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Recursive Estimation for Sensor Systems With One-Step Randomly Delayed and Censored Measurements

ZONGYUAN WANG [1,2] AND WEIDONG ZHOU [2]

[1]School of Mathematics Science, Harbin Engineering University, Harbin 150001, China
[2]College of Automation, Harbin Engineering University, Harbin 150001, China

Corresponding author: Weidong Zhou (zhouweidong@hrbeu.edu.cn)

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ABSTRACT Censored measurements frequently occur in network systems involving censored sensors or saturated sensors. In addition, unreliable network characteristics can produce random measurement delays during signal transmission. In this paper, we investigate the state estimation problem for network systems with the simultaneous appearance of the aforementioned two measurement uncertainties. The occurrences of two random measurement phenomena are described by two Bernoulli random variables in which the censored variable is dependent on the delay variable. The probability of the process signal being uncensored is calculated by the local approximations using a priori and a posteriori of the state estimation. Then, a novel measurement model that incorporates both the censoring random matrix and the signal delay is established. Based on this model, an optimal recursive estimation method is proposed for systems with specified two uncertainties by making use of an innovation analysis approach and a weighted conditional expectation formula. The superior performance of our proposed method is verified through a typical oscillator simulation example.

INDEX TERMS Censored measurement, innovation analysis, random parameter matrices, randomly delayed measurements.

I. INTRODUCTION

In the last few decades, the problem of state estimation for discrete-time linear systems has been extensively studied by researchers owing to its important applications in various fields such as target tracking, navigation and parameter estimation [1]–[10]. The sources of network system uncertainties, such as imperfect transmission channels, network congestion, and sensor saturated mechanisms, will result in the signals being subject to problems of random delays [11]–[13], saturation [14]–[16], and/or packet dropouts [17], [18].

Tobit measurement censoring is a common uncertainty that occurs in many engineering applications such as i) biochemical measurements with limit-of-detection saturation, ii)inexpensive sensors with saturation censoring, and iii) line-of-sight tracking with occlusion. Tobit censoring is also referred to as clipped measurement or limit-of-detection discontinuity, and it arises from limitations in the dynamic range of sensors. Specifically, if the signal to be measured exceeds the sensor inherent threshold, those signals could be censored and cannot be measured correctly [15], [16]. Censoring has the form of a piecewise linear transform, with a zero slope in the censored region, causing significant challenges to the general nonlinear estimators, such as the unscented Kalman filter (UKF) and extended Kalman filter (EKF) [19]. An attempt was previously made in [14] to manage the state estimation of censored measurements. Although the measurement noise is non-Gaussian near the censoring region, a novel Tobit Kalman filter (TKF) was developed based on the formulation of the Kalman filter [20], which was a computationally efficient, unbiased recursive estimator for this special dynamic system with censored measurements. In the literature [19], it was shown that the TKF has more accurate state estimates and state error covariance with censored measurement data, while both EKF and UKF provide unreliable estimates in censored data conditions.

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Later studies have been made to the TKF so as to accommodate the effects of fading measurement, non-Gaussian Lévy measurement noise, multiplicative measurement noise, multiplicative measurement noise under redundant transmission channels protocol, and additive measurement random matrices [21]–[24]. In the area of multisensor information fusion, by setting delayed data packet obeying a Poisson distribution, a distributed federated TKF algorithm over a packet-delaying network was developed in [25], while in [26], a modified TKF with two-side censoring was proposed and applied in event-based multisensor fusion with a deadzone-like measurement.

Additionally, in either signal processing or control theory, discrete-time systems with random transmission delays arise in many real-world communication networks, such as hydraulic processes, temperature processes, chemical systems and large-scale industrial network systems [11], [27]. Noticing the significance of measurement delays to state estimation, existing works [11]–[13] have studied EKF with one-step random measurement delay, and UKF with one-step or two-step random measurement delays for networks with nonlinearity via the least square filtering method. A generalized Gaussian-type filter with a one-step random measurement delay was presented in [28] under a unified framework, which includes the implementation of a cubature Kalman filter (CKF) with a one-step random delay. Based on the Monte Carlo method, a novel particle filtering method for nonlinear systems with random one-step delayed measurements was proposed in [29], with the unknown delay probability being estimated via the maximum likelihood criterion. Then, a particle filter algorithm for network systems with random one- or two-step delayed measurements was investigated in [30]. Considering multiple-step measurement delays, a particle filter was designed in [31] by applying a similar idea as in [29]. The above discussions demonstrate that state estimation, for systems with delayed measurements, is an aspect of deep concern.

Recently, discrete-time systems with mixed uncertainties (e.g., missing measurements, random measurement delays, and packet dropouts) have been increasingly considered. The filtering problem for such systems has been thoroughly studied via different theoretical frameworks. For example, in [27],based on the mean and covariance functions of the signal and noises, recursive least-squares linear estimation algorithms were derived for one-step measurement delay and packet dropout by an innovation approach; in [17], with a state-space method, a unified parameterized augmented model was defined to describe triple uncertainty, and three sequences of Bernoulli variables were used to model the entire uncertain system, based on which the optimal linear filter, predictor and smoother were obtained via an innovation approach; in [32], a particle filter for nonlinear networked systems with random one-step delay and missing measurements was proposed by utilizing two Bernoulli random variables. Under the framework of a Gaussian filter, a Gaussian weighted integral method was developed in [33] for nonlinear systems with a one-step measurement delay and colored noise; in [34], for linear systems influenced by multiplicative and time-correlated additive measurement noise, an optimal linear estimator was proposed without computing the inverse of the state transition matrix.

If the network system contains the censored sensor, the simultaneous consideration of transmission delay and the effect of the censored sensor are significant and unavoidable in practical engineering. Unfortunately, although some approaches have been proposed for systems with censored sensors [20]–[26], a recursive filtering algorithm for censored sensors with the receiving signal subject to one-step measurement delay has not been investigated to date. The reason may be because the calculation process is beyond the existing framework of TKF, and this process cannot be solved by the simple combination of the TKF and the measurement delay model. Also, if the TKF is applied to networks where transmission channels to the censored sensor can induce random delay, the performance of the TKF would inevitably degrade.

Motivated by the above discussions, in this paper, we aim to improve the TKF by developing a novel recursive filtering algorithm, which can deal with estimation problems in the network systems with a one-step delayed signal and censored measurement. The delayed signal and censored measurement are both modeled as sequences of Bernoulli random variables. Furthermore, a new measurement model is set to simultaneously incorporate the above two random phenomena.

The main contributions of this paper are highlighted as follows. (i) To the best of the authors' knowledge, this paper represents the first of the few attempts to deal with the recursive filtering problem for linear systems with a one-step delayed signal and censored measurement in a unified framework by an innovation analysis and the technique of random matrix expectation. (ii) Compared with the TKFs in [20]–[23], the calculation of probability being uncensored includes not only the calculation of the current signal with a one-step state prediction but also the calculation of the delayed signal with the state posteriori. (iii) The proposed filter is of a recursive nature and is thus suitable for online applications.

The rest of the paper is organized as follows. The problem to be investigated is formulated in Section II. The proposed recursion algorithm is derived in Section III. Example of an oscillator is provided in Section IV to demonstrate the applicability and superiority of the proposed state estimation algorithm. Concluding remarks are drawn in Section V.

Notation: The notations used throughout the paper are standard except where otherwise stated. For any matrix $\boldsymbol{A}$ the symbols $\boldsymbol{A}^T$ and $\boldsymbol{A}^{-1}$ represent its transpose and inverse, respectively; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space and $\mathbb{R}^{m+n}$ is the set of $n \times m$ real matrices. $\boldsymbol{I}_n$ and $\boldsymbol{0}$ represent the $n \times n$ identity matrix and zero matrix of appropriate dimensions, respectively. $\boldsymbol{A}^{i,j}$ and $\boldsymbol{A}^i$ represent the (i, j)th subblock and ith row, respectively, of the matrix $\boldsymbol{A}$. diag $([\boldsymbol{x}^m])$ represents a diagonal matrix whose diagonal entry is $\boldsymbol{x}^m$, and col $\{\boldsymbol{x}_1, \cdots, \boldsymbol{x}_m\}$ represents the column vector.

Superscripts "∧" and " ∼" over certain variables represent the estimate and the estimation error, respectively. Moreover, for arbitrary random vectors $\boldsymbol{X}$ and $\boldsymbol{Y}$, we denote $\mathrm{Cov}(\boldsymbol{X},\boldsymbol{Y}) = \mathrm{E}\left[\boldsymbol{X}\boldsymbol{Y}^T\right] - \mathrm{E}[\boldsymbol{X}]\,\mathrm{E}[\boldsymbol{Y}]^T$. and $\mathrm{Var}(\boldsymbol{X}) = \mathrm{Cov}(\boldsymbol{X},\boldsymbol{X})$, where $\mathrm{E}[\cdot]$ represents the mathematical expectation operator. The symbol $\boldsymbol{\delta}_k$ represents the Kronecker delta function, which is equal to one at time k, and zero, otherwise. Let $\lambda(\alpha) = \phi(\alpha)\big/[1-\Phi(\alpha)]$, where $\phi(\cdot)$ and $\Phi(\cdot)$ are the probability density function (pdf) and the cumulative distribution function (cdf), respectively, of a Gaussian random variable.

II. PROBLEM FORMULATION

Consider the following linear discrete time-varying system:

$$\boldsymbol{x}_k = \boldsymbol{A}_{k-1}\boldsymbol{x}_{k-1} + \boldsymbol{w}_{k-1} \tag{1}$$

$$\bar{z}_k = \boldsymbol{C}_k\boldsymbol{x}_k + \boldsymbol{v}_k \tag{2}$$

where $\boldsymbol{x}_k \in \mathbb{R}^{n_x}$ and $\bar{z}_k \in \mathbb{R}^{n_y}$ denote the state vector and the ideal measurement vector, respectively. $\boldsymbol{A}_k \in \mathbb{R}^{n_x\times n_x}$ and $\boldsymbol{C}_k \in \mathbb{R}^{n_y\times n_x}$ are the known time-varying system matrix with appropriate dimensions. $\boldsymbol{w}_k \in \mathbb{R}^{n_x}$ and $\boldsymbol{v}_k \in \mathbb{R}^{n_y}$ are the zero-mean Gaussian noise with covariance $\boldsymbol{Q}_k$ and $\boldsymbol{R}_k$, respectively.

The latent signal transmission model to the censored sensor is given by

$$\bar{\boldsymbol{y}}_k^* = \begin{cases} (1-\gamma_k)\,\bar{z}_k + \gamma_k\bar{z}_{k-1} & k \geq 2 \\ \bar{z}_1 & k = 1, \end{cases} \tag{3}$$

where the random variable sequence $\{\gamma_k, k \geq 1\}$ is mutually independent and denotes the random one-step delay, satisfying the Bernoulli distribution with the statistical property $\mathrm{E}[\gamma_k] = p_k$, and the signal $\bar{z}_1$ is always received by the censored sensor.

The structure of a network system is outlined as follows: the state $\boldsymbol{x}_k$ is evolving through a state equation, the outputs signal $\bar{z}_k$ generated from $\boldsymbol{x}_k$ are sent to the censored sensor via an unreliable network mechanism, and during this period the one-step delay phenomenon may occur; that is, $\bar{z}_{k-1}$ may be received by the censored sensor at time k, and the output $\bar{\boldsymbol{y}}_k$ transformed by the censored sensor with input $\bar{z}_k$ or $\bar{z}_{k-1}$ is transmitted to the fusion center for state estimation (see Fig. 1).

As illustrated by Fig. 1, the transmission signal with delay is censored by the censored sensors, so the actual measurement output for state estimation has the following expression:

$$\bar{\boldsymbol{y}}_k = \begin{cases} \bar{\boldsymbol{y}}_k^* & \bar{\boldsymbol{y}}_k^* > \boldsymbol{\tau} \\ \boldsymbol{\tau} & \bar{\boldsymbol{y}}_k^* > \boldsymbol{\tau}, \end{cases} \tag{4}$$

where $\bar{\boldsymbol{y}}_k$ is the actual measurement output, and $\boldsymbol{\tau} = \left[\boldsymbol{\tau}^1 \cdots \boldsymbol{\tau}^m \cdots \boldsymbol{\tau}^{n_y}\right]^T \in \mathbb{R}^{n_y\times 1}$ denotes the censoring threshold vector.

To have a better predictive description of whether the latent measurement $\bar{\boldsymbol{y}}_k^{*m}\ (m = 1,\cdots,n_y)$ will be censored, we introduce an additional Bernoulli random variable

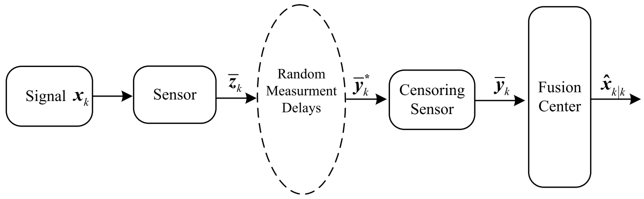

FIGURE 1. Flow chart of the sensor network system.

$\eta_k{}^m\ (m = 1,\cdots,n_y)$ at time $k-1$ as follows:

$$\eta_k{}^m = \begin{cases} 1 & (1-\gamma_k)\,\bar{z}_k^m + \gamma_k\bar{z}_{k-1}^m > \boldsymbol{\tau}^m \\ 0 & (1-\gamma_k)\,\bar{z}_k^m + \gamma_k\bar{z}_{k-1}^m \leq \boldsymbol{\tau}^m, \end{cases}$$
$$\eta_1{}^m = \begin{cases} 1 & \bar{z}_1^m > \boldsymbol{\tau}^m \\ 0 & \bar{z}_1^m \leq \boldsymbol{\tau}^m\ k = 1, \end{cases} \tag{5}$$

where $\eta_k{}^m = 1$ denotes that the latent measurement $\bar{\boldsymbol{y}}_k^{*m}$ will not be censored and the system output for state estimation is $\bar{\boldsymbol{y}}_k^{*m}$, and $\eta_k{}^m = 0$ denotes that threshold $\boldsymbol{\tau}^m$ will be output for the state estimation at time k. So the influence of measurement output on the state $\boldsymbol{x}_k$ will be represented by $(1-\gamma_k)\,\bar{z}_k^m + \gamma_k\bar{z}_{k-1}^m$ with probability $\mathrm{E}[\eta_k{}^m]$. From (5), variables $\eta_k{}^m(m = 1,\cdots,n_y)$ are dependent on the delayed Bernoulli variables γ_k, and the probability of the measurement $\bar{\boldsymbol{y}}_k^{*m}$ being uncensored is identical to that of the occurrence of the event $\{\eta_k{}^m = 1\}$, which is denoted as $c_k{}^m$ with the probability distribution as follows:

$$c_k{}^m = p\left\{\eta_k{}^m = 1\right\} = p\left\{\bar{\boldsymbol{y}}_k^{*m} > \boldsymbol{\tau}^m\right\}, \tag{6}$$

At any given time step k, the probability $c_k{}^m$ is unknown and needs be computed. The probability value will be presented in section III.

Let $\boldsymbol{\Pi}_k = \mathrm{diag}\left\{\eta_k{}^1, \eta_k{}^2, \cdots, \eta_k{}^{n_y}\right\}$ denote the diagonal Bernoulli random matrix, which indicates the censored randomness of latent measurement $\bar{\boldsymbol{y}}_k^{*m}$. With $\boldsymbol{\Pi}_k$ we modify (4) to the following censoring measurement model:

$$\boldsymbol{y}_k = (1-\gamma_k)\,\boldsymbol{\Pi}_k z_k + \gamma_k\boldsymbol{\Pi}_k z_{k-1}. \tag{7}$$

Remark 1: Model (7) represents the rewritten form of (4), which includes the censoring matrix $\boldsymbol{\Pi}_k$ and the translation transformation of the censoring measurement equation $\bar{\boldsymbol{y}}_k = \boldsymbol{\Pi}_k\bar{\boldsymbol{y}}_k^* + \left(\boldsymbol{I}_{n_y} - \boldsymbol{\Pi}_k\right)\boldsymbol{\tau}$ with $z_k = \bar{z}_k - \boldsymbol{\tau}$, $z_{k-1} = \bar{z}_{k-1} - \boldsymbol{\tau}$ and $\boldsymbol{y}_k = \bar{\boldsymbol{y}}_k - \boldsymbol{\tau}$. In addition, model(7) has a similar form as the delay measurement model formulated in [12], [13] and [29], except that there exists the censoring Bernoulli matrix. This transformation helps to make the measurement equation more compact and concise.

Owing to the transformation in (7), the above (3), (4), and (5) are correspondingly rewritten as:

$$\boldsymbol{y}_k^* = \begin{cases} (1-\gamma_k)\,z_k + \gamma_k z_{k-1} & k \geq 2 \\ z_1 & k = 1 \end{cases}$$
$$\boldsymbol{y}_k = \begin{cases} \boldsymbol{y}_k^* & \boldsymbol{y}_k^* > \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{y}_k^* \leq \boldsymbol{0}; \end{cases}$$

$$\eta_k{}^m = \begin{cases} 1 & (1-\gamma_k) z_k^m + \gamma_k z_{k-1}^m > 0 \\ 0 & (1-\gamma_k) z_k^m + \gamma_k z_{k-1}^m \le 0; \end{cases}$$
$$\eta_1{}^m = \begin{cases} 1 & z_1^m > 0 \\ 0 & z_1^m \le 0,\ k=1. \end{cases} \tag{8}$$

The probability $c_k{}^m$ can also be expressed as:

$$c_k{}^m = p\left\{\eta_k{}^m = 1\right\} = p\left\{y_k^m > 0\right\}. \tag{9}$$

To facilitate subsequent developments, we introduce the following definitions:

$$\chi_k = C_k x_k - \tau,\ Y_{k-1} = \{y_i\}_{i=1}^{k-1},\ \hat{\chi}_{k|k-1} = \mathrm{E}\left[\chi_k \left| Y_{k-1}\right.\right],$$
$$\hat{x}_{k|k-1} = \mathrm{E}\left[x_k \left| Y_{k-1}\right.\right], P_{k|k-1} = \mathrm{E}\left[\tilde{x}_k \tilde{x}_k^T \left| Y_{k-1}\right.\right],$$
$$P_{k,k-1|k-1} = \mathrm{E}\left[\tilde{x}_k \tilde{x}_{k-1}^T \left| Y_{k-1}\right.\right], \hat{\chi}_{k-1|k-1} = \mathrm{E}\left[\chi_{k-1} \left| Y_{k-1}\right.\right],$$
$$\hat{x}_{k-1|k-1} = \mathrm{E}\left[x_{k-1} \left| Y_{k-1}\right.\right], \hat{x}_{k-1|k-1} = \mathrm{E}\left[v_{k-1} \left| Y_{k-1}\right.\right],$$
$$\hat{v}_{k|k-1} = \mathrm{E}\left[v_k \left| Y_{k-1}\right.\right], P_{k-1|k-1}^{vv} = \mathrm{E}\left[\tilde{v}_{k-1} \tilde{v}_{k-1}^T \left| Y_{k-1}\right.\right],$$
$$\hat{z}_{k-1|k-1} = \mathrm{E}\left[z_{k-1} \left| Y_{k-1}\right.\right], L_{k|k-1}^m = \left(-\hat{\chi}_{k|k-1}^m\right) \Big/ \sqrt{R_k^{m,m}},$$
$$L_{k-1|k-1}^m = \left(-\hat{\chi}_{k-1|k-1}^m\right) \Big/ \sqrt{R_{k-1}^{m,m}},$$
$$\lambda\left(L_{k|k-1}\right) = \mathrm{col}\left\{\lambda\left(L_{k|k-1}^1\right), \cdots, \lambda\left(L_{k|k-1}^{n_y}\right)\right\}. \tag{10}$$

Next, for the addressed problem, the following two assumptions are made.

Assumption 1: The initial state x_0 and the processes $\{w_k, k \ge 1\}$, $\{v_k, k \ge 1\}$ and $\{\gamma_k, k \ge 1\}$ are mutually independent.

Assumption 2: For small state estimation errors, $\hat{x}_{k|k-1}$ and the estimate $\hat{x}_{k-1|k-1}$ provide a reasonably accurate approximation of x_k and x_{k-1}.

Assumption 2 is made to account for the probability of uncensored measurement, because this probability at time k is a function of distance of latent measurement and the threshold. If the latent measurement is one-step delayed signal z_{k-1}, the posterior estimate $\hat{x}_{k-1|k-1}$ will be used to approximate the probability of signal z_{k-1} being uncensored, which is similar to use the predictive estimate $\hat{x}_{k|k-1}$ for the probability of signal z_k being uncensored at time k.

III. MAIN RESULTS

In this section, we aim to establish a unified framework to solve the addressed recursive filtering problem in the simultaneous presence of delayed transmission signal and censored measurements. Although the TKF was obtained under a similar framework as the Kalman filter (KF), it is insufficient to the actual problem we are addressing. We will establish filter theory via an innovation analysis approach, which has two main differences from the idea proposed in [20]:i) a modified uncensored probability encompassing the probability of occurrence of the delayed signal z_{k-1}, and ii) additional computations of uncensored statistics involved with the delayed signal z_{k-1}.

A. UNCENSORED PROBABILITY AND PROPERTY

First, for the given systems (1)-(4), state prediction is presented, which has the same form as the prediction of the KF due to the Gaussian process noise, and the expression is:

$$\hat{x}_{k|k-1} = A_k \hat{x}_{k-1|k-1}, \quad P_{k|k-1} = A_k P_{k-1|k-1} A_k^T + Q_k. \tag{11}$$

With state prediction, the probability of latent signal z_1 being uncensored can be calculated by the initial state value. As the received latent measurement by the censoring sensor may be delayed signal z_{k-1} at time k; the calculation of probability of measurement being uncensored should contain this possibility. Thus, using the conditional probability and law of total expectation, the probability $c_k{}^m$ can be obtained by the following lemma.

Lemma: The probability $c_k{}^m$ of the measurement y_k^{*m} being uncensored is approximated as:

$$c_k{}^m \approx p_k\left((1-p_{k-1})\delta_k + p_{k-1}\left(1 - \Phi\left(L_{k-1|k-1}^m\right)\right)\right) + (1-p_k)\left(1-\Phi\left(L_{k|k-1}^m\right)\right), \quad k > 1;$$
$$c_1{}^m = 1 - \Phi\left(L_{1|0}^m\right) \tag{12}$$

where δ_k is used to indicate whether the event $\{\hat{z}_{k-1|k-1} > 0 \left| y_{k-1} = z_{k-1}\right.\}$ is necessary or impossible, and $c_1{}^m$ denotes the probability of signal z_1^m being uncensored as $\hat{x}_{1|0}$ has been obtained from (11). The detailed lemma proof is provided in Appendix A.

In virtue of notation definitions in (9) and the probability in (12), the following properties are easily inferred:

Properties:

$$1.\mathrm{E}\left[(1-\gamma_k)\Pi_k \left| Y_{k-1}\right.\right] = (1-p_k)\psi_{k,1};$$
$$2.\mathrm{E}\left[\gamma_k \Pi_k \left| Y_{k-1}\right.\right] = p_k \psi_{k,2}, \quad k > 1;$$
$$3.\mathrm{E}\left[v_k \left| \Pi_k = I_{n_y}, \gamma_k = 0, Y_{k-1}\right.\right] = \hat{v}_{k|k-1}^c = \sqrt{R_k}\lambda\left(L_{k|k-1}\right);$$
$$4.\mathrm{E}\left[v_{k-1} \left| \Pi_k = I_{n_y}, \gamma_k = 1, Y_{k-1}\right.\right] = \hat{v}_{k-1|k-1}^c = (1-p_{k-1})\hat{v}_{k-1|k-1}\delta_k + p_{k-1}\sqrt{R_{k-1}}\lambda\left(L_{k-1|k-1}\right);$$
$$5.\mathrm{E}\left[v_k v_k^T \left| \Pi_k = I_{n_y}, \gamma_k = 0, Y_{k-1}\right.\right] = \Gamma_{k|k-1};$$
$$6.\mathrm{E}\left[v_{k-1} v_{k-1}^T \left| \Pi_k = I_{n_y}, \gamma_k = 1, Y_{k-1}\right.\right] = \Gamma_{k-1|k-1}^c = (1-p_{k-1})\delta_k\left(P_{k-1|k-1}^{vv} + \hat{v}_{k-1|k-1}\hat{v}_{k-1|k-1}^T\right) + p_{k-1}\Gamma_{k-1|k-1}; \tag{13}$$

where $\psi_{k,1} = \mathrm{diag}\left(\left[c_{k,1}^m\right]\right)$, $\psi_{k,2} = p_k \mathrm{diag}\left(\left[c_{k,2}^m\right]\right)$ with $c_{k,1}^m = 1 - \Phi\left(L_{k|k-1}^m\right)$, $c_{k,2}^m = (1-p_{k-1})\delta_k + p_{k-1}$

$\left(1-\Phi\left(L^{m}_{k-1|k-1}\right)\right)$, $k>1$; the definitions of matrices $\Gamma_{k|k-1}$ and $\Gamma_{k-1|k-1}$ together with the detailed proof of the properties are shown in Appendix B.

Remark 2: The condition Y_{k-1} is to indicate that state estimate $\hat{x}_{k-1|k-1}$ and its variance $P_{k-1|k-1}$ have been got. The equality in property 3 holds from the fact that when the noise v_k is conditional on $\Pi_k = I_{n_y}$, $\gamma_k = 0$ and Y_{k-1}, it follows the truncated normal distribution. Additionally, the matrix $\psi_{1,1}$ can give the uncensored probability of the first latent measurement y_1^* only from the initial state and the system transition model. Once y_1 is available, $\psi_{k,2}$ can be computed together with $\psi_{k,1}$, where $\psi_{k,2}$ represents the probability of signal z_{k-1} being uncensored. Compared with the research in [20], the newly emerging items $\hat{v}^c_{k-1|k-1}$ and $\Gamma^c_{k-1|k-1}$ describe the uncensored statistics from the delayed signal z_{k-1}.

Based on the above-established properties 1, 2, 3 and 5, and given measurements Y_{k-1}, the conditional expectations of the measurement model (7) can be computed as follows:

$$\begin{aligned} \mathrm{E}\left[(1-\gamma_k)\,\Pi_k z_k\,|Y_{k-1}\right] &= (1-p_k)\,\psi_{k,1}\hat{z}^c_{k|k-1}, \\ \mathrm{E}\left[\gamma_k \Pi_k z_{k-1}\,|Y_{k-1}\right] &= p_k\psi_{k,2}\hat{z}^c_{k-1|k-1}, \end{aligned} \tag{14}$$

where

$$\begin{aligned} \hat{z}^c_{k|k-1} &= \mathrm{E}\left[z_k\,\middle|\Pi_k = I_{n_y}, \gamma_k = 0, Y_{k-1}\right] \\ &= \hat{\chi}_{k|k-1} + \hat{v}^c_{k|k-1}, \\ \hat{z}^c_{k-1|k-1} &= \mathrm{E}\left[z_{k-1}\,\middle|\Pi_k = I_{n_y}, \gamma_k = 1, Y_{k-1}\right] \\ &= \hat{\chi}_{k-1|k-1} + \hat{v}^c_{k-1|k-1}. \end{aligned} \tag{15}$$

B. STATE ESTIMATION

We proceed to obtain the recursive estimator of the state $\hat{x}_{k|k} = \mathrm{E}\left[x_k\,|Y_k\right]$, $P_{k|k} = \mathrm{Var}\left[x_k\,|Y_k\right]$ by the innovation analysis approach, the principle of which is explained as follows. If $L\left(y_1,\cdots,y_k\right)$ denotes the linear space spanned by the observations $\left\{y_1,\cdots,y_k\right\}$, the linear estimator of the signal x_k based on the observations $\left\{y_1,\cdots,y_k\right\}$ is the orthogonal projection of the vector x_k onto $L\left(y_1,\cdots,y_k\right)$. Hence, the orthogonal projection lemma (OPL) states that the estimator $\hat{x}_{k|k}$ is the only element of the space $L\left(y_1,\cdots,y_k\right)$ satisfying that estimation error, $x_k-\hat{x}_{k|k}$, is orthogonal to $L\left(y_1,\cdots,y_k\right)$. The innovation approach transforms the measurement process $\left\{y_k; k\geq 1\right\}$ into an equivalent process (innovation process) of orthogonal vectors $\left\{\xi_k; k\geq 1\right\}$, which are defined by $\xi_k = y_k - \hat{y}_{k|k-1}$. It is known that $\left\{\xi_k; k\geq 1\right\}$ is a zero mean white process, and each set $\left\{\xi_1,\cdots\xi_k\right\}$ spans the same linear subspace as $\left\{y_1,\cdots,y_k\right\}$ (which are generally non-orthogonal vectors). The orthogonality of the new process enables us to derive the estimators $\hat{x}_{k|k}$ with considerable simplification by calculating the linear combination of the innovations.

To compute the filter estimate of the state, we must first consider the noise filter estimate $\hat{v}_{k|k}$ and its covariance $P^{vv}_{k|k}$ for possibly delayed latent measurement.

Theorem 1: For the system composed of (1), (2) and (7), the filter estimate $\hat{v}_{k|k}$ and its covariance $P^{vv}_{k|k}$ given Y_k at time $k\geq 1$ have the following expressions:

$$\hat{v}_{k|k} = P^{v\xi}_{k|k-1}\left(\Xi_{k|k-1}\right)^{-1}\xi_{k|k-1}, \quad k>1, \tag{16}$$

$$P^{vv}_{k|k} = R_k - P^{v\xi}_{k|k-1}\left(\Xi_{k|k-1}\right)^{-1}\left(P^{v\xi}_{k|k-1}\right)^T, \quad k>1, \tag{17}$$

where $P^{v\xi}_{k|k-1}$ is the cross-covariance matrix between v_k and the innovation ξ_k given Y_{k-1} with the form:

$$P^{v\xi}_{k|k-1} = (1-p_k)\,\psi_{k,1}\Gamma_{k|k-1}, k>1; \tag{18}$$

innovation $\xi_{k|k-1}$ is computed as

$$\xi_{k|k-1} = y_k - \hat{y}_{k|k-1}, \tag{19}$$

with $\hat{y}_{k|k-1}$ as the one-step linear predictor of y_k being

$$\begin{aligned} \hat{y}_{k|k-1} &= (1-p_k)\,\psi_{k,1}\hat{z}^c_{k|k-1} + p_k\psi_{k,2}\hat{z}^c_{k-1|k-1}, \quad k>1, \\ \hat{y}_{1|0} &= \psi_{1,1}\hat{z}^c_{1|0}; \end{aligned} \tag{20}$$

and $\Xi_{k|k-1}$ is denoted as the covariance matrix of $\xi_{k|k-1}$ given Y_{k-1}, whose expressions are:

$$\begin{aligned} &\Xi_{k|k-1} \\ &= (1-p_k)\left(\mathrm{E}\left[\chi_k\chi_k^T\,|Y_{k-1}\right] + \hat{\chi}_{k|k-1}\left(\hat{v}^c_{k|k-1}\right)^T\right) \\ &\quad + (1-p_k)\left(\hat{v}^c_{k|k-1}\,\hat{\chi}^T_{k|k-1} + \Gamma_{k|k-1}\right) \\ &\quad + p_k\left(\mathrm{E}\left[\chi_{k-1}\chi_{k-1}^T\,|Y_{k-1}\right] + \hat{\chi}_{k-1|k-1}\left(\hat{v}^c_{k-1|k-1}\right)^T\right) \\ &\quad + p_k\left(\hat{v}^c_{k-1|k-1}\,\hat{\chi}^T_{k-1|k-1} + \Gamma^c_{k-1|k-1}\right) \\ &\quad - \left((1-p_k)\,\hat{z}^c_{k|k-1} + p_k\hat{z}^c_{k-1|k-1}\right) \\ &\quad \times\left((1-p_k)\,\hat{z}^c_{k|k-1} + p_k\hat{z}^c_{k-1|k-1}\right)^T, \quad k>1; \\ &\Xi_{1|0} \\ &= C_1P_{1|0}C_1^T + \Gamma_{1|0} - \hat{v}^c_{1|0}\left(\hat{v}^c_{1|0}\right)^T, \quad k=1; \end{aligned} \tag{21}$$

with

$$\begin{aligned} \mathrm{E}\left[\chi_k\chi_k^T\,|Y_{k-1}\right] &= C_kP_{k|k-1}C_k^T + \hat{\chi}_{k|k-1}\,\hat{\chi}^T_{k|k-1}, \\ \mathrm{E}\left[\chi_{k-1}\chi_{k-1}^T\,|Y_{k-1}\right] &= C_{k-1}P_{k-1|k-1}C_{k-1}^T \\ &\quad + \hat{\chi}_{k-1|k-1}\,\hat{\chi}^T_{k-1|k-1}. \end{aligned}$$

At time $k=1$, the filter initial values for v_k are approximated as follows:

$$\begin{aligned} \hat{v}_{1|1} &= R_1\left(\Xi_{1|0}\right)^{-1}\xi_{1|0}, \\ P^{vv}_{1|1} &= R_1 - R_1\left(\Xi_{1|0}\right)^{-1}R_1. \end{aligned} \tag{22}$$

Proof: First, given Y_{k-1}, v_k follows a Gaussian distribution and it is independent of Y_{k-1}, thus $\hat{v}_{k|k-1} = \mathbf{0}$ and $P^{vv}_{k|k-1} = R_k$. Based on OPL, equations (16) and (17) are established.

The one-step linear predictor $\hat{y}_{k|k-1}$ in (20) is defined as $\hat{y}_{k|k-1} = \mathrm{E}\left[y_k\,|Y_{k-1}\right]$, and it is obtained by the simple addition of the two expressions in (14).

Equation (19) is obtained from the definition of innovation $\xi_{k|k-1}$.

The cross-covariance $P^{v\xi}_{k|k-1}$ between innovation $\xi_{k|k-1}$ and noise v_k based on Y_{k-1} is calculated as:

$$
\begin{aligned}
P^{v\xi}_{k|k-1} &= \mathrm{E}\left[v_k \xi^T_{k|k-1} \left| Y_{k-1}\right.\right] \\
&= \mathrm{E}\left[v_k \left(\left(1-\gamma_k\right)\Pi_k z_k\right)^T \left| Y_{k-1}\right.\right] \\
&= \mathrm{E}\left[v_k v_k^T \left(1-\gamma_k\right)\Pi_k \left| Y_{k-1}\right.\right] \\
&= p\left[\left(1-\gamma_k\right)\Pi_k \left| Y_{k-1}\right.\right] \\
&\quad \times \mathrm{E}\left[v_k v_k^T \left| \Pi_k = I_{n_y}, \gamma_k = 0, Y_{k-1}\right.\right] \\
&= \left(1-p_k\right)\psi_{k,1}\Gamma_{k|k-1},
\end{aligned} \tag{23}
$$

where the third equality is derived by using the conditional expectation formula and the property 1 and 4. Thus, (18) is obtained.

In terms of the definition of measurement model (7) and the covariance formula of the discrete random variable, the entries of $\Xi_{k|k-1}$ are given as follows:

$$
\begin{aligned}
&\Xi_{k|k-1} \\
&= \mathrm{Cov}\left[\xi_{k|k-1} \left| Y_{k-1}\right.\right] \\
&= \mathrm{Cov}\left[\xi_{k|k-1} \left| Y_{k-1}, \Pi_k = I_{n_y}\right.\right] \\
&\quad + \mathrm{Cov}\left[\xi_{k|k-1} \left| Y_{k-1}, \Pi_k = 0\right.\right] \\
&= \mathrm{Cov}\left[\left(1-\gamma_k\right)\Pi_k z_k, \left(1-\gamma_k\right)\Pi_k z_k \left| \Pi_k = I_{n_y}, Y_{k-1}\right.\right] \\
&\quad + \mathrm{Cov}\left[\left(1-\gamma_k\right)\Pi_k z_k, \gamma_k \Pi_k z_{k-1} \left| \Pi_k = I_{n_y}, Y_{k-1}\right.\right] \\
&\quad + \mathrm{Cov}\left[\gamma_k \Pi_k z_{k-1}, \left(1-\gamma_k\right)\Pi_k z_k \left| \Pi_k = I_{n_y}, Y_{k-1}\right.\right] \\
&\quad + \mathrm{Cov}\left[\gamma_k \Pi_k z_{k-1}, \gamma_k \Pi_k z_{k-1} \left| \Pi_k = I_{n_y}, Y_{k-1}\right.\right] \\
&= \left(1-p_k\right)\mathrm{E}\left[z_k z_k^T \left| \Pi_k = I_{n_y}, \gamma_k = 0, Y_{k-1}\right.\right] \\
&\quad - \left(1-p_k\right)^2 \hat{z}^c_{k|k-1}\left(\hat{z}^c_{k|k-1}\right)^T \\
&\quad - p_k\left(1-p_k\right)\hat{z}^c_{k|k-1}\left(\hat{z}^c_{k-1|k-1}\right)^T \\
&\quad + p_k \mathrm{E}\left[z_{k-1} z_{k-1}^T \left| \Pi_k = I_{n_y}, \gamma_k = 1, Y_{k-1}\right.\right] \\
&\quad - p_k^2 \hat{z}^c_{k-1|k-1}\left(\hat{z}^c_{k-1|k-1}\right)^T \\
&\quad - p_k\left(1-p_k\right)\hat{z}^c_{k-1|k-1}\left(\hat{z}^c_{k|k-1}\right)^T,
\end{aligned} \tag{24}
$$

where Bernoulli statistical properties $\mathrm{E}\left[\left(1-\gamma_k\right)^2\right] = 1 - p_k$, $\mathrm{E}\left[\gamma_k^2\right] = p_k$ and $\mathrm{E}\left[\left(1-\gamma_k\right)\gamma_k\right] = 0$ are used for the derivations. With simple algebraic manipulation, the part for $k > 1$ in (21) can be obtained.

Note that when computing $\Xi_{1|0}$, z_1 does not arrive at time $k = 1$; using $\hat{x}_{1|0}$ obtained from (11) and by the fact that v_1 follows the truncated Gaussian distribution, we obtain the initial innovation covariance:

$$
\begin{aligned}
\Xi_{1|0} &= \mathrm{Cov}\left[\Pi_1 z_1, \Pi_1 z_1 \left| \Pi_1 = I_{n_y}, Y_0\right.\right] \\
&= \mathrm{E}\left[z_1 z_1^T \left| \Pi_1 = I_{n_y}, Y_0\right.\right] - \hat{z}^c_{1|0}\left(\hat{z}^c_{1|0}\right)^T \\
&= C_1 P_{1|0} C_1^T + \hat{\chi}_{1|0}\hat{\chi}^T_{1|0} \\
&\quad + \hat{\chi}_{1|0}\left(\hat{v}^c_{1|0}\right)^T + \hat{v}^c_{1|0}\hat{\chi}^T_{1|0} + \Gamma_{1|0} \\
&\quad - \left(\hat{\chi}_{1|0} + \hat{v}^c_{1|0}\right)\left(\hat{\chi}_{1|0} + \hat{v}^c_{1|0}\right)^T \\
&= C_1 P_{1|0} C_1^T + \Gamma_{1|0} - \hat{v}^c_{1|0}\left(\hat{v}^c_{1|0}\right)^T.
\end{aligned}
$$

Next, we give Theorem 2 for the estimates of the state x_k and its covariance P_k.

Theorem 2: For the system composed of (1), (2) and (7) and the given $\hat{x}_{k-1|k-1}$ and $P_{k-1|k-1}$, the filter estimate of state $\hat{x}_{k|k}$ and its covariance $P_{k|k}$ at time k satisfy the following expressions:

$$
\begin{aligned}
\hat{x}_{k|k} &= \hat{x}_{k|k-1} + P^{x\xi}_{k|k-1}\left(\Xi_{k|k-1}\right)^{-1}\xi_{k|k-1}, \\
&\quad k \geq 1, \quad \hat{x}_{0|0} = x_0.
\end{aligned} \tag{25}
$$

$$
\begin{aligned}
P_{k|k} &= P_{k|k-1} - P^{x\xi}_{k|k-1}\left(\Xi_{k|k-1}\right)^{-1}\left(P^{x\xi}_{k|k-1}\right)^T, \\
&\quad k \geq 1, \quad P_{0|0} = P_0.
\end{aligned} \tag{26}
$$

$$
\begin{aligned}
P^{x\xi}_{k|k-1} &\approx \left(1-p_k\right)P_{k|k-1}C_k^T\psi_{k,1} \\
&\quad + p_k A_{k-1} P_{k-1|k-1} C_{k-1}^T \psi_{k,2}, \quad k > 1, \\
P^{x\xi}_{1|0} &= P_{1|0} C_1^T \psi_{1,1}, \quad k = 1,
\end{aligned} \tag{27}
$$

where $P^{x\xi}_{k|k-1} = \mathrm{Cov}\left[x_k, \xi_{k|k-1} \left| Y_{k-1}\right.\right]$ denotes the conditional cross-covariance matrix between x_k and the innovation $\xi_{k|k-1}$ given Y_{k-1}.

Proof: First, according to **Assumption** 2, the following three approximations hold true:

$$
\begin{aligned}
&\mathrm{E}\left[x_k \Pi_k \left| \gamma_k = 0, Y_{k-1}\right.\right] \\
&\quad \approx \mathrm{E}\left[x_k \left| Y_{k-1}\right.\right]\mathrm{E}\left[\Pi_k \left| \gamma_k = 0, Y_{k-1}\right.\right];
\end{aligned} \tag{28}
$$

$$
\begin{aligned}
&\mathrm{E}\left[x_k v_k^T \Pi_k \left| \gamma_k = 0, Y_{k-1}\right.\right] \approx \mathrm{E}\left[x_k \left| Y_{k-1}\right.\right] \\
&\quad \times \mathrm{E}\left[\Pi_k \left| \gamma_k = 0, Y_{k-1}\right.\right] \\
&\quad \times \mathrm{E}\left[v_k \left| \Pi_k = I, \gamma_k = 0, Y_{k-1}\right.\right]^T;
\end{aligned} \tag{29}
$$

$$
\begin{aligned}
&\mathrm{E}\left[x_{k-1} v_{k-1}^T \Pi_k \left| \gamma_k = 1, Y_{k-1}\right.\right] \approx \mathrm{E}\left[x_{k-1} \left| Y_{k-1}\right.\right] \\
&\quad \times \mathrm{E}\left[\Pi_k \left| \gamma_k = 1, Y_{k-1}\right.\right] \\
&\quad \times \mathrm{E}\left[v_{k-1} \left| \Pi_k = I, \gamma_k = 1, Y_{k-1}\right.\right]^T.
\end{aligned} \tag{30}
$$

The state $\hat{x}_{k|k}$ and error covariance $P_{k|k}$ are obtained from OPL. Then, cross-covariance $P^{x\xi}_{k|k-1}$ is decomposed into the sum of two items:

$$
\begin{aligned}
P^{x\xi}_{k|k-1} &= \mathrm{Cov}\left[x_k, \xi_{k|k-1} \left| Y_{k-1}\right.\right] \\
&= \mathrm{Cov}\left[x_k, \left(1-\gamma_k\right)\Pi_k z_k \left| Y_{k-1}\right.\right] \\
&\quad + \mathrm{Cov}\left[x_k, \gamma_k \Pi_k z_{k-1} \left| Y_{k-1}\right.\right].
\end{aligned} \tag{31}
$$

With the above approximations and the established properties, the first item on the right of (31) is calculated as:

$$
\begin{aligned}
&\mathrm{Cov}\left[x_k, \left(1-\gamma_k\right)\Pi_k z_k \left| Y_{k-1}\right.\right] \\
&\quad = \mathrm{E}\left[\left(x_k x_k^T\right)C_k^T \left(1-\gamma_k\right)\Pi_k \left| Y_{k-1}\right.\right] \\
&\qquad + \mathrm{E}\left[x_k \left(1-\gamma_k\right) v_k^T \Pi_k \left| Y_{k-1}\right.\right] \\
&\qquad - \mathrm{E}\left[x_k \tau^T \left(1-\gamma_k\right)\Pi_k \left| Y_{k-1}\right.\right] \\
&\qquad - \hat{x}_{k|k-1}\mathrm{E}\left[x_k^T C_k^T \left(1-\gamma_k\right)\Pi_k \left| Y_{k-1}\right.\right] \\
&\qquad - \hat{x}_{k|k-1}\mathrm{E}\left[\left(1-\gamma_k\right) v_k^T \Pi_k \left| Y_{k-1}\right.\right] \\
&\qquad + \hat{x}_{k|k-1}\mathrm{E}\left[\tau^T \left(1-\gamma_k\right)\Pi_k \left| Y_{k-1}\right.\right] \\
&\quad \approx P_{k|k-1} C_k^T \mathrm{E}\left[\left(1-\gamma_k\right)\Pi_k \left| Y_{k-1}\right.\right] \\
&\quad = \left(1-p_k\right)P_{k|k-1} C_k^T \psi_{k,1};
\end{aligned} \tag{32}
$$

similarly, the second item on the right of (31) is calculated as:

$$\begin{aligned}
&\mathrm{Cov}\left[\boldsymbol{x}_k, \gamma_k \boldsymbol{\Pi}_k \boldsymbol{z}_{k-1} \,|\boldsymbol{Y}_{k-1}\right] \\
&= \mathrm{E}\left[\boldsymbol{x}_k \boldsymbol{x}_{k-1}^T \boldsymbol{C}_{k-1}^T \gamma_k \boldsymbol{\Pi}_k \,|\boldsymbol{Y}_{k-1}\right] \\
&\quad + \mathrm{E}\left[\boldsymbol{x}_k \boldsymbol{v}_{k-1}^T \gamma_k \boldsymbol{\Pi}_k \,|\boldsymbol{Y}_{k-1}\right] \\
&\quad - \mathrm{E}\left[\boldsymbol{x}_k \boldsymbol{\tau}^T (1-\gamma_k) \boldsymbol{\Pi}_k \,|\boldsymbol{Y}_{k-1}\right] \\
&\quad - \hat{\boldsymbol{x}}_{k|k-1} \mathrm{E}\left[\gamma_k \boldsymbol{x}_{k-1}^T \boldsymbol{C}_{k-1}^T \boldsymbol{\Pi}_k \,|\boldsymbol{Y}_{k-1}\right] \\
&\quad - \hat{\boldsymbol{x}}_{k|k-1} \mathrm{E}\left[\gamma_k \boldsymbol{v}_{k-1}^T \boldsymbol{\Pi}_k \,|\boldsymbol{Y}_{k-1}\right] \\
&\quad + \hat{\boldsymbol{x}}_{k|k-1} \mathrm{E}\left[\boldsymbol{\tau}^T (1-\gamma_k) \boldsymbol{\Pi}_k \,|\boldsymbol{Y}_{k-1}\right] \\
&= \mathrm{E}\left[\boldsymbol{x}_k \boldsymbol{x}_{k-1}^T \,|\boldsymbol{Y}_{k-1}\right] \boldsymbol{C}_{k-1}^T \mathrm{E}\left[\gamma_k \boldsymbol{\Pi}_k \,|\boldsymbol{Y}_{k-1}\right] \\
&\quad - \hat{\boldsymbol{x}}_{k|k-1} \hat{\boldsymbol{x}}_{k-1|k-1}^T \boldsymbol{C}_{k-1}^T \mathrm{E}\left[\gamma_k \boldsymbol{\Pi}_k \,|\boldsymbol{Y}_{k-1}\right] \\
&\quad + \boldsymbol{A}_{k-1} \mathrm{E}\left[\boldsymbol{x}_{k-1} \boldsymbol{v}_{k-1}^T \gamma_k \boldsymbol{\Pi}_k \,|\boldsymbol{Y}_{k-1}\right] \\
&\approx \boldsymbol{P}_{k,k-1|k-1} \boldsymbol{C}_{k-1}^T \mathrm{E}\left[\gamma_k \boldsymbol{\Pi}_k \,|\boldsymbol{Y}_{k-1}\right]. \qquad (33)
\end{aligned}$$

Inserting (32) and (33) into the definition of $\boldsymbol{P}_{k|k-1}^{x\xi}$, we obtain the part at time $k > 1$ in (31); for time $k = 1$,

$$\begin{aligned}
\boldsymbol{P}_{1|0}^{x\xi} &= \mathrm{Cov}\left[\boldsymbol{x}_1, \boldsymbol{\xi}_{1|0} \,|\boldsymbol{Y}_0\right] = \mathrm{Cov}\left[\boldsymbol{x}_1, \boldsymbol{\Pi}_1 \boldsymbol{z}_1 \,|\boldsymbol{Y}_0\right] \\
&= \mathrm{E}\left[\boldsymbol{x}_1 \boldsymbol{x}_1^T \,|\boldsymbol{Y}_0\right] \boldsymbol{C}_1^T \boldsymbol{\psi}_{1,1} - \hat{\boldsymbol{x}}_{1|0} \hat{\boldsymbol{x}}_{1|0}^T \boldsymbol{C}_1^T \boldsymbol{\psi}_{1,1}, \qquad (34)
\end{aligned}$$

with the simple operation, $\boldsymbol{P}_{1|0}^{x\xi}$ is obtained.

Combing Theorems 1 and 2, we have accomplished the filter design for network systems with randomly censored measurement and one-step delayed latent measurement.

Remark 3: Since the measurement noise does not follow a Gaussian distribution around the censoring region, the state estimate $\hat{\boldsymbol{x}}_{k|k}$ and the related $\hat{\boldsymbol{\chi}}_{k|k}$ do not follow a Gaussian distribution.

Remark 4: As analyzed and indicated in [19], compared with the practicality and adequate estimates provided by the TKF, the nonlinear UKF can produce errors in the piecewise uncertainty of the censored measurement. Thus, it is deduced that a Gaussian approximate filter with one-step delay, such as the UKF with a one-step delay [11] or the CKF with a one-step delay [28], cannot efficiently solve the problem of mixed measurement uncertainties with simultaneous signal delay and measurement censoring; Additionally, the TKF lacks the necessary theory to deal with the Tobit measurement problem with the occurrence of transmission delay. In contrast, the proposed approach, in theory, guarantees appropriateness of the problem solved in this paper.

Remark 5: The above discussions and derivations have given the recursive algorithm for the case of the signal delay followed by the measurement censoring. Another case that the measurement censoring happens before the one-step transmission delay can be similarly derived, which indicates that the two algorithms are essentially the same. The detailed explanation of problem formulation of another case is shown in Appendix C.

IV. ILLUSTRATIVE EXAMPLE

In this section, we employ an oscillator example given in [19]–[22] to demonstrate the performance of the proposed filter approach. The system matrices for the oscillator model and the corresponding parameters of the dynamic systems given by (1)-(4) are as follows:

$$\boldsymbol{A} = \begin{bmatrix} \cos(\omega T) & -\sin(\omega T) \\ \sin(\omega T) & \cos(\omega T) \end{bmatrix}, \quad \boldsymbol{C}_k = \begin{bmatrix} 1 & 0 \end{bmatrix},$$

the frequency is $\omega = 0.0052\pi$ with a sampling period of $T = 1$, process noise with variance $\boldsymbol{Q}_k = \mathrm{diag}\left\{\left[\,(0.1)^2 \;\; (0.1)^2\,\right]\right\}$ is uncorrelated with measurement noise with variance $\boldsymbol{R}_k = 10$. The initial conditions are set as $\hat{\boldsymbol{x}}_{0|0} = \left[\,5\;0\,\right]^T$, $\boldsymbol{P}_{0|0} = \boldsymbol{I}_2$, delay probability $p_k = 0.8$ for all k, censored threshold $\boldsymbol{\tau} = 0$, and the simulation time 1,000.

The performance of the proposed method is analyzed by comparison with the TKF proposed in [20] and the CKF with one-step delay given in [28] (denoted as CKF-Delay) in the same conditions. The number of Monte Carlo simulations is 10. The root mean square error (RMSE) criterion and average RMSE (ARMSE) criterion are employed to quantify the performance of the estimators. The RMSE and ARMSE of the filter at time k are computed as follows:

$$\begin{aligned}
\mathrm{RMSE_k} &= \sqrt{\frac{1}{MC} \sum_{i=1}^{MC} \left(\boldsymbol{x}_k^{(i)} - \hat{\boldsymbol{x}}_{k|k}^{(i)}\right)^2}, \\
\mathrm{ARMSE} &= \sqrt{\frac{\sum_{k=1}^{K} \sum_{i=1}^{MC} \left(\boldsymbol{x}_k^{(i)} - \hat{\boldsymbol{x}}_{k|k}^{(i)}\right)^2}{K * MC}}, \qquad (35)
\end{aligned}$$

where $\boldsymbol{x}_{k|k}^{(i)}$ is denoted as the filtering estimate at time k in the i-th simulation run, and $\boldsymbol{x}_k^{(i)}$ is the i-th true values of the state. MC and K represent the number of Monte Carlo simulations and the number of iterations, respectively.

Fig. 2 shows the true state and its corresponding estimates from different filter methods when delay probability p_k is 0.8. RMSE comparison curves between the TKF and the proposed method, CKF-Delay and the proposed method are illustrated in Fig. 3 and Fig. 4, respectively. The specific ARMSE values are summarized in Table 1. For delay probability $p_k = 0.3$, the same comparison curves and ARMSE values as those of $p_k = 0.8$ are presented in Figs. 5 to 7 and Table 2. From these figures, we can see that, except for a few values of k, the proposed method exhibits superior performance for the delay conditions.

For the first state, the proposed method has a slightly better performance than that of the TKF, and both are better than those of the CKF-Delay and the CKF whose state curves and RMSE curves are not plotted to maintain the presentation clarity. For the second state, it is clear that the proposed method tracks the real state more accurately and exhibits a smaller ARMSE. Furthermore, from the Tables, it is also concluded that the TKF exhibits better performance than the CKF-Delay, and the CKF-Delay shows superiority over the CKF when censored measurements and transmission delays

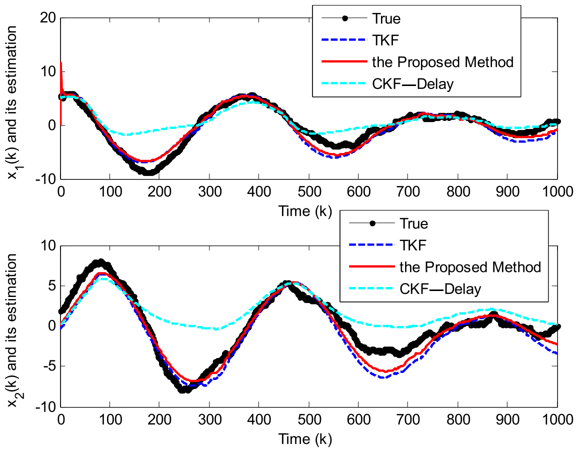

FIGURE 2. True values of state and its corresponding estimates for $p_k = 0.8$.

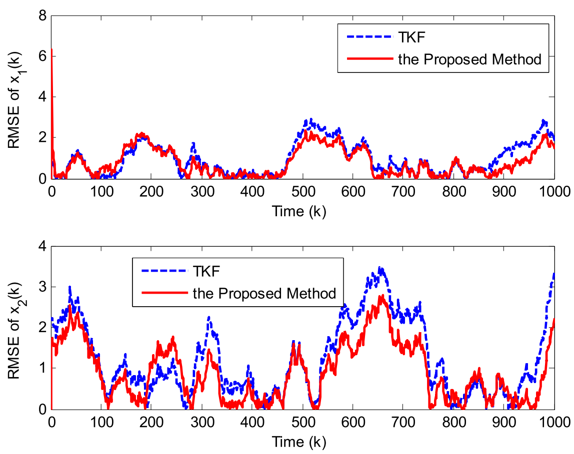

FIGURE 3. RMSE comparisons with the TKF for $p_k = 0.8$.

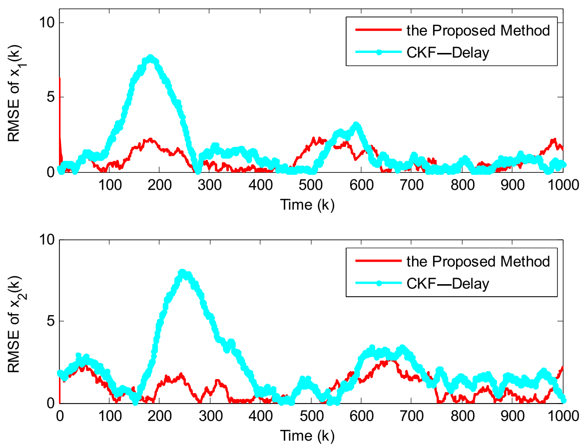

FIGURE 4. RMSE comparisons with the CKF with one-step delay for $p_k = 0.8$.

TABLE 1. ARMSEs of the states between the proposed method and similar methods applied to a stochastic oscillator model for $p_k = 0.8$.

Algorithm	1st ARMSE	2nd ARMSE
TKF	0.9251	1.2805
Proposed Method	0.9028	0.9802
CKF-Delay	1.9857	2.4373
CKF	2.0468	11.8798

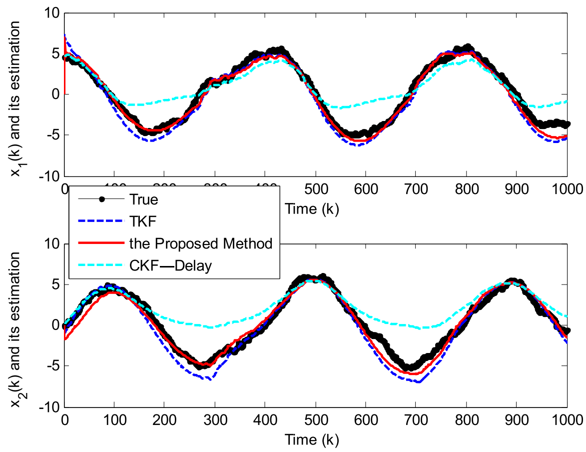

FIGURE 5. True values of state and its corresponding estimates for $p_k = 0.3$.

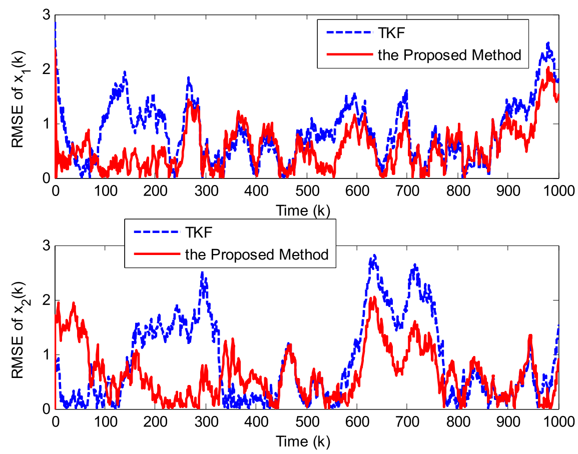

FIGURE 6. RMSE comparisons with the TKF for $p_k = 0.3$.

TABLE 2. ARMSEs of the states between the proposed method and similar methods applied to a stochastic oscillator model for $p_k = 0.3$.

Algorithm	1st ARMSE	2nd ARMSE
TKF	1.2218	1.5121
Proposed Method	1.0837	1.1638
CKF-Delay	2.4251	2.8607
CKF	2.4782	16.5302

occur simultaneously. The RMSE curves are plotted separately to avoid confusion from too many curves in one graph.

Fig. 8-Fig. 9 show the comparisons of the ARMSE curves of the 1st and 2nd state estimations, respectively, for different values of delay probability p_k. From the 1st state comparison in Fig. 8, it can be seen that the performance of the proposed method is slightly better than that of the TKF, and both are smaller than the existing CKF-Delay. From the comparison of the 2nd state in Fig. 9, it can be seen that the performance

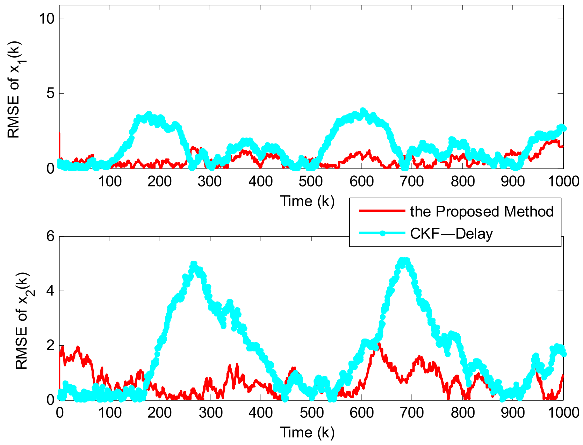

FIGURE 7. RMSE comparisons with the CKF delayed by one-step for $p_k = 0.3$.

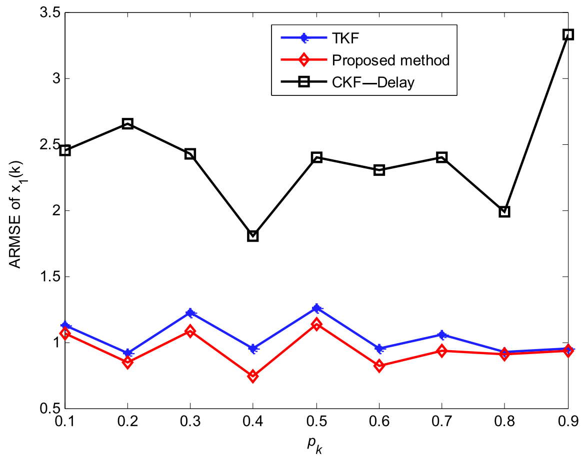

FIGURE 8. ARMSE comparisons of 1st state estimation when $p = 0.1, 0.2, \cdots, 0.9$.

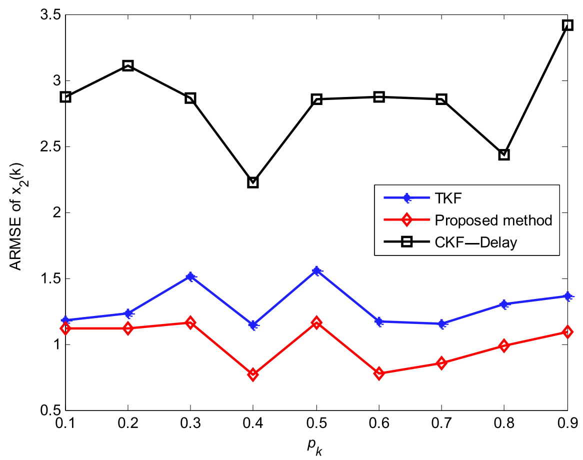

FIGURE 9. ARMSE comparisons of 2nd state estimation when $p = 0.1, 0.2, \cdots, 0.9$.

of the proposed method is obviously better than that of the TKF, and both outperform the existing CKF-Delay.

Overall, according to the values listed in the two tables and graphs for the long simulation performance, it can be concluded that the proposed algorithm presents more robust tracking ability for the considered model.

V. CONCLUSION

In this paper, we have presented an improved method for the TKF under the sensor network circumstances with two important random phenomenons, that is, censored measurements and one-step delayed signal transmission. The probability of latent measurement being uncensored is first modified to account for the delay, and it is provided by a lemma. In a unified framework, the resulting filter formulation is given by two theorems. The behavior of the proposed filter was compared with those of three other methods, namely: the TKF, CKF, and CKF with a one-step delay. Outcomes demonstrate that even when a high proportion of the latent measurements is delayed and censored, the proposed filter can also provide accurate state estimates and can consistently outperform the other three filters under the same conditions. The comparison of the ARMSE for different delay probabilities showed and demonstrated better overall performance of the proposed method. In contrast to the standard TKF, the derived formula does not require an uncorrelation assumption among the measurement components. For further research, the Tobit Kalman filtering problems for linear systems with network-induced phenomena of packet dropouts, and for information fusion of the multisensor with transmission delay and censoring can be investigated.

APPENDIXES

APPENDIX A

DERIVATION OF (12)

Applying the rule of conditional expectation yields

$$\begin{aligned} \boldsymbol{c}_k{}^m &= \mathrm{E}_{\gamma_k}\left[\mathrm{E}_{\eta_k|\gamma_k}\left[\eta_k{}^m\,|\boldsymbol{Y}_{k-1}\right]\right] \\ &= p\left\{\gamma_k = 1\right\}\mathrm{E}\left[\eta_k{}^m\,|\gamma_k = 1, \boldsymbol{Y}_{k-1}\right] \\ &\quad + p\left\{\gamma_k = 0\right\}\mathrm{E}\left[\eta_k{}^m\,|\gamma_k = 0, \boldsymbol{Y}_{k-1}\right]. \end{aligned} \tag{36}$$

Note that the expectation $\mathrm{E}\left[\eta_k{}^m\,|\gamma_k = 0, \boldsymbol{Y}_{k-1}\right]$ is identical to the probability of the measurement z_k being uncensored. Using the notations defined in (9), we then have

$$\mathrm{E}\left[\eta_k{}^m\,|\gamma_k = 0, \boldsymbol{Y}_{k-1}\right] = 1 - \boldsymbol{\Phi}\left(\boldsymbol{L}_{k|k-1}^m\right). \tag{37}$$

The expectation $\mathrm{E}\left[\eta_k{}^m\,|\gamma_k = 1, \boldsymbol{Y}_{k-1}\right]$ in (32) corresponds to the following probability:

$$\mathrm{E}\left[\eta_k{}^m\,|\gamma_k = 1, \boldsymbol{Y}_{k-1}\right] = p\left\{\bar{z}_{k-1}^m > \boldsymbol{\tau}^m\,|\boldsymbol{Y}_{k-1}\right\}.$$

Using the total expectation rule, we can conclude that

$$\begin{aligned} &\mathrm{E}\left[\eta_k{}^m\,|\gamma_k = 1, \boldsymbol{Y}_{k-1}\right] \\ &\quad = p\left\{\bar{\boldsymbol{y}}_{k-1} = \bar{z}_{k-1}\right\}p\left\{\bar{z}_{k-1}^m > \boldsymbol{\tau}^m\,|\boldsymbol{Y}_{k-1}\right\} \\ &\qquad + p\left\{\bar{\boldsymbol{y}}_{k-1} = \bar{z}_{k-2}\right\}p\left\{\bar{z}_{k-1}^m > \boldsymbol{\tau}^m\,|\boldsymbol{Y}_{k-1}\right\} \\ &\quad = (1 - p_{k-1})\,p\left\{\bar{z}_{k-1}^m > \boldsymbol{\tau}^m\left|\boldsymbol{Y}_{k-1}, \bar{\boldsymbol{y}}_{k-1} = \bar{z}_{k-1}\right.\right\} \\ &\qquad + p_{k-1}p\left\{\bar{z}_{k-1}^m > \boldsymbol{\tau}^m\left|\boldsymbol{Y}_{k-1}, \bar{\boldsymbol{y}}_{k-1} = \bar{z}_{k-2}\right.\right\} \end{aligned}$$

According to Assumption 2, the approximation of $\hat{\bar{z}}^m_{k-1|k-1}$ to $\bar{z}^m_{k-1}$ holds true when $\bar{\boldsymbol{y}}_{k-1}$ is equal to $\bar{z}_{k-1}$; thus,

$$\begin{aligned}
&\mathrm{E}\left[\eta_k{}^m \left|\gamma_k = 1, \boldsymbol{Y}_{k-1}\right.\right] \\
&\quad\approx (1-p_{k-1})\, p\left\{\hat{\bar{z}}^m_{k-1|k-1} > \boldsymbol{\tau}^m \left|\boldsymbol{Y}_{k-1}, \bar{\boldsymbol{y}}_{k-1} = \bar{z}_{k-1}\right.\right\} \\
&\qquad + p_{k-1} p\left\{\bar{z}^m_{k-1} > \boldsymbol{\tau}^m \left|\boldsymbol{Y}_{k-1}, \bar{\boldsymbol{y}}_{k-1} = \bar{z}_{k-2}\right.\right\} \\
&\quad= (1-p_{k-1})\, \boldsymbol{\delta}_k + p_{k-1} \boldsymbol{\Phi}\left(\boldsymbol{L}^m_{k-1|k-1}\right).
\end{aligned} \tag{38}$$

We obtain (12) by combining (36), (37) and (38).

APPENDIX B
DERIVATION OF (13)

From the rule of conditional expectation and the results of (37) and (38), we have the following two expressions:

$$\begin{aligned}
&\mathrm{E}\left[(1-\gamma_k)\, \boldsymbol{\Pi}_k \left|\boldsymbol{Y}_{k-1}\right.\right] \\
&\quad= \mathrm{E}_{\gamma_k}\left[\mathrm{E}\left[(1-\gamma_k)\, \boldsymbol{\Pi}_k \left|\boldsymbol{Y}_{k-1}\right.\right]\right] \\
&\quad= p\{\gamma_k = 1\}\, \mathrm{E}\left[(1-\gamma_k)\, \boldsymbol{\Pi}_k \left|\gamma_k = 1, \boldsymbol{Y}_{k-1}\right.\right] \\
&\qquad + p\{\gamma_k = 0\}\, \mathrm{E}\left[(1-\gamma_k)\, \boldsymbol{\Pi}_k \left|\gamma_k = 0, \boldsymbol{Y}_{k-1}\right.\right] \\
&\quad= (1-p_k)\, \mathrm{diag}\left(\left[1 - \boldsymbol{\Phi}\left(\boldsymbol{L}^m_{k|k-1}\right)\right]\right).
\end{aligned} \tag{39}$$

The derivation of **property** 1 is obtained from (39). **Property** 2 can be similarly obtained as follows:

$$\begin{aligned}
&\mathrm{E}\left[\gamma_k \boldsymbol{\Pi}_k \left|\boldsymbol{Y}_{k-1}\right.\right] \\
&= \mathrm{E}_{\gamma_k}\left[\mathrm{E}\left[\gamma_k \boldsymbol{\Pi}_k \left|\boldsymbol{Y}_{k-1}\right.\right]\right] \\
&= p\{\gamma_k = 1\}\, \mathrm{E}\left[\boldsymbol{\Pi}_k \left|\gamma_k = 1, \boldsymbol{Y}_{k-1}\right.\right] \\
&\quad + p\{\gamma_k = 0\}\, \mathrm{E}\left[\gamma_k \boldsymbol{\Pi}_k \left|\gamma_k = 0, \boldsymbol{Y}_{k-1}\right.\right] \\
&= p_k \mathrm{diag}\left(\left[(1-p_{k-1})\, \boldsymbol{\delta}_k + p_{k-1} \boldsymbol{\Phi}\left(\boldsymbol{L}^m_{k-1|k-1}\right)\right]\right).
\end{aligned} \tag{40}$$

From the theorem that if random variable $\boldsymbol{x}$ obeys a standard normal distribution, its probability density function truncated from lower bound L is $p_L(\boldsymbol{x}) = \boldsymbol{\phi}(\boldsymbol{x}) / [1 - \boldsymbol{\Phi}(L)]$; thus, the first two truncated origin moments of $\boldsymbol{x}$, i.e., $\mathrm{E}[\boldsymbol{x} | \boldsymbol{x} > L] = \lambda(L)$ and $\mathrm{E}\left[\boldsymbol{x}^2 | \boldsymbol{x} > L\right] = L\lambda(L) + 1$ can be computed. Therefore, the third equality regarding measurement noise $\boldsymbol{v}_k$ in **property** 3 is easily obtained.

Based on the total expectation law, **property** 4 is calculated as follows:

$$\begin{aligned}
&\mathrm{E}\left[\boldsymbol{v}_{k-1} \left|\boldsymbol{\Pi}_k = \boldsymbol{I}_{n_y}, \gamma_k = 1, \boldsymbol{Y}_{k-1}\right.\right] = \hat{\boldsymbol{v}}^c_{k-1|k-1} \\
&= \mathrm{E}\left[\boldsymbol{v}_{k-1} \left|\bar{z}_{k-1} > \boldsymbol{\tau}, \boldsymbol{Y}_{k-1}\right.\right] \\
&= p\left\{\bar{\boldsymbol{y}}_{k-1} = \bar{z}_{k-1}\right\} \\
&\quad \times \mathrm{E}\left[\boldsymbol{v}_{k-1} \left|\bar{z}_{k-1} > \boldsymbol{\tau}, \boldsymbol{Y}_{k-1}, \bar{\boldsymbol{y}}_{k-1} = \bar{z}_{k-1}\right.\right] \\
&\quad + p\left\{\bar{\boldsymbol{y}}_{k-1} = \bar{z}_{k-2}\right\} \\
&\quad \times \mathrm{E}\left[\boldsymbol{v}_{k-1} \left|\bar{z}_{k-1} > \boldsymbol{\tau}, \boldsymbol{Y}_{k-1}, \bar{\boldsymbol{y}}_{k-1} = \bar{z}_{k-2}\right.\right] \\
&= (1-p_{k-1})\, \hat{\boldsymbol{v}}_{k-1|k-1} \boldsymbol{\delta}_k + p_{k-1} \\
&\quad \times \mathrm{col}\left(\sqrt{\boldsymbol{R}^{m,m}_{k-1}} \mathrm{E}\left[\frac{\boldsymbol{v}^m_{k-1}}{\sqrt{\boldsymbol{R}^{m,m}_{k-1}}} \left|\frac{\boldsymbol{v}^m_{k-1}}{\sqrt{\boldsymbol{R}^{m,m}_{k-1}}} > \boldsymbol{L}^m_{k-1|k-1}\right.\right]\right)
\end{aligned} \tag{41}$$

where $\boldsymbol{v}^m_{k-1} \Big/ \sqrt{\boldsymbol{R}^{m,m}_{k-1}}$ obeys the truncated normal distribution with truncated expectation $\lambda\left(\boldsymbol{L}^m_{k-1|k-1}\right)$, using the notation in (9), the second item in (41) equals $p_{k-1}\sqrt{\boldsymbol{R}_{k-1}}\lambda\left(\boldsymbol{L}_{k-1|k-1}\right)$. Thus, we obtain the equality in **property** 4.

For the fifth equality in (13), the expectation of the diagonal element for square matrix $\left[\boldsymbol{v}_k \boldsymbol{v}_k^T\right]$ is derived as follows:

$$\begin{aligned}
&\mathrm{E}\left[\left(\boldsymbol{v}^m_k\right)^2 \left|\eta_k{}^m = 1, \gamma_k = 0, \boldsymbol{Y}_{k-1}\right.\right] \\
&\quad = \boldsymbol{R}^{m,m}_k \mathrm{E}\left[\left(\frac{\boldsymbol{v}^m_k}{\sqrt{\boldsymbol{R}^{m,m}_k}}\right)^2 \left|\frac{\boldsymbol{v}^m_k}{\sqrt{\boldsymbol{R}^{m,m}_k}} > \frac{\boldsymbol{\tau}^m - \hat{\boldsymbol{\chi}}^m_{k|k-1}}{\sqrt{\boldsymbol{R}^{m,m}_k}}\right.\right] \\
&\quad = \boldsymbol{R}^{m,m}_k \left(\boldsymbol{L}^m_{k|k-1} \lambda\left(\boldsymbol{L}^m_{k|k-1}\right) + 1\right)
\end{aligned} \tag{42}$$

From the fact that $\boldsymbol{R}$ is diagonal and the definition for $\hat{\boldsymbol{v}}^c_{k|k-1}$ in **property** 3, the expectation of the non-diagonal element is expressed as follows:

$$\begin{aligned}
&\mathrm{E}\left[\boldsymbol{v}^m_k \boldsymbol{v}^n_k \left|\eta_k{}^m = 1, \eta_k{}^n = 1, \gamma_k = 0, \boldsymbol{Y}_{k-1}\right.\right] \\
&\quad = \mathrm{E}\left[\boldsymbol{v}^m_k \left|\eta_k{}^m = 1, \gamma_k = 0, \boldsymbol{Y}_{k-1}\right.\right] \\
&\qquad \times \mathrm{E}\left[\boldsymbol{v}^n_k \left|\eta_k{}^n = 1, \gamma_k = 0, \boldsymbol{Y}_{k-1}\right.\right] \\
&\quad = \hat{\boldsymbol{v}}^{m\,c}_{k|k-1} \hat{\boldsymbol{v}}^{n\,c}_{k|k-1}
\end{aligned} \tag{43}$$

where $\hat{\boldsymbol{v}}^{m\,c}_{k|k-1}$ denotes the $m-th$ element of the vector $\hat{\boldsymbol{v}}^c_{k|k-1}$. Define $\boldsymbol{\Gamma}_{k|k-1} = \left[\boldsymbol{\Gamma}^{mn}_{k|k-1}\right]_{n_y \times n_y}$, where

$$\boldsymbol{\Gamma}^{mn}_{k|k-1} = \begin{cases} \boldsymbol{R}^{m,m}_k \left(\boldsymbol{L}^m_{k|k-1} \lambda\left(\boldsymbol{L}^m_{k|k-1}\right) + 1\right) & m = n \\ \sqrt{\boldsymbol{R}^{m,m}_k \boldsymbol{R}^{n,n}_k} \lambda\left(\boldsymbol{L}^m_{k|k-1}\right) \lambda\left(\boldsymbol{L}^n_{k|k-1}\right) & m \neq n. \end{cases} \tag{44}$$

Thus, we obtain **property** 5 in (13).

For the sixth equality in (13), the calculation process is similar to that of (43) and (44).

$$\begin{aligned}
&\mathrm{E}\left[\boldsymbol{v}_{k-1} \boldsymbol{v}^T_{k-1} \left|\boldsymbol{\Pi}_k = \boldsymbol{I}_{n_y}, \gamma_k = 1, \boldsymbol{Y}_{k-1}\right.\right] \\
&\quad = p\left\{\bar{\boldsymbol{y}}_{k-1} = \bar{z}_{k-1}\right\} \\
&\qquad \times \mathrm{E}\left[\boldsymbol{v}_{k-1} \boldsymbol{v}^T_{k-1} \left|\bar{z}_{k-1} > \boldsymbol{\tau}, \boldsymbol{Y}_{k-1}, \bar{\boldsymbol{y}}_{k-1} = \bar{z}_{k-1}\right.\right] \\
&\qquad + p\left\{\bar{\boldsymbol{y}}_{k-1} = \bar{z}_{k-2}\right\} \\
&\qquad \times \mathrm{E}\left[\boldsymbol{v}_{k-1} \boldsymbol{v}^T_{k-1} \left|\bar{z}_{k-1} > \boldsymbol{\tau}, \boldsymbol{Y}_{k-1}, \bar{\boldsymbol{y}}_{k-1} = \bar{z}_{k-2}\right.\right],
\end{aligned} \tag{45}$$

from (42), we have

$$\begin{aligned}
&\mathrm{E}\left[\boldsymbol{v}_{k-1} \boldsymbol{v}^T_{k-1} \left|\boldsymbol{\Pi}_k = \boldsymbol{I}_{n_y}, \boldsymbol{\gamma}_k = 1, \boldsymbol{Y}_{k-1}\right.\right] \\
&\quad = (1-p_{k-1})\, \boldsymbol{\delta}_k \mathrm{E}\left[\boldsymbol{v}_{k-1} \boldsymbol{v}^T_{k-1} \left|\boldsymbol{Y}_{k-1}, \bar{\boldsymbol{y}}_{k-1} = \bar{z}_{k-1}\right.\right] \\
&\qquad + p_{k-1} \boldsymbol{\Gamma}_{k-1|k-1} \\
&\quad = (1-p_{k-1})\left(\boldsymbol{P}^{vv}_{k-1|k-1} + \hat{\boldsymbol{v}}_{k-1|k-1} \hat{\boldsymbol{v}}^T_{k-1|k-1}\right) \boldsymbol{\delta}_k \\
&\qquad + p_{k-1} \boldsymbol{\Gamma}_{k-1|k-1}
\end{aligned}$$

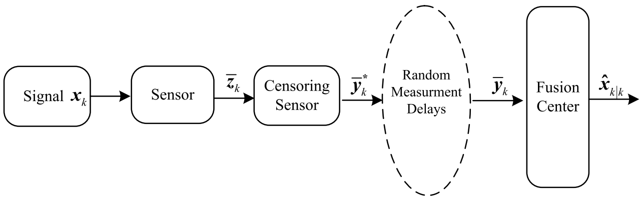

FIGURE 10. Flow chart of the sensor system with the censoring prior to the delay.

where $\boldsymbol{\Gamma}_{k-1|k-1} = \left[\boldsymbol{\Gamma}_{k-1|k-1}^{mn}\right]_{n_y \times n_y}$ with

$$\boldsymbol{\Gamma}_{k-1|k-1}^{mn} = \begin{cases} \boldsymbol{R}_{k-1}^{m,m}\left(\boldsymbol{L}_{k-1|k-1}^{m}\lambda\left(\boldsymbol{L}_{k-1|k-1}^{m}\right)+1\right) & m=n \\ \sqrt{\boldsymbol{R}_{k-1}^{m,m}\boldsymbol{R}_{k-1}^{n,n}}\lambda\left(\boldsymbol{L}_{k-1|k-1}^{m}\right)\lambda\left(\boldsymbol{L}_{k-1|k-1}^{n}\right) & m \neq n. \end{cases} \quad (46)$$

Thus, we obtain **property** 6.

APPENDIX C

The structure of the measurement censoring prior to the signal delay is shown in the Fig.10.

The same assumptions as **Assumptions** 1 and 2 are first made to this case, and the censored output $\bar{\boldsymbol{y}}_k^*$ is:

$$\bar{\boldsymbol{y}}_k^* = \begin{cases} \bar{z}_k & \bar{z}_k > \boldsymbol{\tau} \\ \boldsymbol{\tau} & \bar{z}_k \leq \boldsymbol{\tau}, \end{cases} \quad (47)$$

where the definitions of $\bar{z}_k$ and $\boldsymbol{\tau}$ are same as those in (2) and (4).

Then a Bernoulli indicator variable ${\boldsymbol{\eta}_k}^m$ for $\bar{\boldsymbol{y}}_k^{*m}$ is introduced, whose expression is:

$$ {\boldsymbol{\eta}_k}^m = \begin{cases} 1 & \bar{z}_k^m > \boldsymbol{\tau}^m \\ 0 & \bar{z}_k^m \leq \boldsymbol{\tau}^m. \end{cases} \quad (48)$$

Combining the diagonal random matrix $\boldsymbol{\Pi}_k$ composed of ${\boldsymbol{\eta}_k}^m$, $\bar{\boldsymbol{y}}_k^*$ is rewritten as:

$$\bar{\boldsymbol{y}}_k^* = \boldsymbol{\Pi}_k \bar{z}_k + \left(\boldsymbol{I}_{n_y} - \boldsymbol{\Pi}_k\right)\boldsymbol{\tau}, \quad (49)$$

where $\boldsymbol{\Pi}_k$ models the occurrence of censored output versus an actual output. The transmission signals entering fusion center for state estimation may be subject to one-step random delay, which leads to the following expression of measurement $\bar{\boldsymbol{y}}_k$ as:

$$\bar{\boldsymbol{y}}_k = \begin{cases} (1-\gamma_k)\bar{\boldsymbol{y}}_k^* + \gamma_k \bar{\boldsymbol{y}}_{k-1}^* & k \geq 2 \\ \bar{\boldsymbol{y}}_1^* & k = 1, \end{cases} \quad (50)$$

where properties of $\{\gamma_k, k \geq 1\}$ are the same as those in (3). Likewise, let $z_k = \bar{z}_k - \boldsymbol{\tau}$, $z_{k-1} = \bar{z}_{k-1} - \boldsymbol{\tau}$ and $\boldsymbol{y}_k = \bar{\boldsymbol{y}}_k - \boldsymbol{\tau}$ then (50) is modified as:

$$\boldsymbol{y}_k = \begin{cases} (1-\gamma_k)\boldsymbol{\Pi}_k z_k + \gamma_k \boldsymbol{\Pi}_{k-1} z_{k-1} & k \geq 2 \\ \bar{\boldsymbol{y}}_1^* - \boldsymbol{\tau} & k = 1, \end{cases} \quad (51)$$

with

$$\begin{aligned} \mathrm{E}\left[(1-\gamma_k)\boldsymbol{\Pi}_k \,|\boldsymbol{Y}_{k-1}\right] &= (1-p_k)\boldsymbol{\psi}_{k,1}, \\ \mathrm{E}\left[\gamma_k \boldsymbol{\Pi}_{k-1} \,|\boldsymbol{Y}_{k-1}\right] &= p_k \boldsymbol{\psi}_{k,2}, \quad k > 1. \end{aligned} \quad (52)$$

It is obvious that the equations in (52) are the same as **properties** 1 and 2 .

From the above problem formulation, we can see that the rest of state estimation for the case depicted in Fig.10 will be the same as those for the case depicted in Fig.1.

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ZONGYUAN WANG received the B.S. degree from the College of Science, Harbin Engineering University, Harbin, China, and the M.S. degree from the College of Automation, Harbin Engineering University, where he is currently pursuing the Ph.D. degree in control science and engineering. His current research interests include signal processing, information fusion, and their applications in navigation technology, such as GPS positioning and integrated navigation.

WEIDONG ZHOU received the B.S. degree in automatic control from the Harbin Institute of Technology, in 1988, the M.S. degree in automatic control from the Harbin University of Science and Technology, in 1991, and the Ph.D. degree in navigation from Harbin Engineering University, China, in 2006. Since 2005, he has been a Full Professor with the Department of Automation, Harbin Engineering University. His research interests include the direction of the automatic control theory, integration navigation, estimation theory, computer control, and multisensor data fusion. He is an Editorial Board Member of the Chinese Society of Architects and Marine Engineering, a member of National Marine Boat Standardization Technical Committee, and a member of the Northeast Regional Branch of the Chinese Society of Inertial Technology.