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Disequilibrium Measurement and Analysis of a Road Traffic Network Based on Section Influence Degree and Flow Betweenness

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ABSTRACT Traffic congestion usually occurs in certain areas or sections, which could be regarded as the reflection of road network disequilibrium caused by the traffic flow distribution, network structure, etc. It is found that unbalance analysis of the road network and the unbalanced points study could provide great support for the road network structure optimization, traffic capacity adjustment, and the efficiency improvement of road network resources. Furthermore, it is helpful to reduce the traffic congestion significantly. In order to design a method to analyze the disequilibrium of road networks to find out the unbalanced or bottleneck points, the road network structure, traffic flow distribution and the location of sections are considered in the analysis of road network disequilibrium, and the disequilibrium is measured using Gini coefficient and Theil index from road network attributes of influence and capability in this paper. First, section influence degree and section flow betweenness are employed to separately represent the differences in the influence relationship and capability distribution of a section from others in the network. Second, these two indicators are taken respectively as the input, and the road network equilibrium is analyzed multi-dimensionally using Gini coefficient and Theil index. The level of the disequilibrium is analyzed from the view of the overall network. Meanwhile, the contributions of a certain section to the disequilibrium of the whole network is also analyzed from the perspective of the section. Finally, a case study based on Beijing's regional road network is conducted, and the result shows the similar performance compared with the actual road network traffic situation, which verifies the feasibility and validity of the method in multi-dimension analysis of road network disequilibrium.

INDEX TERMS Road traffic flow, traffic network equilibrium analysis, section influence degree, section flow betweenness, Gini coefficient.

I. INTRODUCTION

Because of the gradual increase in the number of vehicles and the rapid expansion of cities, traffic congestion has become more and more serious in cities such as Beijing, Shanghai, and other large cities. It is generally recognized that traffic congestion usually occurs in certain areas or sections; Then, these certain places will affect the state of the surrounding sections, which, in turn, leads to a large-scale congestion. To a certain extent, traffic congestion can be considered as a reflection of the traffic flow disequilibrium in the road network. Through analyzing the disequilibrium characteristics of the road network to grasp the distribution characteristics and changing rules of traffic flow, we can accurately find out the unbalanced or bottleneck points in the road network. This will provide great supports for more accurate corresponding operational management decisions, for example, optimizing road network structure, adjusting traffic capacity allocation, and so on. Therefore, this has great significance for improving the efficiency of road network resources and reducing traffic congestion.

More recently, many researchers have measured and analyzed the road network equilibrium. A part of the researches has focused on the interaction between the equilibrium of

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road network distribution and economic development differences, income changes, urbanization development, and other factors [1]–[3]. For example, Wang *et al.* [4] analyzed the equilibrium of highway network development using the Gini coefficient. In the field of highway network disequilibrium analysis, the Gini coefficient mainly concentrates on the disequilibrium of structure layout for the highway network. He *et al.* [5] analyzed the imbalance of a road network structural layout based on two indicators: network area density and road transport density. From the point of view of static topology, Barthrlemy [6], Deng *et al.* [7], and Shen [8] separately used complex network theory to measure the disequilibrium of a road network structure by node degree, betweenness, and tightness. In the field of traffic flow analysis, there are also some studies on road network equilibrium. Through the analysis of travel time and occupancy rate, Li *et al.* [9], Sun *et al.* [10], and Zou [11] computed the disequilibrium of the traffic space-time distribution on an expressway from the micro-level. Deng [12] analyzed the unbalanced distribution of traffic volume in a single expressway section or route. Using the knowledge of traffic engineering and information science, Huang [13] described the distribution law of network traffic flow through mathematical and physical models.

In summary, many researchers have analyzed road network equilibria, and many fine methodologies have been proposed. However, most of the studies have focused on road network equilibrium analyses just from the perspective of a static road network structure or just from traffic flow distribution. Studies are relatively lacking that quantifying the traffic distribution disequilibrium in the road network and analyzing its dynamic changes from a global point of view, comprehensively considering the influence of road network topology, traffic distribution, and the differences of nodes/sections on the road network equilibrium. This leads to somewhat inaccurate outcomes. Moreover, there are still some shortcomings in selecting the evaluation index of equilibrium measurements, analyzing the change of road network disequilibria from different dimensions, and finding the causes of these disequilibria.

For this, and with the goal of quantifying the traffic flow disequilibrium of a road network and finding out the unbalanced or bottleneck points, a model for measuring and analyzing the road traffic network disequilibrium is proposed. Considering the influence of road network topology, traffic distribution, and the differences of nodes/sections on the road network equilibrium, the road network disequilibrium is measured from two aspects: network influence and capability separately. Firstly, section influence degree and section flow betweenness are employed to represent the differences in the influence relationship and capability distribution between sections. Section influence degree is to describe the influence relationship between one section and its surrounding sections; Section flow betweenness is to represent the differences in capacity distribution between sections which is resulted from the location and traffic flow distribution. Then, taking these two indicators as the input respectively, the road traffic network disequilibrium is analyzed multi-dimensionally based on the Gini coefficient and Theil index. It not only measures the level of the road traffic network disequilibrium from the view of the overall network, but also analyzes the contributes of a certain section to the disequilibrium of the whole network from the perspective of the section. Finally, the proposed model is validated in a part of Beijing's road network. The results show that the performance of the proposed model is similar with actual road network traffic distribution, which verified the feasibility and validity of the method in multi-dimension analysis of road network disequilibrium. The framework of the model is represented in Figure 1.

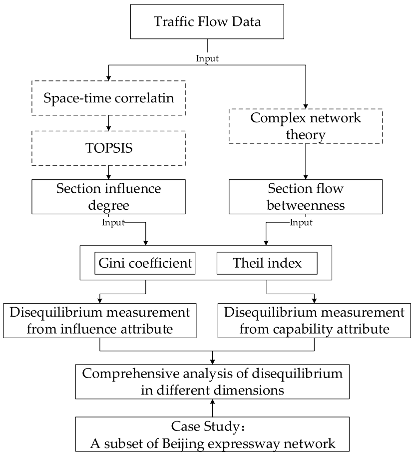


FIGURE 1. The framework and process of the disequilibrium analysis model.

The article is structured as follows: the importance of road traffic network disequilibrium measurement and analysis is stressed in Section 1. In section 2, mathematical descriptions of the section influence degree and section flow betweenness are given to represent the influence relationship and capability distribution of the sections in the network, respectively. Taking the two indicators as inputs, Section 3 gives the modeling process of the road traffic network disequilibrium analysis model based on the Gini coefficient and Theil index. In Section 4, the model has been applied in a subset of Beijing's expressway network in practice, and it is analyzed in detail. Finally, the conclusions and discussions on future directions are made in Section 5.

II. INPUT INDICATORS OF THE DISEQUILIBRIUM ANALYSIS MODEL

For the road network, sections are interrelated, and the connectivity of sections means that the traffic flow attached to section affects each other in the spatial dimension. Meanwhile, traffic flow is a stochastic process that changes continuously with time. Its continuity and periodicity, influenced by social activities, show that traffic flow is also affected in the time dimension. In addition, the roles of each section in the road network are different because of the differences in location and traffic flow distribution in the road network. Therefore, considering the influence of road network structure and traffic flow distribution, the road network disequilibrium is measured from two aspects in this paper. One is from the aspect of network influence by using section influence degree; the other is taking section flow betweenness as an input to measure network disequilibrium from the aspect of capability distribution. This work will discuss the two indicators above in turn.

A. SECTION INFLUENCE DEGREE

For a road network, there are obvious influence relations between sections in time and space. In this work, the section influence degree is calculated by space-time correlation to describe the influence between one section and its surroundings. And then, taking the section influence degree as input, the road network disequilibrium will be measured from the aspect of network influence.

Box *et al.* [14] gave a cross-correlation function (CCF) in 2008 to measure the relationship of two objects at a given delayed time. Taking two traffic flow time series X and Y as the binary stochastic process, the CCF can compute the correlation of the two traffic flow time series at a time lag s. The CCF can be denoted as follows

$$\gamma(s) = \frac{E\left(x\left(t\right) - \mu_X\right)\left(y\left(t+s\right) - \mu_Y\right)}{\sqrt{\sigma_X^2 \sigma_Y^2}} \tag{1}$$

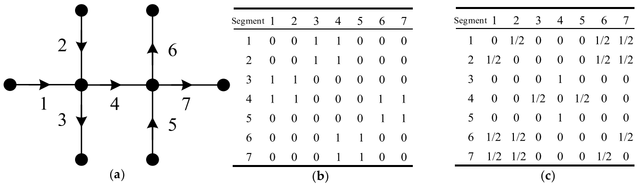
where μ_x, μ_y, σ_x^2, σ_y^2 are the means and variances of the two traffic flow time series. $X(t)$ is the values of traffic flow X at time t, and $y(t+s)$ is the values of traffic flow Y at time $t+s$.

In fact, for a certain section, there is usually more than one neighbor in the network. Therefore, the spatial weight matrix is drawn into the CCF to represent the influence between this section and its surroundings under different time and space delays clearly and accurately.

The spatial weight matrix can measure the adjacency of any two spatial objects of a network [15]. Drawing from graphs and complex network theory, a road network can be considered as a graph $G = (M, N)$ with a set M of m nodes and a set N of n edges. When considering the traffic flow, it should be a directed graph [16], [17]. The adjacency matrix can be expressed as follows [18]: When there is a direct traffic flow that impacts the relationship between edge i and j, it can be viewed that they are first-order neighbors. Therefore, the 1st-order adjacency matrix W_1 is composed of all first-order relationships between all edges in a network, and the 2nd-order adjacency matrix W_2 is the 1st-order relationships of matrix W_1 for the road network. By analogy, the kth-order adjacency matrix W_k can be defined.

Because of the relationship and direction of influence between traffic flows, there is some differences in the adjacency matrix from other common directed graphs [18]:

(1) When the traffic state is free, the direction of influence will be from a section to its downstream section because the traffic flows downstream. However, the direction of influence will be counteracted upstream when the traffic is congested. That is, the relationship and direction of influence between traffic flows is two-way.



Segment	1	2	3	4	5	6	7
1	0	0	1	1	0	0	0
2	0	0	1	1	0	0	0
3	1	1	0	0	0	0	0
4	1	1	0	0	0	1	1
5	0	0	0	0	0	1	1
6	0	0	0	1	1	0	0
7	0	0	0	1	1	0	0

(b)

Segment	1	2	3	4	5	6	7
1	0	1/2	0	0	0	1/2	1/2
2	1/2	0	0	0	0	1/2	1/2
3	0	0	0	1	0	0	0
4	0	0	1/2	0	1/2	0	0
5	0	0	0	1	0	0	0
6	1/2	1/2	0	0	0	0	1/2
7	1/2	1/2	0	0	0	1/2	0

(c)

FIGURE 2. Spatial weight matrices of a road network. (a) An example of road traffic network; (b) First-order spatial weight matrix; c) Second-order spatial weight matrix.

(2) When the traffic out of a section goes into two downstream sections at the same time, or it flows from two sections into their common downstream section, these two sections will be 2nd-order neighbors rather than first-order.

Based on the road network structure, the kth-order adjacency matrix weights are denoted as (2) in this paper, considering the distance between two sections. Taking the road network shown in Figure 2 as an example, the spatial weight matrix can be obtained as in Figure 2.

$$w_{ij} = \begin{cases} \dfrac{1}{k}, & \text{when } i \text{ and } j \text{ are } k\text{th-order neighbors} \\ 0, & \text{else} \end{cases} \tag{2}$$

For a certain section with traffic flow X, the CCF between the section and its kth-order neighbors can be developed as follows.

$$\gamma_i(k, s) = \frac{\sum\limits_{t=1}^{T} \left(x_i(t) - \overline{x_i}\right)\left(W^{(k)}x_i(t+s) - \overline{W^{(k)}x_i}\right)}{\sqrt{\sum\limits_t \left(x_i(t) - \overline{x_i}\right)^2}\sqrt{\sum\limits_t \left(W^{(k)}x_i(t+s) - \overline{W^{(k)}x_i}\right)^2}} \tag{3}$$

where $\gamma_i(k, s)$ is the CCF between section i and its neighbors; k is the space lag; s is the time lags; T is the statistical period; $x_i(t)$ is value of traffic flow X for section i when time is t; $\bar{x_i}$ is the average of traffic flow X; $W^{(k)}x_i(t+s)$ is a space delay operator, representing the weighted value of traffic flow for the kth-order adjacent sections of section i when time lag is s, and $\overline{W^{(k)}}x_i$ is the average of its kth-order adjacent sections' traffic, then it can be denoted as follows

$$W^{(0)}x_i(t) = x_i(t),\ W^{(k)}x_i(t) = \sum_{j=1}^{m} w_{ij}^k x_j(t) \tag{4}$$

where m is the section number of the network, and W_{ij}^k is the value of the adjacency relationship between section i and j in in the kth-order spatial weight matrix.

Note that the CCF measures the correlation between one section and its surroundings. It can be treated as a discrete process that varies with time delay s and space delay k. In this work, section influence degree is employed to describe the influence between one section and its all surroundings, and it can be considered as the comprehensive value of the CCF in space and time dimensions. In order to reflect the influence degree of a certain section in the road network, it is necessary to integrate the CCF in space and time dimensions.

Firstly, an integrated spatial weight matrix is proposed to integrate the CCF in the spatial dimension by the equation (3), then it is integrated in the temporal dimension using the technique for order preference by similarity to an ideal solution (TOPSIS) [19].

The integrated spatial weight matrix can be obtained through the sum of different order spatial weight matrixes as in the following equation (5)

$$W' = \sum_{i=1}^{k} W_i \tag{5}$$

where W_i is the ith-order spatial weight matrix. W' is the integrated spatial weight matrix, representing that the influence of a section on its first-order to kth-order neighbors should be taken into account in the meantime. It is worth noting that W'_{ij} should be row-standardized in the experimental analysis.

The steps of TOPSIS are as follows.

(1) Define the positive and negative ideal points

$$\begin{aligned} A^+ &= \left\{ \max_i r_i(s) \,|\, s \in S, 1 \le i \le N \right\} = \left\{ A^+(s) \,|\, s \in S \right\} \\ A^- &= \left\{ \min_i r_i(s) \,|\, s \in S, 1 \le i \le N \right\} = \left\{ A^-(s) \,|\, s \in S \right\} \end{aligned} \tag{6}$$

where $r_i(s)$ is the CCF of section i at time lag s synthesized in space. S is the set of time delay values. N is the section number of the road network. $A^+(s)$ and $A^-(s)$ are the maximum and minimum values of the CCF. Here, it is usually valued that $A^+(s) = 1$,$A^-(s) = 1$ because of the range of $[-1, 1]$ of the CCF.

(2) Compute the distance

The Euclidean weighted distance is usually used as follows to compute the distance between the CCFs and the positive or negative ideal points.

$$\begin{aligned} E_i^+ &= \sqrt{\sum_s w_s (r_i(s) - A^+(s))^2}, \quad 1 \le i \le N \\ E_i^- &= \sqrt{\sum_s w_s (r_i(s) - A^-(s))^2}, \quad 1 \le i \le N \end{aligned} \tag{7}$$

where w_s is a weighting coefficient. E_i^+ is the distance between the CCF and the positive ideal point. E_i^- is the distance between the CCF and the negative ideal point.

(3) Calculate the influence degree

The relative closeness e_i of section i is calculated to measure the influence degree of section i on other sections in the road network.

$$e_i = \frac{E_i^-}{E_i^- + E_i^+}, \quad 1 \le i \le N \tag{8}$$

(4) Normalize the influence degree

So as to distinguish the differences of the influence of a section on others easily, the influence degree should be normalized as follows.

$$c_i = \frac{e_i - \min e_i}{\max e_i - \min e_i}, \quad 1 \le i \le N \tag{9}$$

where c_i is the influence degree of section i.

B. SECTION FLOW BETWEENNESS

As mentioned above, a road traffic network can be considered as a directed graph $G = (M, N)$ when regarding traffic flow and the topology of the network. Because of the differences in location and traffic flow distribution of the sections, each section plays a different role in a road network. That is, each section has different actual load capacity at different times.

In complex network theory, the betweenness can be used to reflect the importance of each edge in the network in the physical structure. On this basis, by introducing traffic flow, section flow betweenness is proposed to measure this difference in both location and traffic flow distribution. It can be regarded as a reflection of the importance of section of the importance of road capacity distribution. And then, it is used as the input to measure the disequilibrium of the road network from the aspect of network capability based on the Gini coefficient and Theil index.

Section flow betweenness can be defined as the ratio of the total traffic flow of the shortest paths passing through a certain section to the total traffic flow of all the shortest paths between any two nodes in the road network. To a certain extent, the larger the section flow betweenness is, the more traffic the section carries in the whole road network and the greater important role the section plays [20]. Given the traffic flow data of the road network, section flow betweenness can be obtained by the following equation in this paper:

$$f_i = \frac{\sum_{t=1}^{T} \sum_{a,b \in N, a \ne b} x^i_{(a,b)}(t)}{\sum_{t=1}^{T} \sum_{a,b \in N, a \ne b} x_{(a,b)}(t)}, \quad 1 \le i \le N \tag{10}$$

where f_i is the flow betweenness of section i; T is the statistical period; a, b are the nodes in the road network; $x^i_{(a,b)}(t)$ is the traffic flow of section i among the shortest paths passing through section i between node a and b at time t; and $x_{(a,b)}(t)$ is the total traffic flow of the shortest paths between node a and b at time t.

III. ROAD NETWORK DISEQUILIBRIUM ANALYSIS MODEL

In 1921, Italian economist Corrado Gini proposed the Gini coefficient based on a Lorentz curve to measure the equilibrium degree of resource allocations, such as for property, capital, products, markets, and so on [21]. For road network X, it can be divided into n groups according to factors such as section, road, statistical period, and so on. As shown in Figure 3, taking the accumulated percentage of component numbers in each group as the abscissa, and the accumulated percentage of characteristic values as the ordinate, the Lorentz curve forms a square with an area of 1. The diagonal line is the absolute average line. The Gini coefficient is the proportion of the area surrounded by the Lorentz curve and the absolute average line to the area between the absolute fairness line and absolute unfairness line. The greater the Gini coefficient is, the more unbalanced the road network is.

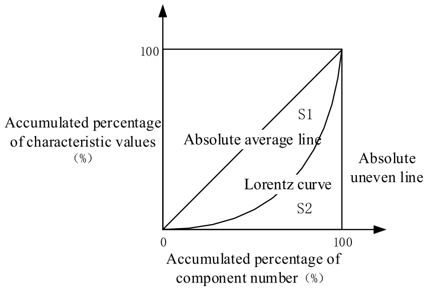


FIGURE 3. **The grading standards of the Gini coefficient.**

The Gini coefficient can reflect the disequilibrium of a road network intuitively from a global perspective. However, it can't show where the imbalance exists in the road network. For this, the Theil index is also employed in the road traffic network disequilibrium analysis as an effective supplement to the Gini coefficient.

The Theil index was proposed by metrology economist Henri Theil in 1967 to measure income inequality using the entropy in information theory [22]. As the effective supplement to the Gini coefficient, Theil index is better in intra-group decomposition than the Gini coefficient, and it can measure the contribution of intra-group and inter-group disparity to the disequilibrium of a road network, respectively [23]. It would be helpful to find out the causes of disequilibrium and verify the results of Gini coefficient measurements.

Therefore, in order to ensure comprehensive and accurate analysis of road network disequilibrium multi-dimensionally, taking the section influence degree and section flow betweenness, respectively, as the input indicators, a road traffic network disequilibrium analysis model is proposed based on the Gini coefficient and Theil index in this work to measure the traffic flow disequilibrium of the road network and to find out the unbalanced points in the network.

For a road network R, analysis of road traffic network disequilibrium based on the Gini coefficient and Theil index can be described as follows.

(1) Divide the network into k groups $R = \{R_i\}=\{(\alpha_m, \beta_m)\}$, $i = 1, 2, \cdots, k$ with an ascending sort order according to the sections. In group R_i, α_m represents the components, which can be a section or a statistical moment; β_m is the section influence degree or section flow betweenness corresponding to α_m and n_i is the number of components, which can be the number of sections or moments in different statistical periods (such as days, hours, minutes, and so on). B_i is the composite value of group R_i, which can be a weighted sum of the section influence degree or section flow betweenness corresponding to α_m.

$$B_i = \sum\nolimits_{m=1}^{n_i} \omega_m \cdot \beta_m \tag{11}$$

(2) Calculate the proportion of component number x_i and accumulated percentage X_i.

$$x_i = n_i \Big/ \sum\nolimits_{i=1}^{k} n_i, \quad X_i = \sum\nolimits_{j=1}^{i} x_i \tag{12}$$

(3) Calculate the proportion of section characteristic value y_i and accumulated percentage Y_i.

$$y_i = B_i \Big/ \sum\nolimits_{i=1}^{k} B_i, \quad Y_i = \sum\nolimits_{j=1}^{i} y_i \tag{13}$$

(4) Taking X_i as the abscissa and Y_i as the ordinate, the expression of the Lorentz curve $Y = f(X)$ is obtained by fitting scatter points (X_i, Y_i).

(5) Compute the Gini coefficient G, as shown in Figure 3.

$$G = S_1/(S_1 + S_2) = S_1 \Big/ 0.5 = 1 - 2\int_0^1 YdX \tag{14}$$

(6) Compute the slope θ_i between two adjacent R_i groups. This means that the bigger θ_i is, the quicker the change of R_i is, and the more unbalanced it is.

$$\theta_i = (Y_i - Y_{i-1})/(X_i - X_{i-1}) = y_i/x_i \tag{15}$$

(7) Compute Theil index T as follows.

$$\begin{cases} T = T_{ra} + \sum\limits_{i=1}^{k} \dfrac{B_i}{\sum_{i=1}^{k} B_i} \cdot T_i \\ T_{ra} = \sum\limits_{i=1}^{k} \dfrac{B_i}{\sum_{i=1}^{k} B_i} \cdot \ln \dfrac{B_i \Big/ \sum_{i=1}^{k} B_i}{n_i \Big/ \sum_{i=1}^{k} n_i} \\ T_i = \sum\limits_{m=1}^{n_i} \dfrac{\beta_{i,m}}{B_i} \ln \dfrac{\beta_{i,m}/B_i}{1/n_i} \end{cases} \tag{16}$$

where T is the Theil index, representing the disequilibrium of road network; T_{ra} is the disequilibrium degree between groups; and T_i is the disequilibrium degree within group R_i.

(8) Calculate the contribution of traffic flow disequilibrium in the road network as follows

$$\eta_{ra} = \frac{T_{ra}}{T}; \quad \eta_i = \frac{B_i}{\sum_{i=1}^{k} B_i} \cdot \frac{T_i}{T}, \; i = 1, 2, \cdots, k \tag{17}$$

where η_{ra} is the contribution rate of the traffic flow disequilibrium between groups to the whole network; and η_i is the contribution rate of the traffic flow disequilibrium within group R_i.

IV. CASE STUDY

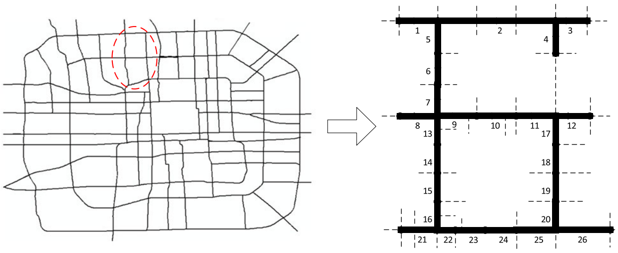
A. EXPERIMENT SCENARIO AND DATA

In this paper, the disequilibrium measurement model is analyzed in practice using traffic flow data from a Beijing regional road network. At present, the Expressway Traffic Information Detection System has been built by Beijing Traffic Management Bureau. The volume, speed and time occupancy of almost all ring roads and their connecting lines are obtained every 2 minutes through microwave detectors in the system. As shown in Figure 4, a subset of Beijing's road network comprising 26 sections (list in table 1) was selected as the test subject in this case.

For the survey data, because of the effects of random elements caused by the noise in the 2-min data, data smoothing was necessary, and the time interval of the traffic data in the modeling was transformed into 10 minutes. Finally, the data

TABLE 1. Information on the 26 sections included in the test network.

Section	Origin	Destination	Section	Origin	Destination
1	Zhan Chun Second Bridge	Xue Yuan Bridge	14	Ming Guang Bridge North	Ming Guang Bridge
2	Xue Yuan Bridge	Jian Xiang Bridge	15	Ming Guang Bridge	Wen Hui Bridge
3	Jian Xiang Bridge	Bei Chen Bridge West	16	Wen Hui Bridge	Xi Zhi Men Bridge
4	Jian Xiang Bridge	Jian Xiang Bridge North	17	Ma Dian Bridge	Bei Jiao Market
5	Xue Yuan Bridge	Hua Yuan North Road	18	Bei Jiao Market	Xin Kang Road
6	Hua Yuan North Road	Xue Zhi Men Bridge	19	Xin Kang Road	An De Road
7	Xue Zhi Men Bridge	Ji Men Bridge	20	An De Road	De Sheng Men Bridge
8	Capital University of Physical Education& Sports	Ji Men Bridge	21	Bei Zhan Bridge	Xi Zhi Men Bridge
9	Ji Men Bridge	Hua Yuan Road	22	Xi Zhi Men Bridge	Huapi Factory Hutong
10	Hua Yuan Road	Bei Tai Ping Zhuang Bridge	23	Huapi Factory Hutong	Xin Jie Kou
11	Bei Tai Ping Zhuang Bridge	Ma Dian Bridge	24	Xin Jie Kou	Ji Shui Tan Bridge
12	Ma Dian Bridge	Yu Min Middle Road	25	Ji Shui Tan Bridge	De Sheng Men Bridge
13	Ji Men Bridge	Ming Guang Bridge North	26	De Sheng Men Bridge	Drum Tower Bridge



(a) (b)

FIGURE 4. A part of Beijing's road network as the test network. (a) The location of the network; (b) The structure of the test network.

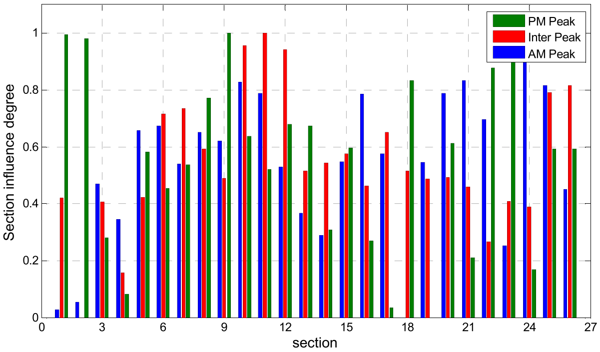


FIGURE 5. Section influence degree of the 26 sections on 21 May 2014.

from 19 May to 29 June 2014 (6 weeks total) were chosen in practice.

In this study, the process of analyzing the road traffic network disequilibrium is threefold. First, taking section influence degree as the input, the road traffic network disequilibrium is measured from the aspect of network influence. Second, taking section flow betweenness as the input, the road traffic network disequilibrium is measured from the aspect of network capability. Lastly, according to the results of the two parts above, the traffic disequilibrium of the test network is analyzed comprehensively in different dimensions.

B. INFLUENCE DISEQUILIBRIUM BASED ON SECTION INFLUENCE DEGREE

Take the Beijing regional road network shown in Figure 4 as the object, the disequilibrium measurement based on section influence degree could be divided into two parts: one is the calculation of section influence degree for the 26 sections in the test network; the other is the quantification of the test network disequilibrium based on the Gini coefficient and Theil index.

In order to analyze traffic disequilibrium of the test network in detail, a case was carried out for three periods in one day: AM peak (7:00–10:00), Inter peak (11:00–14:00), and PM peak (17:00–20:00).

1) SECTION INFLUENCE DEGREE CALCULATION

Based on the traffic data from 19 May to 29 June 2014, the section influence degree in practice for each day of these 26 sections during the 42 days was tested separately in this work. For a certain section, the traffic may flow into the sections that are higher than their 5th-order neighbors in the 10-min time interval because of the length of the sections. Thus, the integrated spatial weight matrix W' is obtained by using the first to fifth-order spatial weight matrices ($W_1 - W_5$) in calculating the section influence degree. That is, the value of k is 5 in equation (5). Besides, it is worth noting that the weights of the spatial weight matrix should be row-standardized at first.

As an example, Figure 5 shows the section influence degree of the 26 sections on 21 May 2014 corresponding to the three periods mentioned above: AM peak, Inter peak, and PM peak. As shown in Table 2, the 42-day average section influence degree of the 26 sections is also given.

In this work, the section influence degree is proposed to describe the influence of the relationship between each section and its surroundings. It is shown that the section influence degree had obvious heterogeneity. For example, the 42-day average section influence degrees of section 2 were 0.434, 0.220, and 0.535, corresponding to AM Peak, Inter Peak, and PM Peak respectively, while they were 0.707, 0.610, and 0.283 for section 19. To a certain extent, this heterogeneity can be considered as a representation of the influence of road traffic network disequilibrium both in time and space. Then, based on the Gini coefficient and Theil index,

TABLE 2. The 42-day average section influence degree.

Section	AM Peak	Inter Peak	PM Peak	Section	AM Peak	Inter Peak	PM Peak
1	0.425	0.303	0.420	14	0.464	0.253	0.260
2	0.434	0.220	0.535	15	0.552	0.326	0.373
3	0.373	0.397	0.239	16	0.671	0.298	0.430
4	0.484	0.370	0.363	17	0.701	0.516	0.327
5	0.602	0.368	0.525	18	0.413	0.570	0.702
6	0.600	0.348	0.427	19	0.707	0.610	0.283
7	0.570	0.296	0.358	20	0.827	0.554	0.413
8	0.514	0.367	0.581	21	0.665	0.365	0.325
9	0.302	0.370	0.632	22	0.609	0.401	0.466
10	0.668	0.347	0.430	23	0.505	0.459	0.689
11	0.637	0.479	0.421	24	0.715	0.409	0.362
12	0.522	0.506	0.439	25	0.597	0.554	0.533
13	0.434	0.284	0.314	26	0.463	0.670	0.468

TABLE 3. The section influence degree of the 26 groups for the statistical period of 42 days.

Section	number	The sum of section influence degree B_i			Section	number n_i	The sum of section influence degree B_i		
R_i	n_i	AM Peak	Inter Peak	PM Peak	R_i		AM Peak	Inter Peak	PM Peak
1	42	17.862	12.727	17.647	14	42	19.490	10.620	10.934
2	42	18.247	9.224	22.481	15	42	23.173	13.691	15.649
3	42	15.675	16.676	10.058	16	42	28.169	12.521	18.059
4	42	20.328	15.529	15.246	17	42	29.440	21.672	13.738
5	42	25.265	15.470	22.048	18	42	17.340	23.957	29.497
6	42	25.216	14.608	17.951	19	42	29.687	25.607	11.900
7	42	23.959	12.416	15.029	20	42	34.742	23.264	17.348
8	42	21.586	15.417	24.385	21	42	27.933	15.326	13.671
9	42	12.691	15.538	26.564	22	42	25.589	16.844	19.586
10	42	28.049	14.578	18.047	23	42	21.215	19.273	28.946
11	42	26.753	20.113	17.691	24	42	30.027	17.174	15.195
12	42	21.912	21.267	18.449	25	42	25.093	23.278	22.372
13	42	18.235	11.921	13.196	26	42	19.467	28.142	19.671

TABLE 4. The results of the Gini coefficient and Theil index in the statistical period of 42 days.

	Lorentz curve	G	T	T_{ra}	η_{ra}	$\sum\eta_i$
AM Peak	$0.352x^2+0.648x-0.004$	0.125	0.153	0.025	16.34%	83.66%
Interpeak	$0.444x^2+0.535x+0.006$	0.156	0.217	0.038	17.51%	82.49%
PM Peak	$0.417x^2+0.566x-0.002$	0.154	0.202	0.037	18.32%	81.68%

the disequilibrium will be quantified multi-dimensionally with the input of the daily section influence degree.

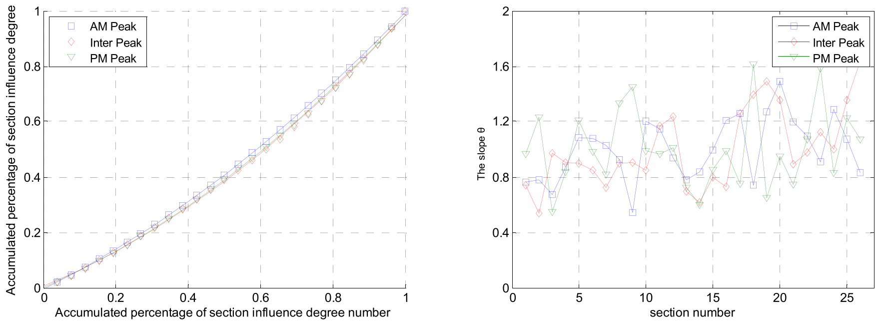
2) THE INFLUENCE DISEQUILIBRIUM MEASUREMENT

In this case, based on the daily section influence degree, the disequilibrium is quantified from two statistical time dimensions. One is the statistical period of 42 days; the other is the same day of each week. As shown in Figure 3, the test network can be divided into 26 groups $R = \{R_i\}=\{(\alpha_m, \beta_m)\}$, $i = 1, 2, \cdots, 26$ according to the sections. In group R_i, β_m is the section influence degree corresponding to section i; α_m is the statistical moment; B_i is the composite value, here it is the sum of the section influence degree; and n_i is the number of section influence degrees for section i in the statistical periods. For the survey data from 19 May to 29 June 2014 (6 weeks total), the section influence degree is calculated daily, that is, the statistical moment α_m is daily. Thus, the number of section influence degrees is 42 for each section in the statistical period of 42 days, and it is 6 on the same day over the 6 weeks.

Taking the statistical period of 42 days as an example, Table 3 gives the section influence degree of the 26 groups. According to the modeling process of the disequilibrium measurements based on the Gini coefficient and Theil index, the 26 groups are sorted in ascending order by the sum of the section influence degree B_i, and then the disequilibrium of the test network is quantified corresponding to AM peak, Inter Peak, and PM peak, respectively. Table 4 gives the outcomes of the Gini coefficient and Theil index in the statistical period of 42 days. Figure 6 shows the fitting process of the Lorentz curve and the slope θ of each group (section) for the 3 periods. Figure 7 gives the intra-group contribution rate η_i of traffic flow disequilibrium.

By examining these figures and tables, the following statements could be drawn:

- Table 4 shows a comparison of the Gini coefficient and Theil index in the statistical period of 42 days. It could be observed that the Gini coefficient calculated during the Inter peak was the largest. That is, compared to the other periods, the test network was the most



(a) (b)

FIGURE 6. The Lorentz curve and the slope for the statistical period of 42 days based on the section influence degree. (a) The results of Lorentz curve; (b) The results of slope.

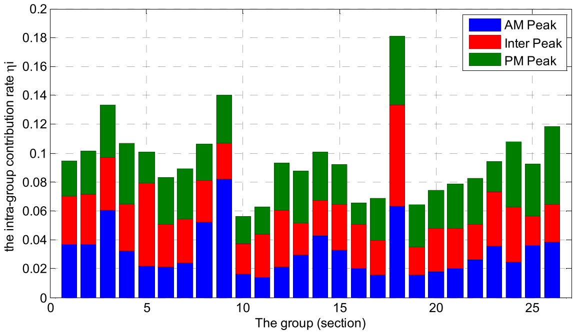


FIGURE 7. The intra-group contribution rates of traffic flow disequilibrium based on the section influence degree.

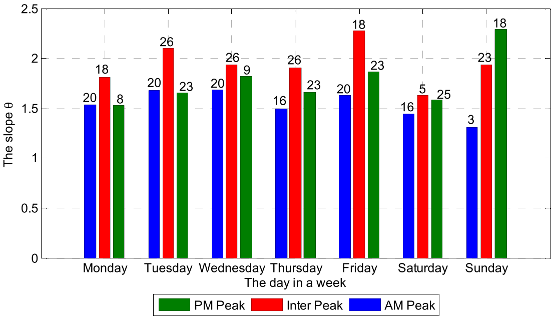


FIGURE 8. The sections with the largest slope θ from Monday to Sunday based on the section influence degree.

unbalanced during the Inter Peak in the statistical period of 42 days from the aspect of the relationship influence. Meanwhile, the values of the Theil index showed the same trend as the Gini coefficient, which confirmed this result.

- The inter-group contribution rate η_{ra} shows how much the disequilibrium between sections contributes to the disequilibrium of the whole network, while the intra-group contribution rate η_i shows how much the disequilibrium between different moments in the corresponding section i contributes to the disequilibrium of the whole network. According to the results in Table 4, disequilibrium in the test network was mainly caused by the intra-group disequilibria of each section, that is, it was mainly caused by the internal differences in influence of a section on others at different moments.
- Figure 6 displays the slope of each group (section) for the 3 periods. The slope of section 20 was the largest during the AM Peak, which was 1.488, while the section with the largest slope in Inter Peak was section 26, whose value was 1.637. For the PM Peak, section 18 had the largest slope with a value of 1.613. This means that there were some differences between these sections and others in the influence on their surroundings. It could be regarded that these sections may be one of the main factors that led to traffic disequilibrium in the corresponding period.
- Based on the Theil index, Figure 7 gives the intra-group contribution rate η_i to explain how much influence the disequilibrium between different moments of each section contributed to the disequilibrium of the whole network. The influence of each section varied greatly at different moments in a time period. For example, during the AM Peak, the influences of section 3, 8, 9, and 18 on their surroundings at different moments had significant differences, while the influences of section 11, 17, and 19 at different times were relatively balanced. In a word, these sections that have the greater intra-group contribution rates could be considered as the key points in the road network corresponding to the statistical period, and they should be focused on in daily management.
- Combining the above conclusion that the test network was mainly caused by the intra-group disequilibrium of each section, the sections that have the greater intra-group contribution rates should be taken priority in what we need to pay attention to daily.

Similarly, we also measured the influence disequilibrium of the test network on the same day of each week during the 42 days. Table 5 gives the section influence degree of the 26 groups on Monday as an example. Table 6 lists the statistical results of the Gini coefficient and Theil index during the AM Peak, Inter Peak, and PM Peak from Monday to Sunday.

In addition, Figure 8 displays the sections with the largest slope θ corresponding to the AM peak, Inter Peak, and PM peak, respectively, from Monday to Sunday. As an example, the sum of intra-group contribution rates η_i in the AM peak, Inter Peak, and PM peak for each section were calculated in this work, and the sum of contribution rates for the 26 sections from Monday to Sunday are shown in Figure 9.

TABLE 5. The section influence degree of the 26 groups on Mondays during the 42 days.

Section R_i	number n_i		B_i		Section R_i	number n_i		B_i	
		AM Peak	Inter Peak	PM Peak			AM Peak	Inter Peak	PM Peak
1	6	2.803	3.148	1.926	14	6	3.925	1.614	1.843
2	6	2.893	1.848	3.556	15	6	3.601	1.484	2.426
3	6	1.446	3.907	0.389	16	6	3.64	1.162	2.562
4	6	2.699	2.503	1.376	17	6	5.050	2.467	2.353
5	6	4.665	2.564	3.575	18	6	1.578	4.551	3.609
6	6	4.300	2.405	3.570	19	6	5.155	3.683	2.396
7	6	3.608	2.046	1.843	20	6	5.536	3.317	3.229
8	6	4.001	1.536	4.107	21	6	3.662	1.328	2.001
9	6	1.361	2.366	3.935	22	6	3.762	1.977	2.847
10	6	4.492	2.262	2.574	23	6	2.828	1.994	3.856
11	6	4.388	3.301	3.203	24	6	4.785	1.426	2.905
12	6	3.761	3.634	2.580	25	6	3.089	3.044	2.617
13	6	3.094	2.023	1.531	26	6	3.367	3.746	2.988

TABLE 6. The statistical results of the Gini coefficient and Theil index based on section influence degree.

	Period	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
G	AM Peak	0.167	0.195	0.211	0.189	0.199	0.127	0.147
	Inter Peak	0.202	0.267	0.239	0.239	0.262	0.19	0.23
	PM Peak	0.184	0.167	0.234	0.186	0.221	0.163	0.178
T	AM Peak	0.132	0.14	0.14	0.114	0.205	0.22	0.093
	Inter Peak	0.173	0.236	0.215	0.21	0.233	0.197	0.245
	PM Peak	0.191	0.181	0.194	0.181	0.179	0.249	0.227
T_{ra} [η_{ra} (%)]	AM Peak	0.049[36.87]	0.066[47.33]	0.073[52.13]	0.064[55.58]	0.066[32.21]	0.027[12.11]	0.053[57.28]
	Inter Peak	0.063[36.64]	0.113[47.58]	0.09[41.85]	0.091[43.12]	0.113[48.28]	0.058[29.39]	0.084[34.27]
	PM Peak	0.061[31.91]	0.044[24.31]	0.085[43.90]	0.057[31.32]	0.078[43.73]	0.041[16.44]	0.056[24.60]

Similar to the results of the Gini coefficient in Table 4, the value of the Gini coefficient in Table 6 was almost the largest during the Inter peak on Monday to Sunday. In the same way, the Theil index in Table 6 also validated this result. This means that the test network was the most unbalanced during the Inter Peak in terms of network influence. According to the results from Monday to Sunday in Table 6, almost all the inter-group contribution rates η_{ra} were a little smaller than the sum of intra-group contribution rates η_i of the 26 sections. That is, the results were the same as those corresponding to the statistical period of the 42 days, and disequilibrium of the test network was also mainly caused by the intra-group disequilibrium of each section.

Figure 8 displays the sections with the largest slope during the AM Peak, Inter Peak, and PM Peak, separately. It could show us which section may have played an important role in road traffic network equilibrium on different days in terms of section influence degree. For example, the slope of section 20 was the largest for the AM Peak on Monday, which was 1.540, while the section with the largest slope in Inter Peak was 18, whose value was 1.811. For PM Peak, section 8 had the largest slope, with a value of 1.530 on Monday. As can be seen from Figure 9, the sum of intra-group contribution rates η_i of the 26 sections varied greatly on different days. Taking Monday as an example, the intra-group contribution rate of section 1 was 0.0964, while the contribution rate of section 18 was 0.2212. Meanwhile, the relationships between sections 4, 9, 18, 24, 26 and their surroundings at different moments in a day had significant internal differences, and these sections could be considered as key points in the road network corresponding to the statistical period. According to the results above, the sections identified by the intra-group contribution rates need to be focused on first in alleviating traffic congestion.

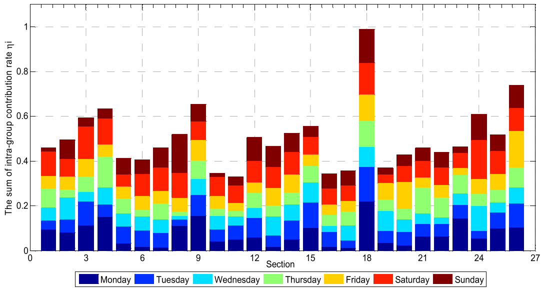


FIGURE 9. The sum of intra-group contribution rates for the 26 sections from Monday to Sunday based on the section influence degree.

C. CAPABILITY DISEQUILIBRIUM BASED ON SECTION FLOW BETWEENNESS

For the test network, disequilibrium measurements based on section flow betweenness are also divided into two parts: one is the calculation of daily section flow betweenness for the 26 sections; the other is the quantification of the test network disequilibrium based on the Gini coefficient and Theil index. The case was also performed according to the three periods mentioned above in one day.

TABLE 7. **The 42-day average section flow betweenness.**

Section	AM Peak	Inter Peak	PM Peak	Section	AM Peak	Inter Peak	PM Peak
1	0.048	0.053	0.052	14	0.072	0.079	0.074
2	0.014	0.013	0.013	15	0.039	0.042	0.038
3	0.004	0.005	0.004	16	0.026	0.028	0.026
4	0.005	0.005	0.005	17	0.050	0.050	0.055
5	0.014	0.014	0.012	18	0.009	0.012	0.014
6	0.042	0.043	0.038	19	0.046	0.046	0.049
7	0.068	0.070	0.066	20	0.028	0.028	0.030
8	0.009	0.009	0.008	21	0.006	0.006	0.006
9	0.086	0.088	0.090	22	0.089	0.080	0.083
10	0.075	0.076	0.079	23	0.053	0.047	0.049
11	0.051	0.054	0.057	24	0.037	0.036	0.036
12	0.009	0.010	0.010	25	0.025	0.016	0.018
13	0.065	0.069	0.066	26	0.030	0.023	0.023

TABLE 8. **The section flow betweenness of the 26 groups for the statistical period of 42 days.**

Section R_i	number n_i	The sum of section influence degree			Section R_i	number n_i	The sum of section influence degree		
		AM Peak	Inter Peak	PM Peak			AM Peak	Inter Peak	PM Peak
1	42	2.007	2.231	2.174	14	42	3.020	3.316	3.089
2	42	0.590	0.543	0.557	15	42	1.657	1.780	1.586
3	42	0.178	0.194	0.179	16	42	1.095	1.180	1.079
4	42	0.223	0.198	0.211	17	42	2.101	2.116	2.302
5	42	0.570	0.569	0.502	18	42	0.374	0.484	0.574
6	42	1.762	1.810	1.590	19	42	1.931	1.916	2.062
7	42	2.841	2.952	2.785	20	42	1.171	1.168	1.259
8	42	0.392	0.377	0.355	21	42	0.253	0.257	0.256
9	42	3.620	3.690	3.768	22	42	3.723	3.366	3.490
10	42	3.134	3.207	3.308	23	42	2.246	1.97	2.063
11	42	2.139	2.266	2.380	24	42	1.545	1.493	1.500
12	42	0.389	0.410	0.424	25	42	1.037	0.655	0.741
13	42	2.725	2.886	2.790	26	42	1.278	0.965	0.976

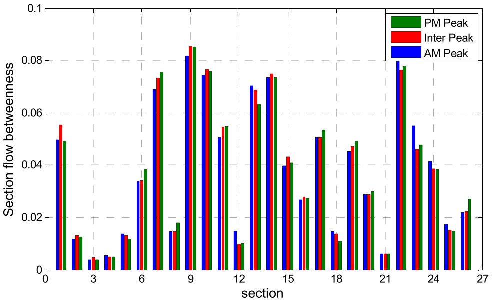


FIGURE 10. **Section flow betweenness of the 26 sections on 21 May 2014.**

1) SECTION FLOW BETWEENNESS CALCULATION

For the traffic data from 19 May to 29 June 2014, section flow betweenness for each day of these 26 sections was tested in practice. As an example, Figure 10 shows the section flow betweenness of the 26 sections on 21 May 2014. Table 7 gives the 42-day average section flow betweenness corresponding to the AM Peak, Inter peak, and PM peak.

In this work, section flow betweenness was shown to describe the differences in location and traffic flow distribution of the sections. Section flow betweenness had obvious heterogeneity, showing that the 26 sections played different roles in the network. For example, the 42-day average values of section flow betweenness of section 18 were 0.009, 0.012, and 0.014, corresponding to the AM Peak, Inter Peak, and PM Peak respectively, while they were 0.089, 0.080, and 0.083 for section 22. These values embody the disequilibrium in the capability of road traffic network. It is noteworthy that the variation of the section flow betweenness for the 26 sections in the AM Peak was similar with that in the Inter Peak and PM Peak. This could be due to the role of each section in the test network being similar or stable during the three periods. Then, based on the Gini coefficient and Theil index, the disequilibrium will be quantified multi-dimensionally with the input of daily section flow betweenness.

2) THE CAPABILITY DISEQUILIBRIUM MEASUREMENT

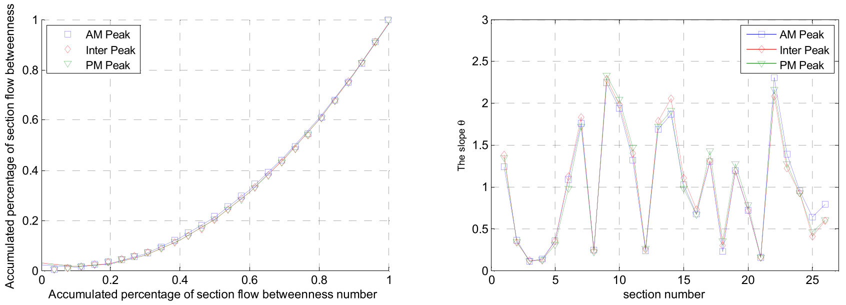
In this case, based on the daily section flow betweenness, disequilibrium measurements were also performed from two statistical time dimensions respectively: over the total 42 days and for the same day of each week. Taking the statistical period of 42 days as an example, Table 8 gives the section flow betweenness of the 26 groups, and then the disequilibrium of the test network was quantified corresponding to the AM peak, Inter Peak and PM peak. Table 9 lists the results of the Gini coefficient and Theil index in the statistical period of 42 days. Figure 11 shows the fitting process of the Lorentz curve and the slope θ of each group (section) for the 3 periods. The intra-group contribution rate η_i of traffic flow disequilibrium is given in Figure 12.

Examining these figures and tables, the following statements could be drawn:

- Taking section flow betweenness as the input, Table 9 shows results of the Gini coefficient and Theil index in the statistical period of 42 days. It could be

TABLE 9. The results of the Gini coefficient and Theil index based on section flow betweenness.

	Lorentz curve	G	T	T_{ra}	η_{ra}	$\sum\eta_i$
AM Peak	$1.152x^2$-$0.189x$+0.021	0.379	0.250	0.241	96.33%	3.67%
Interpeak	$1.216x^2$-$0.258x$+0.030	0.387	0.257	0.252	97.67%	2.43%
PM Peak	$1.209x^2$-$0.253x$+0.029	0.389	0.257	0.250	97.28%	2.72%



(a) (b)

FIGURE 11. The Lorentz curve and the slope for the statistical period of 42 days based on section flow betweenness. (a) The results of Lorentz curve; (b) The results of slope.

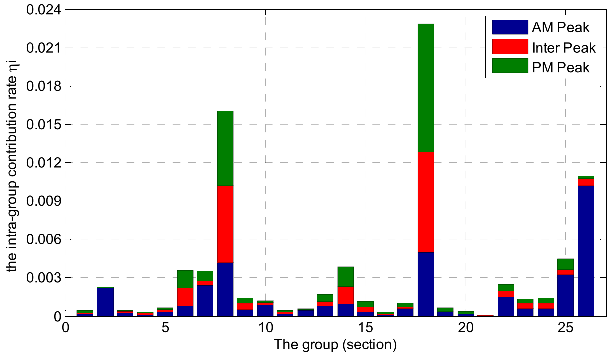


FIGURE 12. The intra-group contribution rate of traffic flow disequilibrium based on section flow betweenness.

observed that the Gini coefficient calculated during the PM peak was the largest. That is, compared to the other periods, the test network was the most unbalanced during the Inter Peak in the statistical period of 42 days, from the aspect of section capability. Meanwhile, the values of the Theil index showed a similar trend as the Gini coefficient, which also confirmed this result.

- Compared to the inter-group contribution rate η_{ra}, the sum of the intra-group contribution rate η_i in the AM Peak, Inter Peak, and PM Peak was much smaller. This means that disequilibrium in the test network was mainly caused by the inter-group disequilibrium between the 26 sections. That is, it was mainly caused by the significant difference in the roles that each section played in the network at the same time, rather than by the intra-section difference in the capability at different moments during the statistical period. There was almost no contribution of the differences in the section capability distribution over time to the disequilibrium of the network.
- Figure 11 displays the slope of each group (section) for the three periods. The slope trends were similar with each other. It can be considered that the capability distributions between different sections were similar in the three different statistical periods. The slope of section 22 was the largest during the AM Peak, which was 2.305. The section with the largest slope in Inter Peak was 9, and its value was 2.285. For the PM Peak, section 9 had the largest value, which was 2.332. This means that there were some differences between these sections and others in the capability distribution of the whole test network. It could be regarded that these sections may be one of the main factors that led to traffic disequilibrium, and they should receive more attention in the corresponding period.
- Figure 12 gives the intra-group contribution rate η_i to explain how much the unbalanced capability distribution of each section in time contributed to the disequilibrium of the whole network. The capability distribution of each section varied greatly at different moments in a time period. For example, during the AM Peak, the capability distributions of sections 8, 18, and 26 at different moments had significant interior differences, while the capability distributions of sections 4, 11, 16, and 21 at different times were relatively balanced. In addition, the intra-group contribution rate of section 26 in the AM Peak was obviously different from that in the Inter Peak or PM peak. It may be because the traffic of section 26 changed greatly in the AM peak, while it was relatively stable in the Inter Peak or PM Peak.
- Combining the above conclusion that disequilibrium of the test network was mainly caused by the significant difference in the roles that each section played in the network at the same time, the sections that have greater slopes should be considered more when just considering the section capability distribution in the whole road network.

TABLE 10. **The section flow betweenness of the 26 groups on Mondays during the 42 days.**

Section	number	The sum of section flow betweenness			Section R_i	Number	The sum of section flow betweenness		
R_i	n_i	AM Peak	Inter Peak	PM Peak		n_i	AM Peak	Inter Peak	PM Peak
1	6	0.287	0.319	0.314	14	6	0.45	0.457	0.421
2	6	0.082	0.076	0.081	15	6	0.235	0.244	0.215
3	6	0.025	0.028	0.025	16	6	0.154	0.163	0.151
4	6	0.033	0.029	0.03	17	6	0.305	0.305	0.335
5	6	0.085	0.082	0.069	18	6	0.064	0.083	0.099
6	6	0.244	0.241	0.216	19	6	0.277	0.277	0.302
7	6	0.406	0.418	0.388	20	6	0.168	0.17	0.185
8	6	0.037	0.04	0.051	21	6	0.036	0.036	0.036
9	6	0.514	0.543	0.553	22	6	0.544	0.496	0.505
10	6	0.45	0.457	0.473	23	6	0.333	0.3	0.308
11	6	0.305	0.318	0.338	24	6	0.236	0.221	0.216
12	6	0.059	0.058	0.061	25	6	0.126	0.096	0.102
13	6	0.398	0.406	0.386	26	6	0.147	0.134	0.137

TABLE 11. **Statistical results of the Gini coefficient and Theil index from Monday to Sunday based on section flow betweenness.**

	Period	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
G	AM Peak	0.387	0.378	0.383	0.369	0.37	0.381	0.388
	Inter Peak	0.390	0.389	0.39	0.39	0.384	0.382	0.394
	PM Peak	0.387	0.390	0.384	0.393	0.393	0.382	0.39
T	AM Peak	0.257	0.244	0.254	0.239	0.252	0.248	0.257
	Inter Peak	0.259	0.255	0.262	0.26	0.253	0.247	0.266
	PM Peak	0.255	0.259	0.25	0.265	0.265	0.247	0.259
T_{ra} $[\eta_{ra}(\%)]$	AM Peak	0.251[97.99]	0.239[98.29]	0.247[97.53]	0.231[97.01]	0.232[92.16]	0.243[98.13]	0.253[98.43]
	Inter Peak	0.254[98.06]	0.252[98.81]	0.257[97.92]	0.256[98.34]	0.248[98.35]	0.244[98.70]	0.260[98.16]
	PM Peak	0.249[97.92]	0.254[98.14]	0.245[98.25]	0.260[98.09]	0.260[97.88]	0.241[97.70]	0.253[97.72]

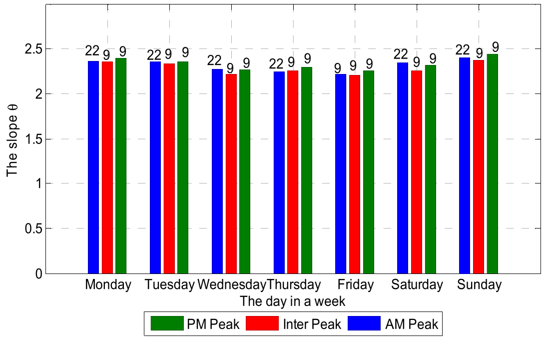


FIGURE 13. **The sections with the largest slope θ from Monday to Sunday based on section flow betweenness.**

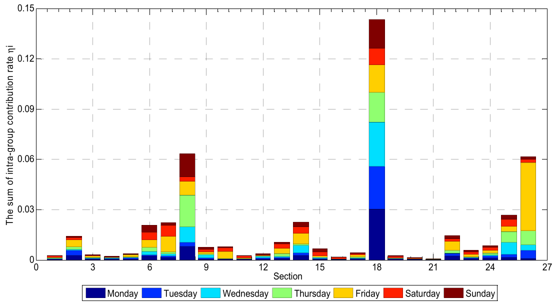


FIGURE 14. **The sum of intra-group contribution rates for the 26 sections from Monday to Sunday based on section flow betweenness.**

Similarly, the disequilibrium of the test network was also measured on the same day of each week during the 42 days. Table 10 gives the section flow betweenness of the 26 groups on Monday as an example. Table 11 lists the statistical results of the Gini coefficient and Theil index.

In addition, taking section flow betweenness as input, Figure 13 displays the sections with the largest slope θ from Monday to Sunday. As an example, the sum of intra-group contribution rates η_i in the AM peak, Inter Peak, and PM peak for each section was calculated in this work, and the sum of contribution rates for the 26 sections from Monday to Sunday are also shown in Figure 14.

Compared to results of the Gini coefficient in Table 6, the results in Table 11 were a little different. It could be observed that the test network was the most unbalanced in the Inter Peak on Monday, Wednesday, and Sunday in terms of network capability, while it was the PM peak on the other days. It is worth noting that the trend of the Theil index in one day was basically the same as that of the Gini coefficient, which confirms the disequilibrium measurements of road traffic network again. From the section capability distribution, the results of T_{ra} indicated the same phenomenon as that shown in table 9, in which the disequilibrium of the test network was mainly caused by the difference between sections, rather than by the different capability distributions of a certain section at different moments.

Based on section flow betweenness, Figure 13 displays the sections with the largest slopes during the AM Peak, Inter Peak, and PM Peak, separately. Section 22 may play an important role in road traffic network equilibrium in the AM peak, except on Friday; however, section 9 played an

TABLE 12. The comprehensive results of the Gini coefficient and Theil index.

	Period	The 42 days	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
G	AM Peak	0.252	0.277	0.287	0.297	0.279	0.285	0.254	0.268
	Inter Peak	0.272	0.296	0.328	0.315	0.315	0.323	0.286	0.312
	PM Peak	0.272	0.286	0.279	0.309	0.290	0.307	0.273	0.284
T	AM Peak	0.202	0.195	0.192	0.197	0.177	0.229	0.234	0.175
	Inter Peak	0.237	0.216	0.246	0.239	0.235	0.243	0.222	0.256
	PM Peak	0.230	0.223	0.220	0.222	0.223	0.222	0.248	0.243
T_{ra}	AM Peak	0.133	0.150	0.153	0.160	0.148	0.149	0.135	0.153
	Inter Peak	0.145	0.159	0.183	0.174	0.174	0.181	0.151	0.172
	PM Peak	0.144	0.155	0.149	0.165	0.159	0.169	0.141	0.155
η_{ra} (%)	AM Peak	66.00	77.12	79.43	81.22	83.57	65.21	57.69	87.43
	Inter Peak	61.18	73.38	74.34	72.75	73.83	74.28	68.02	67.32
	PM Peak	62.53	69.51	67.73	74.32	71.08	76.13	56.85	63.58

important role during the Inter Peak and PM Peak. As shown in Figure 14, the sum of intra-group contribution rates of sections 8, 9, and 26 for the AM Peak, Inter Peak, and PM peak had the larger values on Monday to Sunday. This means that internal differences in the capability distribution of these sections at different moments were more obvious, and the impact of these sections on road network disequilibrium was greater than that of the other sections.

According to the result that the disequilibrium of the test network was mainly caused by the difference in capabilities between sections, the sections with the larger slope should receive more attention in traffic management.

D. ROAD TRAFFIC NETWORK DISEQUILIBRIUM ANALYSIS

In this paper, the disequilibrium of a road traffic network was measured separately with the two aspects of the influence relationship and capability distribution. In order to get a comprehensive evaluation of how disequilibrium in the road traffic network exists and what characteristics the disequilibrium has, the network influence relationship and capability distribution should be considered at the same time when measuring the road traffic network disequilibrium, and the linear weighting method could be employed as follows:

$$z = \omega \cdot x + (1 - \omega) \cdot y \tag{18}$$

where z is the comprehensive measurement of the disequilibrium, both considering the network influence relationship and capability distribution; x is the measurement of the influence disequilibrium based on section influence degree; y is the measurement of the capability disequilibrium based on section flow betweenness; and ω is the adjustment weight, which indicates which factor receives more attention in the comprehensive analysis of road traffic network disequilibrium. For example, when the weight $\omega > 0.5$, it means that the influence is concerned more in the disequilibrium measurement of the road network, while the weight $\omega < 0.5$ means that the network's capability receives more attention in the work.

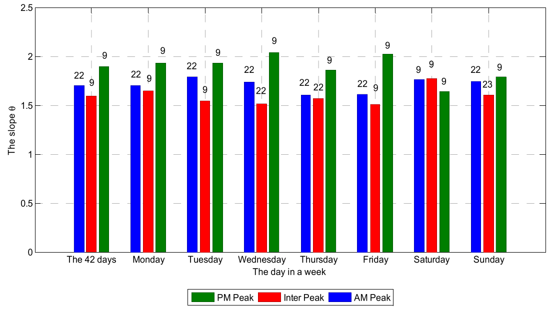


FIGURE 15. **The sections with the largest slope θ considering both attributes.**

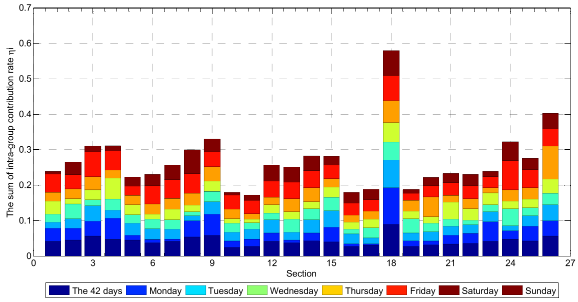


FIGURE 16. **The sum of intra-group contribution rates for the 26 sections considering the two attributes.**

In this case study, the disequilibrium of the test network was also analyzed with a mixture of the network influence relationship and capability distribution. Taking the weight $\omega = 0.5$ as an example, Table 12 gives comprehensive measurements of the disequilibrium based on the statistical period of the 42 days and the same day of each week.

In addition, based on considering the two attributes, Figure 15 displays the sections with the largest slope θ corresponding to the AM peak, Inter Peak, and PM peak, respectively, for the two statistical periods, and the sum of the intra-group contribution rates η_i for the 26 sections are also shown in Figure 16.

According to the results in Table 12, the test network was the most unbalanced basically during the Inter Peak, whether the statistical period is the 42 days or the same day of each week, which was confirmed mutually by the Gini coefficient and Theil index. This was a consequence of both the section influence relationship and capability distribution. It may be because of the nonstationary traffic flow in the Inter Peak. On one hand, travel routes are selected randomly based on people's daily travel habits when traffic is relatively smooth. The traffic flow distribution is stochastic and unbalanced. On the other hand, there are strong differences in the traffic state of sections during the Interpeak, leading to obvious differences in the strength of influence between sections.

Comprehensively considering the actions of network influence and capability attributes, it can be seen from Table 12 that the inter-group contribution rate T_{ra} was a little greater than the total intra-group contribution rate $\sum T_i$, and disequilibrium in the test network was mainly caused by the difference between sections rather than by the interior difference of a certain section at different moments. For this test network, it means that reducing the difference in traffic flow at the same time between sections in the test network should be considered first in alleviating traffic congestion.

Figure 15 displays the sections with the largest slope θ considering the two attributes for the AM Peak, Inter Peak, and PM Peak, separately. Section 22 may have played an important role in road traffic network equilibrium in the AM peak, except on Friday; however, section 9 played an important role during the PM Peak. In addition, during the Inter Peak, the section with the largest slope θ was 9 on Monday, Tuesday, and Saturday, while it was section 22 from Wednesday to Friday and section 23 on Sunday. This means that these sections played a more important role in leading to the disequilibrium of this test network than the other sections. All of this indicates that the traffic flow of the test network changes in a similar way every day, and the rules on one day are relatively more complex and random during the Inter Peak.

As shown in Figure 16, the sum of intra-group contribution rates of the AM Peak, Inter Peak, and PM peak for sections 9, 18, 24, and 26 had the larger values no matter if the 42 days or the same day of each week was measured. The differences within these sections at different moments were more obvious than others. It could be partly regarded that they had some impact on road network balance, and they should also receive attention in daily management.

Therefore, considering the influence relationship and capability distribution of the section, the level of this test network disequilibrium is analyzed from the view of the overall network. It shows that the test network was the most unbalanced basically during Inter Peak, which is consistent with the actual traffic situation. Besides, the disequilibrium contributions of the inter- and intra-section to the whole network during the different time periods is also given in this paper. It could provide effective decision-making supporting information in daily traffic management. The results of Gini coefficient and Theil index are coincident approximately, and agree with the states of the actual road network traffic distribution, which indicates that this proposed method is reasonable and feasible.

V. CONCLUSION

The road traffic network is in apparent disequilibrium due to the traffic flow distribution, network structure, etc., and the traffic congestion always occurred in certain areas or sections. It is found that unbalance analysis of the road network and the unbalanced points study could provide great support for the road network structure optimization. Hence, a road network disequilibrium measurement model based on Gini coefficient and Theil index is proposed in this paper to describe the level of the road network disequilibrium under the action of traffic flow and to give the contributions of sections to the network disequilibrium. First, the difference information on influence and capability between sections generated by the network structure, traffic distribution and section location are represented by introducing the indicators of section influence degree and section flow betweenness. Second, the road network disequilibrium measurement model is established based on the idea of Gini coefficient and Theil index, in which the section influence degree and flow betweenness are taken as the input. In the end, an empirical analysis of the disequilibrium measurement for a subset of Beijing road network is conducted, and this goal was clearly achieved by this effort.

It is verified that the application of section influence degree and section flow betweenness for disequilibrium measurement not only can objectively reveal the disequilibrium level of road network in terms of different characteristics, but also can find out the relatively unbalance places of the road network through the contribution of sections to the network disequilibrium. This is of great theoretical and practical significance to the study of traffic congestion alleviation. It can provide effective decision-making supporting information in daily traffic management to realize the optimization of road network resources, for example, traffic diversion, traffic lights time-assignment, and so on. Meanwhile, through the road network disequilibrium analysis, we can understand the overall development of the road network, and analyze the deficiencies in the development. This will be useful for guiding the road network planning and construction, and we can optimize the road network structure by the transformation of unbalanced or bottleneck points in the road network.

Moreover, it can also be used for other frequency data or experiment scenarios without difficulty because they share a common theoretical foundation. All these characteristics make this method attractive, and there is great potential for improvement. The future steps of this work are as follows:

(1) Improve the method by considering multiple factors to further analyze and discuss the reasons for the imbalance of a road network.

(2) Apply this work to evaluate traffic states, traffic guidance, and other researches, for example, summarizing adaptive adjustment models of the weight of the traffic

state evaluation according to the imbalance of the road network.

CONFLICTS OF INTEREST

The authors declare that there are no conflicts of interest regarding the publication of this article.

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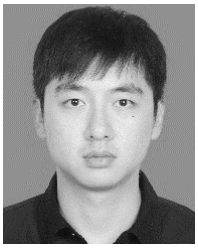

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