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Pignistic Belief Transform: A New Method of Conflict Measurement

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ABSTRACT To measure conflict between two basic probability assignment functions plays the key role of conflict management in Dempster-shafer evidence theory. In this paper, a new conflict measure is proposed. First, the classical pignistic probability transform (PPT) is generalized as pignistic belief transform (PBT). One of the advantages of PBT is that it can assign belief to multiple sets. When the belief is assigned to single element, the proposed PBT is degenerated as classical PPT. Then, the betting distance of two pignistic belief transforms is proposed, which can be used as a new conflict degree of BPAs. Finally, a numerical example is illustrated to show the use of the proposed method to combine conflicting evidence.

INDEX TERMS Dempster-Shafer evidence theory, belief function, conflict, pignistic belief transform.

I. INTRODUCTION

In order to enhance the efficiency of decision system, multi-sensor data fusion is widely used [1]–[5]. There are many methods to deal with information fusion under uncertain environment, such as fuzzy sets [6], rough sets [7], D numbers [8], [9], Dempster-Shafer evidence theory [10], [11] and so on [12]. Many new methods to deal with uncertain information based on fuzzy sets are proposed [13]–[18]. Dempster-Shafer evidence theory takes advantage of handling imprecise and unknown information [19]–[21] since the basic probability assignment (BPA) provides a more flexible way to process uncertainty than probability distribution [22]–[27].

In addition, Dempster rule in evidence theory can combine two or more BPAs and plays an important rule in information fusion. However, Zadeh has found that Dempster combination rule leads to counter-intuitive results in highly conflicting environment [28]. Therefore, Yager [29], Dubois and Prade [30], Smets [31], Muphy [32] and others have proposed some new methods to combine the conflicting BPAs [33]–[35].

However, conflict management is still an open issue [36]–[38]. Even conflict measurement of BPA is argued [39]–[42]. Jousselme et.al proposed the distance between BPAs [43], which is widely used in conflicting data fusion [44]–[46]. But, Liu argued that only the distance function is not enough to measure the conflict. As a result, the distance between betting commitments combined with the classical conflict coefficient k are constructed a two dimensional conflicting measure [47]. However, it still has some problems in some situations. When the set A in the difBetP in [41] is not a single element set, pignistic probability transform(PPT) used in difBetP just sum up the value of BetP of each element and distribute belief to each element rather than its power sets equally.

To address the issue in [47], Smets' Pignistic probability transform (PPT) [48] is generalized as pignistic belief transform(PBT). Based on the presented PBT, a new conflict measure of BPA is proposed. The presented $PBT(A)$ has two parts: (1) belief from subsets of A. (2) belief from those that are not contained in A. Firstly, belief of sets which contain A is assigned to its power sets equally. Then belief of sets that have an intersection with A is assigned to A with the corresponding weight. Conflict is measured by calculating the proposed betting distance between the PBTs. Some numerical examples are used to show the efficiency and rationality of the proposed method.

The rest of the paper is organized as follows. In section 2, the basic concepts and definitions in evidence theory, a typical combination example with high conflict, distance between *BPA*s [43], pignistic possibility transform [48] and distance between betting commitments [47] are reviewed. In section 3, the proposed method is introduced and some numerical

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examples are given to show its efficiency and rationality. In section 4, we show the proposed method's application in data fusion and compare it with some existed methods. Finally, section 5 concludes the paper by showing the proposed method's advantage and limitations.

II. PRELIMINARIES

In this section, some preliminaries are briefly introduced, including Dempster-Shafer evidence theory [10], [11], distance between BPAs [43], Pignistic probability transform [48] and distance between betting commitments [47].

A. DEMPSTER-SHAFER EVIDENCE THEORY

Real world is very complicated [49]–[52]. How to deal with uncertainty is still an open issue [53]–[56]. The basic concepts of evidence theory, including *BPA* and Dempster combination rule, are introduced as follows.

Definition 1: Let Θ be a finite nonempty set of mutually exclusive hypotheses called discernment frame. [10], [11]

$$\Theta = \{H_1, H_2, \cdots, H_i, \cdots, H_N\} \tag{1}$$

The power set of Θ is defined as follows

$$2^{\Theta} = \{\emptyset, \{H_1\}, \cdots, \{H_N\}, \{H_1, H_2\}, \cdots, \{H_1, \cdots, H_n\}\} \tag{2}$$

where $\emptyset$ is an empty set.

Definition 2: A mass function m, is a mapping of 2^{Θ}, which is defined as follows [10], [11].

$$m: \quad 2^{\Theta} \rightarrow [0, 1] \tag{3}$$

which satisfies the following conditions:

$$m(\emptyset) = 0 \quad \sum_{A \in 2^{\Theta}} m(A) = 1 \tag{4}$$

The mass function $m(A)$ represents how strongly the evidence supports A, which is also called the basic probability assignment function (BPA).

Definition 3: The belief function(Bel) is defined as follows [10], [11],

$$Bel(A) = \sum_{B \subseteq A} m(B) \tag{5}$$

It's used to measure one's belief that hypothesis A is true.

Definition 4: The plausibility function(Pl) is defined as follows [10], [11],

$$Pl(A) = \sum_{A \cap B \neq \phi} m(B) \tag{6}$$

It's used to measure the total belief that can be assigned to A. The value of probability that hypothesis A is true should be in the interval $[Bel(A), Pl(A)]$.

Combined evidence can be obtained after using Dempster's combination rule [10], [11].

Definition 5: Given two BPAs m_1 and m_1, Dempster combination rule is defined as follows [10], [11].

$$\begin{cases} m(\emptyset) = 0 \\ m(A) = \frac{\sum_{B \bigcap C = A} m_1(B) m_2(C)}{1-K} \end{cases} \tag{7}$$

where $K = \sum_{B \bigcap C = \emptyset} m_1(B) m_2(C)$.

Combination rule plays an important role in evidential reasoning and decision making [57]. It is markable that K is the coefficient to measure the conflict between evidences, and the combination rules could not be used when $K > 1$.

B. A TYPICAL CONFLICT EXAMPLE

Given two *BPA*s, let

$$m_1(A) = 0.99, \quad m_1(B) = 0.01, \; m_1(C) = 0$$
$$m_2(A) = 0, \quad m_2(B) = 0.01, \; m_1(C) = 0.99$$

Applying Dempster combination rule, then

$$m_1 \oplus m_2(A) = 0$$
$$m_1 \oplus m_2(B) = 1$$
$$m_1 \oplus m_2(C) = 0$$

And $m_{\bigoplus}(\phi) = 0.99$

Evidence fusion with Dempster combination rule gets counterintuitive results when there is high conflict. It's essential to combine evidences when high conflict exists [58] and conflict measure is a key point.

C. DISTANCE BETWEEN BPAs

Definition 6: The distance between BPAs is is defined as [43]

$$d_{BPA} = \sqrt{\frac{1}{2}(m_1 - m_2)^T \underline{\underline{D}}(m_1 - m_2)} \tag{8}$$

where $\underline{\underline{D}}$ is $2^{\Omega} \times 2^{\Omega}$-dimensional matrix with $d(i, j) = \frac{|A \cap B|}{|A \cup B|}$, and $A \in 2^{\Omega}$, $B \in 2^{\Omega}$ are the names of columns and rows respectively (note, we define $|\varnothing \bigcap \varnothing| / |\varnothing \bigcup \varnothing| = 0$).

D. PIGNISTIC PROBABILITY TRANSFORM

Definition 7: Pignistic probability transform(PPT) is defined as follows [48],

$$BetP(A) = \sum_{B \subseteq \Theta} \frac{|A \cap B|}{|B|} \cdot \frac{m(B)}{1 - m(\phi)} \tag{9}$$

where $|B|$ is the cardinality of B and the essence of PPT is to convert a BPA function into a probability distribution. It allocates the belief to each elements equally.

E. DISTANCE BETWEEN BETTING COMMITMENTS

Let m_1 and m_2 be two BPAs,

Definition 8: the distance between betting commitments [47] is defined as follows

$$difBetP_{m_1}^{m_2} = max_{A\subseteq\Omega}(|BetP_{m_1}(A) - BetP_{m_2}(A)|) \quad (10)$$

which is called the distance between betting commitments of the two BPAs.

III. PROPOSED METHOD

The intersection of two sets may occupies different proportion in these two sets, while the relevant components are added together without weight when calculating $BetP$. In this paper, sets are classified into three categories and different weights are given when calculating pignistic belief transform.

A. PIGNISTIC BELIEF TRANSFORM

To calculate the PBT of set A, we divided the BPA into two parts, (1)$B \subseteq A$. (2)$C \nsubseteq A$. For the first part, the belief is added together directly. The first part of PBT is obtained. For the second part, the belief of the set is divided into its power sets equally. After that, these sets are divided into two parts (2.1)$E \subseteq C, E \subseteq A$. (2.2)$D \subseteq C, D \nsubseteq A$.

As mentioned before, belief of set C is divided into its power sets. Cardinality of each set in the power set may be different. And the intersection of each set in the power set and set A may be different.

An asymmetric similarity measure method between set A and set B is defined as follows.

$$S(A, B) = \frac{2^{|A\cap B|}}{2^{|A|}} \quad (11)$$

For part 2.1, actually, E represents the subsets of set $A \cap C$. As a result, $2^{|A\cap C|} - 1$ is used to represent the number of the power sets of the intersection of A and C. The cardinality of $E \bigcap C$ is equal to $E \bigcap A$. Then belief of set E is added directly.

For part 2.2, $S(A, D)$ and $S(A, C)$ are calculated and the quotient of them are used to represent the relevance of D and A. Then the obtained relevance is set as the weight. Belief of D is assigned to its element equally and then multiply the weight. By adding the two parts(part 2.1 and 2.2) together, the second part of PBT is obtained.

Definition 9: Pignistic belief transform(PBT) is defined as follows,

$$B_f(A) = B_{d1}(A) + B_{d2}(A) \quad (12)$$

It is not normalized. While the first part is defined as follows,

$$B_{d1}(A) = \sum_{B\subseteq A} m(B) \quad (13)$$

the second part is defined as follows,

$$\begin{aligned} &B_{d2}(A) \\ &= \sum_{C\nsubseteq A} \frac{m(C)}{2^{|C|}-1} \\ &\quad \cdot \Big(\sum_{E\subseteq C, E\subseteq A} \frac{|E\cap C|}{|E\cap A|} + \sum_{D\subseteq C, D\nsubseteq A} \frac{|A\bigcap D|}{|D|} \cdot \frac{S(A,D)}{S(A,C)}\Big) \\ &= \sum_{C\nsubseteq A} \frac{m(C)}{2^{|C|}-1} \\ &\quad \cdot \Big(2^{|A\cap C|} - 1 + \sum_{D\subseteq C, D\nsubseteq A} \frac{|A\bigcap D|}{|D|} \cdot \frac{2^{|A\cap D|}-1}{2^{|A\cap C|}-1}\Big) \end{aligned} \quad (14)$$

When $|A| = 1$, the first part becomes

$$B_{d1}(A) = m(A) \quad (15)$$

the second part becomes

$$\begin{aligned} B_{d2}(A) &= \sum_{C\nsubseteq A, A\cap C\neq 0} \frac{m(C)}{2^{|C|}-1} \cdot \Big(1 + \sum_{\substack{D\subseteq C,\\ D\nsubseteq A}} \frac{1}{|D|}\Big) \\ &= \sum_{C\nsubseteq A, A\cap C\neq 0} \frac{m(C)}{|C|} \end{aligned} \quad (16)$$

Proof: First of all, a symbol $M(n, i)$ is defined.

$$M(n, i) = \frac{n!}{i! * (n-i)!} \quad (17)$$

And $n! = 1 * 2 * 3 * \cdots * n$.

For the binomial expansion, it's easy to obtain that

$$(x + y)^n = \sum_{i=0}^{n} M(n, i) * x^i * y^{n-i} \quad (18)$$

when $x = 1$ and $y = 1$,

$$(1 + 1)^n = \sum_{i=0}^{n} M(n, i) * 1^i * 1^{n-i} = \sum_{i=0}^{n} M(n, i) \quad (19)$$

and $M(n, 0) = 1$, so

$$\sum_{i=1}^{n} M(n, i) = 2^n - 1 \quad (20)$$

besides,

$$M(n-1, i) \cdot \frac{1}{i+1} = \frac{(n-1)!}{(n-1-i)! \cdot (i+1)!} = \frac{M(n, i)}{n} \quad (21)$$

Then

$$\begin{aligned} &B_{d2}(A) \\ &= \sum_{C\nsubseteq A, A\cap C\neq 0} \frac{m(C)}{2^{|C|}-1} \cdot \Big(1 + \sum_{D\subseteq C, D\nsubseteq A} \frac{1}{|D|}\Big) \\ &= \sum_{C\nsubseteq A, A\cap C\neq 0} \frac{m(C)}{2^{|C|}-1} \cdot \Big(1 + \sum_{i=1}^{|C|-1} \frac{M(|C|-1, i)}{i+1}\Big) \end{aligned}$$

$$= \sum_{C\nsubseteq A, A\cap C\neq 0} \frac{m(C)}{2^{|C|}-1}\cdot\left(\frac{M(|C|,1)}{|C|}+\sum_{i=1}^{|C|-1}\frac{M(|C|,i+1)}{|C|}\right)$$
$$= \sum_{C\nsubseteq A, A\cap C\neq 0} \frac{m(C)}{(2^{|C|}-1)\cdot|C|}\cdot\left(\sum_{i=1}^{|C|}M(|C|,i)\right)$$
$$= \sum_{C\nsubseteq A, A\cap C\neq 0} \frac{m(C)}{(2^{|C|}-1)\cdot|C|}\cdot(2^{|C|}-1)$$
$$= \sum_{C\nsubseteq A, A\cap C\neq 0} \frac{m(C)}{|C|} \tag{22}$$

As a result,

$$\begin{aligned} B_f(A) &= B_{d1}(A)+B_{d2}(A) \\ &= m(A)+\sum_{C\nsubseteq A, A\cap C\neq 0}\frac{m(C)}{|C|} \\ &= BetP(A) \end{aligned} \tag{23}$$

Example 1: Suppose the discernment frame is $\Omega = \{A, B, C\}$ *and*

$$m(B) = 0.3,\quad m(C) = 0.1$$
$$m(A, B) = 0.3,\quad m(\Omega) = 0.3$$

A simple example is given to prove that equivalence of $BetP(A)$ *and proposed function* $B_f(A)$*, results are shown in* $Tab.1$.

TABLE 1. Values of ***BetP*** and proposed function B_f.

$Cases$	$BetP$	B_f
$\{A\}$	0.25	0.25
$\{B\}$	0.55	0.55
$\{C\}$	0.20	0.20

Example 2: Suppose the discernment frame is $\Omega = \{A, B, C, D\}$ *and*

$$m(B) = 0.2,\quad m(C) = 0.1$$
$$m(A, B) = 0.3,\quad m(\Omega) = 0.4$$

and we list Bel function, Pl function, and the possibility function in $Tab.2$.

The proposed method works when the Dempster combination rule works, which means K<1. The value of B_f is in the interval $[Bel(A), Pl(A)]$, which shows that the method for allocation is acceptable.

B. CONFLICT MEASURING

Definition 10: Given two PBTs, the betting distance is defined as follows.

$$difBf_{m_1}^{m_2} = max_{A\subseteq\Omega}(|B_{fm_1} - B_{fm_2}|) \tag{24}$$

The value of $difBf_{m_1}^{m_2}$ is used to measure the conflict.

Then the example in [47] was used and d_{BPA} [43], $difBetP$ [47] and the proposed method were compared.

TABLE 2. Values of belief function ***Bel***, plausibility function ***Pl*** and our possibility function B_f.

$Cases$	Bel	Pl	B_f
$\{A\}$	0	0.7	0.25
$\{B\}$	0.2	0.9	0.45
$\{C\}$	0.1	0.5	0.20
$\{D\}$	0	0.4	0.10
$\{A,B\}$	0.5	0.9	0.6526
$\{A,C\}$	0.1	0.9	0.4026
$\{A,D\}$	0	0.7	0.3026
$\{B,C\}$	0.3	1	0.6026
$\{B,D\}$	0.2	0.9	0.5026
$\{C,D\}$	0.1	0.5	0.2526
$\{A,B,C\}$	0.6	1	0.8352
$\{A,B,D\}$	0.5	0.9	0.7352
$\{A,C,D\}$	0.1	0.8	0.4852
$\{B,C,D\}$	0.3	1	0.6852
$\{A,B,C,D\}$	1	1	1

Example 3: In [47], let Ω *be a frame of discernment with 20 elements. The first BPA,* m_1 *is defined as*

$$m_1(2, 3, 4) = 0.05,\quad m_1(7) = 0.05,$$
$$m_1(\Omega) = 0.1,\quad m_1(A) = 0.8$$

where A is a subset of Ω.

And the second BPA, m_2 is defined as

$$m_2(1, 2, 3, 4, 5) = 1$$

As the subset A changes, we calculate the value and results are shown in $Tab.3$.

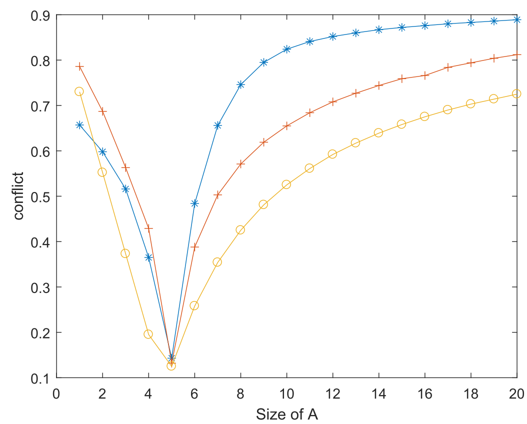

FIGURE 1. Comparison of different methods, $+\rightarrow d_{BPA}$, $\circ \rightarrow difBetP$, $* \rightarrow difBf$.

$Fig.1$ shows that the tend of the value calculated with our method is similar to d_{BPA} [43]. And the minimum conflict appears when $A = \{1, 2, 3, 4, 5\}$. As subset A increments one more element at a time, from $A = \{1\}$ to $A = \{1, 2, \ldots, 20\}$, the value of conflict decreases when the cardinality of A is less than 5, and increases when it's bigger than 5. When the

TABLE 3. Comparison of d_{BPA},$difBetP$,$difB_f$ and $m_{\bigoplus}(\phi)$ values of m_1 and m_2 when subset A changes. Here $m_{\bigoplus}(\phi) = m_1 \oplus m_2$.

$Cases$	d_{BPA}	$difBetP$	$difB_f$	$m_{\bigoplus}(\phi)$
$A = \{1\}$	0.786	0.730	0.657	0.05
$A = \{1, 2\}$	0.687	0.552	0.598	0.05
$A = \{1, 2, 3\}$	0.563	0.373	0.516	0.05
$A = \{1, ..., 4\}$	0.429	0.195	0.365	0.05
$A = \{1, ..., 5\}$	0.132	0.125	0.143	0.05
$A = \{1, ..., 6\}$	0.388	0.258	0.484	0.05
$A = \{1, ..., 7\}$	0.503	0.355	0.656	0.05
$A = \{1, ..., 8\}$	0.571	0.425	0.746	0.05
$A = \{1, ..., 9\}$	0.619	0.481	0.795	0.05
$A = \{1, ..., 10\}$	0.655	0.525	0.824	0.05
$A = \{1, ..., 11\}$	0.684	0.561	0.841	0.05
$A = \{1, ..., 12\}$	0.708	0.592	0.852	0.05
$A = \{1, ..., 13\}$	0.727	0.617	0.860	0.05
$A = \{1, ..., 14\}$	0.744	0.639	0.867	0.05
$A = \{1, ..., 15\}$	0.759	0.658	0.872	0.05
$A = \{1, ..., 16\}$	0.766	0.675	0.876	0.05
$A = \{1, ..., 17\}$	0.784	0.690	0.880	0.05
$A = \{1, ..., 18\}$	0.794	0.703	0.883	0.05
$A = \{1, ..., 19\}$	0.804	0.714	0.886	0.05
$A = \{1, ..., 20\}$	0.812	0.725	0.889	0.05

TABLE 4. Comparison of Dempster's combination rule, Murphy's average combination rule, Deng et.al's combination rule, Proposed modified average combination rule.

	m_1, m_2	m_1, m_2, m_3	m_1, m_2, m_3, m_4	m_1, m_2, m_3, m_4, m_5
Dempster-Shafer's combination rule	m(A)=0 m(B)=0.8750 m(C)=0.0750 m(D)=0.0500	m(A)=0 m(B)=0.8750 m(C)=0.0750 m(D)=0.0500	m(A)=0 m(B)=0.9000 m(C)=0.0375 m(D)=0.0625	m(A)=0 m(B)=0.7339 m(C)=0.1422 m(D)=0.1239
Murphy's average combination rule	m(A)=0.3164 m(B)=0.6140 m(C)=0.0426 m(D)=0.0271	m(A)=0.4848 m(B)=0.4539 m(C)=0.0358 m(D)=0.0255	m(A)=0.7104 m(B)=0.2631 m(C)=0.0111 m(D)=0.0154	m(A)=0.8847 m(B)=0.0964 m(C)=0.0109 m(D)=0.0075
Deng et.al's combination rule	m(A)=0.3164 m(B)=0.6140 m(C)=0.0426 m(D)=0.0271	m(A)=0.6985 m(B)=0.2366 m(C)=0.0382 m(D)=0.0267	m(A)=0.8924 m(B)=0.0857 m(C)=0.0090 m(D)=0.0128	m(A)=0.9633 m(B)=0.0237 m(C)=0.0076 m(D)=0.0052
Proposed modified average combination rule	m(A)=0.3164 m(B)=0.6140 m(C)=0.0426 m(D)=0.0271	m(A)=0.6971 m(B)=0.2388 m(C)=0.0378 m(D)=0.0262	m(A)=0.8923 m(B)=0.0859 m(C)=0.0090 m(D)=0.0128	m(A)=0.9653 m(B)=0.0218 m(C)=0.0076 m(D)=0.0051

cardinality of A is close to 20, the upward trend slows down and the value is finally stable at around 0.9.

It seems unreasonable that the conflict gets the same value when $A = \{1, 2, 3, 4\}$ and $A = \{1, 2, 3, 4, 5\}$ while using *difBetP* [47]. As a result, we proposed *PBT* and measured the conflict based on it. And the results shows that we only obtain the minimum conflict when $A = \{1, 2, 3, 4, 5\}$, which is the same as applying d_{BPA}. The overall trend of the proposed method is similar to d_{BPA}.

However, when the cardinality of A is less than 5, d_{BPA} is larger than $difB_f$. When the cardinality of A is larger than 5, d_{BPA} is smaller than $difB_f$. And the value of $difB_f$ is not equal to d_{BPA} in all cases here, which implies that the proposed method doesn't work for measuring precise conflict between BPAs. It works for calculating the relative distance between conflicting evidence rather than the precise distance.

To test the efficiency, a fictitious example which applies the proposed method for data fusion is given. And numerical results obtained from different combination rules were compared.

IV. APPLICATION IN DATA FUSION

D-S theory is widely used in many fields such as information fusion systems [34], [59]–[63], complex network [64], [65], target identification [66]–[69], decision-making method [70]–[72], fuzzy systems [73]–[77].

An example of data fusion is given here, which applies the proposed method.

A. COMBINATION METHOD

The method proposed by Deng in [78] is adopted for data fusion. But instead of using d_{BPA} [43], the proposed conflict measuring method was used to calculate the similarity of two pieces of evidence. And brief procedure is given as follows.

Firstly, calculate each similarity between different evidences.

$$Sim_{m_i,m_j} = 1 - difBf_{m_i}^{m_j} \tag{25}$$

The remaining part is the same as that in [78]. Pseudo code is as follows. If there are k evidence.

Let $sum \leftarrow 0$, $WAE \leftarrow 0$;
for each $i \in [1, k]$ **do**
  $Sup(i) \leftarrow 0$;
  for each $j \in [1, k]$ and $i \neq j$ **do**
    $Sup(i) \leftarrow Sup(i) + Sim_{m_i,m_j}$;
  end for
  $sum \leftarrow sum + Sup(i)$;
end for
for each $i \in [1, k]$ **do**
  $Crd(i) \leftarrow \frac{Sup(i)}{sum}$;
end for
for each $i \in [1, k]$ **do**
  $WAE \leftarrow WAE + Crd(i) \times m_i$;
end for
Use Dempster's combination rule to combine k WAEs k-1 times;

B. NUMERICAL EXAMPLE

A fictitious example is illustrated to show the use of the proposed combination rule. A is supposed to be the real target in a multisensor-based automatic target recognition system. And from five different sensors, five bodies of evidence are collected which is shown as follows:

$$(R_1, m_1) = ([\{A\}, 0.6], [\{A, B, C\}, 0.1], [\{A, B, C, D\}, 0.3])$$
$$(R_2, m_2) = ([\{A\}, 0], [\{B\}, 0.8], [\{B, C, D\}, 0.2])$$
$$(R_3, m_3) = ([\{A\}, 0.4], [\{A, B, C, D\}, 0.6])$$
$$(R_4, m_4) = ([\{A\}, 0.6], [\{A, B, D\}, 0.2], [\{A, B, C, D\}, 0.2])$$
$$(R_5, m_5) = ([\{A\}, 0.3], [\{A, B\}, 0.2], [\{A, C\}, 0.2], [\{A, C, D\}, 0.3])$$

m_2 is a bad evidence. As shown in *Tab.*4, system with Dempster's combination rule [10], [11] draw the conclusion that the target is B, which is counterintuitive. Murphy's average combination rule [32], Deng et.al's rule [78] and the proposed method all draw the correct conclusion that the target is A. However, when applying Murphy's combination rule [32], the probability that the target is A is always lower than the other two methods in this example. And the proposed method has the same high accuracy as Deng et.al's method [78], which implies that the proposed method is efficient and rational. When bad evidence is involved in the calculation, Dempster's combination rule [10], [11] would get wrong result sometimes, especially when evidence that distribute no belief to the right target. Murphy's average combination rule [32], Deng et.al's rule [78] and the proposed method assign weight to each evidence so that each target is possible to be the final result. When good evidence is more than bad evidence, the right target would be assigned bigger probability. Differently, Deng et.al's rule [78] and the proposed method use similarity to assign the weight rather than treating each evidence equally. As a result, the probability of A in the results obtained from these two methods is higher.

The presented method changes the way for attaining the similarity in order to change the *SMM*. It retains the original advantage in [78] that the weight of bad evidence is decreased by defining the similarity matrix and use the weighed average to combine the evidence.

V. CONCLUSION

Conflict management is still an open issue. Conflict measuring is a key problem in conflict management. By considering adding weight when combining evidence and allocate belief to its power set equally, *PBT* is presented and a new method to measure the conflict has been proposed. A fictitious example is illustrate to show that the proposed method can get true target when a certain amount pieces of good evidence are obtained. Besides, *PBT* is the generalized form of *PPT*, and $B_f(A)$ can be represented by the $BetP(A)$ [48] when the cardinality of A in $B_f(A)$ equals to 1. Differently, *PBT* provides a new way to allocate belief to sets with multiple elements by setting weight. And it leads to different results when compare *difBf* with *difBetP* in some situations. The proposed method has the promising aspect in real engineering. One of our ongoing works is to decrease the complexity of the proposed method.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests regarding the publication of this paper.

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