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H_2/H_∞ Simultaneous Fault Detection and Control for Markov Jump Linear Systems With Partial Observation

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ABSTRACT We focus on the Simultaneous Fault Detection and Control (SFDC) in the context of Markov Jump Linear Systems (MJLS). The main novelty of the paper is the design of H_∞ and H_2 SFDC under the MJLS framework considering partial observation of the Markov chain. Both designs are obtained via Bilinear Matrix Inequalities optimization problem. As secondary results we provide a Mixed H_2/H_∞ SFDC under the same set up, as well as the implementation of a coordinated descent algorithm to solve the optimization problem formulated as Bilinear Matrix Inequalities (BMI). To illustrate the viability of the proposed solution a numerical example is provided.

INDEX TERMS Markovian jump linear systems, simultaneous fault detection and control, hidden markov mode, H_∞ norm, H_2 norm.

I. INTRODUCTION

Over the last decades, the demand for systems with high reliability has increased, and for that reason, there is an increase in the demand for control solutions that aim to optimize not only the performance but also the safety levels. The most recent control solutions developed under this premise are the so-called Fault Detection and Isolation (FDI) approach [1], [13], [25], or Fault-Tolerant Control (FTC) approach [16], [17], [27]. Both solutions aim to increase the reliability using completely different methods, therefore, a straightforward way to increase the reliability would be the implementation of both approaches in parallel. However, the overall complexity of implementing two distinct units may be difficult.

As an alternative to overcome this complexity issue, the design of a single unit that simultaneously works as a stabilizing controller and residue generator has been studied in the literature in the form of the so-called Simultaneous Fault Detection and Control (SFDC). Another aspect that must be considered to increase the system reliability is the communication dropout and communication delay caused by package collision, which are both inherent phenomena to the network communication and have a negative impact on the control system performance.

One strategy to tackle the aforementioned aspects is to use for modelling a class of stochastic systems named Markov jump linear systems (MJLS). In this case, the MJLS role is to model any unpredictable network behavior, with each possible network behavior assigned to a particular Markov chain mode and the transition between modes ruled by the Markov chain.

In this regard, an important premise is that the network state is instantly accessible, which may not be achievable in real implementation. A possible way to model this particular circumstance is the set up presented in [21] and [3], which deals with a detector based approach when the Markov chain modes are partially known. These works allow us to design a SFDC solution that does not depend on the Markov mode, instead it depends only on a detected mode.

The SFDC problem has received a great deal of attention recently. There are plenty of works that tackle similar problems, we can mention [7], [11] for SFDC solutions

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considering the deterministic case, the first using state-feedback controllers, the second one using affine switched systems. [15] presented a deterministic solution for the SFDC based on Linear Matrices Inequalities (LMI). The work [22] presented a solution based on LMI using the performance index H_-/H_∞. The authors in [26] proposed an SFDC for continuous-time MJLS applied to a forging equipment. [14] presented an SFDC for continuous-time MJLS considering uncertain transition rate in the Markov chain. All the aforementioned works consider that the Markov chain is instantly accessible, hence the development of new techniques that do not rely on this premise motivated the present paper.

As previously mentioned, in this paper, the SFDC problem under discrete-time MJLS framework with partial information on the jump parameter is investigated. To provide a solution that works as a controller and a fault detector simultaneously the resulting closed-loop system must be stochastically stable, in which two performance criteria are studied: one regarding the H_∞ norm and the other for the H_2 norm. The contributions are summarized as follows:

- Analysis of the H_∞ SFDC problem under the discrete-time MJLS framework with partial information on the jump parameter, based on Bilinear Matrix Inequalities (BMI).
- Analysis of the H_2 SFDC problem under the discrete-time MJLS framework with partial information on the jump parameter, based on BMI.
- Analysis of the Mixed H_2/H_∞ SFDC problem under the discrete-time MJLS framework with partial information on the jump parameter, based on BMI.
- An illustrative example is presented to demonstrate the usefulness of the proposed approach.

The BMI are solved using a specific type of coordinate decent algorithm, which is also explained in the present paper.

The remainder of this paper is organized in the following manner. Section II presents the notation. Section III formulates the SFDC problem and provides some preliminaries. Section IV introduces the main results. Section V presents an illustrative example. The final comments are given in Section VI.

II. NOTATION

The real n-dimensional Euclidean space is represented by $\mathbb{R}^n$ and the space of $n \times m$ real matrices, by $\mathbb{B}(\mathbb{R}^m, \mathbb{R}^n)$. $(\cdot)'$ indicates the transpose of a matrix, I_n is the identity matrix of size $n \times n$, $0_{n\times m}$ is the null matrix of size $n \times m$, $diag(\cdot)$ is a block diagonal matrix. For partitioned symmetric matrices, the symbol $\bullet$ is a generic symmetric block. For N, a positive integer, we set $\mathbb{N} \triangleq \{1, 2, 3, \ldots, N\}$. The set $\mathbb{H}^{n,m}$ is the linear space of all N-sequence of real matrices $V = (V_1, V_2, \ldots, V_N)$, $V_i \in \mathbb{B}(\mathbb{R}^n, \mathbb{R}^m)$, $i \in \mathbb{N}$ and, for the ease of notation, $\mathbb{H}^n \triangleq \mathbb{H}^{n,n}$ and $\mathbb{H}^{n+} \triangleq \{V \in \mathbb{H}^n; V_i \geq 0,\ i = 1, \ldots, N\}$. For $P, V \in \mathbb{H}^{n+}$, we write that $P > V$ if $P_i > V_i$ for each $i = 1, \ldots, N$. On a probability space $(\Omega, \mathscr{F}, P)$ with filtration $\{\mathscr{F}_k\}$, the expected value operator is represented by $\mathbf{E}(\cdot)$, the conditional expected operator, by $\mathbf{E}(\cdot \mid \cdot)$, and the space of all discrete-time sequences of dimension r, $\mathscr{F}_k$-adapted processes, such that $\|z\|_2^2 \triangleq \sum_{k=0}^{\infty} \mathbf{E}(\|z(k)\|^2) < \infty$, by l_2^r.

III. PRELIMINARIES

Consider the following MJLS in the stochastic space $(\Omega, \mathscr{F}, \mathcal{P})$ with filtration $\{\mathscr{F}_k\}$,

$$\mathcal{G}: \begin{cases} x(k+1) = A_{\theta(k)}x(k) + B_{\theta(k)}u(k) + \ldots \\ \qquad\qquad \ldots J_{w\theta(k)}w(k) + J_{f\theta(k)}f(k) \\ y(k) = L_{\theta(k)}x(k) + H_{w\theta(k)}w(k) + H_{f\theta(k)}f(k) \\ z(k) = C_{\theta(k)}x(k) + D_{\theta(k)}u(k), \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state, $u(k) \in \mathbb{R}^m$ is the control input, $w(k) \in \mathbb{R}^r$ is the disturbance, $f(k) \in \mathbb{R}^f$ is the signature of the failure, $y(k) \in \mathbb{R}^s$ is the measured output, and $z(k) \in \mathbb{R}^q$ is the controlled output. We set $x(0) = 0$ and define $\theta(k)$ as a homogeneous Markov chain taking its values in $\mathbb{N}$ with $\theta(0) = \theta_0$, with θ_0 a random variable, and transition probabilities $\mathcal{P}(\theta(k+1) = j|\theta(k) = i) = p_{ij}$, $\mathbb{P} \triangleq [p_{ij}]$. It is considered, without loss of generality, that $\mathbb{P}$ has no column equal to zero, meaning that $\mathbb{P}$ is nondegenerate, [19].

We would like to design a type of stabilizing controller that simultaneously can act as a residual filter as well. The controller/filter structure is given by

$$\mathcal{C}: \begin{cases} x_c(k+1) = A_{c\hat{\theta}(k)}x_c(k) + B_{c\hat{\theta}(k)}y(k) \\ u(k) = C_{c\hat{\theta}(k)}x_c(k) \\ \hat{f}(k) = C_{f\hat{\theta}(k)}x_c(k) + D_{f\hat{\theta}(k)}y(k), \end{cases} \tag{2}$$

where $x_c \in \mathbb{R}^n$ is the controller state and $\hat{f}(k) \in \mathbb{R}^f$ is an estimate of the signature signal $f(k)$.

One of the main premises in this work is that $\theta(k)$ is not directly accessible but, rather a detector provides an estimation of $\theta(k)$, denoted by $\hat{\theta}(k)$. The estimation $\hat{\theta}(k)$ takes its values on the set $\mathbb{M}_i$, when $\theta(k) = i$. $\mathbb{M}_i$ is a subset of $\mathbb{M} = \{1, \ldots, M\}$, where $\mathbb{M}$ represents all the possible values of the detector $\hat{\theta}(k)$. We consider that the signal $\hat{\theta}(k)$ emitted from the detector depends only on $\theta(k)$. Let $\hat{\mathcal{F}}_0$ be the σ-field generated by $\{x(0),\ \theta(0)\}$ and $\hat{\mathcal{F}}_k$ be the σ-field generated by $\{x(0), \theta(0), \hat{\theta}(0), \ldots, x(k), \theta(k)\}$. We consider that $\hat{\theta}(k) \in \{1, \ldots, M\}$ is associated to $\theta(k)$ as in

$$\mathcal{P}(\hat{\theta}(k) = l \mid \hat{\mathcal{F}}_k) = \mathcal{P}(\hat{\theta}(k) = l|\theta(k)) = \alpha_{\theta(k)l}, \quad l \in \mathbb{M} \tag{3}$$

with $\sum_{l=1}^{M} \alpha_{il} = 1$ for each $i \in \mathbb{N}$.

Consider $\mathcal{F}_k$ as the σ-field generated by $\{x(t), \theta(t), \hat{\theta}(k);\ t = 0, \ldots, k\}$. We have that

$$\mathcal{P}(\theta(k+1) = j|\mathcal{F}_k) = \mathcal{P}(\theta(k+1) = j|\theta(k)) = p_{\theta(k)j}. \tag{4}$$

Therefore, α_{il} denotes the probabilities that the detector will emit the signal $l \in \mathbb{M}$ considering $\theta(k) = i$. The set $\mathbb{M}_i$ can be written as

$$\mathbb{M}_i = \{l \in \mathbb{M}; \alpha_{il} > 0\} = \{k_1^i, \ldots, k_{\tau_i}^i\}, \quad \cup_{i=1}^{N}\mathbb{M}_i = \mathbb{M}. \tag{5}$$

The goal is to stabilize (1) through (2) whilst at the same time the controller acts also as supervisory filter providing estimates of $\hat{f}(k)$ through the residual signal

$$r(k) \triangleq f(k) - \hat{f}(k).$$

By connecting (1) and (2) and defining $\tilde{x}(k)' \triangleq \begin{bmatrix} x(k)' & x_c(k)' \end{bmatrix}$ and, $\tilde{w}(k)' \triangleq \begin{bmatrix} w(k)' & f(k)' \end{bmatrix}$, we get the closed-loop dynamics

$$\mathcal{G}_c : \begin{cases} \tilde{x}(k+1) = \tilde{A}_{\theta(k)\hat{\theta}(k)}\tilde{x}(k) + \tilde{J}_{\theta(k)\hat{\theta}(k)}\tilde{w}(k) \\ z(k) = \tilde{C}_{c\theta(k)\hat{\theta}(k)}\tilde{x}(k), \\ r(k) = \tilde{C}_{f\theta(k)\hat{\theta}(k)}\tilde{x}(k) + \tilde{E}_{f\theta(k)\hat{\theta}(k)}\tilde{w}(k), \end{cases} \tag{6}$$

where

$$\tilde{A}_{il} \triangleq \begin{bmatrix} A_i & B_iC_{cl} \\ B_{cl}L_i & A_{cl} \end{bmatrix}, \quad \tilde{J}_{il} \triangleq \begin{bmatrix} J_{wi} & J_{fi} \\ B_{cl}H_{wi} & B_{cl}H_{fi} \end{bmatrix},$$
$$\tilde{C}_{cil} \triangleq \begin{bmatrix} C_i & D_iC_{cl} \end{bmatrix}, \quad \tilde{C}_{fil} \triangleq \begin{bmatrix} -D_{fl}L_i & -C_{fl} \end{bmatrix},$$
$$\tilde{E}_{fil} \triangleq \begin{bmatrix} -D_{fl}H_{wi} & I_f - D_{fl}H_{fi} \end{bmatrix}.$$

Let us introduce some basic concepts required for properly describing the main goal. The concept of internal stochastic stability and stabilizability are stated next, where $A \triangleq (A_1, \ldots, A_n) \in \mathbb{B}(\mathbb{R}^n)$, $B \triangleq (B_1, \ldots, B_n) \in \mathbb{B}(\mathbb{R}^n, \mathbb{R}^m)$, and $K \triangleq (K_1, \ldots, K_n) \in \mathbb{B}(\mathbb{R}^m, \mathbb{R}^n)$, and for $Q \in \mathbb{H}^n$, $\mathcal{E}_i(Q) \triangleq \sum_{j\in\mathbb{N}} p_{ij}Q_j$.

Definition 1 (Internal Stochastic Stability): System (6) is said to be internally stochastically stable (ISS) if for any $\tilde{x}(0) \in \mathbb{R}^{2n}$ and $\theta_0 \in \mathbb{N}$ we have that $\|\tilde{x}\|_2 < \infty$.

Definition 2 (Internal Stochastic Stabilizability): The pair (A, B) is said to be internally stochastically stabilizable if there exists K and $Y \in \mathbb{H}^{n+}$, $Y > 0$, such that $Y_i - A_i(K_i)'\mathcal{E}_i(Y)A_i(K_i) > 0$ holds for all $i \in \mathbb{N}$, where $A_i(K_i) \triangleq A_i + B_iK_i$.

The class of admissible controllers is given by $\mathscr{C} \triangleq \{\mathcal{C} : \text{(6) is iSS}\}$. Next we introduce the concept of $\mathcal{H}_\infty$ norm of (6) with respect to outputs $z(k)$ and $r(k)$ adapted from [21]. For that, we set $\mathcal{W}_i \triangleq \{\tilde{w} \in l_2^{r+f} : \|\tilde{w}\|_{2i} > 0\}$, where for any signal $g = \{g(k), k = 0, 1, 2, \ldots\}$, $\|g\|_{2i}^2 \triangleq \mathbf{E}(\|g(k)\|^2 \mid \theta_0 = i)$.

Now we define the H_∞ and H_2 norms, which will be used to present later on the mixed formulation. We start with the H_∞ norm definition.

Definition 3 ($\mathcal{H}_\infty$ Norms): Given that $\mathcal{C} \in \mathscr{C}$, the $\mathcal{H}_\infty$ norm of (6) with respect to z is given by

$$\|\mathcal{G}_c\|_\infty^{(\tilde{w}\mapsto z)} \triangleq \sup_{i\in\mathbb{N}} \sup_{\tilde{w}\in\mathcal{W}_i} \frac{\|z\|_{2i}}{\|\tilde{w}\|_{2i}},$$

and the $\mathcal{H}_\infty$ norm of (6) with respect to r by,

$$\|\mathcal{G}_c\|_\infty^{(\tilde{w}\mapsto r)} \triangleq \sup_{i\in\mathbb{N}} \sup_{\tilde{w}\in\mathcal{W}_i} \frac{\|r\|_{2i}}{\|\tilde{w}\|_{2i}}.$$

Consider the following inequalities for given $\gamma_c > 0$ and $\gamma_r > 0$,

$$\begin{bmatrix} P_i & 0 \\ 0 & \gamma_c^2 I \end{bmatrix} > \sum_{l\in\mathbb{M}_i} \alpha_{il} \begin{bmatrix} M_{il} & \bullet \\ N_{il} & S_{il} \end{bmatrix}, \tag{7}$$

$$\begin{bmatrix} M_{il} & \bullet \\ N_{il} & S_{il} \end{bmatrix} > \begin{bmatrix} \tilde{A}_{il} & \tilde{J}_{il} \\ \tilde{C}_{cil} & 0 \end{bmatrix}' \begin{bmatrix} \mathcal{E}_i(P) & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \tilde{A}_{il} & \tilde{J}_{il} \\ \tilde{C}_{cil} & 0 \end{bmatrix}, \tag{8}$$

and

$$\begin{bmatrix} \mathfrak{P}_i & 0 \\ 0 & \gamma_r^2 I \end{bmatrix} > \sum_{l\in\mathbb{M}_i} \alpha_{il} \begin{bmatrix} \mathfrak{M}_{il} & \bullet \\ \mathfrak{N}_{il} & \mathfrak{S}_{il} \end{bmatrix}, \tag{9}$$

$$\begin{bmatrix} \mathfrak{M}_{il} & \bullet \\ \mathfrak{N}_{il} & \mathfrak{S}_{il} \end{bmatrix} > \begin{bmatrix} \tilde{A}_{il} & \tilde{J}_{il} \\ \tilde{C}_{fil} & \tilde{E}_{fil} \end{bmatrix}' \begin{bmatrix} \mathcal{E}_i(\mathfrak{P}) & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \tilde{A}_{il} & \tilde{J}_{il} \\ \tilde{C}_{fil} & \tilde{E}_{fil} \end{bmatrix}, \tag{10}$$

for all $i \in \mathbb{N}$. The following bounded-real lemma is adapted from [21].

Lemma 1 (Bounded-Real Lemma): If there exists $P \in \mathbb{H}^{2n+}$, $P > 0$, $\mathfrak{P} \in \mathbb{H}^{2n+}$, $\mathfrak{P} > 0$, such that (7), (8), (9), and (10) hold, then $\mathcal{C} \in \mathscr{C}$, $\|\mathcal{G}_c\|_\infty^{(\tilde{w}\mapsto z)} < \gamma_c$ and $\|\mathcal{G}_c\|_\infty^{(\tilde{w}\mapsto r)} < \gamma_r$.

Therefore the goal is to design $\mathcal{C} \in \mathscr{C}$ so that $\|\mathcal{G}_c\|_\infty^{(\tilde{w}\mapsto z)} < \gamma_c$ and $\|\mathcal{G}_c\|_\infty^{(\tilde{w}\mapsto r)} < \gamma_r$ for $\tilde{w} \in \mathcal{W}_i$, $i \in \mathbb{N}$. Specifically in this work we focus our efforts in finding

$$\inf_{\mathcal{C}\in\mathscr{C},P,\gamma_r,\gamma_c} \{\gamma_c\beta_c + \gamma_r\beta_r\} \;:\; \text{s. t. (7), (8), (9) and (10)} \tag{11}$$

hold for a given $\beta_c > 0$, $\beta_r > 0$. This particular formulation will be useful later on in this paper. We present next the H_2 norm definition.

Definition 4 (H_2 Norms): Assume that $\mathcal{C} \in \mathscr{C}$. For $\tilde{x}(0) = 0$, define $z^{s,i}$ and $r^{s,i}$, the outputs of (6) for the initial condition $\theta(0) = i$ and the input $\tilde{w}(k) = 0$ for $k \geq 1$ and $\tilde{w}(0) = e_s$, where e_s is the $s-$th vector of the standard basis of $\mathbb{R}^s$. The H_2 norms of (6) with respect to the ouputs z and r are given by

$$\|\mathcal{G}_c\|_2^{(\tilde{w}\mapsto z)} = \sqrt{\sum_{s=1}^{r}\sum_{i=1}^{N} \mu_i \|z^{s,i}\|_2^2} \tag{12}$$

and

$$\|\mathcal{G}_c\|_2^{(\tilde{w}\mapsto r)} = \sqrt{\sum_{s=1}^{r}\sum_{i=1}^{N} \mu_i \|r^{s,i}\|_2^2}, \tag{13}$$

where the initial Markov chain state distribution is given by $\mathcal{P}(\theta(0) = i) = \mu_i \geq 0$ for all $i \in \mathbb{N}$.
Considering the strict inequalities,

$$\tilde{Q}_i > \sum_{l\in\mathbb{M}_i} \alpha_{il}(\tilde{A}_{il}'\mathcal{E}_i(\tilde{Q})\tilde{A}_{il} + \tilde{C}_{cil}'\tilde{C}_{cil}), \quad i\in\mathbb{N},\ l\in\mathbb{M}_i, \tag{14}$$

and

$$\tilde{\mathfrak{Q}}_i > \sum_{l\in\mathbb{M}_i} \alpha_{il}(\tilde{A}_{il}'\mathcal{E}_i(\tilde{\mathfrak{Q}})\tilde{A}_{il} + \tilde{C}_{fil}'\tilde{C}_{fil}), \quad i\in\mathbb{N},\ l\in\mathbb{M}_i, \tag{15}$$

for $\tilde{Q}_i > 0$ and $\mathfrak{Q}_i > 0$, we have that

$$\left(\|\mathcal{G}_c\|_2^{(\tilde{w}\mapsto z)}\right)^2 < \sum_{i=1}^{N}\sum_{l\in\mathbb{M}_i} \alpha_{il}\mu_i Tr(\tilde{J}_{il}'\mathcal{E}_i(\tilde{Q})\tilde{J}_{il}) \tag{16}$$

and

$$\left(\|\mathcal{G}_c\|_2^{(\tilde{w}\mapsto r)}\right)^2 < \sum_{i=1}^{N}\sum_{l\in\mathbb{M}_i} \alpha_{il}\mu_i Tr(\tilde{J}_{il}'\mathcal{E}_i(\tilde{\mathfrak{Q}})\tilde{J}_{il} + \tilde{E}_{fil}'\tilde{E}_{fil}). \tag{17}$$

Following the discussion presented in [3] and [5], we get that if the following inequalities for the filter part

$$\sum_{i=1}^{N}\sum_{l\in\mathbb{M}_i}\mu_i\alpha_{il}Tr(W_{il}) < \lambda_r^2, \tag{18}$$

$$\begin{bmatrix} W_{il} & \bullet & \bullet \\ \tilde{J}_{il} & \mathcal{E}_i(\tilde{Q})^{-1} & \bullet \\ \tilde{E}_{fil} & 0 & I \end{bmatrix} > 0, \tag{19}$$

$$\tilde{Q}_{il} > \sum_{l\in\mathbb{M}_i}\alpha_{il}\tilde{R}_{il}, \tag{20}$$

$$\begin{bmatrix} \tilde{R}_{il} & \bullet & \bullet \\ \tilde{A}_{il} & \mathcal{E}_i(\tilde{Q})^{-1} & \bullet \\ \tilde{C}_{fil} & 0 & I \end{bmatrix} > 0. \tag{21}$$

and for the controller side

$$\sum_{i=1}^{N}\sum_{l\in\mathbb{M}_i}\mu_i\alpha_{il}Tr(\mathfrak{W}_{il}) < \lambda_c^2, \tag{22}$$

$$\begin{bmatrix} \mathfrak{W}_{il} & \bullet \\ \tilde{J}_{il} & \mathcal{E}_i(\tilde{\mathfrak{Q}})^{-1} \end{bmatrix} > 0, \tag{23}$$

$$\tilde{\mathfrak{Q}}_{il} > \sum_{l\in\mathbb{M}_i}\alpha_{il}\tilde{\mathfrak{R}}_{il}, \tag{24}$$

$$\begin{bmatrix} \tilde{\mathfrak{R}}_{il} & \bullet & \bullet \\ \tilde{A}_{il} & \mathcal{E}_i(\tilde{\mathfrak{Q}})^{-1} & \bullet \\ \tilde{C}_{cil} & 0 & I \end{bmatrix} > 0. \tag{25}$$

hold, then $\mathcal{C} \in \mathfrak{C}$, $\|\mathcal{G}_c\|_2^{(\tilde{w}\mapsto z)} < \lambda_c$ and $\|\mathcal{G}_c\|_2^{(\tilde{w}\mapsto r)} < \lambda_r$. Similarly to the $\mathcal{H}_\infty$ case, the main goal is to design $\mathcal{C} \in \mathscr{C}$ so that $\|\mathcal{G}_c\|_2^{(\tilde{w}\mapsto z)} < \lambda_c$ and $\|\mathcal{G}_c\|_2^{(\tilde{w}\mapsto r)} < \lambda_r$ for $\tilde{w} \in \mathcal{W}_i$, $i \in \mathbb{N}$. Specifically in this work we focus our efforts in finding

$$\psi = \{W_{il}, Q_i, R_{il}, \mathfrak{W}_{il}, \mathfrak{Q}_i, \mathfrak{R}_{il},\ i \in \mathbb{N},\ l \in \mathcal{M}_i\} \tag{26}$$

$$\Delta = \{\psi \text{ such that (18)-(25) hold }\}$$

$$\inf_{\mathcal{C}\in\mathscr{C},P,\lambda_r,\lambda_c}\{\lambda_c\zeta_c + \lambda_r\zeta_r\}\ :\ \text{s. t. } \psi \in \Delta, \tag{27}$$

for a given $\zeta_c, \zeta_r > 0$. Similarly to the H_∞ case, we choose this particular formulation in order to derive some results later on.

After the controller in (2) is obtained, the next step is the on-line residual evaluation of the system for detecting faults. As in [28], we define the evaluation function as follows,

$$J(r) \triangleq \sqrt{\sum_{k=k_0}^{k_0+L} r(k)'r(k)}, \tag{28}$$

where k_0 is the initial evaluation time and L is the evaluation duration. The threshold $\bar{J}$ is given by

$$\bar{J} \triangleq \sup_{w\in l_2^r, f=0} \mathbf{E}(J(r)). \tag{29}$$

The idea of (29) is to obtain the value of the residual under nominal operation, that is, without the fault, in a similar way as presented in [22]. The value of (29) can be approximated, for instance, through Monte Carlo simulations and using some knowledge of the nominal process transfer behavior. A deeper discussion about this type of threshold can be found in [2], [9], [12]. The decision process is then characterized by

$$\begin{aligned} J(r) &> \bar{J} \text{ A fault occured,} \\ J(r) &\le \bar{J} \text{ No fault.} \end{aligned} \tag{30}$$

IV. MAIN RESULTS

In this section, we present the main theoretical results proposed in the present work. The first result is the design of a H_∞ SFDC for discrete-time MJLS with partial information, the second result is the design of a H_2 SFDC for discrete-time MJLS with partial information. As secondary results we also present the Mixed H_2 / H_∞ SFDC for MJLS with partial information, as well as the coordinate descent algorithm as a viable way to solve the BMI constraints.

A. H_∞ SFDC

The next result presents BMI constraints regarding the controller design (31), (32), as shown at the bottom of the next page, and for the filter design (33) and (34), as shown at the bottom of the next page.

Theorem 1: There exists an SFDC described as in (2) such that $\mathcal{C} \in \mathscr{C}$, $\|\mathcal{G}_c\|_\infty^{(\tilde{w}\mapsto z)} < \gamma_c$, and $\|\mathcal{G}_c\|_\infty^{(\tilde{w}\mapsto r)} < \gamma_r$ for fixed $\gamma_c > 0$ and $\gamma_r > 0$ if there exist symmetric matrices Z_i, X_i, M_{il}^{11}, M_{il}^{22}, S_{il}^{11}, S_{il}^{22}, $\mathfrak{Z}_i$, $\mathfrak{X}_i$, $\mathfrak{M}_{il}^{11}$, $\mathfrak{M}_{il}^{22}$, $\mathfrak{S}_{il}^{11}$, $\mathfrak{S}_{il}^{22}$, and the matrices M_{il}^{21}, S_{il}^{21}, $\mathfrak{M}_{il}^{21}$, $\mathfrak{S}_{il}^{21}$, N_{il}^{11}, N_{il}^{12}, N_{il}^{21}, N_{il}^{22}, $\mathfrak{N}_{il}^{11}$, $\mathfrak{N}_{il}^{12}$, $\mathfrak{N}_{il}^{21}$, $\mathfrak{N}_{il}^{22}$, G_l, Γ_l, χ_l, Θ_l, Φ_l, and K_l with compatible dimensions such that inequalities (31), (32), (33), and (34) hold $\forall i \in \mathbb{N}, l \in \mathbb{M}$. If a feasible solution is obtained, a suitable SFDC is given by

$$\begin{aligned} A_{cl} &= -G_l^{-1}\Gamma_l, \\ B_{cl} &= -G_l^{-1}\chi_l, \\ C_{cl} &= K_l, \\ C_{fl} &= -\Theta_l, \\ D_{fl} &= -\Phi_l. \end{aligned}$$

Proof: The proof follows the similar reasoning presented in [4] and [10]. We set the structure of matrices P_i and P_i^{-1} of (7)-(8) as

$$P_i = \begin{bmatrix} X_i & \bullet \\ U_i & \hat{X}_i \end{bmatrix}, \quad P_i^{-1} = \begin{bmatrix} Z_i^{-1} & \bullet \\ V_i & \hat{Y}_i \end{bmatrix} \tag{35}$$

and similarly for matrices $\mathfrak{P}_i$ and $\mathfrak{P}_i^{-1}$ of (9)-(10), we set

$$\mathfrak{P}_i = \begin{bmatrix} \mathfrak{X}_i & \bullet \\ \mathfrak{U}_i & \hat{\mathfrak{X}}_i \end{bmatrix}, \quad \mathfrak{P}_i^{-1} = \begin{bmatrix} \mathfrak{Z}_i^{-1} & \bullet \\ \mathfrak{V}_i & \hat{\mathfrak{Y}}_i \end{bmatrix} \tag{36}$$

We also define the matrices τ_i and υ_i as

$$\tau_i = \begin{bmatrix} I & I \\ V_iZ_i & 0 \end{bmatrix}, \quad \upsilon_i = \begin{bmatrix} I & \mathcal{E}_i(X) \\ 0 & \mathcal{E}_i(U) \end{bmatrix} \tag{37}$$

along with

$$\mathfrak{t}_i = \begin{bmatrix} I & I \\ \mathfrak{V}_i\mathfrak{Z}_i & 0 \end{bmatrix}, \quad \mathfrak{u}_i = \begin{bmatrix} I & \mathcal{E}_i(\mathfrak{X}) \\ 0 & \mathcal{E}_i(\mathfrak{U}) \end{bmatrix}. \tag{38}$$

By verifying the diagonal blocks of (31) and also (32), we note that $Her(G_l) > \mathcal{E}_i(X - Z) > 0$ so that G_l is non-singular. Considering the fact that $P_iP_i^{-1} = I$ and $\mathfrak{P}_i\mathfrak{P}_i^{-1} = I$, we rewrite the matrices P_i and P_i^{-1} by setting $U_i = -\hat{X}_i$, and matrices $\mathfrak{P}_i$ and $\mathfrak{P}_i^{-1}$ by setting $\mathfrak{U}_i = -\hat{\mathfrak{X}}_i$, as follows

$$P_i = \begin{bmatrix} X_i & \bullet \\ Z_i - X_i & X_i - Z_i \end{bmatrix}, \tag{39}$$

$$P_i^{-1} = \begin{bmatrix} Z_i^{-1} & \bullet \\ Z_i^{-1} & Z_i^{-1} + (X_i - Z_i)^{-1} \end{bmatrix}, \tag{40}$$

and

$$\mathfrak{P}_i = \begin{bmatrix} \mathfrak{X}_i & \bullet \\ \mathfrak{Z}_i - \mathfrak{X}_i & \mathfrak{X}_i - \mathfrak{Z}_i \end{bmatrix}, \tag{41}$$

$$\mathfrak{P}_i^{-1} = \begin{bmatrix} \mathfrak{Z}_i^{-1} & \bullet \\ \mathfrak{Z}_i^{-1} & \mathfrak{Z}_i^{-1} + (\mathfrak{X}_i - \mathfrak{Z}_i)^{-1} \end{bmatrix}, \tag{42}$$

Besides, Equations (37) and (38) become

$$\tau_i = \begin{bmatrix} I & I \\ I & 0 \end{bmatrix}, \quad \upsilon_i = \begin{bmatrix} I & \mathcal{E}_i(X) \\ 0 & \mathcal{E}_i(Z - X) \end{bmatrix}. \tag{43}$$

and

$$\mathfrak{t}_i = \begin{bmatrix} I & I \\ I & 0 \end{bmatrix}, \quad \mathfrak{u}_i = \begin{bmatrix} I & \mathcal{E}_i(\mathfrak{X}) \\ 0 & \mathcal{E}_i(\mathfrak{Z} - \mathfrak{X}) \end{bmatrix}. \tag{44}$$

Since G_l is non-singular, we set $\Gamma_l = -G_lA_{cl}$, $\chi_l = -G_lB_{cl}$, $K_l = C_{cl}$, $\Theta_l = -C_{fl}$, and $\Phi_l = -D_{fl}$. As presented in [6], [10], we get that $G_l\mathcal{E}_i(X-Z)^{-1}G_l^T \geq Her(G_l)+\mathcal{E}_i(Z-X)$ and $G_l\mathcal{E}_i(\mathfrak{X} - \mathfrak{Z})^{-1}G_l^T \geq Her(G_l) + \mathcal{E}_i(\mathfrak{Z} - \mathfrak{X})$ so that (32) and (34) still hold if the diagonal blocks in which $Her(G_l) + \mathcal{E}_i(Z - X)$ and $Her(G_l) + \mathcal{E}_i(\mathfrak{Z} - \mathfrak{X})$ appear are substituted by $G_l\mathcal{E}_i(X - Z)^{-1}G_l^T$ and $G_l\mathcal{E}_i(\mathfrak{X} - \mathfrak{Z})^{-1}G_l^T$, respectively, resulting in

$$\begin{bmatrix} M_{il}^{11} & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ M_{il}^{21} & M_{il}^{22} & \bullet & \bullet & \bullet & \bullet & \bullet \\ N_{il}^{11} & N_{il}^{12} & S_{il}^{11} & \bullet & \bullet & \bullet & \bullet \\ N_{il}^{21} & N_{il}^{22} & S_{il}^{21} & S_{il}^{22} & \bullet & \bullet & \bullet \\ \Xi^{51} & \mathcal{E}_i(Z)A_i & \mathcal{E}_i(Z)J_{wi} & \mathcal{E}_i(Z)J_{fi} & \mathcal{E}_i(Z) & \bullet & \bullet \\ \Xi^{61} & \Xi^{62} & \Xi^{63} & \Xi^{64} & 0 & \Xi^{66} & \bullet \\ C_i + D_iC_{cl} & C_i & 0 & 0 & 0 & 0 & I \end{bmatrix} > 0, \tag{45}$$

and

$$\begin{bmatrix} \mathfrak{M}_{il}^{11} & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \mathfrak{M}_{il}^{21} & \mathfrak{M}_{il}^{22} & \bullet & \bullet & \bullet & \bullet & \bullet \\ \mathfrak{N}_{il}^{11} & \mathfrak{N}_{il}^{12} & \mathfrak{S}_{il}^{11} & \bullet & \bullet & \bullet & \bullet \\ \mathfrak{N}_{il}^{21} & \mathfrak{N}_{il}^{22} & \mathfrak{S}_{il}^{21} & \mathfrak{S}_{il}^{22} & \bullet & \bullet & \bullet \\ \tilde{\Xi}^{51} & \mathcal{E}_i(\mathfrak{Z})A_i & \mathcal{E}_i(\mathfrak{Z})J_{wi} & \mathcal{E}_i(\mathfrak{Z})J_{fi} & \mathcal{E}_i(\mathfrak{Z}) & \bullet & \bullet \\ \Xi^{61} & \Xi^{62} & \Xi^{63} & \Xi^{64} & 0 & \tilde{\Xi}^{66} & \bullet \\ -C_{fl} - D_{fl}L_i & -D_{fl}L_i & -D_{fl}H_{wi} & I - D_{fl}H_{fi} & 0 & 0 & I \end{bmatrix} > 0, \tag{46}$$

$$\begin{bmatrix} Z_i & \bullet & \bullet & \bullet \\ Z_i & X_i & \bullet & \bullet \\ 0 & 0 & \gamma_{c\,\infty}^2 I & \bullet \\ 0 & 0 & 0 & \gamma_{c\,\infty}^2 I \end{bmatrix} > \sum_{l \in \mathbb{M}_i} \alpha_{il} \begin{bmatrix} M_{il}^{11} & \bullet & \bullet & \bullet \\ M_{il}^{21} & M_{il}^{22} & \bullet & \bullet \\ N_{il}^{11} & N_{il}^{12} & S_{il}^{11} & \bullet \\ N_{il}^{21} & N_{il}^{22} & S_{il}^{21} & S_{il}^{22} \end{bmatrix}, \tag{31}$$

$$\begin{bmatrix} M_{il}^{11} & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ M_{il}^{21} & M_{il}^{22} & \bullet & \bullet & \bullet & \bullet & \bullet \\ N_{il}^{11} & N_{il}^{12} & S_{il}^{11} & \bullet & \bullet & \bullet & \bullet \\ N_{il}^{21} & N_{il}^{22} & S_{il}^{21} & S_{il}^{22} & \bullet & \bullet & \bullet \\ \mathcal{E}_i(Z)(A_i + B_iK_l) & \mathcal{E}_i(Z)A_i & \mathcal{E}_i(Z)J_{wi} & \mathcal{E}_i(Z)J_{fi} & \mathcal{E}_i(Z) & \bullet & \bullet \\ G_l(A_i + B_iK_l) + \Gamma_l + \chi_lL_i & G_lA_i + \chi_lL_i & G_lJ_{wi} + \chi_lH_{wi} & G_lJ_{fi} + \chi_lH_{fi} & 0 & Her(G_l) + \mathcal{E}_i(Z - X) & \bullet \\ C_i + D_iK_l & C_i & 0 & 0 & 0 & 0 & I \end{bmatrix} > 0, \tag{32}$$

$$\begin{bmatrix} \mathfrak{Z}_i & \bullet & \bullet & \bullet \\ \mathfrak{Z}_i & \mathfrak{X}_i & \bullet & \bullet \\ 0 & 0 & \gamma_{r\,\infty}^2 I & \bullet \\ 0 & 0 & 0 & \gamma_{r\,\infty}^2 I \end{bmatrix} > \sum_{l \in \mathbb{M}_i} \alpha_{il} \begin{bmatrix} \mathfrak{M}_{il}^{11} & \bullet & \bullet & \bullet \\ \mathfrak{M}_{il}^{21} & \mathfrak{M}_{il}^{22} & \bullet & \bullet \\ \mathfrak{N}_{il}^{11} & \mathfrak{N}_{il}^{12} & \mathfrak{S}_{il}^{11} & \bullet \\ \mathfrak{N}_{il}^{21} & \mathfrak{N}_{il}^{22} & \mathfrak{S}_{il}^{21} & \mathfrak{S}_{il}^{22} \end{bmatrix}, \tag{33}$$

$$\begin{bmatrix} \mathfrak{M}_{il}^{11} & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \mathfrak{M}_{il}^{21} & \mathfrak{M}_{il}^{22} & \bullet & \bullet & \bullet & \bullet & \bullet \\ \mathfrak{N}_{il}^{11} & \mathfrak{N}_{il}^{12} & \mathfrak{S}_{il}^{11} & \bullet & \bullet & \bullet & \bullet \\ \mathfrak{N}_{il}^{21} & \mathfrak{N}_{il}^{22} & \mathfrak{S}_{il}^{21} & \mathfrak{S}_{il}^{22} & \bullet & \bullet & \bullet \\ \mathcal{E}_i(\mathfrak{Z})(A_i + B_iK_l) & \mathcal{E}_i(\mathfrak{Z})A_i & \mathcal{E}_i(\mathfrak{Z})J_{wi} & \mathcal{E}_i(\mathfrak{Z})J_{fi} & \mathcal{E}_i(\mathfrak{Z}) & \bullet & \bullet \\ G_l(A_i + B_iK_l) + \Gamma_l + \chi_lL_i & G_lA_i + \chi_lL_i & G_lJ_{wi} + \chi_lH_{wi} & G_lJ_{fi} + \chi_lH_{fi} & 0 & Her(G_l) + \mathcal{E}_i(\mathfrak{Z} - \mathfrak{X}) & \bullet \\ \Theta_l + \Phi_lL_i & \Phi_lL_i & \Phi_lH_{wi} & I + \Phi_lH_{fi} & 0 & 0 & I \end{bmatrix} > 0. \tag{34}$$

where

$$\begin{aligned}
\Xi^{51} &= \mathcal{E}_i(Z)(A_i + B_i C_{cl}),\\
\Xi^{61} &= G_l(A_i + B_i C_{cl}) - G_l A_{cl} - G_l B_{cl} L_i,\\
\Xi^{62} &= G_l A_i - G_l B_{cl} L_i,\\
\Xi^{63} &= G_l J_{wi} - G_l B_{cl} H_{wi},\\
\Xi^{64} &= G_l J_{fi} - G_l B_{cl} H_{fi},\\
\Xi^{66} &= G_l \mathcal{E}_i(X - Z)^{-1} G_l',
\end{aligned}$$

and

$$\begin{aligned}
\tilde{\Xi}^{51} &= \mathcal{E}_i(\mathfrak{Z})(A_i + B_i C_{cl}),\\
\tilde{\Xi}^{66} &= G_l \mathcal{E}_i(\mathfrak{X} - \mathfrak{Z})^{-1} G_l'.
\end{aligned}$$

By defining the following matrices

$$\Pi_{il} = \begin{bmatrix} \mathcal{E}_i(Z)^{-1} & I \\ 0 & G_l^{-T}\mathcal{E}_i(X - Z) \end{bmatrix}, \qquad (47)$$

and

$$\tilde{\pi}_{il} = \begin{bmatrix} \mathcal{E}_i(\mathfrak{Z})^{-1} & I \\ 0 & G_l^{-T}\mathcal{E}_i(\mathfrak{X} - \mathfrak{Z}) \end{bmatrix}, \qquad (48)$$

and applying the congruence transformations $diag(I, I, \Pi_{il}, I)$ and $diag(I, I, \tilde{\pi}_{il}, I)$ to (45) and (46), respectively, we get that

$$\begin{bmatrix} \tau_i' M_{il} \tau_i & \bullet & \bullet & \bullet \\ N_{il}\tau_i & S_{il} & \bullet & \bullet \\ \upsilon_i' \tilde{A}_{il} \tau_i & \upsilon_i' \tilde{J}_{il} & \upsilon_i' \mathcal{E}_i(P)^{-1} \upsilon_i & \bullet \\ \tilde{C}_{cil}\tau_i & 0 & 0 & I \end{bmatrix} > 0, \qquad (49)$$

and

$$\begin{bmatrix} \mathfrak{t}_i' \mathfrak{M}_{il} \mathfrak{t}_i & \bullet & \bullet & \bullet \\ \mathfrak{N}_{il}\mathfrak{t}_i & \mathfrak{S}_{il} & \bullet & \bullet \\ \mathfrak{u}_i' \tilde{A}_{il} \mathfrak{t}_i & \mathfrak{u}_i' \tilde{J}_{il} & \mathfrak{u}_i' \mathcal{E}_i(\mathfrak{P})^{-1} \mathfrak{u}_i & \bullet \\ \tilde{C}_{fil}\mathfrak{t}_i & \tilde{E}_{fil} & 0 & I \end{bmatrix} > 0, \qquad (50)$$

hold, for τ_i, υ_i, $\mathfrak{t}_i$, and $\mathfrak{u}_i$ given as in (43) and (44). By applying the congruence transformations $diag(\tau_i^{-1}, I, \upsilon^{-1}, I)$ and $diag(\mathfrak{t}_i^{-1}, I, \mathfrak{u}_i^{-1}, I)$ to (49) and (50), respectively, and the Schur complement to the resulting inequalities, we get that (8) and (10) hold. Finally, by noting that (31) and (33) can be equivalently rewritten as follows

$$\begin{bmatrix} \tau_i' P_i \tau & \bullet \\ 0 & \gamma_c^2 I \end{bmatrix} > \sum_{l \in \mathbb{M}_i} \alpha_{il} \begin{bmatrix} \tau_i' M_{il}\tau_i & \bullet \\ N_{il}\tau_i & S_{il} \end{bmatrix}, \qquad (51)$$

and

$$\begin{bmatrix} \mathfrak{t}_i' \mathfrak{P}_i \mathfrak{t}_i & \bullet \\ 0 & \gamma_r^2 I \end{bmatrix} > \sum_{l \in \mathbb{M}_i} \alpha_{il} \begin{bmatrix} \mathfrak{t}_i' \mathfrak{M}_{il}\mathfrak{t}_i & \bullet \\ \mathfrak{N}_{il}\mathfrak{t}_i & \mathfrak{S}_{il} \end{bmatrix}, \qquad (52)$$

we get, after applying the congruence transformations $diag(\tau_i^{-1}, I)$ and $diag(\mathfrak{t}_i^{-1}, I)$ to (51) and (52), respectively, that (7) and (9) hold. Thus, since (7)-(8) and (9)-(10) hold for the closed-loop system as in (6), we get from Lemma 1 that $\mathcal{C} \in \mathscr{C}$, $\|\mathcal{G}_c\|_{\tilde{w}\mapsto z} < \gamma_c$, and $\|\mathcal{G}_c\|_{\tilde{w}\mapsto r} < \gamma_r$, and the claim follows. ■

B. H_2 SFDC

The next result presents BMI constraints related to the control and filter design of the SFDC system (2).

Theorem 2: There exists an SFDC described as in (2) such that $\mathcal{C} \in \mathscr{C}$, $\|\mathcal{G}_c\|_2^{(\tilde{w}\mapsto z)} < \lambda_c$, and $\|\mathcal{G}_c\|_2^{(\tilde{w}\mapsto r)} < \lambda_r$ for fixed $\lambda_c > 0$ and $\lambda_r > 0$ if there exist symmetric matrices W_{il}^{11}, W_{il}^{22}, T_i, O_i, V_{il}^{11}, V_{il}^{22}, $\mathfrak{W}_{il}^{11}$,, $\mathfrak{V}_{il}^{22}$ $\mathfrak{T}_i$, $\mathfrak{O}_i$, $\mathfrak{V}_{il}^{11}$, $\mathfrak{V}_{il}^{22}$ and the matrices W_{il}^{21}, V_{il}^{21}, $\mathfrak{W}_{il}^{21}$, $\mathfrak{V}_{il}^{21}$ G_l, Γ_l, χ_l, Θ_l, Φ_l, and K_l with compatible dimensions such that inequalities (53), (54), (55), (56), (57), (58), (59), and (60), as shown at the bottom of the next page, hold $\forall i \in \mathbb{N}, l \in \mathbb{M}$. If a feasible solution is obtained, a suitable SFDC is given by

$$\begin{aligned}
A_{cl} &= -G_l^{-1}\Gamma_l,\\
B_{cl} &= -G_l^{-1}\chi_l,\\
C_{cl} &= K_l,\\
C_{fl} &= -\Theta_l,\\
D_{fl} &= -\Phi_l.
\end{aligned}$$

Proof: The proof follows the similar reasoning as the one employed in the proof of Theorem 1. Similarly as presented in [4], [10], the structure of matrices $\tilde{Q}_i$ and $\tilde{Q}_i^{-1}$ of (18)-(21), and $\tilde{\mathfrak{Q}}_i$ and $\tilde{\mathfrak{Q}}_i^{-1}$ of (22)-(25), are

$$\tilde{Q}_i = \begin{bmatrix} O_i & \bullet \\ \bar{U}_i & \hat{O}_i \end{bmatrix}, \quad \tilde{Q}_i^{-1} = \begin{bmatrix} T_i^{-1} & \bullet \\ \bar{V}_i & \hat{T}_i \end{bmatrix}. \qquad (61)$$

and

$$\tilde{\mathfrak{Q}}_i = \begin{bmatrix} \mathfrak{O}_i & \bullet \\ \bar{\mathfrak{U}}_i & \hat{\mathfrak{O}}_i \end{bmatrix}, \quad \tilde{\mathfrak{Q}}_i^{-1} = \begin{bmatrix} \mathfrak{T}_i^{-1} & \bullet \\ \bar{\mathfrak{V}}_i & \hat{\mathfrak{T}}_i \end{bmatrix}. \qquad (62)$$

We also define the matrices η_i and σ_i

$$\eta_i = \begin{bmatrix} I & I \\ \bar{V}_i T_i & 0 \end{bmatrix}, \quad \sigma_i = \begin{bmatrix} I & \mathcal{E}_i(T) \\ 0 & \mathcal{E}_i(\bar{U}) \end{bmatrix}. \qquad (63)$$

along with $\mathfrak{n}_i$ and $\mathfrak{s}_i$,

$$\mathfrak{n}_i = \begin{bmatrix} I & I \\ \bar{\mathfrak{V}}_i \mathfrak{T}_i & 0 \end{bmatrix}, \quad \mathfrak{s}_i = \begin{bmatrix} I & \mathcal{E}_i(\mathfrak{T}) \\ 0 & \mathcal{E}_i(\bar{\mathfrak{U}}) \end{bmatrix}. \qquad (64)$$

We get from (55)-(56) as well as (59)-(60) that G_l is nonsingular. By setting $\bar{U}_i = -\hat{O}_i$ and $\bar{\mathfrak{U}}_i = -\hat{\mathfrak{O}}_i$ in (61) and (62) and using the fact that $\tilde{Q}_i\tilde{Q}_i^{-1} = I$ and $\tilde{\mathfrak{Q}}_i\tilde{\mathfrak{Q}}_i^{-1} = I$, we get that (61)-(64) can be rewritten as

$$\tilde{Q}_i = \begin{bmatrix} O_i & \bullet \\ T_i - O_i & O_i - T_i \end{bmatrix}, \quad \tilde{Q}_i^{-1} = \begin{bmatrix} T_i^{-1} & \bullet \\ T_i^{-1} & \Upsilon_{1i} \end{bmatrix}, \qquad (65)$$

where $\Upsilon_{1i} = T_i^{-1} - (O_i - T_i)^{-1}$, and

$$\tilde{\mathfrak{Q}}_i = \begin{bmatrix} \mathfrak{O}_i & \bullet \\ \mathfrak{T}_i - \mathfrak{O}_i & \mathfrak{O}_i - \mathfrak{T}_i \end{bmatrix}, \quad \tilde{\mathfrak{Q}}_i^{-1} = \begin{bmatrix} \mathfrak{T}_i^{-1} & \bullet \\ \mathfrak{T}_i^{-1} & \Upsilon_{2i} \end{bmatrix} \qquad (66)$$

where $\Upsilon_{2i} = \mathfrak{T}_i^{-1} - (\mathfrak{O}_i - \mathfrak{T}_i)^{-1}$, along with

$$\eta_i = \begin{bmatrix} I & I \\ I & 0 \end{bmatrix}, \quad \sigma_i = \begin{bmatrix} I & \mathcal{E}_i(T) \\ 0 & \mathcal{E}_i(T - O) \end{bmatrix} \qquad (67)$$

and

$$\mathfrak{n}_i = \begin{bmatrix} I & I \\ I & 0 \end{bmatrix}, \quad \mathfrak{s}_i = \begin{bmatrix} I & \mathcal{E}_i(\mathfrak{T}) \\ 0 & \mathcal{E}_i(\mathfrak{T}-\mathfrak{O}) \end{bmatrix}. \tag{68}$$

Recalling the previous reasoning applied in the proof of Theorem 1, we get that $G_l\mathcal{E}_i(O-T)^{-1}G_l' \geq Her(G_l)+\mathcal{E}_i(T-O)$ and $G_l\mathcal{E}_i(\mathfrak{O}-\mathfrak{T})^{-1}G_l' \geq Her(G_l)+\mathcal{E}_i(\mathfrak{T}-\mathfrak{O})$. By performing the change of variables $\Gamma_l = -G_lA_{cl}$, $\chi_l = -G_lB_{cl}$, $K_l = C_{cl}$, $\Theta_l = -C_{fl}$, and $\Phi_l = -D_{fl}$, we can rewrite (55)-(56) and (59)-(60) as follows

$$\begin{bmatrix} W_{il}^{11} & \bullet & \bullet & \bullet \\ W_{il}^{21} & W_{il}^{22} & \bullet & \bullet \\ \mathcal{E}_i(T)J_{wi} & \mathcal{E}_i(T)J_{fi} & \mathcal{E}_i(T) & \bullet \\ G_l[J_{wi}-B_{cl}H_{wi}] & G_l[J_{fi}-B_{cl}H_{fi}] & 0 & G_l\mathcal{E}_i(O-T)^{-1}G_l' \end{bmatrix} > 0, \tag{69}$$

and

$$\begin{bmatrix} V_{il}^{11} & \bullet & \bullet & \bullet & \bullet \\ V_{il}^{21} & V_{il}^{22} & \bullet & \bullet & \bullet \\ \mathcal{E}_i(T)A_i(C_{cl}) & \mathcal{E}_i(T)A_i & \mathcal{E}_i(T) & \bullet & \bullet \\ G_l\Upsilon_{3il} & G_l[A_i-B_{cl}L_i] & 0 & G_l\mathcal{E}_i(O-T)^{-1}G_l' & \bullet \\ C_i+D_iC_{cl} & C_i & 0 & 0 & I \end{bmatrix} > 0, \tag{70}$$

where $A_i(C_c) = A_i+B_iC_{cl}$ and $\Upsilon_{3il} = [A_i(C_{cl})-A_{cl}-B_{cl}L_i]$. along with

$$\begin{bmatrix} \mathfrak{W}_{il}^{11} & \bullet & \bullet & \bullet & \bullet \\ \mathfrak{W}_{il}^{21} & \mathfrak{W}_{il}^{22} & \bullet & \bullet & \bullet \\ \mathcal{E}_i(\mathfrak{T})J_{wi} & \mathcal{E}_i(\mathfrak{T})J_{fi} & \mathcal{E}_i(\mathfrak{T}) & \bullet & \bullet \\ G_l[J_{wi}-B_{cl}H_{wi}] & G_l[J_{fi}-B_{cl}H_{fi}] & 0 & G_l\mathcal{E}_i(\mathfrak{O}-\mathfrak{T})^{-1}G_l' & \bullet \\ -D_{fl}H_{wi} & I-D_{fl}H_{fi} & 0 & 0 & I \end{bmatrix} > 0, \tag{71}$$

and

$$\begin{bmatrix} \mathfrak{V}_{il}^{11} & \bullet & \bullet & \bullet & \bullet \\ \mathfrak{V}_{il}^{21} & \mathfrak{V}_{il}^{22} & \bullet & \bullet & \bullet \\ \mathcal{E}_i(\mathfrak{T})A_i(C_{cl}) & \mathcal{E}_i(\mathfrak{T})A_i & \mathcal{E}_i(\mathfrak{T}) & \bullet & \bullet \\ G_l\Upsilon_{3il} & G_l[A_i-B_{cl}L_i] & 0 & G_l\mathcal{E}_i(\mathfrak{O}-\mathfrak{T})^{-1}G_l' & \bullet \\ -C_{fl}-D_{fl}L_i & -D_{fl}L_i & 0 & 0 & I \end{bmatrix} > 0. \tag{72}$$

By defining the matrices

$$\bar{\Pi}_{il} = \begin{bmatrix} \mathcal{E}_i(T)^{-1} & I \\ 0 & G_l^{-T}\mathcal{E}_i(O-T) \end{bmatrix}$$

and

$$\bar{\pi}_{il} = \begin{bmatrix} \mathcal{E}_i(\mathfrak{T})^{-1} & I \\ 0 & G_l^{-T}\mathcal{E}_i(\mathfrak{T}O-\mathfrak{T}) \end{bmatrix}$$

$$\sum_{i\in\mathbb{N}}\sum_{l\in\mathbb{M}_i}\mu_i\alpha_{il}Tr(W_{il}) < \lambda_c^2, \tag{53}$$

$$\begin{bmatrix} T_i & \bullet \\ T_i & O_i \end{bmatrix} > \sum_{l\in\mathbb{M}_i}\begin{bmatrix} V_{il}^{11} & \bullet \\ V_{il}^{21} & V_{il}^{22} \end{bmatrix}, \tag{54}$$

$$\begin{bmatrix} W_{il}^{11} & \bullet & \bullet & \bullet \\ W_{il}^{21} & W_{il}^{22} & \bullet & \bullet \\ \mathcal{E}_i(T)J_{wi} & \mathcal{E}_i(T)J_{fi} & \mathcal{E}_i(T) & \bullet \\ G_lJ_{wi}+\chi_lH_{wi} & G_lJ_{fi}+\chi_lH_{fi} & 0 & Her(G_l)+\mathcal{E}_i(T-O) \end{bmatrix} > 0, \tag{55}$$

$$\begin{bmatrix} V_{il}^{11} & \bullet & \bullet & \bullet & \bullet \\ V_{il}^{21} & V_{il}^{22} & \bullet & \bullet & \bullet \\ \mathcal{E}_i(T)(A_i+B_iK_l) & \mathcal{E}_i(T)A_i & \mathcal{E}_i(T) & \bullet & \bullet \\ G_l(A_i+B_iK_l)+\Gamma_l+\chi_lL_i & G_lA_i+\chi_lL_i & 0 & Her(G_l)+\mathcal{E}_i(T-O) & \bullet \\ C_i+D_iK_l & C_i & 0 & 0 & I \end{bmatrix} > 0, \tag{56}$$

$$\sum_{i\in\mathbb{N}}\sum_{l\in\mathbb{M}_i}\mu_i\alpha_{il}Tr(\mathfrak{W}_{il}) < \lambda_r^2, \tag{57}$$

$$\begin{bmatrix} \mathfrak{T}_i & \bullet \\ \mathfrak{T}_i & \mathfrak{O}_i \end{bmatrix} > \sum_{l\in\mathbb{M}_i}\begin{bmatrix} \mathfrak{V}_{il}^{11} & \bullet \\ \mathfrak{V}_{il}^{21} & \mathfrak{V}_{il}^{22} \end{bmatrix}, \tag{58}$$

$$\begin{bmatrix} \mathfrak{W}_{il}^{11} & \bullet & \bullet & \bullet & \bullet \\ \mathfrak{W}_{il}^{21} & \mathfrak{W}_{il}^{22} & \bullet & \bullet & \bullet \\ \mathcal{E}_i(\mathfrak{T})J_{wi} & \mathcal{E}_i(\mathfrak{T})J_{fi} & \mathcal{E}_i(\mathfrak{T}) & \bullet & \bullet \\ G_lJ_{wi}+\chi_lH_{wi} & G_lJ_{fi}+\chi_lH_{fi} & 0 & Her(G_l)+\mathcal{E}_i(\mathfrak{T}-\mathfrak{O}) & \bullet \\ \Phi_lH_{wi} & I+\Phi_lH_{fi} & 0 & 0 & I \end{bmatrix} > 0, \tag{59}$$

$$\begin{bmatrix} \mathfrak{V}_{il}^{11} & \bullet & \bullet & \bullet & \bullet \\ \mathfrak{V}_{il}^{21} & \mathfrak{V}_{il}^{22} & \bullet & \bullet & \bullet \\ \mathcal{E}_i(\mathfrak{T})(A_i+B_iK_l) & \mathcal{E}_i(\mathfrak{T})A_i & \mathcal{E}_i(\mathfrak{T}) & \bullet & \bullet \\ G_l(A_i+B_iK_l)+\Gamma_l+\chi_lL_i & G_lA_i+\chi_lL_i & 0 & Her(G_l)+\mathcal{E}_i(\mathfrak{T}-\mathfrak{O}) & \bullet \\ \Theta_l+\Phi_lL_i & \Phi_lL_i & 0 & 0 & I \end{bmatrix} > 0. \tag{60}$$

and applying the congruence transformations $diag(I_{r+f}, \bar{\Pi}_{il})$ and $diag(I_{2n}, \bar{\Pi}_{il}, I_q)$ to (69) and (70) as well as $diag(I_{r+f}, \bar{\pi}_{il}, I_f)$ and $diag(I_{2n}, \bar{\pi}_{il}, I_f)$ to (71)-(72), we get

$$\begin{bmatrix} W_{il} & \bullet \\ \sigma_i' \tilde{J}_{il} & \sigma_i' \mathcal{E}_i(\tilde{Q})^{-1}\sigma_i \end{bmatrix} > 0, \tag{73}$$

$$\begin{bmatrix} \eta_i' \tilde{R}_{il}\eta_i & \bullet & \bullet \\ \sigma_i' \tilde{A}_{il}\eta_i & \sigma_i' \mathcal{E}_i(\tilde{Q})^{-1}\sigma_i & \bullet \\ \tilde{C}_{cil}\eta_i & 0 & I \end{bmatrix} > 0, \tag{74}$$

and

$$\begin{bmatrix} \mathfrak{W}_{il} & \bullet & \bullet \\ \mathfrak{s}_i' \tilde{J}_{il} & \mathfrak{s}_i' \mathcal{E}_i(\tilde{\mathfrak{Q}})^{-1}\mathfrak{s}_i & \bullet \\ \tilde{E}_{fl} & 0 & I \end{bmatrix} > 0, \tag{75}$$

$$\begin{bmatrix} \mathfrak{n}_i' \tilde{\mathfrak{R}}_{il}\mathfrak{n}_i & \bullet & \bullet \\ \mathfrak{s}_i' \tilde{A}_{il}\mathfrak{n}_i & \mathfrak{s}_i' \mathcal{E}_i(\tilde{\mathfrak{Q}})^{-1}\mathfrak{s}_i & \bullet \\ \tilde{C}_{fil}\mathfrak{n}_i & 0 & I \end{bmatrix} > 0. \tag{76}$$

By applying the congruence transformations $diag(I, \sigma_i^{-1})$, $diag(\eta_i^{-1}, \sigma_i^{-1}, I)$, $diag(I, \mathfrak{s}_i^{-1}, I)$, $diag(\mathfrak{n}_i^{-1}, \mathfrak{s}_i^{-1}, I)$ to (73)-(76), we get that (19), (21), (23), and (25) hold with the closed-loop matrices of system (6). Finally, by noting that (54) and (58) can be rewritten as follows

$$\eta_i' \tilde{Q}_i \eta_i > \sum_{l \in \mathbb{M}_i} \alpha_{il} \eta_i' \tilde{R}_{il}\eta_i \tag{77}$$

and

$$\mathfrak{n}_i' \tilde{\mathfrak{Q}}_i \mathfrak{n}_i > \sum_{l \in \mathbb{M}_i} \alpha_{il} \mathfrak{n}_i' \tilde{\mathfrak{R}}_{il}\mathfrak{n}_i \tag{78}$$

and thus, by noting that (53) and (57) are equivalent to (18) and (22), and by applying the congruence transformations η_i^{-1} and $\mathfrak{n}_i^{-1}$ to (77)-(78), respectively, we get that (20)-(24) are also satisfied. Therefore, considering the discussion presented in Section III, see, for instance, [3] and [5], we get that $\mathcal{C} \in \mathfrak{C}$, $\|\mathcal{G}_c\|_2^{(\tilde{w}\mapsto z)} < \lambda_c$, and $\|\mathcal{G}_c\|_2^{(\tilde{w}\mapsto r)} < \lambda_r$, and the claim follows. ■

C. MIXED H_2 / H_∞

We present now the design of mixed H_2/H_∞ SFDC for MJLS with partial information on the jump parameter.

Observing the constraints in Theorems 1 and 2 it is possible to notice that the structure to obtain SFDC is the same, therefore a mixed solution can be formulated.

To increase the overall performance the H_2 norm will be considered in the controller side of the design due to its equivalence to the LQR controllers, which provide good performance in practical solutions. For the fault detection side, we consider the H_∞ norm, which provides an FDI with a lower occurrence of false alarms, [18], [28].

From the aforementioned discussion, we consider the mixed solution with the control side of the SFDC designed using the BMI conditions for Theorem 2 and the fault detection side obtained using the BMI from Theorem 1. Hence, the new rewritten optimization problem is

$$\begin{aligned} \phi &= \{\mathfrak{Z}_i, \mathfrak{X}_i, \mathfrak{M}_{il}, \mathfrak{N}_{il}, \mathfrak{S}_{il}, W_{il}, V_{il}, T_i, O_i \\ &\quad G_l, \Gamma_l, \chi_l, K_l, \Theta_l, \Phi_l\} \end{aligned} \tag{79}$$

$$\begin{aligned} \kappa &= \{\phi \text{ such that (33)-(34) and (53)-(56) hold} \\ &\quad \inf_{\mathcal{C}\in\mathscr{C}, P, \gamma_r, \lambda_c} \{\lambda_c \zeta_c + \gamma_r \beta_r\} \ : \ \text{s. t. } \phi \in \kappa. \end{aligned} \tag{80}$$

for a given $\zeta_c > 0$, $\beta_r > 0$.

Theorem 3: There exists an SFDC described as in (2) such that $\mathcal{C} \in \mathscr{C}$, $\|\mathcal{G}_c\|_\infty^{(\tilde{w}\mapsto r)} < \gamma_r$, and $\|\mathcal{G}_c\|_2^{(\tilde{w}\mapsto z)} < \lambda_c$ for fixed, $\gamma_r > 0$, and $\lambda_c > 0$ if there exist symmetric matrices $\mathfrak{Z}_i$, $\mathfrak{X}_i$, $\mathfrak{M}_{il}^{11}$, $\mathfrak{M}_{il}^{22}$, $\mathfrak{S}_{il}^{11}$, $\mathfrak{S}_{il}^{22}$, W_{il}^{11}, W_{il}^{22}, V_{il}^{11}, V_{il}^{22}, T_i, O_i and the matrices $\mathfrak{M}_{il}^{21}$, $\mathfrak{S}_{il}^{21}$, $\mathfrak{N}_{il}^{11}$, $\mathfrak{N}_{il}^{12}$, $\mathfrak{N}_{il}^{21}$, $\mathfrak{N}_{il}^{22}$, W_{il}^{21}, V_{il}^{21}, G_l, Γ_l, χ_l, Θ_l, Φ_l, and K_l with compatible dimensions such that inequalities, (33), (34), (53), (54), (55), and (56), hold $\forall i \in \mathbb{N}$, $l \in \mathbb{M}_i$. If a feasible solution is obtained, a suitable fault-compensation controller is given by

$$\begin{aligned} A_{cl} &= -G_l^{-1}\Gamma_l, \\ B_{cl} &= -G_l^{-1}\chi_l, \\ C_{cl} &= K_l, \\ C_{fl} &= -\Theta_l, \\ D_{fl} &= -\Phi_l. \end{aligned}$$

Proof: The proof for Theorem 3 is black a direct consequence of Theorems 1 and 2. ■

D. COORDINATE DESCENT ALGORITHM

As explained at the start of this section the constraints in Theorem 1 and 2 are in the form of Bilinear Matrices Inequalities, therefore it is necessary to implement an appropriate procedure to solve such a problem. It can be found in the literature several numerical ways of dealing with BMI as, for instance, a combination of line search and a sequence of LMI as presented in [24]. Although of great interest, an analyzes of the techniques to solve the BMI in Theorems 1 and 2 would fall outside the scope of this paper. Due to that we will focus on a procedure that is extensively used in the literature known as the Coordinate Descent Algorithm (CDA), as implemented in [20], or [23]. The specific approach implemented in the present paper was first introduced in [4].

By inspection, it is possible to observe that all the non-linearities are "caused" by the state-feedback controller K. A usual workaround for those non-linearities is to fix the state-feedback controller and solve the resulting LMI. Assume that there exists a state-feedback controller K, and apply this controller in the constraints (31), (32),(33), and (34) for the H_∞ case, or (53),(54),(55),(56),(57),(58),(59), and (60) for the H_2 case. If a feasible solution is found it may or may not be the optimized solution, due to the choice of the state-feedback controller. The CDA algorithm is described as in Algorithm 1.

V. NUMERICAL EXAMPLE

The coupled tank was chosen as an example. This particular coupled tank parameter and modeling were extracted

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Algorithm 1 Coordinate Descent Algorithm

Input: K_l,γ^{-1},t_{max},ϵ
Output: A_c,B_c,C_c,C_f,D_f

1 Design stabilizing state-feedback controller(e.g. [21]).
2 Fix K in the LMI constraints for the H_∞ case or for the H_2 case, and solve it to obtain the matrices Z_i, $\mathfrak{Z}_i$, and G_l for the H_∞ case, or T_i, $\mathfrak{T}_i$, and G_l for the H_2 case, or $\mathfrak{Z}_i$, T_i, and G_l for the mixed case.
3 Fix Z_i, $\mathfrak{Z}_i$, G_l for H_∞ case, or T_i, $\mathfrak{T}_i$, and G_l for the H_2 case, or Z_i, $\mathfrak{T}_i$, and G_l for the mixed case, and solve the same LMI constraint and now obtain A_{cl}, B_{cl}, C_{cl}, C_{fl}, D_{fl}, and the upper bound values γ_c, γ_r for the H_∞ case and λ_c, λ_r for the H_2 case.
4 If $\frac{\gamma_c^{t-1}-\gamma_c^t}{\gamma_c^{t-1}} \le \epsilon$ or $t \le t_{max}$, go back to step 2.

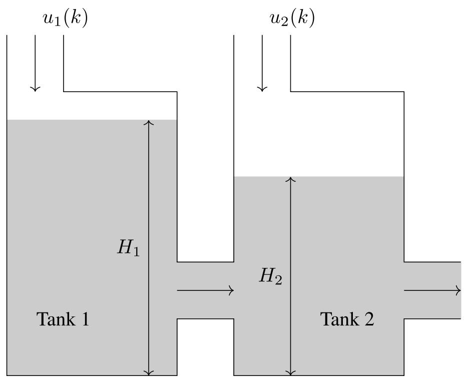

FIGURE 1. Plant scheme.

from [8]. The discrete-time domain space-state model is

$$A_{1,2} = \begin{bmatrix} -0.0239 & -0.0127 \\ 0.0127 & -0.0285 \end{bmatrix}, \quad B_{1,2} = \begin{bmatrix} 0.71 & 0 \\ 0 & 0.71 \end{bmatrix},$$
$$J_{w\ 1,2} = 0.01\, B_{1,2}, \quad J_{f\ 1,2} = I^{2\times 2},$$
$$L_1 = I^{2\times 2}, \quad L_2 = 0^{2\times 2}, \ H_{w\ 1,2} = H_{f\ 1,2} = 0.1\, I^{2\times 2},$$
$$C_1 = I^{2\times 2}, \quad C_2 = 0^{2\times 2}, \ D_1 = I^{2\times 2}, \ D_2 = 0^{2\times 2}.$$

This is the space-state representation for the coupled tank linearized in $h_1 = 0.2$ cm and $h_2 = 0.1$ cm, the sampling time is $T_s = 1$s. The transition matrix, initial distribution, and $\alpha_{k\ell}$ are

$$\mathbb{P} = \begin{bmatrix} 0.8 & 0.2 \\ 0.6 & 0.4 \end{bmatrix}, \quad \mu' = \begin{bmatrix} 0.7 \\ 0.3 \end{bmatrix}, \ \Psi = \begin{bmatrix} 0.7 & 0.3 \\ 0.6 & 0.4 \end{bmatrix}. \quad (81)$$

The SFDC obtained using Theorem 1 is

$$A_{c1} = \begin{bmatrix} 0.5053 & 0.1653 \\ -0.2767 & 0.4161 \end{bmatrix},$$
$$A_{c2} = \begin{bmatrix} 0.2048 & 0.0686 \\ -0.1065 & 0.1725 \end{bmatrix},$$
$$B_{c1} = \begin{bmatrix} -0.8252 & -0.2487 \\ 0.5756 & -0.8252 \end{bmatrix},$$
$$B_{c2} = \begin{bmatrix} -0.7180 & -0.2263 \\ 0.5173 & -0.7661 \end{bmatrix},$$
$$C_{c1} = 10^{-4}\begin{bmatrix} -0.1854 & -0.0811 \\ 0.0043 & -0.1406 \end{bmatrix},$$
$$C_{c2} = 10^{-4}\begin{bmatrix} 0.4957 & 0.3046 \\ -0.0602 & 0.3867 \end{bmatrix},$$
$$C_{f1} = 10^{-6}\begin{bmatrix} -0.1244 & -0.0451 \\ 0.0547 & -0.1130 \end{bmatrix},$$
$$C_{f2} = 10^{-6}\begin{bmatrix} -0.5927 & -0.2846 \\ 0.2542 & -0.6101 \end{bmatrix},$$
$$D_{f1} = 10^{-5}\begin{bmatrix} -0.2573 & -0.0176 \\ -0.0419 & -0.1089 \end{bmatrix},$$
$$D_{f2} = 10^{-5}\begin{bmatrix} 0.6632 & 0.0647 \\ 0.0588 & 0.3256 \end{bmatrix}.$$

We performed Monte Carlo simulation with 2000 rounds. The fault signal is a step signal at $k = 100$[s] applied to the first tank. The noise signal used is the white noise with mean equal to 0 and variance equal to 0.5^2 and multiplied by an exponential. The simulation results are presented in four separated Figs. 2, 3, 4, and 5. Fig. 2 presents the controlled outputs and compares the simulation with (faulty) and without (faultless) the fault signal. An information can be extracted, which is that even with the fault applied to the first tank, the output to the second tank remains almost the same, which means that

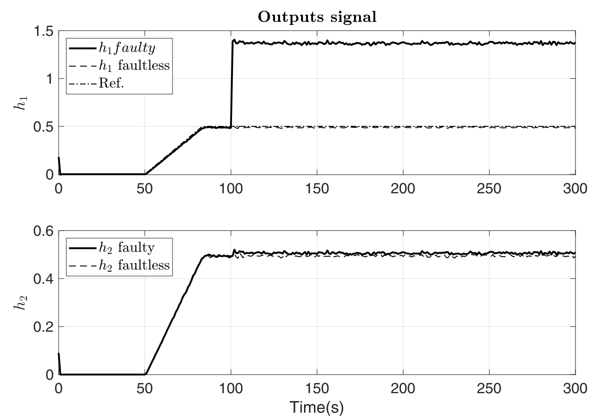

FIGURE 2. Outputs for the H_∞ case.

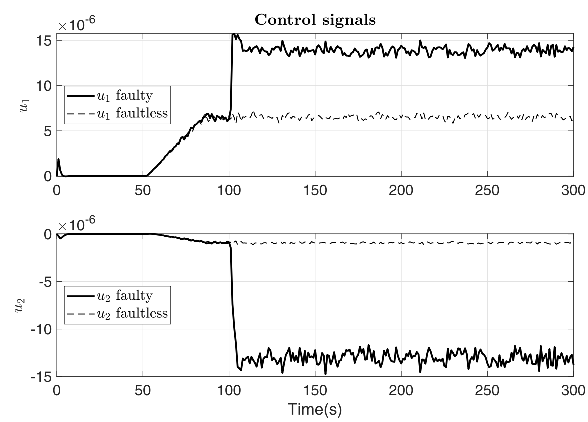

FIGURE 3. Control signal for the H_∞ case.

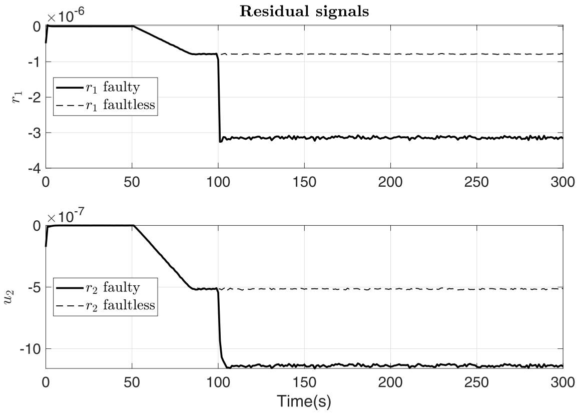

FIGURE 4. Residue signal for the H_∞ case.

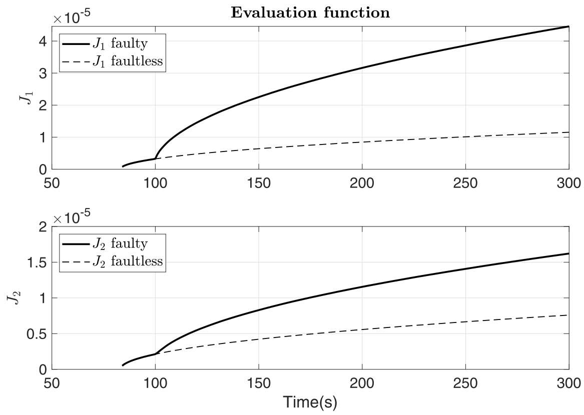

FIGURE 5. Evaluation function for the H_∞ case.

the controller fulfills its purpose. The first output has an offset due to the presence of the fault, as expected. The controller tries to compensate for the fault presence. The first output is stabilized but not compensated, and for the second output the fault is compensated. We should recall that the second state is coupled to the first one, and therefore, any fault occurring in the first tank affects the second one. Observing the control signals in Fig. 3 reinforces the statements made for Fig. 2, where both controllers tried to compensate for the fault occurrence. Fig. 4 shows that the residue signal generated by the SFDC increases near $k = 100$[s], which coincides with the start of the fault signal, meaning that the SFDC almost instantly responds to the fault.

From Fig. 5 we can notice that the fault detection side of the solution works properly, since it is clear the difference between the faulty and faultless evaluation curves.

The setup for the simulation is exactly the same used in the H_∞ case. The SFDC obtained using Theorem 2 is

$$A_{c1} = \begin{bmatrix} 0.5929 & 0.0388 \\ 0.0201 & -0.1255 \end{bmatrix},$$

$$A_{c2} = \begin{bmatrix} -0.5929 & -0.0388 \\ -0.0201 & 0.1255 \end{bmatrix},$$

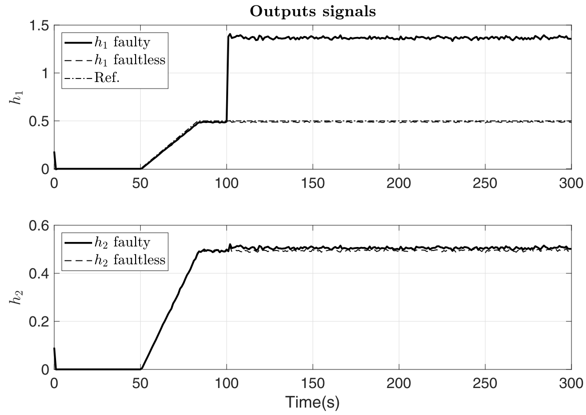

FIGURE 6. Outputs for the H_2 case.

$$B_{c1} = 10^{-6} \begin{bmatrix} -0.2409 & -0.0079 \\ 0.0093 & -0.3303 \end{bmatrix},$$

$$B_{c2} = 10^{-6} \begin{bmatrix} 0.3691 & 0.0010 \\ 0.0044 & 0.0364 \end{bmatrix},$$

$$C_{c1} = \begin{bmatrix} 0.8648 & 0.0728 \\ 0.0108 & -0.1349 \end{bmatrix},$$

$$C_{c2} = \begin{bmatrix} -0.8053 & -0.0366 \\ -0.0460 & 0.2186 \end{bmatrix},$$

$$C_{f1} = 10^{-13} \begin{bmatrix} 0.0748 & -0.0001 \\ 0.0000 & -0.1463 \end{bmatrix},$$

$$C_{f2} = 10^{-13} \begin{bmatrix} -0.0835 & 0.0001 \\ -0.0000 & 0.1375 \end{bmatrix},$$

$$D_{f1} = \begin{bmatrix} 43.2163 & -0.0000 \\ -0.0000 & 7.5839 \end{bmatrix},$$

$$D_{f2} = \begin{bmatrix} -33.2163 & 0.0000 \\ 0.0000 & 2.4161 \end{bmatrix}.$$

The results obtained via simulation are presented in the following Figs. 6, 7, 8, and 9.

In Fig. 6 both output signals are presented, as well as a comparison between the situation with and without the fault signal. As observed in the H_∞ case, the first output has an

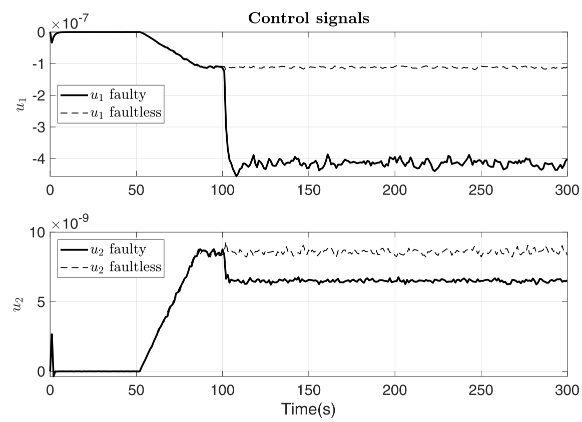

FIGURE 7. Control signal for the H_2 case.

offset caused by the fault and the second output compensates the fault occurrence.

In Fig. 7 for both control signals, it is possible to observe that the controller tries to counterbalance the fault signal applied to the first tank, which was the goal of the designed controller.

In Fig.8 the first residual signal increases right after $k = 100$[s], when the fault signal starts. The presented behavior is the expected behavior for a FDI, which is the goal for the FDI side of the SFDC proposed in this paper.

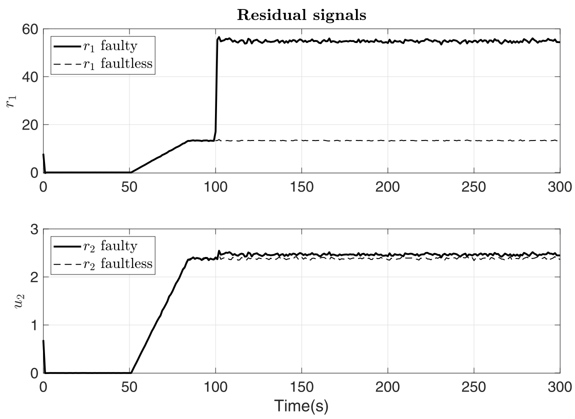

FIGURE 8. Residue signal for the H_2 case.

The evaluation function presented in Fig. 9 shows that the proposed solution responds rapidly after the occurrence of the fault. Another important aspect is that the evaluation function for the second output does not change its behavior.

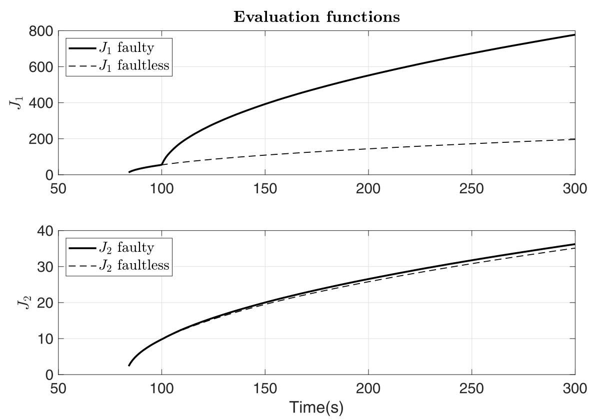

FIGURE 9. Evaluation function for the H_2 case.

VI. CONCLUSION

In the present paper, we focus on the Simultaneous Fault Detection and Control problem under the Markovian Jump Linear Systems with partial observation on the Markov parameter for the discrete-time domain. The main novelties in this paper, presented in Section IV, are the design of H_∞ and H_2 SFDC for MJLS with partial observation based on Bilinear Matrix Inequalities, and the mixed H_2/H_∞ for the SFDC, where the control side of the SFDC considers the H_2 norm and the fault detection part considers the H_∞ norm. We also described the coordinate descent algorithm as a possible method to solve the BMI. In Section V a numerical example was presented to illustrate the viability of the proposed solution. The results presented in Section V indicate that the design of H_∞ / H_2 SFDC for MJLS with partial jump parameter provided in the present paper are viable solutions for the SFDC problem.

Possible future steps along this line of research would be to address the fault compensation problem, or consider H_- index to increase the fault detection performance.

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