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Robust Semipassivity and Practical Stability for Uncertain Switched Nonlinear Systems

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ABSTRACT This paper addresses several issues including robust semipassivity and practical stability analysis for a switched nonlinear system with structural uncertainties. First, robust semipassivity is firstly defined to describe the overall semipassivity property of switched nonlinear systems without requiring the conventional semipassivity property of each active subsystem. Then, the robust semipassive system is shown to be practically stable. Second, a state-dependent switching law is designed to render the switched system robust semipassive. This switching law is a generalization of KYP lemma. Third, robust semipassification for a switched nonlinear system is achieved by the design of a state-dependent switching law and a set of feedback controllers. Finally, a composite state-dependent switching law is designed to render the feedback interconnection of switched nonlinear systems robust semipassive. The effectiveness of the obtained resultsis verified by two numerical examples.

INDEX TERMS Switched nonlinear system, semipassivity, practical stability, robust semipassification.

I. INTRODUCTION

The passivity first proposed by Willems [1] is a useful tool for the analysis and design of nonlinear systems. Since the storage function of a passive system can be selected as a Lyapunov function candidate, a passive system was stabilized by a simple output feedback controller [2]–[6]. Many uncertainties exist in the real world, such as external disturbances, unknown parameters, structural uncertainties. In the past, robust control was often used to address structural uncertainties. Problems concerning the robust passivity, robust passification and stabilization were solved by combining the robust control with passivity theory in [7], [8]. However, it is difficult to achieve passivity or exact feedback passification due to the large number of system uncertainties. The system can be feedback passive outside a ball containing the origin. To describe this property, the semipassivity concept was proposed in [9], [10]. Many physical and biological systems are semipassive [9]–[12]. Compared to passive systems, semipassive systems may produce energy itself. Similar concepts have been proposed, such as almost passivity [13], quasi-passivity [14]–[17] and set passivity [18], [19]. The conditions of quasi-passivity and quasi-passification and the ultimate boundedness of state trajectories of uncertain nonlinear systems were obtained in [14]–[17].

On the other hand, switched systems have received much attention from the control community due to their practical and theoretical applications [20]–[31]. A switched system is composed of a finite number of subsystems and a switching signal. Many practical systems can be effectively modeled as switched systems. Such systems include robotic, mechanical systems, gene regulatory networks, switching power converters, and so on [20]–[26]. Therefore, the analysis and synthesis problems of switched nonlinear systems, especially stability and passivity analysis, have been widely studied in [32]–[35]. For switched nonlinear systems with the structural uncertainties, robust passivity, and robust stabilization problems have been investigated in [35], [37]. So far, there have been no results on semipassivity. However, similar property (i.e. quasi-passivity) of switched nonlinear systems has been investigated in [38]–[40]. In [38], practical stability was achieved by the average dwell time method under the condition that at least one subsystem was quasi-passive. A quasi-passivity concept of switched systems was firstly defined using multiple storage functions and multiple supply rates in [41], where each active subsystem was required to

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be quasi-passive. The ultimate boundedness of trajectories for quasi-passive switched systems are obtained for a given switching signal. A more general quasi-passivity concept was given in [42]. This quasi-passivity property allowed some active subsystem be non-quasipassive. A semipassive system behaves as a passive system outside a sufficiently large ball. Thus, the globally practical stability can be obtained by a simple output feedback. However, a switched system does not necessarily inherit properties of the individual subsystems. Multi-model switching may lead to undesirable or even unbounded trajectories, even if each subsystem is semipassive. To the best of our knowledge, when each subsystem is not semipassive, the problems of semipassivity, semipassification and stabilization for switched nonlinear systems have been not fully investigated and remain open and challenging. This motivates the present study.

In this paper, we will study the robust semipassivity and robust semipassification and practical stability of switched nonlinear systems with structural uncertainties. Compared with the existing results, this paper makes four main contributions. First, a semipassivity concept for a switched system is proposed. This passivity property describes the overall semipassivity property of switched nonlinear systems without requiring the conventional semipassivity property of each active subsystem. Compared with robust passivity, robust semipassivity means that each active subsystem is required to be passive outside some ball. Thus, globally practical stability is obtained. Second, a more general state-dependent switching law is designed to render the switched system robust semipassive. This result is a generalization of the robust KYP lemma in [8]. Compared with the well-known min-switching law proposed in [41], which requires the subsystem corresponding to the smallest Lyapunov function was actived, the designed switching law means subsystem corresponding to the smallest continuous function is selected. This provides more freedom for the design of the switching law. Third, robust semipassification for uncertain switched nonlinear systems is achieved by the design of controllers and a state-dependent switching law. Finally, the semipassivity property is shown to be preserved for feedback interconnected switched nonlinear systems by the design of a composite state-dependent switching law, while practical stability is not preserved in general. This switching law allows interconnected switched systems to switch asynchronously.

Notation: $\|x\| = \left(x^T x\right)^{\frac{1}{2}} = \left(\sum_{i=1}^{n} |x_i|^2\right)^{\frac{1}{2}}$: the norm of a vector $(x_1, x_2 \ldots, x_n)^T$; C^1 functions: continuously differentiable functions. Class K_∞ functions γ: $R^+ \rightarrow R^+$, continuous, strictly increasing functions with $\gamma(0) = 0$ and $\gamma(r) \rightarrow \infty$ as $r \rightarrow \infty$

II. PRELIMINARIES AND PROBLEM FORMULATION

The switched nonlinear system under consideration is described by

$$\begin{aligned} \dot{x} &= f_\sigma(x) + \Delta f_\sigma(x) + g_\sigma(x) u_\sigma \\ y &= h_\sigma(x) \end{aligned} \tag{1}$$

for the state $x(t) \in R^n$ and the switching signal $\sigma(t) : [0, \infty) \rightarrow I = \{1, 2, \cdots, M\}$, which is assumed to be a piecewise constant function depending on time. $u_i \in R^m$ and $h_i(x) \in R^m$ denote the input and output vectors of the i-th subsystem, respectively, and $f_i(x)$, $h_i(x)$, $g_i(x)$ are smooth functions. The structural uncertainty of the i-th subsystem is characterized by $\Delta f_i(x) = e_i(x)\delta_i(x)$, in which $e_i : R^n \rightarrow R^{n\times m}$ is a known matrix whose entries are smooth functions of the state and $\delta_i : R^n \rightarrow R^m$ is an unknown vector-valued function belonging to

$$\Pi_i = \{\delta_i(x) | \, \|\delta_i(x)\| \le \Gamma_i(x), \delta_i(0) = 0\} \tag{2}$$

for a given smooth function $\Gamma_i : R^n \rightarrow R^+$. The switching signal can be characterized by the following switching sequence

$$\Sigma = \{x_0; (i_0, t_0), (i_1, t_1), \ldots, (i_k, t_k), \ldots, | i_k \in I \,, k \in N\}, \tag{3}$$

in which t_0, x_0 and N denote the initial time, the initial state, and the set of nonnegative integers, respectively. When $t \in [t_k, t_{k+1})$, $\sigma(t) = i_k$, i.e. the i_k-th subsystem is active. The assumptions on $f_i(x)$, $h_i(x)$, $\Delta f_i(x)$ ensure existence and uniqueness of the trajectory of system (1) for all initial conditions and switching signals.

Assumption 1 [20]: σ has finite number of switchings on any finite time interval.

Remark 1: This assumption is commonly adopted by [20], [26], [30], [32] to rule out Zeno behavior, which is obviously unacceptable in practice.

Now, we give the definition of robust semipassivity.

Definition 1: System (1) is said to be robust semipassive under the switching signal $\sigma(t)$ if there exists a nonnegative function $S(\sigma(t), x) \; : I \times R^n \rightarrow R^+$, called a storage function, such that

$$\begin{aligned} &S(\sigma(t), x(t)) - S(\sigma(t_0), x(t_0)) \\ &\quad \le \int_{t_0}^{t} y^T u_{\sigma(\tau)}(\tau) d\tau - \int_{t_0}^{t} H_{\sigma(\tau)}(x(\tau)) \, d\tau \end{aligned} \tag{4}$$

holds for all admissible Δf_σ, x_0 and $\forall u_i, t_0 \le t < \infty$, where the continuous function $H_i(x)$ is nonnegative outside the ball with radius ρ, i.e., $\exists \rho > 0$, such that $\|x\| \ge \rho \Rightarrow H_i(x) \ge Q(\|x\|)$ for some nonnegative continuous function $Q(\bullet)$ defined for all $\|x\| \ge \rho$. If $Q(\bullet)$ is positive definite for all $\|x\| \ge \rho$, then, system (1) is said to be robust strictly semipassive.

Remark 2: A semipassive system behaves similar to a passive system for large enough $\|x\|$. If $\rho = 0$ then Definition 1 can degenerate into robust passivity definition of switched nonlinear systems in [36]. Inequality (4) shows that the total stored energy is no more than the total supplied energy from outside and inside the system in any finite interval $[t_0, t]$. This implies that some subsystems may not be semipassive on active time intervals. If the storage functions are common, Definition 1 reduces to the traditional semipassivity definition in [9]–[12].

The concept of practical stability is defined as follows.

Definition 2: System (1) is globally practically stable if for any given constant $\delta \geq 0$, a set of controllers u_i, $i \in I$ and a switching signal $\sigma(t)$ can be designed such that the closed-loop system possesses the following properties

(a) (Uniform boundedness) there exists $\varepsilon > 0$ such that for all $t_0 \geq 0$, $\|x_0\| < \delta$ implies $\|x(t)\| < \varepsilon$.

(b) (Uniform ultimate boundedness) for every initial condition $x(t_0) = x_0$, there is a constant $T = T(x_0, R) \geq 0$ such that $x(t) \in B_R \triangleq \{x \in R^n \mid \|x\| \leq R\}$ holds for $t \geq t_0 + T$.

This paper will investigate robust semipassivity, robust semipassification and globally practical stability for system (1).

III. PRACTICAL STABILITY ANALYSIS

This section will show that robust semipassive switched nonlinear system is globally practically stable.

Theorem 1: Suppose that there exists a storage function $S(\sigma(t), x) = S_{\sigma(t)}(x)$ with nonnegative continuous functions $S_i(x), i \in I$ such that system (1) is robust semi-passive under the switching signal $\sigma(t)$.i) If there exist class K_∞ functions $\beta_1, \ \beta_2$ satisfying

$$\beta_1(\|x\|) \leq S_i(x) \leq \beta_2(\|x\|) \tag{5}$$

for all $i \in I$ then system (1) with $u_i = 0$ is uniformly bounded. ii) If, in addition, system (1) is robust strictly semipassive, then system (1) is globally practically stable.

Proof: i)Uniform boundedness

Consider system (1) with $u_i = 0$. For any $t > t_0$, there exists positive integer k such that $t \in [t_k, t_{k+1})$. It follows from (4) and the conditions of Theorem 1 that

$$\begin{aligned} &S(\sigma(t), x(t)) - S(\sigma(t_0), x(t_0)) \\ &\quad = S_{i_k}(x(t)) - S_{i_0}(x(t_0)) \\ &\quad \leq \int_{t_0}^{t} -H_\sigma(x(\tau))\, d\tau. \end{aligned} \tag{6}$$

First, let $0 < \delta \leq \rho$ and $\|x_0\| \leq \delta$. If $\|x(t)\| \leq \rho, t \geq t_0$, then it follows from (5) that $\|x(t)\| \leq \rho \leq \beta_1^{-1}(\beta_2(\rho)) \leq \beta_1^{-1}(\eta), t \geq t_0, \eta \geq \beta_2(\rho)$. Alternatively, if there exists $T > t_0$ such that $\|x(T)\| > \rho$, then it follows from the continuity of $x(t)$ that there exists $\tau < T$ such that $\|x(\tau)\| = \rho$ and $\|x(t)\| \geq \rho, t \in [\tau, T]$. Since (5) and (6) hold, we have

$$\beta_1(\|x(t)\|) \leq S_{i_k}(x(t)) \leq S_{i_0}(x(t_0)) \leq \beta_2(\delta),$$

which implies that $\|x(t)\| \leq \beta_1^{-1}(\beta_2(\rho)) \leq \beta_1^{-1}(\eta)$. Next, let $\delta > \rho$, assume $\rho < \|x(t_0)\| < \delta$. For every $\bar{t} > 0$ such that $\|x(t)\| \geq \rho, t \in [t_0, \bar{t}]$. Since (5) and (6) hold, we have $\beta_1(\|x(t)\|) \leq S_{i_k}(x(t)) \leq S_{i_0}(x(t_0)) \leq \beta_2(\delta)$, which implies that $\|x(t)\| \leq \beta_1^{-1}(\beta_2(\delta)), t \in [t_0, \bar{t}]$. Next, if there exists $T > t_0$ such that $\|x(T)\| \leq \rho$, then it follows as in the proof of the first case above that $\|x(t)\| \leq \beta_1^{-1}\left(\beta_2^{-1}(\delta)\right)$. Hence, system (1) with $u_i = 0$ is uniformly bounded.

ii) Uniform ultimate boundedness

First, let $0 < \delta \leq \rho$ and $\|x_0\| \leq \delta$. As in the proof of i), we have $\|x(t)\| \leq \rho \leq \beta_1^{-1}(\beta_2(\rho)) \leq \beta_1^{-1}(\eta), t \geq t_0$.

Next, let $\delta > \rho$, assume $\rho < \|x(t_0)\| < \delta$. In this case, it follows from i) that $\|x(t)\| \leq \beta_1^{-1}(\beta_2(\delta)), t \geq t_0$.

Suppose, ad absurdum, that $\|x(t)\| \geq \beta_2^{-1}(\eta) > \rho$, $t \geq t_0$,. i.e., $\beta_2^{-1}(\eta) \leq \|x(t)\| \leq \beta_1^{-1}(\beta_2(\delta)), t \geq t_0$. Since $Q(x)$ is a continuous function, according to Weierstrass' theorem, $k = \min Q(x) > 0$ exists. Therefore,

$$S_{i_k}(x(t)) \leq -k(t - t_0) + S_{i_0}(x(t_0)),$$

which implies that

$$\beta_1(\|x(t)\|) \leq S_{i_k}(x(t)) \leq S_{i_0}(x(t_0)) \leq \beta_2(\delta) - k(t - t_0)$$

It Now, let $t > \beta_2(\delta)/_k + t_0$. It follows that $\beta_1(\|x(t)\|) < 0$, which is a contradiction. Hence, there exists $T = T(\delta, \eta) > 0$ such that $\|x(T)\| < \beta_2^{-1}(\eta)$. Thus, from the proof of i), we have $\|x(t)\| \leq \beta_1^{-1}\left(\beta_2\left(\left(\beta_2^{-1}(\eta)\right)\right)\right) = \beta_1^{-1}(\eta)$, $t \geq T$, which implies system (1) is ultimately bounded. Therefore, system (1) is globally practically stable.

Remark 3: The globally practical stability is obtained in Theorem 1, even if each subsystem is not semipassive. This implies that the conditions given in Theorem 1 are weaker than non-switched result in [11], [12]. When $\rho = 0$, the globally practical stability result can degenerate into the asymptotic stability result in [36]. Therefore, Theorem 1 is a generalization of stability analysis result in [36].

IV. ROUST SEMIPASSIVITY

This section will give a sufficient condition for system (1) to be robust semipassive by the design of a state-dependent switching law. It is a generalization of the "min-switching" law.

Theorem 2: Suppose that there exist C^1 nonnegative functions $S_i(x)$, class K_∞ functions Q_i, continuous functions $V_i(x)$, $\eta_{ij}(x), \beta_{ij}(x) \leq 0$, constants $\alpha_i \geq 0, \lambda_i > 0$ for $i, j \in I$, such that the following inequalities hold

$$\begin{aligned} &L_{f_i} S_i(x) + \left\| \left(L_{e_i} S_i(x)\right)^T \right\| \Gamma_i(x) \\ &\quad + \sum_{j=1}^{M} \beta_{ij}(x) \left(V_i(x) - V_j(x)\right) \leq \alpha_i - Q_i(\|x\|), \end{aligned} \tag{7}$$

$$L_{g_i} S_i(x) = h_i^T(x), \tag{8}$$

$$S_i(x) - S_j(x) = \eta_{ij}(x) \left(V_i(x) - V_j(x)\right). \tag{9}$$

Design the switching law as

$$\sigma(t) = \arg\min_{i \in I} \{V_i(x)\}. \tag{10}$$

Then, system (1) is robust semipassive under the switching law (10).

Proof: Differentiating $S_i(x)$ along the i-th subsystem of system (1) together with (7) and (10) gives

$$\begin{aligned} \dot{S}_i &= \frac{\partial S_i}{\partial x}\left(f_i(x) + \Delta f_i(x) + g_i(x) u_i\right) \\ &\leq L_{f_i} S_i(x) + \left\| \left(L_{e_i} S_i(x)\right)^T \right\| \Gamma_i(x) + y^T u_i \\ &\leq -Q_i + \alpha_i + y^T u_i - \sum_{j=1}^{M} \beta_{ij}(x) \left(V_i(x) - V_j(x)\right). \end{aligned} \tag{11}$$

According to the switching law (10), the switching sequence is described as (3) with the property $V_{i_{k+1}}(x(t_{k+1})) = V_{i_k}(x(t_{k+1}))$.

From (9), we have

$$S_{i_{k+1}}(x(t_{k+1})) = S_{i_k}(x(t_{k+1})). \tag{12}$$

When $t \in [t_p, t_{p+1})$, the i_p-th subsystem is active, i.e., $\sigma(t) = i_p$, so (11) implies

$$\dot{S}_{i_p} \le -Q_{i_p} + \alpha_{i_p} + y^T u_{i_p}. \tag{13}$$

Integrating (13) over $[s, t]$ for $t_{p+1} > t > s \ge t_p$ yields

$$S_{i_p}(x(t)) - S_{i_p}(x(s)) \le \int_s^t [(\alpha_{i_p} + y^T u_{i_p} - Q_i)d\tau. \tag{14}$$

Define the storage function as $S(\sigma(t), x) = S_{\sigma(t)}(x)$. For $t_0 \le t < \infty$, we can find some integer k satisfying $t \in [t_k, t_{k+1})$. From (12) and (14), we have

$$\begin{aligned} &S(\sigma(t), x(t)) - S(\sigma(t_0), x(t_0)) \\ &= S_{i_k}(x(t)) - S_{i_k}(x(t_k)) \\ &\quad + \sum_{p=0}^{k-1} \left(S_{i_p}(x(t_{p+1})) - S_{i_p}(x(t_p))\right) \\ &\quad + \sum_{p=1}^{k} \left(S_{i_p}(x(t_p)) - S_{i_{p-1}}(x(t_p))\right) \\ &\le \int_{t_0}^t (\alpha_{\sigma(\tau)} + y^T u_{\sigma(\tau)} - Q_{\sigma(\tau)}(\|x(\tau)\|))d\tau. \end{aligned} \tag{15}$$

Let $H_i(x) = Q_i(\|x\|) - \alpha_i$. Therefore, system (1) is robust semipassive under the switching law (11).

Remark 4: Inequality (7) and equation (8) imply that each subsystem of system (1) is not required to be semipassive. If $\alpha_i = 0$, $\beta_{ij} = 0$, (7) and (8) imply that the robust KYP conditions in [8] hold on the active time interval. Therefore, (7) and (8) are weaker than the robust KYP conditions. When $V_i = S_i$, the switching law (11) reduces to the well-known "min-switching" law.

V. FEEDBACK SEMIPASSIFICATION

This section will solve the robust semipassification problem by designing a state-dependent switching law and feedback controllers for the switched nonlinear systems.

Consider system (1) with the common output $y = h(x)$.

$$\begin{aligned} \dot{x} &= f_\sigma(x) + \Delta f_\sigma(x) + g_\sigma(x) u_\sigma, \\ y &= h(x). \end{aligned} \tag{16}$$

System (16) is said to be robust feedback semipassive if state feedback controllers $u_i = \alpha_i(x) + \beta_i(x) w_i, i \in I$ and a switching law σ for system (16) can be designed such that the resulting closed-loop system is robust semipassive from w_i to y.

Under some hypotheses in [36], we can find a global diffeomorphism $T(x) = (z^T, y^T)^T = (\phi^T(x), h^T(x))^T$ and feedback controllers $u_i = [a_i(z, y)]^{-1}[v_i - b_i^0(z, y)]$ that transform each subsystem of (16) into robust version of the normal form in [8]:

$$\begin{aligned} \dot{z} &= f_i^0(z, y) + f_i^1(z, y)\tilde{\delta}_i(z, y), \\ \dot{y} &= v_i + b_i^1(z, y)\tilde{\delta}_i(z, y). \end{aligned} \tag{17}$$

Thus, (16) can be presented as follows:

$$\begin{aligned} \dot{z} &= f_\sigma^0(z, y) + f_\sigma^1(z, y)\tilde{\delta}_\sigma(z, y), \\ \dot{y} &= v_\sigma + b_\sigma^1(z, y)\tilde{\delta}_\sigma(z, y), \end{aligned} \tag{18}$$

where $f_i^0, f_i^1, \tilde{\delta}_i$ are smooth and $\left\|\tilde{\delta}_i(z, y)\right\| \le \Gamma_i(T^{-1}(z, y)) = \tilde{\Gamma}_i(z, y)$.

Now, we will provide sufficient conditions of semipassification in the following theorem.

Theorem 3: Consider system (18). Suppose that for any $i, j \in I$, there are continuous functions $U_i(z)$, nonnegative smooth functions $W_i(z)$, class K_∞ functions Q_i, continuous functions $\beta_{ij}(z) \le 0$, $\gamma_i(z) > 0$, $\theta_{ij}(z)$ and constants $c_i \ge 0$ satisfying

$$\begin{aligned} &L_{f_i^0(z,0)} W_i(z) + \frac{\gamma_i(z)}{2}\left\|L_{f_i^1(z,0)} W_i(z)\right\|^2 + \frac{1}{2\gamma_i(z)}\tilde{\Gamma}_i^2(z, 0) \\ &\quad + \sum_{j=1}^M \beta_{ij}(z)\left(U_i(z) - U_j(z)\right) \le c_i - Q_i(\|z\|), \end{aligned} \tag{19}$$

$$W_i(z) - W_j(z) = \theta_{ij}(z)\left(U_i(z) - U_j(z)\right). \tag{20}$$

Then, the robust semipassification problem of system (16) is solvable.

Proof: As in [8], we have

$$\begin{aligned} f_i^0(z, y) &= f_i^0(z, 0) + \tilde{f}_i^0(z, y)y, \\ L_{f_i^1} W_i(z) &= L_{f_i^1(z,0)} W_i(z) + y^T \tilde{f}_i^1(z, y), \\ \tilde{\Gamma}_i(z, y) &= \tilde{\Gamma}_i(z, 0) + P_i(z, y)y. \end{aligned}$$

Design the controllers as

$$v_i = B_i(z, y) - \frac{1}{2}y + w_i, \tag{21}$$

where $B_i(z, y) = -\left[L_{\tilde{f}_i^0} W_i(z)\right]^T - \frac{\gamma_i(z)}{2} B_i^1(z, y) - \frac{1}{2\gamma_i(z)} B_i^2(z, y)$ with

$$\begin{aligned} B_i^1(z, y) &= \left(\tilde{f}_i^1(z, y) + b_i^1(z, y)\right)\left(\tilde{f}_i^1(z, y) + b_i^1(z, y)\right)^T y \\ &\quad + 2\left(\tilde{f}_i^1(z, y) + b_i^1(z, y)\right)\left(L_{f_i^1(z,0)} W_i(z)\right)^T, \\ B_i^2(z, y) &= P_i(z, y)\left[2\tilde{\Gamma}_i(z, 0) + P_i(z, y)y\right]. \end{aligned}$$

Let $S_i(z, y) = W_i(z) + \frac{1}{2}y^T y, i \in I$. Differentiating $S_i(z, y), i \in I$ together with (19) and (21) gives

$$\begin{aligned} &\dot{S}_i(z, y) \\ &= L_{f_i^0(z,0)} W_i(z) + y^T\left(\left[L_{\tilde{f}_i^0(z,y)} W_i(z)\right]^T + v_i\right) \\ &\quad + \left(L_{f_i^1(z,0)} W_i(z) + y^T\left(b_i^1(z, y) + \tilde{f}_i^1(z, y)\right)\right)\tilde{\delta}_i(z, y) \end{aligned}$$

$$
\begin{aligned}
&\leq L_{f_i^0(z,0)} W_i(z) + y^T \left(\left[L_{\tilde{f}_i^0(z,y)} W_i(z) \right]^T + v_i \right) \\
&\quad + \frac{\gamma_i(z)}{2} \left\| \left(L_{f_i^1(z,0)} W_i(z) + y^T \left(b_i^1(z,y) + \tilde{f}_i^1(z,y) \right) \right) \right\|^2 \\
&\quad + \frac{1}{2\gamma_i(z)} \left(\tilde{\Gamma}_i(z,0) + P_i(z,y)\, y \right)^2 \\
&\leq L_{f_i^0(z,0)} W_i(z) + \frac{\gamma_i(z)}{2} \left\| L_{f_i^1(z,0)} W_i(z) \right\|^2 + \frac{1}{2\gamma_i(z)} \tilde{\Gamma}_i^2(z,0) \\
&\quad + y^T \left(\left[L_{\tilde{f}_i^0(z,y)} W_i(z) \right]^T + \frac{\gamma_i(z)}{2} B_i^1(z,y) + \frac{1}{2\gamma_i(z)} B_i^2(z,y) + v_i \right) \\
&= L_{f_i^0(z,0)} W_i(z) + \frac{\gamma_i(z)}{2} \left\| L_{f_i^1(z,0)} W_i(z) \right\|^2 + \frac{1}{2\gamma_i(z)} \tilde{\Gamma}_i^2(z,0) \\
&\quad + y^T \left(\left[L_{\tilde{f}_i^0(z,y)} W_i(z) \right]^T + \frac{\gamma_i(z)}{2} B_i^1(z,y) + \frac{1}{2\gamma_i(z)} B_i^2(z,y) \right. \\
&\quad \left. + B_i(z,y) + w_i - \frac{1}{2} y \right) \\
&= L_{f_i^0(z,0)} W_i(z) + \frac{\gamma_i(z)}{2} \left\| L_{f_i^1(z,0)} W_i(z) \right\|^2 \\
&\quad + \frac{1}{2\gamma_i(z)} \tilde{\Gamma}_i^2(z,0) + y^T w_i - \frac{1}{2} y^T y \\
&\leq -\sum_{j=1}^{M} \beta_{ij}(z) \left(U_i(z) - U_j(z) \right) + c_i - \left(Q_i(z) + \frac{1}{2} y^T y \right) \\
&\leq -\sum_{j=1}^{M} \tilde{\beta}_{ij}(z,y) \left(V_i(z,y) - V_j(z,y) \right) + c_i - \tilde{Q}_i(z,y),
\end{aligned}
$$

where $\tilde{\beta}_{ij}(z,y) = \beta_{ij}(z)$, $V_i(z,y) = U_i(z) + \frac{1}{2} y^T y$ and $\tilde{Q}_i(z,y) = Q_i(z) + \frac{1}{2} y^T y$ are class K_∞ functions.

Since (20) holds, we have

$$S_i^1(z,y) - S_j^1(z,y) = \theta_{ij}(z) \left(V_i(z,y) - V_j(z,y) \right). \tag{22}$$

Design the switching law as $\sigma(t) = \arg\min_{i \in I} \{V_i(z,y)\}$. The rest of the proof is similar to that of Theorem 2.

Remark 5: When $U_i = W_i$, $c_i = \lambda_i = 0$, these conditions reduce to the robust passification conditions of switched nonlinear systems in [36].

VI. SEMIPASSIVITY OF INTERCONNECTED SWITCHED SYSTEMS

In this section, a composite state-dependent switching law is designed to render the feedback interconnection of uncertain switched nonlinear systems robust semipassive.

Consider the switched systems

$$
H_1 : \begin{aligned} \dot{x}^1 &= f_{\sigma_1}^1(x^1) + \Delta f_{\sigma_1}^1(x^1) + g_{\sigma_1}^1(x^1)\, u_{\sigma_1}^1, \\ y^1 &= h_{\sigma_1}^1(x^1), \end{aligned} \tag{23}
$$

where $x^1 \in R^{n_1}$ is the state and $\sigma_1(t) : [0,\infty) \to I_1 = \{1, 2, \cdots M_1\}$ denotes the switching signal of system (23). The switching sequence is described as

$$\Sigma_1 = \{(i_0^1, t_0^1), (i_1^1, t_1^1), \ldots (i_{j^1}^1, t_{j^1}^1), \ldots \big| i_{j^1}^1 \in I_1, j^1 \in N\}$$

and

$$
H_2 : \begin{aligned} \dot{x}^2 &= f_{\sigma_2}^2(x^2) + \Delta f_{\sigma_2}^2(x^2) + g_{\sigma_2}^2(x^2)\, u_{\sigma_2}^2, \\ y^2 &= h_{\sigma_2}^2(x^2), \end{aligned} \tag{24}
$$

in which $x^2 \in R^{n_2}$ is the state; $\sigma_2(t) : [0,\infty) \to I_2 = \{1, 2, \cdots M_2\}$ denotes the switching signal of system (24). The switching sequence is

$$\Sigma_2 = \{(i_0^2, t_0^2), (i_1^2, t_1^2), \ldots, (i_{j^2}^2, t_{j^2}^2), \ldots \big| i_{j^2}^2 \in I_2, j^2 \in N\}.$$

Moreover, $\Delta f_{i^q}^q(x) = e_{i^q}^q(x)\, \delta_{i^q}^q(x)$, $q = 1, 2$, where $e_{i^q}^q$ is a known matrix whose entries are smooth functions of the state, $\left\| \delta_{i^q}^q(x) \right\| \leq \Gamma_{i^q}^q(x)$, $\delta_{i^q}^q(0) = 0$ is an unknown vector-valued function and $\Gamma_{i^q}^q : R^{n^q} \to R^+$ is a given smooth function. The feedback interconnection of H_1 and H_2 is shown in Fig. 1.

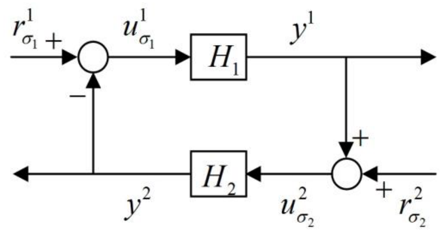

FIGURE 1. Feedback interconnection.

This interconnected switched system H is formed through $u_{\sigma_1}^1 = r_{\sigma_1}^1 - y^2$, $u_{\sigma_2}^2 = r_{\sigma_2}^2 + y^1$, where $\dim r_{\sigma_2}^2 = \dim h_{\sigma_1}^1 = \dim u_{\sigma_2}^2$ and $\dim r_{\sigma_1}^1 = \dim h_{\sigma_2}^2 = \dim u_{\sigma 1}^1$. The input and output of the interconnection system H are $u_\sigma = \begin{pmatrix} r_{\sigma_1}^1 \\ r_{\sigma_2}^2 \end{pmatrix}$ and $y = \begin{pmatrix} y^1 \\ y^2 \end{pmatrix}$. The switching signal is created by the merging switching signal technique, i.e.$\sigma = \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix} : [0,\infty) \to I = I_1 \times I_2 = \left\{ (i^1, i^2) \,\middle|\, i^1 \in I_1; i^2 \in I_2 \right\}$. Thus, system H has $M_1 \cdot M_2$ subsystems. The switching sequence generated by the composite switching signal is described as

$$\Sigma = \left\{ (i_0, t_0), (i_1, t_1), \cdots (i_j, t_j), \cdots \big| i_j \in I, j \in N \right\}, \tag{25}$$

where $t_0 = t_0^1 = t_0^2$, $i_j = (\sigma_1(t_j), \sigma_2(t_j)) = \left(i_{j^1}^1, i_{j^2}^2 \right)$.

In the following, we will discuss robust semipassivity of feedback interconnected system H.

Theorem 4: Suppose that there exist nonnegative smooth functions $V_{i^q}^q(x^q)$, continuous functions $S_{i^q}^q(x^q)$, class K_∞ functions $Q_{i^q}^q$, functions $\beta_{i^q j^q}^q(x^q) \leq 0$, $\delta_{i^q j^q}^q(x^q) \leq 0$, $\eta_{i^q j^q}^q(x^q)$, smooth functions $\mu_{i^q j^q}^q(x^q)$ with $\mu_{i^q i^q}^q(x^q) = 0$, $\mu_{i^q j^q}^q(0) = 0$, $\nu_{i^q j^q}^q(x^q)$ with $\nu_{i^q j^q}^q(0) = 0$, $\nu_{i^q j^q}^q(x^q) = 0$, constants $\alpha_{i^q}^q \geq 0$, $\lambda_{i^q}^q > 0$ such that

$$
\begin{aligned}
&\frac{\partial S_{i^q}^q}{\partial x^q} f_{i^q}^q(x^q) \\
&\quad + \left\| \left(L_{e_{i^q}^q} S_{i^q}^q(x) \right)^T \right\| \Gamma_{i^q}^q(x^q) + Q_{i^q}^q\left(\left\| x^q \right\| \right) \\
&\quad + \sum_{j^1=1}^{M_1} \beta_{i^q j^q}^q(x^q) \left(V_{i^q}^q(x^q) - V_{i^q}^q(x^q) \right) \leq \alpha_{i^q}^q,
\end{aligned} \tag{26}
$$

$$L_{g_{i^q}^q} S_{i^q}^q(x) = h_{i^q}^{qT}(x), \tag{27}$$

$$\left(S_{i^q}^q(x^q) - S_{i^q}^q(x^q) \right) = \eta_{i^q j^q}^q(x^q) \left(V_{i^q}^q(x^q) - V_{i^q}^q(x^q) \right) \tag{28}$$

hold for all $i^q, j^q \in I_q$, $q = 1, 2$. Design the switching law as

$$\sigma(t) = \left(\sigma_1\left(x^1(t)\right), \sigma_2\left(x^2(t)\right)\right), \tag{29}$$

where $\sigma_q(x^q(t)) = \arg\min_{i^q \in I_q}\left\{V_{i^q}^q(x^q)\right\}, q = 1, 2$. Then, the feedback interconnected system H is robust semipassive under the switching law (29).

Proof: Let $S_{(i^1,i^2)}\left(x^1, x^2\right) = S_{i^1}^1\left(x^1\right) + S_{i^2}^2\left(x^2\right)$, $\left(i^1, i^2\right) \in I$.

Differentiating $S_{(i^1,i^2)}\left(x^1, x^2\right)$ gives

$$\begin{aligned}
&\dot{S}_{(i^1,i^2)} \\
&= \frac{\partial S_{i_1}^1}{\partial x^1}\left(f_{i_1}^1\left(x^1\right) + \Delta f_{i_1}^1\left(x^1\right) + g_{i_1}^1\left(x^1\right)u_{i_1}^1\right) \\
&\quad + \frac{\partial S_{i_2}^2}{\partial x^2}\left(f_{i_2}^2\left(x^2\right) + \Delta f_{i_2}^2\left(x^2\right) + g_{i_2}^2\left(x^2\right)u_{i_2}^2\right) \\
&\le L_{f_{i_1}^1} S_{i_1}^1\left(x^1\right) + \left\|\left(L_{e_{i_1}^1} S_{i_1}^1\left(x^1\right)\right)^T\right\| \Gamma_{i_1}^1\left(x^1\right) + y^{1T}u_{i_1}^1 \\
&\quad + L_{f_{i_2}^2} S_{i_2}^2\left(x^2\right) + \left\|\left(L_{e_{i_2}^2} S_{i_2}^2\left(x^2\right)\right)^T\right\| \Gamma_{i_2}^2\left(x^2\right) + y^{2T}u_{i_2}^2 \\
&\le -\sum_{j^1=1}^{M_1} \beta_{i^1j^1}^1\left(V_{i^1}^1\left(x^1\right) - V_{j^1}^1\left(x^1\right)\right) \\
&\quad - \sum_{j^2=1}^{M_2} \beta_{i^2j^2}^2\left(V_{i^2}^2\left(x^2\right) - V_{j^2}^2\left(x^2\right)\right) \\
&\quad - Q_{i^1}^1\left(\left\|x^1\right\|\right) + y^{1T}u_{i^1}^1 + \alpha_{i^1}^1 - Q_{i^2}^2\left(\left\|x^2\right\|\right) + y^{2T}u_{i^2}^2 + \alpha_{i^2}^2.
\end{aligned} \tag{30}$$

Substituting $u_{i^1}^1 = r_{i^1}^1 - y^2$, $u_{i^2}^2 = r_{i^2}^2 + y^1$ into (30) gives

$$\begin{aligned}
&\dot{S}_{(i^1,i^2)} \\
&\le -Q_{(i^1,i^2)} + y^{1T}r_{i^1}^1 + y^{2T}r_{i^2}^2 + \alpha_{i^1}^1 + \alpha_{i^2}^2 \\
&\quad - \sum_{j^1=1}^{M_1} \beta_{i^1j^1}^1\left(V_{i^1}^1\left(x^1\right) - V_{j^1}^1\left(x^1\right)\right) \\
&\quad - \sum_{j^2=1}^{M_2} \beta_{i^2j^2}^2\left(V_{i^2}^2\left(x^2\right) - V_{j^2}^2\left(x^2\right)\right) \\
&\le -Q_{(i^1,i^2)} + y^T u_{(i^1,i^2)} + \alpha_{(i^1,i^2)} \\
&\quad - \sum_{j^1=1}^{M_1} \beta_{i^1j^1}^1\left(V_{i^1}^1\left(x^1\right) - V_{j^1}^1\left(x^1\right)\right) \\
&\quad - \sum_{j^2=1}^{M_2} \beta_{i^2j^2}^2\left(V_{i^2}^2\left(x^2\right) - V_{j^2}^2\left(x^2\right)\right),
\end{aligned} \tag{31}$$

where $\alpha_{(i^1,i^2)} = \alpha_{i^1}^1 + \alpha_{i^2}^2$, $u_{(i^1,i^2)} = \left(r_{i^1}^1, r_{i^2}^2\right)^T$, $y = \left(y^1, y^2\right)^T$, $Q_{(i^1,i^2)} = Q_{i^1}^1\left(\|x^1\|\right) + Q_{i^2}^2\left(\|x^2\|\right)$.

According to the switching law (29), the switching sequence is described as (3) with $i_k = (\sigma_1(t_k), \sigma_2(t_k)) = \left(i_{k^1}^1, i_{k^2}^2\right)$, $\forall k \in N$ and

$$V_{i_{k^q+1}^q}^q\left(x^q\left(t_{k^q+1}^q\right)\right) = V_{i_{k^q}^q}^q\left(x^q\left(t_{k^q+1}^q\right)\right), \quad q = 1, 2.$$

Thus,

$$S_{i_{k^q+1}^q}^q\left(x^q\left(t_{k^q+1}^q\right)\right) = S_{i_{k^q}^q}^q\left(x^q\left(t_{k^q+1}^q\right)\right), \quad q = 1, 2. \tag{32}$$

From (31), we have

$$\dot{S}_{i_k} \le -Q_{i_k} + y^{1T}u_{i_k} + \alpha_{i_k}. \tag{33}$$

The storage function of system H is chosen as

$$S\left(\sigma(t), x^1, x^2\right) = S_{\sigma(t)}\left(x^1, x^2\right) = S_{\sigma_1(t)}^1\left(x^1\right) + S_{\sigma_2(t)}^2\left(x^2\right).$$

For $t_0 \le t < \infty$ there exists nonnegative integer k such that $t \in [t_k, t_{k+1})$. From (32) and (43), we have

$$\begin{aligned}
&S(\sigma(t), x(t)) - S(\sigma(t_0), x(t_0)) \\
&\quad = S_{i_k}(x(t)) - S_{i_0}(x(t_0)) \\
&\quad = S_{i_k}(x(t)) - S_{i_k}(x(t_k)) \\
&\qquad + \sum_{p=0}^{k-1}\left(S_{i_p}\left(x\left(t_{p+1}\right)\right) - S_{i_p}\left(x\left(t_p\right)\right)\right) \\
&\qquad + \sum_{p=1}^{k}\left(S_{i_p}\left(x\left(t_p\right)\right) - S_{i_{p-1}}\left(x\left(t_p\right)\right)\right) \\
&\quad \le \int_{t_0}^{t}\left(y^T u_{\sigma(\tau)} + \alpha_{\sigma(\tau)} - Q_{\sigma(\tau)}\right)d\tau \\
&\qquad + \sum_{p=1}^{k_1}\left(S_{i_p^1}^1\left(x^1\left(t_p^1\right)\right) - S_{i_{p-1}^1}^1\left(x^1\left(t_p^1\right)\right)\right) \\
&\qquad + \sum_{p=1}^{k_2}\left(S_{i_p^2}^2\left(x^2\left(t_p^2\right)\right) - S_{i_{p-1}^2}^2\left(x^2\left(t_p^2\right)\right)\right) \\
&\quad \le \int_{t_0}^{t}\left(y^T u_\sigma - H_\sigma\right)d\tau,
\end{aligned}$$

where $\alpha_\sigma = \alpha_{\sigma_1}^1 + \alpha_{\sigma_2}^2$, $H_\sigma = \alpha_\sigma - Q_\sigma$.

Therefore, the feedback interconnected system H is semipassive under the switching law (29).

Remark 6: If $\beta_{i^qj^q}^q(x^q) = 0$, (26) and (27) imply that H_1 and H_2 are both semipassive. In Theorem 4, H_1 and H_2 are not required to be semipassive. However, H_1 and H_2 are both semipassive under the switching law (29). Thus, the interconnection of H_1 and H_2 are semipassive under the switching law (29). The designed switching law allows interconnected switched systems to switch asynchronously. This provides more design freedom. This result is generalization of the semipassivity Theorem of non-switched system in [12].

VII. EXAMPLES

In this section, we present two numerical examples to demonstrate the effectiveness of the results.

Example 1: Consider system (1) described by

$$
\begin{aligned}
&f_1(x) = \left(-x_1^3+2x_1+x_2+1,\ -x_1-10x_2\right)^T,\ \Delta f_1(x)=x\cos t,\\
&g_1(x) = (0, 0.5x_2)^T,\ g_2(x)=(2x_1, 0)^T,\ \Delta f_2(x)=0.5x\sin t,\\
&f_2(x) = \left(-7x_1-2x_2+4,\ 0.25x_2+x_1-x_2^5\right)^T\\
&\qquad y = \mathrm{h}_1(x) = x_2^2,\ y = \mathrm{h}_2(x) = 2x_1^2.
\end{aligned}
\tag{34}
$$

Let $S_1(x) = x_1^2 + x_2^2$ and $S_2(x) = 0.5x_1^2 + 2x_2^2$. Then, differenting $S_1(x)$ and $S_2(x)$ along the trajectory of system (34) yields

$$
\begin{aligned}
\dot S_1 &\le 16(S_1 - S_2) - \left(x_1^2 + 2x_2^2 - 1\right) + y^T u_1,\\
\dot S_2 &\le 6(S_2 - S_1) - \left(0.5x_1^2 + x_2^2 - 2\right) + y^T u_2.
\end{aligned}
$$

The switching law is designed as $\sigma(t) = \arg\min_{i=1,2}\{S_i(x)\}$. According to Theorem 2, system (34) is robust semipassive. Therefore, based on Theorem 1, system (34) with $u_i = 0$ is practically stable.

A simulation is conducted to verify our method. The simulation results are depicted in Figs.2-5 for the initial states $x(0) = (10.2, -12.5)$. In contrast with the system in [36], the state response of system (34) shown in Fig.4 converges into a ball. The switching signal is shown in Fig.5. Therefore, the closed-loop system is globally practically stable. The simulation results well illustrate the effectiveness of the proposed approach.

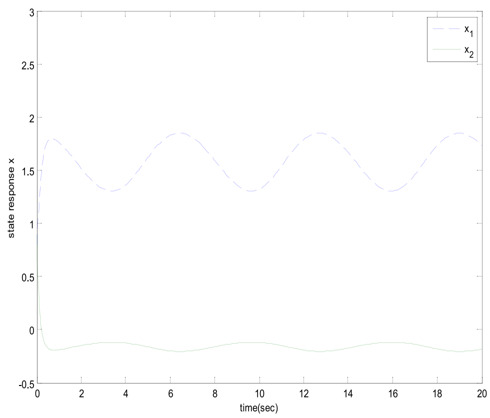

FIGURE 2. State response of subsystem1.

Example 2: Consider system (1) consisting of two subsystems described by

$$
\begin{aligned}
&f_1(x) = \left(-3(x_1+x_2)^3 + (x_1+x_2)x_2^2 - x_2 + 2,\ x_2\right)^T,\\
&\Delta f_1(x) = x\cos t,\quad g_1(x) = (1, -1)^T,\\
&f_2(x) = \left(-3(x_1+x_2) + x_1x_2^2 + 1,\ x_2 + x_1 + x_2^3\right)^T,\\
&\Delta f_2(x) = x(x_1+x_2)^2\sin t,\ g_2(x) = (-1, 1)^T,\ y = x_2,
\end{aligned}
\tag{35}
$$

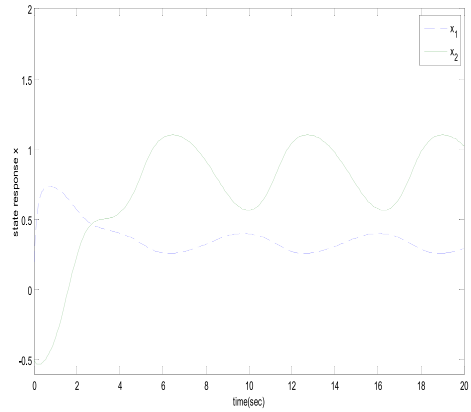

FIGURE 3. State response of subsystem2.

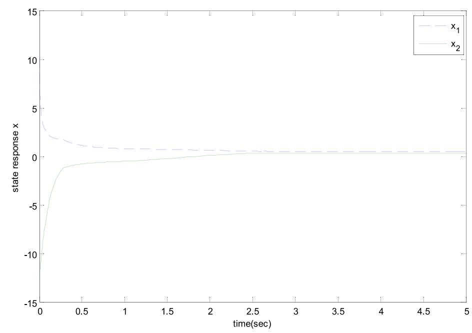

FIGURE 4. State response of the switched system.

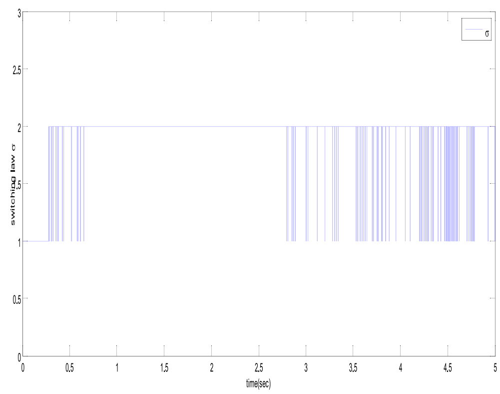

FIGURE 5. Switching law.

By the coordinate transformation $z = x_1 + x_2$, $y = x_2$. (35) can be transformed into the following form:

$$
\begin{aligned}
\dot z &= -3z^3 + zy^2 + 2 + z\cos t,\\
\dot y &= \mathrm{y} + y\cos t - u_1,\\
\dot z &= -2z + zy^2 + 1 + z^3\sin t,\\
\dot y_1 &= z + y^3 + z^2 y\sin t + u_2.
\end{aligned}
$$

Choose the functions $W_1(z) = \frac{1}{2}\mathrm{z}^2$ and $W_2(z) = \frac{1}{2}\mathrm{z}^4$, which satisfy

$$
\begin{aligned}
\dot W_1 &= z\dot z \le 3\left(z^2 - z^4\right) - z^2 + 1 + z^2y^2,\\
\dot W_2 &= 2z^3\dot z \le 2z^2\left(z^4 - z^2\right) - z^4 + 2z^4y^2 + 1.
\end{aligned}
$$

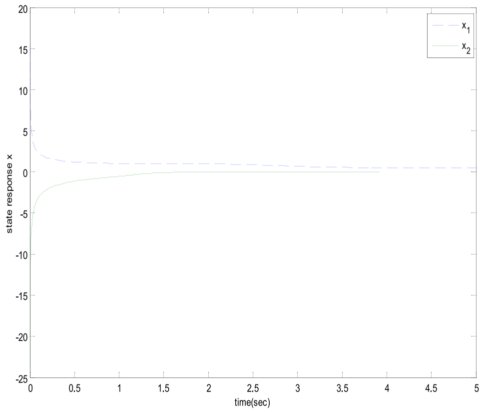

FIGURE 6. State response of the switched system.

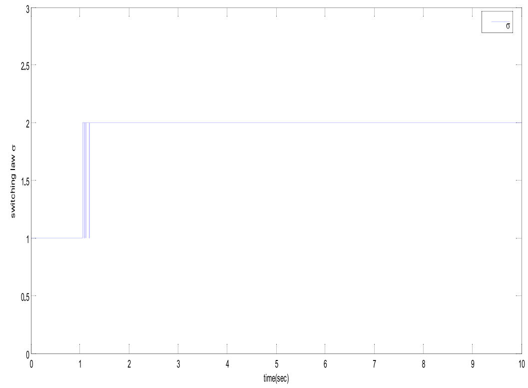

FIGURE 7. Switching law.

Consider the systems involving variables (z, y). Choose the storage functions $S_i(z, y) = W_i(z) + 0.5y^2$. Then,

$$\dot{S}_1(z, y) \leq 3\left(z^2 - z^4\right) - z^2 + 1 + z^2 y_1^2 + 2y_1^2 - y_1 u_1,$$
$$\dot{S}_2(z, y) \leq 2z^2\left(z^4 - z^2\right) - z^4 + 2z^4 y^2 + 1 + z^2 y^2 + y^4 + zy + yu_2.$$

Design the controllers as:

$$u_1 = 3y + z^2 y + v_1,$$
$$u_2 = -\left(2z^4 + z^2 + y^3 + 1\right) y - z + v_2.$$

Then,

$$\dot{S}_1(z, y) \leq 6(S_1 - S_2) - 2S_1 + 1 + yv_1,$$
$$\dot{S}_2(z, y) \leq 4z^2(S_2 - S_1) - 2S_2 + yv_2 + 1.$$

The switching law is designed as $\sigma(t) = 1$, when $S_1 - S_2 \leq 0$, and $\sigma(t) = 2$, when $S_2 - S_1 \leq 0$. Then, system (33) is feedback equivalent to a semipassive system.

Therefore, according to Theorem 1, system (35) with $v_i = -y$ is globally practically stable.

The simulation was performed with the initial state $(z(0), y_1(0)) = (15.4, -23.7)$. The simulation results are shown in Figs. 6,7. In contrast with the system in [36], the state response of system (35) shown in Fig.6 converges into a ball under the switching signal shown in Fig.7, which indicates system (35) is globally practically stable. This verifies the effectiveness of the proposed design method.

Compared with existing results, the method presented in this paper has two distinct features. First, the practical stabilization problem for none of the subsystems of system (34) is solvable, as shown in Figs 2,3 in Example 1. It is impossible to solve this problem by the common Lyapunov function method, the average dwell time approach, adopted in [20], [38]. These methods required that the stabilization problem for at least a subsystem is solvable. Second, the method adopted in this paper required the problem of each subsystem is solvable when it was active. Therefore, the conditions required by the common Lyapunov function method, the average dwell time method and the novel average dwell time methods are not satisfied [29].

VIII. CONCLUSION

This paper has studied robust semi-passivity, robust semi-passification and practical stability for a class of uncertain switched nonlinear systems. The state-dependent designed switching law is more general than the well-known 'min-switching' law. The semipassivity property is shown to be preserved for the feedback interconnected switched nonlinear systems by the design of a composite state-dependent switching law, while practical stability is not preserved in general. This switching law allows interconnected switched systems to switch asynchronously.

There are relevant problems that need to be investigated. One of such problems is how to solve the stabilizatin problem using semipassivity for uncertain switched nonlinear systems with any same relative degree.

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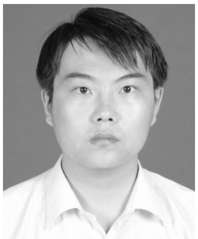

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