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Enhancing the Noise Robustness of the Optimal Computing Budget Allocation Approach

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ABSTRACT Since an optimal computing budget allocation (OCBA) approach maximizes the efficiency of the simulation budget allocation to correctly find the optimal solutions, various OCBA-based procedures, such as OCBA, OCBAm+, and MOCBA+, have been widely applied to solve simulation-based optimization problems. Recently, it has been found that the stochastic noise in a simulation model increases due to the increasing complexity of modern industrial systems. However, the OCBA approach may be inefficient for these practical problems. That is, it is very likely to waste a lot of budget on other candidates that are not truly optimal due to the abnormal simulation results, which occurs frequently in noisy environments. In this paper, we intuitively analyze the causes of this efficiency deterioration of the OCBA approach, and then a simple heuristic adjustment is proposed to enhance the noise robustness of the OCBA approach based on our analysis results. The proposed adjustment allows the OCBA approach to further consider the precision of the simulation results, thereby significantly reducing the wasted budget and increasing the efficiently. In addition, it can be applied to the existing allocation rules without modification and does not require additional computational costs. Many experimental results for the eight OCBA-based procedures clearly demonstrate the effectiveness of this adjustment. In particular, the results of practical problems emphasize its necessity.

INDEX TERMS Discrete-event system, high robustness to noise, optimal computing budget allocation, ranking and selection, simulation-based optimization, stochastic simulation.

I. INTRODUCTION

Discrete-event system simulation is a powerful tool for analyzing modern industrial systems such as telecommunication [1], manufacturing [2], microgrid [3], transportation [4], healthcare [5], and military [6] systems that cannot be described as a closed-form analytic model [7]. As one of the essential applications of simulation, simulation-based optimization (SBO) finds the optimal configurations of the systems' decision variables (i.e., the optimal designs) that satisfy the given optimal requirements of the system performance (e.g., maximize the performance) using simulations [8]. However, the efficiency of SBO is still a concern. For example, suppose we need to find the best design that maximizes the system performance out of k design alternatives. Most simulation models use random variables or processes to capture the uncertainty of the real-world, which results in a stochastic noise appearing in the simulation output. Thus, for one design, its exact system performance, which is represented by the expectation of the output, can be estimated as the mean of the output samples obtained from many simulation replications. Suppose N replications are required to obtain the precise value of the sample mean of the performance for each design, and then kN replications are finally used to correctly select the best design out of the k alternatives. If the number of design alternatives is large (i.e., k is large) and a large stochastic noise exists (i.e., a large N is required to reduce the noise), kN is very large, which may lead to prohibitively high computational costs for SBO [9]. Moreover, due to the higher complexity of modern industrial systems, the increasing costs per simulation run aggravate this problem [10].

Ranking and selection (R&S) is a great way to solve this efficiency problem when the number of design alternatives is a few hundred [11]–[14]. This is because R&S intelligently

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TABLE 1. The OCBA-based R&S procedures for diverse optimal solutions.

Optimal solutions	R&S procedures
Single best design	OCBA [18]
Best and worst designs	OCBAbw [22]
Best subset	OCBAm [23], OCBAm+ [24], OCBA-RM [25][a]
Best and worst subsets	OCBAmn [26]
Complete ranking of all designs	OCBAcr [10], ROCBAcr [27][a]
Subset ranking	OCBAsr [28]
Pareto set in multi-objective	MOCBA [29], MOCBA+ [30][31]
Best feasible design with constraints	OCBA-CO [32]
Best feasible subset with constraints including ranking within the subset	OCBA-CmR [33]

[a] Compared to the others, these procedures consider input uncertainty.

TABLE 2. Notations.

Symbol	Meaning
T	Total number of simulation replications (i.e., simulation budget).
k	Number of design alternatives.
$\boldsymbol{x}_i$	Design (i.e., simulation input), where subscript i represents the design index (i.e., $i \in \{1, \ldots, k\}$).
Θ	Set of designs, $\Theta = \{\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_k\}$.
Y_{ij}	Simulation output of $\boldsymbol{x}_i$ in the jth replication, $Y_{ij} \sim \mathcal{N}(\mu_i, \sigma_i^2)$.
μ_i	Expectation of Y_{ij} (i.e., the performance of $\boldsymbol{x}_i$), $\mu_i = \mathrm{E}[Y_{ij}]$.
σ_i	Variance of Y_{ij}, $\sigma_i = \mathrm{Var}[Y_{ij}]$.
N_i	Number of actually allocated replications at $\boldsymbol{x}_i$ (i.e., number of collected output samples: $Y_{i1}, \ldots, Y_{iN_i}$).
$\bar{\mu}_i$	Sample mean of $Y_{i1}, \ldots, Y_{iN_i}$, $\bar{\mu}_i = 1/N_i \cdot \sum_{j=1}^{N_i} Y_{ij} \sim \mathcal{N}(\mu_i, \sigma_i^2/N_i)$.
s_i^2	Sample variance of $Y_{i1}, \ldots, Y_{iN_i}$, $s_i^2 = \sum_{j=1}^{N_i} (Y_{ij} - \bar{\mu}_i)^2/(N_i - 1)$.

allocates a simulation budget (i.e., the finite number of simulation replications) to the designs based on the statistical inferences for the designs' simulation results, thereby eliminating many of the replications required to correctly select the optimal designs. So far, substantial research has been done to develop an efficient R&S procedure, and they are mainly based on three approaches [15], [16]: the indifference-zone (IZ) [17], the optimal computing budget allocation (OCBA) [18], and the expected value of information (EVI) [19], [20]. These approaches are distinguished based on how they measure the evidence for the correct selection and allocate replications based on the measurements. Both the IZ and OCBA approaches measure the selection quality via the probability of correct selection $P\{\mathrm{CS}\}$, but they define it in different ways. The IZ approach defines $P\{\mathrm{CS}\}$ via the frequentist probability and allocates the replications to provide a guaranteed lower bound of the $P\{\mathrm{CS}\}$. On the other hand, the OCBA approach describes $P\{\mathrm{CS}\}$ using the Bayesian posterior probability and assigns replications to maximize a lower bound of the $P\{\mathrm{CS}\}$. The EVI approach uses the expected opportunity cost E[OC] based on the Bayesian posterior probability as the measure and assigns replications to minimize an upper bound of E[OC] using decision-theory tools. Since E[OC] penalizes particularly bad selections more than mild ones based on the expected linear loss, the EVI approach has advantages when large stochastic noise exists [21]. However, its computational costs can be significantly higher as k increases due to the difficulty of minimizing E[OC].

Among these approaches, this paper focuses on the most popular OCBA approach. As shown in Table 1, many efficient OCBA-based R&S procedures have been proposed to select different kinds of optimal solutions, and they are widely used in various areas, such as healthcare [34], learning automata [35], semiconductor manufacturing [36], [37], simulation-based policy improvement [38], population-based search algorithms [39], [40], inventory problems [41], rare-event simulations [42], etc. Recently, as the complexity of the modern industrial systems to which this OCBA approach is applied increases, the stochastic noise in the simulation model also tends to increase. However, unfortunately, some widely-used OCBA-based procedures, such as OCBA, OCBAm+, MOCBA+, etc., may be significantly inefficient in the presence of large noise, which has been reported in several experimental results [43]–[47].

To resolve such a problem, this paper intuitively analyzes the causes of the efficiency deterioration of the OCBA approach based on the basic OCBA procedure for selecting the single best design [18]. Then, a simple heuristic adjustment is proposed to improve the efficiency based on our analysis results. The proposed adjustment can enhance the robustness to noise for the OCBA approach, thereby considerably increasing $P\{\mathrm{CS}\}$ within a limited simulation budget when large stochastic noise exists. In addition, this adjustment, which requires little computational costs, can be effectively applied to various OCBA-based procedures without modifying their existing allocation rules. The experimental results on several benchmark and practical problems demonstrate the improved efficiency of the OCBA-based procedures via the proposed adjustment.

The remainder of this paper is organized as follows. Section II briefly introduces the OCBA procedure. Section III intuitively analyzes the mentioned inefficiency drawback and proposes the heuristic adjustment. Section IV provides the experimental results, and a conclusion is given in Section V.

II. OPTIMAL COMPUTING BUDGET ALLOCATION PROCEDURE

The notations in this paper are shown in Table 2, where a bold typeface represents a vector. We assume that each simulation replication is independent from one another and that the simulation output Y_{ij} follows a normal distribution with an unknown μ_i and a known σ_i^2. Although the known variance assumption is used for the easy mathematical derivation of the Bayesian posterior distribution, the OCBA procedure can work well, even with the estimated variance (i.e., the sample variance s_i^2) used in practical problems. In addition, this normality assumption is reasonable since the simulation output is typically obtained from an average value or batch means, which means that the central limit theorem holds [48].

The goal of SBO is to select the best design $\boldsymbol{x}_b$ with optimal performance out of k alternatives, where $\boldsymbol{x}_b$ is defined as follows:

$$\boldsymbol{x}_b = \arg\min_{\boldsymbol{x}_i \in \Theta} \mu_i. \tag{1}$$

Although we consider the minimization problem, the OCBA procedure is equally applicable to maximization problems. Due to the limited simulation budget in practice, the exact performance of $\boldsymbol{x}_i$ based on μ_i can only be estimated by the sample mean $\bar{\mu}_i$; thus, instead of $\boldsymbol{x}_b$, we inevitably have to select the estimated best design $\boldsymbol{x}_e$ based on $\bar{\mu}_i$ as follows:

$$\boldsymbol{x}_e = \arg\min_{\boldsymbol{x}_i \in \Theta} \bar{\mu}_i. \tag{2}$$

As mentioned previously, the OCBA procedure evaluates the selection quality of $\boldsymbol{x}_e$ with $P\{\mathrm{CS}\}$ [18]:

$$P\{\mathrm{CS}\} = P\{\boldsymbol{x}_e = \boldsymbol{x}_b\} = P\left\{ \bigcap_{i=1, i \neq e}^{k} \tilde{\mu}_e < \tilde{\mu}_i \right\}, \tag{3}$$

where $\tilde{\mu}_i$ is the Bayesian posterior distribution of μ_i. Supposing no prior knowledge of an unknown μ_i, $\tilde{\mu}_i$ follows a normal distribution $\mathcal{N}\left(\bar{\mu}_i, \sigma_i^2/N_i\right)$.

In order to increase the efficiency of SBO, the OCBA procedure aims to determine the optimal allocation of simulation replications, $N_1^*, \ldots, N_k^*$, for k designs such that $P\{\mathrm{CS}\}$ is maximized, subject to a limited simulation budget T:

$$\arg\max_{N_1^*, \ldots, N_k^*} P\{\mathrm{CS}\} \quad \text{such that} \sum_{i=1}^{k} N_i^* = T \text{ and } N_i^* \geq 0. \tag{4}$$

Here, the constraint $\sum_{i=1}^{k} N_i^* = T$ in (4) implicitly assumes that the computing costs per simulation run are roughly the same across designs. If k is two, $P\{\mathrm{CS}\}$ can be easily calculated, but otherwise, it can only be estimated using a Monte Carlo simulation, which makes this problem intractable. To resolve this problem, the OCBA procedure applied an approximation of $P\{\mathrm{CS}\}$ called APCS based on the Bonferroni inequality [18]:

$$\mathrm{APCS} \equiv 1 - \sum_{i=1, i \neq e}^{k} P\{\tilde{\mu}_e > \tilde{\mu}_i\} \leq P\{\mathrm{CS}\}. \tag{5}$$

The APCS is much simpler to estimate. In addition, since it is a lower bound of $P\{\mathrm{CS}\}$, its maximization can ensure a sufficiently high $P\{\mathrm{CS}\}$. Thus, (4) can be approximated with the APCS as follows [18]:

$$\arg\max_{N_1^*, \ldots, N_k^*} \mathrm{APCS} \quad \text{such that} \sum_{i=1}^{k} N_i^* = T \text{ and } N_i^* \geq 0. \tag{6}$$

As a result, the intelligent allocation rules of the OCBA procedure were derived as an asymptotically optimal solution to (6) as follows (cf. [18] for the detailed mathematical derivations):

$$\frac{N_i^*}{N_j^*} = \left[\frac{\sigma_i/(\bar{\mu}_e - \bar{\mu}_i)}{\sigma_j/(\bar{\mu}_e - \bar{\mu}_j)}\right]^2, \quad i, j \in \{1, \ldots, k\}, \text{ and } i \neq j \neq e, \tag{7}$$

$$N_e^* = \sigma_e \sqrt{\sum_{i=1, i \neq e}^{k} \left(\frac{N_i^*}{\sigma_i}\right)^2}. \tag{8}$$

Here, the asymptotic optimum means that these rules were derived upon the assumption that T is infinite. That is, as T approaches to infinity (i.e., $T \rightarrow \infty$), the APCS can be asymptotically maximized when T is optimally distributed to k designs depending on these rules. In common with the OCBA procedure, the other OCBA-based R&S procedures also use this infinite assumption. However, this assumption can cause the OCBA approach to be inefficient in the presence of large stochastic noise (see Section III-A for the details).

Algorithm 1 Sequential Update Procedure for Selecting the Single Best Design Out of k Design Alternatives [18]

Control parameters: n_0 and Δ
Output: $\boldsymbol{x}_e$ (the estimated best design)

Procedure:
set $l \leftarrow 0$
simulate n_0 times for each $\boldsymbol{x}_i$, $i \in \{1, \ldots, k\}$
set $N_1^l = N_2^l = \ldots = N_k^l \leftarrow n_0$
update $\bar{\mu}_i$ and s_i^2 for $\forall i$, and **select** $\boldsymbol{x}_e$
while $\sum_{i=1}^{k} N_i^l < T$ **do**
  set $T^{l+1} \leftarrow \sum_{i=1}^{k} N_i^l + \min\left(\Delta, T - \sum_{i=1}^{k} N_i^l\right)$
  calculate N_i^* using (7)[a] and (8)[a] for $\forall i$, where $\sum_{i=1}^{k} N_i^* = T^{l+1}$
  simulate $\max\left(N_i^* - N_i^l, 0\right)$ times for each $\boldsymbol{x}_i$
  set $N_i^{l+1} \leftarrow N_i^l + \max\left(N_i^* - N_i^l, 0\right)$ for $\forall i$
  update $\bar{\mu}_i$ and s_i^2 for $\forall i$, and **select** $\boldsymbol{x}_e$
  set $l \leftarrow l + 1$
end while
return $\boldsymbol{x}_e$

[a] In practice, the unknown σ_i^2 is approximated by s_i^2 in (7) and (8).

Contrary to the assumptions used by the OCBA procedure, T is actually limited and σ_i^2 is typically unknown. In order to effectively utilize the allocation rules in such situations, the OCBA procedure applied a heuristic sequential update procedure, as shown in Algorithm 1, where σ_i^2 in (7) and (8) is approximated by the sample variance s_i^2. That is, until a given T is depleted, a small number of simulation replications Δ are allocated in a sequential manner such that the optimal allocation depending on (7) and (8) can be calculated based on more accurate values of $\bar{\mu}_i$ and s_i^2 in each iteration. There are two parameters to control this procedure: n_0 and Δ. n_0 is the initial number of simulation replications allocated to each design to obtain the minimum data of $\bar{\mu}_i$ and s_i^2 for further allocations of Δ. A suitable choice of n_0 is recommended as a number from 5 to 20 [49], [50]. Δ is the one-time increment, which is the number of replications additionally allocated per iteration. A suggested choice for Δ is a number less than 100 or $0.1k$ [11]. In the next section, we intuitively analyze the causes of the efficiency deterioration of the OCBA approach based on this OCBA procedure.

TABLE 3. Benchmark problem descriptions for various OCBA-based R&S procedures.

OCBA-based R&S proc.	Optimal designs	Model[a]	k	# obj.	Benchmark problem description: Model output distribution	Var.	n_0, Δ[b]	Note	Comp.
OCBA	Single best design	SEV	10	1	$Y_i \sim \mathcal{N}(i, 10^2)$[c]	Equal	10, 20	-	OCBA-EOC [51]
		SEV(**H**)	10	1	$Y_i \sim \mathcal{N}(\mathbf{0.25}i, 10^2)$	Equal	10, 20	-	
		SDV	10	1	$Y_i \sim \mathcal{N}(i, (16-i)^2)$	Diff.	10, 20	-	
		SDV(**H**)	10	1	$Y_i \sim \mathcal{N}(\mathbf{0.25}i, (16-i)^2)$	Diff.	10, 20	-	
OCBAm OCBAm+	Best subset (i.e., **top 5** best designs)	LEV	50	1	$Y_i \sim \mathcal{N}(i, 10^2)$	Equal	20, 50	$m = 5$[d]	EOC-m [52]
		LEV(**H**)	50	1	$Y_i \sim \mathcal{N}(\mathbf{0.5}i, 10^2)$	Equal	20, 50	$m = 5$	
		LDV	50	1	$Y_i \sim \mathcal{N}(i, ((51-i)/4)^2)$	Diff.	20, 50	$m = 5$	
		LDV(**H**)	50	1	$Y_i \sim \mathcal{N}(\mathbf{0.5}i, ((51-i)/4)^2)$	Diff.	20, 50	$m = 5$	
OCBAmn	Best and worst subsets (i.e., **top 5** best and worst designs)	LEV	50	1	$Y_i \sim \mathcal{N}(i, 10^2)$	Equal	20, 50	$m = n = 5$[d]	-
		LEV(**H**)	50	1	$Y_i \sim \mathcal{N}(\mathbf{0.5}i, 10^2)$	Equal	20, 50	$m = n = 5$	
		LDV	50	1	$Y_i \sim \mathcal{N}(i, ((51-i)/4)^2)$	Diff.	20, 50	$m = n = 5$	
		LDV(**H**)	50	1	$Y_i \sim \mathcal{N}(\mathbf{0.5}i, ((51-i)/4)^2)$	Diff.	20, 50	$m = n = 5$	
OCBAcr	Complete ranking of all designs	SDV	10	1	$Y_i \sim \mathcal{N}(i, (16-i)^2)$	Diff.	10, 20	-	-
		SDV(**H**)´	10	1	$Y_i \sim \mathcal{N}(\mathbf{0.5}i, (16-i)^2)$	Diff.	10, 20	-	
MOCBA MOCBA+	Pareto set in multi-objective	Steep model	16	2	$Y_{i_o} \sim \mathcal{N}(\mu_{i_o}, 2^2)$ (μ_{i_o} in Table 4(a)), $o \in \{1,2\}$[e]	Equal	10, 20	-	-
		Lee's model	25	3	$Y_{i_o} \sim \mathcal{N}(\mu_{i_o}, 8^2)$ (μ_{i_o} in Table 4(b)) , $o \in \{1,2,3\}$	Equal	10, 20	-	
OCBA-CO	Single best feasible design under constraints	SMEV	26	2	$Y_{i_o} \sim \mathcal{N}(\mu_{i_o}, 6^2)$ (μ_{i_o} in the footnote 'g'), $o \in \{0,1\}$[f]	Equal	20, 50	$\mu_{i_1} < 5.36$[h]	SCORE [53]
		SMEV(**H**)	26	2	$Y_{i_o} \sim \mathcal{N}(\mu_{i_o}, \mathbf{12^2})$ (μ_{i_o} in the footnote 'g'), $o \in \{0,1\}$	Equal	20, 50	$\mu_{i_1} < 5.36$	
		LMEV	101	2	$Y_{i_o} \sim \mathcal{N}(\mu_{i_o}, 6^2)$ (μ_{i_o} in the footnote 'g'), $o \in \{0,1\}$	Equal	20, 50	$\mu_{i_1} < 10.72$	
		LMEV(**H**)	101	2	$Y_{i_o} \sim \mathcal{N}(\mu_{i_o}, \mathbf{12^2})$ (μ_{i_o} in the footnote 'g'), $o \in \{0,1\}$	Equal	20, 50	$\mu_{i_1} < 10.72$	

[a] The abbreviations of the models' names are as follows: small equal variance (SEV), small different variance (SDV), large equal variance (LEV), large different variance (LDV), flight scheduling model (FSM), small multi-objective equal variance (SMEV), and large multi-objective equal variance (LMEV), where the letter '(H)' represents a model with large stochastic noise.

[b] The settings of the parameters n_0 and Δ are the settings used to evaluate the performance of the procedures in the literature.

[c] In every model, $i \in \{1, \ldots, k\}$.

[d] m is the size of the best subset, whereas n is the size of the worst subset.

[e] The sub-subscript o in Y_{i_o}, μ_{i_o}, and $\sigma_{i_o}^2$ means the objective index.

[f] The objective index of 0 indicates the primary objective, whereas the other index numbers represent the secondary objectives. Accordingly, the best feasible design is a design with the best performance on the primary objective while satisfying the stochastic constraints given in the secondary objectives.

[g] Except for $i = 1$, for $i = (a-1)\sqrt{k-1} + 1 + b$, $\mu_{i_0} = 1.34a$ and $\mu_{i_1} = 1.68a$, where $\forall a, \forall b \in \{1,2, \ldots, \sqrt{k-1}\}$. If $i = 1$, $\mu_{1_0} = 0$ and $\mu_{1_1} = 1.34$.

[h] The stochastic constraint given in the secondary objective.

III. ENHANCING THE NOISE ROBUSTNESS OF THE OPTIMAL COMPUTING BUDGET ALLOCATION APPROACH

A. INTUITIVE ANALYSIS

The allocation rules of the OCBA procedure consider s_i^2 and the difference between $\bar{\mu}_i$ and $\bar{\mu}_e$, as shown in (7) and (8), where σ_i^2 is approximated by s_i^2. Concretely, a design with a large s_i^2 and a small difference has a high possibility to be assigned additional simulation replications. Since these rules are dependent on the value of $\bar{\mu}_i$, it can be intuitively inferred that the OCBA procedure can make an incorrect assignment of Δ when $\bar{\mu}_i$ is an inaccurate value. In the presence of large stochastic noise, it is often the case that the value of $\bar{\mu}_i$ based on a small number of collected output samples is unfortunately very poor. Concretely, $\bar{\mu}_i$ may have an abnormal value far from the normal state near μ_i. For example, there are five designs (i.e., $\Theta = \{\boldsymbol{x}_1, \ldots, \boldsymbol{x}_5\}$), where the simulation output of the design follows a normal distribution $\mathcal{N}\left(0.5i, 10^2\right)$. Ten replications are equally allocated to each design, and then the resulting values of $\bar{\mu}_i$ and s_i for the five designs are shown at $l = 0$ of the graph in Fig. 1(a) and (c). Actually, $\boldsymbol{x}_1$ is the best design with the lowest performance (i.e., $\boldsymbol{x}_1 \neq \boldsymbol{x}_b$), but the value of $\bar{\mu}_1$ based on the collected output samples is far from μ_1 and abnormally high (i.e., far from the normal state near $\mu_1 = 0.5$). In this situation, since $\bar{\mu}_4$ is the minimum value of $\bar{\mu}_i$, $\boldsymbol{x}_4$ is chosen as the estimated best design $\boldsymbol{x}_e$, which is a completely incorrect selection (i.e., $\boldsymbol{x}_4 \neq \boldsymbol{x}_b$). In order for $\boldsymbol{x}_1$ to be correctly selected as $\boldsymbol{x}_e$, $\boldsymbol{x}_1$ should be allocated an additional simulation budget to change the abnormal value of $\bar{\mu}_1$ to its normal state.

However, the OCBA's allocation rules in (7) and (8) that rely on the value of $\bar{\mu}_i$ cannot allocate further replications to $\boldsymbol{x}_1$ due to the huge difference between the values of $\bar{\mu}_1$ and $\bar{\mu}_e$ (i.e., $\bar{\mu}_4$). Accordingly, since the abnormal value of $\bar{\mu}_1$ is not updated, whether $\boldsymbol{x}_1$ can be assigned additional replications depends on only the change in the value of other designs' $\bar{\mu}_i$ and s_i^2. That is, the possibility of its further allocation is very uncertain, which leads to a vicious circle in which the actual best design $\boldsymbol{x}_1$ cannot be allocated any additional replications due to the abnormal value of $\bar{\mu}_1$ being maintained. In some cases, even after a lot of the simulation budget has been

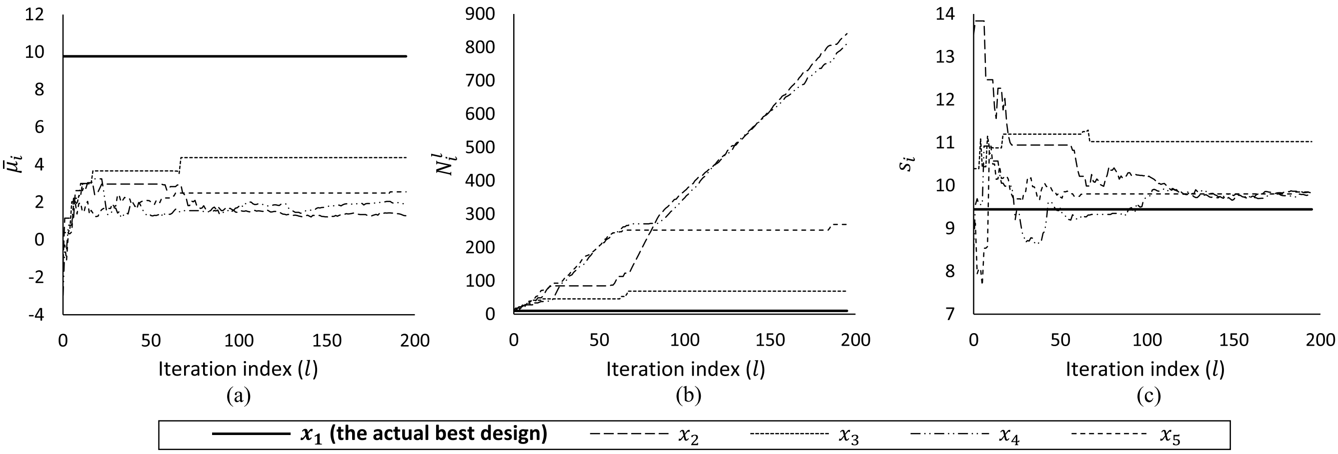

FIGURE 1. An example of the budget-wasted situation via the basic OCBA procedure in the presence of large stochastic noise. Graphs (a)-(c) respectively represent the changes in $\bar{\mu}_i$, N_i^l, and s_i of the five designs while sequentially allocating the 2,000 replications (i.e., $\Delta = 10$) with the existing allocation rules of (7) and (8), where σ_i^2 is approximated by s_i^2. When the actual best design x_1 unfortunately has an abnormal value of $\bar{\mu}_1$ due to the large noise after conducting 10 initial replications (see $l = 0$ in Graph (a)), x_1 has not been allocated any additional replications while allocating 1,950 replications, and these replications have been wasted in the other designs that are not the actual best design.

allocated through the sequential update loop of the OCBA procedure in Algorithm 1, $\boldsymbol{x}_1$ may not be assigned the budget at all. As shown in Fig. 1, even though 2,000 replications were sequentially allocated depending on the OCBA's rules ($\Delta = 10$), none of them was assigned to $\boldsymbol{x}_1$, which means that the abnormal value of $\bar{\mu}_1$ was not recovered. As a result, $\boldsymbol{x}_2$ is wrongly chosen as $\boldsymbol{x}_e$. If T goes to infinity, it will eventually be assigned further replications due to the asymptotic optimal nature of the rules [11]. However, in the meantime, a lot of budget is wasted on other meaningless designs due to the incorrect allocations of the OCBA procedure based on the poor value of $\bar{\mu}_1$.

Actually, this wasted budget situation occurs frequently in the presence of large stochastic noise. For example, consider a practical case such as the MILES design problem in Table 4, which has large noise due to many near optimal designs, as shown in Fig. 5. Then, the occurrence probability of the situation was 0.7285 after allocating 4,000 simulation replications, 0.2396 after assigning 10,000 replications, and still 0.1184 after allocating 100,000 replications. That is, even if a simulation budget of 100,000 was sequentially assigned by the rules of the OCBA procedure, the actual best design whose $\bar{\mu}_i$ unfortunately had a poor value could not be allocated any additional replications to recover its $\bar{\mu}_i$. Consequently, $P\{\text{CS}\}$ was estimated as 0.8643 after allocating 100,000 replications with the OCBA procedure. Therefore, such a wasted budget situation can be a significant cause for the efficiency deterioration of the OCBA procedure when large noise exists. Here, every probability, including $P\{\text{CS}\}$, was estimated over 10,000 independent repeated experiments.

In other OCBA-based R&S procedures, such as OCBAm, MOCBA+, OCBA-CO, etc., the wasted budget situation can sufficiently occur. This is because they also allocate further replications using σ_i^2 and the difference in $\bar{\mu}_i$ similar to the basic OCBA procedure (e.g., the rule of the OCBAm procedure is $N_i^*/N_j^* = \left[\sigma_i\left(\bar{\mu}_j - c\right)/\sigma_j\left(\bar{\mu}_i - c\right)\right]^2$, where $i \neq j$ and variable c is heuristically calculated according to equation (5.18) in [11]). In other words, since their allocation rules also depend on the value of $\bar{\mu}_i$, an incorrect assignment of Δ is made due to an abnormal value of $\bar{\mu}_i$, and thus, the vicious circle of no further allocation in the actual optimal designs lasts. As a result, the efficiency of the OCBA approach for large noise can be improved by reducing the wasted budget.

TABLE 4. The performance of all objectives of all designs in the three multi-objective models: (a) Steep model [11], [54] and (b) Lee's model [29], [55].

(a)

i	μ_{i_1}	μ_{i_2}	i	μ_{i_1}	μ_{i_2}	i	μ_{i_1}	μ_{i_2}
1[a]	0.29	5.71	7	5.0	5.0	13	7.0	7.0
2	3.8	6.2	8	3.0	3.0	14	6.27	3.73
3	9.0	9.0	9	5.9	4.1	15	4.27	1.73
4	4.55	1.45	10	6.7	7.3	16	2.69	3.31
5	1.79	4.21	11	7.9	6.1	-	-	-
6	4.7	5.3	12	3.9	2.1	-	-	-

(b)

i	μ_{i_1}	μ_{i_2}	μ_{i_3}	i	μ_{i_1}	μ_{i_2}	μ_{i_3}
1	8	36	60	14	32	44	64
2	12	32	52	15	26	40	66
3	14	38	54	16	28	42	64
4	16	46	48	17	32	38	66
5	4	42	56	18	30	40	62
6	18	40	62	19	34	42	64
7	10	44	58	20	26	44	60
8	20	34	64	21	28	38	66
9	22	28	68	22	32	40	62
10	24	40	62	23	30	46	64
11	26	38	64	24	32	44	66
12	28	40	66	25	30	40	64
13	30	42	62	-	-	-	-

[a]The shaded designs in each table are the Pareto designs we have to select.

B. A SIMPLE HEURISTIC ADJUSTMENT

Increasing n_0 might be an easy way to decrease the wasted budget by preventing the outbreak of an abnormal value of $\bar{\mu}_i$ in the initialization of the sequential update procedure. However, since σ_i^2 is typically unknown in practice, it is difficult to check in advance whether the simulation model has large stochastic noise. Even though σ_i^2 is known and small, if the spacing of μ_i between designs (which is also unknown depending on the noninformative assumption of μ_i) is relatively smaller, this small σ_i^2 can act as large stochastic noise that precludes the correct selection. That is, it is difficult to decide whether to increase n_0 due to the lack of prior information on the noise. In addition, blindly increasing n_0 may even decrease $P\{CS\}$ by excessively wasting the limited simulation budget for the initialization. Actually, when n_0 increased from 10 to 60 in Lee's model in Table 3, the estimated value of $P\{CS\}$ of the MOCBA+ procedure decreased from 0.992 to 0.951 after allocating 2,000 replications. To summarize, increasing n_0 is limited and not practical.

If it is difficult to prevent the occurrence of an abnormal value of $\bar{\mu}_i$ (which is inevitable in noisy environments), in order to effectively reduce the wasted budget, the actual optimal designs whose value of $\bar{\mu}_i$ is abnormal should be allocated further replications as soon as possible. Thereby, the abnormal value of $\bar{\mu}_i$ can return to its normal state. However, as mentioned before, since the allocation rules of the OCBA approach focus on only the abnormal value of $\bar{\mu}_i$, the possibility of further allocation to such designs is very uncertain. As a result, the further replications cannot be assigned quickly. In order to improve this uncertain possibility of further allocation, it is necessary to take into account not only the value of $\bar{\mu}_i$ but also the precision of $\bar{\mu}_i$, which is not considered in the OCBA approach so far. The precision of $\bar{\mu}_i$ for the actual optimal designs that have an abnormal value of $\bar{\mu}_i$ is typically low. In addition, as the sequential allocation process proceeds, the precision of $\bar{\mu}_i$ for these optimal designs becomes relatively lower than that for other designs that are continuously assigned additional replications. Thus, if the precision can be taken into account, this possibility of further allocation to these optimal designs that are not assigned any additional replication can reliably increase without depending on the uncertain change in the values of other designs' $\bar{\mu}_i$ and s_i^2. As the possibility increases, these designs will eventually be allocated one or two further replications. Since the abnormal value of $\bar{\mu}_i$ is typically based on a small number of samples, a few additional samples can make a significant impact on this value. That is, these few further allocated replications become primers, which break down the vicious circle of continuing the wasted budget situation and result in the allocation of many additional replications to the actual optimal designs.

Based on this intuitive inference, the allocation rules of the OCBA approach should additionally consider the precision of $\bar{\mu}_i$ to reduce wasted budget. Specifically, it is necessary to allocate further replications to designs that have relatively lower precision of $\bar{\mu}_i$. However, why does the OCBA approach not consider the precision that seems quite natural to include in its rules? This can be found in the infinite assumption on which they are based. Since the precision is always 100% under the assumption, there is no need to consider this; thus, during the mathematical derivations, the terms related to the precision become negligible as T approaches to infinity. Intuitively, the number of allocated replications so far should be considered for representing the precision, but it can be ignored when the given budget T is infinite. As a result, the OCBA approach does not take the precision of $\bar{\mu}_i$ into account in its allocation rules. Furthermore, the heuristic sequential update procedure used by almost all R&S procedures, including the various OCBA-based procedures, violates the infinite assumption. This is because the sequential procedure slightly increments T, as shown in Algorithm 1 (i.e., $T^{l+1} = T^l + \Delta$, where l is the iteration index in Algorithm 1), so that the budget allocated so far cannot be negligible. That is, the existing allocation rules of the OCBA approach are not suitable for the sequential update procedure.

In this subsection, we additionally propose a simple heuristic adjustment to consider the precision of $\bar{\mu}_i$ in the existing allocation rules of the OCBA approach. To this end, we paid attention to the standard error of $\bar{\mu}_i$ as follows:

$$s.e.(\bar{\mu}_i) = s_i/\sqrt{N_i}. \tag{9}$$

Without the loss of generality, $s.e.(\bar{\mu}_i)$ can represent the precision of $\bar{\mu}_i$. That is, a lower value of $s.e.(\bar{\mu}_i)$ indicates a higher precision of $\bar{\mu}_i$. In order to allocate more replications to designs with lower precision of $\bar{\mu}_i$, the further allocation for each $\boldsymbol{x}_i$ should be proportional to their $s.e.(\bar{\mu}_i)$. In the existing allocation rules, such as (7) and (8), the allocation is proportional to σ_i, which is be approximated by s_i in practice. In (9), $s.e.(\bar{\mu}_i)$ is also proportional to s_i. Thus, if $s.e.(\bar{\mu}_i)$ replaces σ_i in the rules, we can additionally consider the precision of $\bar{\mu}_i$ while maintaining the characteristics of the existing allocation rules. This is the proposed simple heuristic adjustment. For example, when applying this adjustment, the allocation rules of the basic OCBA procedure are changed as follows:

$$\frac{N_i^*}{N_j^*} = \frac{N_j}{N_i}\left[\frac{s_i/(\bar{\mu}_e - \bar{\mu}_i)}{s_j/(\bar{\mu}_e - \bar{\mu}_j)}\right]^2, \quad i,j \in \{1,\ldots,k\}, \text{ and } i \neq j \neq e, \tag{10}$$

$$N_e^* = s_e\sqrt{\sum_{i=1,i\neq e}^{k} \frac{N_i}{N_e}\left(\frac{N_i^*}{s_i}\right)^2}. \tag{11}$$

Here, N_i, the number of actually allocated replications at $\boldsymbol{x}_i$ (i.e., N_i^l in Algorithm 1), is different from N_i^*, the optimal portion of a given T.

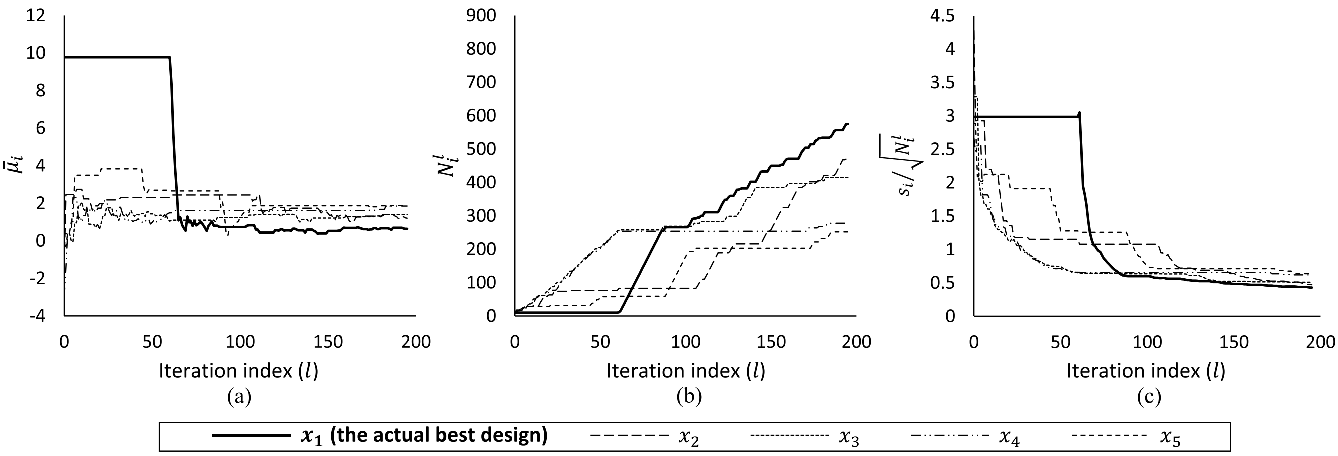

FIGURE 2. An example of the reduction in the wasted budget by applying the proposed adjustment to the basic OCBA procedure under the same initial conditions as in the example in Fig. 1. Graphs (a)-(c) respectively represent the changes in $\bar{\mu}_i$, N_i^l, and $s_i/\text{sqrt}\left(N_i^l\right)$ of the five designs while sequentially allocating 2,000 replications (i.e., $\Delta = 10$) with the improved rules of (10) and (11). Since these rules also consider the precision of $\bar{\mu}_i$, further replications can be quickly allocated to the actual best design x_1 that has a relatively high $s_i/\text{sqrt}\left(N_i^l\right)$. As a result, the wasted budget has been reduced, and x_1 can be correctly selected as the best design after allocating 1,950 replications.

The proposed adjustment is very simple but significantly effective. It can effectively reduce the wasted budget by further considering the precision of $\bar{\mu}_i$ in the existing allocation rules and thus improve the efficiency of the OCBA approach in the presence of large stochastic noise. For example, consider the same example of five designs in the previous subsection again. As shown in Fig. 1, when the actual best design $\boldsymbol{x}_1$ has an abnormally high value of $\bar{\mu}_1$, the existing rules of (7) and (8) could not allocate further replications to $\boldsymbol{x}_1$, and the wasted budget situation continued. However, for the same initial condition, the improved rules of (10) and (11) work differently. Although $\boldsymbol{x}_1$ cannot be allocated further replications immediately due to its abnormal value of $\bar{\mu}_1$, $s.e.(\bar{\mu}_1)$ becomes relatively larger as the allocation of Δ iterates, as shown in Fig. 2(c). That is, the possibility of a further allocation to $\boldsymbol{x}_1$ via the improved rules is also gradually increasing. As a result, a few additional replications can be quickly allocated to $\boldsymbol{x}_1$, which breaks down the vicious circle and results in the allocation of many further replications to $\boldsymbol{x}_1$, as shown Fig. 2(b). After allocating 2,000 replications, $\boldsymbol{x}_1$ is correctly chosen as $\boldsymbol{x}_e$. A comparison of Figs. 1 and 2 demonstrates the effect of the proposed adjustment. Actually, the occurrence probability of the wasted budget situation was 0.1201 in this simple example, whereas the probability significantly lowered to 0.0293 by applying the proposed adjustment. Consequently, $P\{\text{CS}\}$ increased from 0.7661 to 0.8077. As the noise increases, the efficiency improvement by applying this adjustment increases. As mentioned previously, for the practical problem of the MILES design in Table 6, the occurrence probability and $P\{\text{CS}\}$ were 0.1184 and 0.8643, respectively, after assigning 100,000 replications using the existing rules. However, when the same number of replications are allocated using the improved rules, the probability dramatically decreased to 0.0101, and $P\{\text{CS}\}$ increased to 0.9854, which is close to 1.

Although the improved rules maximized $P\{\text{CS}\}$, they are no longer the asymptotically optimal solution to the OCBA problem of (6). However, compared to the existing rules, the improved rules can be more suitable for the sequential update procedure because they use all the information given during the sequential procedure, such as $\bar{\mu}_i$, s_i^2, and N_i (the existing rules use only $\bar{\mu}_i$ and s_i^2). Thus, these rules can appropriately deal with the abnormal value of $\bar{\mu}_i$ of the actual best design that may occur under the sequential procedure due to large stochastic noise. In other words, these modified rules cannot be a mathematical solution to (6), which is an approximation of the original R&S problem of (4), but they can be a good heuristic alternative to more efficiently solve (4). In addition, it can be more efficient since there are little additional computational costs to apply it.

Similar to this example, the proposed adjustment can be applied to other OCBA-based R&S procedures without modifying the existing rules and can significantly improve their $P\{\text{CS}\}$ in the presence of large stochastic noise. However, we clearly point out that the proposed adjustment cannot be applied to all OCBA-based procedures, including the procedures in Table 1. The adjustment is only applicable to the OCBA-based procedures that allocate further replications in proportion to any ratio of $\bar{\mu}_i$ to σ_i^2 (or s_i^2). In addition, their allocation rules should not consider N_i depending on the infinite assumption (i.e., $T \rightarrow \infty$). It is not applicable to some recent OCBA-based procedures that do not satisfy these conditions. For example, the ROCBAcr procedure [27], which identifies the complete ranking of all designs with input uncertainty, allocates further replications equally to only two critical designs. In addition, the OCBA-CmR procedure [33], which ranks the top designs with stochastic constraints, already considers N_i in its heuristic allocation rules. Nevertheless, the proposed adjustment can be applied to the widely used classical procedures in Table 1 and enhance

TABLE 5. Required budget to achieve $P\{CS\}$ of 0.99 for the benchmark problems in OCBA-based procedures when applying the proposed adjustment: (a) OCBA, (b) OCBAm, (c) OCBAm+, (d) OCBAmn, (e) OCBAcr, (f) MOCBA, (g) MOCBA+, and (h) OCBA-CO.

(a)

Model		**OCBA(I)**	OCBA	OCBA-EOC	Equal
SEV	Req. T	2,850	3,600	3,000	10,800
	Ratio[a]	-	***1.26***	*1.05*	*3.79*
SEV(**H**)	Req. T	48,000	72,000	46,480	171,500
	Ratio	-	***1.50***	*0.97*	*3.57*
SDV	Req. T	5,550	7,180	5,800	24,100
	Ratio	-	***1.29***	*1.05*	*4.34*
SDV(**H**)	Req. T	84,600	128,500	90,200	345,000
	Ratio	-	***1.52***	*1.07*	*4.08*

(b)

Model		**OCBAm(I)**	OCBAm	EOC-m	Equal
LEV	Req. T	3,750	30,000	12,500	54,000
	Ratio	-	***8.00***	*3.33*	*14.40*
LEV(**H**)	Req. T	13,350	>>10^6 (0.8406)[b]	46,000	216,000
	Ratio	-	***>>74.91***	*3.45*	*16.18*
LDV	Req. T	4,800	>10^6 (0.9824)	16,750	72,000
	Ratio	-	***>>74.91***	*3.49*	*15.00*
LDV(**H**)	Req. T	17,000	>>10^6 (0.8043)	60,000	252,000
	Ratio	-	***>>74.91***	*3.53*	*14.82*

(c)

Model		**OCBAm+(I)**	OCBAm+	EOC-m	Equal
LEV	Req. T	4,350	4,700	12,500	54,000
	Ratio	-	***1.08***	*2.87*	*12.41*
LEV(**H**)	Req. T	16,750	22,300	46,000	216,000
	Ratio	-	***1.33***	*2.75*	*12.90*
LDV	Req. T	5,300	6,150	16,750	72,000
	Ratio	-	***1.16***	*3.16*	*13.58*
LDV(**H**)	Req. T	19,300	31,100	60,000	252,000
	Ratio	-	***1.61***	*3.11*	*13.06*

(d)

Model		**OCBAmn(I)**	OCBAmn	Equal
LEV	Req. T	7,550	10,150	64,600
	Ratio	-	***1.34***	*8.56*
LEV(**H**)	Req. T	30,900	53,000	267,000
	Ratio	-	***1.72***	*8.64*
LDV	Req. T	5,300	6,400	73,200
	Ratio	-	***1.21***	*13.81*
LDV(**H**)	Req. T	20,750	34,500	256,000
	Ratio	-	***1.66***	*12.34*

(e)

Model		**OCBAcr(I)**	OCBAcr	Equal
SDV	Req. T	12,680	20,000	30,100
	Ratio	-	***1.58***	*2.37*
SDV(**H**)′	Req. T	49,750	103,000	114,000
	Ratio	-	***2.07***	*2.91*

(f)

Model		**MOCBA(I)**	MOCBA	Equal
Steep	Req. T	2,920	>4,000 (0.9420)	12,160
	Ratio	-	***>1.37***	*4.16*
Lee's model	Req. T	1,370	1,970	6,600
	Ratio	-	***1.44***	*4.82*

(g)

Model		**MOCBA+(I)**	MOCBA+	Equal
Steep	Req. T	2,660	>4,000 (0.9257)	12,160
	Ratio	-	***>1.50***	*4.57*
Lee's model	Req. T	1,310	1,930	6,600
	Ratio	-	***1.47***	*5.04*

(h)

Model		**OCBA-CO(I)**	OCBA-CO	SCORE	Equal
SMEV	Req. T	1,340	3,150	1,740	6,770
	Ratio	-	***2.35***	*1.30*	*5.05*
SMEV(**H**)	Req. T	4,970	≈10^5 (0.9874)	16,520	27,220
	Ratio	-	***≈20.12***	*3.23*	*5.48*
SMDV	Req. T	3,340	23,370	5,570	30,670
	Ratio	-	***7.00***	*1.67*	*9.18*
SMDV(**H**)	Req. T	9,670	>10^5 (0.9668)	82,420	>10^5 (0.9751)
	Ratio	-	***>10.34***	*8.52*	*>10.34*

[a] In each table, this is the ratio of T required in the improved procedure denoted by '(I)' and that required in the other procedure (e.g., $T_{OCBA}/T_{OCBA(I)}$).

[b] Even if the simulation budget is sufficient, $P\{\text{CS}\}$ reaches only the value in the parentheses (i.e., more budget than the value on the left of the parentheses is required to achieve $P\{\text{CS}\}$ of 0.99).

their noise robustness. This is demonstrated in the various experimental results for the eight OCBA-based procedures in the next section.

Meanwhile, when applying the proposed adjustment, a simulation budget can be more widely distributed, as shown in Fig. 3. This is because the consideration of $s.e.(\bar{\mu}_1)$ increases the possibility of further allocation for designs that have received relatively small budgets. As mentioned previously, such a conservative allocation can be effective in the presence of large stochastic noise, but otherwise, it might reduce $P\{\text{CS}\}$ by decreasing the replications that need to be allocated to the actual optimal designs and their neighborhood. However, the reduction can be insignificant because the total budget required for correctly selecting the optimal designs is typically small in the small noise cases. That is, the conservative allocation of the proposed adjustment may consume a few more replications for the correct selection, but this increase is meaningless because the required total budget itself is small. Actually, when the variance of the SEV model in Table 3 decreased to 5, the allocation rules of the basic OCBA procedure achieved $P\{\text{CS}\}$ of 0.99 using 740 replications, but the improved rules used 760 replications. After allocating the same 740 replications, the estimated $P\{\text{CS}\}$ of the improved rules was 0.9896, which is slightly reduced in comparison with 0.99. Although the proposed adjustment may be inefficient at small noise, it can still be effective because most of the practical optimization problems have large stochastic noise.

TABLE 6. Summary of the four practical problems.

Problem	Goal of the problem	k	# obj.	Sim. model	Output of sim. model	Appl. proc.	n_0, Δ	Note
MILES[a] design	Select the **best design** of MILES that maximizes the hit rate of MILES.	195	1	Optical engineering simulator [51]	$Y_i \in \{0,1\}$[b] (μ_i in Fig. 5), where $P(Y_i=1)=\mu_i$, $P(Y_i=0)=1-\mu_i$.	**OCBA(I)**, OCBA	20, 100	-
Decoy system design	Select the **best subset** consisting of the top 10 designs of decoy system that maximize the survival rate of warship.	100	1	Anti-torpedo combat simulator [52]	$Y_i \in \{0,1\}$ (μ_i in Fig. 6), where $P(Y_i=1)=\mu_i$, $P(Y_i=0)=1-\mu_i$.	**OCBAm+(I)**, OCBAm+	20, 100	$m=10$[c]
Flight schedule optim.	Select **Pareto set** of flight schedule that optimizes three performance measures in Table 7 simultaneously.	10	3	SIMAIR [53]	$Y_{i_o} \sim \mathcal{N}(\mu_{i_o}, \sigma_{i_o}^2)$ (μ_{i_o} and $\sigma_{i_o}^2$ in Table 7), where $o \in \{1,2,3\}$.	**MOCBA+(I)**, MOCBA	10, 20	-
Military network design	Select **best feasible design** of military network system that minimizes the delay while achieving more than 85% in packet delivery ratio.	65	2	Network -centric warfare simulator [54]	$Y_{i_o} \sim \mathcal{N}(\mu_{i_o}, \sigma_{i_o}^2)$ (μ_{i_o} in Fig. 7), where $o \in \{0,1\}$.	**OCBA-CO(I)**, OCBA-CO	10, 30	$\mu_{i_1} > 0.85$[d]

[a]The multiple integrated laser engagement system (MILES) is a military training gear that simulates engagement using a laser beam attached to an actual weapon and multiple sensors attached to a trainee's body.
[b]In every model, $i \in \{1, \ldots, k\}$.
[c]m is the size of the best subset.
[d]The stochastic constraint given in the secondary objective.

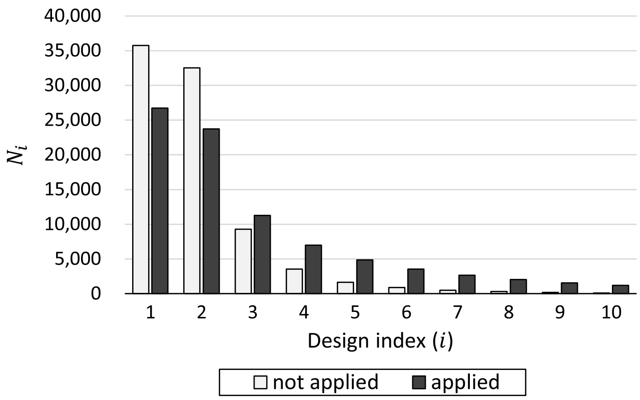

FIGURE 3. Distribution example of a budget of 84,600 to 10 designs for the SDV(H) model in Table 3 using the improved rules of (10) and (11). The application of the proposed adjustment provides a relatively widespread allocation compared to the existing rules of (7) and (8).

IV. EXPERIMENTS

This section presents the many experimental results for the benchmark and practical problems to demonstrate that the proposed adjustment can significantly improve the efficiency of the OCBA approach in the presence of large stochastic noise. We applied the adjustment to the eight OCBA-based R&S procedures in Table 1: OCBA, OCBAm, OCBAm+, OCBAmn, OCBAcr, MOCBA, MOCBA+, and OCBA-CO. That is, we just substituted $s.e.(\bar{\mu}_i)$ for σ_i in their existing rules to additionally consider the precision of $\bar{\mu}_i$. Each procedure improved by applying the proposed adjustment is specified by the letter '(I)' after its name. For example, the OCBA(I) procedure represents the improved OCBA procedure with the application of the proposed adjustment.

A. BENCHMARK PROBLEMS

We used various benchmark problems to evaluate the performance of these OCBA-based procedures in the literature [10], [11], [18], [24], [26], [29], [32]. In addition, for a fair comparison, the same settings of n_0 and Δ were applied as in the literature. Table 3 summarizes these benchmark problems. The models denoted by '(H)' in Table 3 have larger stochastic noise than their original versions, and thus, the experimental results for these models may well illustrate the effectiveness of the proposed adjustment. As mentioned previously, since the EVI approach has high robustness to noise, we compared the results of the OCBA approach with those of the EVI approach for several R&S procedures shown in Table 3 to evaluate the effectiveness of the proposed adjustment. Here, the SCORE procedure compared with the OCBA-CO procedure is not based on the EVI approach, but is the latest R&S procedure proposed to select the single best feasible design under stochastic constraints. In addition, the equal allocation procedure is added as a baseline in every experiment. The source code of the OCBA procedure was referenced in textbook [11], and those of the MOCBA and MOCBA+ procedures were provided by Prof. Lee at the National University of Singapore (the MOCBA procedure is a slightly improved version incorporating a large deviation perspective [31] into the original version in [29]). The rest were implemented based on the code of the OCBA procedure and the referred papers [10], [23], [24], [26], [32].

We estimated the $P\{CS\}$ of each procedure for the corresponding benchmark problems while varying T, and some results are shown in Fig. 4. Here, every $P\{CS\}$ was estimated over 10,000 independent repeated experiments. In addition, to numerically evaluate the improved efficiency of the proposed adjustment, Table 5 exhibits the value of T required for each procedure to correctly select its optimal solutions (i.e., to achieve $P\{CS\}$ of 0.99) for each problem. The experimental results shown in Fig. 4 and Table 5 clearly demonstrate that the proposed adjustment can enhance the noise robustness of the OCBA approach. As shown in Fig. 4, the improved OCBA-based procedures that applied

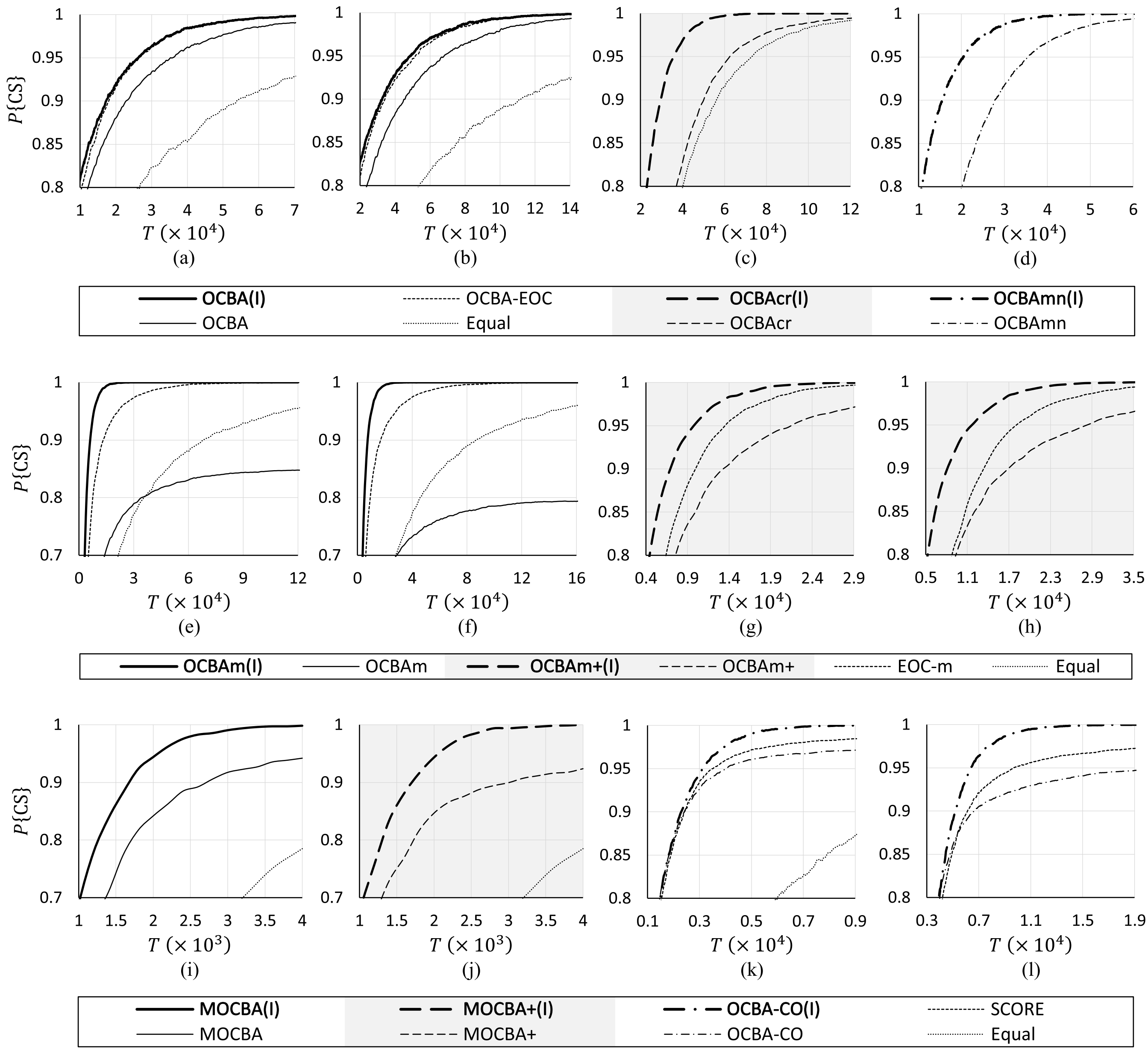

FIGURE 4. Graphs (a)-(l) illustrate the estimated value of $P\{CS\}$ versus the T of the eight OCBA-based procedures for some benchmark problems (i.e., the models denoted by '(H)') in Table 3: [*OCBA*] (a) SEV(H), (b) SDV(H); [*OCBAcr*] (c) SDV(H)'; [*OCBAmn*] (d) LEV(H); [*OCBAm*] (e) LEV(H), (f) LDV(H); [*OCBAm+*] (g) LEV(H), (h) LDV(H); [*MOCBA*] (i) Steep; [*MOCBA+*] (j) Steep; [*OCBA-CO*] (k) SMEV(H), and (l) SMDV(H). In each graph, the thick line (e.g., OCBA(I)) demonstrates the improved efficiency of the eight OCBA-based procedures by applying the proposed adjustment.

the proposed adjustment (i.e., denoted by '(I)', such as the OCBA(I) procedure) converged faster to the maximum value of $P\{CS\}$ than their original versions (see the thick line in each graph). Table 5 indicates that the improved procedures reduced the required T for selecting their optimal solutions compared to the original versions. In particular, the degree of efficiency improvement was significantly large in the presence of large stochastic noise, as shown in the results of the problems denoted by '(H)' in Table 3.

For example, as shown in Fig. 4(e) and Table 5(b), the original version of the OCBAm procedure could not converge to $P\{CS\}$ of 1 in LDV(H), even after allocating 10^6 replications, since it could not break the ongoing budget-wasted situation that occurs more frequently as the stochastic noise increases. Its estimated $P\{CS\}$ after allocating 10^6 replications was just 0.8043. However, the OCBAm(I) procedure, which is the improved version of the existing OCBAm procedure that applied the proposed adjustment, could correctly find the top 5 best designs using 17,000 replications, which is just 1.7% of 10^6. This is because the proposed adjustment enables the existing allocation rules of the OCBAm procedure to additionally consider the precision of $\bar{\mu}_i$, thereby breaking the wasted budget situation by quickly allocating further replications to the actual best designs that have an abnormal value of $\bar{\mu}_i$.

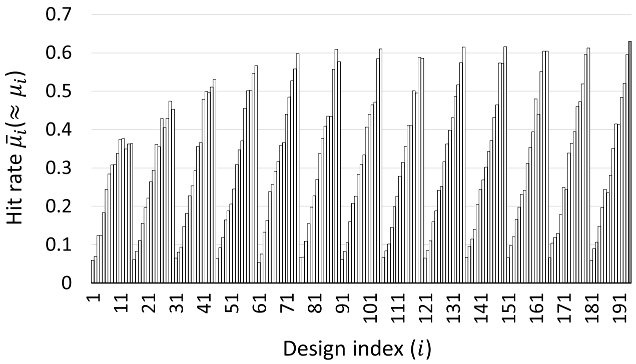

FIGURE 5. **The precisely estimated performance (i.e., the hit rate of MILES) for 195 designs in the MILES design problem. The rightmost dark gray bar represents the performance of the best design (i.e., x_{195}).**

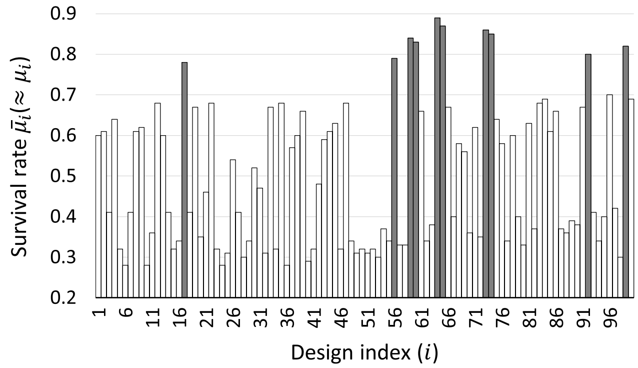

FIGURE 6. **The precisely estimated performance (i.e., the survival rate of a warship) for 100 designs in the decoy system design problem. The 10 dark gray bars represent the performance of the top 10 designs in the best subset (i.e., x_{17}, x_{56}, x_{59}, x_{60}, x_{64}, x_{65}, x_{73}, x_{74}, x_{92}, and x_{99}).**

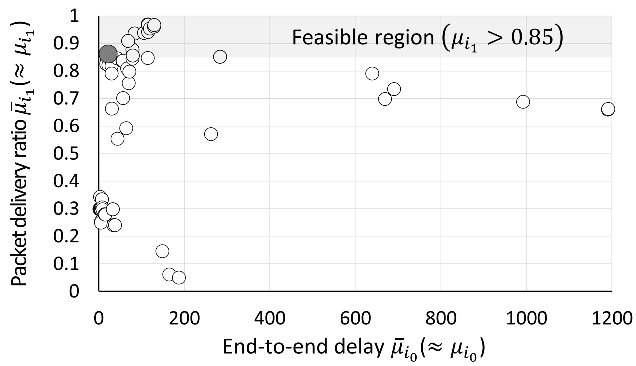

FIGURE 7. **The precisely estimated performance of two objectives (i.e., the end-to-end delay and the packet delivery ratio) for 65 designs in the military network design problem. Among the 15 feasible designs in the shaded area that satisfy the given constraint, the best feasible design (i.e., x_{19}) is represented as the dark gray circle.**

Furthermore, the improved efficiency of the OCBAm(I) procedure is far superior to the EOC-m procedure that is developed to find the best subset based on the EVI approach, as shown in Fig. 4(d)-(g) and Table 5(b) and (c). Since the objective of the EOC-m procedure is to minimize E[OC] rather than maximize $P\{\text{CS}\}$, direct comparisons between them may be unfair. Nevertheless, when considering the results of the original OCBAm and EOC-m procedures for the same problems, these comparisons are meaningful for confirming the effectiveness of the proposed adjustment. The OCBA-CO(I) procedure also exhibits higher efficiency than the SCORE procedure, which is the latest procedure for selecting the best feasible design based on large deviations theory [56]. This efficiency gap is especially pronounced in the '(H)' problems with large stochastic noise, as shown in Fig. 4(j) and (k) and Table 5(g). Meanwhile, the OCBA(I) procedure has similar results as the OCBA-EOC procedure, which is based on the EVI approach, as shown in Fig. 4(a) and (b) and Table 5(a). However, the OCBA-EOC procedure has the disadvantage of the EVI approach in which the computational costs of minimizing E[OC] can be significantly higher as k increases. In addition, since the proposed adjustment has little additional computational costs when applying it, in practice, the OCBA(I) procedure can be more efficient than the OCBA-EOC procedure.

B. PRACTICAL PROBLEMS

As mentioned earlier, as the complexity of modern industrial systems increases, the stochastic noise involved in the simulation model also tends to increase. Thus, we proposed this simple heuristic adjustment in order to efficiently apply the OCBA approach to the SBO of a practical model. To demonstrate the necessity of the proposed adjustment, we applied the OCBA(I), OCBAm+(I), MOCBA+(I), and OCBA-CO(I) procedures to four practical problems: 1) the multiple integrated laser engagement system (MILES) design, 2) the decoy system design, 3) the flight schedule optimization, and 4) the military network design. A brief description of each problem is as follows:

1) MILES design problem: MILES is a military training gear that simulates engagement using a laser beam attached to an actual weapon and multiple sensors attached to a trainee's body. To give trainees an immersive experience, the hit rate of MILES should be maximized, as with an actual weapon [47]. For the design of MILES, its hit rate can be estimated over many simulation replications of the optical engineering simulator [57]. Thus, the first problem is to select the best design of MILES that maximizes the hit rate from 195 design alternatives using the simulation, and the OCBA(I) procedure can be applied to solve this problem.

2) Decoy system design problem: A decoy system can increase the survival rate of a warship by neutralizing the sonar of a torpedo using a noise signal similar to the warship. In order to maximize the survival rate of a warship, which plays key roles in a modern battlefield, it is necessary to find the optimal design of the decoy system [45]. The survival rate for each design can be estimated over many simulation replications of the anti-torpedo combat simulator [58]. However, since the simulation does not consider all the factors, the optimal design should be determined by considering some conditions neglected by the simulation (e.g., qualitative criteria and political feasibility) and the simulation results. To this end, the second problem is to select the top 10 designs (i.e.,

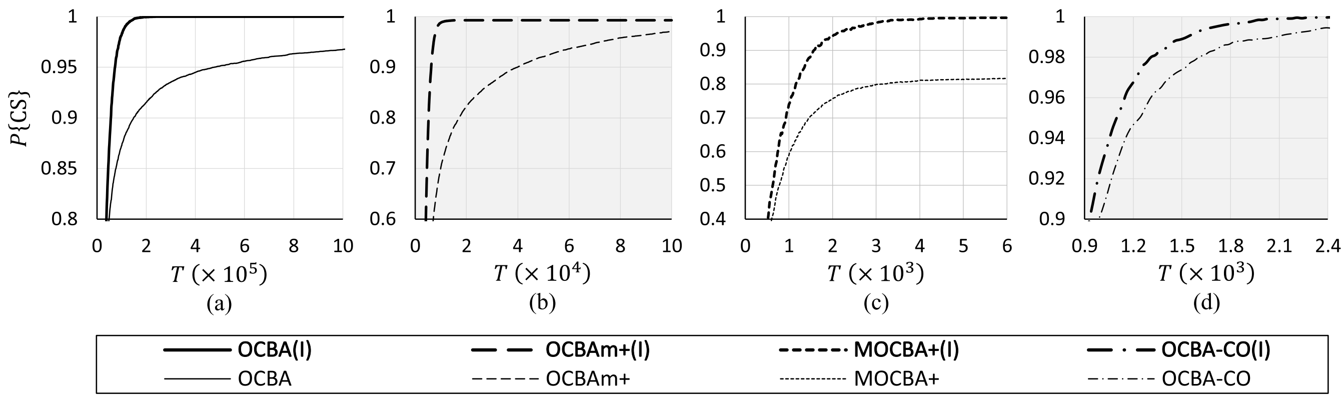

FIGURE 8. Graphs (a)-(d) illustrate the estimated value of $P\{CS\}$ versus the T of the four OCBA-based procedures for the practical problems: (a) the MILES design problem, (b) the decoy system design problem, (c) the flight schedule optimization problem, and (d) the military network design problem.

the best subset) that maximize the survival rate of a warship from 100 design alternatives using the simulation, and the OCBAm + (I) procedure can be applied to solve this problem.

3) Flight schedule optimization problem: Flight schedules are very important for airlines because they directly affect not only the airline's revenues but also the level of service they provide to their passengers. Thus, it is crucial to optimize the flight schedules of airlines [29]. There are various performance measures for evaluating flight schedules, but three representative ones are as follows: the operational crew costs, the percentage of late arrivals, and the number of passengers missing their connections. For the design of flight schedules, these three measures can be estimated over many simulation replications of SIMAIR [59]. Since there are multiple objectives to be considered in the optimization of flight schedules, the third problem is to select the Pareto set from 10 flight schedule design alternatives using the simulation, and the MOCBA + (I) procedure can be applied to solve this problem.

4) Military network design problem: A reliable military network system is essential for modern electronic warfare. For this reliable network, the required operational capability given by the military is that the end-to-end delay should be minimized while achieving a more than 85% packet delivery ratio [46]. For a design of the military network, the delay and the delivery ratio can be estimated over many simulation replications of the network-centric warfare simulator [60]. Thus, the last problem is to select the best feasible design of the military network that minimizes the delay while satisfying the stochastic constraint of the packet delivery ratio from 65 design alternatives. The OCBA-CO(I) procedure can be applied to solve this problem.

Table 6 summarizes these practical problems. Figs. 5-7 and Table 7 illustrate the precisely estimated performance of each design in the four problems obtained with many simulation replications. As shown in these estimation results, the four practical problems have large stochastic noise due to their relatively large variance compared to the small difference in the performance values between the actual optimal designs and many near optimal designs. In order to demonstrate the efficiency improvement of the proposed adjustment in the presence of such large noise, both the four improved procedures and their original versions were applied. The comparative results are shown in Fig. 8 and Table 8. Here, every $P\{CS\}$ was estimated over 1,000 independent repeated experiments.

TABLE 7. The precisely estimated performance and variance of the three objectives for 10 designs in the flight schedule optimization problem [29].

i	Operational crew cost		Late arrivals [%]		# of passenger misconnects	
	μ_{i_1}[a]	$\sigma^2_{i_1}$	μ_{i_2}	$\sigma^2_{i_2}$	μ_{i_3}	$\sigma^2_{i_3}$
1	242.38	1136.48	12.73	11.84	1887.83	582.671
2	241.38	1192.86	12.75	12.42	1876.38	604.822
3	234.43	849.57	12.12	10.92	1735.96	516.557
4	239.81	1172.65	12.96	12.29	1941.82	603.048
5	238.30	1138.36	13.70	12.46	2003.67	605.278
6	242.71	1260.38	13.21	12.94	1946.46	583.636
7	241.86	1274.58	13.57	13.04	1958.28	613.509
8	239.32	1241.66	13.29	12.56	1943.41	633.830
9	229.83	898.30	13.15	11.63	1853.51	534.903
10	241.94	1166.34	13.57	12.51	1970.56	589.607

[a] The two shaded designs are the Pareto designs that we have to select.

TABLE 8. Required budget to achieve $P\{CS\}$ of 0.99 for practical problems in several improved ocba-based procedures by applying the proposed adjustment.

Problem	Results			
MILES design	Proc.	**OCBA(I)**	OCBA	*Ratio*[a]
	Req. T	113,600	2,600,000	***22.89***
Decoy system design	Proc.	**OCBAm+(I)**	OCBAm+	*Ratio*
	Req. T	10,800	>10^5 (0.971)[b]	***>9.26***
Flight schedule optimization	Proc.	**MOCBA+(I)**	MOCBA+	*Ratio*
	Req. T	3,400	>>6,000 (0.817)	***>>1.76***
Military network design	Proc.	**OCBA-CO(I)**	OCBA-CO	*Ratio*
	Req. T	1,520	2,090	***1.38***

[a] This is the ratio of T required in the improved procedure denoted by '(I)' and that required in the original procedure (e.g., $T_{OCBA}/T_{\mathbf{OCBA(I)}}$).

[b] Even if the simulation budget is sufficient, $P\{CS\}$ reaches only the value in the parentheses (i.e., more budget than the value on the left of the parentheses is required to achieve $P\{CS\}$ of 0.99).

The results clearly demonstrate the necessity of the proposed adjustment. In the MILES design problem, the existing OCBA procedure required 2,600,000 replications to correctly select the best design (i.e., achieving $P\{\text{CS}\}$ of 0.99). On the other hand, the OCBA(I) procedure achieved this with only 113,600 replications, which is just 4.3% of the previous amount. In addition, the estimated $P\{\text{CS}\}$ of the existing MOCBA+ procedure was only 0.817 after allocating 6,000 replications in the flight schedule optimization problem, and even the convergence speed is close to zero, as shown in Fig. 8(c). However, the MOCBA + (I) procedure achieved $P\{\text{CS}\}$ of 0.99 within 3,400 replications. Meanwhile, the efficiency improvement in the military network design problem may be insignificant compared to the results of the previous three problems. However, the network-centric warfare simulator has high time-consumption per replication because the simulator interoperates the war game simulator and the ns-3 network simulator using the HLA/RTI [61]. Thus, in this context, even a relatively small improvement in the military network design problem can sufficiently emphasize the necessity of the proposed adjustment.

V. CONCLUSION

Although the OCBA approach has been widely used in various areas, it may be inefficient in the presence of large stochastic noise such as practical SBO problems. This paper intuitively analyzed the causes of the efficiency deterioration of the OCBA approach based on the basic OCBA procedure. Then, a simple heuristic adjustment was proposed to enhance the noise robustness of the OCBA approach based on the analysis. Since the OCBA approach considers only the value of the sample mean to allocate further replications, it is very likely to waste a lot of budget on the designs that are not truly optimal due to the abnormal value of the sample mean in a noisy environment. The proposed adjustment substitutes the variance with the standard error in the existing allocation rules of the OCBA approach, thereby enabling the rules to additionally consider the precision of the sample mean while maintaining the characteristics of the allocation. By applying this adjustment, the improved rules can quickly allocate further replications to the actual optimal designs of which the sample mean has an abnormal value; thus, the wasted budget can be minimized. In addition, the proposed adjustment is more efficient because there are little computational costs to apply it. Compared to the existing rules, the improved rules can no longer be asymptotically optimal solutions to the mathematical OCBA problems, such as (6). However, they become more suitable for the sequential update procedure, which is essential for solving the actual SBO problems in R&S procedures. This is clearly demonstrated in the improved experimental results of the eight OCBA-based R&S procedures for various benchmark problems. In particular, the results of the four practical problems emphasize the necessity of the proposed adjustment. The adjustment is expected to make the OCBA approach more efficient in the fourth industrial revolution, where the complexity of simulation models increases due to the digital twins, etc.

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REFERENCES

[1] L. Wang, R. Iida, and A. M. Wyglinski, "Vehicular network simulation environment via discrete event system modeling," *IEEE Access*, vol. 7, pp. 87246–87264, 2019.

[2] Y. Liu, K. Wang, L. Ge, L. Ye, and J. Cheng, "Adaptive evaluation of virtual machine placement and migration scheduling algorithms using stochastic Petri nets," *IEEE Access*, vol. 7, pp. 79810–79824, 2019.

[3] X. Lu, M. Zhou, A. C. Ammari, and J. Ji, "Hybrid Petri nets for modeling and analysis of microgrid systems," *IEEE/CAA J. Autom. Sinica*, vol. 3, no. 4, pp. 347–354, Oct. 2016.

[4] S. Yang, K. Song, and G. Zhu, "Stochastic process and simulation of traction load for high speed railways," *IEEE Access*, vol. 7, pp. 76049–76060, 2019.

[5] J. Zhou, J. Wang, and J. Wang, "A simulation engine for stochastic timed Petri nets and application to emergency healthcare systems," *IEEE/CAA J. Autom. Sinica*, vol. 6, no. 4, pp. 969–980, Jul. 2019.

[6] B. G. Kang, K.-M. Seo, and T. G. Kim, "Model-based design of defense cyber-physical systems to analyze mission effectiveness and network performance," *IEEE Access*, vol. 7, pp. 42063–42080, 2019.

[7] L. H. Lee, E. P. Chew, P. I. Frazier, Q.-S. Jia, and C.-H. Chen, "Advances in simulation optimization and its applications," *IIE Trans.*, vol. 45, no. 7, pp. 683–684, 2013.

[8] M. C. Fu, "Optimization for simulation: Theory vs. practice," *INFORMS J. Comput.*, vol. 14, no. 3, pp. 192–215, Aug. 2002.

[9] C.-H. Chen, E. Yücesan, L. Dai, and H.-C. Chen, "Efficient computation of optimal budget allocation for discrete event simulation experiment," *IIE Trans.*, vol. 42, no. 1, pp. 60–70, 2010.

[10] H. Xiao, L. H. Lee, and K. M. Ng, "Optimal computing budget allocation for complete ranking," *IEEE Trans. Autom. Sci. Eng.*, vol. 11, no. 2, pp. 516–524, Apr. 2014.

[11] C.-H. Chen and L. H. Lee, *Stochastic Simulation Optimization: An Optimal Computing Budget Allocation*, vol. 1. Singapore: World Scientific, 2011.

[12] S.-H. Kim and B. L. Nelson, "Recent advances in ranking and selection," in *Proc. Winter Simulation Conf.*, Washington, DC, USA, 2007, pp. 162–172.

[13] J. Xu, E. Huang, C.-H. Chen, and L. H. Lee, "Simulation optimization: A review and exploration in the new era of cloud computing and big data," *Asia–Pacific J. Oper. Res.*, vol. 32, no. 03, Jun. 2015, Art. no. 1550019.

[14] H. Xiao, L. H. Lee, and C.-H. Chen, "Optimal budget allocation rule for simulation optimization using quadratic regression in partitioned domains," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 45, no. 7, pp. 1047–1062, Jul. 2015.

[15] J. Branke, S. E. Chick, and C. Schmidt, "Selecting a selection procedure," *Manage. Sci.*, vol. 53, no. 12, pp. 1916–1932, Dec. 2007.

[16] M. Chau, M. C. Fu, H. Qu, and I. O. Ryzhov, "Simulation optimization: A tutorial overview and recent developments in gradient-based methods," in *Proc. Winter Simulation Conf.*, Savannah, GA, USA, 2014, pp. 21–35.

[17] S.-H. Kim and B. L. Nelson, "A fully sequential procedure for indifference-zone selection in simulation," *ACM Trans. Model. Comput. Simul.*, vol. 11, no. 3, pp. 251–273, Jul. 2001.

[18] C.-H. Chen, J. Lin, E. Yücesan, and S. E. Chick, "Simulation budget allocation for further enhancing the efficiency of ordinal optimization," *Discrete Event Dyn. Syst.*, vol. 10, no. 3, pp. 251–270, Jul. 2000.

[19] S. E. Chick and K. Inoue, "New procedures to select the best simulated system using common random numbers," *Manage. Sci.*, vol. 47, no. 8, pp. 1133–1149, Aug. 2001.

[20] S. E. Chick and K. Inoue, "New two-stage and sequential procedures for selecting the best simulated system," *Oper. Res.*, vol. 49, no. 5, pp. 732–743, Oct. 2001.

[21] S. Gao, W. Chen, and L. Shi, "A new budget allocation framework for the expected opportunity cost," *Oper. Res.*, vol. 65, no. 3, pp. 787–803, Jun. 2017.

[22] J. Zhang, L. Zhang, C. Wang, and M. Zhou, "Approximately optimal computing budget allocation for selection of the best and worst designs," *IEEE Trans. Autom. Control*, vol. 62, no. 7, pp. 3249–3261, Jul. 2017.

[23] C.-H. Chen, D. He, M. Fu, and L. H. Lee, "Efficient simulation budget allocation for selecting an optimal subset," *INFORMS J. Comput.*, vol. 20, no. 4, pp. 579–595, Nov. 2008.

[24] S. Zhang, L. H. Lee, E. P. Chew, J. Xu, and C.-H. Chen, "A simulation budget allocation procedure for enhancing the efficiency of optimal subset selection," *IEEE Trans. Autom. Control*, vol. 61, no. 1, pp. 62–75, Jan. 2016.

[25] H. Xiao and S. Gao, "Simulation budget allocation for selecting the top-m designs with input uncertainty," *IEEE Trans. Autom. Control*, vol. 63, no. 9, pp. 3127–3134, Sep. 2018.

[26] H. Xiao, S. Gao, and L. H. Lee, "Simulation budget allocation for simultaneously selecting the best and worst subsets," *Automatica*, vol. 84, pp. 117–127, Oct. 2017.

[27] H. Xiao, F. Gao, and L. H. Lee, "Optimal computing budget allocation for complete ranking with input uncertainty," *IISE Trans.*, pp. 1–11, Sep. 2019, doi: 10.1080/24725854.2019.1659524.

[28] J. Zhang, Z. Li, C. Wang, D. Zang, and M. Zhou, "Approximate simulation budget allocation for subset ranking," *IEEE Trans. Control Syst. Technol.*, vol. 25, no. 1, pp. 358–365, Jan. 2017.

[29] L. H. Lee, E. P. Chew, S. Teng, and D. Goldsman, "Finding the non-dominated Pareto set for multi-objective simulation models," *IIE Trans.*, vol. 42, no. 9, pp. 656–674, Jun. 2010.

[30] J. Li, "Optimal computing budget allocation for multi-objective simulation optimization," Ph.D. dissertation, Dept. Ind. Syst. Eng., Nat. Univ. Singapore, Singapore, 2012.

[31] J. Li, W. Liu, G. Pedrielli, L. H. Lee, and E. P. Chew, "Optimal computing budget allocation to select the nondominated systems—A large deviations perspective," *IEEE Trans. Autom. Control*, vol. 63, no. 9, pp. 2913–2927, Sep. 2018.

[32] L. H. Lee, N. A. Pujowidianto, L.-W. Li, C.-H. Chen, and C. M. Yap, "Approximate simulation budget allocation for selecting the best design in the presence of stochastic constraints," *IEEE Trans. Autom. Control*, vol. 57, no. 11, pp. 2940–2945, Nov. 2012.

[33] H. Xiao, H. Chen, and L. H. Lee, "An efficient simulation procedure for ranking the top simulated designs in the presence of stochastic constraints," *Automatica*, vol. 103, pp. 106–115, May 2019.

[34] J. Song, Y. Qiu, and Z. Liu, "Integrating optimal simulation budget allocation and genetic algorithm to find the approximate Pareto patient flow distribution," *IEEE Trans. Autom. Sci. Eng.*, vol. 13, no. 1, pp. 149–159, Jan. 2016.

[35] J. Zhang, C. Wang, D. Zang, and M. Zhou, "Incorporation of optimal computing budget allocation for ordinal optimization into learning automata," *IEEE Trans. Autom. Sci. Eng.*, vol. 13, no. 2, pp. 1008–1017, Apr. 2016.

[36] B.-W. Hsieh, C.-H. Chen, and S.-C. Chang, "Efficient simulation-based composition of scheduling policies by integrating ordinal optimization with design of experiment," *IEEE Trans. Autom. Sci. Eng.*, vol. 4, no. 4, pp. 553–568, Oct. 2007.

[37] S.-C. Horng, F.-Y. Yang, and S.-S. Lin, "Applying PSO and OCBA to minimize the overkills and re-probes in wafer probe testing," *IEEE Trans. Semicond. Manuf.*, vol. 25, no. 3, pp. 531–540, Aug. 2012.

[38] Q.-S. Jia, "Efficient computing budget allocation for simulation-based policy improvement," *IEEE Trans. Autom. Sci. Eng.*, vol. 9, no. 2, pp. 342–352, Apr. 2012.

[39] D. He, L. H. Lee, C.-H. Chen, M. C. Fu, and S. Wasserkrug, "Simulation optimization using the cross-entropy method with optimal computing budget allocation," *ACM Trans. Model. Comput. Simul.*, vol. 20, no. 1, pp. 1–22, Jan. 2010.

[40] S. Zhang, J. Xu, L. H. Lee, E. P. Chew, W. P. Wong, and C.-H. Chen, "Optimal computing budget allocation for particle swarm optimization in stochastic optimization," *IEEE Trans. Evol. Comput.*, vol. 21, no. 2, pp. 206–219, Apr. 2017.

[41] L. H. Lee, E. P. Chew, S. Teng, and Y. Chen, "Multi-objective simulation-based evolutionary algorithm for an aircraft spare parts allocation problem," *Eur. J. Oper. Res.*, vol. 189, no. 2, pp. 476–491, Sep. 2008.

[42] J. F. Shortle, C.-H. Chen, B. Crain, A. Brodsky, and D. Brod, "Optimal splitting for rare-event simulation," *IIE Trans.*, vol. 44, no. 5, pp. 352–367, May 2012.

[43] S. H. Choi and T. G. Kim, "Efficient ranking and selection for stochastic simulation model based on hypothesis test," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 48, no. 9, pp. 1555–1565, Sep. 2018.

[44] S. H. Choi and T. G. Kim, "Pareto set selection for multiobjective stochastic simulation model," *IEEE Trans. Syst., Man, Cybern., Syst.*, to be published, doi: 10.1109/tsmc.2018.2846680.

[45] S. H. Choi and T. G. Kim, "A heuristic approach for selecting best-subset including ranking within the subset," *IEEE Trans. Syst., Man, Cybern., Syst.*, to be published, doi: 10.1109/tsmc.2018.2870408.

[46] S. H. Choi, B. G. Kang, and T. G. Kim, "A heuristic procedure for selecting the best feasible design in the presence of stochastic constraints," *IEEE Trans. Syst., Man, Cybern., Syst.*, to be published, doi: 10.1109/tsmc.2019.2894523.

[47] S. H. Choi, C. Choi, and T. G. Kim, "An improved budget allocation procedure for selecting the best-simulated design in the presence of large stochastic noise," *IEEE Access*, vol. 7, pp. 154435–154446, 2019.

[48] J. Xu, Y. Ma, and Z. Xu, "A bilevel model for project scheduling in a fuzzy random environment," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 45, no. 10, pp. 1322–1335, Oct. 2015.

[49] A. M. Law, *Simulation Modeling and Analysis*, 4th ed. New York, NY, USA: McGraw-Hill, 2007.

[50] R. E. Bechhofer, T. J. Santner, and D. M. Goldsman, *Design and Analysis of Experiments for Statistical Selection, Screening, and Multiple Comparisons*. New York, NY, USA: Wiley, 1995.

[51] D. He, S. E. Chick, and C.-H. Chen, "Opportunity cost and OCBA selection procedures in ordinal optimization for a fixed number of alternative systems," *IEEE Trans. Syst., Man, Cybern. C, Appl. Rev.*, vol. 37, no. 5, pp. 951–961, Sep. 2007.

[52] S. Gao and W. Chen, "Efficient subset selection for the expected opportunity cost," *Automatica*, vol. 59, pp. 19–26, Sep. 2015.

[53] R. Pasupathy, S. R. Hunter, N. A. Pujowidianto, L. H. Lee, and C.-H. Chen, "Stochastically constrained ranking and selection via SCORE," *ACM Trans. Model. Comput. Simul.*, vol. 25, no. 1, pp. 1–26, Aug. 2014.

[54] Q. C. Zhao, Y. C. Ho, and Q. S. Jia, "Vector ordinal optimization," *J. Optim. Theory Appl.*, vol. 125, no. 2, pp. 259–274, 2005.

[55] L. H. Lee, E. P. Chew, and S. Teng, "Computing budget allocation rules for multi-objective simulation models based on different measures of selection quality," *Automatica*, vol. 46, no. 12, pp. 1935–1950, Dec. 2010.

[56] P. Glynn and S. Juneja, "A large deviations perspective on ordinal optimization," in *Proc. Winter Simulation Conf.*, Washington, DC, USA, 2004, pp. 577–585.

[57] S. H. Choi, C.-B. Choi, and T. G. Kim, "Optical engineering simulator development for efficient design of MILES gear," in *Proc. 12th IM Multi-Conf.*, Bergeggi, Italy, Sep. 2015, pp. 56–64.

[58] K.-M. Seo, H. Sang Song, S. Jung Kwon, and T. Gon Kim, "Measurement of effectiveness for an anti-torpedo combat system using a discrete event systems specification-based underwater warfare simulator," *J. Defense Model. Simul.*, vol. 8, no. 3, pp. 157–171, Jul. 2011.

[59] L. H. Lee, H. C. Huang, C. Lee, E. P. Chew, W. Jaruphongsa, Y. Y. Yong, Z. Liang, C. H. Leong, Y. P. Tan, K. Namburi, E. L. Johnson, and J. Banks, "Discrete event simulation model for airline operations: SIMAIR," in *Proc. Winter Simulation Conf.*, Washington, DC, USA, 2003, pp. 1656–1662.

[60] B. G. Kang, K.-M. Seo, and T. G. Kim, "Machine learning-based discrete event dynamic surrogate model of communication systems for simulating the command, control, and communication system of systems," *Simulation*, vol. 95, no. 8, pp. 673–691, 2019.

[61] *IEEE Standard for Modeling and Simulation (M&S) High Level Architecture (HLA)-Framework and Rules*, IEEE Standard 1516-2000, 2000.

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