The famous paper "The Mathematical Theory of Communication" published by American scientist C.E. Shannon in 1948 laid the theoretical foundation for information theory. In information theory, entropy describes the degree of uncertainty of random variables, and information entropy is often used to reflect the degree of uncertainty in selection or information.

The information entropy can be defined as the average amount of information provided by each representation of source X, or the statistical average of the amount of information contained in various representations sent by information source X in the probability space of information source X. This average value is H(X):

$$H(x) = -\sum_{x \epsilon X} P(x)\, LnP(x) \tag{2}$$

where X represents all solutions.

The distribution degree of individuals in a population can be calculated based on the information entropy. If the population diversity is high, the corresponding information entropy will be high, and vice versa, so that the population can be prevented from falling to local optima.

C. GAME THEORY

Game theory is related to the direct interaction of decision-making behaviours and the equilibrium problem associated with such decisions. Game theory originated in the early

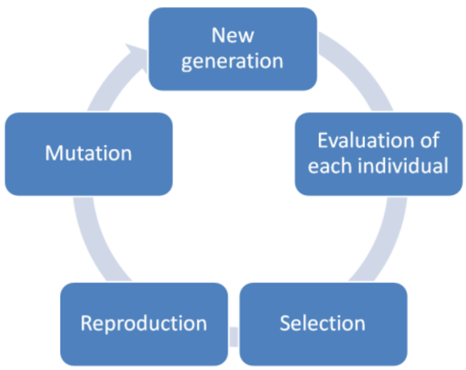

FIGURE 1. Genetic algorithm process.

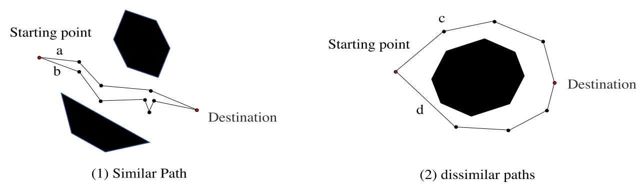

FIGURE 2. Path optimization diagram.

20th century. In 1994, von Neumann and Morgan Stein co-authored "Game Theory and Economic Behaviour" to lay the theoretical foundation for this theory. Since the 1950s, Nash, Zelten, and Haisani expanded on and improved game theory. In the past 20 years, game theory has been widely used to analyse and solve conflicts and cooperative problems in fields such as economics, complex networks, power systems, transportation and path planning.

A typical problem in Game Theory research is that two or more participants (called in-house players) make decisions on a confrontational or competitive basis so that their own party obtains the best possible results. Moreover, the so-called game is a set of rules that stipulate the methods and regulations that should be followed throughout the game (or competition, struggle, etc.), including the players, strategies, outcome after strategy selection, and so on.

The elements of the game include participants, strategies, and utility functions.

Players are the decision-making bodies involved in the game, also called participants. Players are typically represented by the common symbol $Y_i(i = 1, 2, \cdots, n)$.

The strategy involves the actions S of each participant.

The utility function F is a function of the set S and is used to measure the profit of the participants in the game. Notably, this function provides an important basis for the participants to make rational decisions.

II. APPLICATION OF INFORMATION ENTROPY AND GAME THEORY IN GENETIC ALGORITHMS

A. APPLICATION OF INFORMATION ENTROPY IN POPULATIONS

1) CALCULATE THE INFORMATION ENTROPY BASED ON THE FITNESS FUNCTION

In the genetic algorithm, the fitness function value of each individual can be calculated, and the distribution of each function is obtained by statistical analysis. Next, the probability P(x) of different solutions is obtained, and the information entropy is calculated with formula (1). Finally, the degree of diversity of the population is determined.

2) CALCULATE THE INFORMATION ENTROPY BASED ON CODING

In the process of using the genetic algorithm to optimize a route, calculating the information entropy based on solely on the fitness function cannot accurately reflect the degree of diversity of the population. Typically, issues are encountered in the VRP problem. In Figure 2(a), paths a and b are basically the same and can be considered similar, but if the fitness function is based on the length of each path, the two fitness function values will greatly differ. For paths c and d in Fig. 2(b), in a fitness function with the path length as the main criterion, the difference in the resulting values for these two paths may not be large. However, based on individual habits, these two paths may vary greatly. Thus, the fitness function value is not suitable as a measure of the path-based population diversity.

Therefore, a metric of population diversity called the encoding information entropy can be obtained based on the differences in coding.

Let the population consist of N chromosomes of length L. First, the degree of coding similarity for L positions in N chromosomes is determined. Then, chromosomes with similarity values higher than 90% are classified into one class. Next, according to the probability of each class P(x), formula (1) is used to calculate the information entropy. To ensure the accuracy of the information entropy values, the coding information entropy is used to determine the population diversity.

3) INITIALIZE THE POPULATION USING THE INFORMATION ENTROPY

To uniformly distribute the initial population in the solution space, avoid a centralized distribution in the local region of the solution space, and increase the diversity of the initial population, the population can be initialized by calculating the information entropy.

The process of using the information entropy to generate an initial population is as follows.

1) Step 1: Set the critical entropy value $\boldsymbol{S_0}$.
2) Step 2: Randomly generate the first chromosome in the chromosome domain.
3) Step 3: Generate a chromosome each time in the same way and calculate the entropy value S of the chromosome and the existing individual. If $\boldsymbol{S} > \boldsymbol{S_0}$, then accept the chromosome; otherwise, reject the chromosome. In this case, regenerate a new chromosome, and

calculate the entropy value S until the condition of $S > S_0$ is met.

4) Step 4: Repeat Step until the number of chromosomes reaches the specified initial population.

4) SETTING THE INFORMATION ENTROPY THRESHOLD S_0

The execution process of genetic algorithms is similar to a system evolution process. At the beginning of the algorithm, the internal diversity of the population is high, and the algorithm has a wide search space. As the population multiplies from generation to generation, some individuals with large adaptation values and their descendants account for the majority of the population. As a result, the population diversity decreases, the algorithm search space shrinks, and the population tends to be stable. The process of population evolution in the genetic algorithm is consistent with the phylogenetic process; therefore, the entropy threshold S_0 value needs to be gradually reduced according to the evolution of the population. The method of changing S_0 used in this paper is as follows:

$$S_0(\mathrm{K}) = S_0(\mathrm{K}-1) * \gamma, \quad K = 1, 2 \ldots, n \tag{3}$$

where K represents evolutionary algebra and γ is the threshold reduction factor.

B. PARALLEL ALGORITHM

To perform game calculations with the standard genetic algorithm, partheno-genetic algorithm, and standard partheno-genetic hybrid algorithm, three groups of populations must be generated. A coarse-grained model, also known as a distributed model or an island model, can be constructed, which is a type of parallel genetic algorithm.

However, this approach divides a group into several subgroups according to the number of nodes. The algorithm is independently run for each child's generation in parallel based on the respective nodes. During each evolutionary generation time step, each subgroup will exchange individual information. This process can identify and maintain the best individuals, enriches the diversity of the population and prevents early convergence.

C. APPLICATION OF GAME THEORY IN GENETIC OPERATION MODE

Genetic theory can be optimized using game theory. Specifically, classical game theory is based on classical game theory and individual rationality, and the purpose of each participant is to maximize their own income function. This paper assumes that the group population is a participant who emphasizes collective rationality and that the two parties reach a cooperative agreement to maximize the benefit of the entire population. The game at this time is a complete information cooperation game.

In addition, this paper introduces information entropy to evaluate the diversity of populations in the evolution process, calculates the value of information entropy through formula (3), and combines the strategy set to make game

TABLE 1. Element mapping relations for game theory and the hybrid optimization algorithm.

Element	Game Theory	Optimization
Participant	A decision-making body in a game whose purpose is to maximize the level of utility by choosing actions	Refers to the subgroup that participates in independent optimization in the hybrid optimization algorithm. All individuals in the subgroup act as a subject for policy selection.
Strategy	In the game, each participant has an optional action plan given the available information.	The sub-population strategy is determined by the entropy value when the subgroup is optimized.
Income	The result of the game is called income. The payoff of each player at the end of a game is related not only to the strategy chosen by the player but also to the set of strategies selected by all players.	The degree of the solution for each subgroup at parallel nodes based on the fitness value and information entropy.

choices for individuals. The game theory and element mapping relationships in this algorithm are as follows:

1) GAME MODEL IN THE ALGORITHM

We define three subgroups, pop1, pop2 and pop3, participating in independent optimization and three strategies, strategy1, strategy2 and strategy3.Then,for participants N = {1,2,3}, each subgroup i ∈ N is the decision-making subject in the game problem, and it is assumed that all three subgroups have collective rationality. The purpose of the game is to maximize the global benefit. Each subgroup has a separate strategy, $\mathrm{pop_i}$, yielding the strategy set Si = {strategy1, strategy2, strategy3}. The game result is represented by (q1, q2, q3), where q1, q2, and q3 correspond to the fitness values of pop1, pop2 and pop3, respectively, considering the adaptation of the best individual in the group. The degree value is determined by equation (4).

$$q_i = best(pop_i) \tag{4}$$

2) SOLVING THE GAME MODEL

The income of the three subgroups is recorded as P = (P1, P2, $\mathrm{P_3}$), and the income of subgroup i is $\mathrm{P_i}$. This value depends not only on the strategy of each subgroup but also the strategies of other subgroups as function of the combination of strategies.

Classical game theory is based on personal rationality. The objective of each participant is to maximize their own income function. This paper assumes that the three subgroups are participants who emphasize collective rationality and that the three parties reach a cooperative agreement to maximize P∗ for the entire population. That is, each individual implements a single strategy but shares the optimal solution of the combination of strategies, namely, P∗ = best(P).

Under these assumptions, when the subpopulations are searched in parallel, the population strategy at the nodes is updated according to formula (5):

$$X_i^{t+1} = X_i^t + \delta x_i^{t+1}(P^*) \tag{5}$$

where X_i^t representsthe position at node t of subgroup i and $\delta x_i^{t+1}(P^*)$ is the strategy for subgroup updating based on the following three modes.

a: COOPERATION MODE

When the population is optimized to the Kth (K = 1, 2, . . ., n) generation, the entropy values Q_1, Q_2, and Q_3 of the three populations are calculated according to the following formula.

$$\max(\mathrm{Q1}, \mathrm{Q2}, \mathrm{Q3}) < \boldsymbol{S_0} \tag{6}$$

At this time, cooperative mode begins. Moreover, the three populations are merged into one population, and according to the fitness function value f of the individuals in the population, the individuals with function values less than the media value are eliminated, and the remaining are retained and evenly distributed among the three populations. These populations are used to regenerate the population in the same manner as discussed for the initial population.

The cooperative mode is a multi-group self-coordination mechanism. By synergizing with other populations, the performance of at least one population is improved, and the diversity and convergence of the algorithm are balanced.

b: COMPETITION MODE

When the population is optimized to the Kth (K = 1, 2, . . ., n) generation, the entropy values Q_1, Q_2, andQ_3 of the three populations are calculated according to the following formula.

$$\min(\mathrm{Q}_1, \mathrm{Q}_2, \mathrm{Q}_3) > \boldsymbol{S_0} \tag{7}$$

At this time, competition mode begins. The maximum fitness value Fmax of each population is calculated, and the largest function value Fmax is obtained. The individual represented by the function value is copied into the other two populations, and the individuals with the smallest function values are excluded.

Competition mode yields excellent populations in the algorithm and improves the convergence.

c: COORDINATION MODE

Coordination mode is divided into two situations.

a. The population is optimized to the Kth (K = 1, 2, . . ., n) generation, and the entropy values Q_1, Q_2, and Q_3 of the three populations are calculated. When the entropy Q of two of the populations is greater than the critical entropy S_0, the information entropy of one population is lower than the critical entropy S_0. At this time, each of the two populations with information entropy values that meet the threshold requirement account for 1/4 of the population used to obtain the best fitness function. In this case, the information entropy is less than the critical threshold S_0 in the population, and the population excludes individuals whose fitness function value is less than the median.

b. The population is optimized to the Kth (K = 1, 2, . . ., n) generation, and the entropy values Q_1, Q_2, and Q_3 of the three populations are calculated. When the entropy Q of two of the populations is less than the critical entropy $\boldsymbol{S_0}$, the information entropy of onepopulation is greater than the critical entropy $\boldsymbol{S_0}$. At this time, from the population with an information entropy that satisfies the threshold requirement, the individuals with fitness values larger than the median are retained, and other individuals are excluded.

In general, coordination mode is used to improve the diversity of the algorithm and reduce the possibility of local populations being trapped.

III. IMPROVED GENETIC ALGORITHM BASED ON INFORMATION ENTROPY AND GAME THEORY

Based on the above analysis, the steps used to develop the hybrid genetic algorithm in this paper based on information entropy and game theory are as follows.

1) First, determine the coding method according to the relevant requirements. Then, complete the initialization of the three populations based on the population information entropy in the search space. Next, set the search boundary and initialize the maximum iteration number MaxDT and the parallel optimization threshold MaxJT. Finally, initialize the global optimal solution fbest.

2) Implement the parallel genetic algorithm for the three populations according to the standard genetic algorithm,

partheno-genetic algorithm and hybrid genetic algorithm methods. Record the fitness value of each population i.

3) Calculate the global maximum return P∗ and the population information entropy according to the payment utility rule; then, obtain the population updating strategy $\delta x_i^{t+1}(P^*)$.

4) Update the population according to the population updating strategy determined in (3).

5) Determine whether the global maximum return P∗ is better than the global optimal fbest and whether to update fbest.

6) Evolve the MaxJT generation according to the free strategy for the updated population.

7) Determine whether the maximum number of iterations M is reached. If so, the process proceeds to step (8); otherwise, the process proceeds to step (3).

8) Output the results.

The flow chart of this process is shown in Figure 3.

IV. SIMULATION EXPERIMENT

In this section, the hybrid optimization algorithm is tested based on the information entropy and game theory process developed in the previous sections. Three simulation functions, namely, the Rosenbrock function, Rastrigin function and Schaffer function, are used in the numerical simulation experiments. SGA, PGA and SGA-PGA are selected as the reference algorithms to verify the rationality and effectiveness of the proposed algorithm.

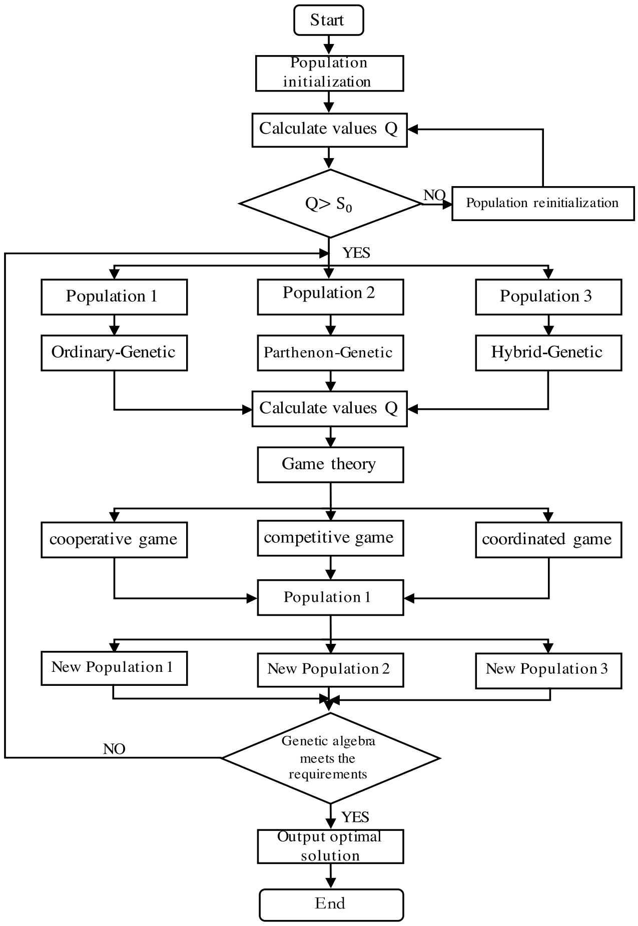

FIGURE 3. Flow chart of the hybrid genetic algorithm for an information entropy game.

To examine the scalability of the algorithm, different variable dimensions are used for each function test, including 10, 20 and 30 dimensions. When performing simulation experiments, the common parameters are set as follows: 100 chromosomes, 300 generations of genetic algebra, a crossover probability of 0.8, a probability of variation of 0.1, an information entropy threshold S_0 of 0.7, and a threshold reduction factor γ of 0.99. All the optimization algorithms were implemented in the MATLAB R2014b environment.

To measure the convergence accuracy, robustness and convergence speed of different optimization algorithms, the optimal value, average optimal value, worst value and standard deviation of each function were determined from 50 independent runs as the final evaluation indexes. The average optimal fitness curve was plotted with the number of iterations as the abscissa and the average optimal value of each function as the ordinate. The average optimal fitness value characterizes the accuracy that the algorithm can achieve for a given number of iterations, reflecting the convergence speed of the algorithm.

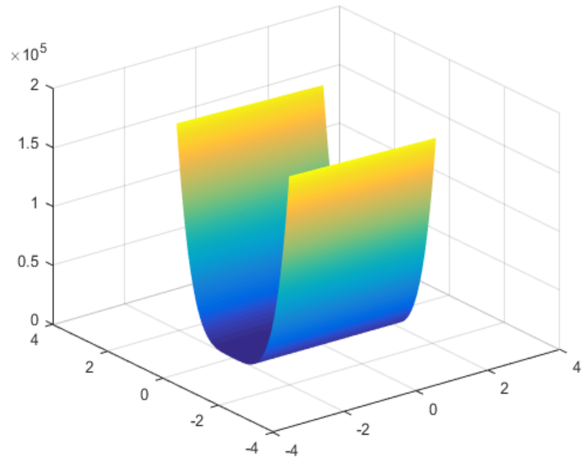

FIGURE 4. 3D diagram of the Rosenbrock function.

A. ROSENBROCK FUNCTION

$$f(X) = \sum_{i=1}^{n} [100 \left(x_{i+1} - x_i^2\right)^2 + (1-x_i)^2] \quad (8)$$

The test function takes a minimum value of 0 at (1, 1, . . ., 1), and a three-dimensional diagram of the test function is shown in Fig. 4.

As shown in Fig. 4, the Rosenbrock function is a unimodal function. The function is very simple in regions farm from the most advantageous area, but the areas near the most advantageous area is banana shaped, with strong correlation between variables and gradient information. Thus, it is often difficult to optimize the search direction of the algorithm and find the extreme values of the function. Selecting this test function can test the optimization ability of the algorithm.

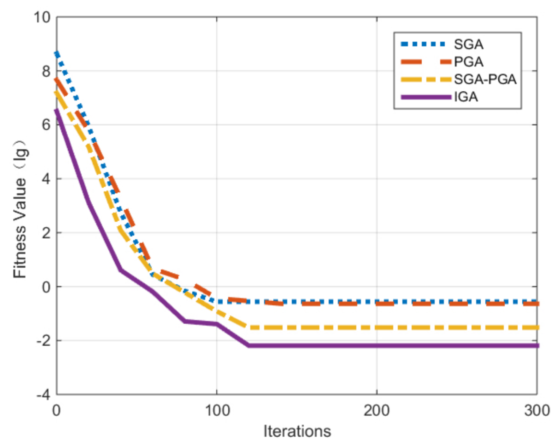

FIGURE 5. Curve of the average optimal fitness value of the Rosenbrock function with the iteration number.

For n = 20, the average optimal fitness value of the Rosenbrock function changes with the number of iterations, as shown in Fig. 5.

TABLE 2. Standard test functions and parameter values.

Function name	Function expression	Dimension	Search area
Rosenbrock function	$f(X)=\sum_{i=1}^{n}[100(x_{i+1}-x_i^2)^2+(1-x_i)^2]$	10/20/30	$[-4, 4]^n$
Rastrigin function	$f(X)=\sum_{i=1}^{n}[x_i^2-10\cos(2\pi x_i)+10]$	10/20/30	$[-5.12,5.12]^n$
Schaffer function	$f(X)=\frac{\sin^2\sqrt{x_1^2+x_2^2}+0.5}{[1+0.001(x_1^2+x_2^2)]^2}-0.5$	10/20/30	$[-5.12,5.12]^n$

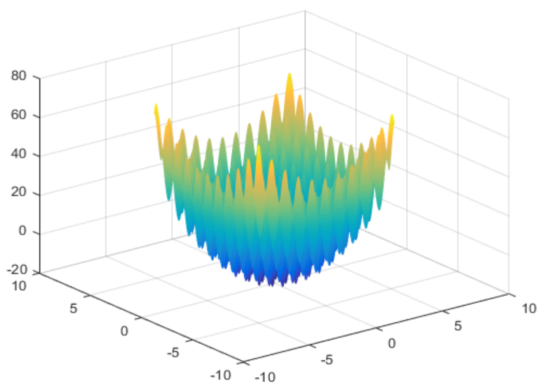

FIGURE 6. 3D diagram of the Rastrigin function.

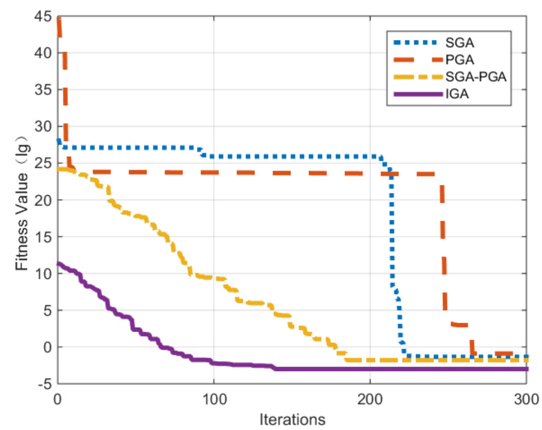

FIGURE 7. The average optimal fitness value of the Rastrigin function varies with the number of iterations.

B. RASTRIGIN FUNCTION

$$f(X)=\sum\nolimits_{i=1}^{n}[x_i^2-10\cos(2\pi x_i)+10] \tag{9}$$

The Rastrigin test function takes the minimum value of 0 at (0, 0, . . ., 0), and a three-dimensional diagram of the test function is shown in Fig. 6.

Fig. 6 shows that the test function contains a plurality of extreme points. As a result, the algorithm easily falls to local optima when the minimum value of the test function is obtained, so the function can be used to verify the optimization ability of the algorithm.

For n = 20, the average fitness value of the Rastrigin function varies with the number of iterations, as shown in Figure 7.

C. SCHAFFER FUNCTION

The mathematical expression of the Schaffer function is shown in equation (10).

$$f(X)=\frac{\sin^2\sqrt{x_1^2+x_2^2}+0.5}{\left[1+0.001\left(x_1^2+x_2^2\right)\right]^2}-0.5 \tag{10}$$

The test function takes a minimum value of 0 at (0,0), and a three-dimensional diagram of the test function is shown in Fig. 8.

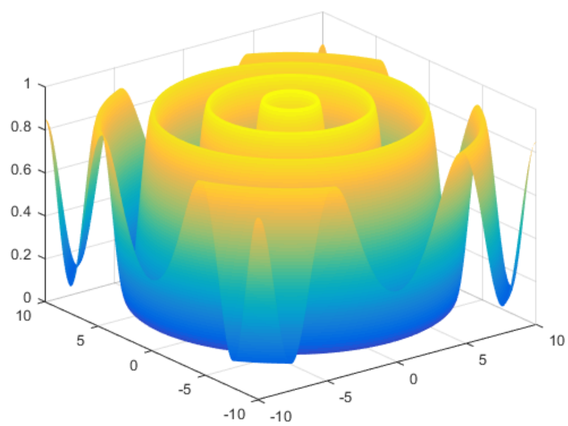

FIGURE 8. 3D diagram of the Schaffer function.

Fig. 8 shows that there are multiple extreme points in the test function and that there is oscillation between the extreme points; therefore, the test function can be selected to verify the optimization ability of the algorithm.

For n = 20, the average fitness value of the Schaffer function varies with the number of iterations, as shown in Figure 9.

By comparing the test function optimization results in Table 2, Table 3 and Table 4 with the fitness value

TABLE 3. Results for the Rosenbrock function under different optimization algorithms.

Algorithm	Dimension	Theoretical optimum	Optimal Value	Worst value	Average optimal value	Standard deviation
SGA	10	0	1.21E-03	3.58E-01	1.68E-02	1.24E + 00
	20	0	1.93E-02	4.79E-01	2.75E-01	1.86E + 00
	30	0	2.65E-01	7.43E+01	3.77E + 00	3.27E+00
PGA	10	0	1.29E-03	2.75E-02	1.60E-02	6.43E-02
	20	0	1.69E-02	4.08E-01	2.32E-01	9.58E-01
	30	0	3.84E-01	1.12E+01	4.37E + 00	1.42E + 00
SGA-PGA	10	0	5.84E-04	1.63E-02	8.20E-03	2.66E-02
	20	0	2.85E-03	8.42E-02	3.03E-02	6.51E-02
	30	0	2.99E-02	7.93E-01	3.55E-01	9.86E-01
Improved genetic algorithm	10	0	6.43E-06	1.31E-03	6.89E-04	1.54E-03
	20	0	5.88E-04	1.67E-02	6.45E-03	2.08E-03
	30	0	2.49E-03	7.29E-02	3.63E-02	2.84E-01

TABLE 4. Results for the Rosenbrock function under different optimization algorithms.

Algorithm	Dimension	Theoretical optimum	Optimal value	Worst value	Average optimal value	Standard deviation
SGA	10	0	1.24E-03	1.26E-02	3.05E-03	1.24E + 00
	20	0	7.02E-03	2.04E-01	4.59E-02	1.86E + 00
	30	0	7.55E + 00	1.25E + 03	1.05E+00	3.27E+00
PGA	10	0	2.21E-03	1.63E-02	2.39E-02	6.43E-01
	20	0	5.44E-03	5.03E-01	1.27E-01	9.58E-01
	30	0	7.78E-02	2.82E+00	5.98E-01	1.42E + 00
SGA-PGA	10	0	9.12E-04	1.05E-02	6.73E-03	2.66E-01
	20	0	1.32E-03	3.19E-02	1.59E-02	6.51E-01
	30	0	2.17E-02	9.57E-01	8.24E-02	9.86E-01
Improved genetic algorithm	10	0	1.03E-06	4.35E-04	8.65E-05	1.54E-02
	20	0	1.46E-04	1.27E-02	1.03E-03	2.08E-01
	30	0	2.10E-03	2.23E-02	1.03E-02	2.84E-01

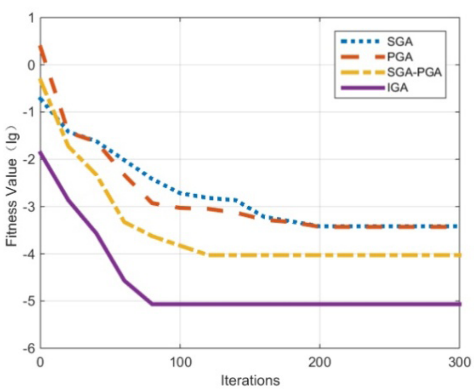

FIGURE 9. The average optimal fitness value of the Schaffer function varies with the number of iterations.

curves in Figure 5, Figure 7 and Figure 9, we find that the algorithm developed in this paper has a strong optimization ability. In the optimization of each of the above three functions, the dimension of the function has a significant influence on the obtained optimal value. As the dimension increases, the optimal value of each algorithm also changes. For the Rosenbrock function, the variable dimension has a considerable influence on each algorithm. For low-dimensional functions, the algorithm can achieve global optimization with other algorithms, but for high-dimensional functions, the algorithm still yields good results. Furthermore, the information entropy game genetic algorithm has the best optimization effect based on the convergence tests with the three test functions, and the convergence speed is faster than that of the SGA optimization, PGA optimization and SGA-PGA optimization methods. For the Rosenbrock and Schaffer functions, in the process of function optimization, the SGA optimization method achieves local optimization quickly due to a premature convergence phenomenon. Although the SGA-PGA optimization algorithm has a fast search speed and overcomes local optima, the convergence speed and convergence accuracy are lower than those of the information entropy game genetic algorithm. Additionally, the information entropy game genetic algorithm also achieves local optimization in the search process. However, due to the introduction of information entropy, the optimization algorithm can quickly avoid local optima and find the global

TABLE 5. Results for the Rosenbrock function under different optimization algorithms.

Algorithm	Dimension	Theoretical optimum	Optimal value	Worst value	Average optimal value	Standard deviation
SGA	10	0	3.86E-07	1.78E-03	6.22E-05	1.24E-02
	20	0	4.08E-05	1.26E-02	3.82E-04	1.86E + 00
	30	0	2.59E-04	6.19E-01	1.28E-02	3.27E+00
PGA	10	0	8.95E-07	4.46E-03	1.80E-04	6.43E-02
	20	0	7.89E-05	4.44E-03	3.72E-04	9.58E-01
	30	0	1.43E-03	9.34E-02	1.40E-02	1.42E + 00
SGA-PGA	10	0	0	7.81E-07	4.24E-05	2.66E-02
	20	0	0	3.39E-04	9.38E-05	6.51E-02
	30	0	0	1.76E-03	4.60E-03	9.86E-01
Improved genetic algorithm	10	0	0	2.77E-07	6.53E-06	1.54E-03
	20	0	0	6.80E-05	8.56E-06	2.08E-03
	30	0	0	2.98E-04	4.75E-05	2.84E-01

solution in the feasible domain. The optimal value indicates that the information entropy game genetic algorithm has good global and local search abilities.

In the comparison of algorithm stability, the experiments in this paper are repeated 50 times. In addition to the strong search ability of the proposed algorithm, the stability of the algorithm is another indicator of performance. Based on the 50 repetitions, the variance of the experimental results was determined to assess the fluctuations in the optimal value obtained by the algorithm. Although the Rosenbrock and Schaffer functions yielded unstable results for most algorithms, the proposed method displayed good stability.

For the function value of the initial population, the information entropy game genetic algorithm yielded a better fitness function can be obtained using traditional genetic score methods, indicating that the method based on information entropy can increase population diversity and optimize the initial population. Additionally, the information entropy game genetic algorithm uses parallel genetic operations; therefore, the efficiency of algorithm optimization can far exceed that of traditional genetic methods. Thus, with the proposed algorithm, the optimal value is found earlier, and the information entropy-based population game operations are associated with genetic nodes, thus avoiding the main disadvantage of the traditional genetic algorithm, which easily falls to local optima.

Based on comprehensive analysis of the above experimental results, the information entropy game genetic algorithm can be applied to complex nonlinear and high-dimensional functions with multiple extreme points and obtain high-precision global optimal values with low computational costs. Notably, the proposed algorithm not only has fast convergence speed but also has better global and local optimization performance than traditional methods.

V. CONCLUSION

In this paper, the genetic algorithm is improved, and multi-group genetic operations are performed in parallel. Information entropy is introduced to quantitatively analyse the diversity in the evolution process and ensure diversity in population genetics. Additionally, combined with game theory, various types of evolutionary processes occur. The changes in group information entropy using the game strategy that is most conducive to the diversity and adaptability of the whole group are considered to strengthen good individuals and eliminate invalid individuals. Three test functions are introduced to assess the validity and convergence of commonly used test algorithms. Based on the Rosenbrock, Rastrigin and Schaffer functions and a coding test, the results show that the proposed improved hybrid genetic algorithm has a considerable advantage over traditional genetic score-based methods in initializing the population and obtaining high fitness values, rapid convergence, and a high optimization speed.

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