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Robust Adaptive Identification of Linear Time-Varying Systems Under Relaxed Excitation Conditions

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ABSTRACT An on-line modified least-squares identification algorithm is proposed for linear time-varying systems with bounded disturbances under relaxed excitation conditions. An extra term which enhances the tracking ability for time-varying parameters is added to the covariance's update law. An indicator of the regressor's excitation level based on the maximum eigenvalue of the covariance matrix is developed. By combining the maximum eigenvalue with its variation trend shown by the sensitivity of the maximum eigenvalue to change in the covariance matrix, a novel identification law, which is switched between a modified least-squares algorithm and a gradient algorithm based on fixed σ-modification, is proposed. The boundedness of the estimation error and the covariance matrix are guaranteed via Lyapunov stability theory. The superiority of the proposed method is verified by simulations.

INDEX TERMS Least-squares identification algorithms, linear time-varying systems, relaxed excitation conditions, covariance matrix.

I. INTRODUCTION

Parameter estimation algorithms are widely used in the field of signal processing [1] and adaptive control [2], [3]. Most of them deal with time-invariant parameters. However, time-varying (TV) behavior of the plant parameters may be unavoidable due to the complex mechanisms, the model-plant mismatch, or unmeasured inputs [2]. The existing results about estimation convergence and robustness analysis are usually derived based on the assumption that the regressor is persistently excited (PE). Unfortunately, the regressor cannot always satisfy PE condition especially in adaptive feedback control systems since control input which forms the regressor is generated from the feedback controller. The robust on-line identification of TV parameters with bounded disturbances and relaxed excitation conditions is still an open problem, which has been attested by numerous textbooks [4], [5] and becomes the key motivation of this paper.

There exist two classical algorithms to estimate unknown parameters: gradient algorithms [6] and least-squares (LS) algorithms [7], [8]. It is generally accepted that LS algorithms achieve faster convergence speed and have better robustness with respect to noise and disturbances than gradient algorithms. However, the standard LS method has poor ability in tracking TV parameters since the covariance matrix becomes arbitrarily small with time, which slows down adaptation [5]. Resetting the covariance matrix is used to ensure the tracking ability for TV parameters, but it is difficult to choose suitable resetting time interval [9]. Exponential forgetting of data [10]–[12] is another effective method to address TV parameters. The main idea is that the past data should be discounted when the current parameters are estimated. Considering TV load disturbances and time invariant plant parameters, Dong et al [13]–[15] introduced two independent forgetting factors to estimate the model parameters and the load disturbances response. This method has the benefit of avoiding the so-called ''wind-up'' effect when expediting the convergence rates. Some other methods, such as polynomial approximation [16], finite-time identification algorithm [17], and fixed-time identification algorithm [17] can also be used for TV parameters. In the former method, polynomials with unknown constant coefficients (e.g. Taylor-series) are used to

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approximate the TV parameters in small time intervals. However, this method increases the number of estimated parameters and leads to large complexity. The latter two methods realize the identification of TV parameters in a finite time or fixed time by introducing the sign function into the adaptation laws. But the inclusion of sign function would induce chatting phenomenon, which excites the unmodeled dynamics when the parameter estimates are used in feedback controller. The above-mentioned methods are all assumed that the regressor should satisfy PE condition. As pointed out in [18], a constant forgetting factor may result in unboundedness of covariance matrix when PE condition is lost. Moreover, the variable forgetting factors are reported to be beneficial for speeding estimation convergence. Therefore, LS algorithms with elaborately constructed variable forgetting factor were proposed [19]–[21]. The main purpose of the above-mentioned methods [19]–[21] is to avoid unboundedness while keeping the ability in tracking TV parameters. Nevertheless, if PE is not satisfied, the estimation errors would drift to infinite under the influence of disturbances [19]. Therefore, under the relaxed excitation condition where whether the regressor satisfies PE condition is unknown (e.g. the regressor is generated form feedback controller), these methods may fail to exhibit the ideal property.

Robustness to the bounded disturbances is another basic requirement of the identification algorithms when being applied in practice. When the regressor is PE, gradient algorithm [19] and LS algorithm [22], [23] can handle bounded disturbances. Furthermore, bounded time-varying disturbances can also be estimated along with system parameters as mentioned in [12]–[15] under the assumption of PE condition. A milder exciting condition that the regressor is sufficiently excited over a finite time interval is introduced to develop an adaptive law for estimation of unknown constant parameters under the influence of bounded disturbances [24]. However, the milder initially exciting condition is also difficult to satisfy in the case of adaptive control. For the time-varying case, not even stability of the recursive estimation algorithms can be guaranteed without some modifications in the absence of PE condition. Some modifications (e.g. dead-zone [25], [26], parameter projection [5] and switching σ-modification [27]) require a priori knowledge of bounds on disturbances or unknown parameters to enhance the robustness of algorithms. However, this requirement can hardly be satisfied in practical systems. Without any prior knowledge, fixed σ-modification [28] and e_1-modification [29] are used to estimate constant unknown parameters at the expense of poor estimation accuracy.

A preliminary method which is available for linear time-varying (LTV) systems with unknown bounded disturbances, variation ranges of the parameters and exciting conditions of the regressor is proposed in our previous work [30]. The minimum eigenvalue of the covariance matrix is intuitively used to detect whether the regressor is persistently excited or not. However, it lacks strict mathematical proof. In addition, the covariance matrix should be calculated in advance to justify the variation trend of the covariance matrix's minimum eigenvalue, which increases the complexity of the algorithm. Another problem is that complex resetting operation causes difficulties in implementing. In order to overcome the aforementioned problems, a novel identification algorithm is proposed in this paper. This algorithm is the combination of the following elements: (a) the inclusion of an extra unit matrix multiplied by μ in the update law of the covariance matrix $P(t)$; (b) the necessary and sufficient condition that the regressor $\varphi(t)$ satisfies PE is that the maximum eigenvalue of $P(t)$ is bounded; (c) the variation trend of the covariance matrix's maximum eigenvalue which can be predicted by calculating the sensitivity of the eigenvalue to change in the matrix; (d) the switching strategy when estimating the parameter vector. Element (a) has been proposed by Wu et al [31] mainly as a variable forgetting factor to enhance the ability in tracking TV parameters. Element (b) together with element (c) can be used to explicitly indicate the excitation level of $\varphi(t)$ and its variation trend. Element (d) is used to switch the parameter estimation algorithm between a gradient algorithm based on fixed σ-modification and a modified least-squares algorithm according to the information provided by element (b) and (c).

This paper is organized as follows. Section 2 introduces the estimation problem. Estimation law for TV parameters and stability analysis are given in Section 3. Section 4 presents simulation results and conclusions are provided in section 5.

II. PRELIMINARIES AND PROBLEM FORMULATION

A. PRELIMINARIES

Before stating the problem, certain definition and notations are introduced here.

Definition 1 (PE Condition): A bounded vector or matrix $\varphi(t)$ satisfies PE condition [3] with a level of excitation $\alpha_0 > 0$, if $T_0 > 0$ exists, such that

$$\frac{1}{T_0}\int_t^{t+T_0} \varphi(\tau)\varphi^T(\tau)\,d\tau \geq \alpha_0 I, \forall t \geq 0. \tag{1}$$

Notations: The following notations are used

(a) $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ are the minimum and maximum eigenvalues of the corresponding matrix, respectively.
(b) If $\nu(t)$ is the function with respect to time t, then $\dot{\nu}(t)$ represents the time-derivative of $\nu(t)$.
(c) $\|\nu(t)\|$ denotes the Euclidean norm of $\nu(t)$ at t.
(d) The $\mathcal{L}_\infty$ is defined as $\|\nu(t)\|_\infty = \sup_{0\leq t}\|\nu(t)\|$, and we say $\nu(t) \in \mathcal{L}_\infty$ when $\|\nu(t)\|_\infty$ exists.

B. PROBLEM FORMULATION

Consider a parametric LTV system with bounded disturbances, i.e.,

$$y(t) = \varphi(t)^T\theta(t) + w(t), \tag{2}$$

where $\theta(t) \in R^m$ is the unknown TV parameter vector to be estimated, $y(t) \in R^n$ is the system output, $\varphi(t) \in R^{m\times n}$ is the known regressor matrix and $w(t) \in R^n$ is an unknown disturbances vector.

Assumption 1: The unknown TV parameter vector $\theta(t)$ exists an upper bound (but may be unknown).

$$\|\theta(t)\|_\infty \le M. \tag{3}$$

Assumption 2: An upper bound of the unknown TV parameter vector's variance ratio $\dot{\theta}(t)$ exists (but may be unknown).

$$\left\|\dot{\theta}(t)\right\|_\infty \le \varepsilon. \tag{4}$$

Assumption 3: The unknown disturbances vector $w(t)$ exists an upper bound (but may be unknown).

$$\|w(t)\| \le d_0. \tag{5}$$

Remark 1: Model (2) is a quite general form. On one hand, any linear system filtered by exponentially stable filters of proper order can be converted into this form [20]. If the order of the studied system is unknown, the method in [32] can be used to change the system to the standard form as (2). On the other hand, by using the so-called X-swapping technique, some nonlinear systems can also be reformulated as the form (2), as seen in [33]. The system output $y(t)$ and the regressor matrix $\varphi(t)$ are assumed to be bounded and accessible for measurement. The assumptions (3-5) have been widely used in the literature [9], [19], [34]. It should be noted that the true values of the upper bounds M, ε and d_0 are not necessarily to be known in advance because they are only used for analytical purpose.

III. ON-LINE ESTIMATION OF TV PARAMETERS

Firstly, we define the covariance matrix as [31]

$$\dot{P}(t) = \beta P(t) - P(t)\varphi(t)\varphi(t)^T P(t) + \mu I, \tag{6}$$

where $P(t)$ denotes the covariance matrix with $P(0)^T = P(0) > 0$, I represents the identity matrix, $\beta > 0$ is the forgetting factor, $\mu > 0$ is a design constant.

$\varphi(t)$ can be chosen to be PE in open-loop systems. However, in feedback control systems, the regressor $\varphi(t)$ is determined by the control input which is generated from feedback controller. Hence, $\varphi(t)$ cannot always be PE. In order to overcome this problem, an on-line explicit indicator of the regressor's excitation level should be developed. Fortunately, the maximum eigenvalue of the covariance matrix can play this role. We will prove that the sufficient and necessary condition for the validation of the regressor vector's PE condition is $\lambda_{\max}(P(t)) \in \mathcal{L}_\infty$.

Lemma 1: Consider the LTV system (2) with adaptive law (6) under assumptions (3-5), then

(a) If the regressor vector $\varphi(t)$ is PE, then $\lambda_{\max}(P(t)) \in \mathcal{L}_\infty$;

(b) If $\lambda_{\max}(P(t)) \in \mathcal{L}_\infty$, then the regressor vector $\varphi(t)$ is PE.

Proof: (a) Define $R(t) = P^{-1}(t)$, $R(0) = P(0)^{-1} > 0$, firstly. Then from (6), it follows that

$$\dot{R}(t) = \frac{d}{dt}P(t)^{-1} = -\beta R(t) + \varphi(t)\varphi(t)^T - \mu R(t)^2.$$

When $\varphi(t)$ is PE, we conclude $\gamma_1 I \le R(t) \le \gamma_2 I$, for some $\gamma_1 > 0$, $\gamma_2 > 0$[31]. Therefore, $\gamma_2^{-1} I \le P(t) \le \gamma_1^{-1} I$ and consequently $\lambda_{\max}(P(t)) \in \mathcal{L}_\infty$. Where γ_1, γ_2 denote the lower and upper bound of $R(t)$, respectively.

(b) The condition $\lambda_{\max}(P(t)) \in \mathcal{L}_\infty$ implies that $\lambda_{\min}(R(t)) > c$ for $c > 0$. Because $\mu > 0$, hence

$$\dot{R}(t) = -\beta R(t) + \varphi(t)\varphi(t)^T - \mu R(t)^2 \le -\beta R(t) + \varphi(t)\varphi(t)^T.$$

The following inequality can be obtained by applying the comparison principle

$$\begin{aligned} R(t+T_0) &\le e^{-\beta(t+T_0)}R(t) + \int_t^{t+T_0} e^{-\beta(t+T_0-\tau)}\varphi(\tau)\varphi^T(\tau)d\tau \\ &\le e^{-\beta(t+T_0)}R(t) + e^{-\beta T_0}\int_t^{t+T_0}\varphi(\tau)\varphi^T(\tau)d\tau. \end{aligned} \tag{7}$$

The inequality (7) can be rewritten as

$$\int_t^{t+T_0}\varphi(\tau)\varphi^T(\tau)d\tau \ge e^{\beta T_0}R(t+T_0) - e^{-\beta t}R(t).$$

Due to the positive definite property of matrix $R(t)$ and its continuity, there exist $\alpha_0 > 0$, $T_0 > 0$ such that $\int_t^{t+T_0}\varphi(\tau)\varphi^T(\tau)d\tau \ge \alpha_0 T_0 I$, $\forall t \ge 0$. Then according to *Definition 1*, the regressor vector $\varphi(t)$ is PE. □

Before putting forward the identification algorithm, two variables are defined as

$$P_d(t) = \beta P(t) - P(t)\varphi(t)\varphi(t)^T P(t) + \mu I, \tag{8}$$

$$\begin{aligned} Q(t) &= \frac{d(\lambda_{\max}(P(t)))}{dt} \\ &= \frac{tr(adj(P(t) - \lambda_{\max}(P(t))\cdot I)\cdot P_d(t))}{tr(adj(P(t) - \lambda_{\max}(P(t))\cdot I))}. \end{aligned} \tag{9}$$

where the formula of $Q(t)$ is derived from [35], $tr(\cdot)$ and $adj(\cdot)$ denote the trace and adjoint of the corresponding matrix, respectively. $P(0)$ can be selected as a diagonal matrix whose diagonal elements should have different values to avoid that $tr(adj(P(t) - \lambda_{\max}(P(t))\cdot I)) = 0$.

Then based on *Lemma 1*, the following on-line LS identification algorithm is proposed for LTV system (2) as follows

$$\dot{P}(t) = \begin{cases} P_d & \text{if } \lambda_{\max}(P(t)) < P_U \text{ or} \\ & \text{if } \lambda_{\max}(P(t)) = P_U \text{ and } Q(t) < 0 \\ 0 & \text{otherwise,} \end{cases} \tag{10}$$

$$\dot{\hat{\theta}}(t) = \begin{cases} P(t)\varphi(t)e(t) & \text{if } \lambda_{\max}(P(t)) < P_U \text{ or} \\ & \text{if } \lambda_{\max}(P(t)) = P_U \text{ and } Q(t) < 0 \\ P(t)\varphi(t)e(t) - \sigma P(t)\hat{\theta}(t) & \text{otherwise,} \end{cases} \tag{11}$$

where $\hat{\theta}(t)$ represents the estimate of the unknown parameter vector, P_U is a designed scalar with $P(0) < P_U I$, $\sigma > 0$ represents the fixed σ-modification. $e(t)$ is the prediction error

$$\begin{aligned} e(t) &= y(t) - \hat{y}(t) = y(t) - \varphi^T(t)\hat{\theta}(t) \\ &= -\varphi^T(t)\tilde{\theta}(t) + w(t), \end{aligned} \tag{12}$$

where $\tilde{\theta}(t) = \hat{\theta}(t) - \theta(t)$ is the estimation error.

Hereafter, we drop in the notation the explicit dependence on t (that is, let $R = R(t)$, $P = P(t)$ and so on), for simplicity. The properties of this algorithm are established in the following theorem.

Theorem 1: For the LTV model (2), when the assumptions (3-5) are satisfied, then the novel estimation algorithm described by the equations (10) and (11) guarantees that

(a) $P, P^{-1}, e, \hat{\theta}, \dot{\hat{\theta}} \in \mathcal{L}_\infty[0, \infty)$;

(b) If φ is independent of w and satisfies PE condition as (1) beforehand, $\tilde{\theta}$ would exponentially converge to the residual set

$$R_{\tilde{\theta}} = \left\{ \tilde{\theta} \,\middle|\, \left\|\tilde{\theta}\right\| \le \sqrt{\frac{(\mu d_0^2 + \varepsilon^2)}{\beta\mu\lambda_{\min}(R)}} \right\}, \tag{13}$$

where ε and d_0 are defined in (4-5).

Proof: From *Lemma 1*, we can conclude that $P \ge \gamma_2^{-1} I$. In order to proof the boundedness of P, define a convex set with a smooth boundary almost everywhere as $\mathcal{S} = \{P \,|\lambda_{\max}(P) - P_U \le 0\}$. We start from an initial point $P(0) \in \mathcal{S}$. The equation for the covariance matrix P is modified such that at the point in the interior of $\mathcal{S}$, P is updated according to (6). If the current point is on the boundary of $\mathcal{S}$ and the direction of search given by the unconstrained algorithm is pointing inside $\mathcal{S}$, then we keep this algorithm. If the direction of search is pointing away from $\mathcal{S}$, then P is a constant matrix, and, therefore, the adaptive law at that point is a gradient algorithm with σ-modification. The above strategy is similar to the adaptive laws with projection proposed in [5]. And the conclusion $\lambda_{\max}(P) \le P_U$ can always be guaranteed, which implies that P, $P^{-1} \in \mathcal{L}_\infty[0, \infty)$.

The Lyapunov-like function is selected as

$$V\left(\tilde{\theta}\right) = \frac{\tilde{\theta}^T R\tilde{\theta}}{2}.$$

Case I: When P is a constant matrix, we have $\dot{R} = 0$. The derivative $\dot{V}$ is derived as

$$\dot{V} = -e^2 + ew - \sigma\hat{\theta}^T\tilde{\theta} - \dot{\theta}^T R\tilde{\theta} \le -\frac{e^2}{2} + \frac{w^2}{2} - \sigma\hat{\theta}^T\tilde{\theta} - \dot{\theta}^T R\tilde{\theta}.$$

We apply the completion of squares

$$-\sigma\hat{\theta}^T\tilde{\theta} \le -\sigma\tilde{\theta}^T\tilde{\theta} + \sigma\left|\theta^T\tilde{\theta}\right| \le -\frac{\sigma\tilde{\theta}^T\tilde{\theta}}{2} + \frac{\sigma\|\theta\|^2}{2}$$

$$-\dot{\theta}^T R\tilde{\theta} = -\left\|\delta R\tilde{\theta} + \frac{\dot{\theta}}{2\delta}\right\|^2 + \delta^2\left\|R\tilde{\theta}\right\|^2 + \left(\frac{\left\|\dot{\theta}\right\|}{2\delta}\right)^2, \tag{14}$$

where $\delta > 0$. It can be shown that

$$\begin{aligned}
\dot{V} &\le -\frac{e^2}{2} + \frac{w^2}{2} - \frac{\sigma\tilde{\theta}^T\tilde{\theta}}{2} + \frac{\sigma\|\theta\|^2}{2} + \delta^2\left\|R\tilde{\theta}\right\|^2 + \left(\frac{\left\|\dot{\theta}\right\|}{2\delta}\right)^2 \\
&\le -\frac{e^2}{2} - \left(\sigma\lambda_{\min}(P) - \frac{2\delta^2}{\lambda_{\min}(P)}\right)V + \frac{d_0^2}{2} + \frac{\sigma M^2}{2} + \frac{\varepsilon^2}{4\delta^2} \\
&\le -\left(\sigma\gamma_2^{-1} - 2\delta^2\gamma_2\right)V + \frac{d_0^2}{2} + \frac{\sigma M^2}{2} + \frac{\varepsilon^2}{4\delta^2}.
\end{aligned} \tag{15}$$

Since d_0, M, $\varepsilon \in \mathcal{L}_\infty$, if $\sigma > 2\delta^2\gamma_2^2$, it can readily be shown that V and $\tilde{\theta}$ are bounded. It needs to be pointed out that δ in (14) can be any constant greater than zero, the stability of V can be guaranteed as long as $\sigma > 0$. But the value of σ is related to the estimation performance, which would be discussed in *Remark 2*.

Case II: When P is updated according to (6), then

$$\dot{V} = \dot{\tilde{\theta}}^T R\tilde{\theta} + \frac{1}{2}\tilde{\theta}^T\frac{d(R)}{dt}\tilde{\theta}, \quad \dot{\tilde{\theta}}^T R\tilde{\theta} = -e^2 + ew - \dot{\theta}^T R\tilde{\theta}.$$

Consider (14) and let $\delta = \sqrt{\mu/2}$, we obtain

$$\begin{aligned}
\dot{V} &= -e^2 + ew - \dot{\theta}^T R\tilde{\theta} + \frac{1}{2}\tilde{\theta}^T(\varphi\varphi^T - \beta R - \mu R^2)\tilde{\theta} \\
&= -e^2 + ew - \dot{\theta}^T R\tilde{\theta} + \frac{1}{2}(\varphi^T\tilde{\theta})^2 - \beta V - \frac{1}{2}\mu\left\|R\tilde{\theta}\right\|^2 \\
&= -\frac{e^2}{2} + \frac{w^2}{2} + \frac{\left\|\dot{\theta}\right\|^2}{2\mu} - \left\|\sqrt{\frac{\mu}{2}}R\tilde{\theta} + \frac{\dot{\theta}}{\sqrt{2\mu}}\right\|^2 - \beta V \\
&\le \frac{w^2}{2} - \frac{e^2}{2} + \frac{\left\|\dot{\theta}\right\|^2}{2\mu} - \beta V \le \frac{d_0^2}{2} + \frac{\varepsilon^2}{2\mu} - \beta V.
\end{aligned} \tag{16}$$

If $V \ge l = \frac{1}{\beta}\left(\frac{d_0^2}{2} + \frac{\varepsilon^2}{2\mu}\right)$ then $\dot{V} \le 0$, which illustrates that V converges to a set with respect to l.

Combining (15) with (16), we can conclude that $V \in \mathcal{L}_\infty$. Hence, $\tilde{\theta} \in \mathcal{L}_\infty$. The boundedness of $\tilde{\theta}$ implies that $\hat{\theta} \in \mathcal{L}_\infty$, which, together with $\varphi \in \mathcal{L}_\infty$, imply that $e, \dot{\hat{\theta}} \in \mathcal{L}_\infty$. So the conclusion (a) in *Theorem 1* is tenable. Since P is bounded as shown in *Lemma 1*, if φ is PE, the conclusion (b) in *Theorem 1* is also tenable. □

Remark 2: The key point of the proposed algorithm is that $\lambda_{\max}(P)$ and its variation tendency are used as an online indicator of φ's PE condition. For the case of PE condition, a novel LS algorithm whose estimation performance can be effectively enhanced is used to estimate the unknown TV parameters. Otherwise the gradient algorithm with fixed σ-modification is used to guarantee the boundedness of the adaptation law. According to *Theorem 1*, the estimation performance of the proposed algorithm depends on the value of forgetting factor β, extra term μ and leakage term σ. A lager β leads to faster convergence and stronger ability in tracking TV parameters at the cost of more oscillations in the estimated parameters. The extra term μ involves a similar tradeoff as β. As for σ, the better robustness against disturbances is reached by a larger value of σ, which, however, may induce larger estimation error. Additionally, unlike algorithm in [30], the value of σ can be chosen as any positive constant, which is independent on μ and another design scalar P_U. It also needs to be pointed out that P_U in [30] should be selected carefully in practice, otherwise the covariance matrix still may become unbounded when PE condition is lost. In contrast, P_U in the proposed method can be arbitrarily selected because it actually represents the level of excitation condition.

Remark 3: t_s, t_{s+1} are defined as the current time-step and the next step, respectively. To recursively compute the

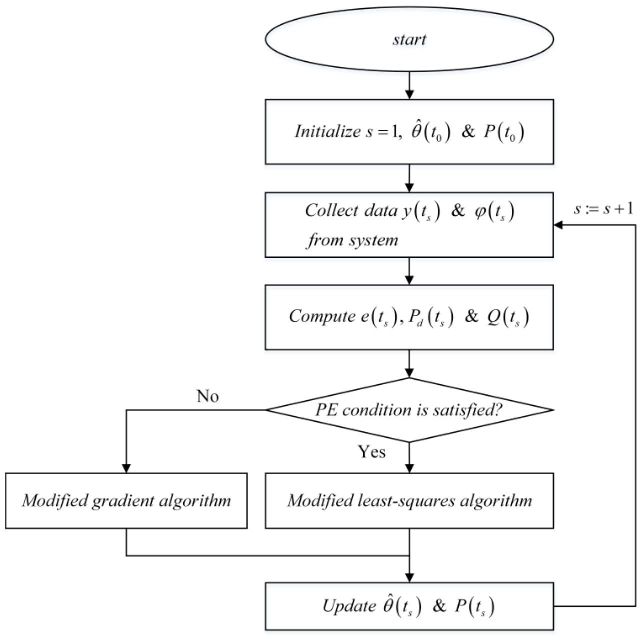

FIGURE 1. Flowchart of computing the parameter estimates.

estimates of unknown parameters, the workflow is listed as following.

1) To initialize, $s = 1$, $\hat{\theta}(t_0)$ and $P(t_0)$ are designed.

2) Collect the input-output data $\varphi(t_s)$ and $P(t_s)$ from system.

3) Compute $P_d(t_s)$, $Q(t_s)$, $e(t_s)$ according to (8), (9), (12), respectively.

4) Identify whether PE condition is satisfied and update $P(t_s)$ and $\hat{\theta}(t_s)$ according to (10) and (11), respectively.

5) Increase s by 1 and go to step 2.

The flowchart of computing the parameter estimate $\hat{\theta}(t_s)$ is shown in Fig. 1.

IV. SIMULATION STUDIES

Consider the following LTV system, in which the regressor can be chosen to be PE or not,

$$y = \theta_1 x_1 + \theta_2 x_2 + w, \tag{17}$$

where y is the system output, x_1 and x_2 are system states that are assumed to be available, $\theta_1 = 2 + \sin t$ and $\theta_2 = 3 + \cos(0.5t)$ are the unknown TV parameters, $w = rand(1) - 0.5$ is the unknown disturbances. This simulation example can be transformed into the standard form as (2), with $\varphi = [x_1 x_2]^T$; $\theta = [\theta_1 \theta_2]^T$. We can easily proof that assumptions (3-5) are satisfied for the LTV system (17).

To illustrate the superiority, LS algorithm with variable forgetting factor [19], modified LS algorithms [20] and [30] are compared with the proposed scheme. The parameters of all algorithms are set to the following values in order to optimize their performance:

The design parameters of the proposed algorithm are set as $P_U = 1000$, $\sigma = 10^{-4}$, $\beta = 5$ and $\mu = 5$. The initial conditions of both algorithms are set as $\hat{\theta}_1(0) = \hat{\theta}_2(0) = 0$,

TABLE 1. Estimation error results using the four algorithms.

Methods	$J_{\tilde{\theta}}(t)\big\|_{t_1=0}^{t_2=10.125}$	$J_{\tilde{\theta}}(t)\big\|_{t_1=20.125}^{t_2=30.125}$	$J_{\tilde{\theta}}(t)\big\|_{t_1=0}^{t_2=40.125}$
The proposed algorithm	2.0818	0.2960	12.8237
The algorithm in [19]	3.0506	0.6211	24.0764
The algorithm in [20]	7.2848	3.4382	28.9737
The algorithm in [30]	3.0293	0.5831	16.6078

$P(0) = [10; 00.5]$. The design coefficients of the algorithm in [19] are set as $\lambda_0 = 5$, $k_0 = 1000$. According to plant (17), the bounds of actual parameters and their variation ratios are calculated as $\varepsilon = 1.12$ and $M = 5$. Thus, the optimal design coefficients of the algorithm in [20] are set as $\sigma = \mu = \varepsilon / M = 0.22$. In order to assure the boundedness of the covariance matrix, the design parameters of the algorithm in [30] are selected as $P_U = 0.75$, $\sigma = 0.1$, $\beta = 5$ and $\mu = 0.05$.

The first element of the regressor φ is chosen as $x_1 = 3\sin(4\pi t)$, the second element of the regressor φ is set as a piecewise continuous function

$$x_2 = \begin{cases} 2.5 & t < 10.125 \\ 2.5\sin(4\pi t) & 10.125 \le t < 20.125 \\ 2.5 & 20.125 \le t < 30.125 \\ 2.5\sin(4\pi t) & 30.125 \le t < 40.125 \end{cases}$$

It is easy to verify that φ satisfies PE condition intermittently. Under this relaxed excitation condition, the estimation performance of the four algorithms is shown in Fig. 2-4, from which one we can see that the proposed algorithm exhibits more superior tracking performance than other algorithms. $D\left[\tilde{\theta}_i(t)\right] = \sqrt{\frac{1}{t}\int_0^t \tilde{\theta}_i^2(\tau)\, d\tau}$ is used to measure the convergence speed of the estimation error. As shown in Fig. 5, the proposed identification law converges faster than the other three algorithms. The maximum eigenvalue of the covariance matrix proposed in this paper is provided in Fig. 6. From this figure, we can see that $\lambda_{\max}(P)$ exceeds the specified value P_U immediately when φ does not satisfy PE condition (i.e., when $10.125 \le t < 20.125$ and $30.125 \le t < 40.125$). Otherwise, $\lambda_{\max}(P)$ would be confined within a small range of changes. This phenomenon demonstrates the effectiveness of *Lemma 1*. In order to compare the four methods in terms of statistical evaluation, the integral of the square norm of the estimation error, $J_{\tilde{\theta}}(t) = \int_{t_1}^{t_2} \left\|\tilde{\theta}(\tau)\right\|^2 d\tau$ is adopted. As shown in Table 1, the proposed algorithm gives smaller estimation errors compared with the algorithm in [19], algorithm in [20] and algorithm in [30] no matter the PE condition is satisfied or not.

Simulations are executed to verify the superiority of the proposed algorithm. The inclusion of an extra unit matrix multiplied by μ in the update law of the covariance matrix enhances the tracking ability of the proposed algorithm. That is the reason why the proposed algorithm and the

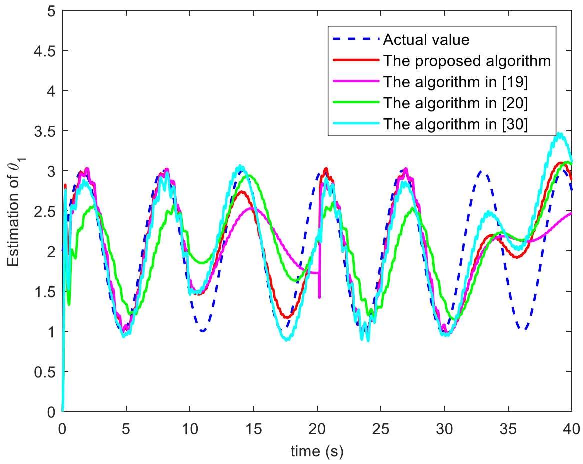

FIGURE 2. Estimation of the unknown TV parameter θ_1.

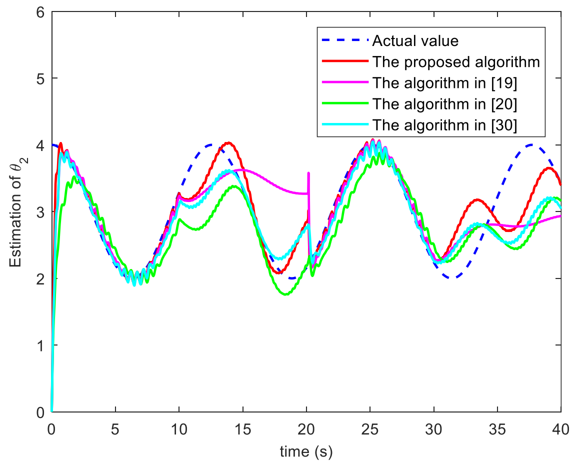

FIGURE 3. Estimation of the unknown TV parameter θ_2.

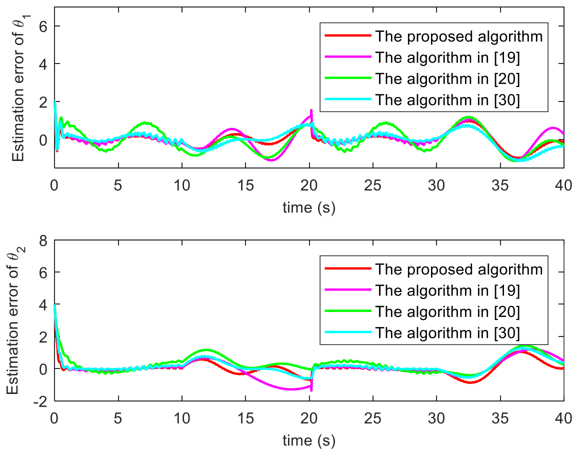

FIGURE 4. Estimation error of unknown TV parameters.

algorithm in [30] achieve faster converge speed than the algorithm in [19] and algorithm in [20]. For the algorithm in [20], the $-\sigma P\hat{\theta}$ of the LS algorithm has the tendency to drive $\hat{\theta}$ to 0. If $\theta \neq 0$, the term may drive $\hat{\theta}$ towards zero and away from the actual value, which may destroy the estimation accuracy. In addition, during the design of the algorithm in [20],

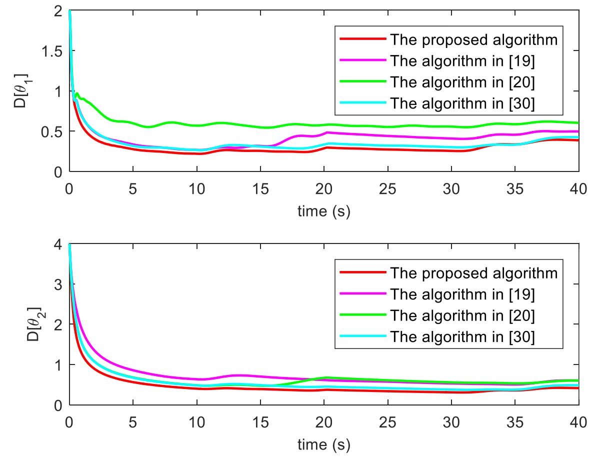

FIGURE 5. The speed of convergence of four algorithms.

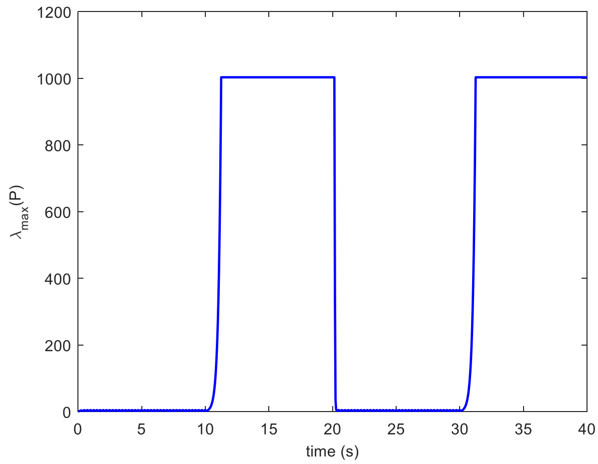

FIGURE 6. The maximum eigenvalue of the covariance matrix proposed in this paper.

the optimal design coefficients are based on the bounds of parameters and their variation ratios, which cannot be easily realized in practical systems. As a comparison, the proposed identification algorithm can be implemented without any priori knowledge on the bounds of the unknown parameters and the disturbances. If the excitation level increases from low to sufficiently high, the proposed method can be switched from gradient algorithm to the modified LS algorithm, which can avoid the adverse effect of σ-modification. Additionally, the selection of P_U in [30] is not arbitrarily, which also restrains the values of μ and σ. This unnecessary limitation is removed in the proposed algorithm so that the optimal value of design parameters can be chosen. So these are the reasons for better simulation results. It should be noted that, when PE condition is not satisfied, the lack of effective information of the regressor influences the estimation performance. Under this condition, the proposed method switches to the gradient algorithm to guarantee the stability of the algorithm.

V. CONCLUSION

In this paper, an on-line identification algorithm is presented to estimate parameters for LTV systems under some relaxed assumptions that the priori knowledge on bounds of the parameters and the disturbances should not be known in advance. The boundedness of estimation error, parameter estimates and their derivatives have been proved. Comparative simulation results validate that the proposed identification algorithm has more superb identification accuracy. Further study will focus on the selection of input signal and the optimal choices of parameters β, σ and μ, through which the estimation performance will be enhanced.

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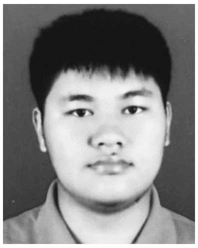

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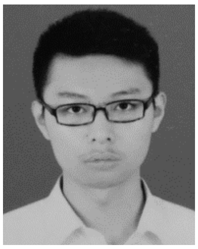

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