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Performance Analysis of NCSs Under Channel Noise and Bandwidth Constraints

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ABSTRACT This paper investigates the performance limitation of multi-input multi-output (MIMO) networked control systems (NCSs). The communication channel is modeled as a power constrained channel with channel noise and the bandwidth constraints. The optimal performance index of the system is expressed by the covariance of the reference input signal and the system output error signal. Some new results about MIMO NCSs are obtained by co-prime factorization, inner-outer factorization and spectral decomposition techniques. The results demonstrate that the tracking performance depends on the non-minimum phase zeros, unstable poles and their directions. At the same time, the network communication parameters constraints, energy constraints and the essential feature of reference input signal also restrict the performance. Some illustrative examples are presented to demonstrate the feasibility of the proposed methods.

INDEX TERMS Bandwidth constrains, channel noise, communication channels, performance limitation.

I. INTRODUCTION

In recent decades, network control systems(NCSs) have attracted people's attention because of its wide application [1]–[6], such as smart grid [7], mobile sensor networks [8], transportation systems [9] and robot control. At the same time, many challenges arise when NCSs have been widely used in practical fields, such as bandwidth [10], [11], packet dropout [12], [13], and quantization [14], [15]. It is well known that stability is the primary condition of the system, and many experts and scholars have attempted to improve the stability of NCSs [16]–[19].

In [17], the stability analysis of improved delay-dependent NCSs have been conducted. The stabilization of NCSs under clock mismatches and quantization have been investigated in [18]. The optimal control and the stabilization of NCSs under packet dropout and input delay constraint have been concerned in [19]. The technologies about modeling of the NCSs and stabilization analysis are now fairly mature. But from the angle of application, we should investigate not only the stability, but also the tracking performance of system. This paper mainly focuses on solving new problems with communication network constraints, such as the best tracking performance of networked control systems, and the performance limitation depends on the essential characteristics of the NCSs and communication parameters. In past years, several scholars have achieved excellent results in the performance limitation about NCSs [22]–[26]. In [22], the modified tracking performance limitation of MIMO NCSs under packet dropouts has been obtained. The performance limitation for MIMO NCSs under bandwidth and quantization constraints have been discussed in [23]. The optimal performance for MIMO discrete-time NCSs under quantization was investigated in [25]. It is well-known that the network signal transmission is bidirectional in networked control systems. Currently, most of the above mentioned research studies only considered the influence factor existed in the feedback channel. Only few studies in the literatures considered the influence factor existed in the forward and the feedback channels simultaneously, and it was shown in [20] that, in order to obtain the minimal tracking error, the channel input of NCSs is often required to have an infinite energy. This requirement cannot be satisfied in general practice. Thus, the channel input energy of NCSs should be considered in the performance index to address this issue. According to analysis above and [26], a novel model with

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channel noise and bandwidth constraint is established for MIMO NCSs.

The relationships among channel noise, energy constraints, bandwidth constraints and performance limitation of NCSs are investigated in this paper. The main objectives are as follows. First, the influence factor both existing in forward and feedback loops are studied, channel noise and bandwidth in forward and feedback loops simultaneously. Second, the performance limitation is affected by several parameters, such as the intrinsic properties of given plant, bandwidth and channel noise, at the same time, the performance limitation has strong connection with the nonminimum phase zeros and unstable poles of the given plant. At last, the expressions for the performance limitation are obtained by inner-outer factorization and the spectral decomposition technique.

The rest of the paper is organized as follows. Section 2 introduces the problem and some regular symbols. Section 3 investigates the performance limitation for NCSs under quantization and bandwidth constraints. Some examples are presented to prove the accuracy of results in Section 4. Section 5 presents the conclusion and the future research directions.

II. PROBLEM FORMULATIONS

For a complex vector v, the complex conjugate transpose is v^H. For any vector u, the transpose and the conjugate transpose are u^T and u^H respectively, and its Euclidean norm is $\|u\|$. The open unit disc, closed unit disc, exterior of the closed unit disc, and unit circle are denoted as $D \triangleq \{z : |z| < 1\}$, $\bar{D} \triangleq \{z : |z| \leq 1\}$, $\bar{D}^c \triangleq \{z : |z| > 1\}$ and $\partial D \triangleq \{z : |z| = 1\}$, respectively. Moreover, L_2 is the Hilbert space and is defined as:

$$\langle F, G\rangle := \frac{1}{2\pi}\int_{-\pi}^{\pi} tr\left[F^H\left(e^{i\theta}\right)G\left(e^{i\theta}\right)\right]d\theta$$

Finally, $\mathbb{R}\mathcal{H}_\infty$ represents all stable, and rational transfer function matrices.

Fig. 1 shows the block diagram of a MIMO NCS with channel noise and bandwidth constraints. In the fig. 1, G, K, F_1, and F_2 represent the given plant, one-parameter compensator, and the bandwidth, with transfer function matrices as $G(z)$, $K(z)$, $F_1(z)$, and $F_2(z)$, respectively. n_1, n_2 denote channel noise. The r and y represent the reference input and the system output signals, respectively, with transfer function matrices as $r(z)$ and $y(z)$, respectively. r is the Brownian motion process and $r(k) = (r_1(k), r_2(k), \cdots r_m(k))^T$, $F_1(z)$, $F_2(z)$ are chosen to be low-pass Butterworth filters of order one that can be denoted as:

$$F(z) = diag\left[f_1(z), f_2(z), \cdots, f_m(z)\right].$$

For channel i, the spectral density of r_i, n_{1i} and n_{2i} are defined as α_i, δ_i and σ_i respectively. The reference signals r, n_1 and n_2 are uncorrelated with each other. The matrices are denoted as:

$$U^2 = diag(\alpha_1^2, \alpha_2^2, \ldots, \alpha_m^2), \quad V^2 = diag(\delta_1^2, \delta_2^2, \ldots, \delta_m^2),$$

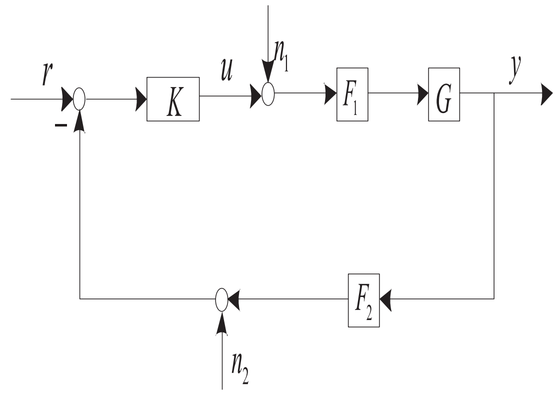

FIGURE 1. Model of one-parameter compensators.

$$A^2 = diag(\sigma_1^2, \sigma_2^2, \ldots, \sigma_m^2).$$

Illustrated by Fig. 1, we have:

$$\begin{aligned} u &= K\left[r - (F_2 y + n_2)\right] = Kr - KF_2 y - Kn_2 \\ y &= GF_1(u + n_1) = GF_1 u + GF_1 n_1 \end{aligned} \tag{1}$$

After calculation, it can be rewritten as:

$$\begin{aligned} u = &(I + KF_2GF_1)^{-1}Kr - (I + KF_2GF_1)^{-1}KF_2GFn_1 \\ &-(I + KF_2GF_1)^{-1}Kn_2 \\ y = &GF_1(I + KF_2GF_1)^{-1}Kr + GF_1(I + KF_2GF_1)^{-1}n_1 \\ &- GF_1(I + KF_2GF_1)^{-1}Kn_2 \end{aligned} \tag{2}$$

Then, following can be calculated:

$$\begin{aligned} e &= r - y \\ &= \left[I - GF_1(I + KF_2GF_1)^{-1}K\right]r \\ &\quad - GF_1(I + KF_2GF_1)^{-1}n_1 \\ &\quad + GF_1(I + KF_2GF_1)^{-1}Kn_2 \\ &= T_1 r - T_2 n_1 + T_3 n_2 \end{aligned} \tag{3}$$

where $T_1 = I - GF_1(I + KF_2GF_1)^{-1}K$, $T_2 = GF_1(I + KF_2GF_1)^{-1}$, $T_3 = GF_1(I + KF_2GF_1)^{-1}K$.

The performance limitation for NCSs under bandwidth constrain can be defined as:

$$J := (1-\varepsilon)E\left\{\|e\|_2^2\right\} + \varepsilon\left\{E\left\{\|y\|_2^2\right\} - \Gamma\right\}$$

where $0 \leq \varepsilon < 1$, represents the trade-off between tracking error and channel input power. The power constraint does not exist when $\varepsilon = 0$.

Based on (2) and (3), it can be obtained:

$$\begin{aligned} E\left\{\|e\|_2^2\right\} &= \|T_1 r - T_2 n_1 + T_3 n_2\|_2^2 = \|T_1 U\|_2^2 \\ &\quad + \|T_2 V\|_2^2 + \|T_3 A\|_2^2 \\ E\left\{\|y\|_2^2\right\} &= \|T_3 U + T_2 V - T_3 A\|_2^2 = \|T_3 U\|_2^2 \\ &\quad + \|T_2 V\|_2^2 + \|T_3 A\|_2^2 \end{aligned}$$

For any transfer function matrix F_2GF_1, it may be factorized as:

$$F_2GF_1 = NM^{-1} \tag{4}$$

where $N, M \in \mathbb{R}\mathcal{H}_\infty$, and satisfy the Bezout identity:

$$MX - NY = I \tag{5}$$

where $X, Y \in \mathbb{R}\mathcal{H}_\infty$, the stabilizing compensators $\mathcal{K}$ can be characterized by Youla parameterization [27]:

$$\begin{aligned} K := \Big\{K : K &= -(X-RN)^{-1}(Y-RM) \\ &= -(Y-RM)(X-RN)^{-1}, R \in \mathcal{H}_\infty\Big\} \end{aligned} \tag{6}$$

A nonminimum phase transfer function may factorize a minimum phase part and an all pass factor [28]:

$$N = F_2L_zN_nF_1, \quad M = B_pM_m \tag{7}$$

where L_z and B_p are all-pass factor, N_n and M_m are minimum phase part. L_z includes all zeros of the plant outside the unit circle $s_k \in \bar{D}^c, k = 1, 2, \cdots, n$, and B_p includes all poles of the plant outside the unit circle $p_k \in \bar{D}^c, k = 1, 2, \cdots m$. L_z and B_p can be co-prime factorized as:

$$L_z(z) = \prod_{i=1}^{n} L_i(z), B_p(z) = \prod_{j=1}^{m} B_j(z) \tag{8}$$

where $L_i(z) = \frac{z-s_i}{1-s_iz}\eta_i\eta_i^H + Y_iY_i^H, B_j(z) = \frac{z-p_j}{1-p_jz}\omega_j\omega_j^H + W_jW_j^H$, η_i and ω_j are the unitary vectors as the direction of nonminimum phase zeros and unstable poles, respectively, and $\eta_i\eta_i^H + Y_iY_i^H = I, \omega_j\omega_j^H + W_jW_j^H = I$.

According to (4) and (6), following can be obtained:

$$T_1 = I + F_2^{-1}N(Y-RM), T_2 = F_2^{-1}N(X-RN),$$
$$T_3 = -F_2^{-1}N(Y-RM).$$

Then, we have:

$$\begin{aligned} J &= (1-\varepsilon)E\left\{\|e\|_2^2\right\} + \varepsilon\left\{E\left\{\|y\|_2^2\right\} - \Gamma\right\} \\ &= (1-\varepsilon)\left\|\left[I + F_2^{-1}N(Y-RM)\right]U\right\|_2^2 \\ &\quad + \varepsilon\left\|F_2^{-1}N(Y-RM)U\right\|_2^2 \\ &\quad + \left\|F_2^{-1}N(X-RN)V\right\|_2^2 \\ &\quad + \left\|F_2^{-1}N(Y-RM)A\right\|_2^2 - \varepsilon\Gamma \end{aligned}$$

OPTIMAL TRACKING PERFORMANCE OF NCSS WITH CHANNEL NOISE AND BANDWIDTH CONSTRAINTS

The tracking performance limitation for NCSs under channel noise and bandwidth constraints is defined as J^*, The performance limitation J^* may be achieved by all the possible stabilizing controllers (denoted by $\mathcal{K}$). Then, J^* can be expressed as:

$$J^* = \inf_{\mathcal{K}\in K} J \tag{9}$$

Then, J^* can be obtained as:

$$\begin{aligned} J^* = \inf_{R\in\mathcal{H}_\infty}\Bigg\{&\left\|\begin{matrix}\sqrt{1-\varepsilon}\left[I + F_2^{-1}N(Y-RM)\right] \\ \sqrt{\varepsilon}F_2^{-1}N(Y-RM)\end{matrix}U\right\|_2^2 \\ &+ \left\|F_2^{-1}N(X-RN)V\right\|_2^2 \\ &+ \left\|F_2^{-1}N(Y-RM)A\right\|_2^2 - \varepsilon\Gamma\Bigg\} \end{aligned}$$

Theorem 1: For given NCSs such those presented in Fig. 1, r, n_1 and n_2 are independent of each other, the performance limitation is given by:

$$\begin{aligned} J^* \geq\ &(1-\varepsilon)\sum_{i,j=1}^{n}\frac{(1-s_i\bar{s}_i)(1-s_j\bar{s}_j)}{\bar{s}_is_j - 1} \\ &\times\sum_{k=1}^{i}|f(s_k)|^2 tr\left(\left[\eta_i\eta_i^HU\right]^H\left[\eta_j\eta_j^HU\right]\right) \\ &+ (1-\varepsilon)\sum_{i,j=1}^{m}\frac{(1-p_j\bar{p}_j)(1-p_i\bar{p}_i)}{\bar{p}_jp_i - 1}tr\left(\gamma_j^H\gamma_i\right) \\ &+ \varepsilon\sum_{i,j=1}^{m}\frac{(1-p_j\bar{p}_j)(1-p_i\bar{p}_i)}{\bar{p}_jp_i - 1}tr\left(\lambda_j^H\lambda_i\right) \\ &+ \sum_{i,j=1}^{n}\frac{(1-s_i\bar{s}_i)(1-s_j\bar{s}_j)}{\bar{s}_is_j - 1}tr\left(\Omega_j^H\Omega_i\right) \\ &+ \sum_{i,j=1}^{m}\frac{(1-p_j\bar{p}_j)(1-p_i\bar{p}_i)}{\bar{p}_jp_i - 1}tr\left(\theta_j^H\theta_i\right) - \varepsilon\Gamma \end{aligned}$$

where

$$\begin{aligned} \gamma_j &= \left(\Theta U + L_z^{-1}(p_j)F_2^{-1}(p_j)U\right)E_j\xi_j\xi_j^H\Phi_j, \\ \lambda_j &= L_z^{-1}(p_j)F_2^{-1}(p_j)UE_j\xi_j\xi_j^H\Phi_j, \\ \Omega_j &= N_n(z_i)F_1(z_i)M^{-1}(z_i)VE_i\eta_i\eta_i^H\Phi_i, \\ \theta_j &= -L_z^{-1}(p_j)F_2^{-1}(p_j)AE_j\xi_j\xi_j^HF_j. \end{aligned}$$

Proof: J^* can be decomposed as:

$$\begin{aligned} J_1^* &= \inf_{R\in\mathbb{R}\mathcal{H}_\infty}\left\|\begin{matrix}\sqrt{1-\varepsilon}\left[I + F_2^{-1}N(Y-RM)\right] \\ \sqrt{\varepsilon}F_2^{-1}N(Y-RM)\end{matrix}U\right\|_2^2, \\ J_2^* &= \inf_{R\in\mathbb{R}\mathcal{H}_\infty}\left\|F_2^{-1}N(X-RN)V\right\|_2^2, \\ J_3^* &= \inf_{R\in\mathbb{R}\mathcal{H}_\infty}\left\|F_2^{-1}N(Y-RM)A\right\|_2^2. \end{aligned} \tag{10}$$

From (7), it can be obtained:

$$J_1^* = \inf_{R\in RH_\infty}\left\|\begin{matrix}\sqrt{1-\varepsilon}\left[I + F_2^{-1}F_2L_zN_nF_1(Y-RM)\right] \\ \sqrt{\varepsilon}F_2^{-1}F_2L_zN_nF_1(Y-RM)\end{matrix}U\right\|_2^2$$

Because L_z is the all-pass factor, J_1^* can be calculated:

$$J_1^* = \inf_{R\in RH_\infty}\left\|\begin{matrix}\sqrt{1-\varepsilon}\left[L_z^{-1} + N_nF_1(Y-RM)\right] \\ \sqrt{\varepsilon}F_1(Y-RM)\end{matrix}U\right\|_2^2$$

By a simple calculation, we have:

$$J_1^* = \inf_{R\in RH_\infty}\left\|v\begin{matrix}\sqrt{1-\varepsilon}\left[L_z^{-1}-\Theta+\Theta+N_nF_1(Y-RM)\right]\\ \sqrt{\varepsilon}N_nF_1(Y-RM)\end{matrix}U\right\|_2^2$$

where $\Theta = \prod_{i=1}^{n} f(s_i)$, $f(s_i) = -\bar{s}_i\eta_i\eta_i^H + Y_iY_i^H$. Since $\left(L_z^{-1}-\Theta\right)$ is in $H_2^\perp$, and $\left[\Theta+N_m(Y-RM)\right]$ is in H_2. Hence:

$$J_1^* = \left\|\begin{matrix}\sqrt{1-\varepsilon}\left(L_z^{-1}-\Theta\right)\\ 0\end{matrix}U\right\|_2^2 + \inf_{R\in RH_\infty}\left\|\begin{matrix}\sqrt{1-\varepsilon}\left[\Theta+N_nF_1(Y-RM)\right]\\ \sqrt{\varepsilon}N_nF_1(Y-RM)\end{matrix}U\right\|_2^2$$

Then:

$$J_{11}^* = \left\|\begin{matrix}\sqrt{1-\varepsilon}\left(L_z^{-1}-\Theta\right)\\ 0\end{matrix}U\right\|_2^2,$$

$$J_{12}^* = \inf_{R\in\mathbb{R}\mathcal{H}_\infty}\left\|\begin{matrix}\sqrt{1-\varepsilon}\left[\Theta+N_nF_1(Y-RM)\right]\\ \sqrt{\varepsilon}N_nF_1(Y-RM)\end{matrix}U\right\|_2^2$$

According to [29], it can be obtained:

$$\left\|\sqrt{1-\varepsilon}\left(L_z^{-1}-\Theta\right)U\right\|_2^2 = (1-\varepsilon)\sum_{i=1}^{n}\prod_{k=1}^{i}|f(s_k)|^2\left\|\left(L_i^{-1}(z)-f(s_i)\right)U\right\|_2^2$$

Next,

$$J_{11}^* = (1-\varepsilon)\sum_{i,j=1}^{n}\frac{(1-s_i\bar{s}_i)(1-s_j\bar{s}_j)}{\bar{s}_is_j-1} \times\sum_{k=1}^{i}|f(s_k)|^2 tr\left(\left[\eta_i\eta_i^HU\right]^H\left[\eta_j\eta_j^HU\right]\right)$$

Then, calculating J_{12}^*, we denote that $MU = M_{\varphi m}B_{\varphi p}$, where $B_{\varphi p} = \prod_{j=1}^{m}B_{\varphi j}$, $B_{\varphi j} = \frac{z-p_j}{1-\bar{p}_jz}\xi_j\xi_j^H + \Lambda_j\Lambda_j^H$, ξ_j is the unitary vector, as the direction of unstable poles, thus $\xi_j\xi_j^H + \Lambda_j\Lambda_j^H = I$, and $\xi_j = \frac{FU^{-1}\omega_j}{\|FU^{-1}\omega_j\|}$. Then,

$$\begin{aligned}J_{12}^* &= \inf_{R\in RH_\infty}\left\|\begin{matrix}\sqrt{1-\varepsilon}\left[\Theta+N_nF_1(Y-RM)\right]\\ \sqrt{\varepsilon}N_nF_1(Y-RM)\end{matrix}U\right\|_2^2\\ &= \inf_{R\in RH_\infty}\\ &\times\left\|\begin{matrix}\sqrt{1-\varepsilon}\left(\Theta U+N_nF_1YU\right)B_{\varphi p}^{-1}-\sqrt{1-\varepsilon}N_nFRM_{\varphi m}\\ \sqrt{\varepsilon}N_nFYUB_{\varphi p}^{-1}-\sqrt{\varepsilon}N_nFRM_{\varphi m}\end{matrix}\right\|_2^2\end{aligned}$$

At the same time, following is denoted:

$$J_a^* = \inf_{R\in RH_\infty}\left\|\sqrt{1-\varepsilon}\left(\Theta U+N_nF_1YU\right)B_{\varphi p}^{-1} - \sqrt{1-\varepsilon}N_nF_1RM_{\varphi m}\right\|_2^2,$$

$$J_b^* = \inf_{R\in RH_\infty}\left\|\sqrt{\varepsilon}N_nFYUB_{\varphi p}^{-1} - \sqrt{\varepsilon}N_nF_1RM_{\varphi m}\right\|_2^2.$$

From partial fraction procedure, it can be obtained:

$$\begin{aligned}&\sqrt{1-\varepsilon}\left(\Theta U+N_nF_1YU\right)B_{\varphi p}^{-1}\\ &= \sqrt{1-\varepsilon}\sum_{j=1}^{m}\left(\Theta U+ N_n(p_j)F_1(p_j)Y(p_j)U\right)E_j\\ &\quad\times\left(B_{\varphi j}^{-1}-B_{\varphi j}^{-1}(\infty)\right)\Phi_j + R_1\sqrt{\varepsilon}N_nF_1YUB_{\varphi p}^{-1}\\ &= \sqrt{\varepsilon}\sum_{j=1}^{m}N_n(p_j)F_1(p_j)Y(p_j)UE_j\\ &\quad\times\left(B_{\varphi j}^{-1}-B_{\varphi j}^{-1}(\infty)\right)\Phi_j + R_2\end{aligned}$$

where $R_1, R_2 \in \mathbb{R}\mathcal{H}_\infty$, and $E_j = \prod_{k=1}^{j-1}\left(B_{\varphi k}(p_j)\right)^{-1}$, $\Phi_j = \prod_{k=j+1}^{m}\left(B_{\varphi k}(p_j)\right)^{-1}$.

So, it can be calculated:

$$\begin{aligned}J_a^* = \inf_{R\in RH_\infty}&\Bigg\|\sqrt{1-\varepsilon}\sum_{j=1}^{m}\left(\Theta U+ N_n(p_j)F_1(p_j)Y(p_j)U\right)\\ &\times E_j\left(B_{\varphi j}^{-1}-B_{\varphi j}^{-1}(\infty)\right)\Phi_j + R_1 - \sqrt{1-\varepsilon}N_nF_1RM_{\varphi m}\Bigg\|_2^2\end{aligned}$$

Because of $\left(B_{\varphi j}^{-1}-B_{\varphi j}^{-1}(\infty)\right)\in H_2^\perp$, and $\left(R_1-\sqrt{1-\varepsilon}N_mF_1RM_{\varphi m}\right)\in H_2$, J_a^* can be rewritten as:

$$\begin{aligned}J_a^* &= \Bigg\|\sqrt{1-\varepsilon}\sum_{j=1}^{m}\left(\Theta U+ N_n(p_j)F_1(p_j)Y(p_j)U\right)\\ &\quad\times E_j\left(B_{\varphi j}^{-1}-B_{\varphi j}^{-1}(\infty)\right)\Phi_j\Bigg\|_2^2\\ &\quad+\inf_{R\in RH_\infty}\left\|R_1-\sqrt{1-\varepsilon}N_nF_1RM_{\varphi m}\right\|_2^2\end{aligned}$$

Because of $R_1, R\in\mathbb{R}\mathcal{H}_\infty$, it can find the proper values of R_1 and R, then,

$$\inf_{R\in\mathbb{R}\mathcal{H}_\infty}\left\|R_1-\sqrt{1-\varepsilon}N_nF_1RM_{\varphi m}\right\|_2^2 = 0$$

So, following is obtained:

$$J_a^* = (1-\varepsilon)\sum_{i,j=1}^{m}\frac{(1-p_j\bar{p}_j)(1-p_i\bar{p}_i)}{\bar{p}_jp_i-1}tr\left(\gamma_j^H\gamma_i\right)$$

where $\gamma_j = \left(\Theta U+N_n(p_j)F_1(p_j)Y(p_j)U\right)E_j\xi_j\xi_j^H\Phi_j$.

In the same way, we have:

$$\begin{aligned}J_b^* &= \left\|\sqrt{\varepsilon}N_n(p_j)F_1(p_j)Y(p_j)UE_j\left(B_{\varphi j}^{-1}-B_{\varphi j}^{-1}(\infty)\right)\Phi_j\right\|_2^2\\ &\quad+\inf_{R\in RH_\infty}\left\|R_2-\sqrt{\varepsilon}N_nF_1RM_{\varphi m}\right\|_2^2\end{aligned}$$

Because of $R \in \mathbb{R}\mathcal{H}_\infty$, it can find the proper values of R, can be selected to make

$$\inf_{R\in\mathbb{R}\mathcal{H}_\infty} \left\| R_2 - \sqrt{\varepsilon} N_n F_1 R M_{\varphi m} \right\|_2^2 = 0$$

Then, one gets:

$$J_b^* = \varepsilon \sum_{i,j=1}^{m} \frac{\left(1 - p_j\bar{p}_j\right)\left(1 - p_i\bar{p}_i\right)}{\bar{p}_j p_i - 1} tr\left(\lambda_j^H \lambda_i\right)$$

where $\lambda_j = N_n\left(p_j\right) F_1\left(p_j\right) Y\left(p_j\right) U E_j \xi_j \xi_j^H \Phi_j$, and since $J_{12}^* = J_a^* + J_b^*$, therefore:

$$J_{12}^* = (1-\varepsilon)\sum_{i,j=1}^{m} \frac{\left(1 - p_j\bar{p}_j\right)\left(1 - p_i\bar{p}_i\right)}{\bar{p}_j p_i - 1} tr\left(\gamma_j^H \gamma_i\right) + \varepsilon \sum_{i,j=1}^{m} \frac{\left(1 - p_j\bar{p}_j\right)\left(1 - p_i\bar{p}_i\right)}{\bar{p}_j p_i - 1} tr\left(\lambda_j^H \lambda_i\right)$$

From (5), and $M\left(p_j\right) = 0$, we get: $Y\left(p_j\right) = -F_1^{-1}\left(p_j\right) N_n^{-1}\left(p_j\right) L_z^{-1}\left(p_j\right) F_2^{-1}\left(p_j\right)$.

Then, $\gamma_j = \left(\Theta U + L_z^{-1}\left(p_j\right) F_2^{-1}\left(p_j\right) U\right) E_j \xi_j \xi_j^H \Phi_j$, $\lambda_j = L_z^{-1}\left(p_j\right) F_2^{-1}\left(p_j\right) U E_j \xi_j \xi_j^H \Phi_j$.

Thus:

$$\begin{aligned} J_1^* = {} & (1-\varepsilon)\sum_{i,j=1}^{n} \frac{\left(1 - s_i\bar{s}_i\right)\left(1 - s_j\bar{s}_j\right)}{\bar{s}_i s_j - 1} \\ & \times \sum_{k=1}^{i} |f(s_k)|^2 tr\left(\left[\eta_i\eta_i^H U\right]^H \left[\eta_j\eta_j^H U\right]\right) \\ & + (1-\varepsilon)\sum_{i,j=1}^{m} \frac{\left(1 - p_j\bar{p}_j\right)\left(1 - p_i\bar{p}_i\right)}{\bar{p}_j p_i - 1} tr\left(\gamma_j^H \gamma_i\right) \\ & + \varepsilon \sum_{i,j=1}^{m} \frac{\left(1 - p_j\bar{p}_j\right)\left(1 - p_i\bar{p}_i\right)}{\bar{p}_j p_i - 1} tr\left(\lambda_j^H \lambda_i\right) \end{aligned}$$

Next, J_2^* and J_3^* are calculated.

$$NV = N_{\varphi n} L_{\varphi z},$$

where $L_{\varphi z} = \prod_{i=1}^{n} L_{\varphi i}$, $L_{\varphi i} = \frac{z - s_i}{1 - s_i z}\eta_j\eta_j^H + Y_i Y_i^H$, then, following is obtained:

$$\begin{aligned} J_2^* &= \inf_{R\in RH_\infty} \left\| N_n F_1 X V - N_n F_1 R N V \right\|_2^2 \\ &= \inf_{R\in RH_\infty} \left\| N_n F_1 X V - N_n F_1 R N_{\varphi n} L_{\varphi z} \right\|_2^2 \\ &= \inf_{R\in RH_\infty} \left\| N_n F_1 X V L_{\varphi z}^{-1} - N_n F_1 R N_{\varphi n} \right\|_2^2 \end{aligned}$$

From partial fraction procedure:

$$\begin{aligned} & N_n F_1 X V L_{\varphi z}^{-1} \\ & = N_n\left(z_i\right) F_1\left(z_i\right) X\left(z_i\right) V E_i \left(L_{\varphi z}^{-1} - L_{\varphi z}^{-1}(\infty)\right) \Phi_i + R_3 \end{aligned}$$

where $R_3 \in \mathbb{R}\mathcal{H}_\infty, E_i = \prod_{k=1}^{i-1}\left(L_{\varphi k}\left(z_i\right)\right)^{-1}, \Phi_i = \prod_{k=i+1}^{n}\left(L_{\varphi k}\left(z_i\right)\right)^{-1}$. From (5), we have $X\left(z_i\right) = M^{-1}\left(z_i\right)$. Then,

$$\begin{aligned} & J_2^* \\ & = \left\| N_n\left(z_i\right) F_1\left(z_i\right) M^{-1}\left(z_i\right) V E_i \left(L_{\varphi z}^{-1} - L_{\varphi z}^{-1}(\infty)\right) \Phi_i \right\|_2^2 \\ & \quad + \inf_{R\in RH_\infty} \left\| R_3 - N_n\left(z_i\right) F_1\left(z_i\right) R N_{\varphi n} \right\|_2^2 \end{aligned}$$

Because of $R_3, R \in \mathbb{R}\mathcal{H}_\infty$, it can find the proper values of R_3, R, can be selected to make,

$$\inf_{R\in\mathbb{R}\mathcal{H}_\infty} \left\| R_3 - N_n\left(z_i\right) F_1\left(z_i\right) R N_{\varphi n} \right\|_2^2 = 0$$

Following can be obtained:

$$J_2^* = \sum_{i,j=1}^{n} \frac{\left(1 - s_i\bar{s}_i\right)\left(1 - s_j\bar{s}_j\right)}{\bar{s}_i s_j - 1} tr\left(\Omega_j^H \Omega_i\right)$$

where $\Omega_j = N_n\left(z_i\right) F_1\left(z_i\right) M^{-1}\left(z_i\right) V E_i \eta_i \eta_i^H \Phi_i$.

At the same time:

$$\begin{aligned} J_3^* &= \inf_{R\in RH_\infty} \left\| L_z N_n F_1 (Y - RM) A \right\|_2^2 \\ &= (1 - 2\varepsilon) \inf_{R\in RH_\infty} \left\| N_n F_1 (Y - RM) A \right\|_2^2 \end{aligned}$$

From J_1^*, following can be obtained:

$$J_3^* = \sum_{i,j=1}^{m} \frac{\left(1 - p_j\bar{p}_j\right)\left(1 - p_i\bar{p}_i\right)}{\bar{p}_j p_i - 1} tr\left(\theta_j^H \theta_i\right)$$

where $\theta = -L_z^{-1}\left(p_j\right) F_2^{-1}\left(p_j\right) A E_j \xi_j \xi_j^H F_j$.

The proof is completed.

Next, the problem of the performance limitation for NCSs under two-parameter compensators is discussed, as shown in Fig. 2. $\left[K_1 \; K_2\right]$ represents the two-parameter compensators, and the transfer function is $\left[K_1(s) \; K_2(s)\right]$. The defined dimension of $\left[K_1 \; K_2\right]$ is the same as the number of communication channels.

From Fig. 2, it is clear that:

$$u = K_1 r + K_2\left(F_2 y + n_2\right), y = G F_1\left(u + n_1\right) \quad (11)$$

A simple calculation can provide the following:

$$\begin{aligned} u &= (I - K_2F_2GF_1)^{-1}K_1 r \\ & \quad + (I - K_2F_2GF_1)^{-1}K_2F_2GF_1 n_1 \\ & \quad + (I - K_2F_2GF_1)^{-1}K_2 n_2 \\ y &= GF(I - K_2F_2GF_1)^{-1}K_1 r \\ & \quad + GF_1(I - K_2F_2GF_1)^{-1} n_1 \\ & \quad + GF_1(I - K_2F_2GF_1)^{-1}K_2 n_2 \\ &= T_7 r + T_5 n_1 + T_6 n_2 \end{aligned} \quad (12)$$

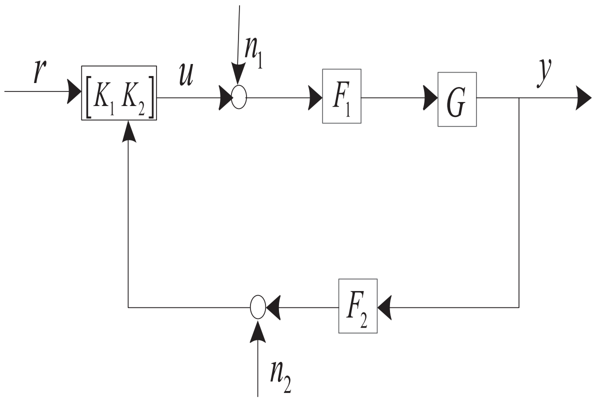

FIGURE 2. Model of constraint with channel noise and bandwidth.

$$\begin{aligned} e &= r - y \\ &= \left[I - GF(I - K_2F_2GF_1)^{-1}K_1\right] r \\ &\quad - GF_1(I - K_2F_2GF_1)^{-1}n_1 \\ &\quad - GF_1(I - K_2F_2GF_1)^{-1}K_2n_2 \\ &= T_4r - T_5n_1 - T_6n_2 \end{aligned} \tag{13}$$

The same as one-parameter compensator, $\mathcal{K}$ can be written as [22]:

$$\begin{aligned} K &:= \{K : K = [K_1 : K_2] \\ &\quad = (X - RN)^{-1} [Q \quad Y - RM], Q, R \in \mathcal{H}_\infty\right\} \end{aligned} \tag{14}$$

According to (4) and (14), one has:

$$\begin{aligned} T_4 &= I - GF(I - K_2F_2GF_1)^{-1}K_1 = I - F_2^{-1}NQ, \\ T_5 &= GF_1(I - K_2F_2GF_1)^{-1} = F_2^{-1}N(X - RN), \\ T_6 &= GF_1(I - K_2F_2GF_1)^{-1}K_2 = F_2^{-1}N(Y - RM), \\ T_7 &= GF(I - K_2F_2GF_1)^{-1}K_1 = F_2^{-1}NQ. \end{aligned}$$

Then,

$$\begin{aligned} J &= (1-\varepsilon) E\left\{\|e\|_2^2\right\} + \varepsilon\left\{E\left\{\|y\|_2^2\right\} - \Gamma\right\} \\ &= (1-\varepsilon)\left\|\left(I - F_2^{-1}NQ\right)U\right\|_2^2 + \varepsilon\left\|F_2^{-1}NQU\right\|_2^2 \\ &\quad + \left\|F_2^{-1}N(X - RN)V\right\|_2^2 \\ &\quad + \left\|F_2^{-1}N(Y - RM)A\right\|_2^2 - \varepsilon\Gamma \end{aligned} \tag{15}$$

According to (5) and (15), it can be calculated:

$$\begin{aligned} J^* = \inf_{\substack{Q\in RH_\infty \\ R\in RH_\infty}} &\left\{\left\| \begin{matrix} \sqrt{1-\varepsilon}\left(I - F_2^{-1}NQ\right)U \\ \sqrt{\varepsilon}F_2^{-1}NQU \end{matrix} \right\|_2^2 \right. \\ &+ \left\|F_2^{-1}N(X - RN)V\right\|_2^2 \\ &\left. + \left\|F_2^{-1}N(Y - RM)A\right\|_2^2 - \varepsilon\Gamma \right\} \end{aligned}$$

Theorem 2: NCSs with channel noise and bandwidth constraints as depicted in Fig. 2, if $\mathcal{K}$ is expressed as (14), the performance limitation can be obtained as:

$$\begin{aligned} J^* &= (1-\varepsilon)\sum_{i,j=1}^{n} \frac{(1 - s_i\bar{s}_i)\left(1 - s_j\bar{s}_j\right)}{\bar{s}_i s_j - 1} \\ &\quad \times \sum_{k=1}^{i} |f(s_k)|^2 tr\left(\left[\eta_i\eta_i^H U\right]^H \left[\eta_j\eta_j^H U\right]\right) \\ &\quad + \varepsilon(1-\varepsilon)\sum_{i=1}^{m}\alpha_m^2 \\ &\quad + \sum_{i,j=1}^{n} \frac{(1 - s_i\bar{s}_i)\left(1 - s_j\bar{s}_j\right)}{\bar{s}_i s_j - 1} tr\left(\Omega_j^H\Omega_i\right) \\ &\quad + \sum_{i,j=1}^{m} \frac{(1 - p_j\bar{p}_j)(1 - p_i\bar{p}_i)}{\bar{p}_j p_i - 1} tr\left(\theta_j^H\theta_i\right) - \varepsilon\Gamma \end{aligned}$$

where $\Omega_j = N_n(z_i)F_1(z_i)M^{-1}(z_i)VE_i\eta_i\eta_i^H\Phi_i$, $\theta_j = -L_z^{-1}(p_j)F_2^{-1}(p_j)AE_j\xi_j\xi_j^HF_j$.

Proof: J^* can be decomposed as:

$$\begin{aligned} J_4^* &= \inf_{Q\in\mathbb{R}\mathcal{H}_\infty} \left\| \begin{matrix} \sqrt{1-\varepsilon}\left(I - F_2^{-1}NQ\right)U \\ \sqrt{\varepsilon}F_2^{-1}NQ \end{matrix} \right\|_2^2, \\ J_5^* &= \inf_{R\in\mathbb{R}\mathcal{H}_\infty} \left\|F_2^{-1}N(X - RN)V\right\|_2^2 \\ J_6^* &= \inf_{R\in\mathbb{R}\mathcal{H}_\infty} \left\|F_2^{-1}N(Y - RM)A\right\|_2^2 \end{aligned}$$

From J_1^*, we have:

$$\begin{aligned} J_4^* &= \left\| \begin{matrix} \sqrt{1-\varepsilon}\left(L_z^{-1} - \Theta\right)U \\ 0 \end{matrix} \right\|_2^2 \\ &\quad + \inf_{Q\in RH_\infty} \left\| \begin{matrix} \sqrt{1-\varepsilon}\left(\Theta - N_nF_1Q\right)U \\ \sqrt{\varepsilon}N_nF_1Q \end{matrix} \right\|_2^2 \end{aligned}$$

Next, calculating J_4^*, denotes:

$$\begin{aligned} J_{41}^* &= \left\| \begin{matrix} \sqrt{1-\varepsilon}\left(L_z^{-1} - \Theta\right)U \\ 0 \end{matrix} \right\|_2^2, \\ J_{42}^* &= \inf_{Q\in RH_\infty} \left\| \begin{matrix} \sqrt{1-\varepsilon}\left(\Theta - N_nF_1Q\right)U \\ \sqrt{\varepsilon}N_nF_1Q \end{matrix} \right\|_2^2 \end{aligned}$$

From Theorem 1., it can be obtained:

$$\begin{aligned} J_{41}^* &= (1-\varepsilon)\sum_{i,j=1}^{n} \frac{(1 - s_i\bar{s}_i)\left(1 - s_j\bar{s}_j\right)}{\bar{s}_i s_j - 1} \\ &\quad \times \sum_{k=1}^{i} |f(s_k)|^2 tr\left(\left[\eta_i\eta_i^H U\right]^H \left[\eta_j\eta_j^H U\right]\right) \end{aligned}$$

From [22], an inner-outer factorization is introduced:

$$\Delta_i\Delta_0 = \begin{pmatrix} -\sqrt{1-\varepsilon}I \\ \sqrt{\varepsilon}I \end{pmatrix} N_nF_1$$

where $\Delta_i = \begin{pmatrix} -\sqrt{1-\varepsilon}I \\ \sqrt{\varepsilon}I \end{pmatrix}$, $\Delta_0 = N_nF_1$.

we select $\psi = \begin{pmatrix} \Delta_i^H \\ I - \Delta_i\Delta_i^H \end{pmatrix}$, and $\psi^T\psi = I$.

Then:

$$
\begin{aligned}
J_{42}^* &= \inf_{Q\in RH_\infty}\left\|\psi\left(\begin{pmatrix}\sqrt{1-\varepsilon I}\\0\end{pmatrix}+\begin{pmatrix}-\sqrt{1-\varepsilon I}\\0\end{pmatrix}N_nF_1Q\right)U\right\|_2^2\\
&=\left\|\left(I-\psi_i\psi_i^H\right)\begin{pmatrix}\sqrt{1-\varepsilon I}\\0\end{pmatrix}U\right\|_2^2\\
&\quad+\inf_{Q\in RH_\infty}\left\|\left(\psi_i^H\begin{pmatrix}\sqrt{1-\varepsilon I}\\0\end{pmatrix}+\psi_0Q\right)U\right\|_2^2
\end{aligned}
$$

Proper Q, can be selected to obtain the following: $\inf_{Q\in\mathbb{R}\mathcal{H}_\infty}\left\|\left(\psi_i^H\begin{pmatrix}\sqrt{1-\varepsilon I}\\0\end{pmatrix}+\psi_0Q\right)U\right\|_2^2=0.$
A simple calculation can provide the following:

$$
\begin{aligned}
&\left\|\left(I-\psi_i\psi_i^H\right)\begin{pmatrix}\sqrt{1-\varepsilon I}\\0\end{pmatrix}U\right\|_2^2\\
&\quad=\left\|\begin{pmatrix}\varepsilon I & \sqrt{\varepsilon(1+\varepsilon)}I\\ \sqrt{\varepsilon(1+\varepsilon)}I & (I-\varepsilon)I\end{pmatrix}\begin{pmatrix}\sqrt{1-\varepsilon I}\\0\end{pmatrix}U\right\|_2^2
\end{aligned}
$$

Thus:

$$
\left\|\left(I-\psi_i\psi_i^H\right)\begin{pmatrix}\sqrt{1-\varepsilon I}\\0\end{pmatrix}U\right\|_2^2=\varepsilon(1-\varepsilon)\sum_{i=1}^{n}\alpha_n^2
$$

From one-parameter compensators, it can be found that:

$$
J_5^*=J_2^*=\sum_{i,j=1}^{n}\frac{(1-s_i\bar{s}_i)\left(1-s_j\bar{s}_j\right)}{\bar{s}_is_j-1}tr\left(\Omega_j^H\Omega_i\right)
$$

$$
J_6^*=J_3^*=\sum_{i,j=1}^{m}\frac{\left(1-p_j\bar{p}_j\right)(1-p_i\bar{p}_i)}{\bar{p}_jp_i-1}tr\left(\theta_j^H\theta_i\right)
$$

The proof is completed.

Corollary. 1 The following corollary. 1 can be obtained by Theorem 2 directly. If the channel has been not considered In Theorem 2, then the performance limitation can be obtained as:

$$
J^*=\sum_{i,j=1}^{n}\frac{(1-s_i\bar{s}_i)\left(1-s_j\bar{s}_j\right)}{\bar{s}_is_j-1}\times\sum_{k=1}^{i}|f(s_k)|^2tr\left(\left[\eta_i\eta_i^HU\right]^H\left[\eta_j\eta_j^HU\right]\right)
$$

Corollary. 2 The following corollary. 2 can be obtained by Theorem 2 directly. In Theorem 2, if $n_1=0$, then the performance limitation can be obtained as:

$$
\begin{aligned}
J^*&=(1-\varepsilon)\sum_{i,j=1}^{n}\frac{(1-s_i\bar{s}_i)\left(1-s_j\bar{s}_j\right)}{\bar{s}_is_j-1}\\
&\quad\times\sum_{k=1}^{i}|f(s_k)|^2tr\left(\left[\eta_i\eta_i^HU\right]^H\left[\eta_j\eta_j^HU\right]\right)\\
&\quad+\varepsilon(1-\varepsilon)\sum_{i=1}^{m}\alpha_m^2\\
&\quad+\sum_{i,j=1}^{m}\frac{\left(1-p_j\bar{p}_j\right)(1-p_i\bar{p}_i)}{\bar{p}_jp_i-1}tr\left(\theta_j^H\theta_i\right)-\varepsilon\Gamma
\end{aligned}
$$

where $\theta=-L_z^{-1}(p_j)F_2^{-1}(p_j)AE_j\xi_j\xi_j^HF_j$.

III. NUMERICAL SIMULATIONS

This section presents some examples to illustrate the results.

Example 1: Consider the given plant as:

$$
G(z)=\begin{pmatrix}\frac{1}{z-5} & 0\\ 1 & \frac{z-k}{(z+0.5)}\end{pmatrix}
$$

From this plant, $|k|>1$, it can be seen the nonminimum phase zeros are located at $z=k$, the zero-direction vector η is $\eta=\begin{pmatrix}1\\0\end{pmatrix}$, the unstable pole is $p=5$, and the pole direction vector ω is $\omega=\begin{pmatrix}0\\1\end{pmatrix}$.

Let suppose $\Gamma=10$, $U=\begin{pmatrix}2&0\\0&2\end{pmatrix}$, $V=\begin{pmatrix}1&0\\0&1\end{pmatrix}$, $N(z)=F_2\begin{pmatrix}1&0\\1&\frac{z-k}{z+0.5}\end{pmatrix}F_1$, $A=\begin{pmatrix}3&0\\0&3\end{pmatrix}$,
so $N_n(z)=\begin{pmatrix}1&0\\0&\frac{1}{z+0.5}\end{pmatrix}$, $M^{-1}(z)=\begin{pmatrix}\frac{1}{z-5}&0\\1&1\end{pmatrix}$.

Next, the bandwidth is assumed as: $F(z)=\begin{pmatrix}\frac{\mu}{z+\mu}&0\\0&\frac{\mu}{z+\mu}\end{pmatrix}$, where μ is the bandwidth rate, if $\mu=10$, following can be obtained: $F_1(z_i)=\begin{pmatrix}\frac{10}{k+10}&0\\0&\frac{10}{k+10}\end{pmatrix}$, $F_2^{-1}(p_j)=\begin{pmatrix}\frac{3}{2}&0\\0&\frac{3}{2}\end{pmatrix}$.

The ε is assumed for three different values of $\varepsilon_1=0$, $\varepsilon_2=\frac{1}{2}$, $\varepsilon_3=\frac{4}{5}$, from Theorem 1, following can be obtained:

$$
\begin{aligned}
J_1^*&=\left(k^2-1\right)k^2+\frac{6\left(2k^2+5k-3\right)^2+1350(1-5k)^2}{(5-k)^2}\\
&\quad+\left(k^2-1\right)\left[\frac{900}{(k+2)^2(k+10)^2}\right.\\
&\quad\left.+\frac{100}{(k+2)^2(k+10)^2(k-5)^2}\right]+1086\\
J_2^*&=\frac{1}{2}\left(k^2-1\right)k^2+\frac{3\left(2k^2+5k-3\right)^2+1377(1-5k)^2}{(5-k)^2}\\
&\quad+\left(k^2-1\right)\left[\frac{900}{(k+2)^2(k+10)^2}\right.\\
&\quad\left.+\frac{100}{(k+2)^2(k+10)^2(k-5)^2}\right]+889\\
J_3^*&=\frac{1}{5}\left(k^2-1\right)k^2+\frac{6\left(2k^2+5k-3\right)^2+6966(1-5k)^2}{5(5-k)^2}\\
&\quad+\left(k^2-1\right)\left[\frac{900}{(k+2)^2(k+10)^2}\right.\\
&\quad\left.+\frac{100}{(k+2)^2(k+10)^2(k-5)^2}\right]+\frac{2990}{5}
\end{aligned}
$$

The performance limitation for NCSs under different ε is shown in Fig. 3. It can be seen from the Fig.3 that the

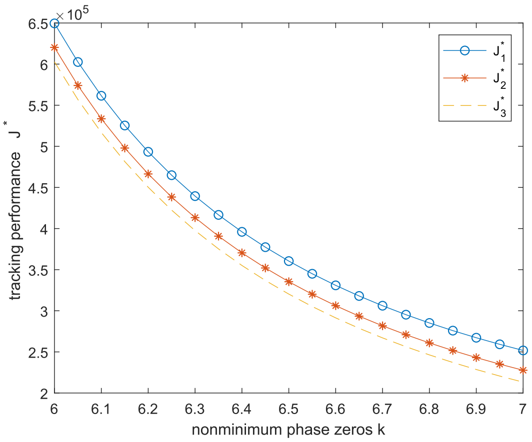

FIGURE 3. The performance limitation under different nonminimum phase zeros.

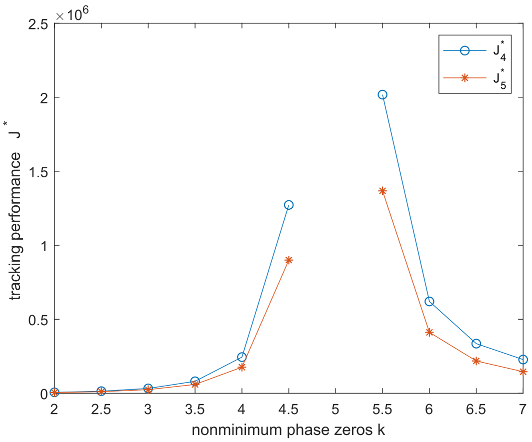

FIGURE 4. Performance limitation under nonminimum phase zeros.

performance limitation will be influenced by ε, the greater ε is, the greater the performance of NCSs will be.

Next, if $\varepsilon = \frac{1}{2}$, and other conditions are unchanged, from Theorem 1 and Theorem 2, following can be obtained:

$$\begin{aligned} J_4^* &= \frac{1}{2}\left(k^2-1\right)k^2 + \frac{3\left(2k^2+5k-3\right)^2 + 1377(1-5k)^2}{(5-k)^2} \\ &\quad + \left(k^2-1\right)\left[\frac{900}{(k+2)^2(k+10)^2} \right. \\ &\quad \left. + \frac{100}{(k+2)^2(k+10)^2(k-5)^2}\right] + 889 \\ J_5^* &= \frac{1}{2}\left(k^2-1\right)k^2 + \frac{1350(1-5k)^2}{(5-k)^2} \\ &\quad + \left(k^2-1\right)\left[\frac{900}{(k+2)^2(k+10)^2} \right. \\ &\quad \left. + \frac{100}{(k+2)^2(k+10)^2(k-5)^2}\right] \\ &\quad + 481 + \frac{5}{4} \end{aligned}$$

The performance limitation for NCSs under one or two-parameter compensator is shown in Fig. 4. It can be seen from Fig. 4 that the optimal performance for NCSs under

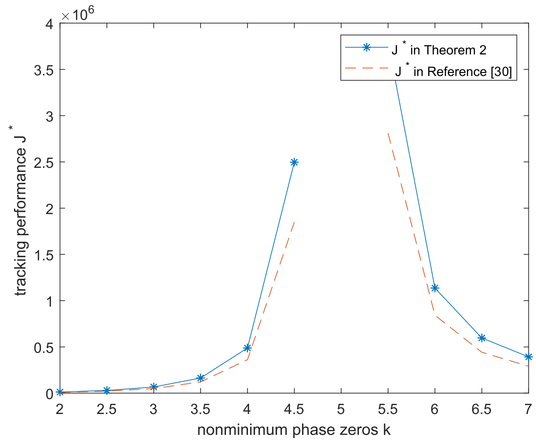

FIGURE 5. Performance limitation under nonminimum phase zeros.

two-parameter compensator is better than one-parameter compensator. It can also be seen from Fig. 4 that the performance will be influenced by the non-minimum phase zeros, unstable poles and their directions of systems, the performance of the systems becomes worse as unstable poles and nonminimum phase zeros are located close to each other.

The bandwidth and two channel noises often appear in NCSs and inevitably degrade or destabilize the control performance of the NCSs. The optimal tracking problem for NCSs over a communication channel with bandwidth is studied in [30] only considers the feedback bandwidth constraint. This paper is aimed at addressing the tracking performance limitation of NCSs with considering the bandwidth and two channel noises. The choice of data is the same as in previous example, thus, according to Theorem 2 and [30], the tracking performance limitation can be obtained in Fig. 5. In Fig. 5, it can be seen that the more communication constraint parameters, the worse the performance is.

Example 2: Consider the given plant as:

$$G(z) = \begin{pmatrix} \frac{1}{(z-5)} & 0 \\ 1 & \frac{z-3}{z+0.5} \end{pmatrix}$$

From this plant, the values of nonminimum phase zeros and unstable poles are known, it can be seen that the nonminimum phase zeros are located at $z = 3$, the zero-direction vector η is $\eta = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, the unstable pole is located at $p = 5$, and the pole direction vector ω is $\omega = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

Let suppose $\Gamma = 10$, $\varepsilon = \frac{1}{2}$, $U = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$, $N(z) = F_2 \begin{pmatrix} 1 & 0 \\ 1 & \frac{z-3}{z+0.5} \end{pmatrix} F_1$, so we have $N_n(z) = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{z+0.5} \end{pmatrix}$, $M^{-1}(z) = \begin{pmatrix} \frac{1}{z-5} & 0 \\ 1 & 1 \end{pmatrix}$.

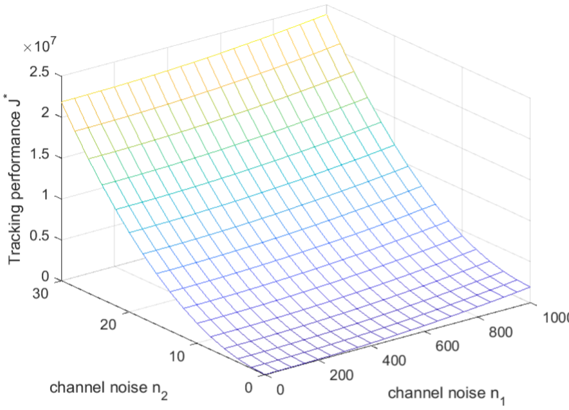

FIGURE 6. The performance for NCSs under different channel noise.

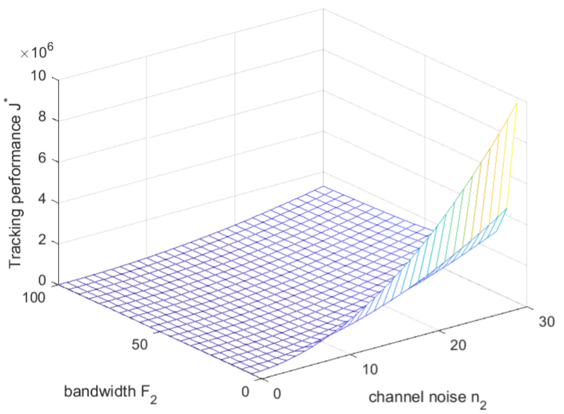

FIGURE 7. The performance limitation for NCSs under channel noise and bandwidth constraints.

At first, the bandwidth is selected as: $F_1(z_i) = \begin{pmatrix} \frac{10}{13} & 0 \\ 0 & \frac{10}{13} \end{pmatrix}$, $F_2^{-1}(p_j) = \begin{pmatrix} \frac{3}{2} & 0 \\ 0 & \frac{3}{2} \end{pmatrix}$, and $V = \begin{pmatrix} \delta & 0 \\ 0 & \delta \end{pmatrix}$, $A = \begin{pmatrix} \sigma & 0 \\ 0 & \sigma \end{pmatrix}$,

From Theorem 2, following can be obtained:

$$J_6^* = \frac{232}{169}\delta^2 + 24300\sigma^2 + 141$$

The performance limitation for NCSs under different channel noise is shown in Fig. 6. It can be seen from Fig. 6 that the performance limitation for NCSs will be influenced by the channel noise, and the greater the channel noise is, the worse the performance limitation for NCSs will be.

Second, the bandwidth is selected as: $F_1(z_i) = \begin{pmatrix} \frac{10}{13} & 0 \\ 0 & \frac{10}{13} \end{pmatrix}$, $F_2^{-1}(p_j) = \begin{pmatrix} \frac{\mu_2+10}{\mu_2} & 0 \\ 0 & \frac{\mu_2+10}{\mu_2} \end{pmatrix}$, and $V = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$, $A = \begin{pmatrix} \sigma & 0 \\ 0 & \sigma \end{pmatrix}$, From Theorem 2, we can obtain:

$$J_7^* = \frac{1200(\mu_2 + 10)^2}{\mu_2^2}\sigma^2 + \frac{4007}{169}$$

The performance limitations for NCSs under channel noise and bandwidth are shown in Fig. 7. The Fig. 7 shows that the channel noise and bandwidth affect the performance limitation for NCSs. The greater the bandwidth is, the greater the performance limitation of NCSs will be, and the greater the channel noise is, the worse the performance of NCSs will be.

IV. CONCLUSION

The performance limitation for NCSs under two-channel constraints is investigated in this paper. Channel noise and bandwidth in forward and feedback loops simultaneously. The obtained results show that the performance limitation for NCSs is related to the intrinsic properties of a given plant such as locations and directions of nonminimum phase zeros and unstable poles. At the same time, the performance limitation for NCSs is influenced by parameters such as channel input power, energy constraints, channel noise and bandwidth. Finally, different influence factors and different Pole-Zero are discussed to validate the feasibility of the proposed methods in the simulation section. The proposed methods in this paper assumes that the parameters of the system models are known. For the systems with unknown parameters, one uses some identification approaches [31]–[33].

This paper has discussed the performance of SISO or MIMO NCSs with communication constraints. However, practical applications also include SIMO NCSs. It is essential to discuss the performance limitation for SIMO NCSs under the constraints used in this paper. This problem will be studied in future work.

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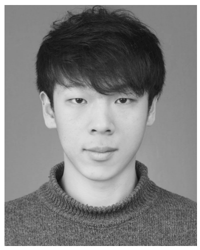

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