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# Robust Continuous Control for a Class of Mechanical Systems Based on Nonsingular Terminal Sliding Mode

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**ABSTRACT** In this paper is proposed a control structure for a class of mechanical systems; this structure consists of a continuous controller based on nonsingular terminal sliding mode control plus uncertainty and disturbance estimator. Closed-loop stability is proved by designing an adequate sliding surface and showing the existence of sliding modes by the fulfillment of the reaching law. A controllers comparison using the nonsingular terminal sliding mode, first-order sliding mode, PID, and the proposed control structure is carried out through numerical simulations of a pendulum system, where the  $l_2$  index is used to measure the performance of the controllers. Moreover, real-time experiments are performed in a mechanical system with a pneumatic actuator. The theoretical, numerical, and experimental results validate the feasibility, performance, and robustness of the proposed control structure.

**INDEX TERMS** Mechanical systems, nonsingular terminal sliding mode control, robust control, uncertainty and disturbance estimator.

## **I. INTRODUCTION**

The design of trajectory tracking controllers has important usages in mechanical systems with electrical, pneumatic, or hydraulic actuators [1]–[4]. In the past decades, several robust control techniques have been established for the tracking control of uncertain mechanical systems such as sliding mode control [5]–[7], fuzzy control [8]–[10], adaptive control [11], [12], PID control [13]–[16], among others. Each controller design has its advantages, and disadvantages such as discontinuous controllers under certain conditions are robust against uncertainties and perturbations. However, they can excite the high-frequency components of the actuators, which heat them, and in long-term, this can reduce their lifetime [17]. Moreover, PID controllers have the advantage of having a continuous signal, thus avoiding the heat in the actuators, but when the PID is implemented to solve the tracking problem, it can only compensate for constant or low-frequency uncertainties and perturbations [18]. More recently fuzzy controllers have been complemented with adaptive control to robustify the closed-loop

Previously, asymptotic sliding mode controllers have shown the feasibility and excellent performance against perturbations and uncertainties, see [25]–[27]. For example, in [28], a second-order asymptotic sliding mode control, which is chattering-free, was designed, and it does not need the derivative of the switching function to ensure the asymptotical convergence to a second-order sliding mode. Moreover, in [29] was developed an adaptive super-twisting algorithm that presents a continuous control law after integration, where an adaptive-tuning controller eliminates the need for the knowledge about the upper bound of the external disturbances and their derivatives. More recently, in [30], [31] is investigated the sliding mode control design for descriptor systems via a linear switching function approach.

In first-order sliding mode control and terminal sliding mode control, the control signal is discontinuous, and the chattering phenomenon affects the system. Moreover, to design the control is necessary to know a priori, the upper

performance before uncertainties and disturbances [19]–[22]. However, adaptive controllers have shown before to be at best uniformly asymptotically stable in the large, and possess an infinite region where the trajectories move arbitrarily slowly, see [23], [24].

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bounds of uncertainties and disturbances. In practice, the control chattering is undesirable since it can damage the actuator and the system [32]; also, the bounds of the uncertainties and disturbances are not always known.

In this paper, the motivation is to propose a control structure that offers a robust closed-loop performance against uncertainties and disturbances while using a continuous control signal. In this way, it could be avoided the heat in the actuators when applying the control structure to mechanical systems, and the life cycle of the actuators could be improved while the robustness property is maintained. These advantages are achieved, first, by replacing the discontinuous gain of a terminal sliding mode control by a continuous expression, and second, the robustness property is kept now by using an uncertainty and disturbance estimator, thereby it can be maintained the robustness of the closed-loop system while the control signal is continuous. The designed controller is based on nonsingular terminal sliding mode [33]–[36] and the uncertainty and disturbance estimator is based on the UDE theory [37]–[40]. The proposed control structure also includes compensation terms where it is necessary to know some of the system's parameters; some examples of parameter identification procedures applied to second-order systems can be found in [41], [42].

The main innovations of this paper are itemized as follows:

- Design and stability analysis of a robust asymptotic control structure: the trajectory tracking controller and the application of the uncertainty and disturbance estimator. Moreover, in the case where the trajectories start on the sliding surface, the closed-loop system's trajectories converge to the origin in finite time.
- The proposed control enforces sliding mode without using discontinuous control and without requiring the knowledge of the uncertainties and disturbances upper bounds.

This paper is organized as follows: Section II states the problem. In Section III is presented a continuous control based on the nonsingular terminal sliding mode. The existence of sliding modes using a continuous controller is proved in Section IV. Section V presents a controller's comparison through numerical simulations. The proposed control structure is tested by performing real-time experiments in a mechanical system with a pneumatic actuator in Section VI. Finally, Section VII gives some conclusions.

#### **II. PROBLEM STATEMENT**

The problem addressed in the paper is the tracking control of a class of mechanical systems through the design and implementation of continuous robust control. The given controller is based on nonsingular terminal sliding mode control and uncertainty and disturbance estimator.

The following state-space equations govern the dynamics of the class of mechanical systems considered in this paper

<span id="page-1-0"></span>
$$
\dot{x} = y,\n\dot{y} = f(x, y) + g(x, y)u + w,
$$
\n(1)

where  $f(x, y)$  and  $g(x, y)$  are nonlinear functions, *w* represents the effect of the uncertainties and external unmeasurable disturbances, if any. For system [\(1\)](#page-1-0) the following control design is as follows

<span id="page-1-1"></span>
$$
u = -g(x, y)^{-1} (f(x, y) - \tau - \ddot{x}_d), \tag{2}
$$

where  $\tau$  is the proposed algorithm, the rest of the terms are considered well known, and they are used for compensation purposes. Substituting [\(2\)](#page-1-1) in [\(1\)](#page-1-0) the remaining closed-loop system stands as follows

<span id="page-1-2"></span>
$$
\begin{aligned}\n\dot{x} &= y, \\
\dot{y} &= \tau + \ddot{x}_d + w.\n\end{aligned} \tag{3}
$$

Let us rewrite system [\(3\)](#page-1-2) in function of the errors, where  $e_1 =$  $x - x_d$  and  $e_2 = \dot{x} - \dot{x}_d$ , where  $x_d$  is the desired trajectory assumed to be twice differentiable. Therefore

<span id="page-1-3"></span>
$$
\dot{e}_1 = e_2,
$$
  
\n
$$
\dot{e}_2 = \tau + w,
$$
\n(4)

the structure of [\(4\)](#page-1-3) corresponds to a double integrator system.

## **III. CONTINUOUS CONTROLLER BASED ON NONSINGULAR TERMINAL SLIDING MODE**

In this section, a tracking controller is designed in the framework of nonsingular terminal sliding mode control. Let us define a sliding surface as follows,

<span id="page-1-9"></span>
$$
\sigma = e_1 + e_2^{p/q},\tag{5}
$$

where *p* and *q* are positive odd integers which satisfy the condition that  $1 \leq p/q \leq 2$ . Now in order to fulfill the reaching condition  $\dot{\sigma} \sigma < 0$  let us obtain  $\dot{\sigma}$ 

<span id="page-1-4"></span>
$$
\dot{\sigma} = e_2 + \frac{p}{q} e_2^{p/q - 1} (\tau + w), \qquad (6)
$$

let the required control be expressed as

<span id="page-1-8"></span>
$$
\tau = u_{eq} + u_n,\tag{7}
$$

selecting by design,

<span id="page-1-5"></span>
$$
u_{eq} = -\frac{qe_2^{2-p/q}}{p} - \rho \sigma,
$$
 (8)

where  $\rho$  is a positive gain constant. From [\(6\)](#page-1-4)-[\(8\)](#page-1-5) let us get

<span id="page-1-6"></span>
$$
\dot{\sigma} = -\frac{p}{q} e_2^{p/q-1} \left( \rho \sigma - u_n - w \right). \tag{9}
$$

Next, the component  $u_n$  will be designed.

## A. COMPENSATION OF UNCERTAINTIES AND **DISTURBANCES**

Through estimation, the lumped uncertainty *w* can be compensated. Assuming that the trajectories are on the sliding surface (in the next section this will be proven), this is  $\sigma =$  $\dot{\sigma} = 0$ , let us rewrite [\(9\)](#page-1-6) as

<span id="page-1-7"></span>
$$
w = \rho \sigma - u_n. \tag{10}
$$

The lumped uncertainty *w* can be computed from the righthand side of [\(10\)](#page-1-7). However, it cannot be done directly. Let the estimate of the lumped uncertainty, denoted by  $\hat{w}$ , be defined as

<span id="page-2-2"></span>
$$
\hat{w} = \rho G_f(s)\sigma - G_f(s)u_n, \qquad (11)
$$

where  $G_f(s)$  is a strictly proper low-pass filter with unity steady-state gain and broad enough bandwidth. With such a filter

<span id="page-2-1"></span>
$$
\hat{w} \simeq w,\tag{12}
$$

which enable the design of  $u_n$  as

<span id="page-2-0"></span>
$$
u_n = -\hat{w} = -\rho G_f(s)\sigma + G_f(s)u_n, \qquad (13)
$$

solving for  $u_n$  gives

<span id="page-2-5"></span>
$$
u_n = -\hat{w} = -\frac{G_f(s)}{1 - G_f(s)}\rho\sigma,\tag{14}
$$

since  $G_f(s)$  is strictly proper, the control signal  $u_n$  in [\(13\)](#page-2-0) is implementable.

#### **IV. EXISTENCE OF SLIDING MODE**

The existence of the sliding mode can be proved as follows, to this end let us define the estimation error as

<span id="page-2-3"></span>
$$
\tilde{w} = w - \hat{w},\tag{15}
$$

using  $(13)$  in  $(9)$ , let us get

$$
\dot{\sigma} = -\frac{p}{q} e_2^{p/q - 1} \left( \rho \sigma - \tilde{w} \right),\tag{16}
$$

which in view of [\(12\)](#page-2-1), leads to

$$
\sigma \dot{\sigma} = -\frac{p\rho}{q} e_2^{p/q - 1} \sigma^2,\tag{17}
$$

for  $e_2 \neq 0$ ,  $p \rho e_2^{p/q-1}$  $\frac{p}{q-1}/q > 0$ . Therefore, for the case  $e_2 \neq 0$ , the reaching condition  $\sigma \dot{\sigma} < 0$  is satisfied. The system states can reach the sliding mode  $\sigma = 0$  exponentially, this can be proved as follows, substituting the control [\(7\)](#page-1-8) into system [\(4\)](#page-1-3) yields

$$
\dot{e}_2 = -\frac{qe_2^{2-p/q}}{p} - \rho \sigma.
$$
 (18)

Then, for  $e_2 = 0$ , it is obtained

$$
\dot{e}_2 = -\rho \sigma. \tag{19}
$$

For both  $\sigma > 0$  and  $\sigma < 0$ , it is obtained  $\dot{e}_2 < 0$  and  $\dot{e}_2 > 0$ , respectively, showing that  $e_2 = 0$  is not an attractor. It also means that there exists a vicinity of  $e_2 = 0$  such that for a small  $\delta > 0$  such that  $|e_2| < \delta$ , there are  $\dot{e}_2 < 0$  for  $\sigma > 0$ and  $\dot{e}_2 > 0$  for  $\sigma < 0$ , respectively. Therefore, the crossing of the trajectory from the boundary of the vicinity  $e_2 = \delta$ to  $e_2 = -\delta$  for  $\sigma > 0$ , and from  $e_2 = -\delta$  to  $e_2 = \delta$  for  $\sigma$  < 0 occurs in finite time. Therefore, it is concluded that the sliding mode  $\sigma = 0$  can be reached from anywhere in the phase plane asymptotically.

Note that in the case where the trajectories  $e_1$  and  $e_2$  start on the sliding surface, this is  $\sigma = 0$ , they converge to the origin in finite time *t<sup>s</sup>* , this time can be obtained by direct integration of [\(5\)](#page-1-9) considering  $\sigma = 0$ , given the following result

$$
t_s = \frac{p}{p - q} |e_1(t_r)|^{1 - \frac{q}{p}},\tag{20}
$$

where  $t_r$  is the reaching time to the sliding surface, in this particular case is zero.

#### A. CRITERIA FOR SELECTING THE APPROPRIATE FILTER

The following result is based on the premise that [\(12\)](#page-2-1) holds. Let us consider the following first-order low-pass filter

<span id="page-2-4"></span>
$$
G_f(s) = \frac{1}{Ts + 1},\tag{21}
$$

where  $T$  is a small positive constant. From  $(10)-(11)$  $(10)-(11)$  $(10)-(11)$ ,  $(15)$ , and [\(21\)](#page-2-4),

<span id="page-2-6"></span>
$$
\tilde{w} = (1 - G_f(s)) [\rho \sigma - u_n],
$$
  
= 
$$
\frac{T_s}{T_s + 1} [\rho \sigma - u_n],
$$
  
= 
$$
T \tilde{w} G_f(s),
$$
 (22)

let us observe that [\(12\)](#page-2-1) will hold, if the term  $\overline{T}$  *w* is sufficiently small. Using the filter [\(21\)](#page-2-4) in [\(14\)](#page-2-5) give us  $u_n$  as follows

<span id="page-2-7"></span>
$$
u_n = -\frac{\rho \sigma}{Ts},\tag{23}
$$

let us note from [\(22\)](#page-2-6)-[\(23\)](#page-2-7) that a smaller *T* implies a smaller estimation error but a larger magnitude of the transient of *u<sup>n</sup>* if  $\sigma$  is not small.

By selecting an appropriate filter  $G_f(s)$  can be reduced the magnitude of the transient. For example, if *T* is sufficiently small, it can be accounted for the following filter:

<span id="page-2-8"></span>
$$
G_f(s) = \frac{1}{T^2 s^2 + 2Tks + 1},\tag{24}
$$

where  $k > 0$  is a real and positive constant. From [\(10\)](#page-1-7)-[\(11\)](#page-2-2), [\(15\)](#page-2-3), and [\(24\)](#page-2-8),

<span id="page-2-9"></span>
$$
\tilde{w} = G_f(s) \left( T^2 \ddot{w} + 2Tk \dot{w} \right), \tag{25}
$$

while  $T^2 \ddot{w} + 2Tk \dot{w}$  is sufficiently small [\(12\)](#page-2-1) will hold. Using the filter [\(24\)](#page-2-8) in [\(14\)](#page-2-5) give us  $u_n$  as follows

<span id="page-2-10"></span>
$$
u_n = -\frac{\rho \sigma}{T^2 s^2 + 2Tks},\qquad(26)
$$

let us note from [\(25\)](#page-2-9)-[\(26\)](#page-2-10) that a smaller *T* implies a smaller estimation error but a larger magnitude of the transient of *u<sup>n</sup>* if  $\sigma$  is not small, this magnitude can be reduced by increasing the value of  $k$ . In this way,  $u_n$  can estimate high-frequency signals with a small magnitude of the transient, unlike filter [\(21\)](#page-2-4) that would give us a larger magnitude of the transient when estimating high-frequency signals.

#### **TABLE 1.** Simulation parameters.

<span id="page-3-0"></span>

## **V. NUMERICAL SIMULATIONS**

The proposed controller  $(2)$ ,  $(5)$ ,  $(7)$ ,  $(13)$ , and filter  $(24)$ is tested through simulations in a double integrator system, whose equations are given by

$$
\begin{aligned}\n\dot{x} &= y, \\
\dot{y} &= u + w,\n\end{aligned} \tag{27}
$$

The initial conditions, plant parameters and controller gains used are shown in Table [1.](#page-3-0)

The gains *p* and *q* were selected according to the guidelines given above.

A comparison is made to test the performance of the controller mentioned above against some other controllers: first-order sliding mode, non-singular terminal sliding mode, PID, finite-time sliding mode control with a disturbance compensator, and a continuous finite-time control based on the terminal sliding mode.

The conventional first-order sliding mode controller (FOSM, red color in Figures) used is

$$
u=-\beta e_2+\ddot{x}_d-\rho\text{sign}(\sigma),
$$

with  $\rho = 2$ , and  $\beta = 1$ ; the sliding surface is  $\sigma = e_1 + e_2$ , where  $e_1 = x - x_d$  and  $e_2 = y - \dot{x}_d$ . The first order sliding mode control can absorb perturbations and uncertainties, as long as *w* remains upper bounded by a positive constant *D*, and the condition  $\rho > D$  must be hold at all time for stability purposes. For more details about first-order sliding mode control, one can see [43].

The non-singular terminal sliding mode control (NTSMC, green color in Figures) is given by

$$
u = \ddot{x}_d - \frac{e_2^{2-\beta}}{\beta} - \rho \text{sign}(\sigma),
$$

where  $\sigma = e_1 + e_2^{\beta}$  $_2^{\beta}$ ,  $\beta = 1.1$ , and  $\rho = 2$ . In this controller as well as in the first order sliding mode controller,  $\rho > D$ must be hold at all time for stability purposes. The controller tuning guidelines are given in [34].

The PID algorithm (magenta color in Figures) used is

$$
u = \ddot{x}_d - k_p e_1 - k_i \int_{t_0}^{t_1} e_1 dt - k_d e_2,
$$

with the gain parameters  $k_p = 40$ ,  $k_i = 18$ ,  $k_d = 10$ , these gains were selected using the PID control toolbox of



<span id="page-3-1"></span>**FIGURE 1.** Position x<sub>1</sub> (simulation).

Simulink with the premise of keeping the closed-loop system with the fastest convergence of the position error towards zero. The PID control algorithm can only compensate for constant or low frequencies uncertainties and perturbations due to the slow rate of change of the integral gain.

Another controller used for comparison purposes, it is a finite-time sliding mode control that uses a disturbance compensator presented in [44]. In Figure [1](#page-3-1) the results using the controller appears with the name FTSMC (finite-time sliding mode control), and from Figure [1](#page-3-1) to Figure [10](#page-5-0) the results appears in cyan color. The controller used is

$$
u = \ddot{x}_d - \mu |e_2|^{\alpha} \text{sign}(e_2) - w|e_1|^{\frac{\alpha}{2-\alpha}} \text{sign}(e_1) - \frac{s}{hs+1} z,
$$

where  $z = y - \int u dt$ ,  $h = 0.01$ ,  $w = 2$ ,  $\mu = 1.5$ , and  $\alpha = 0.2$ . These values were selected according to the tuning rule given in [44].

The last controller used for comparison purposes it is a continuous finite-time control based on terminal sliding mode presented in [45]. In Figure [1](#page-3-1) the results using the controller appears with the name CFTC (continuous finite-time control), and from Figure [1](#page-3-1) to Figure [10](#page-5-0) the results appears in yellow color. The controller used is as follows

$$
u = \ddot{x}_d - \gamma^{-1} \beta^{-1} |e_2|^{2-\alpha} \text{sign}(e_2) - k_1 \sigma - k_2 |\sigma|^p \text{sign}(\sigma),
$$

where  $\sigma = e_1 + \beta |e_2|^\gamma \text{sign}(e_2)$  is the sliding surface,  $\beta = 1$ ,  $\gamma = 1.9, k_1 = 1, k_2 = 30, \text{ and } p = 0.5, \text{ these values were}$ selected according to the guidelines given in [45].

In Figure [1](#page-3-1) are shown the positions *x* and the desired trajectory  $x_d$  in dashed line. In Figure [2](#page-4-0) are shown the velocities *y*. The phase portraits are shown in Figure [3,](#page-4-1) where the asymptotic convergence using the proposed control structure (blue color) can be appreciated.

In Figure [4](#page-4-2) can be appreciated the tracking errors *e*1, where approximately after 3 seconds, the tracking error using the proposed control structure tends to zero. The velocity errors *e*<sup>2</sup> are shown in Figure [5.](#page-4-3)

In Figure [6](#page-4-4) can be seen the control efforts using each one of the different controllers, the control signal of the proposed control structure is continuous and with a low amplitude. In Figure [7](#page-4-5) are displayed the sliding variables of the controllers let us notice that PID control does not have a sliding variable in its design.





<span id="page-4-0"></span>

<span id="page-4-1"></span>FIGURE 3. Phase portrait e<sub>1</sub> vse<sub>2</sub> (simulation).



<span id="page-4-2"></span>

<span id="page-4-3"></span>**FIGURE 5.** Velocity error  $e_2$  (simulation).

The estimated perturbation  $\hat{w}$  is compared with the actual perturbation *w* in Figure [8,](#page-4-6) only the proposed control structure and the proposed in [44] consider uncertainty and disturbance



<span id="page-4-4"></span>**FIGURE 6.** Control signal u (simulation).



<span id="page-4-5"></span>**FIGURE 7.** Sliding variable σ (simulation).



<span id="page-4-6"></span>**FIGURE 8.** Estimated perturbation  $\hat{w}$  (simulation).

compensation. In Figure [9](#page-5-1) is shown the energy signal when applied different controllers to the closed-loop system.

In order to evaluate the closed-loop performance of each controller let us use the  $l_2$  index, this performance index was proposed by [46]–[48]

$$
l_2(e_1) = \sqrt{\frac{1}{T_f} \sum_{t_0}^{t_0 + T_f} |e_1|^2},
$$

which is an average error in finite-time given in the sense of *L*<sub>2</sub> norm, where  $e_1 = x - x_d$  is the position error,  $T_f$  is the time interval, and the initial time is  $t_0$ . Figure [10](#page-5-0) shows the comparative results using the *l*<sup>2</sup> index, where the proposed



**FIGURE 9.** Energy signal  $\int |u|^2 dt$  (simulation).

<span id="page-5-1"></span>

<span id="page-5-0"></span>**FIGURE 10.** Performance index  $I_2$  (e<sub>1</sub>) (simulation).

control structure renders good results compared with other controllers.

## **VI. EXPERIMENTS**

In this Section, for controller design purposes, it is assumed that the plant parameters are unknown, and it is avoided the parameter identification, in this way the plant model used is

<span id="page-5-3"></span>
$$
\begin{aligned}\n\dot{x} &= y, \\
\dot{y} &= u + w,\n\end{aligned} \tag{28}
$$

where *x* and *y* are the mass position and velocity, respectively, and *w* includes the unknown parameters, friction forces, and neglected dynamics of the process. Moreover,  $f(x, y) = 0$ and  $g(x, y) = 1$  according to [\(1\)](#page-1-0). The given model can be considered as a simplification of the model presented in [49].

The system comprises a double-action cylinder model LA-200, a proportional valve 5/3 model MPYE-5-1/8-HF-010-B, and a resistive position sensor MLO-POT-500-TLF, all of these devices are made by Festo Pneumatic Inc, see Figure [11.](#page-5-2)

The proposed controller  $(2)$ ,  $(5)$ ,  $(7)$ ,  $(13)$ , and filter  $(24)$ is implemented in a real-time data acquisition board dSPACE using a sample time of  $T_s = 0.0001$  seconds and the Euler fixed-step solver. The only output of the system is given by the cylinder's position, in order to have access to velocity is used a differentiator based on the model proposed in [25], [50].



**FIGURE 11.** Mechanical system with a pneumatic actuator (experiment).

<span id="page-5-2"></span>

<span id="page-5-4"></span>**FIGURE 12.** Position  $x$ , desired reference  $x_d$  (experiment). The uncertainty and disturbance estimator is turned off after 30 seconds of the experiment have elapsed.



<span id="page-5-5"></span>**FIGURE 13.** Position error  $e_1$  (experiment). The uncertainty and disturbance estimator is turned off after 30 seconds of the experiment have elapsed.

The model of the plant is considered as in [\(28\)](#page-5-3); therefore, no compensation terms were used. The controller gains used are shown in Table [2.](#page-6-0)

Firstly, an experimental comparison is made to appreciate the performance of the uncertainty and disturbance estimator [\(13\)](#page-2-0); the results are shown in Figures [12](#page-5-4) and [13,](#page-5-5) where the estimator is turned off after 30 seconds of the experiment have elapsed. When the estimator is not used, the performance of



<span id="page-6-1"></span>**FIGURE 14.** Position  $x$ , desired reference  $x_d$  (experiment). The transient response can be seen during the first 10 seconds, after this the nominal stage takes place.



FIGURE 15. Position error  $\mathbf{e}_1$  (experiment). The transient response can be seen during the first 10 seconds, after this the nominal stage takes place.

<span id="page-6-2"></span>

**FIGURE 16.** Control signal u (experiment).

**TABLE 2.** Experimental parameters.

<span id="page-6-3"></span><span id="page-6-0"></span>

Notation	Value
$x_d$	$0.08\sin(0.2t)m$
р	19
	$150 \frac{1}{s^2}$
k.	0.5
	$\overline{s}$

the closed-loop system decreases to a position error  $e_1$  of −0.0065 meters approximately.

Now, an experiment is made using the proposed control structure without deactivating the estimator, in Figure [14](#page-6-1) and [15](#page-6-2) are showed the reference tracking results, where it can be seen that the nominal stage takes place after 10 seconds



<span id="page-6-4"></span>FIGURE 17. Estimated disturbance  $\hat{w}$  (experiment).

of been elapsed the experiment approximately. In Figure [16](#page-6-3) is shown the control signal and in Figure [17](#page-6-4) the estimated disturbance.

## **VII. CONCLUSION**

The proposed control structure is based on nonsingular terminal sliding mode control, the main advantage of the proposed control structure is, that the control signal is continuous while the closed-loop system is robust against uncertainties and perturbations. A continuous gain replaces the discontinuous gain of the controller; to not lose robustness, it is designed and implemented an uncertainty and disturbance estimator. Theoretically, the closed-loop system converges to the tracking reference asymptotically, although in the case where the trajectories start on the sliding surface, the closed-loop system converges to zero in a finite time. In a closed-loop simulation comparison, the proposed control approach renders excellent performance according to the  $l_2$  index used. Also, the proposed controller renders reasonable rate of convergence to zero tracking error compared with some robust controllers such: nonsingular terminal sliding mode, first-order sliding mode control, PID control, a continuous finite-time control based on terminal sliding mode, and a finite-time sliding mode control that uses a disturbance compensator.

The experiments carried out in a mechanical system using a pneumatic actuator showed excellent performance and robustness against uncertainties. The present control structure is a robust alternative that can be applied in systems where discontinuous controllers can damage or heat the actuators, thus reducing their lifetime.

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