IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received December 4, 2019, accepted January 4, 2020, date of publication January 7, 2020, date of current version January 15, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.2964654

Design of Double Auction Mechanism Based on Social Network

JUNPING XU AND XIN HE
School of Information Science and Technology, ShanghaiTech University, Shanghai 201210, China
Corresponding author: Junping Xu (e-mail: xujp@shanghaitech.edu.cn).

ABSTRACT In the traditional double auction market, the number of achievable tradable pairs are usually limited by the market size. To solve this problem, we propose new double auction mechanisms based on social networks to realize market expansion. In this paper, we examine the double auction market, which consists of a group of sellers and a group of buyers, where the sellers provide the same type of items, and the buyers respectively purchase one from them. In addition, every buyer can invite other potential buyers to enter the market through social networks. The goal of this paper is to propose mechanisms such that they can encourage buyers already in the market to invite other potential buyers to join the auction through social networks, and achieve an effective allocation of merchandises and increase profits for sellers, which cannot be achieved under the existing double auction mechanism. We found that the extended McAfee auction mechanism cannot motivate existing buyers to invite other potential ones, while the traditional Vickrey-Clark-Groves (VCG) mechanism cannot guarantee to break even. To solve these problems, we propose two mechanisms, which are called information network auction (INA) mechanism and double network auction (DNA) mechanism. Both of these mechanisms can encourage old buyers in the market to invite new potential customers to participate. Moreover, INA focuses on achieving more effective allocation and DNA focuses on ensuring a balance between income and expenditure.

INDEX TERMS Double auction, mechanism design, multiagent systems, algorithmic game theory, artificial intelligence, social network.

I. INTRODUCTION

At present, research on double auction has become a hot topic of great interest in the fields of economics and computer science. With the rise of many emerging electronics markets, computer scientists are increasingly involved in building new market systems. Double auction market can be regarded as a centralized market with its trading rules, where the trading process of the buyers and sellers will be carried out in accordance with its regulations. One idea of double auctions research is to find a dominant strategy that allows sellers and buyers to truly report their respective valuations of merchandises. Certainly, we can apply the well-known VCG (Vickrey-Clark-Groves) [1]–[3] mechanism in this case, but cannot achieve break even. Hence, another mechanism with a dominant strategy proposed by McAfee [4] is mainly used to avoid the deficit problem by sacrificing social welfare. Since then, based on McAfee double auction mechanism, many researchers conducted a study on related mechanisms aimed at improving social welfare while ensuring the balance of payments.

In the traditional double auction theory, we only consider a fixed market with buyer and seller. But if there is no effective double auction mechanism to facilitate the dissemination of information in social networks, potential buyers who can provide high valuations of merchandises may miss the auction information. Besides, in traditional auctions, increasing profits for sellers and improving social welfare are two major and generally conflicting goals [5], [6]. However, by introducing the concept of social networking, we can motivate buyers who originally participated in the auction market to invite more potential buyers in the network to join, so that the two conflicting goals in the traditional auction can be taken care of at the same time. It is important to design an auction mechanism to satisfy the advantages of both in social networks. However, the introduction of social networks means that we not only need to motivate buyers to report their merchandises valuations honestly but also to encourage buyers to spread auction information on social networks.

The associate editor coordinating the review of this manuscript and approving it for publication was Zhan Bu.

This work is licensed under a Creative Commons Attribution 4.0 License. For more information, see http://creativecommons.org/licenses/by/4.0/

Incentives for buyers to disseminate auction information now face the main problem, that is, sellers want to attract more people to join the auction thus they could increase profits, while buyers have no motivation and obligation yet to introduce more competitors into the auction market, which basically reflects the conflict between market optimality and personal interests. Based on the mechanism design of social networks, relevant scholars have obtained some close research results. For example, in 2009, Borgatti *et al.* [7] established basic assumptions, goals, and interpretation mechanisms that are common in the field of social network analysis. Li *et al.* [8] presented a report on related work in 2017, and they focused on the design of single-item auctions on social networks, where buyers were motivated to disseminate auction information to their neighbors. In 2018, Li *et al.* further extended their work to multi-item auctions [9]. The method is based on the fact that if the disseminating behavior of the participants results in a more effective consequence, the participants would be rewarded. Some literature focuses on the application of mechanism design in communication networks [10], [11]. In 2019, Du *et al.* [12] designed a double auction mechanism in a caching network which can maximize the social welfare. After that, they further designed an auction mechanism to solve the issue of data allocation [13]. Other research efforts offer different perspectives in motivating people to share market information. In 2011, Emek *et al.* [14] proposed a theoretical framework for multi-level marketing mechanisms. In 2003, Kempe *et al.* [15] focused on how to select the most influential people and expect them to use social networks to disseminate information, and in 2011, Pickard *et al.* [16] focused on how to design incentives mechanism to inspire buyers who invite more participants to complete the challenges of market trading together. Besides, a review of research related to double auctions can be found in the book by Friedman [17]. The double auction model we studied in this paper is based on the extension of the traditional double auction model. In 2018, Segal-Halevi *et al.* [18] discussed double auctions when buyers and sellers could trade multiple items. With the review by Babichenko *et al.* [19] and the paper by Shen *et al.* [20], we can learn more related research on incentive compatibility mechanisms and network information dissemination.

Our goal is to find a double auction mechanism to motivate all buyers to invite other potential buyers in the social network while keeping the advantages of traditional mechanisms, which can not be achieved under the existing double auction mechanism.

In this paper, firstly, we design a model for double auctions on social networks that allows buyers to interact only with her neighbors about their auction information. Secondly, we demonstrate that an extended McAfee mechanism does not guarantee that buyers have the incentive to spread auction information to all of their neighbors in social networks, and introduce two mechanisms, called information network auction (INA) mechanism and double network auction (DNA) mechanism, to attempt on solving this difficulty. Then, we prove that both of these two mechanisms motivate all buyers to invite neighbors to the auction. The complexity of both mechanisms is in polynomial time and we calculate the different efficiency loss bounds for the two mechanisms. The INA can obtain higher social welfare but will cause a deficit. The DNA mechanism can ensure to guarantee a break-even but will lead to lower social welfare. Finally, we verify the conclusion of theoretical analysis according to the research results of simulations.

An earlier version of this paper was presented at the International Conference on Distributed Artificial Intelligence [21]. The previous paper did not consider the problem of the sacrifice of social welfare in DNA mechanism. This manuscript addresses this problem by introducing another mechanism (INA). We also provide additional simulation experiment and analysis of all three mechanisms. Through these experiments, we further prove the conclusions induced by our theorems.

II. MODEL DESCRIPTION

In a double auction market, there are n sellers whose set is: $N = \{1, 2, \dots, n\}$, all of them would apply to the auction platform to sell a same type of items. The valuation of seller $i (i \in N)$ is recorded as s_i, indicating that i is willing to sell the item at a price no lower than s_i. There are m buyers in the market whose set is: $M = \{1, 2, \dots, m\}$, all of them would apply to the platform to purchase the same type of product. Each buyer $i (i \in M)$ has a valuation b_i on an item, where b_i indicates that buyer i is willing to buy an item at a price no higher than b_i. Buyers also have a set of neighbors $r_i \subseteq M \backslash \{i\}$ with whom i can directly communicate. Buyers need to report the bid to the market, and also could invite her neighbors in social networks to join the double auction market. There are two categories of buyers in the market, one is to join the auction by knowing the auction information of the market herself, and the other is invited by the neighbors to join the auction. Here we refer to the former as the First Buyers and represent them as the set of A. Let $|A|$ be the number of First Buyers.

The sellers in the market hope to expand their market to attract more buyers who will join the auction so as to sell the item at a higher price. However, buyers are generally reluctant to tell other potential buyers in the social networks because it would increase the risk of losing the auction. Therefore, new mechanisms are required to encourage all First Buyers to invite other potential buyers to join the auction. Note that the inviting process will not last for an infinite period. The market holder will set a finite period for all buyers to invite their neighbors. When the period is over, no new buyers can be invited to join in the market.

The seller $i (i \in N)$ is required to report her valuation of the item according to the mechanism, which is represented by $s'_i \in S_i$ $(s'_i \geq 0)$, where S_i is the valuation space of the seller i. Define $s' = (s'_1, \dots, s'_n) \in S$ as the report valuation vector of all sellers, where S represents the valuation space, s'_{-i} as the valuation vector of all sellers except seller i, i.e. $s' = (s'_i, s'_{-i})$, and s_i as the true valuation of seller i.

At the same time, the mechanism requires buyer $j(j \in M)$ to report her valuation of the item and invite her neighbor to join the auction, indicated by $\theta'_j = (b'_j, r'_j)$, where $b'_j \geq 0$ represents the valuation of the buyer j report and r'_j represents the true invited neighbor set of buyer j. In particular, it is expressed as $\theta'_j = null$ when the buyer j is not invited to join the auction. Let Θ_j as the type space of buyer j, joint vector $\theta' = (\theta'_1, \ldots, \theta'_m)$ as the report vector of all buyers, θ'_{-j} as the report vector of all buyers except the buyer j, i.e. $\theta' = (\theta'_j, \theta'_{-j})$, and $\theta_j = (b_j, r_j)$ as the true valuation of the buyer j who honestly invite all her neighbors to join the auction.

Definition 1: The report vector of buyers θ' is feasible if every social network extended by the First Buyers is connected.

This means that, with the exception of the First Buyers, all other buyers must be at least once invited to enter the market by the buyers who have joined the auction. Here we only consider the feasible report vector of buyers. Let $f(\theta')$ denote a function that inputs a report vector of buyers and outputs a feasible report vector, let Θ denote the feasible type spaces of all buyers.

Based on the above discussion, a First Buyer will invite her neighbor to participate in the auction, and her neighbor will also invite neighbor's neighbor again to join. Finally, with the end of the invitation phase, we can expand the auction from $|A|$ buyers to $|A|$ buyer groups. Limited to the complexity of the network structure and the actual calculation problem, here we only consider the case where the intersection of $|A|$ buyer groups is empty. This assumption is reasonable since the initial buyers of the auction can be considered as agencies in different regions and their circle of neighbors may not intersect with each other. As shown in Figure 1, imagining a scenario where some service providers want to sell their computing resource to computation nodes which are consisted of some local area networks and only administrators are aware of those providers. Note that administrators may buy it themselves and use it. Hence, we want to design mechanisms to incentivize these administrators to tell more nodes near them to participant in this market. Generally, we can suppose that these local area networks are independent of each other since these computation nodes are from a different area. Actually, we can also find some other similar scenarios in the real world.

According to the above description, there were only a few buyers initially, thus the incomes of sellers and platforms could be limited. In other words, all sellers and platforms have the incentive to expand the market, but which is bad for the buyer because it increases the risk of bidding failure. To cope with this challenge in this paper, we attempt to find a breakthrough to solve problems from the perspective of mechanism design. First, we define a double auction format for social networks, as follows.

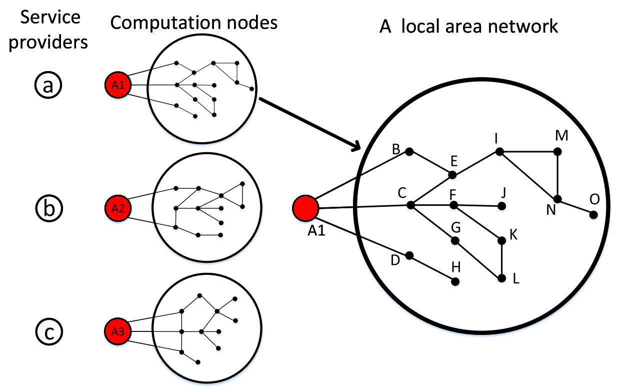


FIGURE 1. **A instance of social networks double auction, where red nodes are administrators and right graph is the detailed version of the local area network that A_1 is the administrator.**

Definition 2: The double auction mechanism $\mathcal{M}$ of social networks consists of a two-tuple group: (π, p), where $\pi = \{\pi_i\}_{i \in M \times N}$ is the allocation scheme and $p = \{p_i\}_{i \in M \times N}$ is the pricing scheme. For all $i \in M \times N$, the allocation scheme is defined as $\pi_i : S \times \Theta \rightarrow \{0, 1\}$ and the pricing scheme is defined as $p_i : S \times \Theta \rightarrow R$.

To simplify the expression, here we divide π_i, p_i into two categories, i) π^s_i, p^s_i represents the allocation and pricing schemes of seller i, and ii) π^b_j, p^b_j represents the allocation and pricing schemes of buyer j, namely $\pi = \{\pi^s_i\}_{i \in N} \cup \{\pi^b_i\}_{j \in M}$ and $p = \{p^s_i\}_{i \in N} \cup \{p^b_i\}_{j \in M}$. $\pi^s_i = 0$ means that seller i sold the item, and $\pi^s_i = 1$, that the seller reserves the item. Similarly, $\pi^b_j = 1$ means that the buyer j win the item and $\pi^b_j = 0$ means that the buyer did not. For all buyers and sellers $i \in M \times N$, $p_i \geq 0$ indicates that seller i needs to pay to platform p_i, and $p_i < 0$ indicates that seller i receives $|p_i|$ from the platform.

Next, we define the relevant economic attributes of this mechanism.

Definition 3: Allocation scheme π is feasible, for all sellers' report valuation vector $s' \in S$ and all buyer's feasible report vectors $\theta' \in \Theta$, we have:

- *for any buyer $i \in M$ while $\theta'_i = null$ is satisfied, $\pi^b_i(s', \theta') = 0$ can be obtained;*
- *for any set G_j extended by one of the First Buyers $j \in A$, $\sum_{i \in G_j} \pi^b_i(s', \theta') \leq 1$ can be obtained;*
- $\sum_{i \in N} \pi^s_i(s', \theta') + \sum_{i \in M} \pi^b_i(s', \theta') = n.$

This means that if an allocation scheme is feasible, sellers can only provide items to somebody who participates in the auction. As mentioned earlier, to be fair, each buyer group can only get at most one item. In addition, the number of items after the auction must be equal to the quantity n held by the sellers at the beginning. After determining a feasible allocation, we can define social welfare as $SW(s', \theta', \pi) = \sum_{i \in N} \pi^s_i(s', \theta')s_i + \sum_{i \in M} \pi^b_i(s', \theta')b_i$.

Definition 4: Allocation scheme π is efficient, for all sellers' report valuation vector $s' \in S$ and all buyer's feasible report vectors $\theta' \in \Theta$, we have:

$$\pi \in \underset{\pi' \in \Pi}{argmax}\, SW(s', \theta', \pi')$$

where Π is the set of all feasible allocations.

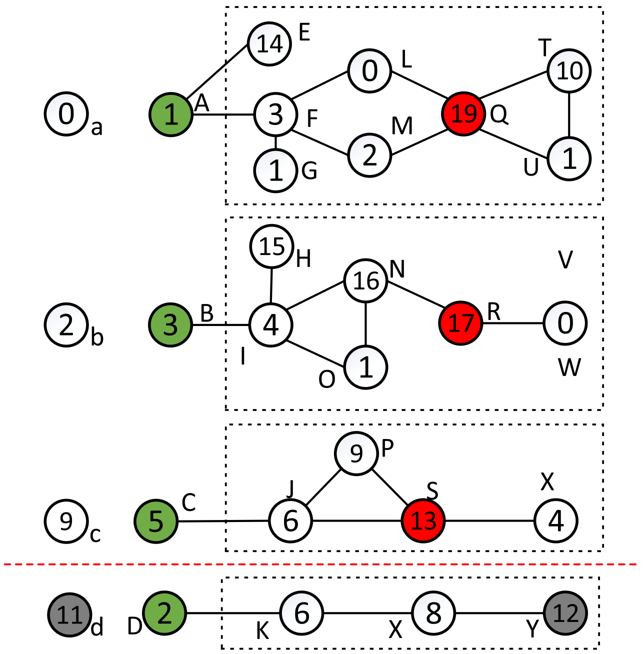


FIGURE 2. **A running instance of extended McAfee's mechanism with social networks, where green nodes represent the First Buyers, red nodes are the final winners and gray nodes are the blocking pair.**

Given a buyer $i \in M$, the feasible report vector $\theta' \in \Theta$ of all buyers and the report valuation vector $s' \in S$ of all sellers, the benefit of buyer i under mechanism $\mathcal{M}$ is quasi-linear and can be defined as:

$$u_i^b(\theta_i, s', \theta', \mathcal{M}) = \pi_i^b(s', \theta')b_i - p_i^b(s', \theta')$$

Similarly, Given a seller $i \in N$, the feasible report vector $\theta' \in \Theta$ of all buyers and the valuation vector $s' \in S$ of all sellers, the benefit of seller i under mechanism $\mathcal{M}$ is quasi-linear and can be defined as:

$$u_i^s(s_i, s', \theta', \mathcal{M}) = (\pi_i^s(s', \theta') - 1)s_i - p_i^s(s', \theta')$$

For all buyers and the seller and a given mechanism $\mathcal{M}$, if the benefits of i are always non-negative when they truthfully report the valuation, we say that mechanism $\mathcal{M}$ satisfies the property of individual rational (IR). This means that the property of individual rationality does not require a buyer to share information with her neighbors.

Definition 5: The mechanism $\mathcal{M}$ is Individually Rational (IR), when

- *given a buyer $i \in M$, the feasible report vector $((b_i, r_i'), \theta_{-i}') \in \Theta$ of all buyers and the report valuation vector $s' \in S$ of all sellers, then $u_i^b(\theta_i, s', ((b_i, r_i'), \theta_{-i}'), \mathcal{M}) \geq 0$,*
- *given a seller $j \in N$, the feasible report vector $\theta' \in \Theta$ of all buyers and the report valuation vector $(s_j, s_{-j}') \in S$ of all sellers, then $u_j^s(s_j, (s_j, s_{-j}'), \theta', \mathcal{M}) \geq 0$.*

After this, we introduce another property called incentive compatibility (IC). That is, the incentive compatibility mechanism for double auctions in social networks not only requires all sellers and buyers to truly report their valuations but also requires all buyers to have the incentive to invite all neighbors to join the auction.

Definition 6: The mechanism $\mathcal{M}$ is Incentive Compatible (IC), when

- *given a buyer $i \in M$, the feasible report vector $(\theta_i, \theta_{-i}') \in \Theta$ of all buyers, the valuation vector $s' \in S$ of all sellers and all $\theta_i' \in \Theta_i$, then*

$$u_i^b(\theta_i, s', (\theta_i, \theta_{-i}'), \mathcal{M}) \geq u_i^b(\theta_i, s', f((\theta_i', \theta_{-i}')), \mathcal{M})$$

- *given a seller $j \in N$, the feasible report vector $\theta' \in \Theta$ of all buyers, the valuation vector $(s_j, s_{-j}') \in S$ of all sellers and all $s_j' \in S_j$, then*

$$u_j^s(s_j, (s_j, s_{-j}'), \theta', \mathcal{M}) \geq u_j^s(s_j, (s_j', s_{-j}'), \theta', \mathcal{M})$$

When buyer i changes her report from θ_i to θ_i', which means that i may not invite all neighbors of her to participate in, report vector $(\theta_i', \theta_{-i}')$ may not be feasible here, so $f((\theta_i', \theta_{-i}'))$ needs to be converted into a feasible report vector. According to the definition of IC, it is a dominant strategy for buyers to report the valuation honestly and invite all neighbors at the same time.

Given a buyer's feasible report vector $\theta' \in \Theta$, a seller's valuation vector $s' \in S$ and a mechanism $\mathcal{M}$, the income of market owner can be defined by the payments sum of all buyers and sellers, denoted by $R^{\mathcal{M}}(s', \theta') = \sum_{i \in N} p_i^s(s', \theta') + \sum_{i \in M} p_i^b(s', \theta')$.

Definition 7: The mechanism $\mathcal{M}$ is weakly budget balanced, for all sellers' valuation vector $s' \in S$ and all buyers' feasible report vectors $\theta' \in \Theta$, $R^{\mathcal{M}}(s', \theta') \geq 0$.

The goal of this paper is to design a mechanism for IR, IC, and weakly budget balanced. In the next section, we first introduce the simple extension of the McAfee *et al.* [4] on social networks, then demonstrate that it does not guarantee incentive compatibility, thus it is necessary for us to develop a new solution for this model.

III. MCAFEE MECHANISM IN SOCIAL NETWORKS

In 1992, McAfee *et al.* [4] proposed a double auction mechanism. The intuitive interpretation of this mechanism is to remove the lowest bid and the highest bid pairing group from many valid pairs of buyers and sellers, which can set a pricing rule for other buyers and sellers. In the case of this paper, since buyers are connected to each other via social networks, we treat each buyer group as a whole single buyer and regard the maximum valuation in a buyer group as the bid of this whole single buyer. The McAfee mechanism can be extended to such a model as follows:

McAfee Reduction Mechanism

1) Given the seller's valuation vector $s' \in S$ and a feasible buyer report vector $\theta' \in \Theta$, then we can obtain $|A|$ social network groups, denoted by G, and each group in G all regard the buyer in A(the First Buyers) as a root.
2) Since the intersection of each buyer group is empty, we can find $|A|$ connected graphs represented by $G = \{G_1, \ldots, G_{|A|}\}$, where each group has a First Buyer as the root node.
3) In a buyer group $G_i \in G$, let $b_{G_i} = max_{k \in G_i}(b'_k)$ denote the highest bid in the buyer group.
4) Sort all buyer groups in descending order according to the value of b_{G_i}. To simplify the representation, we assume that the buyer group possesses the following relation $b_{G_1} \geq b_{G_2} \geq \cdots \geq b_{G_{|A|}}$. Mean while, all sellers are sorted in ascending order according to the valuation, and similarly, we assume the following relation $s'_1 \leq s'_2 \leq \cdots \leq s'_n$.
5) Find a valid transaction quantity q (i.e. q satisfies $b_{G_q} \geq s'_q$ and $b_{G_{q+1}} < s'_{q+1}$). Let the buyer group with the former $q-1$ highest bid makes a deal with the sellers with the former $q-1$ lowest bid, and pay s'_q to each seller who complete the transaction.
6) For all buyers $i \in G_j$, the allocation scheme can be defined as:

$$\pi_i^b(s', \theta') = \begin{cases} 1 & \text{if } b'_i = b_{G_j},\ j < q, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

This mechanism delivers the item to the highest bidder in G_j. If there are multiple buyers in the same G_j that satisfy $\pi^b(s', \theta') = 1$, then randomly assign items to one of the buyers to break-tie. We define $\mathcal{W} = \{w_1, w_2, \ldots, w_{q-1}\}$ denote the winners set of winning the item finally.

For a buyer i, the pricing scheme can be defined as:

$$p_i^b(s', \theta') = \begin{cases} b_{G_q} & \text{if } i \in \mathcal{W}, \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

For a running example in Figure 2, nodes a, b, c, d are sellers. Nodes A, J, Q, V are the First Buyers. Each buyer group is encapsulated by a dotted box. The numbers in circles are the valuation of the item reported by buyers or sellers. Links between nodes represent the relationship of neighborhood. Note that if all the First Buyers do not spread any information, there exists only one seller-buyer pair that can make deal with other according to McAfee's mechanism, for reasons of $s'_a < b'_C, s'_b < b'_B$ and $s'_c > b'_D, s'_d > b'_A$. Applying extended McAfee mechanism, we have $q = 4$, while the seller a, b, c sells the item to the buyer Q, R, S. Seller a, b, c receives $s'_d = 11$, and buyer Q, R, S should pay $b'_Y = 12$.

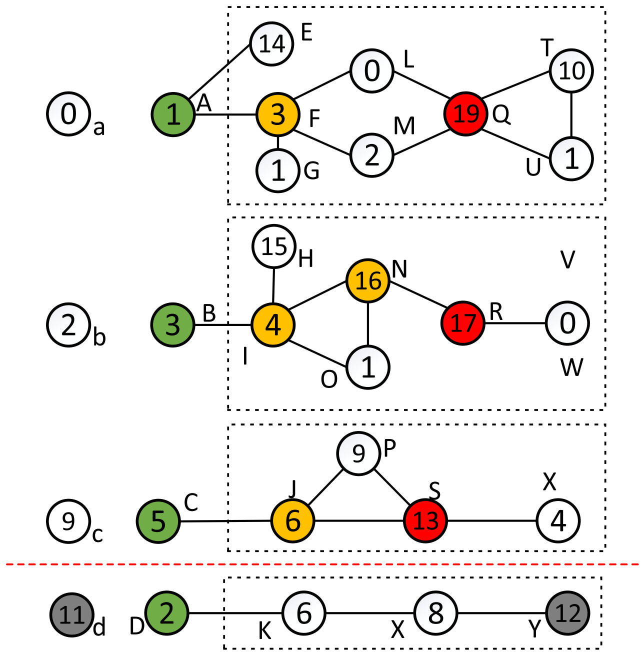


FIGURE 3. A running instance of INA, where red nodes are the final winners, yellow nodes are their cut points and gray nodes are the blocking trade pair.

McAfee *et al.* [4] proved the original trade reduction mechanism at least gets $1 - 1/q$ of the optimal gains-from. Moreover, we can conclude that the McAfee Reduction Mechanism in social networks is also $(1 - 1/q)$ - optimal.

The extended McAfee mechanism does not satisfy incentive compatible. If N does not invite R into the market, then N would get an item. Therefore, buyer N has no incentive to invite her neighbor R. An important reason is that for the buyer, there is no reward for the dissemination of the auction information. In the next section, we attempt to propose two new mechanisms that can be used to motivate buyers to disseminate auction information.

IV. INFORMATION NETWORK AUCTION MECHANISM

The McAfee extension mechanism mentioned above cannot be run directly in our model to motivate buyers to disseminate information to their neighbors. In this section, we propose a new mechanism called Information Network Auction(INA) such that buyers are incentivised to diffuse the sale information.

First, we will introduce an important concept required to describe the mechanism.

Definition 8: In a buyer group $G_i \in G$, given a feasible report vector $\theta' \in \Theta$, for all $j, k \in G_i$ and the First Buyer $a \in A \cap G_i$, if a and j are not connected in the network graph after deleting k, then in the network graph generated by θ', k is the cut point of j.

Intuitively, if all simple paths from a to j in the network pass k, then k is the cut point of j. In particular, a and j themselves are also the cut points of j itself. The cut point represents some important buyers for j. Without these important buyers,

j would never be invited to join the auction. Hence, in this paper, we are more concern about these important buyers. For example, in Figure 3, A, F is the cut point of Q, which means that if A or F decide not to disseminate information, Q will not be able to join the auction. L is not the cut point of Q, because M can still invite Q without propagation of L.

Furthermore, given $G_i \in G$, for all $j, k \in G_i$ and $j \neq k$, if k is the cut point of j, then the definition j is in the set of subsequent buyers of k, denoted by $j \in d_k$. d_k contains all buyers who must obtain auction information through k. Hence, when k does not invite her neighbors to the market, the buyers in d_k would never join the auction. Let $l_k = d_k \cup \{k\}$.

Let $G^{-l_i} = \{G_1^{-l_i}, G_2^{-l_i}, \ldots, G_{|A|}^{-l_i}\}$ denote the set in which the buyer group is reordered in descending order of the highest bid of each group when the buyer i does not join the auction. Here, $-l_i$ means that i does not join the auction, where if i does not participate in, the buyers in the set l_i will have no chance to join the auction. It is worth noting that G_j and $G_j^{-l_i}$ may not represent the same group, because when l_i is removed, the order of all groups will be rearranged accordingly.

Information Network Auction Mechanism (INA)

- As with Mechanism mentioned in Section III, find a valid transaction quantity q (i.e. q satisfies $b_{G_q} \geq s'_q$ and $b_{G_{q+1}} < s'_{q+1}$). Let the buyer group with the former $q-1$ highest bid makes a deal with the seller with the former $q-1$ lowest bid, and pay s'_q to each seller who complete the transaction.
- For all buyers $i \in G_j$, the allocation scheme can be defined as:
  - If $j < q$, then
$$\pi_i^b(s', \theta') = \begin{cases} 1 & \text{if } b'_i = b_{G_j}, \\ 0 & \text{otherwise.} \end{cases} \quad (3)$$
  - If $j \geq q$, then $\pi_i^b(s', \theta') = 0$.

  That is, give the item to the buyer with the highest bid in G_j. If there are multiple buyers i in the same buyer group G_j such that $\pi_i^b(s', \theta') = 1$, allocate the item randomly among them to break the tie.
- For all buyers $i \in G_j$, the pricing scheme can be defined as:
$$p_i^b(s', \theta') = W_{-l_i}^q - (W^q - \pi_i^b b'_i) \quad (4)$$
  where $W_{-l_i}^q = \sum_{k=1}^{q-1} b_{G_k^{-l_i}}$, and $W^q = \sum_{k=1}^{q-1} b_{G_k}$.

Our idea behind the property of INA mechanism is to combine the McAfee mechanism with the VCG mechanism. Intuitively, we can treat each group as a whole. Firstly, we use the McAfee mechanism to calculate the price that each buyer group needs to pay when it obtains the item, which can be seen as a reserve price. Then, within each buyer group, items are allocated through VCG-like mechanism with reserved prices.

For a running instance shown in Figure 3, it is easy to figure out that $q = 4$, and sellers a, b, c sell the item to buyers Q, R, S. Sellers a, b, c get paid $s'_d = 11$. For buyers, Q should pay $(14 + 17 + 13) - (19 + 17 + 13 - 19) = 14$; R should pay 14, S should pay 10. Specially, A would get a payment $(17 + 13 + 12) - (19 + 17 + 13) = -7$ (i.e. A receives a reward of 7), F would get a payment -5. Similarly, B, I get 5 reward; N gets 2 rewards, C, J get 1 rewards.

In this example, we can see that INA is not weakly budget balanced since the total incomes of the market is $11 + 14 + 14 - 7 - 5 - 5 - 5 - 2 - 1 - 1 - 12 - 12 = -11$. Next, we will show that INA mechanism satisfies individually rational (IR) and incentive compatible (IC) and discuss the efficiency loss bounds of INA mechanism.

Theorem 1: The INA mechanism is individual rational (IR).

Proof: Here we will respectively prove that the INA mechanism is individual rational for buyers and sellers.

For sellers: Assume a seller $i \in N$ reports her valuation s_i truthfully. If $i \leq q-1$, that is i is the $q-1$ lowest cost sellers and her payment is $-s_q$ which means she can get $s_q \geq s_i$. Therefore, her utility is $s_q - s_i \geq 0$. If $i > q-1$, it is obvious that i's utility is zero. Therefore, for each seller i, i's utility is non-negative.

For buyers: Assume a buyer $i \in G_j, j \leq q-1$, reports her valuation b_i truthfully. If $b_i = b_{G_j}$, i.e. i is the highest value buyer in G_j ($\pi_i^b = 1$) and her payment is $p_i^b = W_{-l_i}^q - (W^q - b_i)$. Hence i's utility is $u_i = b_i - p_i^b = W^q - W_{-l_i}^q$. Since $M \supseteq M \setminus l_i$, according to the definition of W^q and $W_{-l_i}^q$, we can easily conclude that $W^q \geq W_{-l_i}^q$ which means i's utility is non-negative. If $b_i < b_{G_j}$, i.e. $\pi_i^b = 0$ and her payment is $p_i^b = W_{-l_i}^q - W^q$. Hence i's utility is $u_i = -p_i^b = W^q - W_{-l_i}^q$ which is the same as above, we can still conclude that i's utility is non-negative. Assume a buyer $i \in G_j, j > q-1$, her utility is always zero. Therefore, for each buyer i, i's utility is non-negative.

Hence, the INA is individually rational.

The proof is completed. □

Theorem 2: The INA mechanism is incentive compatible (IC).

Proof: Here we will respectively prove that the DNA mechanism is incentive compatible for buyers and sellers.

For sellers, since their item allocation scheme and pricing scheme are the same as the McAfee mechanism, so for all of seller i, honestly reporting valuation could be a dominant strategy. In all, for each seller $i \in N$, nothing she could do to improve her benefit except report her valuation truthfully.

For buyers, we respectively prove that when fixing r'_i, giving a honest bid is a dominant strategy, i.e. $u_i^b(b_i, r'_i) \geq u_i^b(b'_i, r'_i)$, and when i gives a honestly bid, it becomes a dominant strategy to invite all the neighbors of i to join the auction, i.e. $u_i^b(b_i, r_i) \geq u_i^b(b_i, r_i')$ Combining the above two parts, we can obtain $u_i^b(b_i, r_i) \geq u_i^b(b_i', r_i')$

The fixed r_i':

- If $\pi_i^b = 1$, that is, when i honestly bids, i could win the item. Her utility is $u_i = W^q - W_{-l_i}^q \geq 0$. She can change her valuation report to $b_i' \geq max\{b_{G_q}, b_{G_j}^2\}$, where $b_{G_j}^2$ is the second highest valuation in G_j. In that case she can still receive an item and her utility remains unchanged. However if she reports $b_i' < max\{b_{G_q}, b_{G_j}^2\}$ to lose the item. Then her return becomes $u_i' = W'^{q'} - W_{-l_i}^{q'}$. While lowering the bid can only decreases q and at most only getting smaller to $q-1$, that is, $q-1 \leq q' \leq q$. we can obtain that when $q' = q$, then $\Delta u_i = u_i' - u_i = W'^q - W^q = b_k - b_i \leq 0$ where $b_k = max\{b_{G_q}, b_{G_j}^2\}$; when $q' = q-1$, then $\Delta u_i = u_i' - u_i = W'^{q-1} - W_{-l_i}^{q-1} - (W^q - W_{-l_i}^q) = b_{G_{q-1}^{-l_i}} - b_i$, i.e. the return of i is decreased.
- If $\pi_i^b = 0$, that is, when i honestly bids, i cannot win the item. her return is $u_i = W^q - W_{-l_i}^q \geq 0$. She can change her valuation report to $b_i' \geq max\{b_{G_{q-1}}, b_{G_j}\}$ to get an item. In that case her return becomes $u_i' = W'^{q'} - b_i' + b_i - W_{-l_i}^{q'}$. While raising the bid can only increase q and at most only getting larger to $q+1$, that is, $q \leq q' \leq q+1$. we can obtain that when $q' = q$, then $\Delta u_i = u_i' - u_i = W'^q - b_i' + b_i - W^q \leq 0$; when $q' = q+1$, then $\Delta u_i = u_i' - u_i = W'^{q+1} - b_i' + b_i - W_{-l_i}^{q+1} - W^q + W_{-l_i}^q = b_i - b_{G_q^{-l_i}} \leq 0$, i. e., the return of i is decreased.

Hence, when r_i' is fixed, giving a honest bid is a dominant strategy, i.e. $u_i^b(b_i, r_i') \geq u_i^b(b_i', r_i')$.

The fixed b_i:

When the buyer i honestly bids, the return is $u_i = W^q - W_{-l_i}^q$. Since b_i is fixed, i can only increase the profit by changing q. When the set of neighbors invited by i changes from r_i to r_i', it will cause q to decrease and at most only getting smaller to $q-1$, i.e. $q-1 \leq q' \leq q$. When $q' = q$, $W_{-l_i}^q$ is independent of i, which will not change, while $W_{-l_i}^q$ decreases as i invites fewer neighbors, so the income of i decreases; when $q' = q-1$, then $\Delta u_i = u_i' - u_i = W'^{q-1} - W_{-l_i}^{q-1} - W^q + W_{-l_i}^q$. Since $W_{-l_i}^q - W_{-l_i}^{q-1} = b_{G_{q-1}^{-l_i}} \leq 0$, that is, the income of i decreases.

Hence, when b_i is fixed, inviting all neighbors of i to join the auction is a dominant strategy, i.e. $u_i^b(b_i, r_i) \geq u_i^b(b_i, r_i')$.

Combining the above two parts, we can obtain $u_i^b(b_i, r_i) \geq u_i^b(b_i', r_i')$, that is, the INA mechanism is incentive compatible (IC).

The proof is completed. □

Then we discuss the efficiency bounds of INA mechanism. Note that the efficiency loss of INA is the same as the original trade reduction mechanism and the McAfee mechanism in social networks. Intuitively, the idea behind the INA mechanism is to combine the McAfee mechanism with the VCG mechanism. Therefore only the McAfee process hurts social welfare.

Theorem 3: The INA mechanism has efficiency loss bounded by $1/q$.

Proof: INA allocates items to the buyers with the highest bid in G_j for all $j < q$ (q is the valid transaction quantity of INA).

First the efficiency loss associated with the INA mechanism is $loss = b_{G_q} - s_q$.

For an effecient allocation $\pi \in \Pi$, all sellers' report valuation vector $s' \in S$ and all buyers' feasible report vector $\theta' \in \Theta$, we have

$$\frac{loss}{SW(s', \theta', \pi)} = \frac{b_{G_q} - s_q}{\sum_{i=1}^{q} b_{G_i} + \sum_{i=q+1}^{n} s_i} \leq \frac{b_{G_q} - s_q}{q b_{G_q} + (n-q)s_q}$$
$$= \frac{b_{G_q} - s_q}{q(b_{G_q} - s_q) + n s_q} \leq \frac{b_{G_q} - s_q}{q(b_{G_q} - s_q)}$$
$$= \frac{1}{q}$$

This completes the proof. □

Next, we briefly analysis the implementation complexity of INA. We break down the process of INA into three phases: i) diffusing phase; ii) selecting and sorting phase; iii) allocating phase. In the diffusing phase, the First buyers start to expanded their buyers' groups. The time complexity of this phase is $O(m+E)$, where m is the amount of all buyers and E represents the number of links between all buyers. In the selecting and sorting phase, we select out the maximum valuation of each buyer group and sort all sellers and buyer groups to make them match to each other. The complexity of this phase is $O(nlogn) + O(|A|log|A|)$, where n is the number of sellers and A is the set of First Buyers. In the final phase of allocating, we allocate every item in the winner groups. This phase also takes the time of $O(m+E)$ because we can allocate the item by traversing all graphs. In all, the total complexity of INA is $O(nlogn + m + E)$. Therefore, the INA mechanism can be implemented in polynomial time.

V. DOUBLE NETWORK AUCTION MECHANISM

We have already known that INA is individually rational and incentive compatible but not weakly budget balance. In this section, we propose a new mechanism with the characteristics of individual rationality, incentive compatibility and weakly budget balanced.

First, we will introduce an important concept required to describe the mechanism.

Definition 9: For $G_i \in G$, given a feasible report vector $\theta' \in \Theta$, for all buyers $j \in G_i$ and First Buyers $a \in A \cap G_i$, define $k_1, k_2, \ldots$ as all cut points of j, $CP_j = \{a, k_1, k_2, \ldots, j\}$ as a cut path of j.

To simplify the expression, let $CP_j = \{a, a+1, \ldots, j-1, j\}$ be the cut path of j, obviously, $CP_a \subseteq CP_{a+1} \ldots \subseteq CP_j$.

CP_j represents the process by which a diffuses information to j. That is, if anyone buyer $k \in CP_j$ decides not to invite her neighbor, $k+1, k+2, \ldots, j$ would never join the auction.

For $G_i \in G$, given a feasible report vector $\theta' \in \Theta$ and a buyer $k \in G_i$, let $w = argmax_{i \in d_k} b'_i$. Since k is the cut point of w, then $CP_w = \{a, a+1, \ldots, k, k+1, \ldots, p\}$. In a double auction, w is the candidate in d_k who is most likely to be a winner. In all subsequent nodes d_k of k, the DNA mechanism will focus on those most promising buyer nodes (i.e. w). Let c_k be the successor (i.e. $k+1$) closest to k in CP_w.

Now we can start to describe the DNA mechanism.

Double Network Auction Mechanism (DNA)

- As with Mechanism mentioned in Section III, find a valid transaction quantity q (i.e. q satisfies $b_{G_q} \geq s'_q$ and $b_{G_{q+1}} < s'_{q+1}$). Let the buyer group with the former $q-1$ highest bid makes a deal with the seller with the former $q-1$ lowest bid, and pay s'_q to each seller who complete the transaction.
- For all buyers $i \in G_j$, the allocation scheme can be defined as:
  - If $j \leq q-1$,

$$\pi_i^b(s', \theta') = \begin{cases} 1 & \text{if } b'_i = b_{G_j^{-l_{c_i}}} \geq b_{G_q^{-l_{c_i}}}, \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

  - If $j > q-1$, then $\pi_i^b(s', \theta') = 0$.

  In the former $q-1$ groups, if there is a buyer k on the cut path of the highest valuation buyer, when k does not spread the auction information to c_k, k becomes the highest valuation buyer in the group. Then the DNA mechanism assigns the item to k.

  If there are multiple buyers i in the same buyer group G_j such that $\pi_i^b(s', \theta') = 1$, the items are assigned to the buyer i that makes the CP_i size the smallest.
- Let $\mathcal{W} = \{w_1, w_2, \ldots, w_{q-1}\}$ be the set of all winners. At the same time, let $D_{\mathcal{W}} = \bigcup_{j=1}^{q-1} d_{w_j}$.
- For all buyers $i \in G_j$, the pricing scheme can be defined as:

$$p_i^b(s', \theta') = \begin{cases} 0 & \text{if } i \in D_{\mathcal{W}}, \\ W_{-l_i}^q - (W_{-l_{c_i}}^q - \pi_i^b b'_i) & \text{otherwise.} \end{cases} \tag{6}$$

  where $W_{-l_i}^q = \sum_{k=1}^{q-1} b_{G_k^{-l_i}}$, and $W_{-l_{c_i}}^q = \sum_{k=1}^{q-1} b_{G_k^{-l_{c_i}}}$.

The intuition behind DNA is similar to INA. The difference is how to calculate the marginal contribution of each buyer. In DNA, we use stronger constraints to limit the cut points of winners to get too many rewards. Buyers only get rewards when they are on the cut paths of both winner and another buyer whose valuation is larger than b_{G_q}.

Now, we could still consider the example of Figure 4, where the yellow node has the highest valuation, but it does

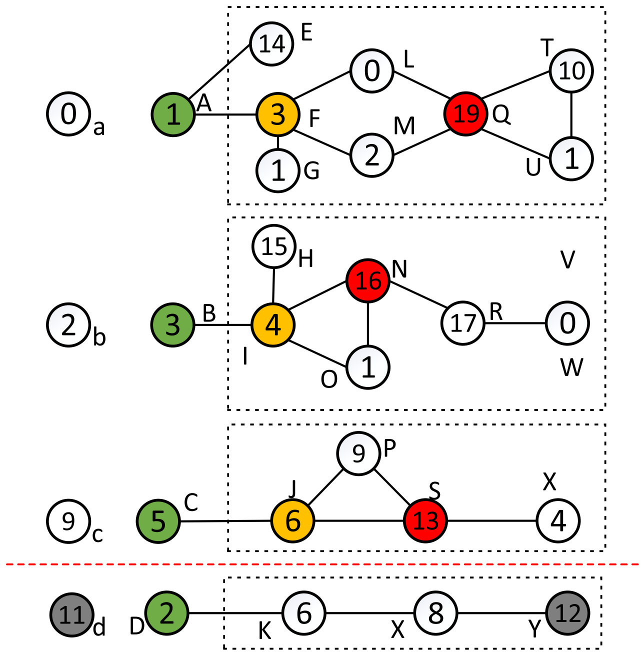


FIGURE 4. **A running instance of DNA, where red nodes are the final winners, yellow nodes are their cut points and gray nodes are the blocking trade pair.**

not mean that the yellow node should be the final winner, while the red nodes are cut points of the yellow node. In the DNA mechanism, we can calculate $q = 4$, then seller a, b, c sells the items to buyer Q, N, S. Buyer N is the winner if N does not tell R about the auction information, then N would be the buyer with the highest valuation in the group. Here, seller a, b, c can receive $s'_d = 11$. As for the buyers, Q should pay $(14+17+13)-(19+17+13-19) = 14$; N should pay 15, while T should pay 12. In particular, A's final payment is $(17+13+12)-(14+17+13) = -2$, and K's final payment is -3.

Next, we attempt to prove that the DNA mechanism is individual rational (IR), incentive compatible (IC), weakly budget balanced, its revenue is no less than the revenue of INA mechanism and equal to the revenue of McAfee Reduction Mechanism and discuss the efficiency loss bounds.

Theorem 4: The DNA mechanism is individual rational (IR).

Proof: Here we will respectively prove that the DNA mechanism is individual rational for buyers and sellers.

For sellers, since their item allocation scheme and pricing scheme are identical with McAfee mechanism, the revenue is non-negative for all of seller i.

For buyers, we assume that the buyer i honestly reports the valuation b_i, and when $i \notin D_{\mathcal{W}}$, the profit of i is $u_i^b = \pi_i^b b_i - p_i^b = W_{-l_{c_i}}^q - W_{-l_i}^q$. As the reason of $l_{c_i} \supseteq l_i$, we can obtain $M \setminus l_{c_i} \subseteq M \setminus l_i$. According to the definition of $W_{-l_i}^q$ and $W_{-l_{c_i}}^q$, we can obtain $W_{-l_{c_i}}^q \geq W_{-l_i}^q$. When $i \in D_{\mathcal{W}}$, the income of i is 0. So, for all of buyer i, the benefits are non-negative.

The proof is completed. □

Theorem 5: The DNA mechanism is incentive compatible (IC).

Proof: Here we will respectively prove that the DNA mechanism is incentive compatible for buyers and sellers.

For sellers, since their item allocation scheme and pricing scheme are the same as the McAfee mechanism, so for all of seller i, honestly reporting valuation could be a dominant strategy.

For buyers, when $i \in D_{\mathcal{W}}$, regardless of the strategy adopted by i, the income of i is 0. For these buyers, it is a dominant strategy to report the valuation honestly and invite all neighbors to join the auction.

When $i \notin D_{\mathcal{W}}$, we respectively prove that when fixing r_i', giving a honest bid is a dominant strategy, i.e. $u_i^b(b_i, r_i') \geq u_i^b(b_i', r_i')$, and when i gives a honestly bid, it becomes a dominant strategy to invite all the neighbors of i to join the auction, i.e. $u_i^b(b_i, r_i) \geq u_i^b(b_i, r_i')$ Combining the above two parts, we can obtain $u_i^b(b_i, r_i) \geq u_i^b(b_i', r_i')$

- The fixed r_i': When the buyer i honestly bids, the return is $u_i = \pi_i^b b_i - p_i^b = W_{-l_{c_i}}^q - W_{-l_i}^q$. We assume that the buyer i bids b_i', and the return of i is $u_i' = W_{-l_{c_i}}^{\prime q'} - \pi_i^{\prime b} b_i' + \pi_i^{\prime b} b_i - W_{-l_i}^{q'}$. Since r_i' is fixed, that is, l_{c_i} and l_i are fixed, i can only increase the profit by changing π_i^b or q.
  If $\pi_i^b = 1$, that is, when i honestly bids, i could win the item. At this time, the raising bid of i cannot bring more income for her; when i decreases the bid and makes $\pi' b_i = 0$, the return of i is $u_i' = W_{-l_{c_i}}^{\prime q'} - W_{-l_i}^{q'}$. While lowering the bid can only decreases q and at most only getting smaller to $q-1$, that is, $q-1 \leq q' \leq q$, we can obtain that when $q' = q$, then $\Delta u_i = W_{-l_{c_i}}^{\prime q} - W_{-l_{c_i}}^{q} \leq 0$; when $q' = q-1$, then $\Delta u_i = b_{G_{q-1}}^{-l_i} - b_i$, i.e. the return of i is decreased.
  If $\pi_i^b = 0$, that is, when i honestly bids, i cannot win the item. At this time, the reducing bid of i cannot bring more income for her; when i raising the bid and makes $\pi' b_i = 1$, the return of i is $u_i' = W_{-l_{c_i}}^{\prime q'} - b_i' + b_i - W_{-l_i}^{q'}$. While raising the bid can only increase q and at most only getting larger to $q+1$, that is, $q \leq q' \leq q+1$, we can obtain that when $q' = q$, then $\Delta u_i = W_{-l_{c_i}}^{q} - b_i' + b_i - W_{-l_{c_i}}^{q}$; when $q' = q+1$, then $\Delta u = b_i - b_{G_{q-1}}^{-l_i} \leq 0$, i. e., the return of i is decreased. Hence, when r_i' is fixed, giving a honest bid is a dominant strategy, i.e. $u_i^b(b_i, r_i') \geq u_i^b(b_i', r_i')$.
- The fixed b_i: When the buyer i honestly bids, the return is $u_i = W_{-l_{c_i}}^q - W_{-l_i}^q$. Since b_i is fixed, i can only increase the profit by changing q. When the set of neighbors invited by i changes from r_i to r_i', it will cause q to decrease and at most only getting smaller to $q-1$, i.e. $q-1 \leq q' \leq q$. When $q' = q$, $W_{-l_i}^q$ is independent of i, which will not change, while $W_{-l_{c_i}}^q$ decreases as i invites fewer neighbors, so the income of i decreases; when $q' = q-1$, then $\Delta u_i = W_{-l_{c_i}}^{\prime q} - b_i' + b_i - W_{-l_{c_i}}^q$, that is, the income of i decreases.
  Hence, when b_i is fixed, inviting all neighbors of i to join the auction is a dominant strategy, i.e. $u_i^b(b_i, r_i) \geq u_i^b(b_i, r_i')$.

Combining the above two parts, we can obtain $u_i^b(b_i, r_i) \geq u_i^b(b_i', r_i')$, that is, the DNA mechanism is incentive compatible (IC).

The proof is completed. □

We can easily conclude that the revenue and transaction number of DNA mechanism is not less than that of McAfee Reduction Mechanism without social networks (only First Buyers participate in the market).

Theorem 6: The DNA mechanism is a weakly budget balanced and its revenue is no less than the revenue of INA mechanism and equal to the revenue of McAfee Reduction Mechanism.

Proof: According to the allocation and pricing schemes of the DNA auction mechanism, since the intersection of each buyer group is empty and consists of $|A|$ connected graphs, firstly, we can calculate the payment within each buyer group G_i. Then, we sum the calculations for all buyer groups, and finally, subtract all sellers' payments. That is,

$$R^{DNA} = \sum_{i \in G_1} p_i^b + \sum_{i \in G_2} p_i^b + \cdots + \sum_{i \in G_{|A|}} p_i^b - (q-1)s_q$$

For any buyer group G_j,

$$\begin{aligned} \sum_{i \in G_j} p_i^b &= \sum_{i \notin l_w} (W_{-l_i}^q - W_{-l_{c_i}}^q) + (W_{-l_w}^q - W_{-l_{cw}}^q - b_w) \\ &= W_{-l_k}^q - W_{-l_{cw}}^q + b_w = b_{G_q} \end{aligned}$$

where $k \in A \cap G_j$. Hence, the total income of the market is:

$$R^{DNA} = (q-1)b_{G_q} - (q-1)s_q = (q-1)(b_{G_q} - s_q) \geq 0$$

Therefore the DNA mechanism is a weakly budget balanced.

According to the allocation and pricing schemes of the INA auction mechanism, the revenue of INA is

$$R^{INA} = \sum_{i \in G_1} p_i^b + \sum_{i \in G_2} p_i^b + \cdots + \sum_{i \in G_{|A|}} p_i^b - (q-1)s_q$$

For any buyer group G_j,

$$\begin{aligned} \sum_{i \in G_j} p_i^b &= \sum_{i \notin l_w} (W_{-l_i}^q - W^q) + (W_{-l_w}^q - W^q - b_w) \\ &= W_{-l_k}^q - W^q + b_w \leq b_{G_q} \end{aligned}$$

where $k \in A \cap G_j$. Hence, the total income of the market is:

$$R^{INA} \leq (q-1)b_{G_q} - (q-1)s_q = R^{DNA}$$

Therefore the revenue of the DNA mechanism is no less than the revenue of the INA mechanism.

In McAfee Reduction Mechanism, the total income of the market is:

$$R^{McAfee} = (q-1)b_{G_q} - (q-1)s_q = (q-1)(b_{G_q} - s_q) = R^{DNA}$$

Therefore the DNA mechanism revenue is equal to the revenue of McAfee Reduction Mechanism.

The proof is completed. □

Then we discuss the efficiency bounds of the DNA mechanism. We can easily conclude that the efficiency of the DNA mechanism is not less than that of McAfee Reduction Mechanism without social networks (only First Buyers participate in the market).

The efficiency loss of DNA mechanism is

$$loss = \sum_{i=1}^{q-1}(b_{G_i} - b_{w_i}) + b_{G_q} - s_q$$

The term $\sum_{i=1}^{q-1}(b_{G_i} - b_{w_i})$ is related to the structure of social network. The more complex the social network is, the more likely it is that the winner will be the highest bidder in his buyer group. When the social network is complex enough (there is no cut point in the network), $\sum_{i=1}^{q-1}(b_{G_i} - b_{w_i})$ will approach zero.

Therefore we introduce a concept of complexity of the social network c. Let

$$\frac{1}{c} = \frac{\sum_{i=1}^{q-1}(b_{G_i} - b_{w_i})}{\sum_{i=1}^{q} b_{G_i}} \in (0, 1)$$

be the simplicity of the network, where the denominator represents the normalization option. Assume that the total number of the First Buyer and buyers are fixed. $\frac{1}{c}$ will approach 1 when the network is simple enough (the number of edges is small) and $\frac{1}{c}$ will approach 0 when the network is complicated enough (the number of edges is large).

Theorem 7: The DNA mechanism has efficiency loss bounded by $1/c + 1/q$.

Proof: For an effecient allocation $\pi \in \Pi$, all sellers' report valuation vector $s' \in S$ and all buyers' feasible report vector $\theta' \in \Theta$, we have

$$\frac{loss}{SW(s', \theta', \pi)} = \frac{\sum_{i=1}^{q-1}(b_{G_i} - b_{w_i}) + b_{G_q} - s_q}{\sum_{i=1}^{q} b_{G_i} + \sum_{i=q+1}^{n} s_i}$$
$$\leq \frac{\sum_{i=1}^{q-1}(b_{G_i} - b_{w_i})}{\sum_{i=1}^{q} b_{G_i}} + \frac{b_{G_q} - s_q}{q b_{G_q} + (n-q)s_q}$$
$$\leq \frac{1}{c} + \frac{b_{G_q} - s_q}{q(b_{G_q} - s_q)} = \frac{1}{c} + \frac{1}{q}$$

This completes the proof. □

Next, we briefly analysis the implementation complexity of DNA. We break down the process of DNA into three phases: i) diffusing phase; ii) selecting and sorting phase; iii) allocating phase. In the diffusing phase, the First buyers start to expanded their buyers' groups. The time complexity of this phase is $O(m+E)$, where m is the amount of all buyers and E represents the number of links between all buyers. In the selecting and sorting phase, we select out the maximum valuation of each buyer group and sort all sellers and buyer groups to make them match to each other. The complexity of this phase is $O(nlogn) + O(|A|log|A|)$, where n is the number of sellers and A is the set of First Buyers. In the final phase of allocating, we allocate every item in the winner groups. This phase also takes the time of $O(m+E)$ because we can allocate the item by traversing all graphs. In all, the total complexity of DNA is $O(nlogn+m+E)$. Therefore, the DNA mechanism can be implemented in polynomial time.

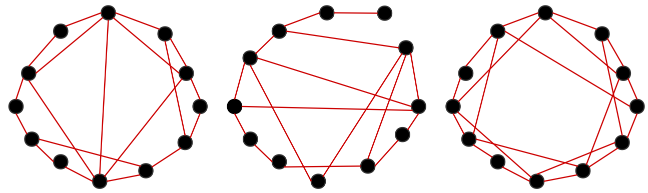


FIGURE 5. **Some examples of small world networks, using networks generated by ($k = 2, p = 0$), ($k = 2, p = 0.5$) and ($k = 4, p = 0$) respectively.**

VI. SIMULATION

In this section, we analyze the process of the simulation the two mechanisms in a randomly generated social network and compare them under different conditions to analyze the performance of them in the auction chain length, seller's income and so on.

In this simulation research, we use the small world network model to replace the real-world social network model. A detailed description of the small world network can be found in the [22]. The small world network is mainly controlled by two parameters, the parameter k and parameter p, where k represents the maximum degree of each node in the network, and $p \in [0, 1]$ represents a probability value. The generation process of a small world network $N(k, p)$ is following: Firstly, we create a ring on n nodes; then, connect each node in the ring with its nearest k neighbors, and then create shortcuts between some points by the following operations, that is, for each edge (u, v) in the above ring, it is replaced by a probability of p with a new edge (u, w), where w is randomly selected from the remaining nodes. Figure 5 shows some examples of small world networks.

In this simulation, we initialize 100 First Buyers and sellers, and then the First Buyers based on random initialization will generate a small world network with a size of 50 nodes. The valuation of each buyer and seller node are $v_i = \mathcal{U}(0, 100)$, which follows a uniform random distribution from 0 to 100. Fix $p = 0.5$ to observe the performance of INA and DNA at different k values, while comparing the social welfare of the McAfee mechanism without expansion. For every value of k, run 1000 times of auctions and take the mean of all results as the final result. The results are shown in Table 1 and Table 2, where ChainL represents the

TABLE 1. Comparison of deal pairs (DP) results with different *k*.

k	ChainL	DP (McAfee)	DP(INA)	DP (DNA)
2	5.99	43.34	60.12	60.12
4	4.34	55.26	65.99	65.99
6	2.17	49.99	56.37	56.37
8	1.0	53.23	59.97	59.97

TABLE 2. Comparison of revenue results with different *k*.

k	Revenue(M)	Revenue (INA)	Revenue (DNA)
2	2168.12	3260.81	2998.41
4	2754.99	3496.73	3299.50
6	2499.01	3139.74	2816.78
8	2714.53	3279.69	3001.04

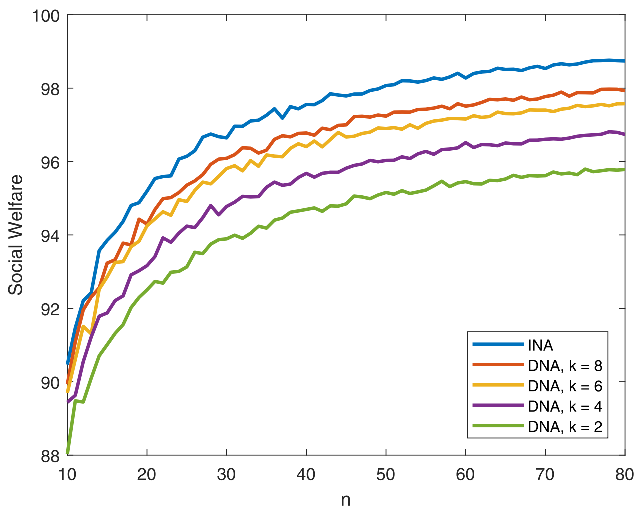


FIGURE 6. Average social welfare under different parameter *k*, where the *n*-axis represents the number of nodes in each group, the *Social Welfare*-axis represents the average social welfare over 1000 rounds simulations.

average transaction chain length of the social network, and DP represents the final volume of double auction market, Revenue (M), Revenue (INA) and Revenue (DNA) represent the final benefits that the market receives from buyers when using McAfee mechanisms without expansion, INA and DNA mechanisms, respectively.

The results show that after using the INA and DNA mechanism for the extended network, we improve the final transaction volume of the market compared to the traditional McAfee mechanism, the reason of which is that more potential buyers have joined the auction market through the spread influence of the network, which gives sellers a greater chance to complete trading. addition, the introduction of too many participants gives the market an opportunity to find buyers with higher valuations, so the expanded double market can complete an auction of items at a higher price, which is also verified in the simulation.

Another result of the simulation is shown in Figure 6. In this simulation, we initialize 100 First Buyers and sellers, and then the First Buyers based on random initialization will generate a small world network with various size from 10 to 80. The valuation of each buyer and seller node are $v_i = \mathcal{U}(0, 1)$, which follows a uniform random distribution from 0 to 1. Fix $p = 0.5$ to observe the performance of INA and DNA at different k values, run 1000 times of auctions and take the mean of all results as the final result.

Note that the result of INA mechanism is independent of the structure of social network which is determined by the parameter k here. Hence, we present just one single social welfare result of INA mechanism in Figure 6. According to the result in Figure 6, the final social welfare obtained by DNA is increasing as the social network gets more and more complicated. We can conclude that when k is enough large, the social welfare of DNA will approximately equal to the social welfare of INA.

VII. CONCLUSION

In this paper, we introduce two mechanisms to solve the information diffusion problem in a double auction market, in which the original buyer is willing to spread the auction information to their neighbors, thus we can expand the market. Firstly, we extend the traditional double auction model to a new model that considers social networks. Secondly, based on the proposed new model, we discuss the performance of the traditional mechanism (McAfee mechanism) in the double market with social network characteristics, and theoretically study the limitations of the traditional double mechanism in the new scenario. Then, we propose two new social-network-based double auction mechanisms (INA and DNA), and theoretically prove that both INA and DNA have good properties such as individual rational (IR), incentive compatible (IC). Moreover, INA can lead to higher social welfare but will cause a deficit, while DNA can avoid deficit but decrease the social welfare compared with INA. Finally, we verify the conclusion of theoretical analysis according to the research results of the simulations.

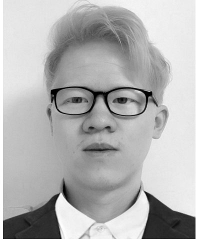
One interesting future work in our setting is to consider the costs of all buyers to invite their neighbors. It's also worth discussing the impact of social network structures on the efficiency and revenue of our mechanisms.

REFERENCES

[1] W. Vickrey, "Counterspeculation, auctions, and competitive sealed tenders," *J. Finance*, vol. 16, no. 1, pp. 8–37, Mar. 1961.
[2] E. H. Clarke, "Multipart pricing of public goods," *Public Choice*, vol. 11, no. 1, pp. 17–33, Sep. 1971.
[3] T. Groves, "Incentives in teams," *Econometrica*, vol. 41, no. 4, p. 617, Jul. 1973.
[4] R. Mcafee, "A dominant strategy double auction," *J. Econ. Theory*, vol. 56, no. 2, pp. 434–450, Apr. 1992.
[5] D. Friedman and J. Rust, *The Double Auction Market: Institutions, Theories, And Evidence*. Boulder, CO, USA: Westview, 1993, vol. 14.
[6] R. Myerson, "Optimal auction design," *Math. Oper. Res.*, vol. 6, no. 1, pp. 58–73, 1981.
[7] S. P. Borgatti, A. Mehra, D. J. Brass, and G. Labianca, "Network analysis in the social sciences," *Science*, vol. 323, no. 5916, pp. 892–895, 2009.
[8] B. Li, D. Hao, D. Zhao, and T. Zhou, "Mechanism design in social networks," in *Proc. 31st AAAI Conf. Artif. Intell.*, 2017, pp. 586–592.

[9] D. Zhao, B. Li, J. Xu, D. Hao, and N. R. Jennings, "Selling multiple items via social networks," in *Proc. 17th Int. Conf. Auton. Agents MultiAgent Syst.*, 2018, pp. 68–76.
[10] J. Du, C. Jiang, H. Zhang, Y. Ren, and M. Guizani, "Auction design and analysis for SDN-based traffic offloading in hybrid satellite-terrestrial networks," *IEEE J. Sel. Areas Commun.*, vol. 36, no. 10, pp. 2202–2217, Oct. 2018.
[11] A. Ndikumana, N. H. Tran, T. M. Ho, Z. Han, W. Saad, D. Niyato, and C. S. Hong, "Joint communication, computation, caching, and control in big data multi-access edge computing," *IEEE Trans. Mobile Comput.*, to be published.
[12] J. Du, C. Jiang, E. Gelenbe, H. Zhang, Y. Ren, and T. Q. S. Quek, "Double auction mechanism design for video caching in heterogeneous ultra-dense networks," *IEEE Trans. Wireless Commun.*, vol. 18, no. 3, pp. 1669–1683, Mar. 2019.
[13] J. Du, E. Gelenbe, C. Jiang, Z. Han, and Y. Ren, "Auction-based data transaction in mobile networks: Data allocation design and performance analysis," *IEEE Trans. Mobile Comput.*, to be published.
[14] Y. Emek, R. Karidi, M. Tennenholtz, and A. Zohar, "Mechanisms for multi-level marketing," in *Proc. 12th ACM Conf. Electron. Commerce*, 2011, pp. 209–218.
[15] D. Kempe, J. Kleinberg, and É. Tardos, "Maximizing the spread of influence through a social network," in *Proc. 9th ACM SIGKDD Int. Conf. Knowl. Discovery Data Mining (KDD)*, 2003.
[16] G. Pickard, W. Pan, I. Rahwan, M. Cebrian, R. Crane, A. Madan, and A. Pentland, "Time-critical social mobilization," *Science*, vol. 334, no. 6055, pp. 509–512, Oct. 2011.
[17] D. Friedman, "The double auction market institution: A survey," in *The Double Auction Market*. Evanston, IL, USA: Routledge, 2018, pp. 3–26.
[18] E. Segal-Halevi, A. Hassidim, and Y. Aumann, "MUDA: A truthful multi-unit double-auction mechanism," in *Proc. 32nd AAAI Conf. Artif. Intell.*, 2018, pp. 1193–1201.
[19] Y. Babichenko, O. Dean, and M. Tennenholtz, "Incentive-compatible diffusion," in *Proc. World Wide Web Conf. World Wide Web (WWW)*, 2018, pp. 1379–1388.
[20] W. Shen, Y. Feng, and C. V. Lopes, "Multi-winner contests for strategic diffusion in social networks," in *Proc. 33rd AAAI Conf. Artif. Intell.*, vol. 33, Aug. 2019, pp. 6154–6162.
[21] J. Xu, X. He, and D. Zhao, "Double auction design on networks," in *Proc. 1st Int. Conf. Distrib. Artif. Intell.*, 2019, pp. 8:1–8:6.
[22] D. J. Watts and S. H. Strogatz, "Collective dynamics of 'small-world' networks," *Nature*, vol. 393, pp. 440–442, Jun. 1998.

JUNPING XU received the B.S. degree in mathematics and applied mathematics from Shanghai University, China, in 2017, and the master's degree in computer science from ShanghaiTech University, China. Her research interests include artificial intelligence and mechanism design.



XIN HE received the B.E. degree in computer science from Anhui University, China, in 2017, and the master's degree in computer science from ShanghaiTech University, China. His research interests include algorithm game theory and artificial intelligence.