

Received December 11, 2019, accepted December 26, 2019, date of publication January 6, 2020, date of current version January 14, 2020. *Digital Object Identifier 10.1109/ACCESS.2020.2963984*

Global Adaptive 4-Points Congruent Sets Registration for 3D Indoor Scenes With Robust Estimation

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This work was supported in part by the National Natural Science Foundation of China under Grant 61771338, and in part by the Yunnan key research program under Grant 2018IB007 and Grant 2018ZF017.

ABSTRACT As a powerful global registration method for point clouds, the 4-points congruent sets (4PCS) algorithm has been wildly used in the 3D scene reconstruction field. In this paper, we propose an adaptive 4PCS (A4PCS), which aims to provide a robustness rigid transformation for two or more overlapping laser scans. The proposed method only incorporates the distance information of the stereoscopic base set and a fast mechanism for congruent base extraction into 4PCS. To ensure the registration accuracy when dealing with restrictive situations, such as point clouds with small overlaps or scenes with symmetrical structures, a non-coplanar 4-points base set is adopted without extra time consumption. Besides, the adaptive set finetuning is introduced to the point pair searching process to accelerate the convergence of the algorithm. In addition, we replace the binary cost function of the original 4PCS with a modified estimator to strengthen the robustness of the proposed method. Experiments on thirteen pairs of point clouds for 3D indoor scenes, including ten regular size models and three scenes of one large-scale model, can demonstrate the accuracy and efficiency of the proposed method.

INDEX TERMS Global registration, 4-points congruent sets (4PCS), rigid transformation, point clouds.

I. INTRODUCTION

With the advancing of laser scanner based applications, 3D registration has attracted great interest in the computer vision [1]–[3] and remote sensing [18], [19] related research fields. As an indispensable component of 3D reconstruction, the main task of the 3D registration is to match multiple point clouds scanned from different orientations of the same scene to a uniform coordinate system [8]–[10]. In this way, 3D registration can provide a basic architecture for 3D reconstruction to restore the entire scene. Therefore, in order to get a more realistic 3D reconstruction result, a precise 3D registration is essential.

To align two scans captured in arbitrary positions with partial overlaps, the first thing to do with 3D registration is to estimate the correspondences between the two scans [5], [6], [16]. The correspondences can be estimated based on various

The associate editor coordinating the [rev](https://orcid.org/0000-0001-9639-7679)iew of this manuscript and approving it for publication was Junchi Yan .

feature descriptors, such as [7], [11]. In order to complete the alignment of the corresponding regions after obtaining the correspondences, the optimal rigid transformation parameters should to be estimated first with the 3D registration. The optimal transformation parameters are defined as the ones that can minimize the distances between correspondences of the two converted point clouds. Since the number of the correspondences obtained by each estimation may be zero or more than one pair, the transformation parameters calculated based on the correspondences also may not exist or are not unique. To solve this problem, random iterations and appropriate evaluation functions are introduced into the registration algorithms to estimate the optimal transformation parameters, which shows better performance [25].

Typically, such transformation parameters are estimated in a two-step procedure: global registration and fine registration. Usually, an initial rigid transformation is estimated by global registration to align point clouds roughly, so as to avoid the fine registration falling into a local optimal

solution [12]–[15]. Besides, good initialization from global registration can speed up the convergence of fine registration, such as the classical iterative closest point (ICP) or its variants [4], [20]. Based on the global registration, the fine registration further refines local regions that are not aligned precisely. Therefore, accurate estimation of global registration parameters is critical for fully automated real-time registration process in real-world applications.

The rest of this paper is organized as follows. In Section II, related work is briefly reviewed. Section III describes the proposed method in details, followed by a performance study in Section IV. The paper is concluded in Section V.

II. RELATED WORK

Recently, a variety of approaches have been proposed to achieve fully automated global registration of the laser scans. Algorithms based the RANdom SAmple Consensus (RANSAC) for global registration have achieved great success. These methods often followed similar strategies: Given two laser scans with partial overlap, repeated the random voting process to match pairs of congruent bases which were extracted from these two scans separately until it found the optimal rigid transformation. Rusu et al. proposed a popular variant named Sample Consensus Initial Alignment (SCA-IA) [21], which used the triplet sets as the congruent bases. Though it was able to complete registration with low errors, the computational complexity usually increased to $O(n^3)$, where *n* was the size of the point sets. As opposed to use a 3-point base, Aiger *et al.* [22], [23] proposed the 4-Points Congruent Sets (4PCS) and Super 4PCS (S4PCS) algorithms with coplanar 4-points set as the bases, which reduced the runtime complexity to $O(n^2)$ and *O*(*n*) respectively.

Based on the 4PCS algorithm, several practical methods have been developed recently [24]–[30]. Theiler *et al.* [26], [27] presented a combined 4PCS method, named Keypointbased 4PCS, in which keypoints were firstly extracted from raw point clouds to be used as the input to the 4PCS algorithm. Mohamad *et al.* [28], [29] proposed the General 4PCS (G4PCS) and Super G4PCS successively. The G4PCS introduces a more general type of 4-points base set that removed the planarity constraint. Huang *et al.* [30] proposed Volumetric 4PCS(V4PCS), which incorporated the volumetric information to the S4PCS to accelerate the extraction of the congruent bases. The MSSF-4PCS, proposed by Xu *et al.* [17], embedded multiscale sparse features (MSSF) with sparse coding into 4PCS to enable efficient global registration of point clouds.

Though the existing methods have made improvements either in terms of the runtime or the registration accuracy, the results of the registration were often not desirable when faced with restrictive situations. For example, when the overlap area between two scans was very small, computation complexity would be very high because it was difficult to extract the congruent base sets within the small overlapping area. Besides, due to the lack of stereo information in the coplanar point sets, the registration accuracy also needed to be further improved in condition of models with symmetrical structures. Considering the above problems, we proposed a novel adaptive 4PCS method and the two main contributions of this paper were as follows.

- 1) Only the distance information of the stereoscopic base set was utilized to the congruent base extraction process, which could make full use of the fast point pairs searching mechanism of the S4PCS algorithm as well as avoiding the time consumption caused by angle calculations to filter the unnecessary base sets.
- 2) To accelerate the extraction of the congruent bases, we introduced a new fast searching mechanism to the proposed method. Besides, the adaptive sets finetuning was incorporated into the point pairs searching process, which gained about a 4 times speedup of the congruent base extraction process.

III. METHODOLOGY

A. GLOBAL REGISTRATION PROCEDURE

The adaptive 4PCS method follows the same framework as the 4PCS but exploiting non-coplanar congruent bases instead of planar ones. The whole framework involves three steps as described in Fig. [1](#page-2-0) and the details are presented below:

- **Sparse Representation:** Given the input sets *S* and *T* with partial overlap, the voxel grid filter is applied to roughly even out the strongly uneven point distribution caused by equipment acquisition.
- **RANSAC Iteration:** After sampled, the two sparse uniform point clouds are used to estimate the optimal transformation. In order to eliminate the variance caused by randomly sampling of the base sets, the RANSAC iteration is introduced in this process. In each RANSAC loop, congruent 4-points base sets $B \in S$ and $M_i \in T$ will be extracted to compute the candidate transformations T_i using the Singular Value Decomposition (SVD) algorithm. The iteration will be terminated if it completes *L* (the maximum number) times iterations or it finds the optimal transformation (*Topt*). In this step, we introduce a new fast method to extract the congruent bases and a modified estimator to find the optimal transformations from the candidate ones, which will be presented in detail in the following Section [III-C](#page-2-1) and [III-D](#page-3-0) separately.
- **Rotation and Translation:** Finally, these two point clouds can be registered by two steps of rotation and translation with the parameter *Topt* .

B. CONGRUENT SETS EXTRACTION WITH 4PCS

As the proposed congruent 4-points sets extraction method is based on the 4PCS algorithm, the basic concept of the 4PCS will be briefly introduced in the first as follows:

Base Set Selection: Randomly select a coplanar 4-points set $B = \{a, b, c, d\}$ from source set *S*, and then calculate the point distances d_1 and d_2 , the intersection point e , and the corresponding ratios r_1 , r_2 as shown in Fig. [2.](#page-2-2)

FIGURE 1. Flowchart of the proposed method.

FIGURE 2. Basic principle of 4PCS with base set $B{a, b, c, d} \in S$ and a corresponding congruent point set $M{a', b', c', d'}$ $\in T$.

Point Pairs Searching: Search for all the point pairs with distances d_1 and d_2 from the target set T .

Congruent Sets Extraction: Match each two point pairs with distances d_1 and d_2 to the base *B*. If they satisfy the following restrictions, the set $M = \{a', b', c', d'\}$ will be considered congruent to *B*.

a) The respective intersection points *e* and *e'* are coincident, i.e.,

$$
r_1 = r'_1, \ r_2 = r'_2. \tag{1}
$$

b) The respective angles between two lines are equivalent,

$$
\theta = \theta'.\tag{2}
$$

Up to this point, the 4PCS method is able to extract the coplanar congruent sets with a $O(n^2 + k + c)$ time complexity, where *n* is size of T , k is the number of the searched point pairs and *c* is the number of the extracted congruent sets. The most time-consuming step is the point pair searching step, which takes $O(n^2)$ time to search all point pairs with a given distance within the set *T* [22]. In order to save this time cost, the S4PCS method introduces a new data structure to organize the points and a smart searching mechanism to find the point pairs in a linear time complexity $O(n)$ [23].

C. CONGRUENT SETS EXTRACTION WITH A4PCS

Since our proposed registration method is for 3D scenes, coplanar congruent bases may not be able to take full

advantage of the information of 3D data. For example, when dealing with scenes with symmetric structures, using coplanar bases may produce incorrect transformation [30]. Moreover, the study in [28] finds that using a coplanar base will result in excess number of congruent bases. Considering the above issues, we select a non-coplanar 4-points set as the base *B*.

As mentioned above, the S4PCS method has developed a fast mechanism to search point pairs with given distances in a linear time. In order to utilize this capability, we propose to extract the congruent bases that only utilizes the distance information. From the knowledge of geometry, two noncoplanar 4-points sets can be considered congruent if all the six distances of their corresponding point pairs are the same. Based on this stereoscopic information, the extraction of the congruent 4-points sets can be converted to extract six corresponding pairs of points.

A similar idea is proposed in [30], which extracts all the point pairs corresponding to the six distances at first, and then stores them into six connective tables to query which six point pairs can be formed into a congruent base. And the authors have derived that the time complexity compared to S4PCS can be decreased from $O(n + k + c)$ to $O(n + k)$, where *n* is the number of points in set T , k is the number of the extracted point pairs and c is the number of congruent base sets. Though this method introduces the S4PCS to accelerate point pairs searching step, searching for all these six group point pairs will still be time-consuming if the size of the target point cloud *n* is too large. In order to further reduce this computation complexity, we propose an adaptive searching strategy to extract the congruent bases. It can be known from the geometry that each vertex in a tetrahedron is connected to its three edges at the same time. So if a point belongs to a congruent base set, it must belong to three of six point pairs of this congruent base at the same time. Therefore, there is no need to search all these six groups of point pairs in the whole set *T*. For this purpose, we extract these six groups of point pairs from the set *T* and its' subsets step by step. By this strategy, the searching pools can be adaptively fine-tuned to

FIGURE 3. Basic principle of the proposed method with the base set $B{a, b, c, d} \in S$ and one of its congruent point set $M_i = \{a'_i, b'_i, c'_i, d'_i\} \in \dot{T}.$

smaller subsets according to the distance information. The details are presented as follows (Fig. [3\)](#page-3-1):

Step1: Base Set Selection Randomly select a non-coplanar 4-points base set $B = \{a, b, c, d\}$ from source set *S*, and then calculate all the six distances $d_1 \sim d_6$ between each two point pairs of the base *B*.

Step2: Point Pairs Searching Instead of searching all six groups point pairs in the set *T* at one time as the V4PCS dose [30], we just first search for two groups with distances of d_1 and d_2 by using the pairs searching algorithm in S4PCS. And then, store them into two respective tables (*Table*¹ and *Table*2) with an effective standard data structure [39].

Step3: Congruent Triangular Set Query Query all the point pairs of the *Table*¹ and *Table*2, if there are two point pairs (one from *Table*¹ and one from *Table*² respectively) that have a common point, then we just need to ensure whether the distance between the rest two points is d_3 . If satisfied, these two point pairs will be stored as the congruent triangular set $\{a'_i, b'_i, c'_i\}$ into *Table*₃.

Step4: Congruent 4-points Base Extraction Based on the extracted triangular sets, the rest three groups point pairs corresponding to distances d_4 , d_5 and d_6 can be searched within small subsets. Specifically, the point pairs with distance of d_4 can be extracted within sets $\{a'_i\}$ and *T*. For each a'_{i} , the searched points from the set *T* are defined as the set O_{sph} . And then, search for all points with a distance of d_5 to the point b'_i within the set \tilde{O}_{sph} and the searched points are defined as the set O_{cir} . Finally, the points d_i' which are at distance d_6 to c'_i can be searched from \hat{O}_{cir} .

Alg. [1](#page-3-2) summarizes the whole process of the congruent base extraction and it is also worth mentioning that the distances of all the point pairs extracted in our method have a given tolerance (ε) . Rather than searching all these six groups of point pairs at one time from the whole set *T* [30], we first query the congruent triangular sets $\{a'_i, b'_i, c'_i\}$ to filter the irrelevant sets. Based on the congruent triangular sets and the distance information of the base set, the searching pools for the rest point pairs can be adaptively adjusted to the subset

Algorithm 1 Extract Congruent Bases by the A4PCS

Input: Source and target point sets *S* and *T* **Output:** The congruent bases *B* and *Mⁱ*

- 1: //*Base Points Selection*
- 2: Select a non-coplanar base set $B = \{a, b, c, d\} \in S$
- 3: Calculate all distances *d*¹ ∼ *d*⁶
- 4: //*Point Pairs Searching*
- 5: Search point pairs with distances d_1 and d_2 from T
- 6: *Table*₁, *Table*₂ \leftarrow Initialize two tables to store the d_1 and *d*² point pairs
- 7: // *Congruent Triangular Set Query*
- 8: **for** $(i = 1; i <$ *Table*₁*.size*; $i + +$) **do**
- 9: **for** $(j = 1; j <$ *Table*₂*.size*; $j + +$) **do**
- 10: $\{a'_i, b'_i, c'_i\} \leftarrow$ Query for congruent triangular sets
- 11: // *Congruent 4-points Base Extraction*
- 12: $\hat{O}_{sph} \leftarrow$ Search for the points in *T* with distance d_4 to a'_i
- 13: $O_{cir} \leftarrow$ Search for the points in O_{sph} with distance d_5 to b_i'
- 14: $\{d'_i\} \leftarrow$ Extract the points in O_{cir} with distance d_6 to c'_i **return** $B = \{a, b, c, d\} \in S$ and $M_i = \{a'_i, b'_i, c'_i, d'_i\}$

 O_{sph} , then to O_{cir} step by step as shown in Fig. [3.](#page-3-1) Through this strategy, the time complexity of the point pairs searching process can be reduced by decreasing the sizes of the searching pools (*n*). Besides, with the decrease of searching pool sizes, the number of the extracted point pairs (*k*) will be decreased as well. As a result, the proposed searching strategy can further decrease the time complexity from $O(n + k)$ to a smaller one. And the experimental results also show that the proposed method has a significant contribution to the reduction of runtime.

D. ESTIMATION WITH ROBUST MODIFIED COST FUNCTION

As the transformation parameters computed in each iteration loop are usually not unique, it is necessary to estimate

FIGURE 4. Examples of successful registration by A4PCS with different parameters settings for each model (Bird: $\delta = 0.8$, Dragon: $\delta = 0.6$, Hippo: $\delta = 0.7$, Head: $\delta = 1.5$, Buddha: $\delta = 0.3$).

the optimal one based on a robust cost function. Given the squared Euclidean residuals e_i^2 between the *i*th point in the transformed set S' and its closest neighbour in the set T , the standard 4PCS algorithm adopts a binary decision as the

cost function.
$$
\rho_F = \frac{1}{n} \sum_{i}^{n} \rho(e_i^2)
$$
, where

$$
\rho(e_i^2) = \begin{cases} 0 & e_i^2 \le \delta^2 \\ 1 & e_i^2 \ge \delta^2 \end{cases}
$$
(3)

This evaluation criterion relies heavily on the threshold settings of δ , which is subject to the density of the point cloud. It has been reveled that incorrect registration results are highly likely to occur with this estimator [25]. Therefore, we modify the robust estimator MSAC (M-estimator Sample Consensus) to be the cost function of our method without additional computation. The function is modified to

$$
\rho(e_i^2) = \begin{cases}\n\frac{e_i^2}{e_i^2 + \delta^2} & e_i^2 \le \delta^2 \\
1 & e_i^2 \ge \delta^2\n\end{cases}
$$
\n(4)

This cost function can weaken the punishment of the inner points and decrease the influence on the threshold settings, so it can be more robust when dealing the point clouds with different densities. In such way, the optical transformation with the lowest score will be estimated by this cost function.

IV. EXPERIMENTAL RESULTS AND DISCUSSION

To verify the efficiency of the proposed method, we have designed three experiments to compare it with the other stateof-art registration methods. We experimentally evaluate the A4PCS with respect to registration accuracy and computational efficiency. And all experiments in this paper are ran on a personal computer equipped with Intel E5400 at 2.69 GHz and 2 GB of RAM.

1) Registration Accuracy: The Root Mean Square Error (RMSE), which is between true correspondences of the transformed input sets, is adopted to evaluate the registration accuracy of these two methods:

$$
RMSE = \sqrt{\frac{1}{m} \sum_{i=1}^{m} (x_i - x'_i)^2 + (y_i - y'_i)^2 + (z_i - z'_i)^2}
$$
 (5)

TABLE 1. The descriptions of the ten measured models.

| Model | Number | | Dimensions $(m \times m \times m)$ | Overlap | |
|-------------|--------|-------|------------------------------------|---------|--|
| | View1 | View2 | | | |
| Bird | 1968 | 2046 | | 0.4 | |
| Head | 83484 | 89811 | | 0.4 | |
| Hippo | 30519 | 21935 | | 0.7 | |
| Buddha | 78212 | 71459 | | 0.7 | |
| Dragon | 41841 | 22092 | | 0.4 | |
| Cups | 1579 | 907 | $0.3 \times 0.15 \times 0.4$ | 0.2 | |
| Shelf | 8294 | 9649 | $0.65 \times 0.4 \times 0.75$ | 0.8 | |
| Table1 | 11306 | 9790 | $1.2 \times 0.6 \times 0.8$ | 0.7 | |
| Table2 | 15219 | 60562 | $1.6 \times 0.6 \times 0.8$ | 0.7 | |
| Lab | 48904 | 96287 | $6 \times 3 \times 3$ | 0.3 | |

where (x_i, y_i, z_i) and (x'_i, y'_i, z'_i) are the coordinates of the *i*th point in the transformed source set *S* and its closest neighbour in the target set *T* , respectively. Besides, *m* is the total number of the correspondent point pairs.

2) Computational Efficiency: Computational efficiency is represented by the average total runtime T_t for the whole process.

A. PERFORMANCE EVALUATION AND COMPARISON WITHOUT LOCAL REGISTRATION

For the first experiment, we compare the proposed method with the S4PCS method (the only open source 4PCS-based method). The testing dataset contains ten pairs of point clouds scanned from ten models. Five data sets of models (Fig. [4\)](#page-4-0) are obtained from [37] [38] and the other five models (Fig. [6\)](#page-6-0) are measured by RPLIDAR scanner produced by SLAMTEC company and we have shared them to [40]. The initial numbers, the estimated overlaps and dimensions of the ten data sets are shown in Table [1.](#page-4-1)

As the proposed method follows the same scheme as S4PCS to set the parameters, we test both algorithms with the same parameters: the threshold of the point pairs distance ε , the threshold of M-estimator δ , the the sample size *N*, the estimated overlap and the maximum number of the iterations *L*. Since both the settings of the threshold δ and ε are subject to the estimated point cloud density, we set δ = ε and we set them to different values for different models as shown in Table [2.](#page-5-0) To average the variances caused by improper sampling densities and make the experimental results more convincing, we use two different sample sizes for each model and we run 30 times for each sample size to report the average values of results. The settings of the two sample sizes for each model are determined by the original size of the dataset. Besides, improper settings of the maximum number of RANSC iterations may affect the alignment performance, for example, the registration method may be difficult to meet the real-time requirements under excessive numerical settings, or it may not be able to reach convergence states within a few iterations. So, we use the same method as 4PCS to calculate *L*.

| Models | δ/ε (cm) | Sample Size | | S ₄ PC _S | | A4PCS | Comparison | |
|---------|---------------------------|-------------|-----------------|--------------------------------|----------------|------------------------|---------------|---------------|
| | | | $T_S(s)$ | RMSE _S (mm) | $T_A(s)$ | RMSE _A (mm) | $TS\%$ | $RA\%$ |
| Bird | 0.8 | 211/407 | 20.37 / 59.06 | 8.25/6.51 | 22.14 / 47.80 | 3.54/3.92 | $-8.69/19.07$ | 57.09 / 39.78 |
| Head | 1.5 | 415/821 | 46.41 / 144.95 | 4.16/6.34 | 28.91/71.10 | 3.94 / 4.52 | 37.71 / 50.95 | 5.29 / 28.71 |
| Hippo | 0.7 | 220/413 | 19.64 / 46.57 | 5.73/7.14 | 16.14/41.63 | 4.39 / 4.88 | 17.82 / 10.61 | 23.39 / 31.65 |
| Buddha | 3.0 | 708/1411 | 116.75 / 354.37 | 33.26 / 24.42 | 86.15 / 268.99 | 22.36 / 18.25 | 26.21/24.09 | 32.77 / 25.27 |
| Dragon | 0.6 | 680/1080 | 133.13/290.75 | 1.61 / 2.18 | 75.9 / 237.52 | 0.61 / 1.60 | 42.99 / 18.31 | 62.11/26.61 |
| Shelf | 0.7 | 253/521 | 18.51/93.23 | 12.66 / 21.00 | 14.91 / 63.38 | 4.12/8.92 | 19.45 / 32.02 | 67.46 / 57.52 |
| Cups | 0.4 | 158/413 | 34.22 / 84.96 | 1.84 / 4.09 | 11.10/31.09 | 1.63 / 3.17 | 67.56 / 63.41 | 11.41 / 22.49 |
| Table1 | 9.0 | 407/833 | 64.90 / 335.45 | 39.49 / 40.26 | 28.90 / 178.84 | 20.02/21.62 | 55.47 / 46.67 | 49.30/46.30 |
| Table2 | 7.0 | 600/1020 | 189.06 / 515.65 | 121.15 / 111.78 | 57.75 / 349.64 | 34.15/41.99 | 69.45 / 32.19 | 71.81/62.44 |
| Lab | 11.0 | 1300 / 2000 | 407.32/910.45 | 92.28 / 72.78 | 268.52/565.22 | 47.47 / 53.83 | 34.08 / 37.92 | 48.56 / 26.04 |
| Average | | | 194.29 | 30.85 | 123.28 | 15.25 | 36.55 | 50.57 |

TABLE 2. Comparison of registration accuracy and runtime between two 4PCS-based methods.

The experimental results of these two methods are shown in Table [2.](#page-5-0) In order to make a comparison, we set $TS = (T_S - T_A)/T_S$ to denote the runtime saving, and $RA =$ $(RMSE_S - RMSE_A)/RMSE_S$ to denote the improvement of the registration accuracy for the A4PCS. According to the last two columns in Table [2,](#page-5-0) it is obvious that the proposed method is superior to the S4PCS under different parameters settings. Compared with the S4PCS, the A4PCS saves 36.55% of the average total runtime. In addition, the average registration accuracy of the A4PCS method is 50.57% higher than that of S4PCS. Since the coplanar bases can not take full advantage of the geometry of 3D data, the non-coplanar bases will perform better when registering the three-dimension point clouds with the stereoscopic information. Especially, when dealing with the symmetrical model (Shelf) or the low overlap model (Cups), our proposed method achieves 67.46% improvement in registration accuracy and 67.56% in runtime saving.

The proposed method has two major contributions for the significant reduction of the total runtime T_t . On one hand, the noncoplanar base B is adopted to take advantage of the stereoscopic information, which can filter the most candidate congruent bases M_i and decreases the runtime of the transformation estimation process to a large extent. On the other hand, to accelerate the congruent base extraction, we first extract the congruent triangle bases in linear time and we adaptively fine tune the pointsets of the congruent base extraction from the whole set T to the smaller subsets step by step as shown in Fig. [3.](#page-3-1) Moreover, with the stereoscopic information of the base set, there is no need to compute the ratios and angles to judge whether the sets are congruent as the S4PCS method does.

In order to validate our improvements in detail, we record the average numbers of congruent bases M_i and the average runtime of the congruent base extraction of each iteration loop. As we set two different sample sizes for each model, we report the recorded data under the bigger sample size (BS) and smaller sample size (SS) separately. As shown in Fig. [5\(](#page-5-1)a), the numbers of the congruent bases extracted

FIGURE 5. Comparison of the congruent bases extracted by the S4PCS and the A4PCS under two different sample sizes of each model. Fig. 5(a) shows the average congruent bases numbers extracted in each iteration. Fig. 5(b) shows the average runtime for the congruent base extraction in each iteration. BS represents bigger sample size and SS represents smaller sample size.

by A4PCS method are much fewer than that by S4PCS both in case of sparse or dense dadasets. Since computation and verification of the transformation parameters accounts for a large part of the total runtime, reducing the congruent bases number can directly lead to a reduction in runtime. Furthermore, as shown in Fig. [5\(](#page-5-1)b), the runtime of the congruent base extraction process for the A4PCS method is also decreased compared with the S4PCS method.

B. PERFORMANCE EVALUATION AND COMPARISON WITH LOCAL REGISTRATION

Since the purpose of the proposed method is to provide a good initial position for fine registration to avoid it falling into local optimal solutions, it is quite feasible to evaluate the effectiveness of the global registration algorithm by com-

FIGURE 6. Registration results of the five measured models with the three registration methods (Go-ICP, S4PCS+ICP and A4PCS+ICP). The two source scans for each model are shown in the first row. The parameters for each model are shown under each column. The colorbar shows the registration error for each model and the unit is centimeters.

FIGURE 7. Visualization for successful registration examples of the six scans (S1-S2, S3-S4, S5-S6) of the ETH Hauptgebaude scene. The extracted NARF keypoints (green) from each scan (red) are displayed blow.

bining local registration algorithms. In order to make a comparison, we perform the ICP fine registration after completing the S4PCS and A4PCS separately in this section. Besides, the developed ICP method Go-ICP [32], is also executed singly to verify the registration performance of the proposed method. As shown in the Fig [6,](#page-6-0) the registration result of the A4PCS followed by ICP outperforms that of the single Go-ICP method, which indicates that the A4PCS method is able to estimate a good initial transformation for fine registration method to gain more desirable registration performance. Moreover, it is clear that our method also shows better performance with less registration error than the S4PCS method.

C. PERFORMANCE EVALUATION AND COMPARISON WITH LARGE-SCALE DATASETS

Although the proposed method has achieved significant improvements in the efficiency of registration,

| Scans $S1-S2$ $S3-S4$ $S5-S6$ | Points | Keypoints | $\tau(m)$ | Time(s) | | | | RMSE(cm) | | | |
|---|--------------------------|-----------|--------------------------|------------|------------|---------|---------|------------|------------|---------|---------|
| | | | | PCL | FGR | K-S4PCS | K-A4PCS | PCL | FGR | K-S4PCS | K-A4PCS |
| | 189982 | 2051 | 0.05 | 263.63 | 275.46 | 227.30 | 219.64 | 3.6 | 4.9 | 3.4 | 3.7 |
| | 186233 | 1144 | 0.1 | 254.76 | 268.73 | 264.62 | 238.22 | 3.6 | 3.9 | 7.2 | 4.9 |
| | 198968 | 3455 | 0.05 | 440.35 | 265.87 | 310.13 | 316.49 | 4.4 | 6.6 | 4.0 | 3.7 |
| | 188904 | 1308 | 0.1 | 433.13 | 272.03 | 354.67 | 276.64 | 4.0 | 6.2 | 6.8 | 3.2 |
| | 188897 | 2241 | 0.05 | 296.79 | 270.00 | 275.64 | 255.74 | 2.4 | 1.9 | 3.2 | 2.9 |
| | 189908 | 1597 | 0.1 | 316.54 | 272.94 | 329.60 | 287.49 | 6.3 | 2.0 | 5.2 | 5.1 |
| Average | $\overline{}$ | | $\overline{}$ | 334.20 | 270.83 | 293.66 | 265.70 | 4.10 | 4.05 | 4.97 | 3.91 |

TABLE 3. Comparison of registration accuracy and runtime between keypoint-based methods with large-scale datasets.

the computational cost will still be quiet large when facing with large-scale point clouds. In such cases, the proposed method can be easily embedded into the keypoint-based method which extracts keypoints from raw point clouds with feature descriptors before the registration process begins [27]. In order to verify the feasibility of this idea, we add an additional experiment as shown in Table [3.](#page-7-0) In this experiment, we test three pairs scans of the ETH Hauptgebaude scene of the Automonous Systems Lab (ASL) dataset [41] as shown in Fig [7.](#page-6-1) Besides, the NARF (Normal Aligned Radial Feature) descriptor [34] is introduced to extract the keypoints in our method and two state-of-art methods are also included to assess the proposed method. As shown in the Table [3,](#page-7-0) PCL is a Point Cloud Library implementation of the algorithms [21], [35] and FGR (Fast Global Registration) is the algorithm of [36]. The trail will be terminated if the the proportion of corresponding points exceeds 80% and it is not necessary to align all points for the overlap is not 100%. Given the expected large influence of the correspondences distance threshold τ , all tests in this part are done with two different threshold values $\tau = \{0.05m, 0.1m\}.$

As shown in the Table [3,](#page-7-0) the time represents the total runtime for the keypoints extraction and registration process. As the results shown, all these four registration methods can obtain satisfactory registration results in a limited time. Besides, from the average value data, the proposed method achieves a higher registration efficiency with the adaptive searching mechanism than the other methods. Moreover, with the stereo information of the non-coplanar bases, the K-A4PCS method achieves more accuracy registration results than what K-S4PCS dose. Therefore, even when dealing with large-scale datasets, the proposed method can achieve the desirable registration efficiency by extracting keypoints to sparse the point clouds density.

V. CONCLUSION

In this paper, we propose a novel fast automated global registration method based on 4PCS algorithm. As the mechanism of the congruent base extraction is more flexible than traditional algorithms, the A4PCS method improves the registration performance both in respect of the runtime and

accuracy. Especially, with fuller stereoscopic information of the non-coplanar bases, our method shows more desirable results than S4PCS when dealing with the laser scans with symmetrical structures or low overlaps. Besides, the proposed method adopts a modified estimator to strengthen the robustness of the algorithm. From the experimental results, the A4PCS is capable of estimating good initial transformations for fine registration method to achieve satisfactory performance.

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