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A General Fast Power Flow Algorithm for Transmission and Distribution Networks

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ABSTRACT Fast power flow calculation is generally required in static security analysis and power system optimal planning. And the classical DC power flow algorithm, due to its simple mathematical model, linearized equation, and rapid solving speed, is widely utilized. However, the classical DC power flow algorithm has been found to be only suitable for high voltage transmission networks with small branch impedance ratio. To address this issue, a general fast power flow algorithm is proposed. Utilizing this method, the classical DC power flow algorithm is firstly performed to obtain the initial values of the branch active power flow. And the nodal voltage angles are then calculated by the established node-injected reactive power equations. Secondly, through the identical transformation and approximate treatment of the node power flow equation, a voltage offset calculation method for PQ nodes is proposed. Finally, the active power flow and active power loss can be calculated based on the corrected phase angles and node voltages. Simulation studies on standard IEEE power systems, such as the IEEE 33-bus and IEEE 118-bus systems, etc., were conducted by the presented algorithm, compared with those by the back/forward sweep, classical DC power flow and Newton-Raphson algorithms. It is indicated that, the proposed power flow algorithm has the superiority in satisfactory calculation speed and non-sensitivity on the network impedance ratio.

INDEX TERMS Power flow algorithm, transmission and distribution network, high impedance ratio of branch, bus voltage amplitude deviation.

I. INTRODUCTION

Power system load flow calculation is the basis of steady state operation analysis of power system. The ac power system load flow calculation methods, such as the Newton-Raphson method and PQ decomposition method can be used to obtain the system's power flow exactly. But the need for multiple iterative operation, and the considerable calculation burden, make them not so applicable in real power system security analysis. Instead, the classical DC flow algorithm, due to its simple mathematical model, linearized equation, rapid solving speed, is widely used in quick calculation occasions, such as power system planning [1], [2], security and stability evaluation of power system [3]–[5], transaction price calculation of electricity market [6]–[9], analysis of transmission capability [10], [11], operation dispatching [5], and probabilistic load flow analysis [12], etc. However, the classical DC power flow algorithm has been found to be mainly suitable for high voltage transmission networks with small branch impedance, with its calculation error usually being within 3%-10%, which is only considered acceptable for occasions with low precision requirements [13].

In order to extend the applicable range of the DC power flow algorithm, a modified DC power flow algorithm based on the equivalent load model of network loss is proposed in [14], [15], where the branch network loss is allocated and iterated until the equivalent load of each node converges. It is worthwhile to point out that the simplified condition could reduce the calculation accuracy of the classical DC power flow. As presented in [16], ignoring the calculation of nodal voltage may lead to the omission and miscalculation of checking results in static security analysis and power

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market calculation. A normal DC power flow of the system is executed to determine voltage angles in [17], with a novel derivation of voltage amplitudes provided. In [18], a new method is presented to improve the accuracy of the DC power flow results while maintaining its computation efficiency and linear formulation. The proposed method uses the empirical knowledge of the system, including observed voltage magnitudes and angles from historical data, to formulate the correction terms. A method to find the linear power flow with the minimum error is presented in [19], where the formulation of the independent variables associated with the minimum linearization error is determined by the distribution of state variables, i.e., v and θ. In [20], a linear model considering the bus voltage magnitudes for radial distribution systems is proposed, while in [21] a novel DC power flow method with reactive power considerations is introduced. In [22], a decoupled fast power flow algorithm based on the traditional ac power flow is proposed, and the voltage amplitude with high accuracy can be obtained.

It is notable that the above literatures mainly focus on improving the DC power flow algorithm from aspects of network loss calculation, node voltage calculation, power flow estimation, etc. As a result, they are only applicable to the distribution network with lower impedance ratio. Hence, as the main contribution of this paper, a general fast power flow algorithm is put forward for transmission and distribution networks, which considers the high impedance ratio of the networks.

The paper is organized as follows. Section II describes the basic concept of the branch equivalent power flow calculations. In Section III, the proposed fast power flow algorithm is presented, while the detailed steps and layout of the proposed method is provided in Section IV. In Section V, case studies are performed on standard IEEE power systems, such as the IEEE 33-bus system, IEEE 14-bus system, IEEE 118-bus systems, etc., to verify the correctness and feasibility of the proposed method. Finally, some useful conclusions are summarized in Section VI.

II. BASIC CONCEPT OF THE BRANCH EQUIVALENT POWER FLOW

In the power network, the unit equivalent circuit between any two nodes can be shown as Fig. 1, where $y_{ij} = 1/z_{ij} = g_{ij} + jb_{ij}$, denotes the branch admittance.

Note that the power lines and transformers are usually be represented by π-type equivalent circuits, whereas the capacitors and reactors can be equivalent to be series or parallel branches, respectively. In the distribution network, if the capacitor is installed in the substation, it can be equivalent to a part of the substation node injecting power according to the power factor; if the capacitor is installed in the middle of a line, it can be regarded as a parallel branch to the ground. The parallel branches of the same node can be equivalent to a branch.

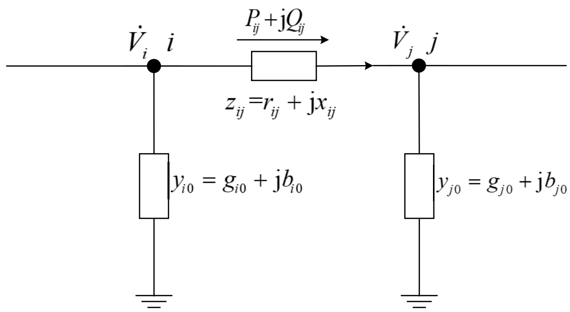

FIGURE 1. The unit equivalent circuit between any two nodes.

According to Fig. 1, the branch (i, j) power flow equation can be obtained by

$$\begin{cases} P_{ij} = V_i^2(g_{i0} + g_{ij}) - V_iV_j(b_{ij}\sin\theta_{ij} + g_{ij}\cos\theta_{ij}) \\ Q_{ij} = -V_i^2(b_{i0} + b_{ij}) + V_iV_j(b_{ij}\cos\theta_{ij} - g_{ij}\sin\theta_{ij}) \end{cases} \tag{1}$$

where P_{ij} and Q_{ij} are the active power and reactive powers of the branch circuit (i, j); V_i is the voltage amplitude of node i; g_{ij} is the conductance parameter of series branch; b_{ij} is the susceptance parameter of series branch; g_{i0} is the conductance parameters of equivalent parallel branch; b_{i0} is the susceptance parameter of equivalent parallel branch; θ_{ij} is the phase angle difference between the sending and receiving ends of the branch (i, j).

The branch power flow equation, i.e., (1) can be written in a matrix form as

$$\begin{bmatrix} P_{ij}/V_i \\ Q_{ij}/V_i \end{bmatrix} = -\begin{bmatrix} b_{ij} & -g_{ij} & -g_{i0} \\ g_{ij} & b_{ij} & b_{i0} \end{bmatrix}\begin{bmatrix} V_j\sin\theta_{ij} \\ V_i - V_j\cos\theta_{ij} \\ V_i \end{bmatrix} \tag{2}$$

According to (2), when carrying out the elementary transformation on the power flow equation matrix, and eliminating the row related to reactive power in the matrix, the active power of the branch can be obtained as

$$P_{ij} = \frac{V_iV_j\sin\theta_{ij}}{x_{ij}} + \frac{r_{ij}}{x_{ij}}Q_{ij} + V_i^2\left(g_{i0} + \frac{r_{ij}}{x_{ij}}b_{i0}\right) \tag{3}$$

Similarly, the reactive power of the branch can be calculated by

$$Q_{ij} = \frac{-V_i\ V_j\sin\theta_{ij}}{r_{ij}} + \frac{x_{ij}P_{ij}}{r_{ij}} - V_i^2\left(b_{i0} + g_{i0}\frac{x_{ij}}{r_{ij}}\right) \tag{4}$$

III. THE PROPOSED FAST POWER FLOW ALGORITHM

In a real power system, the difference of phase angle of node voltage usually includes branch resistance parameters, with the node voltage's amplitude offset being around its rated value. Especially, the resistance of the distribution network is usually much bigger than that of the transmission network. In this section, the calculation error with the classical DC power flow algorithm is firstly discussed. And then an improved fast power flow algorithm is put forward, with the possible influencing factors being taken account.

A. CLASSICAL DC POWER FLOW ALGORITHM

The classical DC power flow algorithm is widely used in power system. For instance, when a power network is in plan, the active power flow distribution is usually concerned, while the voltage amplitude of each node does not need to be considered. In such cases, the DC power flow algorithm is usually adopted. In other words, when the calculation speed rather than the calculation accuracy is put in priority, the DC power flow algorithm shows a satisfactory performance.

In the classical DC power flow algorithm, the parallel branches in the π-type equivalent circuit, as shown in Fig. 1 are usually ignored. And the following assumptions are usually made: ① $V_i = V_j = 1$; ② $\sin\theta_{ij} = \theta_i - \theta_j$; ③ $r_{ij} = 0$. Then, the active power flow equation of the branch circuit (i, j) can be calculated by

$$P_{ij} = \frac{\theta_i - \theta_j}{x_{ij}} \tag{5}$$

For a power network with $n+1$ nodes (including a slack node), the matrix form of the classical DC power flow equation can be represented as

$$\boldsymbol{P}^{\mathrm{SP}} = \boldsymbol{B}_0\boldsymbol{\theta} \tag{6}$$

where $\boldsymbol{P}^{\mathrm{SP}} = [P_1^{\mathrm{SP}}, \cdots, P_i^{\mathrm{SP}}, \cdots, P_n^{\mathrm{SP}}]^{\mathrm{T}}$ is the net injection power of the node, and specified as a known condition; $\boldsymbol{\theta} = [\theta_1, \cdots, \theta_i, \cdots, \theta_n]^{\mathrm{T}}$ represents the voltage phase angle of n nodes; $\boldsymbol{B}_0$ is an $n \times n$ order node admittance matrix based on branch reactance x_{ij}, and any element of the matrix can be expressed as

$$\boldsymbol{B}_0(i,j) = \begin{cases} \sum_{k=1}^{n} \frac{1}{x_{ik}} - \frac{1}{x_{ii}} & i = j \\ -\frac{1}{x_{ij}} & i \neq j \end{cases} \tag{7}$$

B. THE PROPOSED VOLTAGE PHASE ANGLE CALCULATION METHOD CONSIDERING THE EFFECT OF BRANCH RESISTANCE

According to Fig. 1, the voltage drop generated by the series branches can be calculated by

$$\begin{aligned} \dot{V}_j\hat{I} &= P_j + \mathrm{j}Q_j \\ \Delta\dot{V} &= \dot{V}_i - \dot{V}_j = \Delta V + \mathrm{j}\delta V \\ &= \frac{P_j r_{ij} + Q_j x_{ij}}{V_j} + \mathrm{j}\frac{P_j x_{ij} - Q_j r_{ij}}{V_j} \end{aligned} \tag{8}$$

where $\dot{V}_i$ denotes voltage phasor of node i; $\hat{I}$ denotes the conjugate of the current flowing through the impedance of the series branch; ΔV denotes the in-phase component of the voltage drop, which reflects the difference between the ends of the branch; δV is the quadrature component of the voltage drop, indicating the difference of the voltage phase angle between the two ends of the branch; P_j and Q_j are the net injection active power and reactive power of node j; V_j is the voltage amplitude of node j.

Considering that the distribution network contains branches with high impedance ratio, it can be concluded from (8) that the error of applying the classical DC power flow to solve the node voltage phase angle will become larger if the branch resistance or reactive power increases. In order to calculate the phase angle of node voltage more accurately, the effect of parallel branch resistance should be taken into consideration. Assume that $V_i = V_j = 1$, and $\sin\theta_{ij} = \theta_i - \theta_j$, then (3) and (4) can be simplified as

$$\begin{cases} P_{ij} = \frac{\theta_i - \theta_j}{x_{ij}} + \frac{r_{ij}Q_{ij}}{x_{ij}} + \left(g_{i0} + b_{i0}\frac{r_{ij}}{x_{ij}}\right) \\ Q_{ij} = \frac{-(\theta_i - \theta_j)}{r_{ij}} + \frac{x_{ij}P_{ij}}{r_{ij}} - \left(b_{i0} + g_{i0}\frac{x_{ij}}{r_{ij}}\right) \end{cases} \tag{9}$$

Note that the power injected into any node i is always equal to the algebraic sum of the branch power connected to the point. As a consequence, the power balance equation of node i can be obtained as

$$\begin{cases} P_i^{\mathrm{SP}} = \sum_{j=1(j\neq i)}^{n} P_{ij} = \sum_{j=1(j\neq i)}^{n} \frac{\theta_i - \theta_j}{x_{ij}} + \frac{r_{ij}Q_{ij}}{x_{ij}} + \left(g_{i0} + b_{i0}\frac{r_{ij}}{x_{ij}}\right) \\ Q_i^{\mathrm{SP}} = \sum_{j=1(j\neq i)}^{n} Q_{ij} = \sum_{j=1(j\neq i)}^{n} \frac{-(\theta_i - \theta_j)}{r_{ij}} + \frac{x_{ij}P_{ij}}{r_{ij}} + \left(b_{i0} + g_{i0}\frac{x_{ij}}{r_{ij}}\right) \end{cases} \tag{10}$$

According to (10), the node-injected reactive power is coupled with the active power of the branch, the impedance of the series branch and the admittance parameters of the parallel branch as well. As for PQ nodes, the node-injected reactive power equation can be written in matrix form as

$$\boldsymbol{Q}^{\mathrm{SP}} = \boldsymbol{G}_0\boldsymbol{\theta} + \boldsymbol{P}_{\mathrm{cp}} + \boldsymbol{C}_{\mathrm{Q}} \tag{11}$$

where $\boldsymbol{Q}^{\mathrm{SP}} = [Q_1^{\mathrm{SP}}, \cdots, Q_i^{\mathrm{SP}}, \cdots, Q_n^{\mathrm{SP}}]^{\mathrm{T}}$ represents the net injected reactive power into the nodes, and specified as a known condition; $\boldsymbol{G}_0$ is an $n \times n$ order nodal conductance matrix based on the branch resistance r_{ij}; $\boldsymbol{P}_{\mathrm{cp}} = [P_{\mathrm{cp}1}, \cdots, P_{\mathrm{cp}i}, \cdots, P_{\mathrm{cp}n}]^{\mathrm{T}}$ is defined as the node-branch power coupling term, which couples the node reactive injection power and branch active power; $\boldsymbol{C}_{\mathrm{Q}} = [C_{\mathrm{Q}1}, \ldots, C_{\mathrm{Q}i}, \ldots, C_{\mathrm{Q}n}]^{\mathrm{T}}$ denotes the node power-branch parameter coupling term, which couples the node reactive power and branch parameter.

It is notable that any element of the matrix $\boldsymbol{G}_0$ can be expressed as

$$\boldsymbol{G}_0(i,j) = \begin{cases} \sum_{k=1}^{n} \frac{-1}{r_{ik}} + \frac{1}{r_{ii}} & i = j \\ \frac{1}{r_{ij}} & i \neq j \end{cases} \tag{12}$$

Similarly, any element of the coupling term $\boldsymbol{P}_{\mathrm{cp}}$ between the node reactive power and branch active power can be obtained as

$$P_{\mathrm{cp}i} = \sum_{j=1(j\neq i)}^{n} \frac{x_{ij}}{r_{ij}} \cdot P_{ij}^{0} \tag{13}$$

where P_{ij}^{0} is the initial value of the active power flowing into the branch (i, j), which can be calculated by the classical DC power flow algorithm.

In addition, any element of the coupling term $\mathbf{C}_{\mathrm{Q}}$ between the node reactive power and branch parameters can be calculated by the following equation.

$$C_{\mathrm{Q}i} = \sum_{j=1(j\neq i)}^{n} b_{i0} + g_{i0}\frac{x_{ij}}{r_{ij}} \tag{14}$$

Note that in (11), $\boldsymbol{Q}^{\mathrm{SP}}$ is given, $\boldsymbol{P}_{cp}$ can be obtained by DC load flow algorithm, and $\boldsymbol{C}_{\mathrm{Q}}$ can be obtained by branch parameter. Consequently, the phase angle of the node voltage can be obtained by solving (11), with the impedance of parallel branch and branch being taken into account.

Similarly, based on (10), the node-injected active power and reactive power equation can be written as

$$\begin{cases} \boldsymbol{P}^{\mathrm{SP}} = \boldsymbol{B}_0\boldsymbol{\theta} + \boldsymbol{Q}_{\mathrm{cp}} + \boldsymbol{C}_{\mathrm{P}} \\ \boldsymbol{Q}_{\mathrm{cp}i} = \sum\limits_{j=1(j\neq i)}^{n} \dfrac{r_{ij}}{x_{ij}} \cdot Q_{ij} \\ \boldsymbol{C}_{\mathrm{P}i} = \sum\limits_{j=1(j\neq i)}^{n} g_{i0} + b_{i0}\dfrac{r_{ij}}{x_{ij}} \end{cases} \tag{15}$$

where $\boldsymbol{Q}_{\mathrm{cp}} = [Q_{\mathrm{cp}1}, \cdots, Q_{\mathrm{cp}i}, \cdots, Q_{\mathrm{cp}n}]^{\mathrm{T}}$ denotes the node-branch power coupling term, which couples with the node injected active power and branch reactive power; $\boldsymbol{C}_{\mathrm{P}} = [C_{\mathrm{P}1}, \cdots, C_{\mathrm{P}i}, \cdots, C_{\mathrm{P}n}]^{\mathrm{T}}$ denotes the node power-branch parameter coupling term.

As a result, if the branch reactive power $\boldsymbol{Q}_{ij}$ is given, the phase angle of node voltage can be obtained by solving (15), with the impedance of parallel branch and branch being considered.

C. VOLTAGE AMPLITUDE CORRECTION METHOD BASED ON NODAL POWER FLOW EQUATION

During normal operation conditions, the voltage amplitude of each node is usually near its rated value. In order to obtain the precise value, the assumption, i.e., $V_i = V_j = 1$ needs to be corrected by node voltage deviation.

As known, the injected power equation of a node can be represented as

$$\frac{P_i + \mathrm{j}Q_i}{\dot{V}_i} = \sum_{j=1}^{n} V_j(G_{ij} - jB_{ij})(\cos\theta_j - \mathrm{j}\sin\theta_j) \tag{16}$$

where P_i, Q_i are the net injected active and reactive powers of node i; $\dot{V}_i$, V_j are the node voltage phase and magnitude, respectively; G_{ij}, B_{ij} are the real and imaginary parts of the node admittance element, respectively; n is the node number in the system.

Since the voltage amplitudes of the PV and Vθ nodes are known, it is only necessary to modify the voltage amplitude of the PQ nodes. By sorting out the imaginary part of (16), (17) can be obtained as

$$\frac{\omega_i}{1+\Delta V_i} = -\sum_{j\in M}\eta_{ij}\left(1+\Delta V_j\right) - \sum_{j\in N}\eta_{ij}V_j \quad (i \in \boldsymbol{M}) \tag{17}$$

where ΔV_i denotes the voltage offset of node i; $\boldsymbol{M} = \{1, 2, \cdots\cdots, m\}$ represents the PQ node sets; ω_i, η_{ij} are the intermediate variables and can be defined as

$$\omega_i = Q_i\cos\theta_i - P_i\sin\theta_i. \tag{18}$$

$$\eta_{ij} = B_{ij}\cos\theta_j + G_{ij}\sin\theta_j. \tag{19}$$

Since the node voltage deviation of a power system, i.e., ΔV, is usually within $\pm 10\%$ in its steady-state operation, it can be assumed that $1/(1+\Delta V_i) \approx 1 - \Delta V_i$, which would result in an error less than $\pm 0.999\%$.

Regarding that the voltage amplitudes of the PV and V θ nodes are known, it is only necessary to modify the node voltage amplitude of PQ nodes. For m PQ nodes, the left side of (17) can be linearized and approximated to the matrix form as

$$\begin{bmatrix} -\eta_{11}+\omega_1 & \cdots & -\eta_{1j} & \cdots & -\eta_{1m} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ -\eta_{i1} & \cdots & -\eta_{ij}+\omega_i & \cdots & -\eta_{im} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ -\eta_{m1} & \cdots & -\eta_{mj} & \cdots & -\eta_{mm}+\omega_m \end{bmatrix} \cdot \begin{bmatrix} \Delta V_1 \\ \cdots \\ \Delta V_i \\ \cdots \\ \Delta V_m \end{bmatrix} = \begin{bmatrix} \omega_1 + \sum\limits_{j\in \boldsymbol{M}}\eta_{1j} + \sum\limits_{j\in \boldsymbol{N}}\eta_{1j}V_j \\ \cdots \\ \omega_i + \sum\limits_{j\in \boldsymbol{M}}\eta_{ij} + \sum\limits_{j\in \boldsymbol{N}}\eta_{ij}V_j \\ \cdots \\ \omega_m + \sum\limits_{j\in \boldsymbol{M}}\eta_{mj} + \sum\limits_{j\in \boldsymbol{N}}\eta_{mj}V_j \end{bmatrix} \tag{20}$$

The m-order coefficients matrix $\boldsymbol{A}$ and m-dimensional independent vector $\boldsymbol{b}$ are defined as

$$\boldsymbol{A}(i,j) = \begin{cases} -\eta_{ij} & i \neq j \\ -\eta_{ij}+\omega_i & i = j \end{cases}$$

$$\boldsymbol{b}(i) = \omega_i + \sum_{j\in \boldsymbol{M}}\eta_{ij} + \sum_{j\in \boldsymbol{N}}\eta_{ij}V_j \quad (i \in \boldsymbol{M}) \tag{21}$$

Then substituting (21) into (20), the node voltage correction equation can be obtained as

$$\boldsymbol{A} \cdot \Delta\boldsymbol{V} = \boldsymbol{b} \tag{22}$$

where $\Delta\boldsymbol{V}$ is m-dimensional voltage offset of the PQ nodes and can be obtained by solving (22). Then the voltage amplitude of PQ nodes can be modified as

$$\boldsymbol{V} = \boldsymbol{1} + \Delta\boldsymbol{V} \tag{23}$$

where $\boldsymbol{V}$ denotes the corrected voltage amplitude of m PQ nodes; $\boldsymbol{1}$ represents m-dimension all unit column vector.

IV. STEPS AND ARRANGEMENT OF THE PROPOSED FAST POWER FLOW CALCULATION

On the basis of the classical DC power load flow algorithm, the calculation method of node voltage phase angle and the correction method of node voltage amplitude are proposed by modifying the assumptions of DC power load flow calculation. Without iteration, the power flow distribution can be solved quickly. The implementation steps and process of the algorithm are as follows.

Step I: initialize the network parameters

According to the network topology, the nodes are numbered, and the branch is represented by ij, where i and j denote the starting and receiving points of the branch, respectively, with $i < j$. Then the network parameters, including the resistance, reactance and admittance of each branch, can be initialized.

Step II: set the initial value of the variables

Since the injected power of PQ and PV nodes are given, the equilibrium node and the initial voltage amplitude of each node can be set conveniently.

Step III: calculate the initial power of the branch

Establish the nodal admittance matrix considering the branch resistance and the nodal admittance matrix neglecting the branch resistance, respectively. The classical DC power flow algorithm is used to calculate the branch active power flow, and set as the initial value of the branch power.

Step IV: calculate the phase angle of the node voltage

The node-branch power coupling term $\boldsymbol{P}_{\mathrm{cp}}$ and the node power-branch parameter coupling term $\boldsymbol{C}_{\mathrm{Q}}$ are calculated. The node conductance matrix $\boldsymbol{G}_0$ can then be formed. Hence, based on these coupling terms ($\boldsymbol{P}_{\mathrm{cp}}$, $\boldsymbol{C}_{\mathrm{Q}}$) and node admittance matrix ($\boldsymbol{G}_0$), the node voltage phase angle ($\boldsymbol{\theta}$) can be calculated.

Step V: calculate the node voltage offset and correct the voltage amplitude

For m PQ nodes, the m-dimensional coefficient matrix $\boldsymbol{A}$ and the m-dimensional independent vector $\boldsymbol{b}$ can be formed by calculating the intermediate variables. The voltage amplitude offset $\Delta\boldsymbol{V}$ of PQ node is obtained by solving the linear voltage correction equation. And then the voltage amplitude $\boldsymbol{V}$ of the PQ node is modified.

Step VI: calculate the injected reactive power of PV node

As for PV nodes, the injected reactive power is calculated. If the reactive power exceeds the limit, the PV nodes will be converted into PQ nodes, and update the known input variables of the network. Then return back to **Step IV.**

Step VII: calculate the power flow of the branches and the total network power loss

Based on the obtained node voltage phase angle, i.e., $\boldsymbol{\theta}$ and the modified node voltage amplitude $\boldsymbol{V}$, the branch active and reactive power flow can be obtained.

Finally, the branch power loss and the whole network power loss can be calculated by the difference of the active power at both ends of the branch, which is given as

$$\begin{cases} \Delta P_{ij} = P_{ij} - P_{ji} \\ \Delta P = \sum \Delta P_{ij} \end{cases} \tag{24}$$

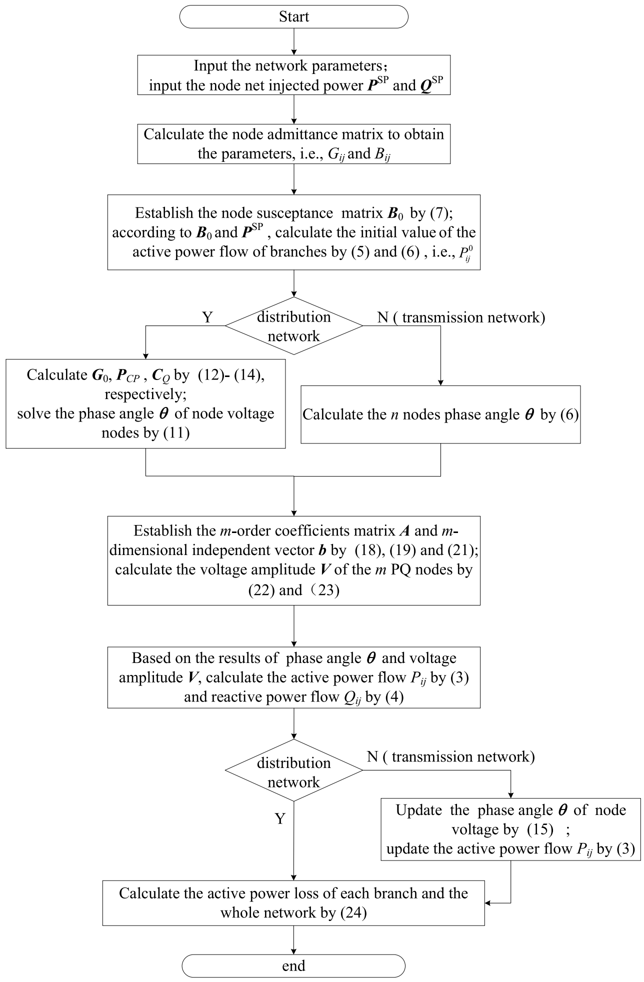

FIGURE 2. Flow chart of the fast power flow algorithm.

V. CASE STUDIES AND DISCUSSION

The effectiveness and feasibility of the proposed algorithm were tested and validated by simulation studies. Firstly, the standard IEEE 33-bus system (open distribution network) and IEEE 118-bus system (closed transmission network) were selected as tested examples. And the classical DC power plow algorithm, back/forward sweep algorithm, Newton-Raphson algorithm and other algorithms were also carried out for comparison.

Note that, for the open distribution network with high impedance ratio, the back/forward sweep algorithm is selected as the standard power flow algorithm. And for the closed power transmission networks with low impedance ratio, the Newton-Raphson algorithm is chosen as the standard power flow algorithm. The calculation results of the

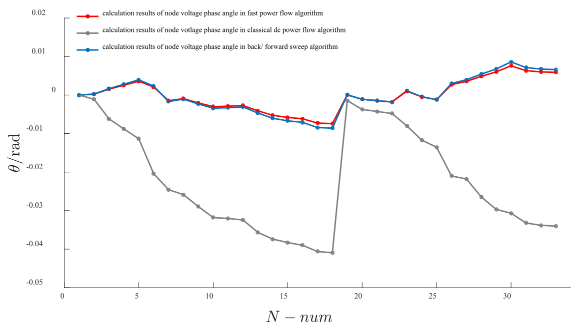

FIGURE 3. Comparison of node voltage phase angle in the IEEE 33-bus system.

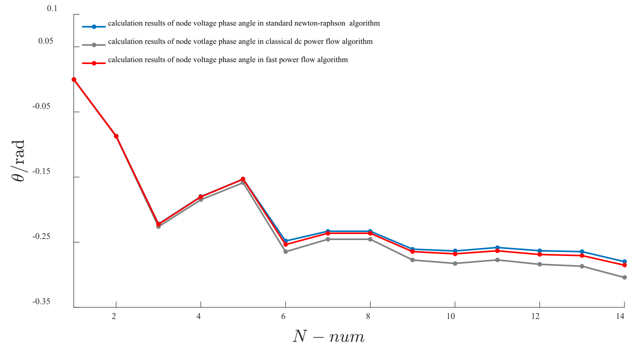

FIGURE 4. Comparison of node voltage phase angle in the IEEE 14-bus system.

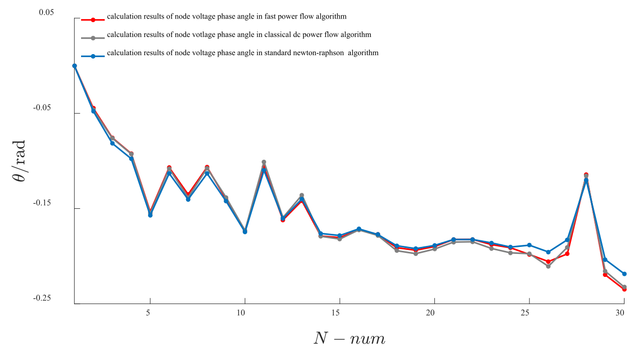

FIGURE 5. Comparison of node voltage phase angle in the IEEE 30-bus system.

standard power flow algorithms are then selected as the reference values.

Definite $\delta_{i(j)}$ to be the relative error between the calculated and the referred power flow. And suppose σ to represent the total deviation between the calculated and the referred value of power flow, i.e.,

$$\delta_{i(j)} = \left| \frac{\alpha_{i(j)} - \alpha_{i(j)}^{AC}}{\alpha_{i(j)}^{AC}} \right| \tag{25}$$

$$\sigma = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (\delta_{i(j)})^2} \tag{26}$$

where $\alpha_{i(j)}$ represents the calculated power flow of node i or branch (i, j), while $\alpha_{i(j)}^{AC}$ denotes the power flow obtained by the standard power flow algorithm; and n is the number of nodes.

A. ANALYSIS OF THE PHASE ANGLE CORRECTION OF THE NODE VOLTAGE

In order to study whether the proposed node voltage phase angle calculation method is applicable to the distribution network with high impedance ratio, the IEEE 33-bus distribution network was studied, with its node number being shown in Appendix. Note that in such network, about 78% of the branch impedance ratio is greater than 1, and almost 10% of the branch impedance ratio is greater than 3. The calculation results were compared with those from the classical DC power flow algorithm and the back/forward sweep algorithm as shown in Fig. 3.

As can be seen from Fig. 3, for the distribution network with high impedance ratio, the calculated node voltage phase angle with the classical DC power flow algorithm deviates greatly from those by the back/forward sweep algorithm, indicating that the classical DC power flow algorithm is not suitable for the distribution network with high impedance ratio. In contrast, the calculation results obtained through the proposed algorithm are quite similar to those with the standard algorithm, which means that the proposed algorithm is applicable in such high impedance ratio networks.

It is notable that the node voltage phase angle calculation method assumes that the node voltage amplitude is its rated value, i.e., 1p.u., and the deviation of the node voltage, i.e., ΔV is within $\pm 10\%$. However, the real distribution networks are usually with tree structures, and the node voltage of the tree branch will decline with the distance increasing from the first node. The node voltage offset at the end of the branch, which is far from the head of the power supply, may exceed -10%, such as the nodes numbered 14 to 18, 28 and 33 in the IEEE 33-bus system.

In order to validate whether the proposed algorithm is also suitable for the calculation of transmission networks with low impedance ratio, testes on the IEEE 14-bus and IEEE 30-bus transmission systems were then performed. And the calculation results are compared with those from the classical DC power flow algorithm, as shown in Figs. 4 and 5.

Furthermore, the IEEE 118-bus transmission system is studied with the proposed method and the classic DC flow algorithm. Considering that the nodes and routes are relatively more, the results obtained by the Newton-Raphson algorithm are regard as the reference values, as shown in Fig. 6.

Based on the tests aforementioned, the calculation errors of the phase angle are then summarized in Tab. 1, where the ‘‘average relative error’’ denotes the average value of all $\delta_{i(j)}$ in the tested system.

As can be seen from Tab. 1, compared with the reference value of the standard Newton-Rapson algorithm, the average relative error and total deviation degree of the voltage phase

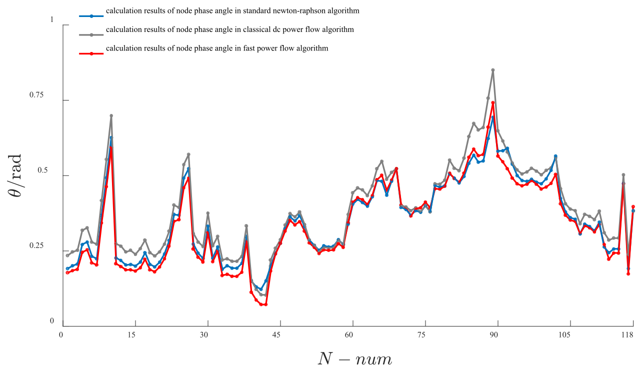

FIGURE 6. Comparison of voltage phase angle in the IEEE 118-bus system.

TABLE 1. Calculation errors of phase angle in example system.

Tested systems	The proposed power flow algorithm		The classical DC power flow algorithm	
	Average relative error	Total deviation	Average relative error	Total deviation
IEEE 14	0.0122	0.0098	0.0515	0.0435
IEEE 30	0.0299	0.0405	0.0324	0.0391
IEEE 33	0.0995	0.0530	7.6860	3.0758
IEEE 118	0.0609	0.0063	0.1027	0.1133

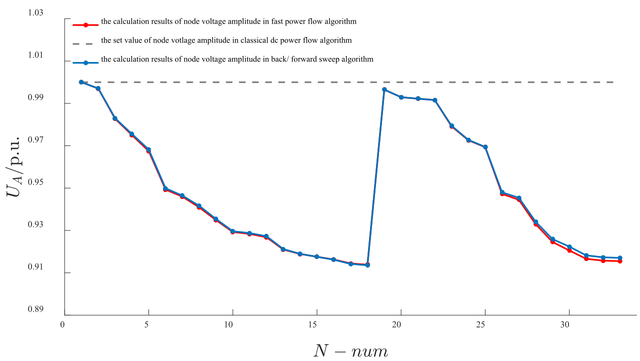

FIGURE 7. Comparison of node voltage amplitude in the IEEE 33-bus system.

angles by the proposed algorithm in all the IEEE 14-bus system, IEEE 30-bus system, IEEE 33-bus system, IEEE 118-bus systems are smaller than that of the classical DC power flow algorithm. In other words, the fast power flow algorithm presented in this paper can obtain a more accuracy power flow result.

B. ANALYSIS OF THE NODE VOLTAGE AMPLITUDE CALCULATION RESULTS

The proposed algorithm is firstly performed on the IEEE 33-bus system with high impedance ratio, and the results are compared with those from the classical DC power algorithm and back/forward sweep algorithm, as shown in Fig. 7.

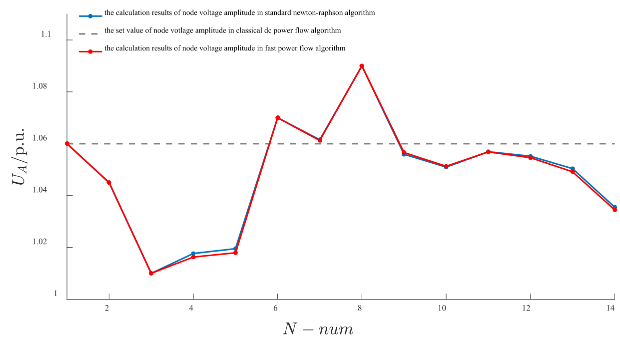

FIGURE 8. Comparison of node voltage amplitude in the IEEE 14-bus system.

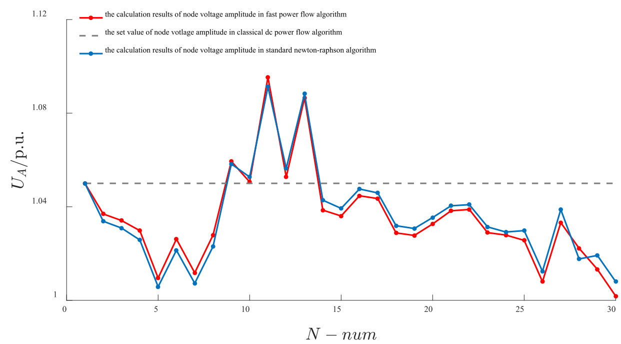

FIGURE 9. Comparison of node voltage amplitude in the IEEE 30-bus system.

It can be observed from Fig. 7 that, compared with the voltage amplitude reference value of back/forward sweep algorithm, the voltage amplitude calculation error by the fast power flow algorithm proposed in this paper is slightly smaller than that of the classical DC power flow algorithm, especially in the high number nodes.

In order to demonstrate the applicability of this algorithm in transmission network, the IEEE 14-bus system and IEEE 30-bus systems are then used as tested networks. The node voltage amplitudes obtained by the fast power flow algorithm were compared with those by the standard Newton-Raphson algorithm, as shown in Figs. 8 and 9.

Further studies are carried out on the voltage amplitude of each PQ nodes of the IEEE 118-bus transmission system, comparing the proposed fast power flow algorithm and the classical DC algorithm, as depicted in Fig. 10. From Fig. 10, it can be concluded that the voltage amplitude of each node obtained by fast power flow algorithm is consistent very well with the reference value that acquired by the standard Newton-Raphson algorithm.

To quantitatively evaluate the correctness of the proposed algorithm, the node-voltage calculation errors compared with the back/forward sweep algorithm and Newton-Raphson algorithm were summarized in Tab. 2. As shown, the maximum relative error, minimum relative error, average relative error and total deviation of node voltage amplitude by the fast power flow algorithm are all under the level of 10^{-3},

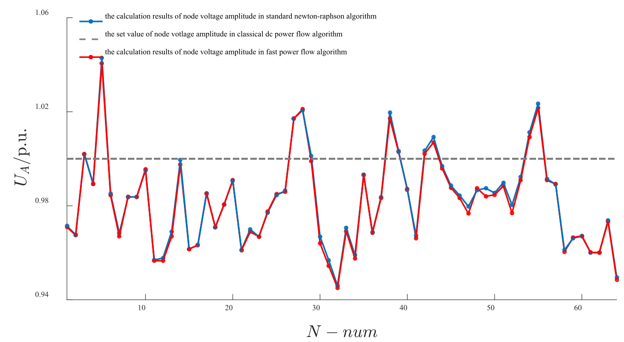

FIGURE 10. Comparison of node voltage amplitude in the IEEE 118-bus system.

TABLE 2. Calculation error of voltage amplitude in example system.

Tested systems	Maximum relative error	Minimum relative error	Mean relative error	Total deviation
IEEE 14	0.0010	0	0.0005	0.0005
IEEE 30	0.0061	0.0011	0.0033	0.0036
IEEE 33	0.0018	1.5106×10^{-5}	0.0005	0.0005
IEEE 118	0.0034	5.9115×10^{-6}	0.0009	0.0008

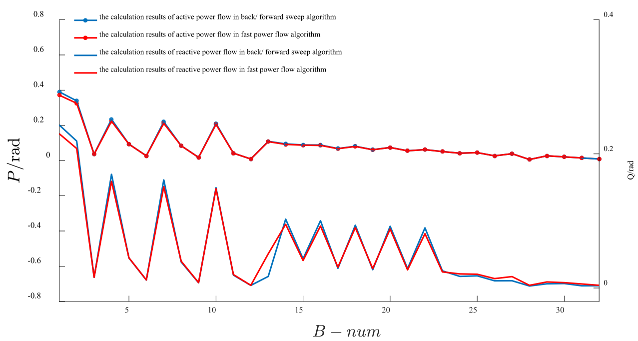

FIGURE 11. Active and reactive power flow in the IEEE 33-bus system.

indicating that the proposed method has a satisfactory calculation accuracy.

C. ANALYSIS OF THE CALCULATION RESULTS OF THE ACTIVE AND REACTIVE POWER FLOW OF BRANCH CIRCUITS

Firstly, to validate the effectiveness of the proposed method, the fast power flow algorithm is applied to calculate the active and reactive power flow of the IEEE 33-bus system, compared with the back/forward sweep algorithm, as shown in Fig. 11. It can be seen from the figure that the calculation results of active power flow by proposed algorithm are in accordance very well with the reference values by the back/forward sweep algorithm, so do that of the reactive power flow.

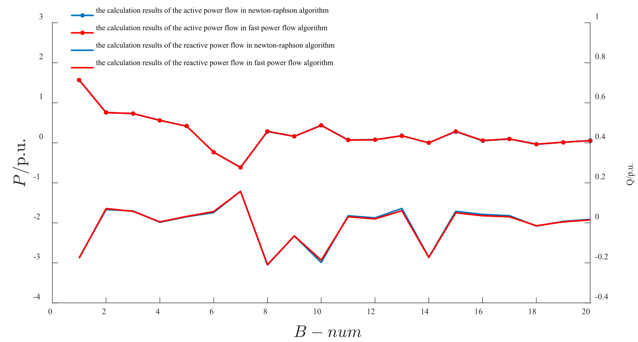

FIGURE 12. Active and reactive power flow in the IEEE 14-bus system.

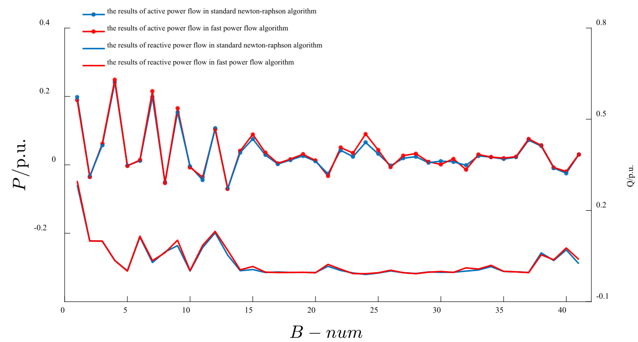

FIGURE 13. Active and reactive power flow in the IEEE 30-bus system.

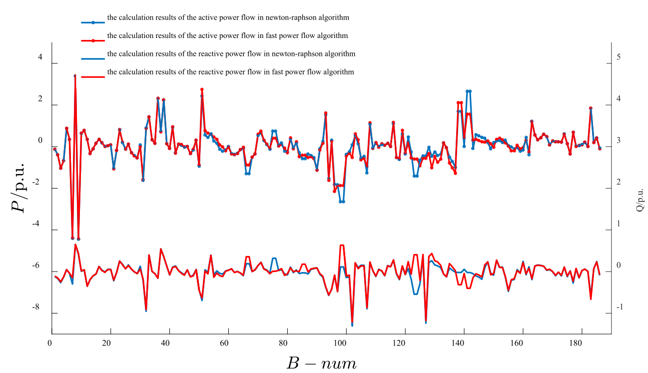

FIGURE 14. Active and reactive power flow in the IEEE 118-bus system.

Furthermore, tests are carried out on the IEEE 14-bus system, IEEE 30-bus system and IEEE 118-bus systems, with the results obtained by the standard Newton-Raphson algorithm being the reference values, as shown in Figs. 12 to 14. As shown in the figures, the calculation results acquired by the proposed method coincide very well with those from the Newton-Raphson algorithm.

Similarly, to give a qualitatively comparison, the calculation results in Fig. 14 are further handled and summarized in Tab. 3. It shows that compared with the classical DC power flow algorithm, the total deviation of the calculated active power flow calculated by the fast power flow algorithm is much smaller. And the error of the active power flow in heavy active power branch is also smaller than that of the classical DC power flow algorithm. Besides, the calculation results of

TABLE 3. Calculation error of active and reactive power flow in IEEE 118-bus system.

Algorithms	Active power flow		Reactive power flow	
	Average maximum relative error	Total deviation	Average maximum relative error	Total deviation
Classical DC power flow algorithm	0.0966	0.1920	—	—
Fast power flow algorithm	0.0877	0.1679	0.0175	0.0676

Note: the mean value of the maximum relative error corresponds to the calculation results of five active and heavy load branches and five reactive and heavy load branches, respectively.

TABLE 4. Comparison of network loss in IEEE 118-bus system.

Algorithm	Run time	Iterations	Maximum relative error	Minimum relative error	Mean relative error	Total deviation
Fast power flow algorithm	0.0086 s	1	0.0423	0	0.0027	0.0063
Network loss iteration algorithm [14]	0.0183 s	7	0.0587	0	0.0040	0.0095
Newton-Raphson algorithm	0.0522 s	3	—	—	—	—

TABLE 5. Comparison of computational efficiency in IEEE systems.

Algorithm	IEEE 33-bus system		IEEE 118-bus system	
	Run time	Iterations	Run time	Iterations
Fast power flow algorithm	0.0051s	1	0.0086 s	1
Back/forward sweep algorithm	0.0390s	5	—	—
Newton-Raphson algorithm	—	—	0.0522 s	3

reactive power flow obtained by the proposed algorithm also have a high accuracy.

D. ANALYSIS OF THE BRANCH NETWORK LOSS

In order to analyze the branch network loss based on the fast power flow algorithm, the IEEE118-bus system is selected. Again, the results by the Newton-Raphson algorithm are regarded as reference values. Then the proposed algorithm is compared with that based on network loss equivalent load model in [14]. The operational efficiency and calculation error of branch network loss are concluded in Tab. 4. It can be seen that the proposed fast power flow algorithm saves more computing resources since it does not need the iterative calculation. And the maximum relative error, the minimum relative error, the mean relative error and the total deviation of the branch network loss, achieved by the proposed algorithm are much smaller than those of the network loss iterative algorithm in [14].

To illustrate the in computational efficiency of the proposed algorithm, comparison is performed between the proposed algorithm and the back/forward sweep algorithm by taking the IEEE 33-bus system as an example of distribution network. Meanwhile, the IEEE 118-bus system is selected as an example to compare the computational efficiency between the standard Newton-Raphson algorithm and the proposed one. The results are summarized in Tab. 5, where the computational efficiency of the proposed algorithm in both transmission and distribution networks can be verified.

VI. CONCLUSION

Through analyzing the factors that affects the accuracy of the classical DC power flow algorithm, a fast power flow algorithm is proposed which adopts the linear equation structure of the classical DC power flow algorithm. While maintaining the characteristics of fast calculation speed, the proposed method improves the calculation accuracy and shows non-sensitivity on the impedance ratio. Conclusions can be summarized as follows.

(1) The node-injected reactive power matrix based on the branch reactive power flow equation is re-established with the branch active power and impedance parameters being considered, which makes up for the neglect of resistance and parallel branch in the classical DC power flow equation.

(2) The proposed approaches for calculating the voltage phase angle and correcting the voltage offset are both based on linearized equations, and thus the calculation process does not include iteration. Consequently, the algorithm can be easily applied to the on-line quasi-real-time power flow analysis.

(3) The proposed fast power flow algorithm does not depend by the network topologies and branch impedance ratios, and is suitable for both the distribution and the transmission networks. Test results on the standard IEEE power systems indicate that, the proposed method has satisfactory accuracy and good engineering application prospect as well.

(4) It should be noted that, the proposed algorithm does not consider the case of PV nodes in medium and low voltage distribution network. With more and more DGs connected to the distribution network, the influence of PV operation mode needs to be considered and will be studied in our future work.

APPENDIX

See Figure 15.

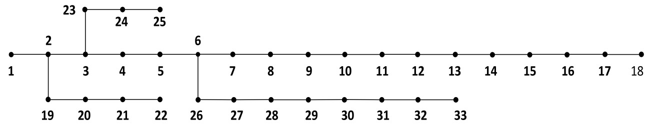

FIGURE 15. Topology diagram of IEEE 33-bus system with renumbered node number.

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