IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received January 13, 2020, accepted January 24, 2020, date of publication January 29, 2020, date of current version February 6, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.2970159

An Extended Intuitionistic Fuzzy Cognitive Map via Dempster-Shafer Theory

ZHUOSHENG JIA [1,2], YINGJUN ZHANG [2,3], (Member, IEEE), AND XUEMIN DONG [2,3]

[1] Information Technology Center, Beijing Jiaotong University, Beijing 100044, China
[2] School of Computer and Information Technology, Beijing Jiaotong University, Beijing 100044, China
[3] Beijing Key Laboratory of Traffic Data Analysis and Mining, Beijing Jiaotong University, Beijing 100044, China

Corresponding author: Yingjun Zhang (zhangyj@hit.edu.cn)

This work was supported in part by the National Key Research and Development Program of China under Grant 2017YFF0108303, and in part by the National Natural Science Foundation of China under Grant 51827813.

ABSTRACT Fuzzy cognitive map has gradually emerged as a powerful paradigm for uncertain knowledge representation and a simulation mechanism that is applicable in dealing with complex artificial reasoning problems. To better model uncertain inference reasoning problems, we propose an extended intuitionistic fuzzy cognitive map via Dempster-Shafer theory. First of all, some new operations on IFSs are introduced from the perspective of Dempster-Shafer theory. Then, the extended intuitionistic fuzzy cognitive map is established via the proposed new operations. Next, we investigate the problem of modeling complex system from multiple decision makers using extended intuitionistic fuzzy cognitive maps and present a method to aggregate a number of maps. Particular emphases are put on defining the augmented connection matrices, determining the importance levels of different extended intuitionistic fuzzy cognitive maps and aggregating them. Finally, the performances of extended intuitionistic fuzzy cognitive maps have been validated through a number of simulations. The simulations indicate that the theory of extended intuitionistic fuzzy cognitive map not only provides much more choices to model complex system but also reduces the computational complexity by comparison with intuitionistic fuzzy cognitive map.

INDEX TERMS System modeling, extended intuitionistic fuzzy cognitive map (EIFCM), intuitionistic fuzzy sets, Dempster-Shafer theory.

I. INTRODUCTION

Since Kosko [1] introduced the fuzzy cognitive map (FCM), the theory has gained considerable research interests and has been widely utilized in a number of fields, such as time series modeling [2], decision making [3], prediction [4], cooperative sensing in cognitive radio [5], etc. To enhance the performance of modelling the complex system, a number of granules, such as interval-valued fuzzy sets, grey sets, rough sets and intuitionistic fuzzy sets (IFSs), have been successfully employed to design high-order cognitive maps [5]–[9].

The existing studies indicate that various granular cognitive maps are more effective in system modeling with uncertainty than FCM. Among all granular theories, IFS belongs to an effective generalization of fuzzy set and extends fuzzy set by the membership degree, the non-membership degree and the uncertain degree to depict the uncertain information, where the three values are located in the interval [0, 1] and their sum equals to 1 [10], [11]. In view of its performance to depict and handle uncertain information, IFS has been successfully employed in a number of fields, such as decision making [12], patter classification [13], image processing [14], uncertain information fusion [15], [16] and cognitive learning [9]. Note that intuitionistic fuzzy cognitive map (IFCM) [8], [9], a generalization of FCM, provides more choices to model uncertain problems, such as process control, medical decision support and social economic problems. In particular, IFCM covers three key aspects [8]–[10]: (1) IFCM represents the state of nodes and the connections via intuitionistic fuzzy number (IFN) to describe the uncertain information. (2) IFCM employs conventional addition operation, multiplication operation and sigmoid functions in the exploration process. (3) IFCM's transformation function just depends on the membership degree and the non-membership degree to determine the state of all the nodes in each iteration. Despite the better performance of modeling uncertain system than FCM, IFCM may induce unreasonable results in system inference due to its operations in iterative

The associate editor coordinating the review of this manuscript and approving it for publication was Fan-Hsun Tseng.

process [9]. What's more, the problems of both aggregating a number of different IFCMs and quantifying their importance levels (weights) have not been fully considered [8], [9].

To overcome the limitations and to enhance the performances of IFCM, we propose an extended intuitionistic fuzzy cognitive map (EIFCM) via Dempster-Shafer (D-S) theory [17]. The main contributions of the EIFCM can be categorized as follows.

- We propose an aggregation operator, a new multiplication and a similarity degree between intuitionistic fuzzy matrices as supplements of conventional basic operations on IFSs. Note that an IFN (μ, ν, π) may be regarded as a piece of evidence and D-S theory belongs to an effective fusion scheme to aggregate evidences. Thus, D-S theory can be effectively utilized within the framework of IFS. Then, an aggregation operator and a new multiplication are introduced from the perspective of D-S theory [17]. By comparison with conventional basic operations, both the aggregation operator and the multiplication consider both the membership degree and the non-membership degree with equal status which reflects the essence of the concept of IFS. The result is that the two operations can overcome some unreasonable results derived from conventional operations on IFSs in processing some uncertain fusion or decision making problems. In addition, the similarity degree presents an effective way to describe the divergence between two intuitionistic fuzzy matrices.
- We present a complete mathematical frame of the EIFCM via the proposed operations. In addition, the EIFCM fully considers three elements of IFSs while the IFCM just employs the membership degree and the non-membership degree during the activation process of concepts. Thus, the EIFCM not only has much more choices to choose the transformation function but also can meet more system modeling problems than the IFCM.
- We propose a scheme for solving the problem of aggregating a number of EIFCMs. To better model inference reasoning problems in complex system, it is of great importance to aggregate knowledge from different maps. This scheme mainly solves three challenges including augmenting connection matrices, determining the importance levels of different maps and aggregating them. What's more, this scheme can be employed in aggregating IFCMs.

In brief, in view of above three aspects, it is certain that that the theory of EIFCM delivers a new vision of system modeling.

This paper is organized as follows. In Section II, we recall some preliminaries to be utilized in the whole paper. In Section III, we present some new operations on IFSs as supplements of conventional theories. In Section IV, we establish a complete mathematical frame of the EIFCM via the new proposed operations. Meanwhile, we investigate the problem of aggregating knowledge in the framework of the EIFCMs. In Section V, we utilize a number of simulations to validate the performance of the EIFCM by comparison with conventional models. Section VI concludes the paper.

II. PRELIMINARIES

A. INTUITIONISTIC FUZZY SET

In 1986, Atanassov [10] introduced the concept of IFS which utilize the membership degree, the non-membership degree and the uncertain degree to depict the uncertain information.

Definition 1 [10]: Let X be a set, an IFS A on X is defined as $A = \{(x, \mu_A(x), \nu_A(x)) | x \in X\}$, where $\mu_A : X \to [0, 1]$ and $\nu_A : X \to [0, 1]$ are two maps satisfying $0 \le \mu_A(x) + \nu_A(x) \le 1$ for all $x \in X$. $\mu_A(x)$ and $\nu_A(x)$ denote the membership degree and the non-membership degree of x to A, respectively. For each IFS A in X, we designate $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ an intuitionistic index of x in A. In order to facilitate the description, we adopt $\alpha = (\mu_\alpha, \nu_\alpha, \pi_\alpha)$ or $\alpha = (\mu_\alpha, \nu_\alpha)$ to denote an intuitionistic fuzzy number (IFN).

Definition 2 [10]: For two IFNs $\alpha = (\mu_\alpha, \nu_\alpha)$ and $\beta = (\mu_\beta, \nu_\beta)$, the basic operations between them are with the following forms,

$$\alpha \oplus \beta = (\mu_\alpha + \mu_\beta - \mu_\alpha\mu_\beta, \nu_\alpha\nu_\beta), \tag{1}$$

$$\alpha \otimes \beta = (\mu_\alpha\mu_\beta, \nu_\alpha + \nu_\beta - \nu_\alpha\nu_\beta), \tag{2}$$

$$\neg\alpha = (\nu_\alpha, \mu_\alpha). \tag{3}$$

B. DEMPSTER-SHAFER THEORY

As a generalization of the Bayesian theory of subjective probability, D-S theory allows one to combine those evidence from different sources and get a degree of belief which considers all the available information [17]. Due to good performance in aggregating uncertain information, D-S theory has been deeply discussed and successfully utilized in decision making [18], [19], pattern classification [20], spectrum sensing [21], etc. In what follows, we briefly recall D-S theory.

Definition 3 [17]: Let Θ be a finite set called the discernment frame and 2^Θ be the power set of Θ. A basic belief assignment (bba) is a mapping $m : 2^\Theta \to [0, 1]$ which satisfies $\sum_{A \subseteq \Theta} = 1$.

Definition 4 [17]: Let Θ be a finite set called the discernment frame and 2^Θ be the power set of Θ. A belief (credibility) function and a plausibility functions o $A \subseteq \Theta$ are defined by

$$Bel(A) = \sum_{B \in 2^\Theta, B \subseteq A} m(B), \tag{4}$$

$$Pl(A) = \sum_{B \in 2^\Theta, B \cap A \neq \emptyset} m(B) = 1 - Bel(\bar{A}), \tag{5}$$

where $\bar{A}$ denotes the complement of A in Θ.

Definition 5 [17]: Let m_1 and m_2 be two bbas defined on Θ which are derived from two different sources. Then the combined bba is defined as

$$m_1 \oplus m_2(A) = \begin{cases} 0 & A = \emptyset \\ \dfrac{\sum_{B,C \subseteq \Theta, B \cap C = A} m_1(B)m_2(C)}{1 - \sum_{B,C \subseteq \Theta, B \cap C = \emptyset} m_1(B)m_2(C)} & A \neq \emptyset \end{cases} \tag{6}$$

when $\sum_{B,C\subseteq\Theta,B\cap C=\emptyset} m_1(B)m_2(C) \neq 1$. $K = \sum_{B,C\subseteq\Theta,B\cap C=\emptyset} m_1(B)m_2(C)$ *is called the degree of conflict between the two mass sets.*

III. THE NEW OPERATIONS ON INTUITIONISTIC FUZZY SETS

A. OPERATIONS

On the basis of D-S theory [17], [22], we introduce a special fusion rule as below.

Definition 6: Let $\Theta = \{A_1, A_2, A_3\}$ *be the frame of discernment. Mass function from different J information sources are denoted by* m_j $(j = 1, 2, \ldots, J)$ *satisfying* $m_j(A_1) + m_j(A_2) + m_j(A_3) = 1$, $0 \leq m_j(A_1), m_j(A_2), m_j(A_3) \leq 1$. *Then the fusion rule* $\bar{\oplus}$ *is with the following methematical form*:

$$m(A_i) = \bar{\oplus}_{j=1}^{J} m_j(A_i) = \begin{cases} \dfrac{\sum_{B_1\cap\cdots\cap B_J=A_i,\ B_j\in\Theta} \prod_{j=1}^{J} m_j(B_j)}{1-K} & (i) \\ \dfrac{1}{J}\sum_{j=1}^{J} m_j(A_i) & (ii) \end{cases} \tag{7}$$

$$K = \sum_{B_j\in\Theta,\ B_1\cap\cdots\cap B_J=\emptyset} \prod_{j=1}^{J} m_j(B_j) \tag{8}$$

where (i) : $K \neq 1$ *and* $\prod_{j=1}^{J} m_j(A_i) \neq 0$ $(i = 1, 2)$; (ii) : $K = 1$ *or* $\prod_{j=1}^{J} m_j(A_i) = 0$.

Following Definition 6 and relevant works [17], we present an aggregation operator on IFS.

Definition 7: For n IFNs $\alpha_i = (\mu_i, \nu_i)$ $(i = 1, 2, \ldots, n)$, *a D-S-theory-based intuitionistic fuzzy aggregation (DSIFA) operator is defined by*

$$DSIFA_w(\alpha_1, \alpha_2, \ldots, \alpha_n) = (\bar{\oplus}_{i=1}^{n} m_i(A_1), \bar{\oplus}_{i=1}^{n} m_i(A_2)), \tag{9}$$

where $m_i(A_1) = w_i\mu_i$, $m_i(A_2) = w_i\nu_i$, $w_i \in [0, 1]$ *and* $\max_{i\in 1,2,\ldots,n}\{w_i\} = 1$.

Remark 1: As indicated in Definition 7, the DSIFA operator not only provides a new way to aggregate a number of IFNs but also describes their importance levels. Different from the existing aggregation operators, the weights of all IFNs are located in the interval $[0, 1]$ *and the maximum value equals to 1 for the DSIFA operator. Clearly, the weights can be transferred between the DSIFA operators and others.*

As can be observed from [10], the three elements of three IFS have not been considered equally in the multiplication operation. As proved in [25], the operation may induce some unreasonable in some fusion problems. To equally reflect the three elements of IFS, we present a new multiplication operation as supplements of conventional operations on IFS.

Definition 8: For two IFNs $\alpha = (\mu_\alpha, \nu_\alpha)$ *and* $\beta = (\mu_\beta, \nu_\beta)$, *a new multiplication rule* $\bar{\otimes}$ *is defined by*

$$\alpha\bar{\otimes}\beta = DSIFA_w(\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6), \tag{10}$$

where $w = \frac{1}{C}[\mu_\beta\ \ \nu_\beta\ \ 1-\mu_\alpha-\nu_\beta\ \ \mu_\alpha\ \ \nu_\alpha\ \ 1-\mu_\alpha-\nu_\alpha]^T$, $C = \max\{\mu_\beta, \nu_\beta, 1-\mu_\alpha-\nu_\beta, \mu_\alpha, \nu_\alpha, 1-\mu_\alpha-\nu_\alpha\}$, $\gamma_1 = (\mu_\alpha, \nu_\alpha)$, $\gamma_2 = (\mu_\alpha, \nu_\alpha)$, $\gamma_3 = (\frac{\mu_\alpha+\nu_\alpha}{2}, \frac{\mu_\alpha+\nu_\alpha}{2})$, $\gamma_4 = (\mu_\beta, \nu_\beta)$, $\gamma_5 = (\mu_\beta, \nu_\beta)$ *and* $\gamma_6 = (\frac{\mu_\beta+\nu_\beta}{2}, \frac{\mu_\beta+\nu_\beta}{2})$.

In what follows, we employ two examples to validate the performance of the DSIFA operator and the multiplication rule.

Example 1: There are 10 IFNs to be aggregated via the DSIFA operator and a number of conventional operators [23]–[27] as TABLE 1.

Example 2: For two IFNs $\alpha = (0.5, 0.5)$ *and* $\beta = (0.5, 0.5)$, *we get*

$$\alpha \otimes \beta = (0.25, 0.75),$$
$$\alpha\bar{\otimes}\beta = (0.5, 0.5).$$

Remark 2: As can be observed from TABLE 1, the aggregated IFN of a number of IFNs is $(1, 0)$ *if there is one equaling to* $(1, 0)$ *for some operators. Similarly, the aggregated IFN is* $(0, 1)$ *of a number of IFNs from some aggregation operators [23]–[27] if one is* $(0, 1)$. *In contrast with conventional operators, the fusion value derived from DSIFA describes the majority of aggregated IFNs. From the perspective of IFS,* $(0.5, 0.5)$ *implies that we can not distinguish the importance levels of the membership degree or the non-membership degree. As shown in Example 2,* $\bar{\otimes}$ *better reflects the essence of IFS than* $\otimes$.

B. SIMILARITY ON INTUITIONISTIC FUZZY MATRICES

As well known, the similarity degree (or the distance) presents an effective way to describe the divergence between two vectors or matrices [12]. In order to describe the divergence between two intuitionistic fuzzy matrices, we define a similarity degree on intuitionistic fuzzy matrices as follows.

Definition 9: Let A and B be two $n \times n$ *intuitionistic fuzzy matrices, where*

$$A = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{n1} & \alpha_{n1} & \cdots & \alpha_{nn} \end{pmatrix}, \tag{11}$$

$$B = \begin{pmatrix} \beta_{11} & \beta_{12} & \cdots & \beta_{1n} \\ \beta_{21} & \beta_{22} & \cdots & \beta_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{n1} & \beta_{n1} & \cdots & \beta_{nn} \end{pmatrix}. \tag{12}$$

Then the similarity $Sim(A, B)$ *between A and B is defined by*

$$Sim(A, B) = 1 - \frac{1}{2n^2}\sum_{j=1}^{n}\sum_{j'=1}^{n}\theta(\alpha_{jj'}, \beta_{jj'}), \tag{13}$$

where

$$\theta(\alpha_{jj'}, \beta_{jj'}) = |\mu(\alpha_{jj'}) - \mu(\beta_{jj'})| + |\nu(\alpha_{jj'}) - \nu(\beta_{jj'})| + |\pi(\alpha_{jj'}) - \pi(\beta_{jj'})|. \tag{14}$$

Note that $\alpha_{jj'} = (\mu(\alpha_{jj'}), \nu(\alpha_{jj'}), \pi(\alpha_{jj'}))$ *and* $\beta_{jj'} = (\mu(\beta_{jj'}), \nu(\beta_{jj'}), \pi(\beta_{jj'}))$ *are two IFNs [10].*

Obviously, the proposed similarity between two intuitionistic fuzzy matrices satisfies the following theorem.

TABLE 1. Aggregation results based on different operators.

Case	Operators	Relevant parameters α_i	Relevant parameters w	Results
1	IFWA	$\alpha_1=(1,0)$ $\alpha_j=(0.1,0.9),\ j=2,\ldots,10$	$w_i=0.1$ $i=1,2,\ldots,10$	$(1,0)$
	IFOWA	$\alpha_1=(1,0)$ $\alpha_j=(0.1,0.9),\ j=2,\ldots,10$	$w_i=0.1$ $i=1,2,\ldots,10$	$(1,0)$
	IFPWA	$\alpha_1=(1,0)$ $\alpha_j=(0.1,0.9),\ j=2,\ldots,10$	$w_i=0.1$ $i=1,2,\ldots,10$	$(1,0)$
	IFHWA	$\alpha_1=(1,0)$ $\alpha_j=(0.1,0.9),\ j=2,\ldots,10$	$w_i=0.1$ $i=1,2,\ldots,10$	$(1,0)$
	IFHOWA	$\alpha_1=(1,0)$ $\alpha_j=(0.1,0.9),\ j=2,\ldots,10$	$w_i=0.1$ $i=1,2,\ldots,10$	$(1,0)$
	IFEWA	$\alpha_1=(1,0)$ $\alpha_j=(0.1,0.9),\ j=2,\ldots,10$	$w_i=0.1$ $i=1,2,\ldots,10$	$(1,0)$
	IFEOWA	$\alpha_1=(1,0)$ $\alpha_j=(0.1,0.9),\ j=2,\ldots,10$	$w_i=0.1$ $i=1,2,\ldots,10$	$(1,0)$
	IFWGA	$\alpha_1=(1,0)$ $\alpha_j=(0.1,0.9),\ j=2,\ldots,10$	$w_i=0.1$ $i=1,2,\ldots,10$	$(1,0)$
	IFOWGA	$\alpha_1=(1,0)$ $\alpha_j=(0.1,0.9),\ j=2,\ldots,10$	$w_i=0.1$ $i=1,2,\ldots,10$	$(1,0)$
	DSIFA	$\alpha_1=(1,0)$ $\alpha_j=(0.1,0.9),\ j=2,\ldots,10$	$w_i=1$ $i=1,2,\ldots,10$	$(0.19,0.81)$
2	IFWG	$\alpha_1=(0,1)$ $\alpha_j=(0.9,0.1),\ j=2,\ldots,10$	$w_i=0.1$ $i=1,2,\ldots,10$	$(0,1)$
	IFHGWA	$\alpha_1=(0,1)$ $\alpha_j=(0.9,0.1),\ j=2,\ldots,10$	$w_i=0.1$ $i=1,2,\ldots,10$	$(0,1)$
	IFHGOWA	$\alpha_1=(0,1)$ $\alpha_j=(0.9,0.1),\ j=2,\ldots,10$	$w_i=0.1$ $i=1,2,\ldots,10$	$(0,1)$
	IFEGA	$\alpha_1=(0,1)$ $\alpha_j=(0.9,0.1),\ j=2,\ldots,10$	$w_i=0.1$ $i=1,2,\ldots,10$	$(0,1)$
	IFEOGA	$\alpha_1=(0,1)$ $\alpha_j=(0.9,0.1),\ j=2,\ldots,10$	$w_i=0.1$ $i=1,2,\ldots,10$	$(0,1)$
	DSIFA	$\alpha_1=(0,1)$ $\alpha_j=(0.9,0.1),\ j=2,\ldots,10$	$w_i=1$ $i=1,2,\ldots,10$	$(0.81,0.19)$

Theorem 1: Let A and B be two $n\times n$ intuitionistic fuzzy matrices defined as Equations (13) and (14), then $Sim(A,B)$ satisfies the following three properties.

(1) $0\le Sim(A,B)\le 1$;

(2) $Sim(A,B)=Sim(B,A)$;

(3) $Sim(A,B)=1$ if and only if $A=B$.

Proof: (1) Since $0\le|\mu(\alpha_{jj'})-\mu(\beta_{jj'})|+|\nu(\alpha_{jj'})-\nu(\beta_{jj'})|+|\pi(\alpha_{jj'})-\pi(\beta_{jj'})|\le\mu(\alpha_{jj'})+\nu(\alpha_{jj'})+\pi(\alpha_{jj'})+\mu(\beta_{jj'})+\nu(\beta_{jj'})+\pi(\beta_{jj'})$ and $\mu(\alpha_{jj'})+\nu(\alpha_{jj'})+\pi(\alpha_{jj'})+\mu(\beta_{jj'})+\nu(\beta_{jj'})+\pi(\beta_{jj'})=2$ hold, we have $0\le Sim(A,B)\le 1$.

(2) As shown in (14), $Sim(A,B)=Sim(B,A)$ obviously holds.

(3) $\Rightarrow$ $Sim(A,B)=1$ implies that $\theta(\alpha_{jj'},\beta_{jj'})=0$ $(j,j'\in\{1,2,\ldots,n\})$. $\theta(\alpha_{jj'},\beta_{jj'})=0$ is equivalent to $\mu(\alpha_{jj'})=\mu(\beta_{jj'})$, $\nu(\alpha_{jj'})=\nu(\beta_{jj'})$ and $\pi(\alpha_{jj'})=\pi(\beta_{jj'})$, i.e., $A=B$. $\Leftarrow$ $A=B$ means that $\theta(\alpha_{jj'},\beta_{jj'})=0$ $(j,j'\in\{1,2,\ldots,n\})$. Clearly, $Sim(A,B)=1$ holds. ■

IV. EXTENDED INTUITIONISTIC FUZZY COGNITIVE MAP

Based on FCM and its generalizations [1], [5], [6], [8], [9], we present the EIFCM via the proposed operations. What's more, a scheme is proposed in dealing with the problem of aggregating fuzzy knowledge networks within the framework of EIFCM.

A. THE CONCEPT OF EXTENDED INTUITIONISTIC FUZZY COGNITIVE MAP

An EIFCM is characterized by its connection matrix E. It is the crux of the EIFCM. Connections between any two concepts in an EIFCM are denoted by IFNs. Input and output information (activations and responses) in an EIFCM is also expressed by IFNs.

To clearly state the concept of the EIFCM, some abbreviations are employed as below.

- $IFSs(X)$ - all the IFSs on X
- $N=\{N_1,N_2,\ldots,N_n\}$ - the set of n concepts forming the nodes of a map
- w_{ij} - a weight of directed edge from N_i to N_j
- $E=[w_{ij}]_{n\times n}$ - connection matrix
- $C(t)=[C_1(t)\ C_2(t)\ \ldots\ C_n(t)]^T$ - all concept values from vector C at the moment t

EIFCM exploration is based on activations $C(t)$, which are processed with the connection matrix E according to the formula

$$C_i(t+1)=F\big(DSIFA(C_i(t),C_{1'}(t)\bar{\otimes}w_{1'i},\ldots,C_{s'}(t)\bar{\otimes}w_{s'i})\big) \tag{15}$$

where $w_{j'i}$ means all non-(0, 0) connections from the j'th ($j'\in\{1,2,\ldots,n\}$) concept to the ith concept. $F: IFSs(X)\to L$ is a transformation function with the following form,

$$F(\alpha)=\Big(\frac{f(\mu)}{\Upsilon},\frac{f(\nu)}{\Upsilon}\Big), \tag{16}$$

$$\Upsilon=f(\mu)+f(\nu)+f(\pi). \tag{17}$$

Remark 3: By comparison with IFCM, the differences of EIFCM cover the following three aspects. (1) Both the DSIFA operator and $\bar{\otimes}$ are employed to construct the EIFCM instead of conventional operations (see $\oplus$ and $\otimes$) from the IFCM. (2) The EIFCM represents the default connection through

(0, 0) *instead of 0 in IFCM. (3) EIFCM has more freedom to choose the transformation function than IFCM.*

B. AGGREGATING FUZZY KNOWLEDGE NETWORKS

In general, aggregating knowledge from multiple experts can provide much more reliable or reasonable decision results than single expert. It can be concluded that it is of great importance to consider the problem of aggregating knowledge in the establishment of cognitive models [7], [9]. As indicated in [9], any set of cognitive maps can be naturally aggregated through adding their pointwise augmented connection matrices. The key points of aggregating knowledge within the framework of EIFCM cover quantifying the augmented connection matrices, determining the importance levels of different EIFCMs and aggregating them. In what follows, we present the proposed scheme of aggregating fuzzy knowledge within the framework of EIFCM. The flowchart of this scheme is shown as FIGURE 1.

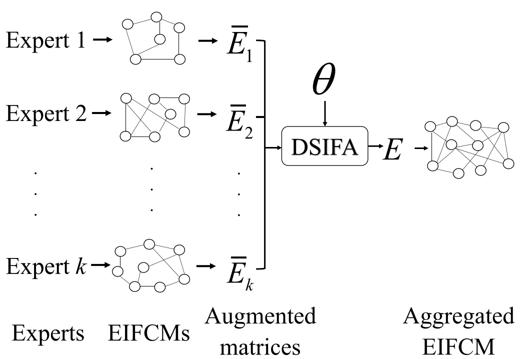


FIGURE 1. The flowchart of aggregating EIFCMs.

1) PROBLEM DESCRIPTION

Suppose there are k experts each draw an EIFCM. The *ith* expert's EIFCM is equivalent to an $n_i \times n_i$ connection matrix E_i.

2) QUANTIFYING THE AUGMENTED CONNECTION MATRICES

Transform connection matrices E_i $(i = 1, 2, \ldots, k)$ to augmented connection matrices $\bar{E}_i$ $(i = 1, 2, \ldots, k)$. In general, these different connection matrices are not likely to be conformable for aggregating directly. Suppose the first EIFCM utilize a concept N_1 that is not utilized in the second one. It implies that there are not any causal relationships between N_1 and every concept in the second one. Then E_2 can be augmented to include N_1 by adding a row and column of all (0, 0). If the total number of distinct concepts of k EIFCMs is n, then each connection matrix E_i is augmented to a $n \times n$ matrix $\bar{E}_i$ as

$$\bar{E}_i = \begin{pmatrix} w_{11}^{(i)} & w_{12}^{(i)} & \cdots & w_{1n}^{(i)} \\ w_{21}^{(i)} & w_{22}^{(i)} & \cdots & w_{2n}^{(i)} \\ \vdots & \vdots & \ddots & \vdots \\ w_{n1}^{(i)} & w_{n1}^{(i)} & \cdots & w_{nn}^{(i)} \end{pmatrix}. \quad (18)$$

3) DETERMINE THE WEIGHTS OF ALL EIFCMs

For aggregating above k maps, the first task is to determine their weights w. If w is known, we just need to aggregate all the augmented connection matrices $\bar{E}_i$ $(i = 1, 2, \ldots, k)$ through the DSIFA operator. Concerning the situation that the weights are completely unknown, we propose a consensus-based method to assess the importance levels of different maps. From the perspective of consensus, those maps with higher similarities to others will be given larger values and vice versa. As analyzed above, the weights of different maps are defined by

$$\theta_i = \frac{\Gamma_i}{\max\{\Gamma_1, \Gamma_2, \ldots, \Gamma_k\}}, \quad (19)$$

where $\theta_i \in [0, 1]$ and $\Gamma_i = \sum_{i'=1, i' \neq i}^{k} Sim(\bar{E}_i, \bar{E}_{i'})$.

4) AGGREGATING ALL EIFCMs

Let E be the aggregated connection matrix as

$$E = \begin{pmatrix} w_{11} & w_{12} & \cdots & w_{1n} \\ w_{21} & w_{22} & \cdots & w_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ w_{n1} & w_{n1} & \cdots & w_{nn} \end{pmatrix}. \quad (20)$$

On the basis of both (18) and (19), we get

$$w_{jj'} = DSIFA_{\theta}(w_{jj'}^{(1)}, w_{jj'}^{(2)}, \ldots, w_{jj'}^{(k)}), \quad j, j' = 1, 2, \ldots, n \quad (21)$$

where $\theta = [\theta_1, \theta_2, \ldots, \theta_k]^T$ is the weight vetor of all maps to be aggregated.

V. SIMULATION AND ANALYSIS

Two examples are employed to validate the performance of the proposed EIFCM by comparison with IFCM.

A. MEDICAL DECISION SUPPORT PROBLEM

Here we consider a pneumonia risk decision making problem [9], [28] which contains seven symptoms of cough (N_1), fever (N_2), rigor (N_3), the radiological evidences of pneumonia (N_4), the dyspnoea (N_5), the immunosuppression (N_6) and the risk of infection (N_7). Above seven symptoms are the concepts of the considered cognitive models as FIGURE 2.

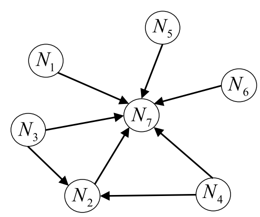


FIGURE 2. A cognitive map for pneumonia risk decision making [28].

1) EIFCM VS. IFCM

Here we employ both EIFCM and IFCM to model above problem [9]. Suppose all the concept values are denoted by IFNs, where the membership degree, the non-membership

TABLE 2. **The connection matrix.**

	N_1	N_2	N_3	N_4	N_5	N_6	N_7
N_1							$(0.70, 0.10)$
N_2							$(0.60, 0.30)$
N_3		$(0.65, 0.10)$					$(0.60, 0.15)$
N_4		$(0.60, 0.30)$					$(0.60, 0.10)$
N_5							$(0.60, 0.30)$
N_6							$(0.50, 0.40)$

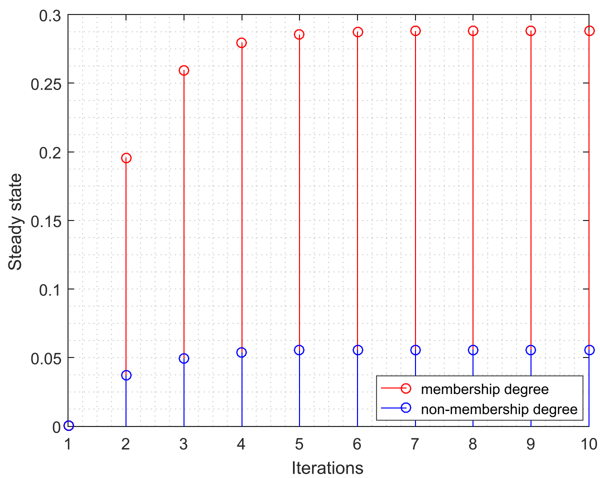


FIGURE 3. **The change tendency of C_2 for Case 1.**

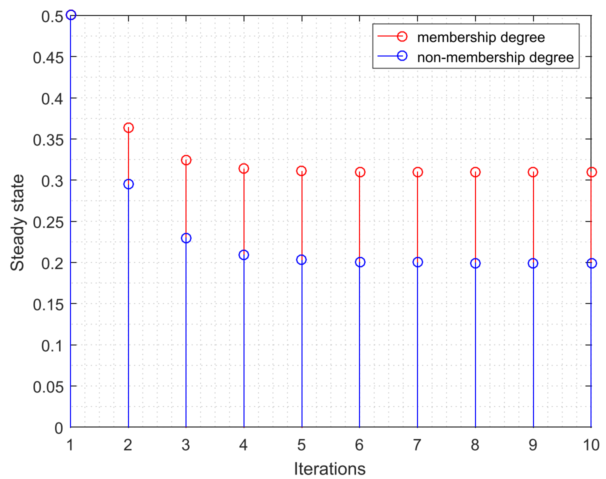


FIGURE 5. **The change tendency of C_2 for Case 2.**

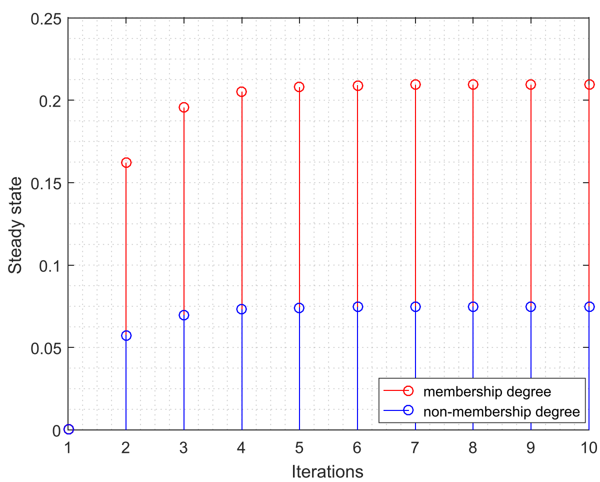


FIGURE 4. **The change tendency of C_7 for Case 1.**

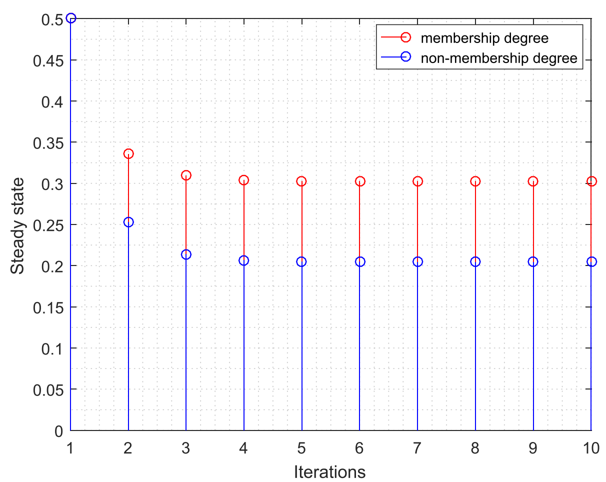


FIGURE 6. **The change tendency of C_7 for Case 2.**

degree and the hesitant degree denote the serious intensity, the good degree and the uncertain degree of the symptoms, respectively. The connection matrix is shown as TABLE 2, in which the default connections are $(0, 0)$ in EIFCM. Once $|\mu_i(t+1) - \mu_i(t)| < 10^{-4}$ and $|\nu_i(t+1) - \nu_i(t)| < 10^{-4}$ $(i = 1, 2, \ldots, 7)$ $(C_i(t) = \{\mu_i(t), \nu_i(t)\}$ and $C_i(t+1) = \{\mu_i(t+1), \nu_i(t+1)\})$ hold, we think EIFCM or IFCM reach the steady states. Both IFCM and EIFCM utilize tanh as the transformation function. The results of the reasoning process derived from both IFCM and EIFCM are shown as TABLE 3 and FIGUREs 3-8.

From the simulation results, we can get the following results: (1) As shown in TABLE 3, the iterative times of three cases from EIFCM are 0.86%, 0.77% and 1.32% from IFCM's, and the executing time (MATLAB 2016R; Intel(R) Core (TM) i5-3470 CPU; 4.00 GB RAM) of three cases from EIFCM are 8.74%, 8.91% and 35.25% from IFCM's. The results indicate that EIFCM has lower computational complexity in modeling the pneumonia risk decision making problem than IFCM. (2) The initial state is $[(1, 0)\ (0, 0)\ (1, 0)\ (1, 0)\ (0, 0)\ (0, 0)\ (0, 0)]^T$ for Case 1. The steady state of IFCM is $[(0.0379, 0)\ (0.4181, 0)\ (0.0379, 0)$

TABLE 3. Simulation results via IFCM and EIFCM.

Cases	Concepts	Initial State	Steady state		Iterative times		Executing time (second)	
			IFCM	**EIFCM**	**IFCM**	**EIFCM**	**IFCM**	**EIFCM**
Case 1	C_1	$(1, 0)$	$(0.0379, 0)$	$(1, 0)$	1043	9	0.4256	0.0372
	C_2	$(0, 0)$	$(0.4181, 0)$	$(0.2883, 0.0559)$				
	C_3	$(1, 0)$	$(0.0379, 0)$	$(1, 0)$				
	C_4	$(1, 0)$	$(0.0379, 0)$	$(1, 0)$				
	C_5	$(0, 0)$	$(0, 0)$	$(0, 0)$				
	C_6	$(0, 0)$	$(0, 0)$	$(0, 0)$				
	C_7	$(0, 0)$	$(0.6309, 0)$	$(0.2094, 0.0749)$				
Case 2	C_1	$(0.5, 0.5)$	$(0.0379, 0.0379)$	$(0.5, 0.5)$	1040	8	0.4141	0.0369
	C_2	$(0.5, 0.5)$	$(0.4180, 0)$	$(0.3101, 0.1996)$				
	C_3	$(0.5, 0.5)$	$(0.0379, 0.0379)$	$(0.5, 0.5)$				
	C_4	$(0.5, 0.5)$	$(0.0379, 0.0379)$	$(0.5, 0.5)$				
	C_5	$(0.5, 0.5)$	$(0.0379, 0.0379)$	$(0.5, 0.5)$				
	C_6	$(0.5, 0.5)$	$(0.0379, 0.0379)$	$(0.5, 0.5)$				
	C_7	$(0.5, 0.5)$	$(0.6414, 0)$	$(0.3025, 0.2046)$				
Case 3	C_1	$(0, 1)$	$(0, 0.0531)$	$(0, 1)$	530	7	0.0959	0.0338
	C_2	$(0, 0)$	$(0, 0)$	$(0.1366, 0.2109)$				
	C_3	$(0, 1)$	$(0, 0.0531)$	$(0, 1)$				
	C_4	$(0, 1)$	$(0, 0.0531)$	$(0, 1)$				
	C_5	$(0, 0)$	$(0, 0)$	$(0, 0)$				
	C_6	$(0, 0)$	$(0, 0)$	$(0, 0)$				
	C_7	$(0, 0)$	$(0, 0)$	$(0.1326, 0.1529)$				

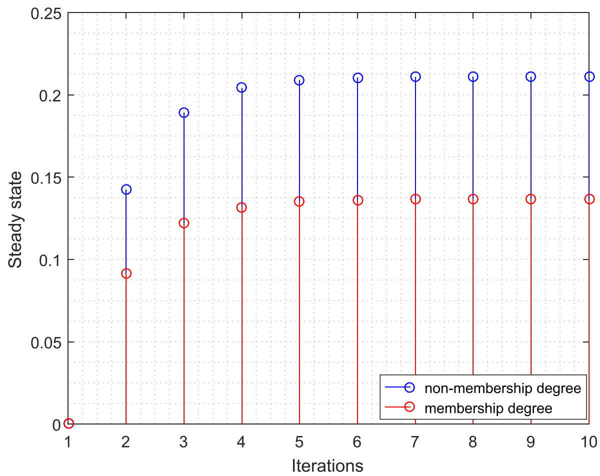


FIGURE 7. The change tendency of C_2 for Case 3.

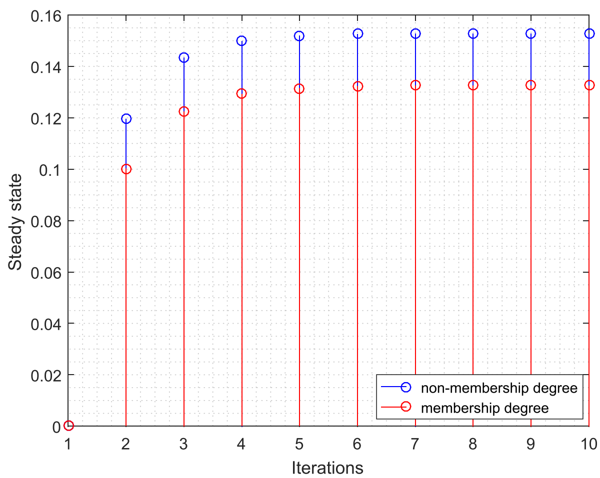


FIGURE 8. The change tendency of C_7 for Case 3.

$(0.0379, 0)$ $(0, 0)$ $(0, 0)$ $(0.6309, 0)]^T$ and the steady state of EIFCM is $[(1, 0)$ $(0.2823, 0.0559)$ $(1, 0)$ $(1, 0)$ $(0, 0)$ $(0, 0)$ $(0.2094, 0.0749)]^T$. Considering that there are not any connections to N_1, N_3, N_4, N_5 and N_6, the steady state values of the five nodes should be maintained. As indicated in TABLE 3, the simulation results from EIFCM are more reasonable than the results from IFCM with respect to above five nodes. The steady state values of N_2 from both IFCM and EIFCM are $(0.4181, 0)$ and $(0.2823, 0.0559)$ respectively. The patient has a big probability to get a fever and the risk of infection since $0.4181 > 0$, $0.2823 > 0.0559$, $0.6309 > 0$ and $0.2094 > 0.0749$ hold. In addition, $0.4181 > 0.2823$ and $0.6309 > 0.2094$ are due to the reason that IFCM just considers the membership degree and the non-membership degree in the construction of transformation function and adopts conventional operations [10] while EIFCM fully considers three elements of IFS and employs both the new proposed DSIFA operator and the multiplication rule $\tilde{\otimes}$. (3) The initial state is $[(0.5, 0.5)$ $(0.5, 0.5)$ $(0.5, 0.5)$ $(0.5, 0.5)$ $(0.5, 0.5)$ $(0.5, 0.5)$ $(0.5, 0.5)]^T$ for Case 2. The steady state of IFCM is $[(0.0379, 0.0379)$ $(0.4180, 0)$ $(0.0379, 0.0379)$ $(0.0379, 0.0379)$ $(0.0379, 0.0379)$ $(0.0379, 0.0379)$ $(0.6414, 0)]^T$ and the steady state of EIFCM is $[(0.5, 0.5)$ $(0.3101, 0.1996)$ $(0.5, 0.5)$ $(0.5, 0.5)$ $(0.5, 0.5)$ $(0.5, 0.5)$ $(0.3025, 0.2046)]^T$. From the perspective of IFS, $(0.5, 0.5)$ implies that we can not distinguish the importance levels of every nodes. As discussed in (2), the steady state values of N_1, N_3, N_4, N_5 and N_6 should be $(0.5, 0.5)$ which are the same as the results from EIFCM. The steady state values of N_2 from both IFCM and EIFCM are $(0.4180, 0)$ and $(0.3101, 0.1996)$ respectively. Since $0.4180 > 0.3101 > 0.1996 > 0$ holds, IFCM thinks that the patient has higher probability to get a fever by comparison with EIFCM. The steady state values of N_7 from both IFCM and EIFCM are $(0.6414, 0)$ and $(0.3025, 0.2046)$ respectively. $0.6414 > 0.5 > 0.3025 > 0.2046 > 0$ means that IFCM thinks that the patient has higher probability to get the risk of infection than EIFCM. $0.6414 > 0.5$ indicates that the patient is with great probability to have the risk of infection which contradicts the assumption of initial state. (4) The initial state is $[(0, 1)$ $(0, 0)$ $(0, 1)$ $(0, 1)$ $(0, 0)$ $(0, 0)$

TABLE 4. Simulation results via EIFCM with three different transformation functions.

Cases	Concepts	Initial State	Steady state			Iterative times		
			f_1	f_2	f_3	f_1	f_2	f_3
Case 1	C_1	$(1,0)$	$(0.3333,0.3333)$	$(1,0)$	$(1,0)$	6	9	9
	C_2	$(0,0)$	$(0.3241,0.3168)$	$(0.2883,0.0559)$	$(0.2380,0.0439)$			
	C_3	$(1,0)$	$(0.3333,0.3333)$	$(1,0)$	$(1,0)$			
	C_4	$(1,0)$	$(0.3333,0.3333)$	$(1,0)$	$(1,0)$			
	C_5	$(0,0)$	$(0.3333,0.3333)$	$(0,0)$	$(0,0)$			
	C_6	$(0,0)$	$(0.3333,0.3333)$	$(0,0)$	$(0,0)$			
	C_7	$(0,0)$	$(0.3225,0.3142)$	$(0.2094,0.0749)$	$(0.1649,0.0579)$			
Case 2	C_1	$(0.5,0.5)$	$(0.3333,0.3333)$	$(0.5,0.5)$	$(0.5,0.5)$	5	8	11
	C_2	$(0.5,0.5)$	$(0.3241,0.3168)$	$(0.3101,0.1996)$	$(0.2942,0.1818)$			
	C_3	$(0.5,0.5)$	$(0.3333,0.3333)$	$(0.5,0.5)$	$(0.5,0.5)$			
	C_4	$(0.5,0.5)$	$(0.3333,0.3333)$	$(0.5,0.5)$	$(0.5,0.5)$			
	C_5	$(0.5,0.5)$	$(0.3333,0.3333)$	$(0.5,0.5)$	$(0.5,0.5)$			
	C_6	$(0.5,0.5)$	$(0.3333,0.3333)$	$(0.5,0.5)$	$(0.5,0.5)$			
	C_7	$(0.5,0.5)$	$(0.3225,0.3142)$	$(0.3025,0.2046)$	$(0.2888,0.1901)$			
Case 3	C_1	$(0,1)$	$(0.3333,0.3333)$	$(0,1)$	$(0,1)$	6	7	10
	C_2	$(0,0)$	$(0.3241,0.3168)$	$(0.1366,0.2109)$	$(0.1092,0.1713)$			
	C_3	$(0,1)$	$(0.3333,0.3333)$	$(0,1)$	$(0,1)$			
	C_4	$(0,1)$	$(0.3333,0.3333)$	$(0,1)$	$(0,1)$			
	C_5	$(0,0)$	$(0.3333,0.3333)$	$(0,0)$	$(0,0)$			
	C_6	$(0,0)$	$(0.3333,0.3333)$	$(0,0)$	$(0,0)$			
	C_7	$(0,0)$	$(0.3225,0.3142)$	$(0.1326,0.1529)$	$(0.1026,0.1196)$			

$(0,0)]^T$ for Case 3. The steady state of IFCM is $[(0,0.0531)\,(0,0)\,(0,0.0531)\,(0,0.0531)\,(0,0)\,(0,0)\,(0,0)]^T$ and the steady state of EIFCM is $[(0,1)\ (0.1366,0.2109)\ (0,1)\ (0,1)\ (0,0)\ (0,0)\ (0.1326,0.1529)]^T$, respectively. Same as both Case 1 and Case 2, the steady state values of N_1, N_3, N_4, N_5 and N_6 from EIFCM have maintained while the steady state values of N_1, N_3 and N_4 from IFCM are completely different from initial values. In addition, $0.1366 < 0.2109$ and $0.1326 < 0.1529$ from EIFCM indicate that the patient have maintained good situations while IFCM can not make a definite decision since the steady state values of both N_2 and N_7 are $(0,0)$. (5) FIGUREs 3-8 reflect the change trend of both N_2 and N_7 and the gap between the membership degree and the non-membership degree.

Remark 4: As discussed above, EIFCM can not only provide more reasonable inference results in system modeling but also reduce the computational complexity than IFCM .

2) EIFCM WITH DIFFERENT TRANSFORMATION FUNCTIONS

We respectively utilize three different transformation functions including $f_1(x) = \frac{1}{1+\exp(-x)}$, $f_2(x) = \tanh(x)$ and $f_3(x) = \exp(x) - \exp(-x)$ in the establishment of EIFCM to model the above medical decision support problem. Using f_1, f_2 and f_3, the simulation results via EIFCM are shown as TABLE 4.

Remark 5: As proved in TABLE 4, EIFCM presents a flexible way to choose different functions as transformation function while IFCM just chooses sigmoid functions. As a result, we have more choices in system modeling.

B. SOCIAL ECONOMIC PROBLEM

Here we employ the proposed scheme of aggregating EIFCMs to model a social economic inference system via five factors including *Population* (N_1), *Crime* (N_2), *Economic condition* (N_3), *Poverty* (N_4), and *Unemployment* (N_5) [28]. Three experts present their respective EIFCMs based on some of above five factors shown as FIGUREs 9(a)-9(c). In what follows, we present two cases: (1) the known weights of three EIFCMs; (2) completely unknown weights of the three EIFCMs.[1]

1) THREE EIFCMs ARE WITH KNOWN WEIGHTS

Step 1: Let [0.2 0.5 0.3] be the weight vector of three EIFCMs. As shown in FIGURE 9, the connection matrices of three respective EIFCMs E_1, E_2 and E_3 are denoted by (22)-(24).

$$\begin{pmatrix} & N_1 & N_3 & N_5 \\ N_1 & (0,0) & (0.1,0.8) & (0.8,0.1) \\ N_3 & (0,0) & (0,0) & (0.1,0.8) \\ N_5 & (0,0) & (0,0) & (0,0) \end{pmatrix}, \tag{22}$$

$$\begin{pmatrix} & N_1 & N_2 & N_3 & N_4 \\ N_1 & (0,0) & (0,0) & (0.1,0.7) & (0,0) \\ N_2 & (0,0) & (0,0) & (0,0) & (0.1,0.7) \\ N_3 & (0,0) & (0.1,0.8) & (0,0) & (0,0) \\ N_4 & (0.1,0.8) & (0.9,0) & (0,0) & (0,0) \end{pmatrix}, \tag{23}$$

$$\begin{pmatrix} & N_1 & N_3 & N_4 & N_5 \\ N_1 & (0,0) & (0.2,0.7) & (0,0) & (0.7,0.2) \\ N_3 & (0,0) & (0,0) & (0,0) & (0.2,0.7) \\ N_4 & (0.1,0.7) & (0,0) & (0,0) & (0.7,0.2) \\ N_5 & (0,0) & (0,0) & (0.8,0.1) & (0,0) \end{pmatrix}. \tag{24}$$

Next, we get their augmented connection matrices $\bar{E}_1$, $\bar{E}_2$ and $\bar{E}_3$ as (25)-(27).

$$\begin{pmatrix} (0,0) & (0,0) & (0.1,0.8) & (0,0) & (0.8,0.1) \\ (0,0) & (0,0) & (0,0) & (0,0) & (0,0) \\ (0,0) & (0,0) & (0,0) & (0,0) & (0.1,0.8) \\ (0,0) & (0,0) & (0,0) & (0,0) & (0,0) \\ (0,0) & (0,0) & (0,0) & (0,0) & (0,0) \end{pmatrix}, \tag{25}$$

[1] The problem of aggregating knowledge regarding IFCMs has not been considered in [8], here we just employ EIFCMs to mode this aggregating knowledge problem.

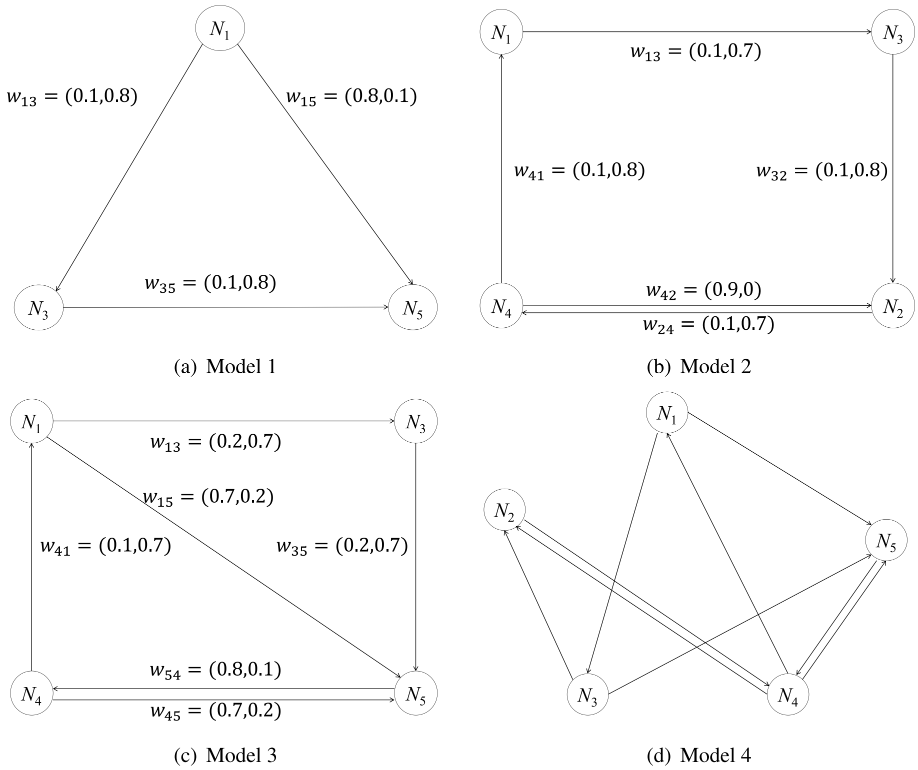


FIGURE 9. Three EIFCMs and their aggregated map.

$$\begin{pmatrix} (0,0) & (0,0) & (0.1,0.7) & (0,0) & (0,0) \\ (0,0) & (0,0) & (0,0) & (0.1,0.7) & (0,0) \\ (0,0) & (0.1,0.8) & (0,0) & (0,0) & (0,0) \\ (0.1,0.8) & (0.9,0) & (0,0) & (0,0) & (0,0) \\ (0,0) & (0,0) & (0,0) & (0,0) & (0,0) \end{pmatrix}, \tag{26}$$

$$\begin{pmatrix} (0,0) & (0,0) & (0.2,0.7) & (0,0) & (0.7,0.2) \\ (0,0) & (0,0) & (0,0) & (0,0) & (0,0) \\ (0,0) & (0,0) & (0,0) & (0,0) & (0.2,0.7) \\ (0.1,0.7) & (0,0) & (0,0) & (0,0) & (0.7,0.2) \\ (0,0) & (0,0) & (0,0) & (0.8,0.1) & (0,0) \end{pmatrix}. \tag{27}$$

Step 2: Based on the known weight vector [0.2 0.5 0.3], we get $\theta = [0.4\ 1\ 0.6]$. Then we get the aggregated connection matrix as (28), shown at the bottom of this page.

Step 3: Let $C(0) = [(0.1, 0.8)\ (0.1, 0.1)\ (0.4, 0.1)\ (0.1, 0.2)\ (0.1, 0.1)]^T$ be the initial state values. In addition, we have selected the function $f(x) = \frac{1}{1+\exp(-x)}$ as the transformation function for the simulations. Once $|\mu_i(t+1) - \mu_i(t)| < 10^{-4}$ and $|\nu_i(t+1) - \nu_i(t)| < 10^{-4}$ $(i = 1, 2, \ldots, 5)$ $(C_i(t) = \{\mu_i(t), \nu_i(t)\}$ and $C_i(t+1) = \{\mu_i(t+1), \nu_i(t+1)\})$ hold, the aggregated model is considered to reach the steady states. The results of the reasoning process that are obtained with the aggregated model at each

$$E = \begin{pmatrix} (0,0) & (0,0) & (0.0005, 0.1002) & (0,0) & (0.2467, 0.0533) \\ (0,0) & (0,0) & (0,0) & (0.0333, 0.2333) & (0,0) \\ (0,0) & (0.0333, 0.2667) & (0,0) & (0,0) & (0.0533, 0.2467) \\ (0.0533, 0.4067) & (0.3000, 0) & (0,0) & (0,0) & (0.1400, 0.0400) \\ (0,0) & (0,0) & (0,0) & (0.1600, 0.0200) & (0,0) \end{pmatrix}. \tag{28}$$

$$E = \begin{pmatrix} (0,0) & (0,0) & (0.0023, 0.4534) & (0,0) & (0.4944, 0.0984) \\ (0,0) & (0,0) & (0,0) & (0.0305, 0.2137) & (0,0) \\ (0,0) & (0.0305, 0.2443) & (0,0) & (0,0) & (0.0984, 0.4944) \\ (0.0631, 0.4720) & (0.2748, 0) & (0,0) & (0,0) & (0.2277, 0.0651) \\ (0,0) & (0,0) & (0,0) & (0.2603, 0.0325) & (0,0) \end{pmatrix}. \tag{30}$$

TABLE 5. Reasoning using the aggregated EIFCM.

t	$C_1(t)$	$C_2(t)$	$C_3(t)$	$C_4(t)$	$C_5(t)$
0	(0.1000,0.8000)	(0.1000,0.1000)	(0.4000,0.1000)	(0.1000,0.2000)	(0.1000,0.1000)
1	(0.3013,0.3456)	(0.3076,0.2887)	(0.3429,0.2981)	(0.3047,0.2889)	(0.3096,0.2885)
2	(0.3306,0.3147)	(0.3401,0.2923)	(0.3350,0.3039)	(0.3357,0.2923)	(0.3431,0.2889)
3	(0.3348,0.3123)	(0.3448,0.2925)	(0.3339,0.3031)	(0.3402,0.2923)	(0.3479,0.2888)
4	(0.3354,0.3121)	(0.3455,0.2925)	(0.3338,0.3029)	(0.3409,0.2923)	(0.3486,0.2887)
5	(0.3355,0.3121)	(0.3456,0.2925)	(0.3338,0.3029)	(0.3410,0.2923)	(0.3487,0.2887)
6	(0.3355,0.3121)	(0.3456,0.2925)	(0.3338,0.3029)	(0.3410,0.2923)	(0.3487,0.2887)
7	(0.3355,0.3121)	(0.3457,0.2925)	(0.3338,0.3029)	(0.3410,0.2923)	(0.3487,0.2887)
8	(0.3355,0.3121)	(0.3457,0.2925)	(0.3338,0.3029)	(0.3410,0.2923)	(0.3487,0.2887)

TABLE 6. Reasoning using the aggregated EIFCM.

t	$C_1(t)$	$C_2(t)$	$C_3(t)$	$C_4(t)$	$C_5(t)$
0	(0.1000,0.8000)	(0.1000,0.1000)	(0.4000,0.1000)	(0.1000,0.2000)	(0.1000,0.1000)
1	(0.3014,0.3513)	(0.3072,0.2887)	(0.3429,0.2991)	(0.3059,0.2889)	(0.3155,0.2890)
2	(0.3309,0.3175)	(0.3393,0.2922)	(0.3349,0.3137)	(0.3388,0.2923)	(0.3535,0.2902)
3	(0.3351,0.3146)	(0.3440,0.2924)	(0.3338,0.3143)	(0.3439,0.2923)	(0.3590,0.2900)
4	(0.3357,0.3143)	(0.3447,0.2924)	(0.3336,0.3142)	(0.3446,0.2923)	(0.3598,0.2900)
5	(0.3358,0.3143)	(0.3448,0.2924)	(0.3336,0.3142)	(0.3447,0.2923)	(0.3599,0.2900)
6	(0.3358,0.3143)	(0.3448,0.2924)	(0.3336,0.3142)	(0.3447,0.2923)	(0.3599,0.2900)

iteration t till convergence at a steady state are displayed in TABLE 5.

2) THREE EIFCMs ARE WITH COMPLETELY UNKNOWN WEIGHTS

Step 1: Similarity as above example, the connection matrices of three respective EIFCMs are the same as (22)-(24), and their augmented connection matrices are the same as (25)-(27).

Step 2: On the basis of Eq. (19), we get

$$\theta = [1.0000\ 0.9161\ 0.9760]. \tag{29}$$

Next, we get the aggregated connection matrix as (30), shown at the bottom of the previous page.

Step 3: Let $C(0)=[(0.1, 0.8)\ (0.1, 0.1)\ (0.4, 0.1)\ (0.1, 0.2)\ (0.1, 0.1)]^T$ be the initial state values. In addition, we have selected the function $f(x) = \frac{1}{1+\exp(-x)}$ as the transformation function for the simulations. Once $|\mu_{C_i(t+1)} - \mu_{C_i(t)}| < 10^{-4}$ and $|\nu_{C_i(t+1)} - \nu_{C_i(t)}| < 10^{-4}$ $(i = 1, 2, \ldots, 5)$ hold, the aggregated model is considered to reach the steady states. The results of the reasoning process that are obtained with the aggregated model at each iteration t till convergence at a steady state are displayed in TABLE 6.

Remark 6: Firstly, the augmented matrices of EIFCMs have been defined in the construction of the aggregated map as (25)-(27). Secondly, the problem of objectively determining the weights (importance levels) of different EIFCMs has been solved. Note that (29) describes the importance levels of different cognitive models. Thirdly, the aggregated EIFCM has been established via the proposed DSIFA operator as (28) (or (30)) which describes the mutual causal relationships among different maps. Fourthly, the difference between (28) and (30) and the variation tendency of all five factors (see TABLE 5 and TABLE 6) indicate the weights of different models play an important role in system modeling. Overall, the proposed new scheme of aggregating EIFCMs delivers a new vision to establish reasonable map in system modeling.

VI. CONCLUSION

In this paper, an extended intuitionistic fuzzy cognitive map via D-S theory (EIFCM) is proposed. While the proposed extension retains the key advantage of IFCM, i.e., quantifying the uncertain information in system modeling, it helps to accurately capture the uncertain state to be modeled. Additionally, a new structure of EIFCM has been established. Specifically, the DSIFA operator, a new multiplication operation and a similarity degree between two intuitionistic fuzzy matrices have been proposed in succession as supplements of basic theory on IFS. Basing on the DSIFA operator and the new multiplication operation, the mathematical structure of EIFCM was defined. What's more, the problem of aggregating knowledge has been solved via the DSIFA operator, especially on the key point of objectively determining the weights of different maps. The experiments indicate that EIFCM has lower computational complexity and better generalization ability when compared with the performance of IFCM.

EIFCM has not been fully validated by more practical problems in the current work. In addition, it is necessary to consider the combination between EIFCM and other machine learning models. The studies on afore-mentioned two aspects will be the future research hotspots.

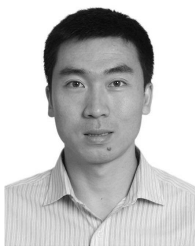
REFERENCES

[1] B. Kosko, "Fuzzy cognitive maps," *Int. J. Man-Mach. Stud.*, vol. 24, no. 1, pp. 65–75, 1986.

[2] A. Jastrzębska and A. Cisłak, "Interpretation-aware cognitive map construction for time series modeling," *Fuzzy Sets Syst.*, vol. 361, pp. 33–55, Apr. 2019.

[3] P. Hajek and W. Froelich, "Integrating TOPSIS with interval-valued intuitionistic fuzzy cognitive maps for effective group decision making," *Inf. Sci.*, vol. 485, pp. 394–412, Jun. 2019.
[4] S. Yang and J. Liu, "Time-series forecasting based on high-order fuzzy cognitive maps and wavelet transform," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 6, pp. 3391–3402, Dec. 2018.
[5] J. Wang and Q. Guo, "Ensemble interval-valued fuzzy cognitive maps," *IEEE Access*, vol. 6, pp. 38356–38366, 2018.
[6] W. Pedrycz and W. Homenda, "From fuzzy cognitive maps to granular cognitive maps," *IEEE Trans. Fuzzy Syst.*, vol. 22, no. 4, pp. 859–869, Aug. 2014.
[7] G. Nápoles, C. Mosquera, R. Falcon, I. Grau, R. Bello, and K. Vanhoof, "Fuzzy-rough cognitive networks," *Neural Netw.*, vol. 97, pp. 19–27, Jan. 2018.
[8] E. I. Papageorgiou and D. K. Iakovidis, "Intuitionistic fuzzy cognitive maps," *IEEE Trans. Fuzzy Syst.*, vol. 21, no. 2, pp. 342–354, Apr. 2013.
[9] Y. Zhang, J. Qin, P. Shi, and Y. Kang, "High-order intuitionistic fuzzy cognitive map based on evidential reasoning theory," *IEEE Trans. Fuzzy Syst.*, vol. 27, no. 1, pp. 16–30, Jan. 2019.
[10] K. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets Syst.*, vol. 20, no. 1, pp. 87–96, Aug. 1986.
[11] Y. Li and Y. Deng, "Intuitionistic evidence sets," *IEEE Access*, vol. 7, pp. 106417–106426, 2019.
[12] C. Cheng, F. Xiao, and Z. Cao, "A new distance for intuitionistic fuzzy sets based on similarity matrix," *IEEE Access*, vol. 7, pp. 70436–70446, 2019.
[13] J. F. B. Valderrama and D. J. L. B. Valderrama, "On LAMDA clustering method based on typicality degree and intuitionistic fuzzy sets," *Expert Syst. Appl.*, vol. 107, pp. 196–221, Oct. 2018.
[14] F. Zhao, Y. Chen, H. Liu, and J. Fan, "Alternate PSO-based adaptive interval type-2 intuitionistic fuzzy C-means clustering algorithm for color image segmentation," *IEEE Access*, vol. 7, pp. 64028–64039, 2019.
[15] L. Fei, H. Wang, L. Chen, and Y. Deng, "A new vector valued similarity measure for intuitionistic fuzzy sets based on OWA operators," *Iranian J. Fuzzy Syst.*, vol. 16, no. 3, pp. 113–126, 2019.
[16] F. Xiao, "Multi-sensor data fusion based on the belief divergence measure of evidences and the belief entropy," *Inf. Fusion*, vol. 46, pp. 23–32, Mar. 2019.
[17] A. Dempster, "The Dempster–Shafer calculus for statisticians," *Int. J. Approx. Reasoning*, vol. 48, no. 2, pp. 365–377, Jun. 2008.
[18] L. Fei, Y. Deng, and Y. Hu, "DS-VIKOR: A new multi-criteria decision-making method for supplier selection," *Int. J. Fuzzy Syst.*, vol. 21, no. 1, pp. 157–175, Feb. 2019.
[19] L. Fei, J. Xia, Y. Feng, and L. Liu, "An ELECTRE-based multiple criteria decision making method for supplier selection using Dempster–Shafer theory," *IEEE Access*, vol. 7, pp. 84701–84716, 2019.
[20] T. Denœux, "Logistic regression, neural networks and Dempster–Shafer theory: A new perspective," *Knowl.-Based Syst.*, vol. 176, pp. 54–67, Jul. 2019.
[21] J. Wang, Q. Guo, W. X. Zheng, and Q. Wu, "Robust cooperative spectrum sensing based on adaptive reputation and evidential reasoning theory in cognitive radio network," *Circuits Syst. Signal Process.*, vol. 37, no. 10, pp. 4455–4481, Oct. 2018.
[22] L. Zhou and F. Xiao, "A new matrix game with payoffs of generalized Dempster–Shafer structures," *Int. J. Intell. Syst.*, vol. 34, no. 9, pp. 2253–2268, Sep. 2019.
[23] Z. Xu and R. R. Yager, "Some geometric aggregation operators based on intuitionistic fuzzy sets," *Int. J. Gen. Syst.*, vol. 35, no. 4, pp. 417–433, Aug. 2006.
[24] Z. Xu, "Intuitionistic fuzzy aggregation operators," *IEEE Trans. Fuzzy Syst.*, vol. 15, no. 6, pp. 1179–1187, Dec. 2007.
[25] S.-M. Chen and C.-H. Chang, "Fuzzy multiattribute decision making based on transformation techniques of intuitionistic fuzzy values and intuitionistic fuzzy geometric averaging operators," *Inf. Sci.*, vols. 352–353, pp. 133–149, Jul. 2016.
[26] Y. D. He, H. Chen, L. Zhou, J. Liu, and Z. Tao, "Intuitionistic fuzzy geometric interaction averaging operators and their application to multi-criteria decision making," *Inf. Sci.*, vol. 259, pp. 142–159, Feb. 2014.
[27] Y. D. He, H. Y. Chen, L. G. Zhou, B. Han, Q. Y. Zhao, and J. Liu, "Generalized intuitionistic fuzzy geometric interaction operators and their application to decision making," *Expert Syst. Appl.*, vol. 41, no. 5, pp. 2484–2495, Apr. 2014.
[28] B. Kang, Y. Deng, R. Sadiq, and S. Mahadevan, "Evidential cognitive maps," *Knowl.-Based Syst.*, vol. 35, pp. 77–86, Nov. 2012.

ZHUOSHENG JIA is currently a Senior Engineer, a Supervisor of master's degree in computer science, and the Director of the Information Technology Center, Beijing Jiaotong University, and the Chairman of the Network Application Branch, China Computer Users Association. His research interests include computer network and application, data analysis, and information technology.



YINGJUN ZHANG (Member, IEEE) received the Ph.D. degree in computer science and technology from the Harbin Institute of Technology, China, in 2012. He is currently an Associate Professor and a Ph.D. Supervisor with the School of Computer and Information Technology, Beijing Key Laboratory of Traffic Data Analysis and Mining, Beijing Jiaotong University. His current research interests include machine learning, artificial intelligence, and decision theory and applications.

XUEMIN DONG received the bachelor's degree in computer science and technology from the Inner Mongolia University of Technology, in 2018. She is currently pursuing the master's degree in computer technology with Beijing Jiaotong University. Her research interests include machine learning and cognitive radio.