IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received November 24, 2019, accepted December 26, 2019, date of publication January 10, 2020, date of current version January 27, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.2965601

High-Speed Railway EMUs' Circulation Plan Optimization: A Two-Stage Optimization Algorithm Based on Column Generation

WEN-JUN LI[1,2], **YU-TING WANG**[1], **LEI NIE**[2], **YINGHUI WU**[1], **AND LIU PENG**[1]
[1]School of Economics and Management, Jiangsu University of Science and Technology, Zhenjiang 212003, China
[2]School of Traffic and Transportation, Beijing Jiaotong University, Beijing 100044, China

Corresponding author: Lei Nie (lnie8509@yahoo.com)

This work was supported in part by the Project Entitled One Belt and One Road of the Landway International Intermodal Research and Exchange Center under Grant 2016YFE0201700, and in part by the National Natural Science Foundation of China, High-Speed Railway Travel Demand Analysis and Service Network Design under Grant U1434207. The work of Yinghui Wu was supported in part by the National Science Foundation of China under Grant 71601088, and in part by the Project Funded by the China Postdoctoral Science Foundation under Grant 2017 M610287. The work of Liu Peng was supported by the National Science Foundation of China under Grant 71871108.

ABSTRACT Considering the characteristics of China's high-speed railway network and the allocation structure of Electric Multiple Units (EMUs), the EMUs operation management mode provides an effective technical route for EMUs route planning optimization. To reduce the number of EMUs, key factors such as the EMUs mileage, time limit, maintenance model, maintenance capability, night accommodation capacity, maintenance management mode, and train tasks are considered as constraints. The number of maintenance tasks is also the objective so that the EMUs route planning solution method is more integrated with actual production needs, and an optimization model of EMUs route planning is established to overcome the shortcomings of previous studies that did not sufficiently consider EMUs maintenance. To verify the feasibility and effectiveness of the proposed model and method of the proposed two-stage optimization algorithm based on column generation, a case study on the preparation of EMUs circulation planning for the Beijing-Shanghai high-speed railway network is conducted, and the results are analyzed.

INDEX TERMS Branch and price algorithm, column generation, EMUs circulation plan, high-speed railway, two-stage optimization method.

I. INTRODUCTION

The EMUs (Electric Multiple Units) utilization problem studies the management measures that improve the operation efficiency of a high-speed railway by considering EMUs management regulation, train timetable requirement, etc. In addition to considering the modes of EMUs allocation, operation, maintenance, backup and train unit composition, the constraints of the EMUs maintenance mileage, maintenance time interval, EMUs type restrictions, maintenance capability, night accommodation capacity, and train tasks given in a timetable are considered to optimize EMUs utilization efficiency, reasonably exploit the maintenance capabilities of each EMUs depot, and plan EMUs circulation schemes.

The EMUs circulation plan optimization problem provides EMUs with the train connection schedules and daily maintenance location plans that satisfy the train operation demand and the EMUs maintenance regulations by considering the EMUs allocation and management modes. China's high-speed railway companies have purchased a large number of EMUs. By 2020, the operating mileage of China's high-speed railways is expected to reach more than 30,000 kilometers, and the number of EMUs will exceed 4,500 standard train units. At present, the operating mileage of the high-speed railways has exceeded 25,000 kilometers, the number of EMUSs in use is more than 2,900 standard train units, and approximately 40 EMUs depots are running, which form a rapid railway network, including interregional express railways and

The associate editor coordinating the review of this manuscript and approving it for publication was Jesus Felez.

intercity railways. The rapid railway network consists of high-speed railway lines that cover cities with a population of more than 500,000. All the passenger trains that operate on high-speed railways use EMUs. Over 4,600 high-speed trains operate every day to transport more than 5 million passengers. These high-speed trains have transported more than 6 billion passengers. The optimization of EMUs utilization should consider not only the train operation demand, EMUs accommodation capacity, maintenance capability, EMUs maintenance regulation and EMUs management mode but also the large size of the solution space of the mathematical model. It is very difficult to conduct a comprehensive optimization of the EMUs utilization problem by simultaneously integrating the optimization problems of the EMUs assignment plan and the high-class maintenance plan. To practically and feasibly study the problem of EMUs operation, first, the EMUs circulation planning problem, which is the core foundation of the entire EMUs utilization problem, is studied, which establishes a good foundation for the follow-up study of the EMUs assignment plan and maintenance plan. Therefore, this paper focuses on the problem of EMUs circulation plan optimization.

II. LITERATURE REVIEW

This section mainly discusses and analyzes the research related to the EMUs utilization optimization problem.

A. OPTIMIZATION STUDY OF EMUs CIRCULATION PLAN CONSIDERING MAINTENANCE

When EMUs circulation schemes are being planned, the maintenance constraint influences the EMUs operating procedures as the EMUs traveled mileage and time cannot be exceeded; this constraint cannot be directly and linearly expressed by common mathematical programming models. A heuristic method or an intelligent optimization algorithm is an effective method to address this constraint [1]–[7]. Zhou et al. (2017) proposed an optimization model to optimize train connection times and maintenance costs and designed an efficient multi-group genetic algorithm to solve it [1]. Lai et al. (2017) put forward an optimization model and a solution algorithm for a problem with the maintenance schedule and constraints of maintenance capability to prepare the train unit routing plan for Taiwan's high-speed railway [2]. Zhong et al. (2019) proposed a heuristic approach to decompose the problem into two main stages that can be iteratively performed to avoid generating all schedules [3]. Lusby et al. (2017) presented a mathematical model based on a path description that can easily consider and deal with maintenance-related constraints [4]. Lai et al. (2015) considered the problem of locating maintenance facilities in a railway setting. Because of the strategic nature of facility location, the opened facilities should be able to handle the current maintenance demand for any of the scenarios that capture changes such as changes to the line plan and the introduction of new rolling stock types. Several scenarios are allowed in the form of opening additional facilities, closing facilities, and increasing the facility size, and a two-stage robust programming formulation is provided [5].

B. OPTIMIZATION STUDY OF EMUs CIRCULATION PLAN WITHOUT MAINTENANCE

Without the consideration of maintenance, mathematical programming models based on multi-commodity flow theory can generally be established and can be solved by classic exact algorithms or commercial software [8]–[9]. Valentina et al. (2012a) studied a train unit assignment problem that considered the need for train seats and proposed two optimization models: one is a linear relaxation problem and introduces an effective inequality, and the other model uses a Lagrangian relaxation method that introduces a local search algorithm to improve the solution until a satisfactory solution is obtained [8]. Valentina et al. (2013) considered passenger demand for train seats, determined vehicle assignments to different train tasks, and performed a Lagrangian relaxation decomposition to quickly obtain high-quality solutions [9].

In addition, some studies do not consider maintenance-adopted heuristic methods [10]–[17]. Valentina et al. (2010) (2012b) used a step-by-step construction method to generate feasible solutions [10], [11]. Nishi et al. (2017) proposed a short-term locomotive vehicle turnover plan optimization problem with daily maintenance. This problem is described as several shortest basic circuits to operate all trains, and the designed solution method was combined with a heuristic method, column generation and Lagrangian relaxation [12]. Lin et al. (2016) proposed a branch-and-price algorithm that includes a number of branching strategies that can meet practical applications and can solve large-scale train unit routing problems, which can consider the vehicle matching compatibility limit and the process of coupling and decoupling at some stations [13]. Valentina et al. (2019) considered the train unit assignment problem as a competitive bid process for a train operator to win a contract for providing rolling stock circulation [14].

C. DISRUPTION MANAGEMENT OF TRAIN UNITS AND EMUs

A railway system often suffers from random damage caused by external factors that interrupt the operation, disturb the operation order of the EMUs circulation plan, and prevent follow-up plans from being implemented smoothly. The circulation plan of EMUs must then be quickly rescheduled according to the actual situation [18]–[26]. Because the occurrence of emergencies has always led to the infeasibility of railway timetables, train unit circulation plans and crew shift plans, Wagenaar et al. (2017) introduced empty vehicle transfers during the rescheduling of rolling stock and adjustment of passenger travel demand based on the given operational plans [19]. Kroon et al. (2015) described a real-time rolling stock rescheduling model for the disruption management of passenger railways that considered these dynamic passenger flows, which contrasts with most traditional rolling stock rescheduling models that consider passenger flows either as static or as known input [20]. Veelenturf et al. (2017) described a real-time disruption management approach that integrates the rescheduling of the rolling stock and the timetable by considering the changed passenger demand. The timetable decisions are limited to the additional stops of trains at stations at which they normally would not call [21].

Usually, a number of train units have a scheduled maintenance appointment during the day, and these appointments must be considered while rescheduling the rolling stock. Wagenaar et al. (2017) proposed three mixed-integer programming models for this purpose [22].

In cases of disruption, the passengers will receive route advice, which they are not required to follow: passengers' route choice depends on the route advice and the timetable information available to them. Simultaneous to providing advice, rolling stock is rescheduled to accommodate the anticipated passenger demand. Van et al. (2018) present an optimization-based algorithm that aims to minimize passenger inconvenience through the provision of route advice and rolling stock rescheduling, where the advice optimization and rolling stock rescheduling modules are supported by a passenger simulation model [23].

D. COMPREHENSIVE OPTIMIZATION RESEARCH INCORPORATING MULTIPLE-STAGED PLANS

The tendency of integration is very obvious because recent rapid development of computation capacity and modeling technologies has enabled a wide range of possibilities for a systematic integration of different railway planning aspects across decision layers [27]–[30]. Specifically, the multiple components, stakeholders, and interconnected layers of a railway system define many challenges that range from line planning, train timetabling, and railway traffic administration to rolling stock and crew scheduling and train control, including extra aspects that are also vital for a good business, such as passenger satisfaction or maintenance [27].

Borndorfer et al. (2016) proposed a highly integrated generic hypergraph-based mixed-integer programming model for a rolling stock rotation problem in the context of long-distance passenger traffic between cities and an integrated algorithm for its solution, which can handle train composition, train unit maintenance, infrastructure capacities, and regularity aspects [28].

Haahr et al. (2018) integrated the Rolling Stock Scheduling Problem and the Train Unit Shunting Problem and proposed two similar branch-and-cut-based approaches to solve the integrated problem [29].

Wang et al. (2018) studied the integration of train scheduling and rolling stock circulation planning under time-varying passenger demand for an urban rail transit line, where the practical train operation constraints, e.g., the capacity of the trains, the number of available rolling stocks, and the entering/exiting depot operations, are considered. A benders-decomposition-based heuristic to obtain better and more robust circulations of rolling stock train units is proposed, while accounting for the train routing problem [30].

Because of the complexity of the EMUs utilization optimization problem and the problem size under network ground, heuristic algorithms or intelligent optimization algorithms are generally used. Although these algorithms can find a satisfactory and feasible solution in a short time, their design and implementation strongly depend on the specific parameter values used in the problem. The solution algorithm is mostly intentionally coded; therefore, the scalability of the algorithm is weak. When the problem changes slightly, the solution algorithm will not be applicable and cannot stably maintain good solution quality.

Compared with existing studies, the potential innovations of our research are as follows.

(1) In this paper, an integer programming model and a two-stage algorithm for EMUs circulation planning based on a path description that considers the complex conditions in China are constructed to improve the adaptability of mathematical models and algorithms to practical problems. Based on existing research, the model is constructed for EMUs route optimization that considers key factors such as the EMUs maintenance mileage, time limit, vehicle maintenance mode, maintenance management mode, maintenance capability, accommodation capacity, and train tasks.

(2) Furthermore, a branch-and-price algorithm based on column generation that can decompose the problem and assign these decomposed problems to different modules of the algorithm is proposed, which effectively reduces the complexity of the algorithm and makes it clear and extensible for studying EMUs route planning. Decomposing the problem into two stages can reduce the difficulty of modeling the problem so that the algorithm can be implemented efficiently in practice when the corresponding algorithm is redesigned.

III. PROBLEM DESCRIPTION

A. EMUs MANAGEMENT

Railway transportation operation has considerable complexity. The operational plans of railways are generally generated stage by stage. For passenger railways, from a strategic plan to a micro operational plan, the related operation organizing and planning include railway network planning, train service timetabling, train unit scheduling, crew shift plans, and real-time train dispatching.

A line plan is the basis for a subsequent series of transportation plans. The quality of the line plan directly affects the implementation results of the follow-up plans and plays a decisive role in the transportation organizing procedure. The train timetable is an important input for the preparation of the EMUs circulation plan.

Here, we briefly introduce the EMUs assignment management mode, the EMUs maintenance management mode, and the usage mode of backup EMUs.

1) EMUs ALLOCATION MANAGEMENT

China's EMUs adopt a flexible composition mode including long and short combinations of EMUs. Currently, the following are the three types of high-speed railway trains: 16-unit trains; 8-unit trains; and 8+8-coupled trains. The appropriate train compositions are decided based on the changes in the passenger flow characteristics in different time periods.

2) EMUs MAINTENANCE MANAGEMENT

To maintain a stable maintenance quality and clearly divide the safety responsibility, the maintenance tasks of EMUs are undertaken by their home depots. For a special situation where EMUs cannot return to their home depots within the regulated mileage or time cycle during operation, some maintenance tasks can be performed in other depots.

The maintenance capability of a depot is one of the key factors that affects EMUs utilization efficiency. EMUs maintenance capacity is related to the number of pieces of maintenance equipment, the types of EMUs that can be repaired, and the number of maintenance technicians. Daily maintenance is usually conducted at night.

3) EMUs OPERATION MANAGEMENT

Based on the assignment and maintenance management modes, train task assignment is scheduled by individual railway companies; that is, the EMUs that belong to one company must not be used by other companies.

B. PROBLEM SOLUTION

The EMUs routing problem studied in this paper is obviously suitable for a solution by using multi-commodity flow theory. Many engineering problems are solved by multi-commodity flow theory, especially in the transportation problem field. The purpose of the EMUs routing problem is to establish cost-optimal paths for EMUs to cover all train tasks while meeting maintenance and management rules. Compared with a large number of similar studies such as for aircraft scheduling and vehicle routing, the characteristics of the EMUs routing problem are obviously suitable for adopting multi-commodity flow theory.

Based on the abstraction of the EMUs routing problem, the Dantzig-Wolfe (D-W) decomposition principle is used to establish a mathematical model, and a two-stage method is proposed based on a column generation approach. The EMUs routing process is designed to assign train tasks to EMUs in the master problem and to ensure the rational allocation and use of maintenance resources. In this algorithm, the resource-constrained shortest path theory is used according to the column generation approach to decompose the model. The original problem is divided into a series of pricing subproblems that correspond to each EMU. At the subproblem level, the constraints of a single EMU are considered. The algorithm used in this study is based on progressive column generation to achieve an iterative improvement of the master problem.

The linear relaxation subproblem of the main problem is solved by optimization software based on the resource-constrained shortest path theory according to the maintenance rules and management rules required. The advantage of an optimization process based on column generation is that the algorithm solution framework has strong scalability.

IV. MATHEMATICAL MODEL

A. CONNECTION NETWORK BASED ON THE TRAIN TIMETBALE

As shown in Figure 1, the connection network is established based on a time-space diagram derived from the train timetable. The connection network is a directed graph that describes the activities involved in the EMUs operation process, where $\boldsymbol{N} = \boldsymbol{T} \cup \boldsymbol{D}$ is the vertex set and $\boldsymbol{T}$ is a train in the given train timetable, $i, j \in \boldsymbol{T}$. Set $\boldsymbol{D}$ represents depots on the high-speed railway line, where $d \in \boldsymbol{D}$; $\boldsymbol{A} = \boldsymbol{A}_0 \cup \boldsymbol{A}_1 \cup \boldsymbol{A}_2 \cup \boldsymbol{A}_3$ is the collection of arcs, where $\boldsymbol{A}_0$ is the collection of train connection arcs, $\boldsymbol{A}_1$ is the collection of empty vehicles, $\boldsymbol{A}_2$ is the collection of continuation arcs, and $\boldsymbol{A}_3$ is the entry arcs. Exit segments are not marked in the figure due to the coincidence of the stations. As shown in Figure 1, train station 1 serves stations 1 and 2, and train station 3 simultaneously serves stations 3 and 4.

The train trajectory extracted from the train trip line shown in Figure 1 is abstracted as virtual originating and destination nodes, which can be transformed into an abstract connection network, as shown in Figure 2.

The conditions for the establishment of the various arcs shown in Figure 2 are as follows:

1) The arrival station of train i is the same as the departure station of train j;

2) The arrival time of train i and the departure time of train j meet the requirements of the connection time;

3) The two connected trains use the same EMUs type; and

4) The two connected trains are served by one company.

$\boldsymbol{A}_0$ includes the train connection arcs; if the trains meet conditions 1-4, a train connection arc is established. This type of EMUs connection generally has a short distance, and the traveled mileage can be ignored.

$\boldsymbol{A}_1$ is a deadheading arc set of trains that cannot meet condition 1 and need a long connection time but can meet conditions 3 and 4 if the connection can be realized by a deadheading transfer within a reasonable distance and time. This limitation is a known constant (in this paper, only deadheading within 30 minutes is considered, and a certain redundancy time is given to ensure feasibility).

$\boldsymbol{A}_2$ is the night accommodation arc set; if trains i and j meet conditions 1, 3, and 4, after the train tasks are executed, the EMUs need to enter the depot to stay for one night until the next day, and this process will require arranging a track. The destination station and the originating station of a train may be the same, or trains of different stations may be

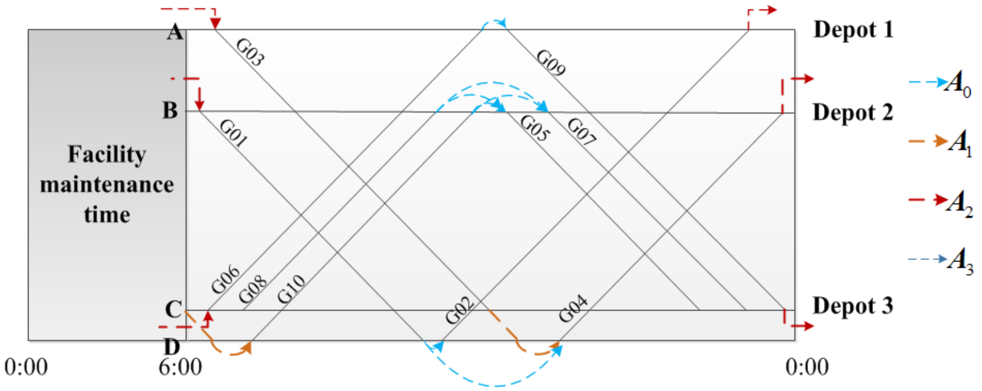


FIGURE 1. Connection network based on the train timetable.

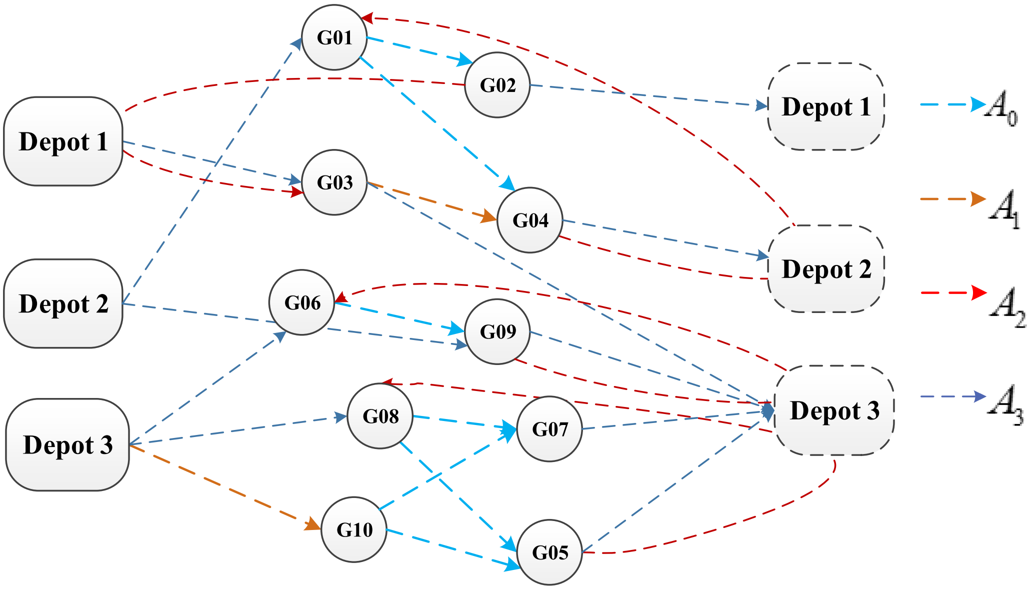


FIGURE 2. Abstract converted connection network.

connected, and a resident arc is established, where DIS_{ij} is the travel distance.

A_3 is the set of depot station arcs. Each such arc corresponds to an EMU that is sent from destination station s to depot d, provided that the EMU satisfies the EMUs type restrictions at depot d.

CT_{ij} is calculated by equation (1) as follows:

$$CT_{ij}=\begin{cases} TSD_j-TSA_i & TSD_j-TSA_i\geq TC \quad \forall(i,j)\in \boldsymbol{A}_0\cup\boldsymbol{A}_1 \\ TSD_j-TSA_i+1440 & TSD_j-TSA_i<TC \quad \forall(i,j)\in\boldsymbol{A}_2 \end{cases} \tag{1}$$

In equation (1), TSD_i is the departure time of the train at station SD_i, TSA_i is the time when the train arrives at station SA_i, and TC is the standard train turnaround time, which comprises the immediate return time of the EMUs and the time needed for compartment cleaning and seat direction adjustment.

Taking Figure 2 as an example, a connection network that is established based on the train timetable includes the various types of arcs described above and explains various activities in the operation process. The legend specifies the specific activities represented by the various arcs. As shown in Figure 2, trains 1 and 2 serve stations A and B, respectively, to provide the EMUs required for the originating train, and EMU 3 provides the EMUs required for the originating trains from adjacent stations C and D.

By abstracting the trains into customer nodes and the depots into originating and terminating nodes, Figure 2 can be transformed into a description of the multi-depot vehicle routing problem (MDVRP) with multiple stations as the destination. To clearly observe the relationships between the nodes shown in Figure 3, only a part of the entry and exit arcs are marked in the figure, and to facilitate drawing a transformed schematic diagram, another virtual node is established in the node of the moving train segment. Thus, two nodes correspond to the same depot.

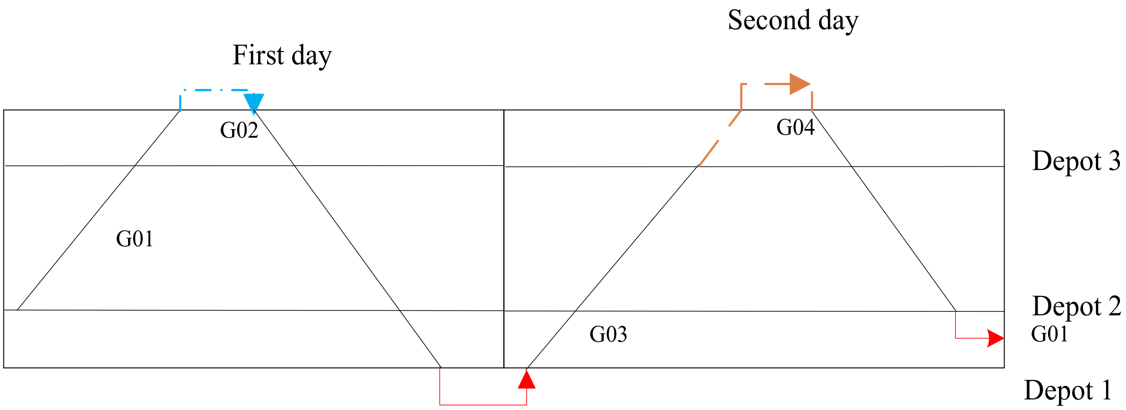


FIGURE 3. Illustration of the EMUs circulation plan.

B. ANALYSIS OF THE MODEL

There are two important optimization objectives in the EMUs circulation planning problem, namely, the number of EMUs used and the number of EMUs maintenance tasks. The purchasing cost of the EMUs is much higher than the cost of maintenance.

Before establishing the model, we first need to define the decision variables. Most previous research is based on using connection arcs to define the decision variables, constructing a nonlinear model, and then using heuristic or intelligent optimization algorithms. Because the general definition form is not applicable to this problem, we use path formulation to model the problem. If the maintenance of EMUs is not considered, the EMUs route planning problem can be solved by multicommodity flow theory. The EMUs maintenance constraints are also referred to as resource constraints and are the most difficult constraints to address in this study. Resource constraints limit the amounts of certain types of resources that are allowed to be consumed along a path. The resource constraints in this problem mainly include two types: the maintenance time cycle and maintenance mileage. For each train node, the cumulative time and mileage of a route from the departure at the origin until the arrival at the destination are limited by EMUs maintenance regulations for the time and mileage. Such constraints that influence a series of train task assignments will result in the destruction of the problem structure that would have been described by the multicommodity flow model. Consequently, the maintenance constraints are unable to be expressed by the connection arc variables.

Through the above analysis, the problem of EMUs route planning can be expressed as a vehicle routing problem. The problem variable is generally defined based on flow variable representation, i.e., an arc in the directed graph. The post-master problem decision variable is expressed as a path variable that corresponds to a path in the graph. According to flow decomposition theory, any model based on arcs can be transformed into a model based on a path formulation. To facilitate modeling the problem and overcome the difficulty of expressing the resource constraints based on the arcs, this study introduces a path formulation, because its related expression satisfies the definition of a path; thus, this study is mainly based on using a path variable to represent the optimization model.

The advantage of a model formulated by a path is that the cumulative constraints of the simulated resource constraints can be conveniently expressed, and some constraints on the routes can be directly translated into the generation of routes. However, the transformation model is large in scale. Since a given connection network can generate a large number of EMUs routes, determining how to effectively generate the required routes is extremely important. Based on a large number of studies, the specific details of the algorithm for this problem are discussed in the dynamic programming algorithm section of the pricing problem.

Ideal EMUs Routes and Real EMUs Routes: This paper uses the following definitions of EMUs routes as the basis for modeling.

Ideal EMUs route: only the train tasks form an **ideal EMUS route**.

Real EMUs route: various maintenance constraints, the train task nodes, entry and exit activity, and night accommodation compose a **real EMUs route**.

An **ideal EMUs route** considers the problem of train connection optimization under ideal conditions, that is, without considering influencing factors such as maintenance, parking, and the entry and exit activity of the EMUs and can be used to solve the lower bound of the number of EMUs. A **real EMUs route** considers all of the relevant factors that affect EMUs operation.

Based on the network shown in Figure 2, the two-day cycle can be obtained, as shown in Figure 3. According to the definition of the ideal EMUs route, an ideal EMUs circulation plan can be obtained, as shown in Figure 4.

According to the definition of the real EMUs route shown in Figure 4, taking station 2 as the origin and then operating trains G01 and G02, according to the mileage of the accumulated travel, it is sometimes necessary for a train to immedi-

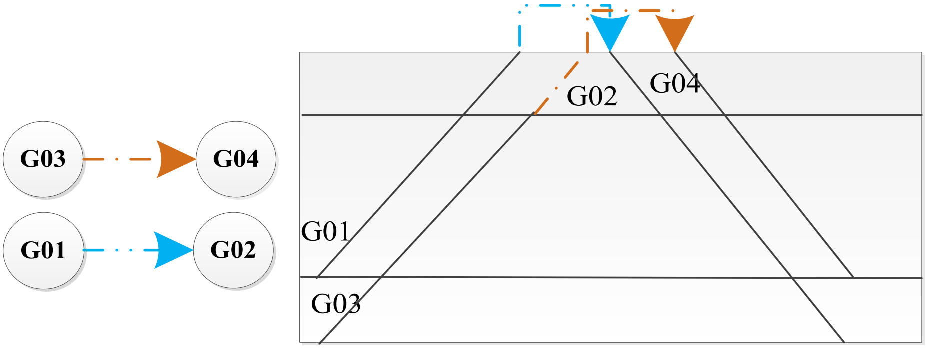


FIGURE 4. An ideal EMUs circulation plan segment.

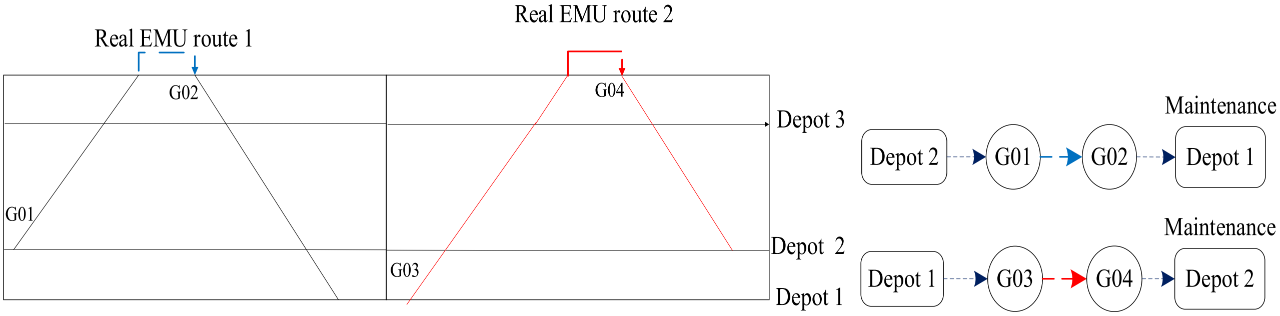


FIGURE 5. One-day real EMUs route.

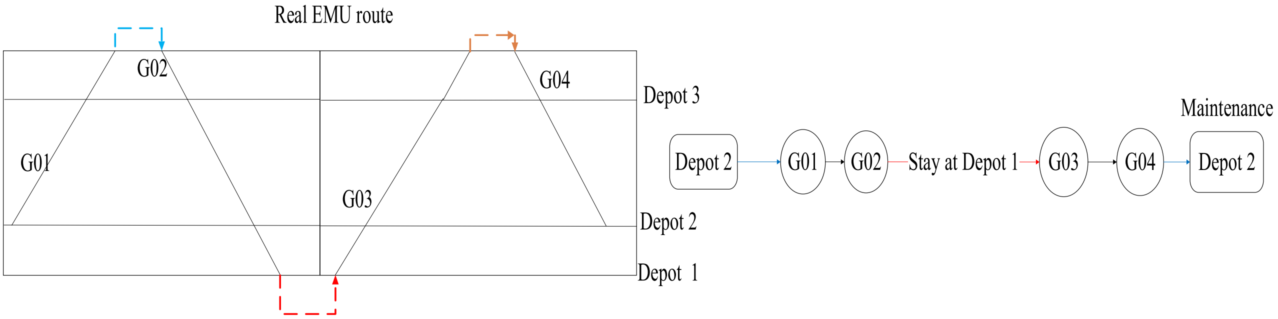


FIGURE 6. Two-day real EMUs route.

ately be repaired, and then station 1, where the EMU arrives, is the end point, as shown in Figure 5. If there is no need to arrange for maintenance immediately and maintenance is required after two days, a 2-day real EMUs route can be formed, as shown in Figure 6.

C. MODEL FORMULATION

The definitions of the sets, parameters, and decision variables used later in this article are presented here.

This section proposes two models: M1 is a model of a set partition of train tasks and is used to calculate the optimal number of EMUs, and M2 is a model formulated by the multi-commodity flow and the known number of EMUs that are used to optimize the EMUs maintenance tasks. The two objectives are optimized and solved in an algorithm with separate stages of solution. The benefit of the phased solution algorithm is that it can decompose the different factors of the problem into different parts.

$$\min N_{\min} = \sum_{p \in \boldsymbol{P}} c_p x_p \tag{2}$$

$$s.t.$$

$$\sum_{p \in \boldsymbol{P}} \theta_{ip} x_p = 1 \quad \forall i \in \boldsymbol{T} \tag{3}$$

Objective function (2) is used to solve the minimum number of EMUs $N_{\min}$, where $x_p \in \{0, 1\}$ is used to indicate whether an intersection is used by the optimal solution and c_p is the cost of the ideal EMUs route, that is, an EMU. Constraint (3) requires each train task to be uniquely covered by an EMUs route.

TABLE 1. Definitions of the set symbols.

Set	Definition
$\boldsymbol{T}$	set of trains in a given timetable, $i \in \boldsymbol{T}$
D	set of depots, $d \in \boldsymbol{D}$
$\boldsymbol{T}_d$	set of trains that can be operated from depot d , $d \in \boldsymbol{D}$
$\boldsymbol{S}$	set of stations, $s \in \boldsymbol{S}$
$\boldsymbol{P}$	set of EMUs routes without considering maintenance, $P \in \boldsymbol{P}$
$\boldsymbol{R}$	set of EMUs routes considering maintenance, $r \in \boldsymbol{R}$
$\boldsymbol{BS}(i)$	predecessor node set of node i, $\boldsymbol{BS}(i)=\{u:(u,i)\in \boldsymbol{A}\}$
$\boldsymbol{FS}(i)$	successor node set of node i, $\boldsymbol{FS}(i)=\{j:(i,j)\in \boldsymbol{A}\}$
$\boldsymbol{AS}(r)$	set of connection arcs used in route r
$\boldsymbol{NS}(r)$	set of train nodes visited by route r
$\boldsymbol{M}$	types of restricted resources on a route, $m \in \boldsymbol{M}$

The EMUs route plan optimization model based on the known number of EMUs is M2 Model M2 consists of two parts, namely, the main problem model M2 and a pricing problem model. $\boldsymbol{R'}$ is the set of real EMUs routes generated by the pricing problem, where $\boldsymbol{R'} \subset \boldsymbol{R}$. In M2, y_r only indicates whether a real EMUs route will be used in the optimal route plan. Some important related information, such as the maintenance and residence locations, cannot be expressed by y_r; therefore, a series of auxiliary parameters are associated with y_r. α_{dr} and β_{dr} are important parameters whose definitions are given in Table 2.

The main contribution is to solve the problem based on column generation. The established model based on set partitioning mainly includes the master problem model and the subproblem model. The main problem is that M2 solves the optimal real EMUs routes and the reduced cost. If there is a reduced cost that does not satisfy the problem, the current basis is not optimal, and the optimal solution has not been found. It is then necessary to reselect the base column, and the subproblem specifically needs to continue to generate real EMUs routes that can further improve the master problem. This specific process is the second stage of the algorithm.

The existing number of EMUs is considered to be $N_{\min}$, and ΔN is introduced as the number of adjustable EMUs. This parameter is introduced to temporarily relax the number of EMUs when model M2 is not feasible. The model obtains the minimum number of EMUs required for the practical situation. In addition, because it is not guaranteed that some EMUs will return to the depots, an Euler loop constraint condition is introduced in the model to ensure the necessary connection relationship to form an EMUs turnaround loop.

TABLE 2. Definitions of the parameter symbols.

Parameter	Definition
SD_i	origin station of train i
SA_i	destination station of train i
TSD_i	departure time of train i at origin station SD_i
TSA_i	time when train i arrives at destination station SA_i
TT_i	travel time of train i
TD_i	travel distance of train i
CT_{ij}	connection time between trains i and j, $i \neq j$, $i, j \in \boldsymbol{T}$
DIS_{ij}	distance traveled between trains i and j, $i \neq j$, $i, j \in \boldsymbol{T}$
TC	standard train connection time
ML	maintenance-based mileage limit for an EMU
MT	maintenance-based time interval limit for an EMU
P	a binary vector $P=(\theta_1,\ldots,\theta_i,\ldots,\theta_n)$ that indicates which train nodes are covered, $i \in \boldsymbol{T}$
θ_{iP}	a binary parameter that is equal to 1 if train i is covered by a route and is equal to 0 otherwise, $r \in \boldsymbol{R}$
α_{dr}	a binary parameter that is equal to 1 if an EMU goes to depot d for EMUs maintenance after completing EMUs maintenance route r and is equal to 0 otherwise, $r \in \boldsymbol{R}$
β_{dr}	a binary parameter that is equal to 1 if an EMU goes to depot d for overnight accommodation while operating on EMUs maintenance route r and is equal to 0 otherwise, $r \in \boldsymbol{R}$
c_p	cost of EMUs route p
c_r	cost of EMUs maintenance route r
η_r	number of EMUs used on EMUs maintenance route r
MC_d	EMUs maintenance capacity of depot d, $d \in \boldsymbol{D}$
AC_d	accommodation capacity of depot d, $d \in \boldsymbol{D}$
$NEMU$	maximum number of EMUs available

TABLE 3. Definitions of the decision variables.

Decision variable	Definition
x_{ij}	if trains i and j are connected, =1; otherwise, =0
y_r	if EMUs maintenance route r is used in the optimal solution, $y_r=1$; otherwise, $y_r=0$
z_P	if p is used in the optimal solution, then =1; otherwise, =0

1) MASTER PROBLEM MODEL M2

$\boldsymbol{R}$ is the set of real EMUs routes, and $y_r \in \{0, 1\}$ indicates whether an intersection will be used in the optimal route plan, where 1 means adopted and 0 means not adopted; the model of the main problem is as follows:

$$\text{Min } Z = \sum_{r \in \boldsymbol{R}} c_r y_r \tag{4}$$

$s.t.$

$$\sum_{r \in \boldsymbol{R}} \theta_{ir} y_r = 1 \quad \forall i \in \boldsymbol{T} \tag{5}$$

$$\sum_{r\in \boldsymbol{R}} \alpha_d y_r \leq MC_d \quad \forall d \in \boldsymbol{D} \tag{6}$$

$$\sum_{r\in \boldsymbol{R}} \beta_d y_r \leq AC_d \quad \forall d \in \boldsymbol{D} \tag{7}$$

$$\sum_{r\in \boldsymbol{R}} \eta_r y_r \leq N_{\min} + \Delta N \tag{8}$$

$$\sum_{r\in \boldsymbol{R}} u_{dr} y_r - \sum_{r\in \boldsymbol{R}} v_{dr} y_r = 0 \quad \forall d \in \boldsymbol{D} \tag{9}$$

Equation (4) is the objective function, that is, the minimum number of daily EMUs maintenance tasks. After optimizing the number of EMUs in the first stage, we mainly optimized the number of maintenance tasks. Each maintenance task costs tens of thousands of yuan, and hundreds of maintenance tasks are performed every day; thus, the total cost will exceed one hundred million yuan RMB. Constraint (5) requires that each train be covered by only one real EMUs route, and parameter $\theta_{ir} \in \{0, 1\}$ indicates whether a real EMUs route covers the train node, where 0 means no and 1 means yes. It ensures that each train is served by only one EMU. If two EMUs run this train simultaneously, it will lead to conflicts in the task allocation plan. Constraint (6) limits the number of EMUs scheduled for maintenance to not exceed the maintenance capacity MC_d of the depot, and parameter $\alpha_{dr} \in \{0, 1\}$ indicates whether an EMU is going to depot d for maintenance at the end, where 0 means no and 1 means yes. The maintenance capacity of an EMU depot is limited. You cannot assign more EMUs than the capacity; otherwise, congestion will occur, and if the EMU cannot operate the trains on time, it will need additional EMUs, which results in an unreasonable number of train unit increases. Constraint (7) means that the number of EMUs scheduled to stop at night cannot exceed the night accommodation capacity, and $\beta_{dr} \in \{0, 1\}$ indicates whether an EMU is going to the depot to stop, where 0 means no and 1 indicates yes. The night stopping capacity of an EMU is limited, and it is not possible to assign an EMUs number that exceeds the capacity; otherwise, the EMU will stop at the stations. The safety of the EMU during the night cannot be guaranteed. The station has no responsibility for the safety management of the EMU. Constraint (8) refers to the maximum number of EMUs that serve as train tasks and here indicates the number of slack EMUs, where $N_{\min} + \Delta N$ is the number of EMUs required. This is a heuristic constraint for problem solving that helps the model to use and manually adjust the parameters to help analyze the problem. Constraint (9) imposes that the number of EMUs circulated between the entrances and exits is conserved, which ensures the establishment of a continuous EMUs turnaround cycle, where $u_{dr} \in \{0, 1\}$ indicates whether the origin depot is d and $v_{dr} \in \{0, 1\}$ indicates whether the end depot is d.

2) PRICING PROBLEM

Here, the pricing problem is a shortest path problem with elementary constraints and resource constraints. The pricing issues can be described by the following model. For each r, a given depot is the starting point. π_i is a dual variable associated with the set partition constraint (5), and λ_d, ρ_d, ω_d and μ are dual variables that correspond to constraints (6), (7), (8), and (9), respectively. For any r, we need to solve the resource-constrained shortest path problem as follows:

Min

$$RC = c_r - \sum_{i\in \boldsymbol{NS}(r)} \pi_i\theta_i - \sum_{d\in \boldsymbol{D}} \lambda_d\theta_d - \sum_{d\in \boldsymbol{D}} \rho_d\alpha_d - \sum_{d\in \boldsymbol{D}} \omega_d\beta_d - \mu \tag{10}$$

$s.t.$

$$\sum_{j\in \boldsymbol{BS}(i)} x_{ij} = \sum_{j\in \boldsymbol{FS}(i)} x_{ji} = \theta_i \quad \forall i \in \boldsymbol{T} \tag{11}$$

$$\sum_{j\in \boldsymbol{BS}(d)} x_{dj} = \sum_{i\in \boldsymbol{FS}(d)} x_{id} = 1 \tag{12}$$

$$\sum_{i\in \boldsymbol{NS}(r)} TD_i\theta_i + \sum_{(i,j)\in \boldsymbol{AS}(r)} DIS_{ij}x_{ij} \leq MM \tag{13}$$

$$\sum_{i\in \boldsymbol{NS}(r)} TT_i\theta_i + \sum_{(i,j)\in \boldsymbol{AS}(r)} CT_{ij}x_{ij} \leq MT \tag{14}$$

In equation (10), RC is the reduced cost calculated from the dual variable values (shadow prices of the resources) for each constraint in the main problem model. Constraint (11) indicates that the train nodes of the network can only be accessed once during the path search. Each train only needs to be operated by one EMU every time. If multiple EMUs are used to run the train, conflicts between EMUs will occur. Constraint (12) limits that EMUs can only return to their assigned sections for maintenance. In the previous problem description, we mentioned that the maintenance management of EMUs is strictly restricted. Constraint condition (13) represents that the accumulated mileage traveled from the end of the last inspection should not exceed the mileage MM specified in the maintenance regulations. For safety, the maintenance task of the EMU is implemented in accordance with safety regulations, and it needs to be repaired immediately after running a certain mileage. Constraint (14) imposes that EMUs should not exceed the maintenance time limit. For safety reasons, not only the limitations of the EMU run mileage but also the run periods need to be considered for immediate maintenance after operation.

An EMU can be repaired at any depot that meets the restrictions of the EMU type. If the restrictions of the EMU maintenance location during the EMU inspection can be ignored, constraint (9) can be replaced by constraint (15) as follows:

$$\sum_{j\in \boldsymbol{BS}(d)} x_{dj} = 1 \tag{15}$$

V. ALGORITHMS

A. FLOWCHART OF THE ALGORITHM

In this paper, the algorithm shown in Figure 7 is used to solve the EMUs routing plan. The algorithm is divided into two stages. The first stage is to generate a set based on the ideal EMUs route and solve it with model M1. The second

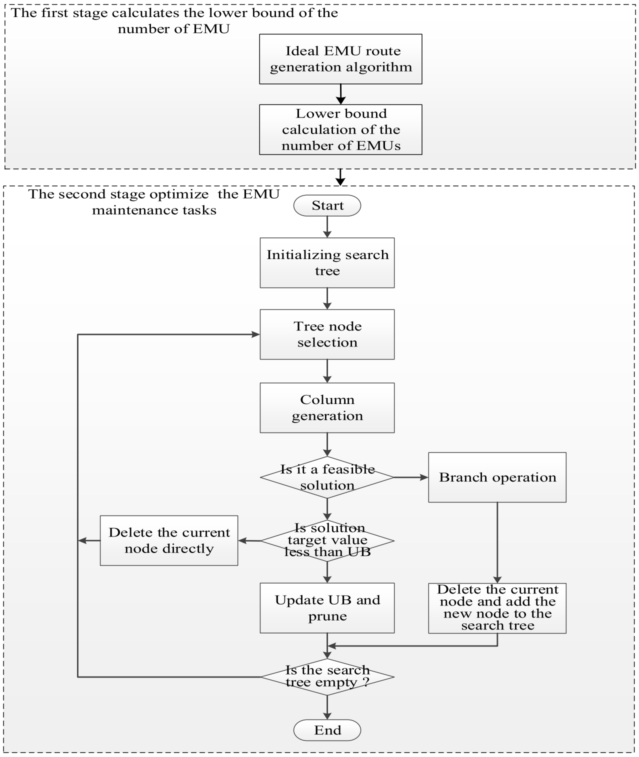


FIGURE 7. Flowchart of the EMUs circulation plan generation algorithm.

TABLE 4. Definitions of the parameter symbols and decision variables.

Parameter or variable	Definition
o	the start node given when searching for a route
Stack	stack recording train node order visited by a route
top	the top pointer of the stack, used to record the top position
Stack(*top*)	the last visited train node recorded on the top of the stack
(*Stack*(*top*),*j*)	a directed arc starting from Stack(top) with j being the end point
CD	the mileage traveled by the EMU at the intersection
ES(*i*,*j*)	an array for recording whether the arc has been accessed. If the arc (i, j) is a used edge in the route, it is marked as 1; otherwise, 0.
SN(*i*)	an array used to record whether node i has been accessed. If it has been accessed by the segment, it is marked as 1; otherwise, it is 0.
SNC	a collection of nodes that includes all nodes associated with the top-of-stack node

TABLE 5. Pseudocode of the search algorithm.

Pseudocode

```
$top$=1; $stack(top)$=$o$;  //push node $o$ into the stack//
      $CD$=$td_o$;
      $NS[o]$=1;
      While(Stack ≠ø)
         $j$=$Findnode(Stack, top, SN)$;
         while($j$≠null&($Stack(top)$,$j$) $\notin A_3$&$CD$+$TD_j$≤$ML$)
           $j$=$SelectingEdge(G, Stack, top,$ **SNC**);
           $ES(Stack(top),j)$=1;
           $top$=$top$+1; $Stack(top)$=$j$; $SN(j)$=1;
           $CD$=$CD$+ $TD_j$;
         end while
         record the path in $Stack$;
          for all $i$ in **SNC**
            if $SN(i)$==0
              $ES(Stack(top),i)$=0;
            end if
          end for
          $NS(Stack(top))$=0; $CD$=$CD$-$TD_{(Stack(top))}$; $top$=$top$-1;
      end while
```

stage is to solve the EMUs circulation plan for M2 based on the principle of a branch-and-price algorithm. The branch-and-price algorithm is divided into two layers. The outer layer is a branch-and-bound tree. The integer programming problem that corresponds to each branch-and-bound node on the branch-and-bound search tree is relaxed, and a branch search is performed to treat the generated fractional solution. The inner layer is column generation, which is the core process that the branch-and-price algorithm needs to use.

B. GENERATION ALGORITHM FOR IDEAL EMU ROUTES

By comparing the definition of a route with the definition of a path in graph theory, it is found that an ideal route is just a special form of the path definition, and their main difference is whether there is a limit on the length of the path. In considering this similarity with the path definition, this paper proposes a route enumeration algorithm based on the definition of a route to generate routes with a given node as the starting point and to use these routes as the input of model M1 to calculate the optimal route and the number of EMUs used.

Condition (Stack(top), j) $\notin \boldsymbol{A}_3$A3 indicates that the intersection must end at the given time.

The function *Findnode* determines which nodes have not been visited, and if no unvisited node is found, this function returns null. The function *SelectingEdge* finds the node where the segment is currently connected and does not exceed the maintenance limit. The time approximation complexity of the algorithm is $O(\prod_{i=1}^{k} w(i))$, which shows that the time required for the solution mainly depends on the structure of the connected network, the length limit of the intersection segment, and parameters such as the running mileage and time given by the train. w(i) is the out-degree of the node, and k is the average number of train nodes that need to be accessed when generating the routes.

In the connection network, due to the limited number of train vertices and according to various constraints, a large number of infeasible arcs have been eliminated, and the number of arcs is not large. In addition, the length of the intersection segment is strictly limited. Therefore, the enumeration based on the ideal intersection segment does not have general meaning. It is based on a network with a limited number of vertices and a limited length of the segment, and efficiency is guaranteed.

C. SOLUTION ALGORITHM FOR THE MASTER PROBLEM

1) BRANCH-AND-PRICE ALGORITHM

In this algorithm, the initialization solution of the search tree root node is very important and is the global lower bound, which plays a pivotal role. We must initialize the root node, find the corresponding dual variable value from the initial problem, and pass this value to the pricing subproblem to start the column generation method. In the subsequent branch process, the subsequent child node initialization is relatively simple. Usually, the child node inherits the feasible solutions from the parent node as its initial solution to start calculation. The other key issue with branch algorithms is to develop effective branching strategies. An effective branching strategy should produce a balanced search tree while clipping the fractional solution and maintaining the stable size and depth of the search tree.

The conventional integer programming problem is usually branched based on variables, but this method is not applicable to the branch-and-price algorithm because it will destroy the working process of the entire problem structure and disturb the pricing problem. As mentioned earlier, the branch-and-price method is applied to the linear relaxation of the main problem of a node, and the main problem variable often corresponds to a solution of a node, and the main problem variable often corresponds to a solution of the subsystem, which is a Boolean variable. The pricing problem is only used to determine the optimal solution of a subsystem. The pricing problem does not pay attention to any restrictions taken in the main problem. The pricing problem is still solved according to the set optimization algorithm, which has no effect on the pricing issue. Additionally, deleting a column is the only requirement for the values of the variables of the main problem, which does not contain any qualified information on the value of the pricing problem variable. In the pricing problem, the optimal basis of the linear relaxation just from the main problem deleted in the column is likely to produce a reduced cost with a value less than 0, which is selected again in the base column. The lack of an intermediate coordination mechanism between the main problem and the pricing problem causes the column generation process to fall into a degenerate state, and a final integer solution cannot be obtained.

To overcome this problem, some researchers have developed other branching strategies. The vehicle routing problem variable is usually defined on the arc o and is usually called a flow variable; a main problem variable decomposed by the D-W principle corresponds to the graph of a path and is called a path variable. Branching the flow variable instead of a path variable can make up for the abovementioned deficiencies caused by the branching of the main problem variables and decompose the original problem solution space into two parts that are relatively balanced, and some of the variables are prohibited from passing through the arc on the directed graph. The other part of the path on the directed graph forces the path to pass from the arc, and the measure has a reducing effect on the pricing problem search space.

TABLE 6. **Algorithm symbol definitions.**

Symbol	Definition
$\boldsymbol{L}(i)$	a set of valid labels on node i, $\forall \boldsymbol{i} \in \boldsymbol{N}$
l_i	$l_i = (RC_i, AD_i, AT_i, P_i)$ is a label on node *i*, $l_i \in \Gamma_i$. Each of the labels *h=1,…,Hi* represents a route *r* from the origin depot to node *i*.
RC_i	reduced cost accumulated from the starting node up to node *i*
AT_i	the accumulated time from the last overhaul to node *i*
AD_i	the mileage accumulated from the last overhaul until node *i*
P_i	$P_i = (\theta_1, \theta_2, \ldots, \theta_n, \ldots, \theta_{\lvert N \rvert})$, $n=1,\ldots,\lvert \boldsymbol{N} \rvert$, is a Boolean vector that indicates the state of the visited train node, where $\lvert \boldsymbol{N} \rvert$ is the number of nodes included in set N, $\forall \boldsymbol{i} \in \boldsymbol{N}$

2) SOLVING THE MASTER MODEL

In this paper, the master problem is first linearly relaxed, and then, the linear relaxation model of the main problem is solved by CPLEX. The master problem is initialized with the identity matrix with a large penalty to start the calculation. After linear relaxation, the node-branching and node selection strategies are applied according to the solution until the optimal integer solution is obtained.

D. PRICING SUBPROBLEMS

This part is the core part of the column generation method, which is an important process for real EMU routes. This paper uses a dynamic programming algorithm to solve the problem.

If the resource constraints are ignored, the problem is a common shortest path problem that can be solved quickly in polynomial time, but the emergence of resource constraints directly causes the problem to become an NP-hard problem. Consequently, this resource-constrained shortest path problem is generally solved by dynamic programming algorithms in many references. The advantage of using a dynamic programming algorithm is that its state definition is equivalent to an open container that can flexibly consider multiple targets, and the algorithm can accommodate a variety of different complex constraints in the solution process, judge each condition in each step of the extended solution process and solve multiple targets simultaneously.

$\boldsymbol{L}(i)$ is a set of valid labels of vertex i, $l_i = (RC_i, AD_i, AT_i, P_i)$, and represents a real EMU route r from the starting node to vertex i. H is the number of valid labels that can be stored and can be dynamically adjusted to meet the needs of optimization and the constraints. However, the solving time increases as parameter H increases.

Feasibility Definition of Labels: A real EMUs route (or label) is feasible. When the conditions $AT_i \leq MT$ and $AD_i \leq ML$ are met, $i \in \boldsymbol{NS}(r)$, where $\boldsymbol{NS}(r)$ is the set of train vertices included in the real EMU route. AT_i is the time interval from the last maintenance to node *i* of the EMU home depot, and AD_i is the mileage of the EMUs from the last maintenance to node *i*.

To avoid enumerating all feasible labels, a dominant criterion is introduced below, and the labels that cannot be expanded into the optimal labels are discarded in advance.

The Dominant Rule Definition: $l_i^1 = (RC_i^1, AD_i^1, AT_i^1, P_i^1)$ and $l_i^2 = (RC_i^2, AD_i^2, AT_i^2, P_i^2)$ are the two valid labels of the vertex, and there are the following inequalities: $RC_i^1 \leq RC_i^2$; $AD_i^1 \geq AD_i^2$; and $AT_i^1 \leq AT_i^2$. If at least one inequality is strictly satisfied, label l_i^1 is better than l_i^2.

The list *LN* is used to store the nodes with labels that need to be extended. The functio $Extend(l_i^h, j)$ extends the feasible label *lj* from the outgoing arcs to node *j* according to the resource constraint. If label l_j is not feasible, the function will not return any result. The functions $START(r)$ and $ARRIVE(r)$ are used to obtain the start and end nodes of route *r*, respectively.

The following rules control the label stop expansion in different management modes.

1) When an EMU has to return to its home depot, *li* extends to the depot node *j*, and $START(\text{r}) == ARRIVE(r) == j$, *lj* stops expanding. Otherwise, *lj* is deleted.

2) When an EMU can undergo maintenance at any depot, *li* extends to a depot node *j*. If *j* is a depot node that can provide appropriate maintenance services, *lj* stops expanding. Otherwise, *lj* is deleted.

2) When an EMU can undergo maintenance at any depot, *li* extends to a depot node *j*. If *j* is a depot node that can provide appropriate maintenance services, *lj* stops expanding. Otherwise, *lj* is deleted.

E. NODE SELECTION AND THE NODE BRANCHING METHOD

1) NODE SELECTION STRATEGY

At the beginning, a depth-first strategy is used to select a node until a feasible solution is found. Then, according to this feasible solution, the branch-and-price search tree is pruned. Next, the algorithm will use the optimal-bound-first strategy to select the most suitable node for calculation and repeatedly use the node selection strategy to accelerate the branch solution process.

2) BRANCH METHOD

When we establish a connection network, it is formed by splicing a plurality of subnets, and branch operations are performed inside each subnet without affecting one another. With this feature, a batch of arcs can be deleted in each branch process. The batch treatment of branch arcs in the network greatly improves efficiency. When the solution obtained by a node is not an integer solution, the calculated Ru* optimal intersection can be used to determine which key nodes lead to the generation of non-integer solutions, because the key nodes are associated with a series of arcs that will cause different real EMUs routes to simultaneously access certain train nodes. As long as these nodes and the arc that causes the conflict are identified, the conflicting arc will be deleted, the search space can be effectively decomposed, and a column that is not an integer solution can be correspondingly deleted in the model. For each node TN_u, if the solution to TN_u is a non-integer, Ru* will be used to determine the key nodes that are simultaneously accessed by different segments. The function

Determine_branch_arcs is responsible for determining the branch arcs that may need to be processed. These arcs will be deleted or reserved on the connected network of the child nodes. Finally, the newly produced child nodes are added to the branch-and-price tree.

To improve efficiency, by referring to the characteristics of China's high-speed train operations and the EMU circulation planning, for some long-travel EMUs routes, the connections between some trains can be fixed in advance. The EMUs that meet the requirements can be used to remove redundant, unnecessary train connection arcs, and this approach can

TABLE 7. Dynamic programming labeling algorithm.

```
Labeling algorithm
set $l_d^1 = (RC_d^1, AD_d^1, AT_d^1, p_d^1)$ with $RC_d^1 = 0$, $AD_d^1 = 0$, $AT_d^1 = 0$, $p_d^1 = (0,...,0)$
$\boldsymbol{LAB}(d) = l_d^1 \quad \forall d \in \boldsymbol{D}$
$\boldsymbol{LAB}(i) = \phi \quad \forall i \in \boldsymbol{T}$
set $LN = \boldsymbol{D}$
repeat
    select a node $i$ from $LN$;
    for each $l_i^h = (RC_i^h, AD_i^h, AT_i^h, p_i^h) \in \boldsymbol{LAB}(i)$ do
        for each $j \in \boldsymbol{FS}(i) \& \theta_j^h < 1$, where $\theta_j^h \in P_i^h$, do
            $l_j \leftarrow Extend(l_i^h, j)$
            if $l_j$ is not null and $l_j$ is not dominated by any label in $\boldsymbol{LAB}(j)$
                set $\boldsymbol{LAB}(j) = \boldsymbol{LAB}(j) \cup \{l_j\}$;
                remove all labels that are dominated by $l_j$ from $\boldsymbol{LAB}(j)$;
                add node $j$ to the list $LN$ if $j$ is not already in it;
            end if
        end for each
    end for each
until $LN = \varnothing$
return $\boldsymbol{LAB}(d)$ for all $d \in \boldsymbol{D}$;
```

TABLE 8. Definitions of the symbols used in the algorithm.

Parameter	Definition
$\boldsymbol{BA}$	a collection of arcs for node branches
TN_u	a node of the branch pricing search tree, where u is an identifier used to distinguish the nodes
TN_{u1}, TN_{u2}	two new child nodes obtained by node TN_u through the branch operation
$\boldsymbol{R}_u^*$	the optimal set of route segments calculated by node TN_u

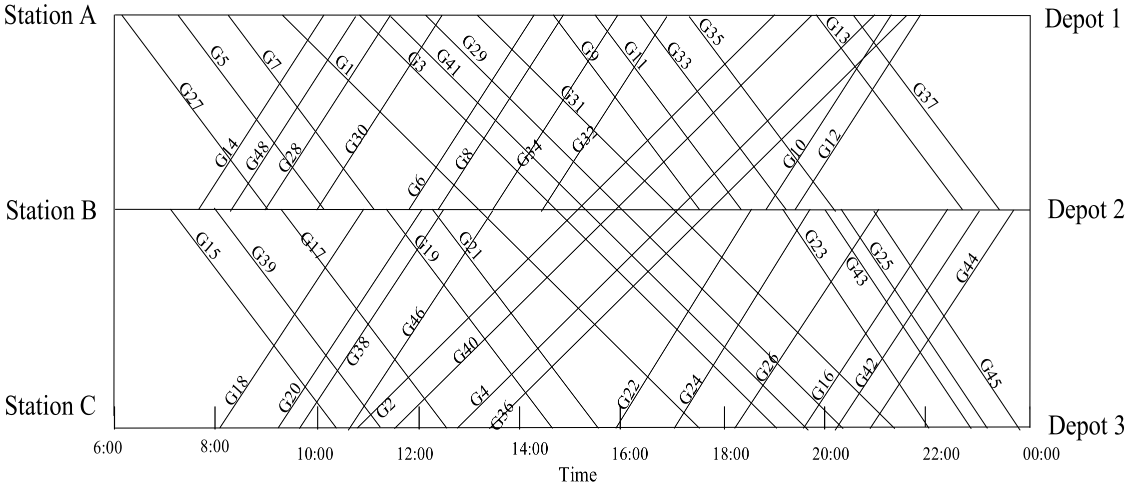


FIGURE 8. Example flight-line diagram for high-speed railway trains.

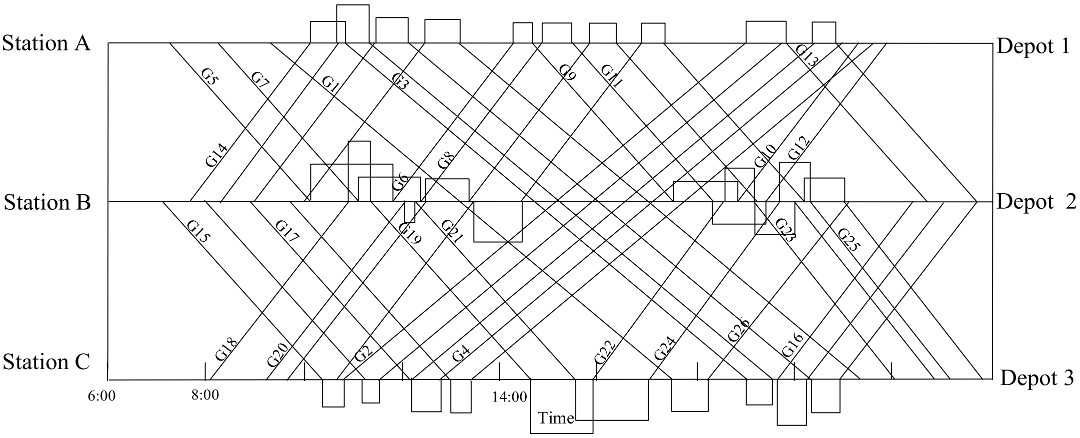


FIGURE 9. EMUs circulation plan diagram.

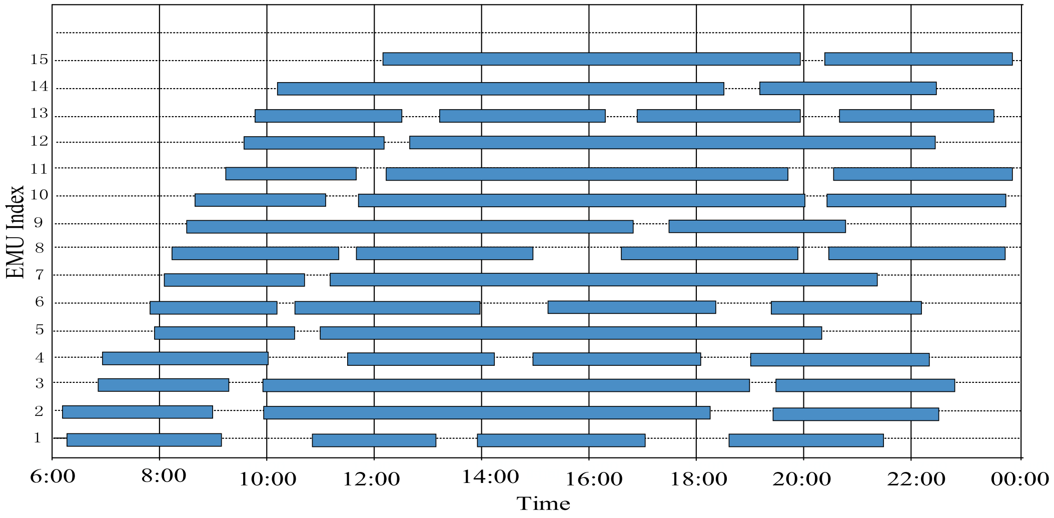


FIGURE 10. Implementation of the train tasks assigned to EMUs under conventional conditions.

quickly reduce the size of the train connection network after several iterations.

The function *Nodeget* obtains all the train nodes of EMUs route r. If the solution of node TN_u is a fraction, the function

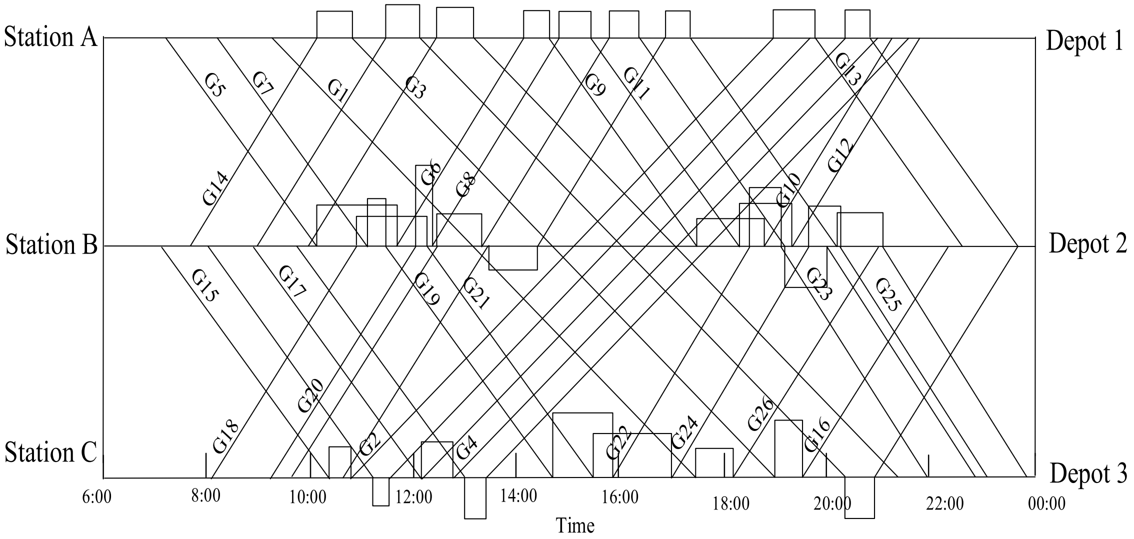


FIGURE 11. EMUs circulation plan in the collaborative mode.

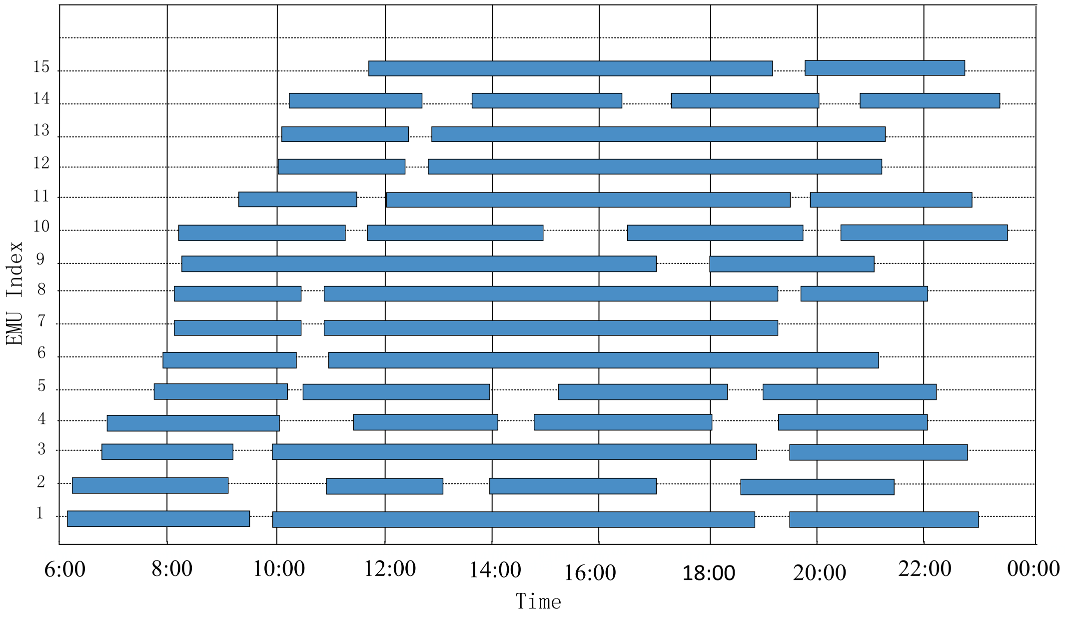


FIGURE 12. Train tasks assigned to EMUs in the collaborative mode.

Create creates a new child node for node TN_u to prepare for the next branching process.

The introduced branching method is a method for the batch processing of branch arcs, and this method improves the branching efficiency by simultaneously retaining and deleting a batch of arcs at the child tree nodes.

VI. CASE STUDY

A. ILLUSTRATIVE CAS

1) EMUs CIRCULATION PLAN IN A CONVENTIONAL MODE

The specific train information of the given illustrative case is shown in Table 10. According to Table 10, when planning the EMUs circulation plan, we do not consider the maintenance capacity limit and vehicle type restrictions of each application. However, EMUs maintenance must be performed at home depots, and the EMUs travel distance and time cannot exceed 4,400 km and 48 h, respectively. Figure 8 shows the train's time-space diagram from the train timetable. Figure 8 illustrates the change in the position of the train over time. The abscissa is the time, the ordinate is the train space position, and the location of the station and the positions of the trains are also shown in the figure. A, B and C are high-speed railway stations, and depots 1, 2 and 3 provide the needed EMUs and EMUs maintenance services and correspond to stations A, B and C, respectively.

TABLE 9. Node branching algorithm.

```
Node branching algorithm
repeat
  for all $r_1$ in $\boldsymbol{R}_u^*$
    for all $r_2$ in $\boldsymbol{R}_u^*$
      if $r_1 \neq r_2$&($Nodeget$ ($r_1$)∩$Nodeget$ ($r_2$))≠ø
        $r_1 \leftarrow r_1 \bigcup r_2$
        remove $r_2$ from $\boldsymbol{R}_u^*$;
      end if
    end for
  end for
until no merger
  $\boldsymbol{BA} \leftarrow Determine\_branch\_arcs(\boldsymbol{R}_u^*)$;
  ($TN_{u1}$, $TN_{u2}$) ←$Create$($TN_u$);
  repeat
    select an arc ($i$,$j$) from $\boldsymbol{BA}$;
    Delete ($i$,$j$) in $G$ of $TN_{u1}$;
Delete the incoming arcs toward node $j$ in $G$ of $TN_{u2}$ except ($i$,$j$);
  until $\boldsymbol{BA}$=ø
```

The train tasks assigned to each of the EMUs shown in Figure 9 are converted into the diagram shown in Figure 10, where each row is an EMU, the vertical axis is the EMUs index, and the abscissa is time. This figure shows that the connection among the trains is optimal and ensures that the minimum number of EMUs is used to cover the given trains. Because it can be directly seen from this simple picture, it is easy to verify from the point of view of any train that although the trains can be adjusted, it is not possible to obtain a better connection relationship to reduce the number of EMUs used.

2) EMUs CIRCULATION IN THE COLLABORATIVE MAINTENANCE MODE

In the above case, it is easy to return to the home depot for maintenance under the given distance conditions, and maintenance is not needed at other depots. We redesign the cases to make the trains travel farther away, then schedule the EMUs route plan, as shown in Table 11, and impose the constraint that depot 3 only undertakes the maintenance of certain EMUs.

An EMU circulation plan that allows for collaborative maintenance is constructed. The specific process is shown in Figure 11. An EMU departs from depot 3 and undergoes maintenance at depot 1, and the next day, after completing the train tasks given in the route, the EMU returns to its home depot. By liberalizing the maintenance location restrictions, more route combinations can be created to provide more optimization options for practical problems.

B. REAL-WORLD CASE STUDY

We select the Beijing-Shanghai high-speed railway as a case study. According to the proposed theory and method, this section systematically develops a practical case study to

TABLE 10. Train information in the virtual case.

Train number	Origin station	Destination station	Departure time	Arrival time	EMU type	Travel distance (km)
G1	A	C	08:15	17:02	CRH380BL	1,090
G2	C	A	09:55	18:50	CRH380BL	1,090
G3	A	C	09:55	18:55	CRH380BL	1,090
G4	C	A	10:58	21:03	CRH380BL	1,090
G5	A	B	06:15	09:05	CRH380BL	505
G6	B	A	10:55	13:05	CRH380BL	505
G7	A	B	07:45	10:11	CRH380BL	505
G8	B	A	11:26	14:02	CRH380BL	505
G9	A	B	13:55	17:00	CRH380BL	505
G10	B	A	18:32	21:25	CRH380BL	505
G11	A	B	14:50	18:00	CRH380BL	505
G12	B	A	19:16	22:00	CRH380BL	505
G13	A	B	19:28	22:58	CRH380BL	505
G14	B	A	06:50	09:10	CRH380BL	505
G15	B	C	06:10	09:30	CRH380BL	585
G16	C	B	19:26	22:45	CRH380BL	585
G17	B	C	07:54	10:25	CRH380BL	585
G18	C	B	06:55	09:58	CRH380BL	585
G19	B	C	10:30	13:55	CRH380BL	585
G20	C	B	08:10	11:20	CRH380BL	585
G21	B	C	12:14	15:40	CRH380BL	505
G22	C	B	15:52	18:28	CRH380BL	505
G23	B	C	19:16	22:00	CRH380BL	505
G24	C	B	17:15	19:40	CRH380BL	505
G25	B	C	20:16	23:10	CRH380BL	505
G26	C	B	18:12	21:03	CRH380BL	505
G27	A	B	08:32	9:12	CRH380BL	585
G28	B	A	08:24	10:40	CRH380BL	585
G29	A	B	12:08	20:18	CRH380BL	585
G30	B	A	10:00	12:45	CRH380BL	585
G31	A	C	13:06	21:24	CRH380BL	1,090
G32	B	A	14:25	15:55	CRH380BL	585
G33	A	B	16:24	19:12	CRH380BL	585
G34	C	A	10:56	19:22	CRH380BL	1,090
G35	C	B	17:15	20:09	CRH380BL	505
G36	A	C	11:30	19:56	CRH380BL	1,090
G37	A	B	20:48	23:43	CRH380BL	585
G38	C	B	09:39	12:32	CRH380BL	505
G39	B	C	09:48	13:01	CRH380BL	505
G40	C	A	11:25	19:44	CRH380BL	1,090
G41	C	A	13:23	21:40	CRH380BL	1,090
G42	C	B	20:18	23:06	CRH380BL	505
G43	A	C	19:45	22:26	CRH380BL	1,090
G44	C	B	20:21	23:44	CRH380BL	505
G45	B	C	21:02	23:28	CRH380BL	505
G46	C	B	10:39	13:29	CRH380BL	505
G48	B	A	8:20	10:44	CRH380BL	585

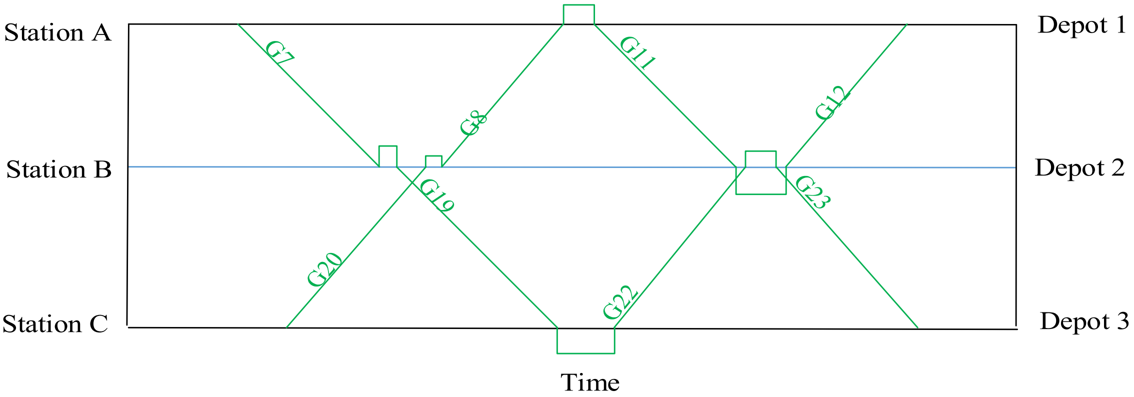


FIGURE 13. EMUs circulation plan depicted by a train space-time diagram.

FIGURE 14. Illustration of a high-speed railway network diagram.

verify the validity of the model and the optimization method and analyzes the impact of different management measures and optimization theory methods on the efficiency of EMUs.

The Beijing-Shanghai high-speed railway is an important part at the core of the high-speed railway network. The use and maintenance of EMUs in the Beijing-Shanghai high-speed railway involves a number of EMUs, and there are many types of EMUs in use. The railway line layout is shown in Figure 14. Eleven depots directly provide the relevant EMUs maintenance services. Table 12 shows the high-speed rail stations and associated vehicles served by the main train depots.

C. OPTIMIZATION RESULTS

The Beijing-Shanghai high-speed railway is selected as the research case to study the EMU circulation planning in different modes, such as EMUs inspection in the home depot mode, collaborative maintenance mode, and 5,000-km maintenance mileage. The case includes 406 trains for a certain year as the input.

The case returned the different effects of maintenance and collaborative maintenance to improve the mileage limit of the first-level maintenance cycle and provided a quantitative basis for optimizing the maintenance mode to improve the efficiency of the EMUs.

A relatively high proportion of medium to long range high-speed trains travel 1,000-2,000 km. The first-level maintenance period is basically 4,000 km/48 h, which restricts EMUs circulation. According to statistics, the average daily travel of a Beijing-Shanghai high-speed train EMU is 2,500-3,500 km. The impact of a higher mileage limit on improving the efficiency of EMUs needs a quantitative analysis. EMUs route planning under a 5,000-km maintenance mileage is introduced as the third mode to be analyzed.

1) OPTIMIZATION RESULTS OF THE NUMBER OF EMUs

The calculation results in Table 14 show the number of EMUs required in different maintenance modes.

The case study requires a minimum of 169 EMUs in the conventional mode, collaborative maintenance mode and 5,000-km maintenance mileage limitation mode. According to the needs, the distribution of the configuration of the EMUs will change. The EMUs circulation plan is mainly affected by the train timetable structure. At present, China's high-speed trains exhibit the characteristics of long-, medium- and short-distance trains with low density, and train arrival and running times are concentrated at certain times, which results in a large number of trains that need to arrange EMUs simultaneously.

2) OPTIMIZATION RESULTS OF EMUs MAINTENANCE TASKS

In this section, we analyze the overall maintenance tasks and situations of the average maintenance mileage changes in different maintenance modes.

TABLE 11. Train-related information in the virtual case.

Train number	Origin station	Destination station	Departure time	Arrival time	EMU type	Travel distance (km)
G1	A	C	08:08	17:02	CRH380BL	1,120
G2	C	A	09:45	18:50	CRH380BL	1,120
G3	A	C	09:55	19:02	CRH380BL	1,120
G4	C	A	10:58	21:09	CRH380BL	1,120
G5	A	B	06:15	09:12	CRH380BL	520
G6	B	A	10:55	13:12	CRH380BL	520
G7	A	B	07:37	10:11	CRH380BL	520
G8	B	A	11:26	14:09	CRH380BL	520
G9	A	B	13:55	17:07	CRH380BL	520
G10	B	A	18:32	21:32	CRH380BL	520
G11	A	B	14:43	18:00	CRH380BL	520
G12	B	A	19:16	22:07	CRH380BL	520
G13	A	B	19:20	22:58	CRH380BL	520
G14	B	A	06:50	09:17	CRH380BL	520
G15	B	C	06:10	09:37	CRH380BL	600
G16	C	B	19:19	22:45	CRH380BL	600
G17	B	C	07:54	10:33	CRH380BL	600
G18	C	B	06:48	09:58	CRH380BL	600
G19	B	C	10:22	13:55	CRH380BL	600
G20	C	B	08:10	11:27	CRH380BL	600
G21	B	C	12:14	15:40	CRH380BL	600
G22	C	B	15:52	18:28	CRH380BL	600
G23	B	C	19:16	22:00	CRH380BL	600
G24	C	B	17:15	19:40	CRH380BL	600
G25	B	C	20:16	23:10	CRH380BL	600
G26	C	B	18:12	21:03	CRH380BL	600
G27	A	B	08:32	9:12	CRH380BL	600
G29	A	B	12:08	20:18	CRH380BL	520
G30	B	A	10:00	12:45	CRH380BL	520
G31	A	C	13:06	21:24	CRH380BL	1,120
G32	B	C	14:25	15:55	CRH380BL	600
G33	A	B	16:24	19:12	CRH380BL	600
G34	C	A	10:56	19:22	CRH380BL	1,120
G35	C	B	17:15	20:09	CRH380BL	600
G36	B	C	13:23	21:40	CRH380BL	600
G37	A	B	20:48	23:43	CRH380BL	600
G38	C	B	09:39	12:32	CRH380BL	520
G39	B	C	09:48	13:01	CRH380BL	520
G40	C	A	11:25	19:44	CRH380BL	1,120
G41	A	C	11:30	19:56	CRH380BL	1,120
G42	C	B	20:18	23:06	CRH380BL	600
G43	A	C	19:45	22:26	CRH380BL	1,120
G44	C	B	20:21	23:44	CRH380BL	600
G45	B	C	21:02	23:28	CRH380BL	520
G46	C	B	10:39	13:29	CRH380BL	520
G48	B	A	8:20	10:44	CRH380BL	600

TABLE 12. Depots and stations on the Beijing-Shanghai high-speed railway.

Depot	Served stations	EMU type	Speed level	Composition type
Beijing South depot	Beijing South Station, Tianjin West Station	CRH2A	200-250 km/h	Short
		CRH2E	200-250 km/h	Long
		CRH380A	350 km/h	Short
		CRH380L	350 km/h	Long
		CRH380L	350 km/h	Long
		CRH380L	350 km/h	Long
		CRH3C	300-350 km/h	Short
		CRH5A	200-250 km/h	Short
Jinan depot	Jinan West Station, Jinan Station, Zaozhuang Station	CRH380L	350 km/h	Long
Qingdao depot	Qingdao Station	CRH380A	350 km/h	Short
		CRH380L	350 km/h	Long
		CRH380A-coupled	350 km/h	Long
		CRH5A	200-250 km/h	Short
Nanjing South depot	Nanjing South Station, Weinan Station, Xuzhou East Station, Hongqiao Station, Nanjing Station	CRH2C	300-350 km/h	Short
		CRH380A	350 km/h	Short
		CRH1B	200-250 km/h	Long
		CRH1E	200-250 km/h	Short
		CRH2B	200-250 km/h	Long
		CRH380L	350 km/h	Long
Hangzhou depot	Hangzhou Station, Hangzhou East Station, Ningbo East Station	CRH1A	200-250 km/h	Short
		CRH1B	200-250 km/h	Long
		CRH380L	350 km/h	Long
		CRH1E	200-250 km/h	Long
		CRH2E	200-250 km/h	Long
		CRH2B	200-250 km/h	Long
Hongqiao depot	Xuzhou Station, Hongqiao Station, Nanjing South Station, Nanjing Station, Hangzhou East Station, Hangzhou Station	CRH2C	300-350 km/h	Short
		CRH380L	350 km/h	Long
Hefei depot	Hefei	CRH380A	300-350 km/h	Short
		CRH380L	300-350 km/h	Long
Zhengzhou East depot	Zhengzhou Station	CRH380L	350 km/h	Long
Hankou depot	Hankou Station	CRH2A	200-250 km/h	Short
		CRH5A	200-250 km/h	Short
Wuhan depot	Wuhan Station	CRH2C	300-350 km/h	Short
		CRH380L	350 km/h	Long
Changsha depot	Changsha South Station	CRH3C	300-350 km/h	Short

TABLE 13. **Daily maintenance capacity.**

Depot name	Level 1 maintenance		
	Tracks for maintenance	Maintenance capability	Accommodation capability
Beijing South depot	12	56	68
Hongqiao depot	14	60	68
Hefei depot	4	20	25
Hangzhou depot	6	30	13
Nanjing South depot	8	24	31
Jinan depot	6	18	17
Qingdao depot	3	10	21
Changsha depot	2	6	17
Hankou depot	6	42	42
Wuhan depot	9	48	49
Zhengzhou East depot	4	18	20

A comparison of the number of maintenance tasks in the plans is shown in Table 14. Compared with the existing circulation plan, the total maintenance tasks are gradually reduced through optimization; in particular, under the condition of 5,000 km, the total number of these tasks dropped to 97, a decrease of 29.7%, which shows that the collaborative mode can effectively reduce the number of maintenance tasks.

As shown in Table 15, the average maintenance mileage gradually improves; under the condition of 5,000 km, the index further improves. In the networked mode, the EMUs maintenance resource sharing mode is beneficial to improve the efficiency of the EMUs. The reason is that when the number of maintenance tasks is reduced with the relaxation of maintenance management restrictions, the problem's train combination space continues to increase, and trains can form better routing plans. When the maintenance restrictions are relaxed, more connections can be used between trains, and when increasingly more feasible solutions are reproduced, the variables of the solutions will increase accordingly. In the end, we can find more suitable and flexible routing plans to make the problem obtain a better solution, thereby reducing the cost of EMUs operations.

(2) An analysis of the EMUs maintenance tasks in Figure 15 shows the number of maintenance tasks in the different plans. As the mileage limit increases, most of the maintenance tasks undertaken by the depots tend to decrease.

TABLE 14. **Number of EMUs used by each depot in different maintenance modes.**

Depot	EMU type	Real case	Conventional mode	Collaborative mode	Collaborative mode with 5,000 km
Beijing South depot	CRH380A	7	7	7	7
	CRH380L	23	23	23	23
Hangzhou depot	CRH380L	4	4	5	4
	CRH380L	11	11	12	15
	CRH1A	1	1	1	1
Hefei depot	CRH380A	3	3	3	3
	CRH380L	2	3	2	0
Nanjing South depot	CRH2B	7	7	7	7
	CRH1B	3	3	2	2
	CRH1E	2	2	2	2
	CRH380A	1	1	1	1
	CRH380L	16	15	14	15
Shanghai South depot	CRH2A	15	11	11	11
	CRH1A	1	1	1	1
	CRH1A coupled	4	4	4	4
	CRH1B	10	9	9	9
	CRH1E	6	6	6	6
	CRH2A coupled	8	8	8	8
Hongqiao depot	CRH380L	27	26	26	25
	CRH2A coupled	3	3	3	3
	CRH2B	1	1	1	1
Wuhan depot	CRH380L	1	1	1	1
Wuhan depot	CRH2A	2	2	2	2
Zhengzhou depot	CRH2A	1	1	1	1
	CRH2A coupled	1	1	1	1
Qingdao depot``	CRH380A	3	3	3	3
	CRH380L	2	2	2	2
	CRH380A coupled	2	2	2	2
Jinan depot	CRH380L	13	13	13	13
Total	-	171	170	169	169

(3) An analysis of the maintenance task changes in different maintenance modes in Table 16 shows that the number of maintenance tasks for the different travel mileage ranges of an EMU as a percentage of the total number of maintenance tasks. In the different maintenance modes, maintenance is mostly conducted within 2,000-3,000 km, which accounts for more than 60% of the total maintenance tasks. Most of the plans are scheduled to have a maintenance task every day. Under the condition of a 5,000-km maintenance mileage, the number of maintenance tasks is greatly reduced, and the EMUs are mainly maintained once every two days. This result indicates that increasing the maintenance mileage limit is effective.

TABLE 15. **Comparison of the maintenance workloads in different maintenance management modes.**

EMU maintenance mode	Number of EMU maintenance tasks	Average mileage before maintenance (km)	Percent decrease in the number of EMU maintenance tasks (%)
Real case	138	2,517.78	—
Conventional mode	132	2,632.23	-4.3
Collaborative mode	124	2,802.05	-10.1
Collaborative mode with 5,000 km	97	4,267.48	-29.7

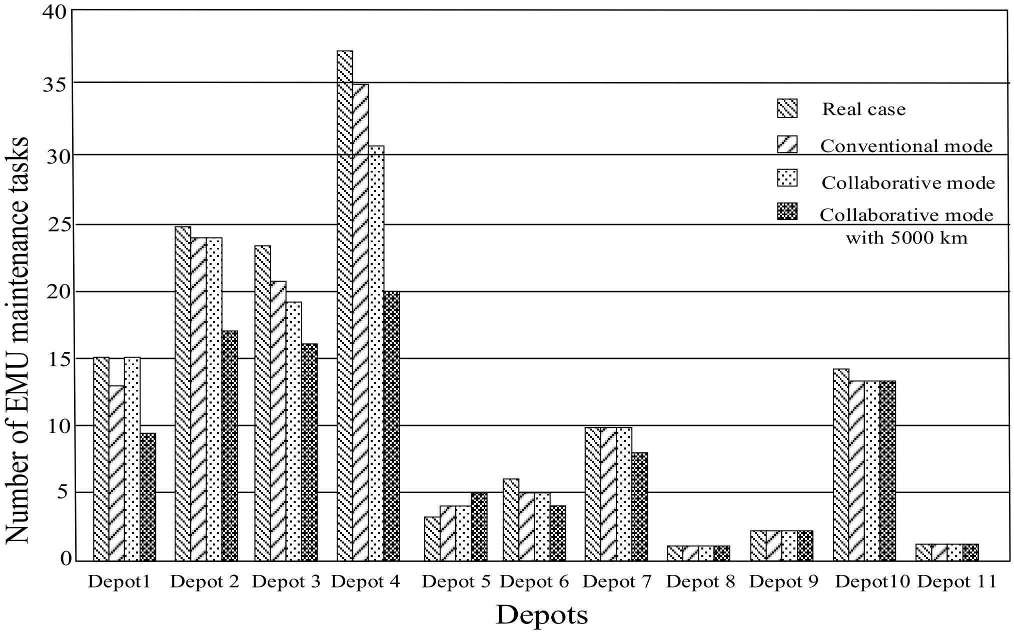


FIGURE 15. **Comparison of the maintenance workloads of each depot on the Beijing-Shanghai high-speed railway.**

TABLE 16. **Distribution of first-class EMUs maintenance mileage when operated in different modes (%).**

Maintenance mode	Maintenance mileage ranges (km)				
	0-2,000	2,000-3,000	3,000-4,000	4,000-5,000	>5,000
Real case	25.3	52.3	12.7	9.7	0
Conventional mode	18.2	53.2	18.3	10.3	0
Collaborative mode	16.9	50.8	18.5	12.9	0
Collaborative mode with 5,000 km	14.6	20.9	16.7	26.0	21.9

When different EMUs maintenance modes and more flexible connection modes are considered, taking the Beijing-Shanghai high-speed railway as a case study, different EMUs circulation plans in the three maintenance modes of the home depot mode, collaborative mode and improved mileage mode are generated by the proposed method. Due to the limited number of EMUs and based on the given train timetable with a fixed structure under large-scale network conditions, the possibility for the further optimization of the number of EMUs is limited, but the number of maintenance tasks can be reduced by optimizing the maintenance mode. The collaborative maintenance mode for this real-world case can reduce the EMUs maintenance tasks by 10.1%, and collaborative maintenance with the maintenance mileage improved to 5,000 km can reduce the maintenance work by 29.7%, which can greatly reduce the maintenance costs, release a large amount of time used for maintenance, and can offer favorable conditions for improving the efficiency of EMUs. Through an actual case analysis, it is found that the most critical constraint in the model proposed in this research is the mileage limit of the EMUs, because the experiments show that this constraint not only affects the maintenance cost but also has a great impact on the solution time, which will increase several times. If the actual maintenance mileage increases in the future, there may be the possibility of not finding the optimal solution smoothly within a reasonable time.

VII. CONCLUSION

By combining the current EMUs resource allocation conditions and management modes of China's high-speed railway system, this paper constructs a train connection network that considers various practical operation management measures based on considerations such as breaking through the restrictions of a maintenance management system, introducing a collaborative maintenance mode and extending the mileage limit.

Regarding the minimum number of EMUs and the number of maintenance tasks as objectives, we consider the constraints of the maintenance mode, maintenance capability, and flexible EMUs route optimization to model the problem, and an optimization algorithm based on a two-stage concept is proposed. A multi-commodity flow problem model with side constraints is proposed in which the first stage is an ideal EMUs route generation algorithm, and the second stage is a branch-and-price algorithm. This method realizes an EMUs circulation plan that considers the return of the EMUs for maintenance and collaborative maintenance and increasing the mileage limit of the first-level maintenance cycle to optimize the preparation of the EMUs circulation.

Through an analysis of the case of the high-speed railway network associated with the Beijing-Shanghai high-speed railway, the management mode proposed by this research is explained. The mathematical modeling and optimization method for the current EMUs circulation planning problem proposed in this paper are systematically analyzed. Through a statistical analysis, the results show that the main objectives of the case study are improved to some extent, and the case study verifies the effectiveness of the method.

REFERENCES

[1] Y. Zhou, L. Zhou, Y. Wang, Z. Yang, and J. Wu, "Application of multiple-population genetic algorithm in optimizing the train-set circulation plan problem," *Complexity*, vol. 2017, pp. 1–14, 2017.

[2] Y.-C. Lai, S.-W. Wang, and K.-L. Huang, "Optimized train-set rostering plan for taiwan high-speed rail," *IEEE Trans. Autom. Sci. Eng.*, vol. 14, no. 1, pp. 286–298, Jan. 2017.

[3] Q. Zhong, R. M. Lusby, J. Larsen, Y. Zhang, and Q. Peng, "Rolling stock scheduling with maintenance requirements at the Chinese High-Speed Railway," *Transp. Res. B, Methodol.*, vol. 126, pp. 24–44, Aug. 2019.

[4] R. M. Lusby, J. T. Haahr, J. Larsen, and D. Pisinger, "A branch-and-price algorithm for railway rolling stock rescheduling," *Transp. Res. B, Methodol.*, vol. 99, pp. 228–250, May 2017.

[5] Y.-C. Lai, D.-C. Fan, and K.-L. Huang, "Optimizing rolling stock assignment and maintenance plan for passenger railway operations," *Comput. Ind. Eng.*, vol. 85, pp. 284–295, Jul. 2015.

[6] J. Wu, B. Lin, H. Wang, X. Zhang, Z. Wang, and J. Wang, "Optimizing the high-level maintenance planning problem of the electric multiple unit train using a modified particle swarm optimization algorithm," *Symmetry*, vol. 10, no. 8, p. 349, Aug. 2018.

[7] D. Tönissen and J. Arts, "Economies of scale in recoverable robust maintenance location routing for rolling stock," *Transp. Res. B, Methodol.*, vol. 117, pp. 360–377, Nov. 2018.

[8] V. Cacchiani, A. Caprara, and P. Toth, "Models and algorithms for the train unit assignment problem," *Combinat. Optim.*, vol. 7422, pp. 24–35, 2012.

[9] V. Cacchiani, A. Caprara, and P. Toth, "A Lagrangian heuristic for a train-unit assignment problem," *Discrete Appl. Math.*, vol. 161, no. 12, pp. 1707–1718, Aug. 2013.

[10] V. Cacchiani, A. Caprara, and P. Toth, "Solving a real-world train-unit assignment problem," *Math. Program.*, vol. 124, nos. 1–2, pp. 207–231, Jul. 2010.

[11] V. Cacchiani, A. Caprara, and P. Toth, "A fast heuristic algorithm for the train unit assignment problem," in *Proc. 12th Workshop Algorithmic Approaches Transp. Modeling, Optim., Syst. (ATMOS)*, vol. 25, 2012, pp. 1–9.

[12] T. Nishi, A. Ohno, M. Inuiguchi, S. Takahashi, and K. Ueda, "A combined column generation and heuristics for railway short-term rolling stock planning with regular inspection constraints," *Comput. Oper. Res.*, vol. 81, pp. 14–25, May 2017.

[13] Z. Lin and R. S. Kwan, "A branch-and-price approach for solving the train unit scheduling problem," *Transp. Res. B, Methodol.*, vol. 94, pp. 97–120, Dec. 2016.

[14] V. Cacchiani, A. Caprara, L. Galli, L. Kroon, G. Maróti, and P. Toth, "Railway rolling stock planning: Robustness against large disruptions," *Transp. Sci.*, vol. 46, no. 2, pp. 217–232, May 2012.

[15] H. Joudrier and F. Thiard, "A greedy approach for a rolling stock management problem using multi-interval constraint propagation: ROADEF/EURO challenge 2014," *Ann. Oper. Res.*, vol. 271, no. 2, pp. 1165–1183, Dec. 2018.

[16] M. V. Buyanov and A. V. Naumov, "Optimizing the operation of rolling stock in organizing cargo transportation at a railway network segment," *Automat. Remote Control*, vol. 79, no. 9, pp. 1661–1672, Sep. 2018.

[17] V. Cacchiani, A. Caprara, and P. Toth, "An effective peak period heuristic for railway rolling stock planning," *Transp. Sci.*, vol. 53, no. 3, pp. 746–762, 2019.

[18] G. Budai, G. Maróti, R. Dekker, D. Huisman, and L. Kroon, "Rescheduling in passenger railways: The rolling stock rebalancing problem," *J. Scheduling*, vol. 13, no. 3, pp. 281–297, Jun. 2010.

[19] J. Wagenaar, L. Kroon, and I. Fragkos, "Rolling stock rescheduling in passenger railway transportation using dead-heading trips and adjusted passenger demand," *Transp. Res. B, Methodol.*, vol. 101, pp. 140–161, Jul. 2017.

[20] L. Kroon, G. Maróti, and L. Nielsen, "Rescheduling of railway rolling stock with dynamic passenger flows," *Transp. Sci.*, vol. 49, no. 2, pp. 165–184, May 2015.

[21] L. P. Veelenturf, L. G. Kroon, and G. Maróti, "Passenger oriented railway disruption management by adapting timetables and rolling stock schedules," *Transp. Res. C, Emerg. Technol.*, vol. 80, pp. 133–147, Jul. 2017.

[22] J. C. Wagenaar, L. G. Kroon, and M. Schmidt, "Maintenance appointments in railway rolling stock rescheduling," *Transp. Sci.*, vol. 51, no. 4, pp. 1138–1160, Nov. 2017.

[23] E. Van Der Hurk, L. Kroon, and G. Maróti, "Passenger advice and rolling stock rescheduling under uncertainty for disruption management," *Transp. Sci.*, vol. 52, no. 6, pp. 1391–1411, Dec. 2018.

[24] D. Canca and E. Barrena, "The integrated rolling stock circulation and depot location problem in railway rapid transit systems," *Transp. Res. E, Logistics Transp. Rev.*, vol. 109, pp. 115–138, Jan. 2018.

[25] J. T. Haahr, J. C. Wagenaar, L. P. Veelenturf, and L. G. Kroon, "A comparison of two exact methods for passenger railway rolling stock (re)scheduling," *Transp. Res. E, Logistics Transp. Rev.*, vol. 91, pp. 15–32, Jul. 2016.

[26] Y. Yin, D. Li, N. Bešinović, and Z. Cao, "Hybrid demand-driven and cyclic timetabling considering rolling stock circulation for a bidirectional railway line," *Comput.-Aided Civil Infrastruct. Eng.*, vol. 34, no. 2, pp. 164–187, Feb. 2019.

[27] R. Borndörfer, M. Reuther, T. Schlechte, K. Waas, and S. Weider, "Integrated optimization of rolling stock rotations for intercity railways," *Transp. Sci.*, vol. 50, no. 3, pp. 863–877, Aug. 2016.

[28] J. Haahr and R. M. Lusby, "Integrating rolling stock scheduling with train unit shunting," *Eur. J. Oper. Res.*, vol. 259, no. 2, pp. 452–468, Jun. 2017.

[29] Y. Wang, A. D'Ariano, J. Yin, L. Meng, T. Tang, and B. Ning, "Passenger demand oriented train scheduling and rolling stock circulation planning for an urban rail transit line," *Transp. Res. B, Methodol.*, vol. 118, pp. 193–227, Dec. 2018.

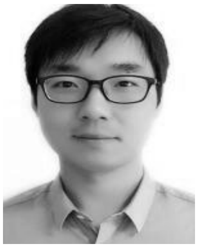
[30] L. Meng, F. Corman, X. Zhou, and T. Tang, "Special issue on Integrated optimization models and algorithms in rail planning and control," *Transp. Res. C, Emerg. Technol.*, vol. 88, pp. 87–90, Mar. 2018.

WEN-JUN LI received the bachelor's degree in computer science and technology and the master's degree in transportation planning and management from Lanzhou Jiaotong University, and the Ph.D. degree in transportation planning and management from Beijing Jiaotong University. He is currently a Lecturer with the School of Economics and Management, Jiangsu University of Science and Technology. His main research interests include high-speed railway transportation organization optimization, logistics system network planning and modeling, and operational planning optimization theory.

YU-TING WANG received the bachelor's degree in management from Anhui Jianzhu University, Hefei, China, in 2019. She is currently pursuing the master's degree with the Jiangsu University of Science and Technology, Zhenjiang, China.

Her research interests include transportation optimization, logistics optimization, and supply chain management.

LEI NIE received the Ph.D. degree in transportation planning and management from Beijing Jiaotong University, in 2000. Since 2000, she has been involved in teaching and research work with the School of Traffic and Transportation, Beijing Jiaotong University, where she is currently a Professor. Her main research interests include transportation planning and management, transportation organization theory and technology, transportation and logistics theory and technology, and comprehensive transportation theory and technology.



YINGHUI WU received the Ph.D. degree in systems engineering from Northeastern University, in 2015. He is currently an Associate Professor with the School of Economics and Management, Jiangsu University of Science and Technology. His research interests include smart city bus system optimization, logistics system and network optimization, optimization theory, and algorithm design.

LIU PENG received the Ph.D. degree from the Dalian University of Technology, in 2016. He is currently a Lecturer with the School of Economics and Management, Jiangsu University of Science and Technology. His main research interest includes dynamics in complex social networks (collective behavior evolution and knowledge dissemination).

• • •