

Received December 19, 2019, accepted January 10, 2020, date of publication January 14, 2020, date of current version January 21, 2020. Digital Object Identifier 10.1109/ACCESS.2020.2966530

# **Exploring the Impact of Node Correlation** on Transmission Reuse in MANETs

## **RIHENG JIA<sup>(D)</sup>, FEILONG LIN<sup>(D)</sup>, AND ZHONGLONG ZHENG<sup>(D)</sup>** College of Mathematics and Computer Science, Zhejiang Normal University, Jinhua 321004, China

Corresponding author: Riheng Jia (rihengjia@zjnu.edu.cn)

This work was supported in part by the National Natural Science Foundation of China under Grant 61902358, Grant 61877055, and Grant 61672467, and in part by the National Natural Science Foundation of Zhejiang Province of China under Grant LQ19F020007 and Grant LY18E030013

**ABSTRACT** The main advantage of multicast over multi-unicast is the existence of transmission reuse, i.e., the cooperation among destinations. This intrinsic characteristic benefits the multicast regarding the enhancement of transmission efficiency. Consequently, multicast is extensively studied under various wireless environments. Yet people still have limited understanding on the impact of node mobility on the transmission reuse when multicast is applied instead of multiple unicast. In this paper, we focus on the correlated mobility which captures the feature of real mobility processes, to study its influence on transmission reuse in mobile ad hoc networks (MANETs). Specifically, we quantify the transmission reuse as multicast gain, i.e., the capacity ratio of multicast and multi-unicast under certain delay constraint. We design a multi-layer routing protocol and propose different kinds of causal scheduling policies, under which the overall multicast capacity-delay tradeoff is derived by exploring various correlation degrees of node mobility. Compared with the capacity-delay tradeoff in unicast case, we calculate the multicast gain. Results show that the correlation of node mobility greatly influences the multicast gain, and in certain cases network can achieve the upper bound of multicast gain regardless of the logarithmic factor.

**INDEX TERMS** Correlated mobility, multicast, transmission scheduling.

### I. INTRODUCTION

Multicast, an one-to-many traffic pattern, enables network links to be reused when a single source serves multiple destinations. Therefore, multicast plays an important role in many applications like group communications within military networks, content dissemination in online social networks, emergency alarming for earthquake disaster, etc. Compared with the multi-unicast mechanism, the capacity gain is obtained in multicast via sharing links for the destinations and cooperation among them. Li et al. [1] analyzed the capacity of wireless static networks where each source is associated with k destinations. They showed that each node can achieve  $\Theta(1/\sqrt{nk\log n})$  and  $\Theta(1/n)$  multicast capacity when k = $O(n/\log n)$  and  $k = \Omega(n/\log n)$  respectively. This result indicates that the transmission benefits from multicast instead of multi-unicast, as  $\Theta(1/\sqrt{k})$  links are saved and reused for transmission.

With the increasing development of mobile wireless devices including smartphone, pad, portable entertainment device, etc., the research on the impact of node mobility on the network performance draws much attention within network communities. Grossglauser and Tse [2] first incorporated mobility into the study of ad hoc networks and showed that the unicast capacity can be raised to O(1) at the expense of unbounded delay based on random i.i.d mobility model. Wang et al. [3] first defined the multicast in mobile networks as "MotionCast" and studied the delay and capacity tradeoff for multicast in MANETs. Since then, research on multicast performance in wireless mobile networks was extensively developed including the multicast performance under various mobility models [4]–[7] and realistic application scenarios such as buffer constraint [8] and cognitive radio networks [9]. Further, some literatures focused on improving the multicast capacity and delay via infrastructure support [10], network coding [11] and cooperation among nodes [12], [13]. In these works, results demonstrate that mobility can also increase the multicast capacity and different mobility models bring different capacity gains, compared with the static case.

The associate editor coordinating the review of this manuscript and approving it for publication was Jose Saldana<sup>10</sup>.

In static networks, Li *et al.* [1] proved that better transmission efficiency is realized in multicast compared with multi-unicast. Meanwhile, due to the stable topology of static networks, the network scheduler can conveniently collect the associated state information of each node such as location, buffer load and power consumption. Therefore, the link reuse as well as the cooperation among nodes is easy to manage, which can better enhance the transmission efficiency. As for the mobile situation where the topology changes over time, we doubt if the capacity gain can still be achieved when operating multicast instead of multi-unicast? In this paper, we study the above problem by utilizing the correlated mobility model [14], [15], which covers a broad range of mobility models and captures the feature of real mobility processes [16]–[19].

According to the correlated mobility model, nodes are organized into different groups with one center node in each group. Then, each center node moves over slot around the network according to the *i.i.d.* mobility model, dragging behind all the nodes belonging to the same group. Note that in the long term, all nodes will visit the entire network. The trajectories of individual nodes are not independent but constrained to jointly follow the same or similar motion path, which reflects the correlation of node movements. The correlated node mobility is common in the real world. For example, many buses travel in the city with each bus carrying a certain number of passengers and soldiers are organized into different troops marching in the battlefield. With various degrees of node correlations, the network topology changes and influence the network performance. Thus we try to figure out how the correlation of node mobility affect the performance gain between multicast and multi-unicast. In particular, we focus on the following two open questions in this paper:

- What is the upper bound of the optimal capacity-delay tradeoff for multicast under correlated mobility?
- How does the correlation of node mobility impact the transmission reuse in mobile ad-hoc networks?

To address the above questions, we quantify the transmission reuse as *multicast gain*, i.e., the capacity ratio of multicast and multi-unicast under certain delay constraint. By designing a layered routing protocol and different causal scheduling schemes, we first derive the upper bound of optimal multicast capacity-delay tradeoff under correlated mobility. Then we calculate the multicast gain under various correlation degrees of node mobility, compared with the capacity-delay tradeoff in unicast case.

Based on the multicast gain we study, we have the following interesting observations: 1) The correlation of node mobility is an essential feature that affects the node cooperation and link reuse during multicast communications in a wide range. 2) Very strong node correlation can greatly improve the multicast gain. It can even achieve the upper bound of multicast gain up to a logarithmic factor, i.e., the performance of multicast is almost equivalent to that of multiunicast, which can hardly be found in other mobility models. 3) Very weak node correlation can also improve the multicast

#### TABLE 1. Notations.

n	number of nodes
m	number of clusters
$n_d$	number of destinations in each multicast session
v	growth exponent of $m$ : $m = \Theta(n^v), 0 < v \le 1$
q	average number of node per cluster, $q = n/m = \Theta(n^{1-v})$
R	cluster radius
$\beta$	growth exponent of $R$ : $R = \Theta(n^{\beta})$
$\gamma$	growth exponent of $n_d$ : $n_d = \Theta(n^{\gamma})$
$C_s$	the cluster containing the source node of packet $b$
$C_r$	the cluster containing the copy of packet $b$
$C_d$	the cluster containing the destination node of packet $b$

gain to a certain extent. 4) The distinction between multicast and multi-unicast is weak when nodes show medium correlation.

The rest of the paper is organized as follows. In Section II, we introduce our system model and basic notations. In Section III, we briefly analyze the multicast gain and give a summary of main results. The multicast capacity-delay tradeoff under various correlation degrees of node mobility is studied in Section IV. We conclude the paper in Section V.

## **II. SYSTEM MODEL AND NOTATIONS**

We consider the network extension as a  $\sqrt{n} \times \sqrt{n}$  square region with wrap-around conditions (i.e., a tours), to avoid border effects. All the *n* nodes are organized into  $m = \Theta(n^{\nu})$  $(0 \le \nu < 1)$  groups. Each group has a group leader and covers a circular area with radius  $R = \Theta(n^{\beta})$   $(0 \le \beta \le 1/2)$ . We call such groups as *clusters* and the group leader as the *cluster center*. According to the node partitioning process, each cluster contains  $q = \Theta(n/m)$  nodes.<sup>1</sup> According to the definition of multicast, we assume each source node is designated for  $n_d$  destinations, where  $n_d = n^{\gamma}$  and  $0 \le \gamma < 1$ . In addition, the  $n_d$  destinations are randomly selected among max $\{m-1, 1\}$  other clusters. The definitions of system parameters are summarized in Table 1.

#### A. TIME SCALE

Time is divided into slots of equal unit duration. All nodes move over slots and keep static during a particular slot. In addition, we consider slow mobility time scale here, i.e., the speed of node movement is much slower than that of packet transmission. Thus the multi-hop transportation can be realized within one slot.

### **B. CORRELATED MOBILITY**

The correlated mobility process of a given node is described by the combination of two movements, i.e., a group (cluster) movement and a node movement:

• *The Group Movement:* For group movement, we assume that the location of each cluster center is updated at each slot by choosing a new location uniformly at random in

<sup>1</sup>It will not change our result if the number of nodes within each cluster is not exactly the same but remains  $\Theta(n/m)$ .

the network area, independently from any other cluster center. Once the new location of a cluster center has been selected, all nodes belonging to this cluster have to move to a place close to it (i.e., inside a region of area  $\pi R_2$  around the cluster center). The degree of correlation in the node mobility process can be increased either by reducing the area of each cluster-region or reducing the number of clusters.

• *The Node Movement:* Once a node of a group reaches the place close to the new location of the respective cluster center, it moves to a position chosen uniformly at random within the cluster-region of area  $\pi R_2$  around the cluster center.

The above movement describes the correlated mobility in our work. We observe that either reducing the number of clusters or the area each cluster covers will achieve strong correlation of node mobility. Depending on the values of  $\beta$  and v, we can divide our analysis into two different regimes: 1) *cluster sparse regime* ( $v + 2\beta < 1$ ): the total areas  $mR^2$  that all clusters cover is o(n), which shows strong correlation of node mobility; 2) *cluster dense regime* ( $v + 2\beta \ge 1$ ): the total areas  $mR^2$  that all clusters cover is  $\omega(n)$ , which shows weak correlation of node mobility.<sup>2</sup>

#### C. TRAFFIC PATTERN

We assume each node is a source node associated with  $n_d = o(n)$  destinations, which are randomly and independently chosen among all the other nodes in the network. We also assume the  $n_d$  destinations are uniformly chosen among all the clusters excluding the cluster of the source. Then sources send packets to their  $n_d$  destinations respectively via a common wireless channel and we utilize the protocol model in [20] to reduce the interference.

#### D. DEFINITION OF CAPACITY AND DELAY

Suppose that all sources communicate with their  $n_d$  destinations at the same rate  $\lambda$ . Let  $\overline{D}$  denote the average delay over all messages among all source-destination pairs. Let  $\lambda_i$  (i = 1, ..., n) denote the sustainable rate of data flow  $df_i$  (i = 1, ..., n) for node *i*.  $D_{b,k}$   $(b = 1, ..., \lambda nT, k = 1, ..., n_d)$  represents the delay for packet *b*. Assume that  $\lambda = \min{\{\lambda_1, \lambda_2, ..., \lambda_{n-1}, \lambda_n\}}$  and  $\overline{D} = \sum_{b=1}^{\lambda nT} (\sum_{k=1}^{n_d} D_{b,k} / n_d) / \lambda nT$ . Then  $\lambda = \Theta(f(n))$  is defined as the asymptotic capacity if there exists constants c > c' > 0, such that

$$\lim_{n \to \infty} \mathbf{Pr}(\lambda = cf(n) \text{ is achievable}) < 1,$$
$$\lim_{n \to \infty} \mathbf{Pr}(\lambda = c'f(n) \text{ is achievable}) = 1.$$

Meanwhile,  $\overline{D} = \Theta(g(n))$  is defined as the asymptotic delay as well.



FIGURE 1. Multicast gain of correlated mobility.

#### **III. MULTICAST GAIN AND MAIN RESULTS**

In this section, we introduce the multicast gain and show how the correlation of node mobility influences the multicast gain based on our results.

Regarding one multicast session with one source and  $n_d$  destinations as  $n_d$ -multi-unicast transmission, i.e., we separate the  $n_d$ -destination multicast into  $n_d$  independent one to one unicast. Note that, the multi-unicast mechanism makes the source node repeatedly sending copies of the same packet, this behavior wastes many network resources (e.g., bandwidth) which could have been reused for transmitting packets from other sources. As network links are usually shared by multiple data flows and the network capacity is limited, repeatedly transmitting the same data packets decreases the average throughput of not only a single multicast session but also the whole network. Thus, transmission reuse basically illustrates how much network resources can be preserved to deliver as many different packets as possible rather than being wasted on the duplicated packets. To give the quantitative analysis of transmission reuse, we define the multicast gain as the capacity ratio of multicast and multi-unicast under a certain delay constraint. Because high ratio of transmission reuse implies that more network resources are preserved for delivering different multicast packets wither from the same source or from other sources, which results in high average network throughput. Specifically, we set capacity-delay tradeoff as a function of capacity  $\lambda$  with variable of delay D, i.e.,  $\lambda(D) = f(D)$ .  $\lambda_m$  and  $\lambda_{mu}$  denote the capacity of multicast and multi-unicast, respectively. Given a delay  $D_{\Delta}$ , the multicast gain G is

$$G = \frac{\lambda_m(D_\Delta)}{\lambda_{mu}(D_\Delta)}.$$

In Fig.1, we clearly show the variation of multicast gain versus the node correlation index  $v + 2\beta$ . First, we briefly explain the upper bound and lower bound of the multicast gain, which are  $\theta$  ( $n_d$ ) and  $\theta$  (1) respectively. For a generic multicast session and packet b, the best case is that b can be successfully delivered to all the  $n_d$  destinations within one unicast slot, then the total amount of throughput for multicast is  $bn_d$  compared with b in the unicast. Thus, the upper bound  $\theta$  ( $n_d$ ) is achieved. Similarly, the worst case is that during one unicast slot only one of the  $n_d$  destinations receives packet b.

<sup>&</sup>lt;sup>2</sup>The degree of correlation of node mobility can be adjusted by changing the values of  $\beta$  and  $\nu$ 



**FIGURE 2.** The red node and yellow node denote the source and destination. Green line indicates the successful delivery of packet and gray line indicates the unsuccessful case.

Then, we easily get the lower bound of multicast gain as  $\theta$  (1). We show the two cases in Fig.2.

Based on the multicast capacity-delay tradeoff in both cluster sparse regime  $\lambda^s = O(f(D))$  and cluster dense regime  $\lambda^d = O(f(D))$ , we can easily get the multicast gain compared with the results in [22]. The variation tendency of the multicast gain gives a whole picture about the impact of correlation of node mobility on the transmission reuse. Specifically, when  $0 < v + 2\beta < 1$ , i.e., the node mobility shows strong correlation, the multicast gain is relatively high. As the value of  $v + 2\beta$  tends to 0, the multicast gain continues to increase until it approaches the upper bound of  $\theta$  (*n<sub>d</sub>*). Under the strong correlation of node mobility, traces of certain amount of mobile nodes tend to be regular and tractable. Therefore, the network topology is easier to control by the scheduler, which is important for managing the node cooperation and link reuse. When  $1 < v + 2\beta < 2$ , i.e., the node mobility shows weak correlation, the multicast gain is smaller than that of the strong correlation case. It is intuitive that the multicast gain should decrease as the correlation of node mobility weakens, i.e.,  $v + 2\beta \rightarrow 2$ . However, from Fig.1 we can find that the multicast gain increases as  $v + 2\beta$  ranges from 1 to 2. It can be justified as follows. When the correlation weakens, different clusters overlap with each other w.h.p. and nodes of different clusters achieve much better effect of interactions, although the cooperation within each cluster falls off. Furthermore, the multicast gain is smallest when  $v+2\beta = 1$ , which can be easily explained that when  $v+2\beta = 1$ 1 (i.e., node mobility shows medium correlation), neither the cooperation within each cluster or the interaction between clusters is active.

## **IV. CAPACITY-DELAY TRADEOFF FOR MULTICAST**

According to the definition of multicast gain, i.e., the capacity ratio of multicast and multi-unicast under certain delay constraint, we need to study the capacity-delay tradeoff of both multicast and unicast for correlated mobility. As the unicast case has already been addressed in [22], in this paper we focus on the tradeoff in the multicast case. Based on this, we calculate the multicast gain and obtain the result.

#### A. CLUSTER SPARSE REGIME: $V + 2\beta < 1$

1) ROUTING SCHEME FOR CLUSTER SPARSE REGIME

Consider a particular packet b and a particular traffic flow, i.e.,  $s \rightarrow d$ , we show how packet b traverses from the source to its destination. According to the feature of correlated mobility, we divide the transmission into two parts: 1) *inter-cluster transmission*, i.e., the transmission among clusters; 2) *intra-cluster transmission*, i.e., the transmission within a cluster. In the following, we analyze the routing process illustrated in Fig.3.

#### a: INTER-CLUSTER TRANSMISSION

In this part, we establish a three-layer routing scheme to highlight the different stages of the packet routing.

- Layer 1: In this layer, when the cluster of source node  $C_s$  (i.e., the red circle) meets any circle including the cluster of destination  $C_d$  (i.e., the yellow circle), the source node delivers packet b to one relay (i.e., the green node) in the cluster which it came across through one-hop unicast transmission.
- Layer 2: The main function of layer 2 is to spread b to as many clusters as possible. In Fig.3, we color a cluster without b grey (i.e., the grey circle). When the cluster containing b (i.e.,  $C_r$ , the green circle) comes across a grey circle, the relay node in  $C_r$  hands over b to another relay node of the grey circle. After that, the grey circle turns green, i.e., becomes  $C_r$ . The whole process can be viewed as a virus disseminating. The Clusters  $C_r$  will not stop infecting the grey circles until all clusters of destination  $C_d$  received packet b.
- Layer 3: After a period of time, the number of  $C_r$  is getting larger. When  $C_r$  meets a cluster of destination  $C_d$ , then the relay node in  $C_r$  delivers b to  $C_d$ . In addition, considering the multicast case, the transmission between any two clusters of destination is allowed, i.e., when a  $C_d$  containing b meets another  $C_d$  without b, then packet b can be delivered from one to another. Thus, the transmission reuse is activated compared with the multi-unicast.

*Remark 1:* It is worth noting that the transmission within the three layers do not have to operate in sequence, i.e., the transmission in different layers like  $C_r$  to  $C_r$ ,  $C_r$  to  $C_d$ ,  $C_d$  to  $C_d$  can happen in any slot, on condition that the network exists certain amount of clusters containing *b* (This can be realized when the system tends to be stable).

## b: INTRA-CLUSTER TRANSMISSION

We present the two-layer transmission as follows:

• Layer 1: When packet b arrives at  $C_d$ , the relay node first broadcasts b to other nodes within the transmission range. Then the new relay nodes who carry b move around and deliver b to other nodes which they meet until



FIGURE 3. Transmission in cluster sparse regime.

TABLE 2.	Illustrations of	of	parameters in	n cluster	sparse regime.
----------	------------------	----	---------------	-----------	----------------

k	the $k$ -th destination of $b$ in one multcast session
$l_{b,k}^s$	capture range of the $k$ -th destination
$h_{b,k}^s$	number of hops to reach the destination after being captured
$r_{b,k}^h$	transmission range of each hop that needed to reach the destination after being captured
$R_{db,k}^{s}$	number of intra-cluster duplication when $b$ is captured by the $k$ -th destination
$R_{db}^{s}$	number of total intra-cluster duplication until b is captured by the last destination
$R_{cb,k}^{s}$	number of inter-cluster duplication when $b$ enters the cluster of destination $k$
$R_{cb}^{s}$	number of total inter-cluster duplication when $b$ enters the last cluster containing destination

any one of the relays enters the *capture region* of the destination. The capture region is defined as a circular area that each destination covers. The size of this area can be adjusted by trading off the capacity and delay.

• *Layer 2:* Once the relay node enters the capture region, packet *b* is transmitted from the relay node to the final destination via multi-hop transmission. The whole process is finished within one slot, as we use the slow mobility time scale.

Similarly, the transmission within the two layers do not have to operate in sequence after the system get stable.

## 2) SCHEDULING POLICY FOR CLUSTER SPARSE REGIME

The network scheduler has the whole information of current and past network status, which can schedule any radio transmission in current and future slot. This casual scheduling covers a certain amount of scheduling schemes and is helpful to derive the capacity-delay tradeoff. In each slot, the scheduler needs to make either of the following two decisions:

• Duplication: The scheduler needs to decide whether to duplicate packet *b* to other nodes and what kind of duplication it should create. Under the cluster sparse regime, there are two kinds of duplication. The first one is inter-cluster duplication, i.e., to transmit packet *b* to the clusters of destinations. Former work [22] showed that increasing the number of inter-cluster duplication in each cluster cannot decrease the transmitting delay.

VOLUME 8, 2020

The second one is intra-cluster duplication. Its function is to transmit packet b to the destinations located in the same cluster. The scheduler also needs to consider how to create the duplication, using one-hop, multi-hop, or broadcast transmission.

• Capture: The scheduler needs to decide whether to transmit packet *b* to destination *k* in the current slot. If yes, the scheduler needs to choose a relay node which arrives within the capture region or the source itself at the beginning of that slot, and forward packet *b* to the destination via multi-hop transmission. We define the radius of the capture region as the capturing range.

Compared with the unicast case in [22], the derivation of capacity-delay tradeoff for multicast is much more complicated since there are multiple destinations which are associated with one source node. The formula derivation is complex and needs more related scheduling parameters, which are given in Table 2. Note that each cluster contains at most an inter-cluster duplication for each packet. In addition, the total delay is denoted as  $D^s$ , which is divided into two parts.  $D_I^s$  is denoted as the delay needed for packet *b* to create inter-cluster duplication and transmission among clusters.  $D_{II}^s$  is denoted as the delay of transmission within the cluster containing destinations.

*Remark 2:* Before studying the capacity-delay tradeoff, we briefly outline the logic flow of calculating the tradeoff. First we explore the fundamental relationship between the

delay, capacity and various scheduling parameters including the number of relays, the size of capture region, the number of hops, etc. These scheduling parameters correlate closely to the network performance. Then, we establish formulas to depict the quantitative relationship between them, which can be utilized to derive the upper bound of capacity-delay tradeoff later. In addition, the deducing process is complex for many mathematical tools being involved. We try hard to simplify it and emphasize the key issues.

#### 3) CAPACITY-DELAY TRADEOFFS I

In this section, we assume  $n_d \leq m$ , i.e., there are  $n_d$  clusters with  $\Theta(1)$  destinations in each of them. We first introduce some basic tradeoffs and then use them to derive the upper bound of optimal capacity-delay tradeoff for multicast.

Proposition 1: Under cluster sparse regime and  $n_d \leq m$ , the delay for packet b and its scheduling parameters comply with the following inequalities.

$$c_{1}^{s} \log n \mathbb{E}[D_{I,k}^{s}] \geq \frac{n}{R^{2} \mathbb{E}[R_{cb,k}^{s}](n_{d}-k+1)}$$

$$c_{2}^{s} \log n \mathbb{E}[D_{II}^{s}] \geq \frac{R^{2}}{\mathbb{E}[R_{db,k}^{s}] \mathbb{E}[l_{b,k}^{s} + \frac{mR^{2}}{n^{2}}]^{2}}$$
(1)

where  $D_I^s = \sum_{k=1}^{n_d} D_{I,k}^s$ ,  $\mathbb{E}[D^s] = \mathbb{E}\left[\sum_{k=1}^{n_d} D_{I,k}^s\right] + \mathbb{E}[D_{II}^s]$ ,  $c_1^s = 6\pi$  and  $c_2^s = 8\pi$ . As the two constants would not affect the asymptotic properties of the above two inequalities in Equation (1), we just use two symbols to represent them, which is more convenient for later calculation when Equation (1) is involved.

Proposition 2: Under cluster sparse regime and  $n_d \leq m$ , the capacity per multicast session and its scheduling parameters comply the following inequality,  $c_3^s$  is a constant.

$$\mathbb{E}\bigg[\sum_{b=1}^{\lambda^{s}nT} \frac{\pi \,\Delta^{2}}{4} \Big(\sum_{k=1}^{n_{d}} \sum_{h=1}^{h_{b,k}^{s}} \frac{r_{b,k}^{h-2}}{mR^{2}} + \sum_{h=1}^{\frac{nhc_{b,k}}{mR^{2}}} \frac{r_{b}^{h}}{mR^{2}}\Big)\bigg] \\ + \sum_{b=1}^{\lambda^{s}nT} \frac{\Delta^{2}}{4} \frac{\sum_{k=1}^{n_{d}} \mathbb{E}[R_{d}^{s}_{b,k}] - n_{d}}{n} \le c_{3}^{s} WT \log n \quad (2)$$

The proof of above two propositions are similar as [21], so we omit them here. Proposition 1 and Proposition 2 jointly depict the fundamental relationship between the delay, capacity and some key radio resources. Based on Inequality 1, we can find that the delay can be reduced if we increase either the number of relays or the capturing range. Because a larger number of relays results in higher probability of the packet being captured by the destination. This reason also holds for increasing the capturing range. However, more relays generated, more bandwidth resources consumed, which would decrease the network capacity. In addition, as the capturing range increases, the number of concurrent transmission within the area that the capture region covers is reduced, which poses the negative impact on capacity. Some other related tradeoffs can be simply derived as in [22], which we omit here.

When  $D_{II}^s \ge D_I^s$ ,  $D_{II}^s$  dominates when *n* is large. Hence, for *n* large enough, we focus on the delay of transmission within the cluster of destinations, which is shown in Lemma 1.

Lemma 1: When  $D_{II}^s \ge D_I^s$  and  $n_d \le m$ , under cluster sparse regime, let  $\overline{D}_1^s$  be the mean delay averaged over all packets and let  $\lambda_1^s$  be the capacity per multicast session. The following upper bound holds for any causal scheduling policy,

$$(\lambda_1^s)^3 \le O\left(\frac{m\bar{D_1^s}}{n(n_d)^3}\log^3 n\right) \tag{3}$$

*Proof:* When  $\sum_{k=1}^{n_d} h > \frac{nR_c}{mR^2}$  and  $\lambda_1^s = O(\log n/n_d)$ , according to Proposition 2 and Cauchy-Schwarz Inequality,

$$\frac{\pi \Delta^2}{8nmR^2} \Big( \sum_{b=1}^{\lambda_1^s nT} \sum_{k=1}^{n_d} \mathbb{E}[l_{b,k}^s] \Big)^2 + \frac{\Delta^2}{8n} \sum_{b=1}^{\lambda_1^s nT} \sum_{k=1}^{n_d} \mathbb{E}[R_d^{s}_{b,k}] \le WT \log n \quad (4)$$

According to Inequality (1) and Holder Inequality

$$\frac{1}{\left(\frac{\sum_{k=1}^{n_d} \mathbb{E}[l_{b,k}^s]}{\sum_{k=1}^{n_d} 1}\right)^2} \le \left(\frac{\sum_{k=1}^{n_d} \frac{1}{\mathbb{E}[l_{b,k}^s]}}{\sum_{k=1}^{n_d} 1}\right)^2 \le \frac{\sum_{k=1}^{n_d} \frac{1}{(\mathbb{E}[l_{b,k}^s])^2 \mathbb{E}[D_{II,k}^s]}}{\sum_{k=1}^{n_d} 1} \frac{\sum_{k=1}^{n_d} \mathbb{E}[D_{II,k}^s]}{\sum_{k=1}^{n_d} 1}$$

We assume  $D_{II}^s = \frac{\sum_{k=1}^{n_d} D_{II,k}^s}{\sum_{k=1}^{n_d} 1}$ , then

$$\sum_{k=1}^{n_d} \mathbb{E}[R_d^{s}_{b,k}] \ge \frac{R^2}{\log n} \frac{(n_d)^3}{(\sum_{k=1}^{n_d} \mathbb{E}[l^{s}_{b,k}])^2 D_{II}^{s}}$$

Applying the similar process and assuming that  $\bar{D}_{II}^{s} = \frac{\sum_{b=1}^{\lambda^{s}nT} D_{II}^{s}}{\sum_{b=1}^{\lambda^{s}nT} 1}$ , we can obtain that

$$\sum_{b=1}^{\lambda_1 n I} \sum_{k=1}^{n_d} \mathbb{E}[R_d^{s}_{b,k}] \ge \frac{R^2 (n_d)^3}{\log n} \frac{(\lambda_1^s n T)^3}{(\sum_{b=1}^{\lambda_1^s n T} \sum_{k=1}^{n_d} \mathbb{E}[l_{b,k}^s])^2 \bar{D_{II}^s}}$$
(5)

Substituting (5) into (4), we have

$$\frac{\Delta^2 R^2(n_d)^3}{8n \log n} \frac{(\lambda_1^s n T)^3}{(\sum_{b=1}^{\lambda_1^s n T} \sum_{k=1}^{n_d} \mathbb{E}[l_{b,k}^s])^2 \bar{D}_{II}^s} + \frac{\pi \Delta^2}{8nmR^2} \left(\sum_{b=1}^{\lambda_1^s n T} \sum_{k=1}^{n_d} \mathbb{E}[l_{b,k}^s]\right)^2 \le WT \log n$$
$$\sqrt{\frac{\pi \Delta^4(n_d \lambda_1^s T)^3 n}{64m \log n \bar{D}_1^s}} \le WT \log n$$

In order to prove the existence of equality of maximum throughput, all inequalities in the former proof should hold with equality. By studying the conditions under which these

**TABLE 3.** The order of optimal values of the scheduling parameters under cluster sparse regime when  $D_{II}^s \ge D_I^s$  and  $n_d \le m$ .

$R_{db}^{s}$	$\Theta(n^{rac{1-d-v}{3}})$
$R_{cb}^{s}$	$\Theta(n^{1-d-2\beta}/\log n)$
$l_{b,k}^s$	$\Theta(n^{\frac{v+6\beta-2d-1}{6}}/\log^{\frac{1}{2}}n)$
$r_b^h$	$\Theta(n^{\frac{v-1+2\beta}{2}}\log^{\frac{1}{2}}n)$

inequalities are tight, we can identify the optimal values of various key parameters of the scheduling policy. We assume  $\bar{D}_1^s = n^d$  and results are summarized in Table 3.

Similarly, the next lemma illustrates the capacity-delay tradeoff of transmission among clusters.

Lemma 2: When  $D_{II}^{s} < D_{I}^{s}$  and  $n_{d} \leq m$ , under cluster sparse regime, let  $\overline{D_{2}^{s}}$  be the mean delay averaged over all packets and let  $\lambda_{2}^{s}$  be the capacity per multicast session. The following upper bound holds for any causal scheduling policy,

$$\lambda_2^s \le O\left(\frac{mR^4\bar{D}_2^s}{n^2\log n_d}\log^3 n\right) \tag{6}$$

*Proof:* According to Inequality (1), we have:

$$c_{1}^{s} \log n \sum_{k=1}^{nd} \mathbb{E}[D_{I,k}^{s}] \geq \sum_{k=1}^{n_{d}} \frac{n}{R^{2} \mathbb{E}[R_{cb,k}^{s}](n_{d}-k+1)}$$
$$\mathbb{E}[R_{cb}^{s}] \geq \frac{n \log n_{d}}{c_{1}^{s} \mathbb{E}[D_{I}^{s}]R^{2} \log n}$$
$$\sum_{b=1}^{\lambda_{2}^{s}nT} \mathbb{E}[R_{cb}^{s}] \geq \frac{n \log n_{d}}{c_{1}^{s}R^{2} \log n} \sum_{b=1}^{\lambda_{2}^{s}nT} \frac{1}{\mathbb{E}[D_{I}^{s}]}$$
$$\geq \frac{n \log n_{d}}{c_{1}^{s}R^{2} \log n} \frac{(\sum_{b=1}^{\lambda_{2}^{s}nT} 1)^{2}}{\sum_{b=1}^{\lambda_{2}^{s}nT} \mathbb{E}[D_{I}^{s}]}$$
$$= \frac{n \log n_{d}}{R^{2} \log n \overline{D^{s}}}.$$

Based on Proposition 2, assuming  $\sum_{k=1}^{n_d} h_b^s = n^{\xi} n R_{cb}^s / (m R^2)$  and  $\lambda_2^s = O(1/n_d)$ , we obtain

$$\frac{\pi \Delta^2 n}{4m^2 R^4} \sum_{b=1}^{\lambda_2^s nT} \frac{(1+n^\gamma) \mathbb{E}[R_{c_b}^s] \mathbb{E}[r_b^h]^2}{\log n} \le 2c_2^s WT \log n$$
$$\frac{\pi \Delta^2 n (1+n^\gamma) \log n_d \mathbb{E}[r_b^h]^2}{4m^2 R^4 \log^2 n} \frac{n\lambda_2^s nT}{c_1^s R^2 \bar{D}_2^s} \le 2c_2^s WT \log n \quad (7)$$

We assume  $\mathbb{E}[r_b^h] = \Theta(\sqrt{m/nR})$  to ensure the network connectivity. Therefore

$$\lambda_{2}^{s} \leq \frac{8c_{1}^{s}c_{2}^{s}WT}{\pi\Delta^{2}} \frac{mR^{4}\bar{D}_{2}^{s}\log^{3}n}{n^{2}\log n_{d}} \leq O\left(\frac{mR^{4}\bar{D}_{2}^{s}}{n^{2}\log n_{d}}\log^{3}n\right)$$

Similarly, we are able to identify the optimal choices of various key parameters of the scheduling policy. We assume  $D_2^s = n^d$ . Results are summarized in Table 4.

**TABLE 4.** The order of optimal values of the scheduling parameters under cluster sparse regime when  $D_{II}^{s} < D_{I}^{s}$  and  $n_{d} \le m$ .

$R_{db}^{s}$	$\Theta(n^{2-\nu-4\beta-d-\gamma}\frac{\log n_d}{\log^2 n})$
$R_{cb}^{s}$	$\Theta(n^{1-d-2\beta} \frac{\log n_d}{\log^2 n})$
$l^s_{b,k}$	$\Theta(\min\{R, n^{\frac{3-v-6\beta-2d}{2}}\frac{\log n_d}{\log^2 n}\})$
$r_b^h$	$\Theta(n^{rac{v-1+2eta}{2}})$

#### 4) CAPACITY-DELAY TRADEOFFS II

In this section, we discuss the case of  $n_d > m$ , i.e., each cluster contains  $\Theta(n_d/m)$  destinations. The derivation of the upper bound of capacity-delay tradeoff follows the same logic as in Section IV-A-3.

Proposition 3: Under cluster sparse regime and  $n_d > m$ , the delay for packet b and its scheduling parameters comply with the following inequalities

$$c_{4}^{s} \log n\mathbb{E}[D_{I,k}^{s}] \geq \frac{n}{R^{2}\mathbb{E}[R_{c_{b,k}}^{s}](m-k+1)}$$

$$c_{5}^{s} \log n\mathbb{E}[D_{II,k}^{s}] \geq \frac{R^{2}}{\mathbb{E}[R_{d_{b,k}}^{s}]\mathbb{E}[l_{b,k}^{s} + \frac{mR^{2}}{n^{2}}]^{2}(\frac{n_{d}}{m} - k + 1)}$$
(8)

where  $D_I^s = \sum_{k=1}^m D_{I,k}^s$ ,  $D_{II}^s = \sum_{k=1}^{n_d/m} D_{II,k}^s$ ,  $\mathbb{E}[D^s] = \mathbb{E}\left[\sum_{k=1}^m D_{I,k}^s\right] + \mathbb{E}\left[\sum_{k=1}^{n_d/m} D_{II,k}^s\right]$ , and  $c_4^s$  and  $c_5^s$  are two positive constant.

Proposition 4: Under cluster sparse regime and  $n_d > m$ , the capacity per multicast session and its scheduling parameters comply with the following inequality

$$\mathbb{E}\bigg[\sum_{b=1}^{\lambda_1^s nT} \frac{\pi \, \Delta^2}{4} \Big(\sum_{k=1}^{n_d} \sum_{h=1}^{h_b^s} \frac{r_{b,k}^{h-2}}{mR^2} + \sum_{h=1}^{\frac{nkc_b^s}{mR^2}} \frac{r_b^{h^2}}{mR^2}\Big)\bigg] \\ + \sum_{b=1}^{\lambda_1^s nT} \frac{\Delta^2}{4} \frac{\sum_{k'=1}^m \mathbb{E}[R_d_{b,k'}^s] - n_d}{n} \le c_6^s WT \log n$$

Similarly, we omit the proofs here for simplicity. Proposition 3 and Proposition 4 illustrate the basic tradeoff of the delay, capacity, number of relays and the size of capture region. Different from Inequality (1) in Proposition 1, Inequality (8) is much more complex. In the case of  $n_d > m$ , each cluster contains more than one destinations. Within each cluster, packet *b* is supposed to deliver to  $\theta \left(\frac{n_d}{m}\right)$  destinations, which causes severe competition for the limited radio resources. Thus, the delay issue becomes the major concern. However, a larger number of destinations may result in a high probability of packet *b* being captured.

Following the same logic of analysis in Section IV-A-3, we divide the proof into two parts. The first part illustrates the transmission within the cluster of destinations, i.e.,  $D_{II}^s \ge D_I^s$ . The second part illustrates the transmission among clusters, i.e.,  $D_{II}^s < D_I^s$ .

Lemma 3: When  $D_{II}^s \ge D_I^s$  and  $n_d > m$ , under cluster sparse regime, let  $\overline{D}_1^s$  be the mean delay averaged over all packets and let  $\lambda_1^s$  be the capacity per multicast session.

The following upper bound holds for any causal scheduling policy,

$$(\lambda_1^s)^3 \le O\left(\frac{D_1^s}{n_a^2 n} \log^3 n\right) \tag{9}$$

Proof: According to Inequality (8),

$$\begin{split} \mathbb{E}[l_{b,k}^{s}] &\geq \sqrt{\frac{R^{2}}{c_{1}^{s}\mathbb{E}[D_{II,k}^{s}]\mathbb{E}[R_{d_{b}}^{s}](\frac{n_{d}}{m}-k+1)\log n}} \\ \sum_{\hat{k}=1}^{n_{d}/m} \mathbb{E}[l_{b,k}^{s}] &\geq \frac{R}{\sqrt{c_{1}^{s}\mathbb{E}[R_{d_{b}}^{s}]\log n}} \sum_{\hat{k}=1}^{n_{d}/m} \frac{1}{\sqrt{\mathbb{E}[D_{II,k}^{s}](\frac{n_{d}}{m}-k+1)}} \\ &\geq \frac{R}{\sqrt{c_{1}^{s}\mathbb{E}[R_{d_{b}}^{s}]\log n}} \frac{\left[\frac{\sum_{\hat{k}=1}^{n_{d}/m}(\frac{n_{d}}{m}-k+1)^{-\frac{1}{4}}\right]^{2}}{\sum_{\hat{k}=1}^{n_{d}/m}\sqrt{\mathbb{E}[D_{II,k}^{s}]}} \\ &\geq \frac{R}{\sqrt{c_{1}^{s}\mathbb{E}[R_{d_{b}}^{s}]\log n}} \frac{\left(\frac{n_{d}}{m}\right)^{\frac{3}{2}}}{\sqrt{\sum_{\hat{k}=1}^{n_{d}/m}\mathbb{E}[D_{II,k}^{s}]\sum_{\hat{k}=1}^{n_{d}/m}1}} \end{split}$$

Then

$$\left(\sum_{k=1}^{n_d/m} \mathbb{E}[l_{b,k}^s]\right)^2 \\ \geq \frac{R^2(n_d)^2}{m^2 \mathbb{E}[R_d_b^s] \mathbb{E}[D_H^s] \log n} \\ \mathbb{E}[R_d_b^s] \geq \frac{R^2(n_d)^2}{m^2 (\sum_{k=1}^{n_d/m} \mathbb{E}[l_{b,k}^s])^2 \mathbb{E}[D_H^s] \log n} \\ \sum_{k'=1}^m \mathbb{E}[R_d_b^s] \\ \geq \frac{R^2(n_d)^2}{m^2 \log n} \sum_{k'=1}^m \frac{1}{(\sum_{k=1}^{n_d/m} \mathbb{E}[l_{b,k}^s])^2 \mathbb{E}[D_H^s]} \\ \geq \frac{R^2(n_d)^2}{m^2 \log n} \frac{(\sum_{k'=1}^m 1)^3}{(\sum_{k'=1}^m \sum_{k'=1}^{n_d/m} \mathbb{E}[l_{b,k}^s])^2 \sum_{k'=1}^m \mathbb{E}[D_H^s]} \\ = \frac{R^2(n_d)^2}{m^2 \log n} \frac{m^3}{(\sum_{k=1}^{n_d} \mathbb{E}[l_{b,k}^s])^2 \sum_{k'=1}^m \mathbb{E}[D_H^s]}$$

Similarly,

$$\sum_{b=1}^{\lambda_1^s nT} \sum_{k'=1}^m \mathbb{E}[R_d_b^s] \ge \frac{R^2 (n_d)^2 m (\lambda_1^s nT)^3}{(\sum_{b=1}^{\lambda_1^s nT} \sum_{k=1}^{n_d} \mathbb{E}[l_{b,k}^s])^2 \bar{D^s} \log n}$$
(10)

When  $\sum_{k=1}^{n_d} h > \frac{nR_c}{mR^2}$  and  $\lambda_1^s = O(\log n/n_d)$ , we substitute Inequality (10) and obtain that

$$\sqrt{\frac{1}{nmR^2}} \frac{R^2(n_d)^2 m(\lambda_1^s nT)^3}{n\bar{D_1^s} \log n} \le WT \log n$$
(11)

The optimal values of various key parameters of scheduling policy are summarized in Table 5.  $\hfill \Box$ 

Next, we study the case when  $D_{II}^s < D_I^s$ , i.e., the transmission among clusters.

**TABLE 5.** The order of optimal values of the scheduling parameters under cluster sparse regime when  $D_{II}^{S} \ge D_{I}^{S}$  and  $n_{d} > m$ .

$R_{db}^{s}$	$\Theta(n^{rac{1+2\gamma-d-3v}{3}})$
$R_{cb}^{s}$	$\Theta(n^{1-d-2\beta}/\log n)$
$l_{b,k}^s$	$\Theta(n^{\frac{3\nu+6\beta-2d-1-2\gamma}{6}}/\log^{\frac{1}{2}}n)$
$r_b^h$	$\Theta(n^{\frac{v-1+2\beta}{2}}\log^{\frac{1}{2}}n)$

Lemma 4: When  $D_{II}^{s} < D_{I}^{s}$  and  $n_{d} > m$ , under cluster sparse regime, let  $\overline{D_{2}^{s}}$  be the mean delay averaged over all packets and let  $\lambda_{2}^{s}$  be the capacity per multicast session. The following upper bound holds for any causal scheduling policy,

$$\lambda_2^s \le O\left(\frac{mR^4D_2^s}{n^2\log m}\log^3 n\right) \tag{12}$$

Proof: According to Inequality (8),

$$c_{4}^{s} \log n \sum_{k=1}^{m} \mathbb{E}[D_{I,k}^{s}] \geq \sum_{k=1}^{m} \frac{n}{R^{2} \mathbb{E}[R_{c}^{s}_{b,k}](m-k+1)}$$
$$\mathbb{E}[D_{I}^{s}] \geq \frac{n \log m}{R^{2} \mathbb{E}[R_{c}^{s}_{b}]}$$
$$\sum_{b=1}^{\lambda_{2}^{s} nT} \mathbb{E}[R_{c}^{s}_{b}] \geq \frac{n \log m}{c_{4}^{s} R^{2} \log n} \sum_{b=1}^{\lambda_{2}^{s} nT} \frac{1}{\mathbb{E}[D_{I}^{s}]}$$
$$\geq \frac{n \log m}{c_{4}^{s} R^{2} \log n} \frac{(\sum_{b=1}^{\lambda_{2}^{s} nT} 1)^{2}}{\sum_{b=1}^{\lambda_{2}^{s} nT} \mathbb{E}[D_{I}^{s}]}$$
$$= \frac{n \log n_{d} \sum_{b=1}^{\lambda_{2}^{s} nT} 1}{c_{4}^{s} R^{2} \log n \overline{D^{s}}}$$

We assume  $\sum_{k=1}^{n_d} h_b^s = n^{\xi} n R_{cb}^s / (mR^2)$  and  $\lambda_2^s = O(1/n_d)$ . According to Proposition 4, we obtain

$$\begin{split} &\frac{\pi\,\Delta^2 n}{4m^2R^4}\sum_{b=1}^{\lambda_2^s nT}\frac{(1+n^\gamma)\mathbb{E}[R_{cb}^{\ s}]\mathbb{E}[r_b^h]^2}{\log n} \leq 2c_6^sWT\log n\\ &\frac{\pi\,\Delta^2 n(1+n^\gamma)\log m\mathbb{E}[r_b^h]^2}{4m^2R^4\log^2 n}\frac{n\lambda_2^s nT}{c_4^sR^2\bar{D}_2^s} \leq 2c_6^sWT\log n\\ &\lambda_2^s \leq \frac{8c_4^sc_6^sWT}{\pi\,\Delta^2}\frac{m^2R^6\bar{D}_2^s}{n^3\log m}\times\frac{\log^3 n}{(1+n^\gamma)\mathbb{E}[r_b^h]^2} \end{split}$$

We assume  $\mathbb{E}[r_b^s] = \Theta(\sqrt{m/nR})$  to ensure the network connectivity. Therefore

$$\lambda_2^s \le \frac{8c_4^s c_6^s WT}{\pi \, \Delta^2} \frac{mR^4 \bar{D}_2^s \log^3 n}{n^2 \log m}$$

We assume  $\overline{D_2^s} = n^d$ , the optimal values of various key parameters of scheduling policy are summarized in Table 6

Theorem 1: Under cluster sparse regime, let  $\overline{D^s}$  be the mean delay averaged over all packets and let  $\lambda^s$  be the

**TABLE 6.** The order of optimal values of the scheduling parameters under cluster sparse regime when  $D_{II}^{s} < D_{I}^{s}$  and  $n_{d} > m$ .

$R_{db}^{s}$	$\Theta(n^{2-2v-4\beta-d}\frac{\log m}{\log^2 n})$
$R_{cb}^{s}$	$\Theta(n^{1-d-2\beta} \frac{\log m}{\log^2 n})$
$l_{b,k}^s$	$\Theta(\min\{R, n^{\frac{3-v-6\beta-2d}{2}}\frac{\log m}{\log^2 n}\})$
$r_b^h$	$\Theta(n^{rac{v-1+2eta}{2}})$

capacity per multicast session. The following upper bound holds for any causal scheduling policy,

$$\lambda^{s} = O\left(\min\left\{\lambda_{1}^{s}, \lambda_{2}^{s}\right\}\right), \quad \bar{D}_{1}^{s} = \bar{D}_{2}^{s} = \bar{D}^{s}.$$

$$\lambda_{1}^{s} \leq \begin{cases} O\left(\sqrt[3]{\frac{m\bar{D}_{1}^{s}}{n(n_{d})^{3}}\log^{3}}n\right) D_{II}^{s} \ge D_{I}^{s}, n_{d} \le m; \\ O\left(\sqrt[3]{\frac{\bar{D}_{1}^{s}}{2}\log^{3}}n\right), \quad D_{II}^{s} \ge D_{I}^{s}, n_{d} > m; \end{cases}$$
(13)

$$\lambda_{2}^{s} \leq \begin{cases} O\left(\frac{mR^{4}\bar{D}_{2}^{s}}{n^{2}\log n_{d}}\log^{3}n\right) & D_{II}^{s} < D_{I}^{s}, n_{d} \leq m; \\ O\left(\frac{mR^{4}\bar{D}_{2}^{s}}{n^{2}\log m}\log^{3}n\right), D_{II}^{s} < D_{I}^{s}, n_{d} > m; \end{cases}$$
(14)

*Proof:* According to the value of system parameters, i.e.,  $n_d$  and m, we can choose suitable expression of capacity-delay tradeoff for  $\lambda_1^s$  and  $\lambda_2^s$  in (8) and (9), which are concluded from lemma 1, 2, 3 and 4. We assume  $\overline{D_1^s} = \overline{D_2^s} = \overline{D^s}$  and obtain the values of  $\lambda_1^s$  and  $\lambda_2^s$  needed for our theorem.

## B. LOWER BOUND OF THE CLUSTER SPARSE REGIME

We have obtained the upper bound as well as the optimal values of scheduling parameters, so we construct an achievable lower bound in this section.

We divide our normal time slot into three subslots. The operations of each slot are shown below:

- 1) The nodes (source node and relays) create inter-cluster duplications and the destination cluster  $C_d$  receives messages from inter-cluster duplications via one hop unicast with transmission range  $r_b^h$ .
- 2)  $R_{d_b}^s$  intra-cluster duplications are created via broadcast.
- 3) Intra-cluster duplication is captured by range  $l_{b,k}^s$  and transmitted to the destination via  $h_b^s$ -hop unicast with single-hop transmission range  $r_b^h$ .

The scheduling parameters in our scheme use the optimal values in Table 3-6.

In each subslot, we tessellate the network into several cells and employ a cellular time-division multi-access (TDMA) transmission scheme so that each cell is scheduled to be active regularly. When a cell is activated, nodes within it are allowed to transmit to nodes inside the same cell or neighbouring cells. The TDMA transmission scheme allow each cell to have a  $1/c_6^s$  fraction of subslot to transmit, where  $c_6^s$  is a constant being independent of the tessellation information. We describe our tradeoff achieving scheme and the network tessellation then.

1) In the 1st subslot, we divide each cluster  $\Theta(R^2)$  into  $\mathbb{T}_1^s = q = n^{1-\nu}$  equal-area cells. Assume that each message has a length of  $\lambda^s/\log^2 n \leq mR^2/(nR_{cb})$ , and all transmissions are employed by one-hop unicast. So each node can transmit at least  $nR_{cb}^s/(mR^2)$  messages when it is scheduled to be active. Each cluster has at least a chance of  $\Theta(mR^2/(n\log n))$  per slot to communicate with other clusters, which indicates at least  $R_{cb}^s/\log n$  messages can be sent per slot and network can sustain  $\lambda^s/\log^2 n$  per slot capacity. If each time the network cannot sustain  $\Theta(mR^2/(n\log n))$  per-node capacity of inter-cluster transmission, we denote it as *Error*<sub>I</sub><sup>s</sup>. If the network falls to forward a message to all clusters containing destinations during  $\Theta(D_I^s \log^2 n)$  slots, we denote it as *Error*<sub>I</sub><sup>s</sup>.

2) & 3) In the 2th and 3th subslot, all messages are transmitted in clusters containing destinations. Nodes in a certain cluster follow the uniform distribution. The achievable lower bound under uniform condition have been studied widely that the network can achieve  $\Theta(\lambda^s/\log n)$  capacity with  $\Theta(\overline{D^s})$  delay. But there exists a problem. If different clusters overlap at a certain area, they will take turns to transmit (under all three subslots). *Error*<sup>s</sup><sub>III</sub> denote that more that  $c_4^s$  overlap at a certain area, where  $c_4^s$  is a positive number. So each cluster will take at least  $1/c_4^s$  length of subslot to transmit.

Theorem 2:  $Error_I^s \to 0$ ,  $Error_{II}^s \to 0$ , and  $Error_{III}^s \to 0$ as  $n \to \infty$ , So Our lower bound under cluster sparse regime can achieve the per-node throughput of  $\Theta(\lambda^s/\log^2 n)$  with  $\Theta(\overline{D}^s \log^2 n)$  delay.

*Proof:* Since we obtain the optimal values of scheduling parameters, we can easily get this theorem. We omit the proof here for simplification.  $\Box$ 

## *C.* CLUSTER DENSE REGIME: $V + 2\beta \ge 1$

## 1) ROUTING SCHEME FOR CLUSTER DENSE REGIME

The routing in cluster dense regime is similar to that of the cluster sparse regime, about which you can get the whole picture in Fig.3. However, we make some adjustment to the scheduling policy in line with the variation of correlation of node mobility.

## 2) SCHEDULING POLICY FOR CLUSTER DENSE REGIME

Under cluster dense regime, the area that all clusters cover is larger than that of the whole network and clusters overlap with each other w.h.p.. Suppose that the network scheduler has the whole information on the current and past status of the network, which can schedule any radio transmission in the current and future slot. We revise the scheduling policy of cluster sparse regime and propose two new scheduling schemes for the cluster dense regime.

## a: SCHEDULING POLICY A

In each slot, the scheduler needs to make either of the following two decisions:

- Duplication: The scheduler still needs to decide whether to duplicate packet b to other nodes and what kind of duplication it creates. Under the cluster dense regime, each cluster is covered by certain amount of clusters at any time within the network, which distinguishes from the cluster sparse regime. The revised scheme of inter-cluster duplication is as follows: once packet benters the network, the source node broadcasts b to nearby nodes instead of waiting to meet another cluster. Then, the source node as well as relays containing bkeeps broadcasting until b is captured by all clusters of destinations. The scheme of intra-cluster duplication is the same as that of the cluster sparse regime. In addition, the scheduler needs to consider how to create these duplications, using single-hop, multi-hop, or broadcast transmission.
- *Capture:* In the cluster dense regime, we incorporate the capturing process into the transmission among clusters. The reason why we abandon the capturing process in the inter-cluster transmission of the cluster sparse regime is that, the inter-cluster connectivity can hardly be realized due to the strong correlation of node mobility. However this problem no longer exists in the cluster dense regime. The capturing process of intra-cluster transmission is still applied here.

#### b: SCHEDULING POLICY B

In each slot, the scheduler needs to make either of the following two decisions:

- *Duplication:* Scheduling policy B is designed for the situation when the correlation degree of node mobility tends to be weak. In this case, clusters are highly overlapped and the transmission among clusters contributes little to the enhancement of network performance but only consumes much more radio resources. Thus, we assume that the source and relays cooperate to broadcast the packet until *b* is captured by all the destinations.
- *Capture:* When *b* is captured by the destination, it will be sent to the destination through multi-hop unicast transmission.

## 3) CAPACITY-DELAY TRADEOFFS FOR SCHEDULING POLICY A

As the derivation is kind of complex compared with the cluster sparse regime, we divide the analysis into two parts: 1) *Part I:* transmission among clusters; 2) *Part II:* transmission within the cluster containing destinations.  $D_I^d$  and  $D_{II}^d$  denote the delay of the two parts respectively and  $D^d = D_I^d + D_{II}^d$ . Moreover  $R_{db}^d$  and  $R_{cb}^d$  denote the the number of duplications when the packet is captured by the last destination or the last cluster containing destination.

#### Part I: The Transmission Among Clusters

In this part, we derive the capacity-delay tradeoff for the transmission among clusters, including creating inter-cluster duplication and forwarding packets to clusters that

12616

contain destinations. We first give some fundamental tradeoff of the radio resources, number of relays, capturing range, etc.

Proposition 5: Under cluster dense regime and  $n_d \leq m$ , the delay for packet b transmitted among different clusters and its scheduling parameters comply with the following inequality

$$c_{1}^{d} \log n \mathbb{E}[D_{I,\hat{k}}^{d}] \geq \frac{m}{\mathbb{E}[R_{cb,k}^{d}] \mathbb{E}[l_{1b,\hat{k}}^{d} + \frac{1}{n}]^{2}(n_{d} - \hat{k} + 1)}$$
(15)

where  $c_1^d$  is a positive constant,  $D_{I,\hat{k}}^d$  is the delay for transmitting packet to the  $\hat{k}$  cluster that containing destinations, and  $D_I^d = \sum_{\hat{k}=1}^{n_d} D_{I,\hat{k}}^d$ . Proposition 6: Under cluster dense regime and  $n_d \leq m$ ,

Proposition 6: Under cluster dense regime and  $n_d \leq m$ , the capacity of transmission among different clusters,  $\lambda_1^d$ , and its scheduling parameters comply with the following inequality

$$\mathbb{E}\left[\sum_{b=1}^{\lambda_{1}^{d}nT}\sum_{\hat{k}=1}^{n_{d}}\sum_{h=1}^{h_{1}^{d}}\frac{\pi\,\Delta^{2}r_{b,\hat{k}}^{h-2}}{4n}\right] + \sum_{b=1}^{\lambda_{1}^{d}nT}\frac{\Delta^{2}}{4}\frac{\mathbb{E}[R_{cb}^{d}]-1}{n} \le c_{2}^{d}WT\log n$$

where  $c_2^d$  is a positive constant.

The proofs of Proposition 5 and Proposition 6 follow the same logic as in [21], which are omitted here for simplicity. We mainly focus on the capacity-delay tradeoff using these basic tradeoffs.

Lemma 5: Under cluster dense regime, when  $v - \gamma \geq d \geq \frac{3-v-6\beta-2\gamma}{2}$  and  $n_d \leq m$ , let  $D_1^d$  be the mean delay averaged over all packets and let  $\lambda_1^s$  be the capacity per multicast session for inter-cluster communication. The following upper bound holds for any causal scheduling policy,

$$(\lambda_1^d)^3 \le O\left(\frac{\bar{D_1^d}}{m(n_d)^2}\log^3 n\right)$$

Proof: According to Inequality (15),

$$\begin{split} \mathbb{E}[l_{1}^{s}_{b,\hat{k}}] &\geq \sqrt{\frac{m}{c_{3}^{d}\log n\mathbb{E}[D_{I,\hat{k}}^{d}]\mathbb{E}[R_{c}^{d}_{b,\hat{k}}](n_{d}-\hat{k}+1)}} \\ \sum_{\hat{k}=1}^{n_{d}} \mathbb{E}[l_{1}^{d}_{b,\hat{k}}] &\geq \sqrt{\frac{m}{c_{3}^{d}\log n\mathbb{E}[R_{c}^{d}_{b}]}} \sum_{\hat{k}=1}^{n_{d}} \frac{1}{\sqrt{\mathbb{E}[D_{I,\hat{k}}^{d}]}\sqrt{n_{d}-\hat{k}+1}} \\ &\geq \sqrt{\frac{m}{c_{3}^{d}\log n\mathbb{E}[R_{c}^{d}_{b}]}} \frac{\left(\sum_{\hat{k}=1}^{n_{d}}(n_{d}+1-\hat{k})^{-\frac{1}{4}}\right)^{2}}{\sum_{\hat{k}=1}^{n_{d}}\sqrt{\mathbb{E}[D_{I,\hat{k}}^{d}]}} \\ &= n_{d}\sqrt{\frac{m}{c_{3}^{d}\log n\mathbb{E}[R_{c}^{d}_{b}]D_{1}^{d}}} \end{split}$$

VOLUME 8, 2020

TABLE 7. Illustrations of parameters in cluster dense regime.

k	the $k$ -th destination of $b$ in one multcast session
$\hat{k}$	a cluster containing destination k
$l_{1_{b,\hat{k}}^{s}}$	the capture range of cluster $\hat{k}$
$l_{2b,k}^{s}$	the capture range of $k$
$h_{1b,\hat{k}}^{d}$	number of hops to reach $\hat{k}$ after being captured by $\hat{k}$
$h_{2b,k}^{d}$	number of hops to reach $k$ after being captured by $k$
$r^h_{b,k}$	transmission range of each hop that needed to reach $k$ after being captured by $k$
$r^h_{b,\hat{k}}$	transmission range of each hop that needed to reach $\hat{k}$ after being captured by $\hat{k}$
$\begin{bmatrix} R_{db,k} \end{bmatrix}$	number of intra-cluster duplication before being captured by $k$
$R_{c_{b,\hat{k}}}^{d}$	number of inter-cluster duplication before being captured by $\hat{k}$

#### Therefore

$$\mathbb{E}[R_{cb}^{d}] \geq \frac{m(n_{d})^{2}}{c_{3}^{d} \log n(\sum_{k=1}^{n_{d}} \mathbb{E}[l_{1}_{b,\hat{k}}^{d}])^{2}D_{1}^{d}}$$

$$\sum_{b=1}^{\lambda_{1}^{d}nT} \mathbb{E}[R_{cb}^{d}] \geq \frac{m(n_{d})^{2}}{c_{3}^{d} \log n} \sum_{b=1}^{\lambda_{1}^{d}nT} \frac{1}{(\sum_{\hat{k}=1}^{n_{d}} \mathbb{E}[l_{1}_{b,\hat{k}}^{d}])^{2}D_{1}^{d}}$$

$$\geq \frac{m(n_{d})^{2}}{c_{3}^{d} \log n} \frac{(\sum_{b=1}^{\lambda_{1}^{d}nT} \sum_{k=1}^{n_{d}} \mathbb{E}[l_{1}_{b,\hat{k}}^{d}])^{2} \sum_{b=1}^{\lambda_{1}^{d}nT} D_{1}^{d}}{(\sum_{b=1}^{\lambda_{1}^{d}nT} \sum_{\hat{k}=1}^{n_{d}} \mathbb{E}[l_{1}_{b,\hat{k}}^{d}])^{2} \sum_{b=1}^{\lambda_{1}^{d}nT} D_{1}^{d}}$$

$$= \frac{m(n_{d})^{2} (\sum_{b=1}^{\lambda_{1}^{d}nT} \sum_{k=1}^{n_{d}} \mathbb{E}[l_{1}_{b,\hat{k}}^{d}])^{2} \overline{D_{1}^{d}} \log n}$$

According to Proposition 6 and Cauchy-Schwarz Inequality,

$$\frac{\Delta^2 m(n_d)^2 (\sum_{b=1}^{\lambda_1^d nT} 1)^3}{4c_3^d n (\sum_{b=1}^{\lambda_1^d nT} \sum_{\hat{k}=1}^{n_d} \mathbb{E}[l_1 \frac{d}{b, \hat{k}}])^2 \bar{D_1^d} \log n} \\ + \frac{\pi \Delta^2}{4n^2} \left( \sum_{b=1}^{\lambda_1^d nT} \sum_{\hat{k}=1}^{n_d} \mathbb{E}[l_1 \frac{d}{b, \hat{k}}] \right)^2 \le c_2^d WT \log n \\ \sqrt{\frac{\pi \Delta^4 m(n_d)^2 (\sum_{b=1}^{\lambda_1^d nT} 1)^3}{16c_3^d n^3 \bar{D_1^d} \log n}} \le c_2^d WT \log n$$

In order to prove the existence of equality of maximum capacity, all inequalities in the former proofs should hold with equality. Thus we are able to identify the optimal choices of various key parameters of scheduling policy A. Assuming  $D_1^d = n^d$ , results are summarized in Table 8. According to these optimal values, we can obtain the constraint for this tradeoff.  $1 \le l_1 \frac{d}{b,\hat{k}} \le \sqrt{mR^2/n}$ , *i.e.*  $v - \gamma \ge d \ge \frac{2-v-6\beta-2\gamma}{2}$ .

Lemma 6: Under cluster dense regime, when  $d < \frac{3-\nu-6\beta-2\gamma}{2}$  and  $n_d \leq m$ , let  $D_1^d$  be the mean delay averaged over all packets and let  $\lambda_1^s$  be the capacity per multicast session for inter-cluster communication. The following upper

**TABLE 8.** The order of optimal values of the scheduling parameters under cluster dense regime inter-cluster communication and  $n_d \le m$ , I.

$R_{cb}^{\ d}$	$\Theta(n^{(v-d+2\gamma)/3}/\log n)$
$l_{1b,\hat{k}}^{d}$	$\Theta(n^{(v-d-\gamma)/3}/\log^{rac{1}{2}}n)$
$r_b^h$	$\Theta(\log^{\frac{1}{2}} n)$

bound holds for any causal scheduling policy,

$$\lambda_1^d \le O(\frac{R^2 \bar{D_1^d}}{n \log n_d} \log^3 n)$$

*Proof:* When  $d < \frac{3-\nu-6\beta-2\gamma}{2}$ ,  $l_1^d_{b,\hat{k}} = \sqrt{mR^2/n}$ . Therefore Inequality (15) turns into

$$c_{1}^{d} \log n\mathbb{E}[D_{I,\hat{k}}^{d}] \geq \frac{n}{\mathbb{E}[R_{c_{b,\hat{k}}}^{d}]R^{2}(n_{d}-\hat{k}+1)}$$

$$\sum_{\hat{k}=1}^{n_{d}} c_{1}^{d} \log n\mathbb{E}[D_{I,\hat{k}}^{d}] \geq \sum_{\hat{k}=1}^{n_{d}} \frac{n}{\mathbb{E}[R_{c_{b,\hat{k}}}^{d}]R^{2}(n_{d}-\hat{k}+1)}$$

$$c_{1}^{d} \log n\bar{D}_{1}^{d} \geq \frac{n\log n_{d}}{\mathbb{E}[R_{c_{b}}^{d}]R^{2}}$$

According to Proposition 6,

$$\frac{\pi\Delta^2}{2WTn^2} \Big(\sum_{b=1}^{\lambda_1^d nT} \sum_{\hat{k}=1}^{n_d} \sqrt{\frac{mR^2}{n}}\Big)^2 + \sum_{b=1}^{\lambda_1^d nT} \frac{\Delta^2}{4} \frac{\mathbb{E}[R_{cb}^d] - 1}{n} \le c_2^d WT \log n$$
$$O\Big(\frac{\lambda_1^d n \log n_d}{R^2 \bar{D_1^d} \log^2 n} + \frac{(\lambda_1^d n_d)^2 mR^2}{n \log^2 n}\Big) \le \log n$$

which leads to our result directly. Similarly, we are able to identify the optimal choices of various key parameters of the scheduling policy. We assume  $D_1^d = n^d$  and results are summarized in Table 9.

Next, we discuss the case of  $n_d > m$ .

Proposition 7: Under cluster dense regime and  $n_d > m$ , the delay for packet b transmitted among different clusters

TABLE 9. The order of optimal values of the scheduling parameters und	der
cluster dense regime inter-cluster communication and $n_d \leq m$ , II.	

$R_{cb}^{\ d}$	$\Theta(n^{1-2\beta-d}\log n_d/\log n)$
$l_{1b,\hat{k}}^{d}$	$\Theta(n^{(v+2\beta-1)/2})$
$r_b^h$	$\Theta(\log^{\frac{1}{2}}n)$

and its scheduling parameters comply with the following inequality

$$c_4^d \log n \mathbb{E}[D_{I,\hat{k}}^d] \ge \frac{m}{\mathbb{E}[R_c^d_{b,\hat{k}}] \mathbb{E}[l_1^d_{b,\hat{k}} + \frac{1}{n}]^2 (m - \hat{k} + 1)}$$
(16)

where  $c_4^d$  is a positive constant,  $D_{I,\hat{k}}^d$  is the delay for transmitting packet to the  $\hat{k}$  cluster that containing destinations, and  $D_{I}^{d} = \sum_{\hat{k}=1}^{n_{d}} D_{I,\hat{k}}^{d}.$ Proposition 8: Under cluster dense regime and  $n_{d} > m$ ,

the capacity of transmission among different clusters,  $\lambda_1^d$ , and its scheduling parameters comply with the following inequality

$$\mathbb{E}\left[\sum_{b=1}^{\lambda_{1}^{d}nT}\sum_{\hat{k}=1}^{m}\sum_{h=1}^{h_{1,\hat{k},\hat{k}}}\frac{\pi\,\Delta^{2}r_{b,\hat{k}}^{h-2}}{4n}\right] + \sum_{b=1}^{\lambda_{1}^{d}nT}\frac{\Delta^{2}}{4}\frac{\mathbb{E}[R_{d_{c}}^{d}]-1}{n} \le c_{5}^{d}WT\log n$$

where  $c_5^d$  is a positive constant Lemma 7: Under cluster dense regime, when  $n_d > m$  or  $n_d \leq m, d \geq v - \gamma$ , let  $D_1^d$  be the mean delay averaged over all packets and let  $\lambda_1^s$  be the capacity per multicast session for inter-cluster communication. The following upper bound holds for any causal scheduling policy,

$$\lambda_1^d \le O\left(\frac{\bar{D_1^d}}{\min\{n_d, m\}\log\min\{n_d, m\}}\log^3 n\right)$$

*Proof:* Here we omit the proof of case  $n_d \le m, d \ge v - \gamma$ for similarity.

$$\begin{split} \sum_{\hat{k}=1}^{m} \mathbb{E}[l_{1b,\hat{k}}^{d}] &\geq \sqrt{\frac{m}{c_{6}^{d}\log n\mathbb{E}[R_{cb}^{d}]}} \sum_{\hat{k}=1}^{m} \frac{1}{\sqrt{\mathbb{E}[D_{I,\hat{k}}^{d}]}\sqrt{m-\hat{k}+1}} \\ &\geq \sqrt{\frac{m}{c_{6}^{d}\log n\mathbb{E}[R_{cb}^{d}]}} \frac{\left(\sum_{\hat{k}=1}^{m}(m+1-\hat{k})^{-\frac{1}{4}}\right)^{2}}{\sum_{\hat{k}=1}^{m}\sqrt{\mathbb{E}[D_{I,\hat{k}}^{d}]}} \\ &\geq \sqrt{\frac{m}{c_{6}^{d}\log n\mathbb{E}[R_{cb}^{d}]}} \frac{(m)^{\frac{3}{2}}}{\sqrt{\sum_{\hat{k}=1}^{m}\mathbb{E}[D_{I,\hat{k}}^{d}]\sum_{\hat{k}=1}^{m}1}} \\ &= m\sqrt{\frac{m}{c_{6}^{d}\log n\mathbb{E}[R_{cb}^{d}]D_{1}^{d}}} \end{split}$$

TABLE 10. The order of optimal values of the scheduling parameters under cluster dense regime inter-cluster communication and  $n_d > m$ , I.

$R_{cb}^{\ d}$	$\Theta(n^{(3v-d)/3}/\log n)$
$l_{1b,\hat{k}}^{d}$	$\Theta(n^{-d/3}/\log^{\frac{1}{2}}n)$
$r_b^h$	$\Theta(\log^{\frac{1}{2}}n)$

Therefore

$$\sum_{b=1}^{d nT} \mathbb{E}[R_{cb}^{\ d}] \ge \frac{m^3}{c_6^d \log n} \sum_{b=1}^{\lambda_1^d nT} \frac{1}{(\sum_{\hat{k}=1}^m \mathbb{E}[l_1_{b,\hat{k}}^d])^2 D_1^d}$$
$$\ge \frac{m^3}{c_6^d \log n} \frac{(\sum_{b=1}^{\lambda_1^d nT} 1)^4}{(\sum_{b=1}^{\lambda_1^d nT} \sum_{\hat{k}=1}^m \mathbb{E}[l_1_{b,\hat{k}}^d])^2 \sum_{b=1}^{\lambda_1^d nT} D_1^d}$$
$$= \frac{m^3 (\sum_{b=1}^{\lambda_1^d nT} 1)^3}{c_6^d (\sum_{b=1}^{\lambda_1^d nT} \sum_{\hat{k}=1}^m \mathbb{E}[l_1_{b,\hat{k}}^d])^2 \overline{D_1^d} \log n}$$

According to Proposition 8 and Cauchy-Schwarz Inequality,

$$\frac{\Delta^2 m^3 (\sum_{b=1}^{\lambda_1^d nT} 1)^3}{4c_6^d n (\sum_{b=1}^{\lambda_1^d nT} \sum_{\hat{k}=1}^m \mathbb{E}[l_1_{b,\hat{k}}^d])^2 \bar{D_1^d} \log n} + \frac{\pi \Delta^2}{4n^2} \left( \sum_{b=1}^{\lambda_1^d nT} \sum_{\hat{k}=1}^m \mathbb{E}[l_1_{b,\hat{k}}^d] \right)^2 \le c_5^d WT \log n$$

$$\sqrt{\frac{\pi \Delta^4 m^3 (\sum_{b=1}^{\lambda_1^d nT} 1)^3}{16c_3^d n^3 \bar{D_1^d} \log n}} \le c_5^d WT \log n$$

which leads to our result directly. Similarly, we are able to identify the optimal choices of various key parameters, which are shown in Table 10. However, we find the constraint  $l_{1_{b,\hat{k}}}^{d} \geq 1$ , leading to that this situation is impossible.

Thus we assume that  $l_{1b,\bar{k}}^d = 1$ , and inequality (16) turns into

$$c_{1}^{d} \log n\mathbb{E}[D_{I,\hat{k}}^{d}] \geq \frac{m}{\mathbb{E}[R_{c_{b,\hat{k}}}^{d}](m-\hat{k}+1)}$$
$$\sum_{\hat{k}=1}^{m} c_{1}^{d} \log n\mathbb{E}[D_{I,\hat{k}}^{d}] \geq \sum_{\hat{k}=1}^{m} \frac{n}{\mathbb{E}[R_{c_{b,\hat{k}}}^{d}]R^{2}(m-\hat{k}+1)}$$
$$c_{1}^{d} \log n\bar{D_{1}}^{d} \geq \frac{m\log m}{\mathbb{E}[R_{c_{b}}^{d}]}$$

According to Proposition 8,

$$\frac{\pi\Delta^2}{2WTn^2} \Big(\sum_{b=1}^{\lambda_1^d nT} \sum_{\hat{k}=1}^m 1\Big)^2 + \sum_{b=1}^{\lambda_1^d nT} \frac{\Delta^2}{4} \frac{\mathbb{E}[R_{cb}^d] - 1}{n} \le c_2^d WT \log n$$
$$O\Big(\frac{\lambda_1^d m \log m}{\bar{D}_1^d \log^2 n} + \frac{(\lambda_1^d m)^2}{\log^2 n}\Big) \le \log n$$

VOLUME 8, 2020

**TABLE 11.** The order of optimal values of the scheduling parameters under cluster dense regime inter-cluster communication and  $n_d > m$ , II  $n_d \le m$ , III.

$R_{cb}^{\ d}$	$\Theta(n^{v-d}\log m/\log n)$
$l_{1}{}^{d}_{b,\hat{k}}$	$\Theta(1)$
$r_b^h$	$\Theta(\log^{\frac{1}{2}} n)$

**TABLE 12.** The order of optimal values of the scheduling parameters under cluster dense regime intra-cluster communication and  $n_d \le m$ , I.

$R_{db}^{\ d}$	$\Theta(n^{(2-2v-2\beta-d)/2}/\log n)$
$l_{2b,k}^{d}$	$\Theta(n^{(v+4\beta-1-d)/3}/\log^{\frac{1}{2}}n)$
$r_b^h$	$\Theta(\log^{\frac{1}{2}}n)$

Similarly, we are able to identify the optimal choices of various key parameters of the scheduling policy. The results are summarized in Table 11.  $\Box$ 

Part II: The Transmission Within Cluster Containing Destinations

We first give some basic tradeoffs of radio resources, number of relays, capturing range, etc.

Proposition 9: Under cluster dense regime and  $n_d \leq m$ , the delay for packet b transmitted within a generic cluster and its scheduling parameters comply with the following inequality

$$c_{7}^{d} \log n \mathbb{E}[D_{II}^{d}] \ge \frac{R^{2}}{\mathbb{E}[R_{db,k}^{d}] \mathbb{E}[l_{2b,k}^{d} + \frac{1}{n}]^{2}}$$
(17)

where  $c_7^d$  is a positive constant.

Proposition 10: Under cluster dense regime and  $n_d \leq m$ , the capacity of transmission within a generic cluster,  $\lambda_2^d$ , and its scheduling parameters comply with the following inequality

$$\sum_{b=1}^{\lambda_2^n nT} \frac{\Delta^2}{4} \frac{mR^2/n(\sum_{k=1}^{n_d} \mathbb{E}[R_d_{b,k}^d] - n_d)}{n} + \mathbb{E}\bigg[\sum_{b=1}^{\lambda_2^d nT} \sum_{k=1}^{n_d} \sum_{h=1}^{h_2^d} \frac{\pi \Delta^2}{4} \frac{r_{b,k}^h}{n}\bigg] \le c_8^d WT \log n$$

where  $c_8^d$  is a positive constant.

Lemma 8: Under cluster dense regime, when  $d \leq 2 - 2\nu - 2\beta$  and  $n_d \leq m$ , let  $D_2^d$  be the mean delay averaged over all packets and let  $\lambda_2^d$  be the capacity per multicast session for intra-cluster communication. The following upper bound holds for any causal scheduling policy,

$$(\lambda_2^d)^3 \le O\left(\frac{nD_2^d}{mR^4(n_d)^3}\log^3 n\right)$$

*Proof:* Optimal values of parameters is shown in Table 12. According to these optimal values, we can obtain the constraint for this tradeoff.  $R_{db}^d \ge 1$ , *i.e.*  $d \le 2 - 2v - 2\beta$ .

**TABLE 13.** The order of optimal values of the scheduling parameters under cluster dense regime intra-cluster communication and  $n_d \le m$ , II.

$R_{db}^{\ \ d}$	$\Theta(1)$
$l_{2b,k}^{\ d}$	$\Theta(n^{(2\beta-d)/2}/\log^{\frac{1}{2}}n)$
$r_b^h$	$\Theta(\log^{\frac{1}{2}}n)$

Lemma 9: Under cluster dense regime, when  $d > 2 - 2\nu - 2\beta$  and  $n_d \le m$ , let  $D_2^d$  be the mean delay averaged over all packets and let  $\lambda_2^d$  be the capacity per multicast session for intra-cluster communication. The following upper bound holds for any causal scheduling policy,

$$(\lambda_2^d)^2 \le O\left(\frac{D_2^d}{R^2(n_d)^2}\log^3 n\right)$$

*Proof:* When  $d > 2 - 2v - 2\beta$ ,  $R_d^{\ d}_{b,k} = 1$ . Therefore Inequality (17) turns into

$$c_7^d \log n \mathbb{E}[D_{II}^d] \ge \frac{R^2}{\mathbb{E}[l_{2b,k}^d + \frac{1}{n}]^2}$$

Then Proposition 10 turns into

$$\sum_{b=1}^{\lambda_{2}^{d}nT} \frac{\Delta^{2}}{4} \frac{mR^{2}/n(\sum_{k=1}^{n_{d}} 1 - n_{d})}{n} + \frac{\pi\Delta^{2}}{4n^{2}} \left(\sum_{b=1}^{\lambda_{2}^{d}nT} \sum_{k=1}^{n_{d}} \mathbb{E}\left[l_{2}^{d}_{b,k} + \frac{1}{n}\right]\right)^{2} \le c_{8}^{d}WT \log n$$

which leads to our result directly. Then we are able to identify the optimal choices of various key parameters of the scheduling policy. We assume  $D_2^d = n^d$ . The results are summarized in Table 13.

Proposition 11: Under cluster dense regime and  $n_d > m$ , the delay for packet b transmitted within a generic cluster and its scheduling parameters comply with the following inequality

$$c_{9}^{d} \log n \mathbb{E}[D_{H,k}^{d}] \geq \frac{R^{2}}{\mathbb{E}[R_{db,k}^{d}] \mathbb{E}[l_{2b,k}^{d} + \frac{mR^{2}}{n^{2}}]^{2}(\frac{n_{d}}{m} - k + 1)}$$

where  $D_{II}^d = \sum_{k=1}^{n_d} D_{II,k}^d$  and  $c_9^d$  is a positive constant. Proposition 12: Under cluster dense regime and  $n_d > m_{r_s}$ 

Proposition 12: Under cluster dense regime and  $n_d > m$ , the capacity of transmission within a generic cluster,  $\lambda_2^d$ , and its scheduling parameters comply with the following inequality

$$\sum_{b=1}^{\lambda_2^d nT} \frac{\Delta^2}{4} \frac{mR^2 / n(\sum_{k=1}^m \mathbb{E}[R_d_{b,k}^d] - n_d)}{n} + \mathbb{E}\bigg[\sum_{b=1}^{\lambda_2^d nT} \sum_{k=1}^{n_d} \sum_{h=1}^{h_2^d, k} \frac{\pi \Delta^2}{4} \frac{r_{b,k}^{h-2}}{n}\bigg] \le c_{10}^d WT \log n$$

where  $c_{10}^d$  is a positive constant.

Lemma 10: Under cluster dense regime, when  $d \le 2-v-2\beta - \gamma$  and  $n_d > m$ , let  $\overline{D}_2^d$  be the mean delay averaged over

**TABLE 14.** The order of optimal values of the scheduling parameters under cluster dense regime intra-cluster communication and  $n_d > m$ , I.

$R_{db}^{\ \ d}$	$\Theta(n^{(2-4v-2\beta-d+2\gamma)/3}/\log n)$
$l_{2b,k}^{d}$	$\Theta(n^{(2v+4\beta-1-d-\gamma)/3}/\log^{\frac{1}{2}}n)$
$r_b^h$	$\Theta(\log^{\frac{1}{2}}n)$

all packets and let  $\lambda_2^d$  be the capacity per multicast session for intra-cluster communication. The following upper bound holds for any causal scheduling policy,

$$(\lambda_2^d)^3 \le O\left(\frac{nD_2^d}{m^2R^4(n_d)^2}\log^3 n\right)$$

*Proof:* Since the proof is similar as Lemma 3.7, we omit it here. The optimal values of scheduling parameter are summarized in Table 14. The constraint  $R_{db}^d \ge n_d/m$  leads to  $d \le 2 - v - 2\beta - \gamma$ .

Lemma 11: Under cluster dense regime, when  $d > 2-v - 2\beta - \gamma$  and  $n_d > m$ , let  $D_2^d$  be the mean delay averaged over all packets and let  $\lambda_2^d$  be the capacity per multicast session for intra-cluster communication. The following upper bound holds for any causal scheduling policy,

$$(\lambda_2^d)^2 \le O\left(\frac{\bar{D_2^d}}{R^2 m n_d} \log^2 n\right)$$

*Proof:* When  $d > 2 - v - 2\beta - \gamma$ ,  $R_{db}^d = n_d/m$ . Therefore Inequality (17) turns into

$$\left(\sum_{b=1}^{\lambda_2^d nT} \sum_{k=1}^{n_d} \mathbb{E}[l_{b,k}^d]\right)^2 \ge \frac{R^2 m n_d (\lambda_2^d nT)^2}{c_7^d \bar{D_2^d} \log n}$$

Then Proposition 10 turns into

$$\sum_{b=1}^{\lambda_2^d nT} \frac{\Delta^2}{4} \frac{mR^2 / n(\sum_{k=1}^m n_d / m - n_d)}{n} + \frac{\pi \Delta^2}{4n^2} \left( \sum_{b=1}^{\lambda_2^d nT} \sum_{k=1}^{n_d} \mathbb{E} \left[ l_{2b,k}^d + \frac{1}{n} \right] \right)^2 \le c_8^d WT \log n$$

which leads to our result directly. Similarly, we are able to identify that the optimal choices of various key parameters of the scheduling policy. The results are summarized in Table 15.  $\hfill \Box$ 

## 4) CAPACITY-DELAY TRADEOFFS FOR SCHEDULING POLICY B

We first give some basic tradeoffs.

Proposition 13: Under cluster dense regime, the delay for packet b with Scheme B and its scheduling parameters comply with the following inequality

$$c_{11}^d \log n \mathbb{E}[D_{B,k}^d] \geq \frac{n}{\mathbb{E}[R_{b,k}^d] \mathbb{E}[l_{Bb,k}^d + \frac{1}{n^2}]^2 (n_d - k + 1)}$$
  
where  $D_B^d = \sum_{k=1}^{n_d} D_{B,k}^d$  and  $c_{11}^d$  is a positive constant.

**TABLE 15.** The order of optimal values of the scheduling parameters under cluster dense regime intra-cluster communication and  $n_d > m$ , II.

$R_{db}^{\ \ d}$	$\Theta(1)$
$l_{2b,k}^{d}$	$\Theta(n^{(2\beta+v-d-\gamma)/2}/\log^{\frac{1}{2}}n)$
$r_b^h$	$\Theta(\log^{\frac{1}{2}}n)$

Proposition 14: Under cluster dense regime, the capacity with Scheme B,  $\lambda_B^d$ , and its scheduling parameters comply with the following inequality

$$\mathbb{E}\left[\sum_{b=1}^{\lambda_{B}^{d}nT}\sum_{k=1}^{n_{d}}\sum_{h=1}^{h_{B}^{d}_{b,k}}\frac{\pi\,\Delta^{2}}{4}\frac{r_{b,k}^{h-2}}{n}\right] + \sum_{b=1}^{\lambda_{B}^{d}nT}\frac{\Delta^{2}}{4}\frac{\left(\mathbb{E}[R_{b}^{d}]-n_{d}\right)}{n} \le c_{12}^{d}WT\log n$$

where  $c_{12}^d$  is a positive constant.

Lemma 12: Under cluster dense regime, when Scheme B is applied, let  $D_B^d$  be the mean delay averaged over all packets and let  $\lambda_B^d$  be the capacity per multicast session. The following upper bound holds for any causal scheduling policy,

$$(\lambda_B^d)^3 \le O\left(\frac{\bar{D_B^d}}{n(n_d)^2}\log^3 n\right)$$

*Proof:* The proof is similar as the former proofs, so we omit it. Similarly, we are able to identify the optimal choices of various key parameters But we omit them here.  $\Box$ 

Theorem 3: Under cluster dense regime, let  $D^d$  be the mean delay averaged over all packets and let  $\lambda^d$  be the capacity per multicast session. The following upper bound holds for any causal scheduling policy,

$$\lambda^{s} = O\left(\max\left\{\lambda_{B}^{d}, \min\{\lambda_{1}^{d}, \lambda_{2}^{d}\}\right\}\right)$$

*Proof:* We assume  $D_1^d = D_2^d = D_B^d = D^d$ . According to the value of system parameter, such as  $n_d$ , m,  $D^d$ , and R, we can choose suitable expression of delay-throughput trade-off for  $\lambda_1^d$ ,  $\lambda_2^d$ , and  $\lambda_B^d$  from former lemmas. We then obtain the value of  $\lambda_1^d$ ,  $\lambda_2^d$ , and  $\lambda_B^d$  needed for our theorem.

#### D. LOWER BOUND OF THE CLUSTER DENSE REGIME

Lower bound of the cluster dense regime is omitted here for simplification. Bound can be easily constructed since we obtained optimal values of scheduling parameters.

#### **V. CONCLUSION**

The correlation of node mobility have huge impacts on the transmission reuse performance of mobile ad hoc networks (MANETs). In this paper, we have outlined the general characterization of the impact of correlation of node mobility on the multicast gain, an important indicator of the transmission reuse. Our study reveals that the various correlation degrees of node mobility brings different multicast gain in MANETs. Strong correlation of node mobility significantly improves

the multicast gain and even achieves the upper bound up to a logarithmic factor. Weak correlation of node mobility can also improve the multicast gain to a certain extent. The multicast gain is smallest when the node mobility shows medium correlation, where the distinction between multicast and multi-unicast is weak.

## REFERENCES

- X.-Y. Li, S.-J. Tang, and O. Frieder, "Multicast capacity for large scale wireless ad hoc networks," in *Proc. 13th Annu. ACM Int. Conf. Mobile Comput. Netw. (MobiCom)*, Montreal, QC, Canada, Sep., 2007.
- [2] M. Grossglauser and D. Tse, "Mobility increase the capacity of ad hoc wireless networks," *IEEE/ACM Trans. Netw.*, vol. 10, no. 4, pp. 477–486, Aug. 2002.
- [3] X. Wang, W. Huang, S. Wang, J. Zhang, and C. Hu, "Delay and capacity tradeoff analysis for MotionCast," *IEEE/ACM Trans. Netw.*, vol. 19, no. 5, pp. 1354–1367, Oct. 2011.
- [4] Z. Wang, H. Sadjadpour, and J. J. Garcia, "A unifying perspective on the capacity of wireless ad hoc networks," in *Proc. IEEE INFOCOM*, Phoenix, AZ, USA, Apr. 2008.
- [5] U. Lee, S. Y. Oh, K. W. Lee, and M. Gerla, "RelayCast: Scalable multicast routing in delay tolerant networks," in *Proc. IEEE ICNP*, Orlando, FL, USA, Oct. 2008.
- [6] S. Zhou and L. Ying, "On delay constrained multicast capacity of largescale mobile ad hoc networks," *IEEE Trans. Inf. Theory*, vol. 61, no. 10, pp. 5643–5655, Oct. 2015.
- [7] J. Zhang, X. Wang, X. Tian, Y. Wang, X. Chu, and Y. Cheng, "Optimal multicast capacity and delay tradeoffs in MANETs," *IEEE Trans. Mobile Comput.*, vol. 13, no. 5, pp. 1104–1117, May 2014.
- [8] Y. Xu, J. Liu, Y. Shen, X. Li, and X. Jiang, "On throughput capacity of large-scale ad hoc networks with realistic buffer constraint," *Wireless Netw.*, vol. 23, no. 1, pp. 193–204, Jan. 2017.
- [9] J. Zhang, Y. Li, Z. Liu, F. Wu, F. Yang, and X. Wang, "On multicast capacity and delay in cognitive radio mobile ad hoc networks," *IEEE Trans. Wireless Commun.*, vol. 14, no. 10, pp. 5274–5286, Oct. 2015.
- [10] Z. Qian, X. Tian, X. Chen, W. Huang, and X. Wang, "Multicast capacity in MANET with infrastructure support," *IEEE Trans. Parallel Distrib. Syst.*, vol. 25, no. 7, pp. 1808–1818, Jul. 2014.
- [11] Z. Luo, X. Gan, X. Wang, and H. Luo, "Optimal throughput-delay tradeoff in MANETs with supportive infrastructure using random linear coding," *IEEE Trans. Veh. Technol.*, vol. 65, no. 9, pp. 7543–7558, Sep. 2016.
- [12] B. Yang, Y. Shen, X. Jiang, and T. Taleb, "Generalized cooperative multicast in mobile ad hoc networks," *IEEE Trans. Veh. Technol.*, vol. 67, no. 3, pp. 2631–2643, Mar. 2018.
- [13] X. Wang, Q. Peng, and Y. Li, "Cooperation achieves optimal multicast capacity-delay scaling in MANET," *IEEE Trans. Commun.*, vol. 60, no. 10, pp. 3023–3031, Oct. 2012.
- [14] M. Garetto, P. Giaccone, and E. Leonardi, "Capacity scaling in delay tolerant networks with heterogeneous mobile nodes," in *Proc. ACM MobiHoc*, Montreal, QC, Canada, Sep. 2007.
- [15] D. Ciullo, V. Martina, M. Garetto, and E. Leonardi, "Impact of correlated mobility on delay-throughput performance in mobile ad hoc networks," *IEEE/ACM Trans. Netw.*, vol. 19, no. 6, pp. 1745–1758, Dec. 2011.
- [16] J. Yoon, B. D. Noble, M. Liu, and M. Kim, "Building realistic mobility models from coarse-grained traces," in *Proc. ACM MobiSys*, Uppsala, Sweden, Jun. 2006.
- [17] R. Jia, F. Yang, S. Yao, X. Tian, X. Wang, W. Zhang, and J. Xu, "Optimal capacity-delay tradeoff in MANETs with correlation of node mobility," *IEEE Trans. Veh. Technol.*, vol. 66, no. 2, pp. 1772–1785, Feb., 2017.
- [18] V. Naumov, R. Baumann, and T. Gross, "An evaluation of inter-vehicle ad hoc networks based on realistic vehicular traces," in *Proc. ACM MobiSys*, Uppsala, Sweden, Jun. 2006.
- [19] M. Musolesi and C. Mascolo, "Designing mobility models based on social network theory," ACM SIGMOBILE Mobile Comput. Commun. Rev., vol. 11, no. 3, pp. 59–70, Jul. 2007.
- [20] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Trans. Inf. Theory*, vol. 46, no. 2, pp. 388–404, Mar. 2000.

- [21] X. Lin and N. B. Shroff, "The fundamental capacity-delay tradeoff in large mobile ad hoc networks," in *Proc. MedHoc*, Bodrum, Turkey, Jun. 2004.
- [22] S. Yao, X. Wang, X. Tian, and Q. Zhang, "Delay-throughput tradeoff with correlated mobility of ad-hoc networks," in *Proc. IEEE INFOCOM*, Toronto, ON, Canada, Apr. 2014.



**RIHENG JIA** received the Ph.D. degree in computer science and technology from Shanghai Jiao Tong University, Shanghai, China, in 2018. He is currently a Lecture with the School of Mathematics and Computer Science, Zhejiang Normal University (ZJNU). His current research interests include energy harvesting networks and wireless charging technology.



**FEILONG LIN** received the Ph.D. degree in automation from Shanghai Jiao Tong University, Shanghai, China, in 2016. He is currently a Lecture with the School of Mathematics and Computer Science, Zhejiang Normal University (ZJNU). His current research interests include the Internet of Things and blockchain.



**ZHONGLONG ZHENG** received the Ph.D. degree in automation from Shanghai Jiao Tong University, Shanghai, China, in 2004. He is currently a Professor with the School of Mathematics and Computer Science, Zhejiang Normal University (ZJNU). His current research interests include artificial intelligence and pattern recognition.

...