IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received December 17, 2019, accepted January 27, 2020, date of publication January 29, 2020, date of current version February 7, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.2970385

Combining Facility-Location Approaches for Public Schools Expansion

CLAHILDEK MATOS XAVIER [1], MARLY GUIMARÃES FERNANDES COSTA [2], AND CÍCERO FERREIRA FERNANDES COSTA FILHO [2]

[1]Operations and Management Center of the Amazonian Protection System (CENSIPAM), Manaus 69041-010, Brazil
[2]Department of Electronic and Computation, Federal University of Amazonas, Manaus 69080-900, Brazil

Corresponding author: Clahildek Matos Xavier (clahildek@gmail.com)

This work was supported in part by the Amazonas State Research Support Foundation under Grant FAPEAM PAPAC 015/2014, in part by the Samsung Electronica da Amazonia Ltda., through the terms of the Brazilian Federal Law and an Agreement Signed with CETELI/UFAM, under Grant 8.387/91, and in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES), Brasil, under Grant 001.

ABSTRACT This study proposes determining optimal locations for expanding a higher education system by using populational and social criteria. With this aim, this work evaluates single objective location models in determining the optimal distribution of higher education facilities in Amazonas State, Brazil. Three optimization options are evaluated and made available to decision makers: 1) prioritize cities with a lower United Nation Human Development Index; 2) prioritize cities with a higher population, 3) favor both criteria. Also, the location must equalize student distribution between the regions of Amazonas State. With this aim, three discrete location models were evaluated: p-center model, p-median model, and p-dispersion model. The location models were designed using a Genetic Algorithm metaheuristic. A state-of-art implementation of the Genetic Algorithm that optimizes the solution and converge time was used. The expansions proposed here present lower mean values of United Nation Human Development Index compared with 0.619 from the existing distribution of campuses. The best results were obtained with the p-median model.

INDEX TERMS p-median modeling, locating higher education schools, single objective location, genetic algorithm.

I. INTRODUCTION

According to a report by [1], the percentage of the population in Brazil with access to college education is only 16.3% in the age group between 25-34 years old, and 11.2% in the group between 55-64 years old, so there is much room for growth of higher education in Brazil. In Brazil, earning an undergraduate degree may ensure an income 80% greater than that of an average non-graduate worker [2].

In the northern region of Brazil, in Amazonas state, the largest area of the federation, 1,559,146.876 km^2, this problem is aggravated. As shown in Figure 1, the state of Amazonas comprises 62 cities distributed in four mesoregions: Central, North, South and South-West, see Figure 2. The distances between locations can reach 1,500 km. Access to such cities is typically managed by inland waterway navigation (blue-colored lines in Figure 1). The fluvial network (blue-colored lines in Figure 1) of Amazonas State is the most important one. In the river sides are located the majority of Amazonas State cities. The railroad network does not exist and the highway network is very limited and poorly maintained. Half of the students eligible (Secondary School Students (SSS), over 105,000) to join a Higher Education Institution (HEI) are in the capital of Amazonas state, Manaus, and the other half are distributed over the interior (another 61 cities of the state of Amazonas) [3]. Transportation of students from one city to the nearest city with an HEI is time-consuming and usually done by river, which is not always the most modernized form of transportation.

Amazonas State has two public HEI systems and eighteen private HEIs. One of the public systems is linked to the federal government and the other linked to the state government. As shown in Figure 2, the HEIs are located in only ten cities of the four mesoregions. In the Central, North, South-West and South mesoregions, there are 5, 1, 2 and 2 cities with an HEI, respectively.

For each mesoregion, Table 1 shows the following fields: the number of cities; the number of SSS; the SSS ratio,

The associate editor coordinating the review of this manuscript and approving it for publication was Laxmisha Rai.

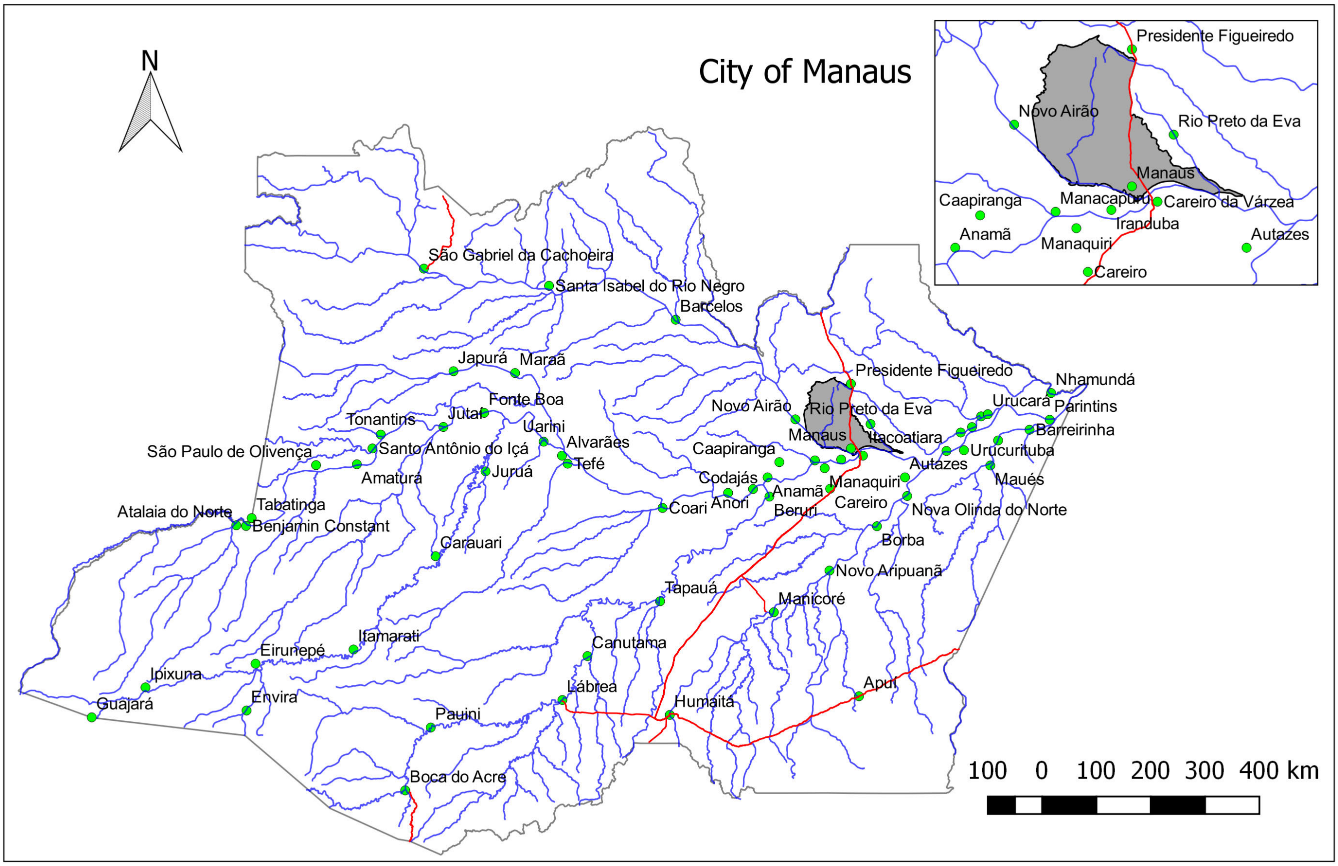

FIGURE 1. Amazonas state with 62 cities, rivers (blue) and roads (red). Shapefiles – files with geospatial data – obtained through [4], [5] and [6].

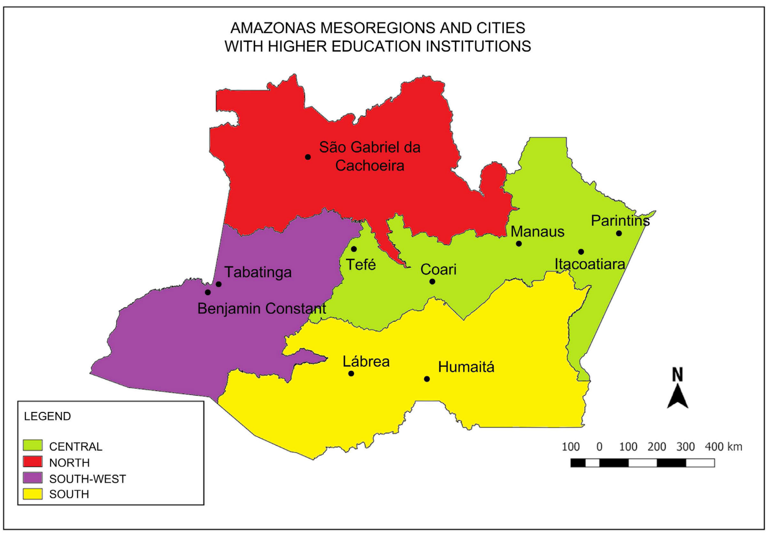

FIGURE 2. Amazonas state map showing mesoregions of the state and cities having at least one higher education unit.

defined as the number of SSS in each mesoregion divided by the total number of SSS in Amazonas state; the number of Higher Education Students, HES; the HES ratio, defined as the number of HES in each mesoregion divided by the total number of HES of the Amazonas state; the number of HEIs; the HEI ratio, defined as the number of HEIs in each mesoregion divided by the total number of HEIs in Amazonas state.

The private HEIs are located only in the Capital. In Table 2, we show a comparison between the mean value of United Nation Human Development Index - UN-HDI and mean number of SSS of the 10 cities with HEIs and of all 62 cities.

TABLE 1. Comparing populational and educational data of Amazonas State mesoregions, Brazil, [8].

Mesoregion	Cities	SSS	SSS ratio*	HES	HES ratio**	Number of HEIs	HEI Ratio***
Central	30	165,525	0.783	133,645	0.949	20	0.800
North	6	6,109	0.0289	623	0.004	1	0.040
South-West	16	25,651	0.121	4,044	0.029	2	0.080
South	10	13,993	0.0662	2,596	0.018	2	0.080

*SSS/Total of SSS(211,278) **HES/Total of HES(140,908) *** HEIs/Total of HEIs(25)

TABLE 2. Mean number of SSS and mean UN-HDI of cities with HEIs and all cities of Amazonas state, [9] and [10].

Cities	mean number of SSS	mean UN-HDI
10 cities currently with HEIs	14,170.40	0.619
All cities of Amazonas State	3,407.70	0.565

The UN-HDI Index takes into account life expectancy at birth, mean years of schooling and expected years of schooling and gross national income per capita [7].

From Tables 1 and 2, the following conclusions can be drawn:

1) In Amazonas State, Brazil, HES are concentrated in the Central mesoregion, the most populous one and where Manaus, the State capital, is located. However, the HES ratio of that mesoregion compared to the same ratio of other mesoregions is greater than the SSS ratio of that mesoregion compared to same ratio of other mesoregions, suggesting that more SSS of this mesoregion have access to higher education;
2) The mean UN-HDI of the cities with an HEI is higher than the mean UN-HDI of all cities, suggesting that the current HEI distribution favors cities with a higher UN-HDI.

To equalize the HES ratio and the SSS ratio of all mesoregions, there are two options: increase the number of students in the HEIs located in the mesoregions with a lower student ratio or increase the number of cities with HEIs in these mesoregions. The first option is an administrative decision. The second option can be achieved by expanding the number of HEIs, seeking a better distribution of them. This can be accomplished using discrete location models. We intend to evaluate solutions obtained with the following models:

1) PMM (P-Median Model): One that minimizes the average distance between a demand node (city with no HEI) and a nearest facility node (city with HEI);
2) PCM (P-Center Model): One that minimizes the maximal distance between demand nodes and a nearest facility node;
3) PDM (P-Dispersion Model): One that maximizes the minimum distance between pairs of facilities;

The PCM model [11] is a type of covering-based model. This type of model that minimizes the maximal distance between demand nodes and a facility, assumes that there is some critical coverage distance or time within which demands need to be met. Such models are typically used in designing emergency services, such as fire services or emergency health services. The PMM model [12] is a type of median-based model that minimizes the average distance between demand nodes and assigned facilities. Such models are typically used in distribution planning contexts in which minimizing the total transport cost is essential. The PDM model [11] maximizes the minimum distance between pairs of facilities.

These three models are the discrete location models most used in the literature [13]. As stated in the last paragraph, each one optimizes a given criterium. The choice of these models aims to evaluate which one achieves better performance in terms of the parameters evaluated in this work: HEI ratio and SSS ratio.

In order to provide the decision maker with multiple options, we intend also to model the expansion problem in such a way as to provide the following strategies:

a) Strategy A: An expansion that favors cities with lower UN-HDI;
b) Strategy B: An expansion that favors most populous cities;
c) Strategy C: An expansion that equally favors both criteria (UN-HDI and most populous cities).

Facility location problems (FLP) are usually employed for solving public, commercial, industrial and military problems. In these problems, service demand points must be attended by a limited number of facilities. The computational complexity theory classifies the PMM as a non-polynomial hard problem - NP-hard problem [14]. Meta-heuristic methods such as Greedy Interchange (GI), Neighborhood (N) and Exchange [15], Semi-Lagrangean relaxation [16], Simulated Annealing (SA) [17], Tabu Search (TS) [18], and Genetic Algorithm (GA) ([19], [20]) are usually used for solving NP-hard problems for which an optimal solution method is not known or does not exist. In the comparisons made in [18] and [21], the GA heuristic stands out as the best one in terms of time and precision of solution.

The GA implementation, proposed by [22], when used for solving PMM problems of the Operational Research (OR) library [23], outperformed the implementations proposed in [19] and [20], concerning time and precision. In this paper, we adapt this GA implementation for solving the three types of location problems previously mentioned.

II. LITERATURE REVIEW

Several authors used the PMM formulation in optimizing single objective school location problems ([24]–[26]).

In [24], a variant of the PMM was employed in order to determine p schools in Coimbra, Portugal. The optimization criterion maximizes the accessibility of students to schools, with constraints on maximum and minimum capacity occupation. The optimization problem considered 11 existing secondary schools, 43 population centers, and the total student-to-school distance of 21,097 km. An XPRESS-MP optimizer was used and an optimized value of 10,861 km was achieved for the sum of distances, for 14 schools.

A single objective optimization problem was solved in [25]. The optimization criterion was the average distance traveled by pupils from home to school. The application was on a network of secondary suburban schools of Dakar, Senegal, with 15 centers. The authors considered 8 candidate sites for establishing schools, including 5 sites with existing schools, so that the average student-school distance was limited to 2,000 km. The authors used an IBM-CPLEX solver to perform the experiments. They achieved the best results for p = 5. Four of five candidate sites already had schools and only one candidate site did not. The average student-to-school travel distance was 1,620 km.

In [26], the authors proposed location of new municipal schools in the Guaratiba area - assessment region 5, Rio de Janeiro, Brazil, relying on capacitated p-median and maximum coverage models. In the first model, they sought to minimize total student travel distance to the nearest school. Each school had a 1,300-student maximum capacity and the total number of new schools was 15. For the second model, they sought to maximize population coverage within a pre-set 1,500m maximum distance. Both approaches were carried out using an Advanced Interactive Multidimensional Modeling System tool based on a CPLEX solver.

Other studies conducted by [27] and [28] solve multi objective school location problems. A model for determining the location of schools aiming at minimizing fixed and overhead school costs such as per-hour rates was proposed in [26]. Such work took into consideration the cost for opening and closing existing school facilities. The problem was optimized using CPLEX 9.1. They managed to reduce over 30% of total costs for a 5-year period and they saved more than $6,000,000 per year.

In [28], the authors proposed a multi-objective model for locating secondary schools in rural areas in Chile, considering 45 population centers and 34 existing schools. The objective function minimizes operating and investment costs in the education system, minimizes the average travel time of student-to-school, maximizes the average number of students enrolled per school and minimizes the number of multilevel schools. These authors obtained a decrease of 4.5% in cost. GAMS/MINOS (Modular In-core Nonlinear Optimization System) and Tabu Search were used to solve the model. These authors set aside the p-median approach and adopted the procedure of optimizing a weighted function.

A model for the school location-allocation problem to minimize the aggregated travel impedance (time or distance) student-to-school was formulated by [29], for maximizing the number of students sent to their closest school and for minimizing the number of students sent to schools so far away from their home. These authors applied their model in the Charlotte-Mecklenburg School System, in North Carolina - United States), and its surrounding county, considering that 20 schools could have been open out of a feasible set of 25 sites with 37,851 students between 14 to17 years old. The model was solved by CPLEX and Interactive Graphical Location-Allocation System for Schools (iGLASS), an approach proposed by these authors that integrates Tabu Search, Greedy, and Genetic Algorithms. For iGLASS and CPLEX, respectively, these authors obtained a total impedance of 144.8 km and 138.3 km; the percentage of students assigned to their closest school was 80.61% and 75.45%, and the percentage of students assigned to a school that is twice as far as the closest school of 5.13% and 2.68%).

The following works did comparative studies using two or more methods to solve location problems. The authors in [30] compared the classic PMM, maximal covering location problem and PCM models applied in an emergency service location problem. These models were evaluated with respect to multiple performance criteria, varying p = {10, 15, 20}, and using n = 200. The authors solve the optimization models in the General Algebraic Modeling System (GAMS©), using CPLEX 12.2.0.2. The authors achieved the best results for the PMM. This model outperformed other models in four out of seven criteria.

The PMM, maximum coverage and PCM location models were applied by [31] for locating emergency facilities in a mine. The mathematical model was solved using GAMS© and 7 locations were selected from among 46 candidates.

None of the papers found in the literature deal with location of higher education schools, nor do they solve problems encompassing schools located in several cities, taking into account social parameters of these cities, such as the UN-HDI.

Therefore, concerning HEI locations, the present study deals with a different problem compared with papers previously published in the literature.

III. DATA AND PARAMETER NORMALIZATION

A. DATA

The location models proposed in this study use the following variables: distance between cities, SSS population and UN-HDIs. SSS Populations and UN-HDIs were acquired from [32] and [3]. The distances considered between cities were those corresponding to fluvial distances, which refer to the main means of transportation used in Amazon region. Some fluvial distances were established through data from [33]. Other distances missing from this database were acquired by means of one of the following methods: 1) subtraction between known routes along the same river; 2) using Google Maps API to measure the route between two cities located on different rivers or tributaries.

B. PARAMETER NORMALIZATION

As will be shown in section 4, the location models proposed in this study combines in the same expression the variables cited in section 3.1. As these variables have values with different dynamic ranges, they must be normalized. In a previous study about the same topic [34], we evaluated normalization in three different ways: a) using maximum and minimum values, Eq. (1) [35]; b) using median values, Eq. (2); and c) using standard deviation values, Eq. (3).

It was shown that, with respect to normalization, the median leads to lower values for fluvial distance between cities with and without an HEI and to a higher number of SSS for cities with an HEI. Both standard deviation and maximum-minimum normalization lead to lower UN-HDI values for cities with an HEI. In this paper, we will use the median normalization.

$$x_i = \frac{x_i^* - min(x_i^*)}{max(x_i^*) - min(x_i^*)} \tag{1}$$

$$x_i = \frac{x_i^*}{X_i} \tag{2}$$

$$x_i = \frac{x_i^*}{s_i} \tag{3}$$

where

- x_i^* stands for non-normalized parameters: distance, population or UN-HDI for cities;
- x_i is the normalized parameter value;
- X_i is the median [36] of a parameter x_i^*;
- s_i is the standard deviation [37] of a parameter x_i^*.

IV. LOCATION MODELING

In the introduction, it was mentioned that we will offer nine expansion options for decision makers. These options are the following: a) PMM, PCM and PDM that favor cities with lower UN-HDI, by setting $\beta = 0$ and $\gamma = 1$ in equations (4), (5) and (6), respectively; b) PMM, PCM and PDM that equally favor most populous cities, by setting $\beta = 1$ and $\gamma = 0$ in equations (4), (5) and (6), respectively; c) PMM, PCM and PDM that equally favor both criteria (UN-HDI and most populous cities), by setting $\beta = 0.5$ and $\gamma = 0.5$ in equations (4), (5) and (6), respectively. The equations (4), (5), (6) are submitted to restrictions (7), (8), (9) and (10). In equations (4), (5) and (6) the number of SSS of the cities with a facility is maximized (minus signal) and the UN-HDI is minimized (positive signal).

$$Minimize\, f_1 = \alpha \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} x_{ij}}{n-p} - \beta \frac{\sum_{j=1}^{n} p_j z_j}{p} + \gamma \frac{\sum_{j=1}^{n} HDI_j z_j}{p} \tag{4}$$

$$Minimize\, f_2 = \alpha \max(d_{ij}^{w}) - \beta \frac{\sum_{j=1}^{n} p_j z_j}{p} + \gamma \frac{\sum_{j=1}^{n} HDI_j z_j}{p} \tag{5}$$

$$Minimize\, f_3 = \alpha \frac{1}{\max(d_{ij}^{f})} - \beta \frac{\sum_{j=1}^{n} p_j z_j}{p} + \gamma \frac{\sum_{j=1}^{n} HDI_j z_j}{p} \tag{6}$$

$$Subjectto: \sum_{i=1}^{n} x_{ii} = p \tag{7}$$

$$\sum_{j=1}^{n} x_{ij} = 1, \quad \forall i \in \{1, \ldots, n\} \tag{8}$$

$$\sum_{i=1}^{n} x_{ij} \leq n x_{jj}, \quad \forall j \in \{1, \ldots, n\} \tag{9}$$

$$x_{ii} \text{ and } x_{ij} \in \{0, 1\}, \quad \forall i \in I, j \in J \tag{10}$$

$$x_{ij} = \begin{cases} 1, & \text{if } \exists \text{ a school in city } j \\ & \text{and if city } i \text{ is nearest} \\ & \text{to } j \text{ than any other city} \\ & \text{with an allocated school;} \\ 0, & \text{otherwise} \end{cases} \tag{11}$$

where

- d_{ij} is the normalized river navigation distance between cities i and j;
- p_j is the normalized SSS population for city j;
- z_j is 1 for city j with an HEI and 0, otherwise;
- HDI_j is the normalized UN-HDI for city j;
- α, β and γ are weights for distance, population and UN-HDI, respectively;
- d_{ij}^{w} is the normalized river navigation distance between cities i without an HEI and j with an HEI;
- d_{ij}^{f} is the normalized river navigation distance between cities i and j with an HEI;
- I is the set of demand locations;
- J is the set of candidate locations;

In this work, we have three single objective problems that are optimized separately by the genetic algorithm. The first single objective problem optimizes a PMM model, Eq. (4). The second single objective problem optimizes a PCM model, Eq. (5). The third single objective optimizes a PDM model, Eq. (6). These models are optimized separately by the genetic algorithm. In each one of these equations we have three parameters, one related to distance, other related to population and the last one related to UN-HDI.

The p-median modeling just presented is a classical one. The objective function of Eq. (4) minimizes average distance between cities and their closest facility, including the number of SSS and UN-HDI criteria. The constraint of Eq. (7) requires that only p facilities may be established. Constraint of Eq. (8) requires that each city be assigned a single HEI. Constraint of Eq. (9) requires cities with no allocated HEIs be associated only with cities with allocated HEIs. Eq. (10) indicates the binary nature of the decision variables.

In p-center modeling, the objective function of Eq. (5) minimizes the maximum distance between a city with no HEI and its closest city with an HEI. Constraints (7), (8), (9) and (10) are the same as for p-median modeling.

In p-dispersion modeling, the objective function of Eq. (6) maximizes the distance between the two closest cities with allocated HEIs. Constraints (7), (8), (9) and (10) are the same as for p-median modeling.

V. METRICS

In this study, we evaluated the results of location solutions using seven types of metrics, divided into three groups.

1) Group 1: to evaluate if the solutions obtained by combining location models and strategies cited in section 1 are providing a better distribution of HEIs we will use the following metrics:
   a) Metric 1: mean fluvial distance between a city without an HEI and a city with an HEI;
   b) Metric 2: maximal fluvial distance between a city without an HEI and a city with an HEI;
   c) Metric 3: maximal fluvial distances between cities with an HEI;
   d) Metric 4: HEI ratio;
2) Group 2: to evaluate if the solutions obtained by combining location models and strategies cited in section 1 are providing better equalization of the SSS of all mesoregions, we will use the following metric:
   a) Metric 5: SSS ratio;
3) Group 3: to evaluate if the solutions obtained by combining location models and some strategies cited in section 1 could favor cities with lower UN-HDI or with a higher number of SSS, we will use the following metrics
   a) Metric 6: mean SSS, computed by the summation of the SSS of cities with a HEI divided by the number of these cities;
   b) Metric 7: mean UN-HDI, computed by the summation of the UN-HDI of cities with HEI divided by the number of these cities.

VI. GENETIC ALGORITHM MODELING

A genetic Algorithm is a stochastic optimization algorithm, inspired by the theory of evolution of Charles Darwin [38]. Since its initial proposition, it has been effectively applied in the solution of complex problems, such as the Traveling Salesman Problem (TSP) [39] and PMM ([19]–[21]).

The genetic algorithm starts with a population P(0) of N chromosomes, randomly generated. The fitness value of each chromosome is evaluated through an objective function of the problem. The implementation of GA usually consists of three steps: building the chromossome model, defining the objective function and parameterization of genetic operators. In this work, the fitness function is one of the equations (4), (5) or (6), used in problems modeled with PMM, PCM and PDM, respectively. The chromosome model uses the facility indices: {1, 2,.., 62}.

In this study, we used an adapted version of the GA algorithm proposed in [22]. It is presented below in the eight steps of Algorithm 1.

Algorithm 1 Adapted Genetic Algorithm [21]

Steps
1 Randomly generate the initial population.
2 Compute fitness of population.
Repeat for x generations.
3 Roulette wheel selection of 2 parents
4 One-point crossover, at a 95% probability
5 One-gene random mutation, at a 5% probability
6 Compute fitness
7 Replace the parents with better fitness in the children.
Until population has converged, go to Step 3
8 Stop.

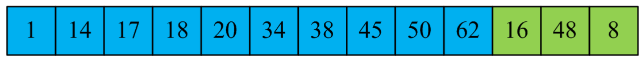

FIGURE 3. Example of an encoded chromosome with 13 facilities. The blue indicates the fixed genes. The other genes (green) are those that can be replaced.

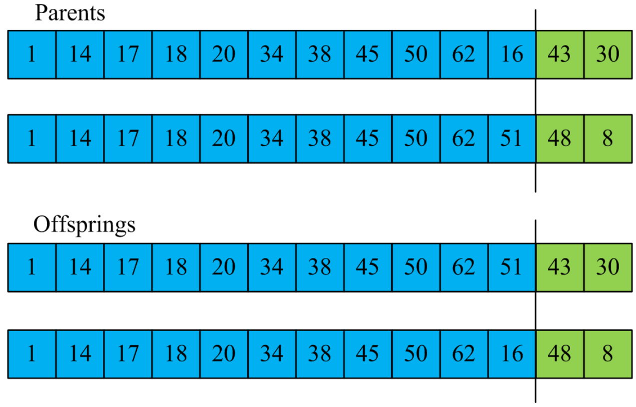

FIGURE 4. Example of one-point crossover operation. Blue-colored boxes indicate the fixed genes. The other genes (green) are those that can be replaced.

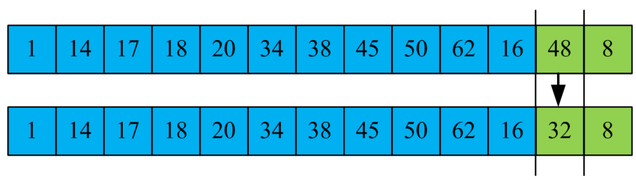

FIGURE 5. Example of one-gene mutation operation. Blue-colored boxes indicate the fixed genes. The other genes (green) are those that can be replaced.

As stated before, the genetic codification uses the facility indices. Figure 3 shows an encoded chromosome used in a PMM problem with 13 facilities. The first ten genes, in blue color, represent the existing cities with a HEI. The other genes, indicated with green color, are those that can be replaced.

The selection operator used is the roulette wheel operator [40]. In this study, the selection operator assigns a probability value to each individual, proportional to its fitness

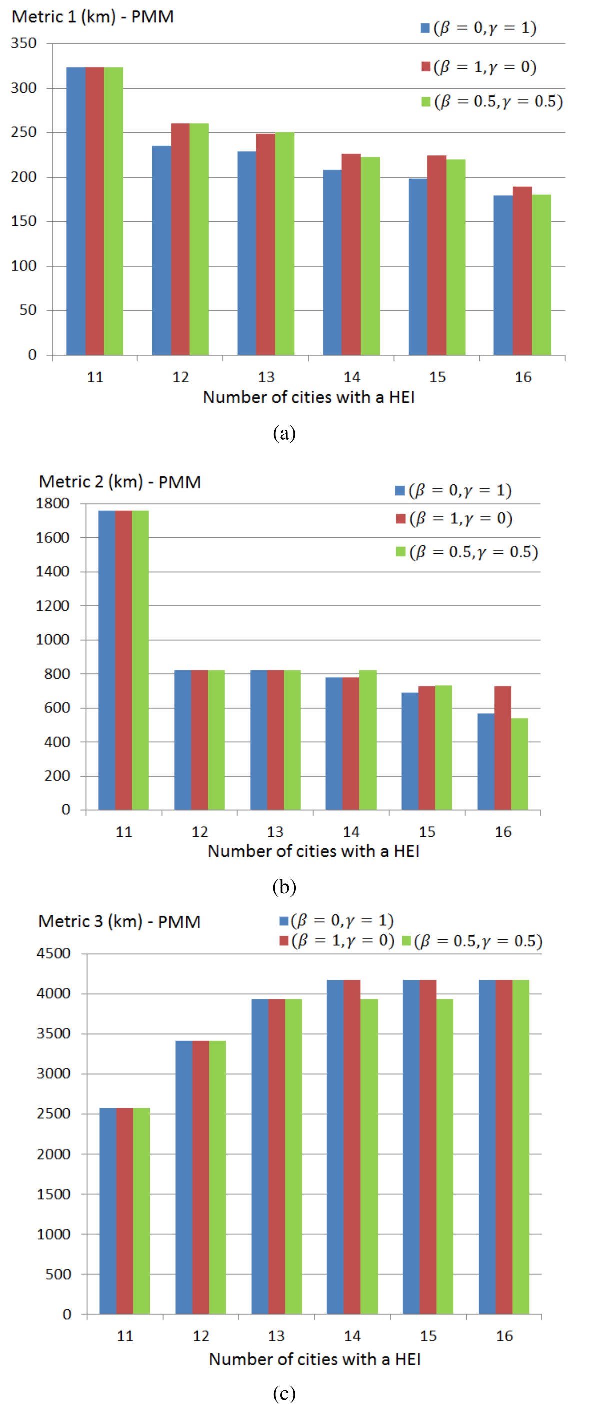

FIGURE 6. Results obtained by using the PMM model for expanding from 10 to 11-16 cities with HEIs: (a) metric 1, (b) metric 2 (c) metric 3. All distances are in kilometers.

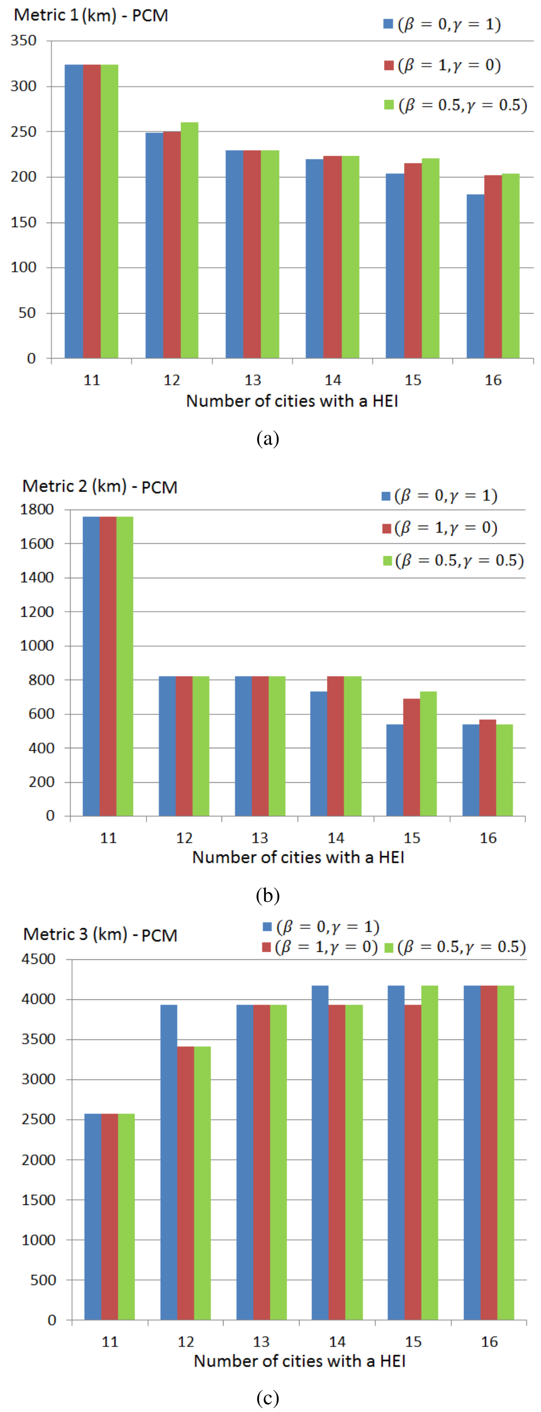

FIGURE 7. Results obtained by using a PCM model for expanding from 10 to 11-16 cities with HEIs: (a) metric 1, (b) metric 2 (c) metric 3. All distances are in kilometers.

value. This probability is given in Eq. (12).

$$p(c_i) = \frac{rank(c_i)}{\sum_{i=1}^{n} rank(c_i)} \tag{12}$$

where

- c_i is i-th chromosome in the population;
- $rank(c_i)$ is the position of the c_i ordered by decreasing values of its fitness function;
- $p(c_i)$ is the selection probability of the chromosome c_i;

As we have a minimization problem, the fitness function values were ordered by decreasing values. The fitness function value was not used for the probability calculation, but rather by rank, because high values of the fitness function dominate the population, generating a premature convergence of GA [41].

A one-point crossover operator is used in this study [40]. This operator randomly generates a reference point to

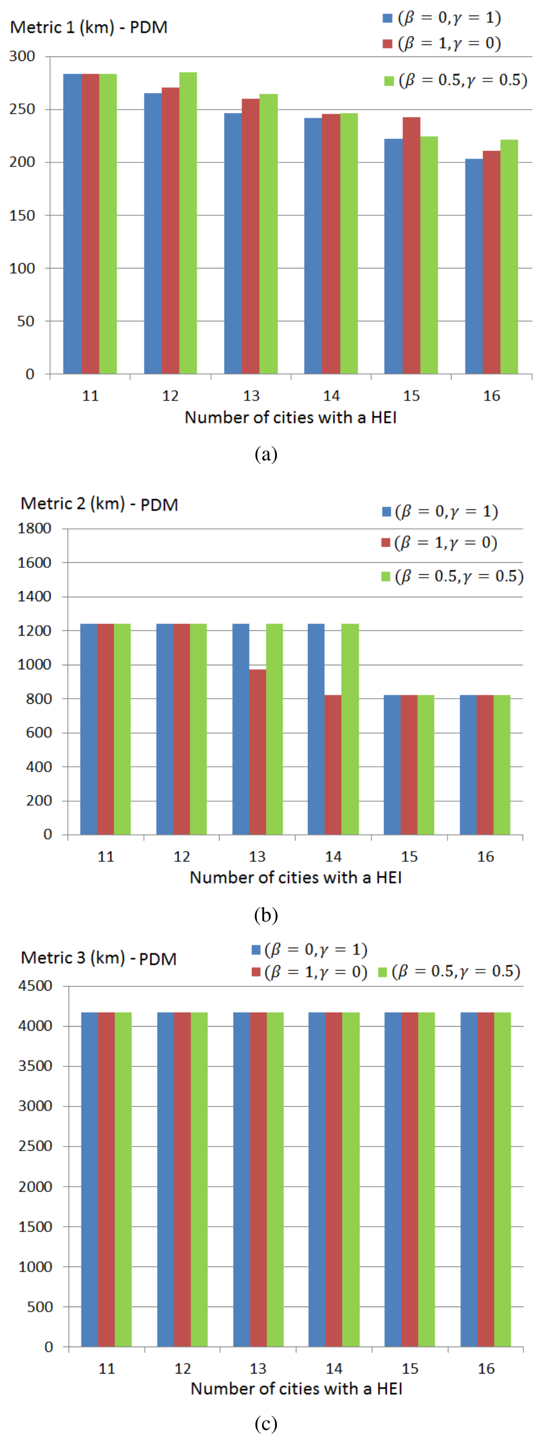

FIGURE 8. Results obtained by using the PDM model for expanding from 10 to 11-16 cities with HEIs: (a) metric 1, (b) metric 2 (c) metric 3. All distances are in kilometers.

permutate genes between parents. The crossover probability used is 95%. For the chromosome shown in Figure 3, with 13 genes, the crossover point is chosen randomly in a point between genes 11 and 12, or between genes 12 and 13. These genes correspond to cities without an HEI. Figure 4 illustrates the genetic permutation performed by the one-point crossover operator. To avoid repeated indices in the

TABLE 3. Values of metric 4 with parameter setting ($\beta = 0$, $\gamma = 1$), for expanding from 10 to 16 cities with HEIs.

MicroRegion	Methods that favor cities with lower UN-HDI					
	PMM		PCM		PDM	
	HEIs	HEI ratio	HEIs	HEI ratio	HEIs	HEI ratio
Central Region	6	0.3750	6	0.3750	7	0.4375
North	1	0.0625	1	0.0625	1	0.0625
South-West	6	0.3750	6	0.3750	6	0.375
South	3	0.1875	3	0.1875	2	0.125

TABLE 4. Values of metric 4 with parameter setting ($\beta = 1$, $\gamma = 0$), for expanding from 10 to 16 cities with HEIs.

MicroRegion	Methods that favor most populous cities					
	PMM		PCM		PDM	
	HEIs	HEI ratio	HEIs	HEI ratio	HEIs	HEI ratio
Central Region	6	0.3750	7	0.4375	8	0.500
North	1	0.0625	1	0.0625	1	0.0625
South-West	6	0.3750	5	0.3125	5	0.3125
South	3	0.1875	3	0.1875	2	0.125

offspring, we scanned the genes of each child, and replaced the repeated index with another randomly selected value.

The mutation operator randomly selects one gene [20], with probability of 0.05, and performs a mutation. Figure 5 illustrates the mutation operator. One gene with an index value of 48 is selected and replaced with an index value of 32. The replacing value is randomly selected.

The parameters used in this study for the GA are described in the sequence. Population size = 10. Maximum number of iterations = 600. The simulation for this heuristic was performed at MATLAB R2014a on a ASUS Intel(R) Xeon(R) Processor @1.80-2.40 GHz computer with 8 GB of RAM and Windows 10.

VII. RESULTS

The results will be broken down to the metric groups shown previously in section 5. We will also show results concerning the Genetic Algorithm performance.

A. GROUP 1 METRIC RESULTS

Figures 6(a), 6(b) and 6(c) show results for metrics 1, 2 and 3, respectively, for PMM model, for the three parameter settings: ($\beta = 0$, $\gamma = 1$), ($\beta = 1$, $\gamma = 0$) and ($\beta = 0.5$, $\gamma = 0.5$), and for an expansion from 10 to 16 cities with HEIs.

Figures 7(a), 7(b) and 7(c) show results for metrics 1, 2 and 3, respectively, for the PCM model and for the three parameter settings: ($\beta = 0$, $\gamma = 1$), ($\beta = 1$, $\gamma = 0$) and ($\beta = 0.5$, $\gamma = 0.5$), and for expanding from 10 to 16 cities with HEIs.

Figures 8(a), 8(b) and 8(c) show results for metrics 1, 2 and 3, respectively, for PDM model and for the three parameter settings: ($\beta = 0$, $\gamma = 1$), ($\beta = 1$, $\gamma = 0$) and ($\beta = 0.5$, $\gamma = 0.5$), and for expanding from 10 to 16 cities with HEIs.

Tables 3, 4, and 5 show the number of HEIs and values of metric 4 for the Amazonas State mesoregions, for the expansion from 10 to 16 cities with HEIs for the three parameter settings, respectively: ($\beta = 0$, $\gamma = 1$), ($\beta = 1$, $\gamma = 0$) and ($\beta = 0.5$, $\gamma = 0.5$).

TABLE 5. Values of metric 4 with parameter setting ($\beta = 0.5$, $\gamma = 0.5$), for expanding from 10 to 16 cities with HEIs.

MicroRegion	Methods that equally favor both criteria (UN-HDI and most populous cities)					
	PMM		PCM		PDM	
	HEIs	HEI ratio	HEIs	HEI ratio	HEIs	HEI ratio
Central Region	6	0.3750	7	0.4375	8	0.500
North	1	0.0625	1	0.0625	1	0.0625
South-West	6	0.3750	5	0.3125	5	0.3125
South	3	0.1875	3	0.1875	2	0.125

TABLE 6. Values of metric 5 with parameter setting ($\beta = 0$, $\gamma = 1$), for expanding from 10 to 16 cities with HEIs.

MicroRegion	Methods that favor cities with lower UN-HDI					
	PMM		PCM		PDM	
	SSS	SSS ratio	SSS	SSS ratio	SSS	SSS ratio
Central Region	129,176	0.867	129,914	0.873	129,914	0.872
North	2,707	0.0181	2,707	0.0182	2,707	0.0182
South-West	11,281	0.076	10,219	0.069	11,885	0.080
South	6,037	0.040	6,037	0.041	4,414	0.030

TABLE 7. Values of metric 5 with parameter setting ($\beta = 1$, $\gamma = 0$), for expanding from 10 to 16 cities with HEIs.

MicroRegion	Methods that favor most populous cities					
	PMM		PCM		PDM	
	SSS	SSS ratio	SSS	SSS ratio	SSS	SSS ratio
Central Region	131,581	0.868	130,377	0.876	135,159	0.889
North	2,707	0.018	2,707	0.0182	2,707	0.0182
South-West	11,281	0.074	9,783	0.0657	9,777	0.0643
South	6,037	0.040	6,037	0.0405	4,414	0.0290

B. GROUP 2 METRIC RESULTS

Tables 6, 7, and 8 show the values of SSS and values of metric 5 for the Amazonas State mesoregions, for expanding from 10 to 16 cities with HEIs for the three parameter settings, respectively: ($\beta = 0$, $\gamma = 1$), ($\beta = 1$, $\gamma = 0$) and ($\beta = 0.5$, $\gamma = 0.5$).

C. GROUP 3 METRIC RESULTS

Figures 9 and 10 show values of metrics 6 and 7, respectively, obtained for PMM, PCM and PDM models, for expanding cities with HEI from 10 to 11-16 cities and for the three parameter settings: ($\beta = 0$, $\gamma = 1$), ($\beta = 1$, $\gamma = 0$) and ($\beta = 0.5$, $\gamma = 0.5$).

D. RESULTS CONCERNING GA

Concerning the GA employed in this work, the simulation times using the three models, for expansions from 10 to 11, 12, 13, 14, 15, and 16 cities with HEIs, are shown in Table 9. We emphasize that, in this work, we used an adapted version of the GA state-of-art solution proposed by [22]. In [34], the authors used GA for designing a multi-objective PMM for locating new schools with a chromosome

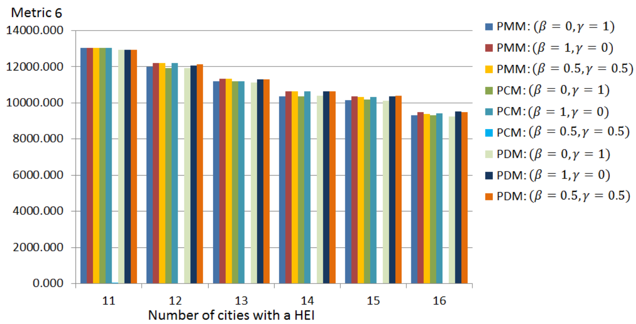

FIGURE 9. Values for metric 6 obtained for PMM, PCM and PDM, for expanding from 11 to 16 cities with HEIs.

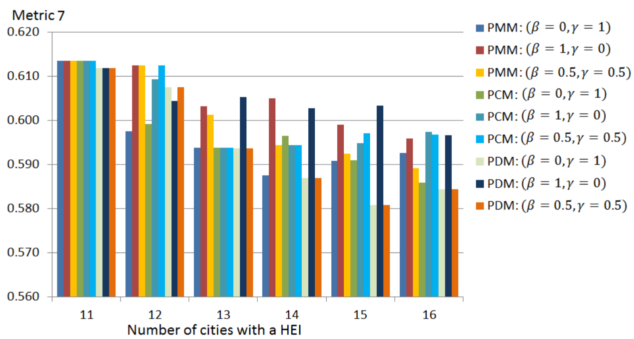

FIGURE 10. Values for metric 7 obtained for PMM, PCM and PDM, for expanding from 11 to 16 cities with HEIs.

composed of 62 binaries genes, population size equal to 300, and it varied the number of iteractions. These authors achieved the best execution time for 10 schools with 275 seconds. The best execution time in this work was achieved for PCM model with 11 schools, about 0.006 seconds, as shown in Table 9.

Applying the t-test to evaluate the differences between fluvial distance, of SSS and UN-HDI criteria achieved for GA simulation with 10, 20, and 30 initial population size,

TABLE 8. Values of metric 5 with parameter setting ($\beta = 0.5$, $\gamma = 0.5$), for expanding from 10 to 16 cities with HEIs.

MicroRegion	Methods that equally favor both criteria (UN-HDI and most populous cities)					
	PMM		PCM		PDM	
	SSS	SSS ratio	SSS	SSS ratio	SSS	SSS ratio
Central Region	129,914	0.866	131,711	0.877	135,112	0.889
North	2,707	0.018	2,707	0.0182	2,707	0.0178
South-West	11,281	0.075	9,783	0.065	9,777	0.0643
South	6,037	0.0403	6,037	0.0402	4,414	0.0290

TABLE 9. Simulation times obtained with parameter setting ($\beta = 0.5, \gamma = 0.5$) for expanding from 10 to 11, 12, 13, 14, 15, and 16 cities with HEIs.

Method	Simulation time (seconds)					
	Number of cities with HEI					
	11	12	13	14	15	16
PMM	0.046	0.279	0.047	0.071	0.011	0.104
PCM	0.006	0.193	0.066	0.169	0.202	0.283
PDM	3.549	0.105	0.232	0.043	0.217	0.208

we conclude that they are not statistically significant at a 90% confidence level. As we can observe, the best simulation times were obtained for the PMM model. The worst simulation times were obtained for the PDM Model.

VIII. DISCUSSION AND CONCLUSION

This section is divided in four sub-sections, in accordance with the sub-sections of section 7 of the work. In the end we will summarize the main contributions and present the limitations of this work.

A. DO THE SOLUTIONS OBTAINED BY COMBINING LOCATION MODELS AND STRATEGIES CITED IN SECTION 1 PROVIDE A BETTER DISTRIBUTION OF HEIs?

As observed in Figure 6(a), the combination of PMM with parameter setting ($\beta = 0, \gamma = 1$) results in smaller values for metric 1. The worst performance is obtained with the combination PMM with parameter setting ($\beta = 1, \gamma = 0$). In these expansions, the mean fluvial distance decreases with the increasing of the number of cities with an HEI. In Figure 6(b), the PMM with parameter setting ($\beta = 0$, $\gamma = 1$) and the PMM with parameter setting ($\beta = 0.5$, $\gamma = 0.5$) results in lower and slightly lower values for metric 2, for expansions to 11-15 HEIs and to 16 HEIs, respectively. In Figure 6(c), PMM with parameter setting ($\beta = 0.5, \gamma = 0.5$) results in minimum values for maximum fluvial distance between cities with HEIs.

As observed in Figure 7(a), the PCM with parameter setting ($\beta = 0, \gamma = 1$) results in smaller values for metric 1. The worst performance is obtained with PCM with parameter setting ($\beta = 0.5, \gamma = 0.5$). In these expansions, the mean fluvial distance decreases with the increasing of the number of cities with a HEI. In Figure 7(b), the PCM with parameter setting ($\beta = 0, \gamma = 1$) results in lower values for metric 2. In Figure 7(c), PCM with parameter setting ($\beta = 1, \gamma = 0$) results in minimum values for metric 3 and the worst results are obtained with the PCM with parameter setting ($\beta = 0, \gamma = 1$).

As observed in Figure 8(a), the PDM with parameter setting ($\beta = 0, \gamma = 1$) results in smaller values of metric 1. The worst performances were obtained with PCM with parameter setting ($\beta = 0.5, \gamma = 0.5$), except for expanding to 15 HEI. In these expansions, the mean fluvial distance decreases with the increasing of the number of cities with a HEI. In Figure 8(b), the PDM with parameter setting ($\beta = 1$, $\gamma = 0$) results in lower values for metric 2. In Figure 8(c), all parameter settings result in the same minimum value for metric 3.

In Table 3, the best HEI ratio was obtained by PMM and PCM models. The worst HEI ratio was achieved by PDM. Both PMM and PCM prioritize cities in South-West and South mesoregions. In Tables 4 and 5, the best HEI ratio was obtained by PMM. The worst HEI ratio was achieved by PDM. PMM prioritize cities in South-West and South mesoregions. In these tables, the PMM results in the same HEI ratio.

From Tables 3, 4 and 5 we draw an important conclusion of this work: all location methods (PMM, PCM and PDM) obtained, for expanding from 10 to 16 cities with HEIs, a better HEI ratio for North, North-West and South mesoregions is obtained compared with the values shown in Table 1. The only exception is the Central mesoregion. Indeed, from Table 1, we have that the HEI ratio of Central mesoregion, the most populous one, is 0.8, while in Tables 3, 4 and 5, the HEI ratio of Central mesoregion are less than or equal to 0.5. Therefore, the location models increase the number of cities with HEIs in the mesoregions with a lower student ratio.

B. DO THE SOLUTIONS OBTAINED BY COMBINING LOCATION MODELS AND STRATEGIES CITED IN SECTION 1 PROVIDE BETTER EQUALIZATION OF THE SSS RATIO OF ALL MESOREGIONS?

For metric 5, Tables 6, 7, and 8 show the values of the SSS ratio for the three models with strategies A, B, and C, respectively. As we can observe, for the Central mesoregion, the existing HEIs in Manaus, Parintins, Itacoatiara, Coari, and Tefé represent the total of the 128,501 SSS and the new HEI in Beruri, 1,413 SSS, as shown in Figure 11. The other mesoregions have 20,005 SSS. Thus, the number of SSS and SSS ratio in the first is higher than the former. This SSS ratio of the Central mesoregion decreases with the increase of the HEIs in the other mesoregions. The best SSS ratios were obtained by PMM with parameter setting ($\beta = 0.5, \gamma = 0.5$) for the South-West and South mesoregions, 0.075 and 0.0403, respectively, compared with the other models.

It is important to note that, to increase the HES ratio from the South, South-West and North mesoregions in relation to the Central mesoregion, it is not only necessary to increase the number of HEIs in these mesoregions, but also, to increase the number of students of the HEIs located in them. However, with small cities, to increase the number of HES, it is strongly recommended to increase the number of cities with HEIs. Increasing the number of HEIs implies decreasing the distances between cities without an HEI and cities with a HEI, because it facilitates student access. Indeed, in all the locations with 11, 12, 13, 14, 15, and 16 cities, we have a decrease of metric 1, as shown in Figures 6(a), 7(a) and 8(a), and in the metric 2, as shown in Figures 6(b), 7(b) and 8(b), between a city without an HEI and a city with an HEI.

FIGURE 11. Results obtained by using the PMM model combined with strategy C for expanding from 10 to 16 cities with HEIs.

C. DO THE SOLUTIONS OBTAINED BY COMBINING LOCATION MODELS AND SOME STRATEGIES CITED IN SECTION 1 FAVOR CITIES WITH LOWER UN-HDI OR CITIES WITH A HIGHER NUMBER OF SSS?

Figure 9 shows values for metric 6 obtained for PMM, PCM and PDM, for expanding the number of cities with HEI from 10 to 11-16 cities. As shown in this figure, if the decision maker chooses one of the strategies A, B, or C and one of the optimization methods, PMM, PDM, or PCM, the values for metric 6 are almost the same.

Figure 10 shows values for metric 7 obtained for PMM, PCM, and PDM, for expanding the number of cities with HEI from 10 to 11-16 cities. As shown in this figure, in some expansions, to obtain minimum values for metric 7, the decision makers must select a PMM model combined with strategy A, while in other expansions, they must select the PDM model combined with strategy C. Figure 11 shows the cities with HEIs for expanding from 10 to 16 cities with HEIs obtained for the PMM model combined with strategy C. As we can observe in this figure, the new cities with HEIs are represented by a cross marker. Four new cities with HEIs are in the South-West mesoregion. One new city with an HEI is in the Central mesoregion and the other is in the South mesoregion.

From Figure 10, we draw another important conclusion of this work: all allocation methods used in this work, PMM, PCM and PDM, obtained, for expanding from 10 to 11-16 cities, lowered values for metric 7. Indeed, from Table 2, we notice that the existing distribution presents a mean UN-HDI value of 0.619. All the expansions shown in Figure 10 present lower mean values of UN-HDI compared with 0.619. The better performances in this work were obtained with parameter setting ($\beta = 0$, $\gamma = 1$) or with parameter setting ($\beta = 0.5, \gamma = 0.5$). The worst performance was again obtained with parameter setting ($\beta = 1$, $\gamma = 0$).

Finally, we observe that, for all expansions, from 10 to 11-16 cities, there is a decrease in the values for metric 6. From Table 2, we notice that, for the existing distribution, the mean number of SSS is 14,170.40. For expansions from 10 to 16, the mean number of SSS varied from 13,019.94 to 9,229.91, for the three models. Therefore, although the model favors an expansion to cities with higher populations, there was a decrease in the mean number of SSS, when the number of cities with HEIs increase. Concerning this issue, the following considerations may be taken into account: the existing HEI distribution already favors cities with a higher number of SSS. Therefore, the inclusion of more cities with HEIs tends to drop the mean number of SSS of cities with HEIs.

D. COMMENTS ON THE GENETIC ALGORITHM RESULTS

Table 9 shows the simulation times using the three models, for expansions from 10 to 11, 12, 13, 14, 15 and 16 cities with HEIs, for the parameter setting ($\beta = 0.5, \gamma = 0.5$). As can be seen, the best simulation times were obtained for the PCM model for expanding to 11 HEI in 0.006 seconds. The worst simulation times were obtained for the PDM Model.

Some limitations of our work are the following: an optimum set of weights is not proposed for each term in equations (4), (5) and (6); no border treatment is proposed for cities that could be served by institutions of neighboring states; cities with high populations could generate spatial aggregation. However, we would like to emphasize that two terms in equations (4), (5) and (6) are conflicting: population and UN-HDI. The conflict arises because very small cities have lower UN-HDIs. This conflict minimizes the problem of spatial aggregation observed in other papers ([42] and [43]). In other words, it minimizes the danger that the local medians in the PMM solutions will be located wherever particular centers exist with their populations having more inhabitants than other centers.

The methodology developed in this work can be applied to any problem of locating public resources. Although the problem of locating government resources, in general, involves a political dimension, this does not invalidate a technical analysis of the problem, as we did in this work. The analysis presented herein aims primarily to contribute to the evaluation of a solution for locating HEIs in Amazonas State, Brazil.

We would also like to point out that this work differs from other works already published in the literature, because the focus here is on expanding an existing system of HEIs. We believe that this study is novel in the sense that it is the only study that compares classic multiple criteria facility location models.

Finally, we would like to point out that the main contribution of this study underscores the necessity of social development of cities with lower UN-HDI values, and we propose future expansion of the Amazonas Higher Education system that favors mesoregions with small student concentrations, preventing an oversized number of students in the mesoregion with a higher population.

ACKNOWLEDGEMENT

The authors would like to thank J. Hesson proofread the manuscript (https://www.academicenglishsolutions.com).

REFERENCES

[1] *Brazilian Ministry of Education and Culture*, Census, Brasilia, Brazil, 2017.

[2] *Education at a Glance*, OECD, Paris, France, 2018, doi: 10.1787/19991487.

[3] Secondary School Students Achievements. (2018). *Brazilian Institute of Geography and Statistics—IBGE*. Accessed: Jun. 6, 2019. [Online]. Available: https://cidades.ibge.gov.br/brasil/am/panorama

[4] HidroWeb—Hydrological Information Systems, Shapefiles Amazon Basin. (2016). *Brazilian Agency of Water—ANA*. Accessed: May 5, 16. [Online]. Available: http://hidroweb.ana.gov.br/baixar/mapa/Bacia1.zip

[5] Brazilian Transport Infrastructure Department—DNIT. (2016). *Shapefiles Multimodal Road Maps*. Accessed: Feb. 5, 2016. [Online]. Available: https://189.9.128.64/mapas-multimodais/shapefiles/shapefiles.rar

[6] Brazilian Institute of Geography and Statistics—IBGE. (2016). *Shapefiles Municipal Meshes of the State of Amazonas*. Accessed: Feb. 5, 2016. [Online]. Available: ftp://geoftp.ibge.gov.br/organizacaodoterritorio/malhasterritoriais/malhasmunicipais/municipio 2014/AM/ammunicipios.zip

[7] *Human Development Indices and Indicators 2018: Statistical Update*, New York, NY, USA. 2018, doi: 10.18356/656a3808-en.

[8] *Map of higher education in Brazil*, Union Entities Maintaining Higher Educ. Inst. State S~ao Paulo—SEMESP, São Paulo, Brazil 2018.

[9] Federal University of Amazonas—UFAM. (2016). *Institutional Development Plan 2016–2025*. Accessed: May 25, 19. [Online]. Available: https://www.proplan.ufam.edu.br/index.php/pdi

[10] Univerty of the State of Amazonas—UEA. (2012). *Institutional Development Plan 2012–2016*. Accessed: May 25, 19. [Online]. Available: http://www.pdi.uea.edu.br/data/area/c33/download/1-1.pdf

[11] G. K. Tutunchi and Y. Fathi, "Effective methods for solving the bi-criteria *p*-center and *p*-dispersion problem," *Comput. Oper. Res.*, vol. 101, pp. 43–54, Jan. 2019.

[12] M. S. Daskin and K. Lee Maass, "The *p*-median problem," in *Location Science*. Cham, Switzerland: Springer, 2015.

[13] M. S. Daskin, "What you should know about location modeling," *Naval Res. Logistics*, vol. 55, no. 4, pp. 283–294, Jun. 2008.

[14] O. Kariv and S. L. Hakimi, "An algorithmic approach to network location problems. II: Thep-medians," *SIAM J. Appl. Math.*, vol. 37, no. 3, pp. 539–560, Dec. 1979.

[15] R. Whitaker, "ERrata: A fast algorithm for the greedy interchange for large-scale clustering and median location problems," *Inf. Syst. Oper. Res.*, vol. 22, no. 1, pp. 70–71, Feb. 1984.

[16] C. Beltran, C. Tadonki, and J. P. Vial, "Solving the p-median problem with a semi-lagrangian relaxation," *Comput. Optim. Appl.*, vol. 35, no. 2, pp. 239–260, Oct. 2006.

[17] F. Chiyoshi and R. D. Galv ao, "A statistical analysis of simulated annealing applied to the p-median problem," *Ann. Oper. Res.*, vol. 96, pp. 61–74, Nov. 2000.

[18] S. Salhi, "Defining tabu list size and aspiration criterion within tabu search methods," *Comput. Oper. Res.*, vol. 29, no. 1, pp. 67–86, Jan. 2002.

[19] O. Alp, E. Erkut, and Z. Drezner, "An efficient genetic algorithm for the p-median problem," *Annals Operational Research*, vol. 122, pp. 21–42, Sep. 2003.

[20] S. Satoglu, M. Oksuz, G. Kayakutlu, and K. A. Buyukozkan, "Genetic algorithm for the p-median facility location problem," in *Proc. GJCI Global Joint Conf. Ind. Eng.*, İstanbul, Turkey, 2016.

[21] Z. Drezner, J. Brimberg, N. Mladenović, and S. Salhi, "New heuristic algorithms for solving the planar p-median problem," *Comput. Oper. Res.*, vol. 62, pp. 296–304, Oct. 2015.

[22] S. Silva, M. Costa, and C. C. Filho, "Customized genetic algorithm for facility allocation using p-median," in *Proc. Federated Conf. Comput. Sci. Inf. Syst.*, Sep. 2019, pp. 165–169.

[23] J. E. Beasley, "OR-library: Distributing test problems by electronic mail," *J. Oper. Res. Soc.*, vol. 41, no. 11, pp. 1069–1072, Nov. 1990.

[24] J. Teixeira, A. Antunes, and D. Peeters, "An optimization-based study on the redeployment of a secondary school network," *Environ. Planning B, Planning Des.*, vol. 34, no. 2, pp. 296–315, Apr. 2007.

[25] F. Ndiaye, B. M. Ndiaye, and I. Ly, "Application of the p-median problem in school allocation," *Amer. J. Oper. Res.*, vol. 2, pp. 253–259, Apr. 2012.

[26] R. C. Menezes and N. D. Pizzolato, "Locating public schools in fast expanding areas: Application of the capacitated p-median and maximal covering location models," *Pesqui. Oper.*, vol. 34, no. 2, pp. 301–317, Aug. 2014.

[27] J. Amaya, D. Peeters, P. Uribe, and J. P. Valenzuela, "Optimization modeling for resource allocation in the chilean public education system," *Int. Regional Sci. Rev.*, vol. 39, no. 2, pp. 155–176, Apr. 2016.

[28] I. Castillo-LÓpez and H. A. LÓpez-Ospina, "School location and capacity modification considering the existence of externalities in students school choice," *Comput. Ind. Eng.*, vol. 80, pp. 284–294, Feb. 2015.

[29] M. Chen, J.-C. F. Thill, and E. Delmelle, "iGLASS: An open source SDSS for public school location-allocation," in *GeoComputational Analysis and Modeling of Regional Systems*. Cham, Switzerland: Springer, 2018, pp. 325–353.

[30] M. Karatas, N. Razi, and H. Tozan, "A multi-criteria assessment of the p-median, maximal coverage and p-center location models," *Tehnicki Vjesnik*, vol. 24, no. 2, pp. 399–407, 2017.

[31] R. Lotfian and M. Najafi, "Optimal location of emergency stations in underground mine networks using a multiobjective mathematical model," *J. Int. Soc. Child Adolescent Injury Prevention*, vol. 25, no. 4, pp. 264–272, Aug. 2019.

[32] *XII Demographic Census*, Brazilian Inst. Geography Statist., Rio de Janeiro, Brazil, 2010.

[33] *Characterization of Supply and Demand in the Inland Waterways of the Amazon Region Passengers*, National Waterway Transportation Agency, Brasília, Brazil, 2013.

[34] C. M. Xavier, M. G. F. Costa, and C. F. F. C. Filho, "Optimizing Universities allocation with integer programming using a new distance measure," in *Proc. 5th Int. Conf. Eng. Optim. (EngOpt)*, vol. 1. Rio de Janeiro, Brazil: e-Papers, 2016, pp. 1–10.

[35] R. Xu and D. C. Wunsch, "Clustering," *IEEE Press Series on Computational Intelligence. Section 2.2: Definition of Proximity Measures*, 2nd ed. Hoboken, NJ, USA: Wiley, 2008.

[36] W. O. Bussab and P. A. Morettin, *Basic Statistics* (Section 3.1: Position Measures), 6th ed. São Paulo, Brazil: Saraiva., 2009.

[37] M. R. Spiegel, "Statistics," *Standard deviation* (Wiley Series on Parallel and Distributed Computing), 3rd ed. New York, NY, USA: Makron Books, 2004, pp. 106–107.

[38] J. Holland, *Adaptation in Natural and Artificial Systems*, 2nd ed. Ann Arbor, MI, USA: The Univ. of Michigan Press, 1975.

[39] G. Reinelt, "TSPLIB—A traveling salesman problem library," *ORSA J. Comput.*, vol. 3, no. 4, pp. 376–384, 1991.

[40] P. Fernando, S. Katkoori, D. Keymeulen, R. Zebulum, and A. Stoica, "Customizable FPGA ip core implementation of a general-purpose genetic algorithm engine," *IEEE Trans. Evol. Comput.*, vol. 14, no. 1, pp. 133–149, Feb. 2010.

[41] G. Zapfel, R. Braune, and M. Bögl, *Metaheuristic Search Concepts: A Tutorial With Applications to Production and Logistics*. New York, NY, USA: Springer, 2010.

[42] R. L. Francis, T. J. Lowe, G. Rushton, and M. B. Rayco, "A synthesis of aggregation methods for multifacility location problems: Strategies for containing error," *Geograph. Anal.*, vol. 31, no. 1, pp. 67–87, Jan. 1999.

[43] E. Erkut and B. Bozkaya, "Analysis of aggregation errors for the p-median problem," *Comput. Oper. Res.*, vol. 26, nos. 10–11, pp. 1075–1096, Sep. 1999.

CLAHILDEK MATOS XAVIER received the Graduate degree in data processing technology from State University, UEA, in 2004, the master's degree in electrical engineering from the Federal University of Amazonas (UFAM), Brazil, in 2012, and the Ph.D. degree from the Institute of Computation, UFAM, in 2019. He is currently an Analyst in science and technology, working in the database administration area with the Management and Operational Center of the Amazonian Protection System (Censipam), where he is also a Research Project Guide. His areas of research are optimization, pattern recognition, and digital image processing. His work has been documented in more than five articles from 2012 to 2019.

MARLY GUIMARÃES FERNANDES COSTA received the degree in electrical engineering from the Federal University of Amazonas (UFAM), in 1983, the master's degree in electrical engineering, telecommunications concentration area, from the State University of Campinas (UNICAMP), in 1988, and the Ph.D. degree in electrical engineering, automation concentration area, from UNICAMP, in 1996. She is currently a Professor with the Department of Electrical Engineering, UFAM. She has experience in digital image processing, pattern recognition, and biomedical engineering, acting on the following subjects: image automation, development of diagnostic aid tools, and expert systems.

CÍCERO FERREIRA FERNANDES COSTA FILHO received the degree in electrical engineering from the Federal University of Pernambuco (UFPE), in 1982, the master's degree in electrical engineering from the State University of Campinas (UNICAMP), in 1985, in the area of microeletronics, and the Ph.D. degree in automation area from UNICAMP, in 1996. He worked with UNICAMP and the Center for Analysis, Research, and Technologial Innovation Foundation-FUCAPI. Since 1990, he has been a Professor with the Federal University of Amazonas. He is currently a Full Professor, working in undergraduate (electrical engineering and computer engineering) and postgraduate (master in electrical engineering - PPGEE and Ph.D. in computer engineering - PPGI) courses. His research topics are artificial intelligence, pattern recognition, digital image processing, and optimization.