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Disturbance Observer Based Tracking Control of Quadrotor With High-Order Disturbances

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ABSTRACT In this paper, the flight control of an unmanned aerial vehicle (UAV), quadrotor, in the presence of high-order disturbances is presented. Since during flight, UAV faces enormous, and various kinds of disturbance, the effect of such disturbances becomes vital for consideration during the development of disturbance observer (DO) based tracking control scheme. To obtain the desired tracking performance, standard sliding mode control (SMC) method is utilized while for disturbance estimation, DO based on Simpson's approximation is developed and incorporated with SMC. Furthermore, in this paper, both matched and mismatched disturbances are considered. Hence, a matrix associated with disturbances is invoked in the system model. To show the effective and desired control performance of developed disturbance observer-based control (DOBC) scheme, extensive simulations are conducted, followed by presenting the results in the paper.

INDEX TERMS Disturbance observer, high-order disturbances, quadrotor, sliding mode control, tracking control

I. INTRODUCTION

Without automatic control and theory, the evolution in science and technology would never have been the same. Because, it provides necessary helping hand resulting in a change in manpower, efficiency and reduction of time required to complete a particular task. To achieve accurate and efficient control performance, extensive research has been conducted in this area for several decades. However, there still exists the possibility of improving the performances of control systems even further. In this paper, for the development of automatic flight control, the model of a quadrotor is considered. The quadrotor is a kind of robot built using four rotors installed equidistant from each other. With the rotation of the rotors, thrust is produced achieving flight in the air [1]. When all the four rotors yield equal thrust by the equal rotation speed of the rotors, the lifting phenomenon occurs labelled as vertical take-off and landing (VTOL) [2]. By producing a mismatch in the rotation of the rotors, rolling, pitching and yawing, motions of the quadrotors are obtained during the flight in the air [3]–[6]. Before the development of the control algorithms and schemes for the flight control of quadrotor, it is necessary to obtain the mathematical model of the quadrotor. In literature, researchers have worked on developing the model of UAV, quadrotor, the model adopted in this article is developed in [7]. Following the development of a mathematical model, researchers worked on the control design using different linear control methodologies. The methods included in those linear methods were proportional derivative, proportional integral derivative and linear quadratic regulator in [4], [8]–[12] and references therein. However, the control schemes developed using linear control theory had limitations. That is, the control development was based on linear control theory, while the model of a quadrotor is highly nonlinear. Hence, it was necessary to work on the nonlinear control scheme to obtain improved performance.

Therefore, researchers worked on the nonlinear control scheme for a quadrotor with the abilities of robustness, disturbance rejection and achieving the control performance asymptotically, exponentially and in finite time. For obtaining excellent asymptotic performance, sliding mode control (SMC) method was studied for flight control in [13]–[16]. Also, backstepping control synthesis in [17], [18] and a modified version of SMC algorithm labelled as super twisting

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SMC in [19] are available in the literature. Furthermore, hybrid control techniques were developed with the ability of SMC and other nonlinear control schemes in [20]–[22]. However, regardless of these developments, the practical implementation of control algorithms did not achieve efficient performances by using the known bound of unknown disturbances.

To solve the problem of disturbances affecting the control performances, during the last couple of decades, the major highlight and sight of research is on the development of disturbance observers based control (DOBC) theory. Because in practice, the disturbances regress and oppose the control performance. Therefore, it is important to estimate and reject the disturbance effect on the systems by developing the control schemes integrated with disturbance observers (DO) [23]. DOBCs for different kinds of disturbances have been investigated in [24]–[30]. Disturbances can be sub-divided into matched and mismatched disturbances while considering its effect in the state-space model of the system [31], [32]. In [23], DO with the ability to reject disturbances as well as estimate the unmodelled uncertainties was presented. The DOs integrated separately with different types of nonlinear control schemes, i.e. SMC, backstepping, adaptive, fuzzy control schemes and many more were discussed in [33]–[48] for different kinds of disturbances. Nevertheless, in the research, as mentioned earlier, the quadrotor affected by disturbances with a model of high-order disturbances is not considered. Since practically the disturbances and their models are unknown before the development of the control algorithms, hence, it is required to consider higher-order disturbances, which includes disturbances of various behaviours. To estimate these kinds of disturbances, author in [29] developed a DO for high-order disturbances using integrator. However, integrator can lead to saturation, instability and adversely affect the control system. Thus, in this paper, a criteria to solve the problem of using integrator for DO development is presented. In this technique, the idea is to develop a DO without an integrator for the disturbance of higher-order function models. Following that it is patched with a robust SMC to obtain desired DOBTC for tracking control. The major contributions of this paper are summarized as follows:

- Integrator free DO is designed for higher-order disturbances to avoid the requirement of additional integrator for each increasing order in disturbance model.
- Higher-order disturbances with variable frequency, i.e. chirp modelled disturbances, are investigated with the designed DO.

The paper is organized as follows. In section 2, the problem statement is presented. In section 3, the disturbance observer-based tracking control scheme is developed along with the necessary stability proof of the control scheme. In section 4, proposed DO based control algorithm is simulated and discussed for quadrotor followed by a conclusion in section 5.

II. PROBLEM STATEMENT AND PRELIMINARIES

In this paper, a criteria to solve the problem of using integrator for DO development is presented. The idea is to develop a DO without an integrator for the disturbance of higher-order function models. Now, prior to control development, following lemmas and assumptions are necessary for the design of DOBTC based on SMC.

Lemma 1 [7]: According to [7], a quadrotor model can be derived as $\dot{X}(t) = f(X, U)$, where $X = [x_1\ x_2 \ldots\ x_{12}]^T$ and $U = [U_\phi \quad U_\theta \quad U_\psi \quad U_h]^T$. Assuming that the disturbance exists in each channel of the model, a mathematical model of quadrotor is written as follows:

$$f(X,U) = \begin{bmatrix} x_2 + F_{\phi_{11}} d_\phi \\ x_4\, x_6\, k_{1_\phi} + x_4 \Omega_r k_{2_\phi} + l_\phi U_\phi + F_{\phi_{21}} d_\phi \\ x_4 + F_{\theta_{11}} d_\theta \\ x_2\, x_6\, k_{1_\theta} - x_2 \Omega_r k_{2_\theta} + l_\theta U_\theta + F_{\theta_{21}} d_\theta \\ x_6 + F_{\psi_{11}} d_\psi \\ x_2\, x_4\, k_{1_\psi} + l_\psi U_\psi + F_{\psi_{21}} d_\psi \\ x_8 + F_{h_{11}} d_h \\ g - \frac{U_h}{m}(\cos x_1 \cos x_3) + F_{h_{21}} d_h \end{bmatrix} \tag{1}$$

where x_2, x_4 and x_6 are representing rate of change of roll angle (x_1), pitch angle (x_3) and yaw angle (x_5), respectively. x_8 represent the rate of change in height (x_7). d_ϕ, d_θ, d_ψ and d_h are disturbances in each subsystem of roll, pitch, yaw and height along with $F_{\phi_{11}}$, $F_{\phi_{21}}$, $F_{\theta_{11}}$, $F_{\theta_{21}}$, $F_{\psi_{11}}$, $F_{\psi_{21}}$, $F_{h_{11}}$ and $F_{h_{21}}$ as known positive constants [23]. Furthermore, U_ϕ, U_θ, U_ψ and U_h are representing control inputs, Ω_r is quadrotor parameter, g is representing gravitational constant. k_{1_ϕ}, k_{2_ϕ}, k_{1_θ}, k_{2_θ}, k_{1_ψ}, l_ϕ, l_θ and l_ψ are simplified version of constants written as $k_{1_\phi} = (I_{yy} - I_{zz})/I_{xx}$, $k_{2_\phi} = J_r/I_{xx}$, $k_{1_\theta} = (I_{zz} - I_{xx})/I_{yy}$, $k_{2_\theta} = J_r/I_{yy}$, $k_{1_\psi} = (I_{xx} - I_{yy})/I_{zz}$, $l_\phi = l/I_{xx}$, $l_\theta = l/I_{yy}$, $l_\psi = 1/I_{zz}$, with l representing the length, J_r denotes rotor inertia, I_{xx}, I_{yy} and I_{yy} are air-frame inertia of roll, pitch and yaw, respectively. In addition, for the development of DOBTC, the model of a quadrotor is decoupled in roll, pitch and yaw. To obtain a decoupled quadrotor model, geometrical technique has been applied in [58], a method introduced for nonlinear system in [57]. Furthermore, in [7], [59], it was assumed that the flight of quadrotor is either in hovering state or in motion based on small changes in angles. Hence, the gyroscopic effects can be neglected, and thus the cross-coupling can be removed, resulting in a decoupled model of roll, pitch and yaw. In order to design a control scheme, the decoupled models of a quadrotor in the presence of disturbances can be written in a generalized form defined by a class of second-order nonlinear systems as follows:

$$\dot{x} = \gamma(x, u; t) + d(t) \tag{2}$$

where $\dot{x} = [\dot{x}_1 \quad \dot{x}_2]^T$, $\gamma(x, u; t) = [x_2 \quad f(x) + g(x)u]^T$ and $d(t) = [d_1(t) \quad d_2(t)]^T$. In this paper, it is assumed that disturbance appearing in each channel of the model is same but with different magnitude, hence, a constant is invoked with the disturbance as presented in (1). Furthermore, $f(x)$ and $g(x)$ are smooth continuous functions and u is control input.

Lemma 2 [29]: A class of non-affine nonlinear systems suffering from high order disturbance can be written as follows:

$$\dot{x}_\iota = f(x_\iota, u_\iota; t) + F d_\iota(t) \quad (3)$$

where $x_\iota \in R^n$ represents state vector of the system, $u_\iota \in R^m$ represents control input vector, t represents time and $F \in R^n$ is a vector and $d_\iota(t)$ is disturbance. Furthermore, according to [29], a disturbance observer can be developed for the aforementioned class of nonlinear systems as follows:

$$\begin{cases} \dot{z}_\iota = F^+ f(x_\iota, u_\iota; t) + \Gamma_0 g_0(t) + \cdots + \Gamma_q g_q(t) \\ \hat{d}_\iota(t) = \Gamma_0 g_0(t) + \cdots + \Gamma_q g_q(t) \end{cases} \quad (4)$$

with

$$g_k(t) = \begin{cases} F^+ x_\iota - z_\iota (k = 0); & \int_0^t g_{k-1}(\tau) d\tau (k \geq 1) \end{cases} \quad (5)$$

where F^+ is pseudo inverse of F and it is obtained by using Moore-Penrose method, z_ι represent auxiliary variable and subscript k is a constant such that $0 \leq k \leq q$. Furthermore, $\Gamma_q = diag[\gamma_{01}, \ldots, \gamma_{0r}]$ has elements $\gamma_{0j} > 0$ with $j = 1, \ldots, r$ and should be chosen such that the following polynomial is Hurwitz stable.

$$\rho_i(s) = s^{q+1} + \gamma_{0j} s^q + \cdots + \gamma_{(q-1)j} s + \gamma_{qj} \quad (6)$$

where $j = 1, \ldots, r$ and r is representing the dimension of a square matrix. Hence, $\hat{d}_l$ will estimate $d_l(t)$, if the diagonal constant elements of matrix Γ_q are chosen according to (6). Furthermore, defining $\tilde{d}(t) = \hat{d}_\iota(t) - d_\iota(t)$ followed by taking derivative and combining it with (4), (5) and (6) yields error dynamics as follows:

$$\dot{\tilde{d}}_\iota(t) = -\Gamma_0 \tilde{d}(t) + \Gamma_1 g_0(t) + \cdots + \Gamma_q g_{q-1}(t) - \dot{d}_\iota(t) \quad (7)$$

Similarly, the qth derivative is obtained as $\tilde{d}_\iota^{[q]}(t) = -\Gamma_0 \tilde{d}_\iota^{[q-1]}(t) - \Gamma_1 \tilde{d}_\iota^{[q-2]}(t) - \cdots - \Gamma_q \tilde{d}_\iota(t)$. The dynamics of the observer can be decoupled for the aforementioned equations because Γ_k with $k \in (0, \ldots, q)$ are diagonal matrices with elements obtained using (6).

Lemma 3 [54]: According to Simpson's rule, a numerical integration for a definite integral can be estimated using following equation [53], [54]

$$\int_a^b h(x) = \frac{\Delta x}{3} \Bigg[h(a) + \sum_{j=1}^{(n/2)-1} 2h(t_{2j}) + \sum_{j=1}^{(n/2)} 4h(t_{2j-1}) + h(b) \Bigg] - O(\Delta^4) \quad (8)$$

where $O(\Delta^4) = \frac{b-a}{180} \Delta^4 \; h(4)(\mu)$ is negligibly small estimation error term with Δ^4 representing fourth and higher order terms after estimation and μ is a constant. Furthermore, it is assumed that estimation error and its rate of change are bounded such that $|O(\Delta^4)| \leq \kappa$ and $|\frac{\partial}{\partial t}(|O(\Delta^4)|)| \leq \varrho$.

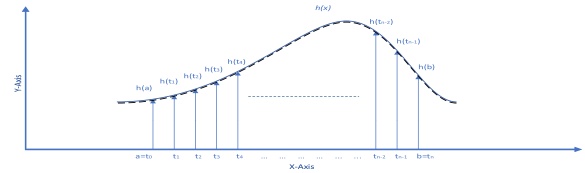

FIGURE 1. Simpson's rule for approximation of a function $h(x)$.

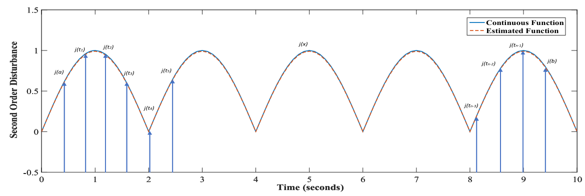

FIGURE 2. Continuous and estimated parabolic functions using Simpson's rule.

a and b are limits of the integral such that $0 < a < b$. Moreover, $\Delta x = (b-a)/n$ and $t_j = a + j\Delta x$ where $j \in (0, 1, 2, \ldots, n)$. Fig. 1 illustrates the Simpon's approximation method.

It should be noted that the estimation of Simpson's rule is dependent on the interval decided by n. When n is set to a large value, the approximation error is less and vice-versa.

Assumption 1: The model of quadrotor can be written in a class of nonlinear system (2), where the disturbance is assumed to be modelled using second-order function as $d(t) = d_0 + d_1\, t + d_2\, t^2$ with $d_0 > 0$, $d_1 > 0$ and $d_2 < 0$ as constants.

Assumption 2 [23]: It is assumed that a positive constant bounds, d^* and ϱ, can be defined for the disturbances effecting the quadrotor such that $d^* = \sup|d(t)|$ and $\varrho = \sup|\dot{d}(t)|$ for $t > 0$.

Assumption 3 [28]: It is assumed that the roll angle, ϕ and pitch angle, θ lies in interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

Remark 1: By setting $d_1 = d_2 = 0$ and $d_2 = 0, d_1 < 0$ in $d(t)$ yields disturbance model for constant and ramp, respectively. The former disturbances can be during the flight of UAV drone when the pressure of air vary slowly, however, rate of change in disturbances can be taken as zero [23]. While, the latter and parabolic disturbance incurs because of the noise induced from the rotors creating the vibrations.

Remark 2: In order to find out whether the proposed DO can estimate disturbances which can not be modeled in high-order function but have similar and identical behavior, the model of chirp function is considered for simulation purpose. In Fig. 2, a second order function is presented. It can be seen from Fig. 3 that chirp function is similar to the second order function, however, it has variable frequency. Mathematically, the chirp function can be written as as $d_c(t) = \upsilon + kt$ with υ representing a constant, $k = \frac{f_1 - f_0}{T}$, and f_1 and f_0 representing final frequency and initial frequencies, respectively. T is time constant.

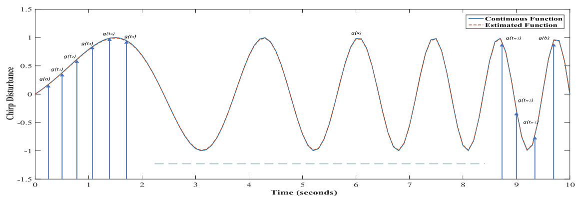

FIGURE 3. Continuous and estimated chirp functions using Simpson's rule.

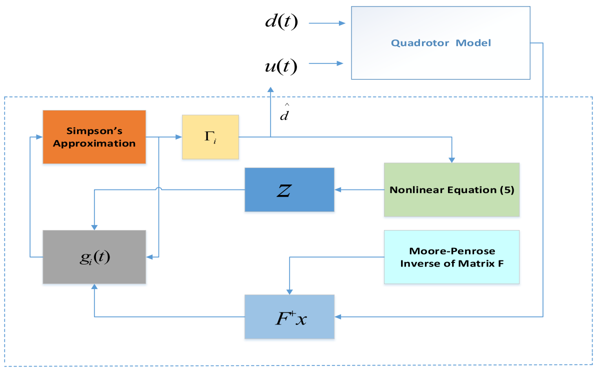

FIGURE 4. Proposed disturbance observer.

III. DISTURBANCE OBSERVER BASED TRACKING CONTROL FOR QUADROTOR

The development of DOBTC requires two steps. Firstly a DO is designed according to a criterion shown in Fig. 4 followed by control design in the second step. Following section presents an in detail derivation of DOBTC for decoupled models of roll, pitch, yaw and height.

A. DISTURBANCE OBSERVER BASED ROLL TRACKING CONTROL USING SMC

Using (1) and (3), a nonlinear model of roll is written as $\dot{x}_1 = x_2 + F_{\phi_{11}} d_\phi$ and $\dot{x}_2 = x_4\, x_6\, k_{1_\phi} + x_4 \Omega_r k_{2_\phi} + F_{\phi_{21}} d_\phi + l_\phi U_\phi$ where $F_{\phi_{11}}$ and $F_{\phi_{12}}$ are the elements of the matrix F_ϕ associated with high-order disturbances affecting roll subsystem.

1) DO FOR PARABOLIC DISTURBANCES (SECOND ORDER)

Let us assume a disturbance with second order function model as $d_\phi(t) = d_{0_\phi} + d_{1_\phi} t + d_{2_\phi} t^2$ where d_{0_ϕ}, d_{1_ϕ} and d_{2_ϕ} are unknown bounded constants. Furthermore, $F_\phi = [F_{\phi_{11}}, \quad F_{\phi_{21}}]^T$ with its Moore-Penrose inverse written as $F_\phi^+ = [F_{\phi_{11}}^+ \; F_{\phi_{12}}^+]$. Now, a DO for estimation of second-order disturbance can be designed as follows:

$$\begin{cases} \dot{z}_\phi = F_\phi^+ f_\phi(x_\phi, U_\phi; t) + \Gamma_{0_\phi} g_{0_\phi}(t) \\ \qquad + \Gamma_{1_\phi} g_{1_\phi}(t) + \Gamma_{2_\phi} g_{2_\phi}(t) \\ \hat{d}_\phi = \Gamma_{0_\phi} g_{0_\phi}(t) + \Gamma_{1_\phi} g_{1_\phi}(t) + \Gamma_{2_\phi} g_{2_\phi}(t) \end{cases} \tag{9}$$

where $g_{0_\phi}(t) = F_{\phi_{11}}^+ x_1 + F_{\phi_{12}}^+ x_2 - z_\phi$, $g_{1_\phi}(t) = \Delta h/3\big(g_{0_\phi}(t_0) + \sum_{j=1,3,\ldots}^{m-1} 4g_{0_\phi}(t_i) + \sum_{j=2,4,\ldots}^{m-1} 2g_{0_\phi}(t_i) + g_{0_\phi}(t_m)\big)$ and $g_{2_\phi}(t) = \Delta h/3\big(g_{1_\phi}(t_0) + \sum_{j=1,3,\ldots}^{m-1} 4g_{1_\phi}(t_i) + \sum_{j=2,4,\ldots}^{m-1} 2g_{1_\phi}(t_i) + g_{1_\phi}(t_m)\big)$ with $\Delta h = (b-a)/n$, $0 < 0 < b$ and $n > 0$ according to Lemma 3. Now, simplifying yields

$$\begin{cases} \dot{z}_\phi = \Gamma_{0_\phi} F_{\phi_{11}}^+ x_1 + (F_{\phi_{11}}^+ + \Gamma_{0_\phi} F_{\phi_{12}}^+) x_2 \\ \quad + F_{\phi_{12}}^+ \big(x_4 x_6 k_{1_\phi} + x_4 \Omega_r k_{2_\phi} + l_\phi U_\phi\big) \\ \quad - \Gamma_{0_\phi} z_\phi + \Gamma_{1_\phi} \frac{\Delta h}{3} \big(g_{0_\phi}(t_0) \\ \quad + \sum_{j=1,3,\ldots}^{m-1} 4g_{0_\phi}(t_j) + \sum_{j=2,4,\ldots}^{m-1} 2g_{0_\phi}(t_j) \\ \quad + g_{0_\phi}(t_m)\big) + \Gamma_{2_\phi} \frac{\Delta h}{3} \big(g_{1_\phi}(t_0) \\ \quad + \sum_{j=1,3,\ldots}^{m-1} 4g_{1_\phi}(t_j) + \sum_{j=2,4,\ldots}^{m-1} 2g_{1_\phi}(t_j) + g_{1_\phi}(t_m)\big) \\ \hat{d}_\phi = \Gamma_{0_\phi} \big(F_{\phi_{11}}^+ x_1 + F_{\phi_{12}}^+ x_2 - z_\phi\big) \\ \quad + \Gamma_{1_\phi} \frac{\Delta h}{3} \big(g_{0_\phi}(t_0) + \sum_{j=1,3,\ldots}^{m-1} 4g_{0_\phi}(t_j) \\ \quad + \sum_{j=2,4,\ldots}^{m-1} 2g_{0_\phi}(t_j) + g_{0_\phi}(t_m)\big) \\ \quad + \Gamma_{2_\phi} \frac{\Delta h}{3} \big(g_{1_\phi}(t_0) + \sum_{j=1,3,\ldots}^{m-1} 4g_{1_\phi}(t_j) \\ \quad + \sum_{j=2,4,\ldots}^{m-1} 2g_{1_\phi}(t_j) + g_{1_\phi}(t_m)\big) \end{cases} \tag{10}$$

where $\hat{d}_\phi$ and z_ϕ represent disturbance estimation and auxiliary variable of DO, respectively.

2) ROLL TRACKING CONTROL DESIGN

To obtain roll tracking control, a robust SMC technique is used. However, before proceeding, firstly the state space model of roll subsystem is assumed to have no disturbance to obtain error-based state space model. Hence, we assume that $\dot{x}_1 = x_2$; $\dot{x}_2 = x_4 x_6 k_{1_\phi} + x_4 \Omega_r k_{2_\phi} + l_\phi U_\phi$. Now, let us define a tracking error $\xi_{1_\phi} = x_1 - x_{\phi D}$ with $x_{\phi D}$ as desired reference. Taking first and second derivative and assuming $\dot{\xi}_{1_\phi} = \xi_{2_\phi}$, followed by substitution of $\dot{x}_1$ and $\dot{x}_2$, respectively, yields:

$$\begin{aligned} \dot{\xi}_{1_\phi} &= \xi_{2_\phi} + F_{\phi_{11}} d_\phi \\ \dot{\xi}_{2_\phi} &= x_4 x_6 k_{1_\phi} + x_4 \Omega_r k_{2_\phi} + l_\phi U_\phi - \ddot{x}_{\phi D} + F_{\phi_{21}} d_\phi \end{aligned} \tag{11}$$

where d_ϕ is representing disturbance, $F_{\phi_{11}}$ and $F_{\phi_{12}}$ are constant. Now, choosing sliding mode surface as $\zeta_\phi = \delta_{1_\phi} \xi_{1_\phi} + \delta_{2_\phi} \xi_{2_\phi}$ where $\delta_{1_\phi} > 0$ and $\delta_{2_\phi} > 0$ are SMC parameters. Now, taking time derivative

$$\dot{\zeta}_\phi = \delta_{1_\phi} \dot{\xi}_{1_\phi} + \delta_{2_\phi} \dot{\xi}_{2_\phi} \tag{12}$$

substituting (11)

$$\begin{aligned} \dot{\zeta}_\phi = \delta_{1_\phi}\big(\xi_{2_\phi} + F_{\phi_{11}} d_\phi\big) + \delta_{2_\phi}\big(x_4 x_6 k_{1_\phi} \\ + x_4 \Omega_r k_{2_\phi} + l_\phi U_\phi - \ddot{x}_{\phi D} + F_{\phi_{21}} d_\phi\big) \end{aligned} \tag{13}$$

Next step is to invoke $\hat{d}_\phi$ for d_ϕ, i.e. disturbance estimation, followed by setting $\dot{\zeta}_\phi = 0$. And, also introduce a switching control and a criterion to reduce chattering [48], thus, the to obtain tracking performance, following SMC law can be designed

$$U_\phi = -\frac{1}{l_\phi \delta_{2_\phi}}\big[\delta_{1_\phi}\xi_{2_\phi} + \delta_{2_\phi}\big(x_4x_6k_{1_\phi} + x_4\Omega_r k_{2_\phi} - \ddot{x}_{\phi D}\big) + \eta_\phi \hat{d}_\phi + K_\phi \zeta_\phi + L_\phi sgn(\zeta_\phi)\big] \quad (14)$$

where $\eta_\phi = \big(\delta_{1_\phi}F_{\phi_{11}} + \delta_{2_\phi}F_{\phi_{21}}\big) > 0$, and $K_\phi > 0$, $L_\phi > 0$ are sufficiently large constants to obtain robust SMC controller. Now, before proceeding ahead, it is necessary to ensure the stability of the designed DOBC. Thus, we choose a Lyapunov candidate as follows:

$$V_r = V_\phi + V_{\phi D} \quad (15)$$

where $V_\phi = \frac{1}{2}\zeta_\phi^2$ and $V_{\phi D} = \frac{1}{2}\tilde{d}_\phi^2$ with $\tilde{d}_\phi = \hat{d}_\phi - d_\phi$. Therefore, we have $V_r = \frac{1}{2}\zeta_\phi^2 + \frac{1}{2}\tilde{d}_\phi^2$. Now, after taking derivative of $V_{\phi D}$, we have $\dot{V}_{\phi D} = \tilde{d}_\phi \dot{\tilde{d}}_\phi$. Thus,

$$\dot{V}_{\phi D} = \tilde{d}_\phi\big(\Gamma_{0_\phi}\dot{g}_{0_\phi}(t) + \Gamma_{1_\phi}\dot{g}_{1_\phi}(t) + \Gamma_{2_\phi}\dot{g}_{2_\phi}(t) - \dot{d}_\phi\big) \quad (16)$$

Using Lemma 2, it can be written that $g_{0_\phi}(t) = F_\phi^+ x_\phi - z_\phi$. Taking time derivative yields

$$\dot{g}_{0_\phi}(t) = F_\phi^+ \dot{x}_\phi - \dot{z}_\phi \quad (17)$$

From (3), the roll model can be written as $\dot{x}_\phi = f_\phi(x_\phi, U_\phi; t) + F_\phi d_\phi$. Thus, substituting $\dot{x}_\phi$ and (9) yields

$$\begin{aligned}\dot{g}_{0_\phi}(t) &= F_\phi^+(f_\phi(x_\phi, U_\phi; t) + F_\phi d_\phi) - (F_\phi^+ \times f_\phi(x_\phi, U_\phi; t) + \hat{d}_\phi)\\ &= -\tilde{d}_\phi \end{aligned}\quad (18)$$

Similarly, according to Lemma 2, it can be written that $g_{1_\phi} = \int_0^t g_{0_\phi}(t)dt$ and $g_{2_\phi}(t) = \int_0^t g_{1_\phi}(t)dt$. Thus, after taking derivative, it can be derived that

$$\dot{g}_{1_\phi}(t) = g_{0_\phi}(t), \quad \dot{g}_{2_\phi}(t) = g_{1_\phi}(t) \quad (19)$$

However, $g_{1_\phi}(t)$ and $g_{2_\phi}(t)$ are approximated using the criterion presented in Lemma 3, i.e. (8). Thus, an error may exist in the approximation. Hence,

$$\dot{g}_{1_\phi}(t) = g_{0_\phi}(t) + e_{1_\phi}, \quad \dot{g}_{2_\phi}(t) = g_{1_\phi}(t) + e_{2_\phi} \quad (20)$$

where e_{1_ϕ} and e_{2_ϕ} are the errors incurred due to the approximation technique of $g_{1_\phi}(t)$ and $g_{2_\phi}(t)$, respectively. It should be noted that according to (5), $g_{0_\phi}(t)$ does not require any approximation. Now, substituting (18) and (20) into (16)

$$\begin{aligned}\dot{V}_{\phi D} &= \tilde{d}_\phi\big(-\Gamma_{0_\phi}\tilde{d}_\phi + \Gamma_{1_\phi}\big(g_{0_\phi}(t) + e_{1_\phi}\big) + \Gamma_{2_\phi} \times \big(g_{1_\phi}(t) + e_{2_\phi}\big) - \dot{d}_\phi\big)\\ &\le -\Gamma_{0_\phi}\tilde{d}_\phi^2 + |\Gamma_{1_\phi}|\big(|g_{0_\phi}(t)| + |\kappa_{1_\phi}|\big)|\tilde{d}_\phi| + |\Gamma_{2_\phi}|\big(|g_{1_\phi}(t)| + |\kappa_{2_\phi}|\big)|\tilde{d}_\phi| + |\varrho_\phi||\tilde{d}_\phi|\\ &\le -\Gamma_{0_\phi}\tilde{d}_\phi^2 + |\Gamma_{1_\phi}||g_{0_\phi}(t)||\tilde{d}_\phi| + |\Gamma_{1_\phi}||\kappa_{1_\phi}| \times |\tilde{d}_\phi| + |\Gamma_{2_\phi}||g_{1_\phi}(t)||\tilde{d}_\phi| + |\Gamma_{2_\phi}||\kappa_{2_\phi}||\tilde{d}_\phi| + |\varrho_\phi||\tilde{d}_\phi| \end{aligned}\quad (21)$$

where $\varrho_\phi \ge \dot{d}_\phi$, $\kappa_{1_\phi} \ge e_{1_\phi}$ and $\kappa_{2_\phi} \ge e_{2_\phi}$. Further simplification yields

$$\begin{aligned}\dot{V}_{\phi D} &\le -\Gamma_{0_\phi}\tilde{d}_\phi^2 + \big(\frac{1}{2}\Gamma_{1_\phi}^2\tilde{d}_\phi^2 + \frac{1}{2}g_{0_\phi}^2(t)\big) + \big(\frac{1}{2}\Gamma_{1_\phi}^2 \times \tilde{d}_\phi^2 + \frac{1}{2}\kappa_{1_\phi}^2\big) + \big(\frac{1}{2}\Gamma_{2_\phi}^2\tilde{d}_\phi^2 + \frac{1}{2}g_{1_\phi}^2\big)\\ &\quad + \big(\frac{1}{2}\Gamma_{2_\phi}^2\tilde{d}_\phi^2 + \frac{1}{2}\kappa_{2_\phi}^2\big) + \frac{1}{2}\varrho_\phi^2 + \frac{1}{2}\tilde{d}_\phi^2\\ &\le -\big(\Gamma_{0_\phi} - \Gamma_{1_\phi}^2 - \Gamma_{2_\phi}^2 - \frac{1}{2}\big)\tilde{d}_\phi^2 + \frac{1}{2}\big(g_{0_\phi}^2 + g_{1_\phi}^2\big) + \frac{1}{2}\big(\kappa_{1_\phi}^2 + \kappa_{2_\phi}^2 + \varrho_\phi^2\big)\\ &\le -2\bar{\Gamma}_\phi V_{\phi D} + \frac{1}{2}\big(B_{1_\phi} + B_{2_\phi}\big) \end{aligned}\quad (22)$$

where $\bar{\Gamma}_\phi \ge \Gamma_{0_\phi} - \Gamma_{1_\phi}^2 - \Gamma_{2_\phi}^2 - \frac{1}{2}$, $B_{1_\phi} = \max\{g_{0_\phi}^2 + g_{1_\phi}^2\}$ and $B_{2_\phi} = \max\{\kappa_{1_\phi}^2 + \kappa_{2_\phi}^2 + \varrho_\phi^2\}$. Now, since the DO gain constants are designed according to Lemma 2 and if $\bar{\Gamma}_\phi \ge \big(\Gamma_{0_\phi} - \Gamma_{1_\phi}^2 - \Gamma_{2_\phi}^2 - \frac{1}{2}\big) > 0$ holds, then the bounds $B_{1_\phi} > 0$ and $B_{2_\phi} > 0$ are legit and the states will not escape. Therefore, the convergent dynamics are obtained and it can be stated that $\dot{V}_{\phi D} \le 0$. Thus, the designed DO is stable.

Next is to take derivative of $V_\phi = \frac{1}{2}\zeta_\phi^2$ as follows:

$$\dot{V}_\phi = \zeta_\phi \dot{\zeta}_\phi \quad (23)$$

To obtain $\dot{\zeta}_\phi$, substituting (14) into (13)

$$\begin{aligned}\dot{\zeta}_\phi &= -\big(\delta_{1_\phi}F_{\phi_{11}} + \delta_{2_\phi}F_{\phi_{21}}\big)\big(\tilde{d}_\phi(t) + \Delta_\phi\big) - K_\phi\zeta_\phi - L_\phi sgn(\zeta_\phi)\\ &= -\eta_\phi\big(\tilde{d}_\phi(t) + \Delta_\phi\big) - K_\phi\zeta_\phi - L_\phi sgn(\zeta_\phi) \end{aligned}\quad (24)$$

where Δ_ϕ is an error. Now, substituting (24) into (23)

$$\begin{aligned}\dot{V}_\phi &= \zeta_\phi\big(-\eta_\phi\big(\tilde{d}_\phi(t) + \Delta_\phi\big) - K_\phi\zeta_\phi - L_\phi sgn(\zeta_\phi)\big)\\ &\le -K_\phi\zeta_\phi^2 + \big(\frac{1}{2}L_\phi^2\zeta_\phi^2 + \frac{1}{2}\big) + \big(\frac{1}{2}\zeta_\phi^2 + \frac{1}{2}\eta_\phi^2(\tilde{d}_\phi + \Delta_\phi)^2\big)\\ &\le -\big(K_\phi - \frac{1}{2}(L_\phi^2 + 1)\big)\zeta_\phi^2 + \frac{1}{2}\big(\eta_\phi^2(\tilde{d}_\phi + \Delta_\phi)^2 + 1\big)\\ &\le -(2K_\phi - (L_\phi^2 + 1))V_\phi + B_{3_\phi} \end{aligned}\quad (25)$$

where $B_{3_\phi} \ge \max\frac{1}{2}\{\eta_\phi^2(\tilde{d}_\phi + \Delta_\phi)^2 + 1\}$. Thus, with the appropriate choice of K_ϕ and L_ϕ, the asymptotic stability can be achieved. Now, taking derivative of (15) and substituting (22) and (25)

$$\begin{aligned}\dot{V}_r &= \dot{V}_\phi(t) + \dot{V}_{\phi D}\\ &\le -2\bar{\Gamma}_\phi V_{\phi D} - (2K_\phi - (L_\phi^2 + 1))V_\phi + 1/2\\ &\le -2\bar{\Gamma}_\phi V_{\phi D} - (2K_\phi - (L_\phi^2 + 1))V_\phi + B_\phi^* \end{aligned}\quad (26)$$

where $B_\phi^* = \frac{1}{2}\big(B_{1_\phi} + B_{2_\phi}\big) + B_{3_\phi}$. Now, by designing $\bar{\Gamma}_\phi \ge \Gamma_{0_\phi} - \Gamma_{1_\phi}^2 - \Gamma_{2_\phi}^2 - \frac{1}{2}$, $K_\phi \ge \frac{1}{2}(L_\phi^2 + 1)$ and with appropriate design of L_ϕ such that the states stay at the sliding manifold, the Lyapunov criteria for stability can be ensured. Thus, the developed DOBC is stable. This completes the proof.

B. DISTURBANCE OBSERVER BASED PITCH TRACKING CONTROL USING SMC

From (1), the pitch model can be written as follows:

$$\begin{aligned}\dot{x}_3 &= x_4 + F_{\theta_{11}} d_\theta \\ \dot{x}_4 &= x_2 x_6 k_{1_\theta} - x_2 \Omega_r k_{2_\theta} + F_{\theta_{21}} d_\theta + l_\theta U_\theta\end{aligned} \tag{27}$$

where $F_\theta = [F_{\theta_{11}} \;\; F_{\theta_{21}}]^T, f_\theta(x_\theta, U_\theta : t) = [x_4 \;\; x_2\, x_6\, k_{1_\theta} - x_2\Omega_r k_{2_\theta} + l_\theta U_\theta]^T$. Furthermore, we have $F_\theta^+ = [F_{\theta_{11}}^+ \;\; F_{\theta_{12}}^+]$.

1) DO FOR PARABOLIC DISTURBANCES (SECOND ORDER)

A second order disturbance can be assumed as $d_\theta(t) = d_{0_\theta} + d_{1_\theta} t + d_{2_\theta} t^2$ where d_{0_θ}, d_{1_θ} and d_{2_θ} are constants. Similar to the previous section, the DO can be constructed as follows:

$$\begin{cases} \dot{z}_\theta = F_\theta^+ f_\theta(x_\theta, U_\theta : t) + \Gamma_{0_\theta} g_{0_\theta}(t) + \Gamma_{1_\theta} g_{1_\theta}(t) \\ \qquad +\Gamma_{2_\theta} g_{2_\theta}(t)\,; \\ \hat{d}_\theta = \Gamma_{0_\theta} g_{0_\theta}(t) + \Gamma_{1_\theta} g_{1_\theta}(t) + \Gamma_{2_\theta} g_{2_\theta}(t) \end{cases} \tag{28}$$

where Γ_{0_θ}, Γ_{1_θ}, Γ_{2_θ} are chosen according to Lemma 1. Furthermore, we have $g_{0_\theta}(t) = F_{\theta_{11}}^+ x_3 + F_{\theta_{12}}^+ x_4 - z_\theta$, $g_{1_\theta}(t) = \Delta h/3\big(g_{0_\theta}(t_0) + \sum_{j=1,3,\ldots}^{m-1} 4g_{0_\theta}(t_j) + \sum_{j=2,4,\ldots}^{m-1} 2g_{0_\theta}(t_j) + g_{0_\theta}(t_m)\big)$ and $g_{2_\theta}(t) = \Delta h/3\big(g_{1_\theta}(t_0) + \sum_{j=1,3,\ldots}^{m-1} 4g_{1_\theta}(t_j) + \sum_{j=2,4,\ldots}^{m-1} 2\, g_{1_\theta}(t_j) + g_{1_\theta}(t_m)\big)$. Now, $\dot{z}_\theta$ and $\hat{d}_\theta$ can be obtained as follows:

$$\begin{cases} \dot{z}_\theta = \Gamma_{0_\theta} F_{\theta_{11}}^+ x_3 + \big(F_{\theta_{11}}^+ + \Gamma_{0_\theta} F_{\theta_{12}}^+\big) x_4 \\ \qquad + F_{\theta_{11}}^+ \big(x_2 x_6 k_{1_\theta} - x_2 \Omega_r k_{2_\theta} + l_\theta U_\theta\big) \\ \qquad - \Gamma_{0_\theta} z_\theta + \Gamma_{1_\theta} \frac{\Delta h}{3} \Bigg(g_{0_\theta}(t_0) \\ \qquad + \sum_{j=1,3,\ldots}^{m-1} 4g_{0_\theta}(t_j) + \sum_{j=2,4,\ldots}^{m-1} 2g_{0_\theta}(t_j) \\ \qquad + g_{0_\theta}(t_m) \Bigg) + \Gamma_{2_\theta} \frac{\Delta h}{3} \Bigg(g_{1_\theta}(t_0) \\ \qquad + \sum_{j=1,3,\ldots}^{m-1} 4g_{1_\theta}(t_j) + \sum_{j=2,4,\ldots}^{m-1} 2g_{1_\theta}(t_j) + g_{1_\theta}(t_m) \Bigg) \\ \hat{d}_\theta = \Gamma_{0_\theta} \big(F_{\theta_{11}}^+ x_3 + F_{\theta_{12}}^+ x_4 - z_\theta\big) \\ \qquad + \Gamma_{1_\theta} \frac{\Delta h}{3} \Bigg(g_{0_\theta}(t_0) + \sum_{j=1,3,\ldots}^{m-1} 4g_{0_\theta}(t_j) \\ \qquad + \sum_{j=2,4,\ldots}^{m-1} 2g_{0_\theta}(t_j) + g_{0_\theta}(t_m) \Bigg) \\ \qquad + \Gamma_{2_\theta} \frac{\Delta h}{3} \Bigg(g_{1_\theta}(t_0) + \sum_{j=1,3,\ldots}^{m-1} 4g_{1_\theta}(t_j) \\ \qquad + \sum_{j=2,4,\ldots}^{m-1} 2g_{1_\theta}(t_j) + g_{1_\theta}(t_m) \Bigg) \end{cases} \tag{29}$$

where $\hat{d}_\theta$ and z_θ represent disturbance estimation and auxiliary variable for high-order DO, respectively.

2) PITCH TRACKING CONTROL DESIGN

Define $\xi_{3_\theta} = x_3 - x_{\theta_D}$ and $\xi_{4_\theta} = x_4 - \dot{x}_{\theta_D}$ where x_{θ_D} is desired reference, an error based system is obtained as follows:

$$\begin{aligned}\dot{\xi}_{\theta_3} &= \xi_{4_\theta} + F_{\theta_{11}} d_\theta \\ \dot{\xi}_{4_\theta} &= x_2 x_6 k_{1_\theta} - x_2 \Omega_r k_{2_\theta} - \ddot{x}_{\theta_D} + F_{\theta_{21}} d_\theta + l_\theta U_\theta\end{aligned} \tag{30}$$

Next, sliding mode surface is defined as $\zeta_\theta = \delta_{1_\theta}\xi_{3_\theta} + \delta_{2_\theta}\xi_{4_\theta}$. Using SMC technique, control scheme is obtained as follows:

$$\begin{aligned} U_\theta = -\frac{1}{l_\theta \delta_{2_\theta}} \big[& \delta_{1_\theta}\xi_{4_\theta} + \delta_{2_\theta}\big(x_4 x_6 k_{1_\theta} - x_4 \Omega_r k_{2_\theta} - \ddot{x}_{\theta_D}\big) \\ & + \eta_\theta \hat{d}_\theta + K_\theta \zeta_\theta + L_\theta sgn(\zeta_\theta) \big] \end{aligned} \tag{31}$$

where $\hat{d}_\theta$ is disturbance estimation, $K_\theta > 0$, $L_\theta > 0$ is switching gain constant and $\eta_\theta = \big(\delta_{1_\theta} F_{\theta_{11}} + \delta_{2_\theta} F_{\theta_{21}}\big)$. Next step is to analyze stability of pitch subsystem. Hence, defining Lyapunov function for pitch as follows:

$$V_p = V_\theta(t) + V_{\theta_D} = \frac{1}{2}\zeta_\theta^2 + \frac{1}{2}\tilde{d}_\theta^2 \tag{32}$$

where $\tilde{d}_\theta = \hat{d}_\theta - d_\theta$. Now, following the same procedure of roll stability, it can be derived that

$$\dot{V}_p \le -2\bar{\Gamma}_\theta V_{\theta_D} - (2K_\theta - (L_\theta^2 + 1))V_\theta + B_\theta^* \tag{33}$$

where $\bar{\Gamma}_\theta$ is a constant, $B_\theta^* = \frac{1}{2}\big(B_{1_\theta} + B_{2_\theta}\big) + B_{3_\theta}$ with $B_{1_\theta} = \max\{g_{0_\theta}^2 + g_{1_\theta}^2\}$, $B_{2_\theta} = \max\{\kappa_{1_\theta}^2 + \kappa_{2_\theta}^2 + \varrho_\theta^2\}$ and $B_{3_\theta} \ge \max\frac{1}{2}\{\eta_\theta^2(\tilde{d}_\theta + \Delta_\theta)^2 + 1\}$ are representing bounds. ϱ_θ is a bound on rate of change of disturbance. κ_{1_θ} and κ_{2_θ} are bounds on approximation errors introduced because of using Simpson's technique for the derivation of $g_{1_\theta}(t)$ and $g_{2_\theta}(t)$, respectively. Δ_θ represents an error. Now, by designing the $\bar{\Gamma}_\theta \ge \big(\Gamma_{0_\theta} - \Gamma_{1_\theta}^2 - \Gamma_{2_\theta}^2 - \frac{1}{2}\big) > 0$, $K_\theta \ge \frac{1}{2}(L_\theta^2 + 1)$ and choosing appropriate gain for L_θ will ensure $\dot{V}_p \le 0$, i.e. the stability of DOBC developed for pitch model.

C. DISTURBANCE OBSERVER BASED YAW TRACKING CONTROL USING SMC

From (1), the yaw subsystem can be written as follows:

$$\begin{aligned}\dot{x}_5 &= x_6 + F_{\psi_{11}} d_\psi \\ \dot{x}_6 &= x_2 x_4 k_{1_\psi} + l_\psi U_\psi + F_{\psi_{21}} d_\psi\end{aligned} \tag{34}$$

where d_ψ is disturbances and we can write $F_\psi = [F_{\psi_{11}}, \;\; F_{\psi_{21}}]^T$.

1) DO FOR PARABOLIC DISTURBANCES (SECOND ORDER)

A second order model for disturbances affecting yaw subsystem can be assumed as $d_\psi(t) = d_{0_\psi} + d_{1_\psi} t + d_{2_\psi} t^2$ where d_{0_ψ}, d_{1_ψ} and d_{2_ψ} are constants. Next is to design a DO as follows:

$$\begin{cases} \dot{z}_\psi = F_\psi^+ f_\psi(x, u; t) + \Gamma_{0_\psi} g_{0_\psi}(t) + \Gamma_{1_\psi} g_{1_\psi}(t) + \Gamma_{2_\psi} g_{2_\psi}(t); \\ \hat{d}_\psi = \Gamma_{0_\psi} g_{0_\psi}(t) + \Gamma_{1_\psi} g_{1_\psi}(t) + \Gamma_{2_\psi} g_{2_\psi}(t) \end{cases} \tag{35}$$

where $F_{\psi}^{+} = [F_{\psi_{11}}^{+} \quad F_{\psi_{12}}^{+}]$, $\Gamma_{0_\psi}, \Gamma_{1_\psi}, \Gamma_{2_\psi}$ are chosen according to Lemma 2. Furthermore, we have $g_{0_\psi}(t) = F_{\psi_{11}}^{+}x_5 + F_{\psi_{12}}^{+}x_6 - z_\psi$, $g_{1_\psi}(t) = \Delta h/3\big(g_{0_\psi}(t_0) + \sum_{i=1,3,\ldots}^{n-1} 4g_{0_\psi}(t_i) + \sum_{i=2,4,\ldots}^{n-1} 2g_{0_\psi}(t_i) + g_{0_\psi}(t_n)\big)$ and $g_{2_\psi}(t) = \Delta h/3\big(g_{1_\psi}(t_0) + \sum_{i=1,3,\ldots}^{n-1} 4g_{1_\psi}(t_i) + \sum_{i=2,4,\ldots}^{n-1} 2g_{1_\psi}(t_i) + g_{1_\psi}(t_n)\big)$. Now, $\dot{z}_\psi$ and $\hat{d}_\psi$ can be obtained as follows:

$$\begin{cases} \dot{z}_\psi = \Gamma_{0_\psi}F_{\psi_{11}}^{+}x_5 + \big(F_{\psi_{11}}^{+} + \Gamma_{0_\psi}F_{\psi_{12}}^{+}\big)x_6 \\ \quad +F_{\psi_{12}}^{+}\big(x_2x_4k_{1_\psi} + l_\psi U_\psi\big) - \Gamma_{0_\psi}z_\psi \\ \quad +\Gamma_{1_\psi}\frac{\Delta h}{3}\Bigg(g_{0_\psi}(t_0) + \sum_{j=1,3,\ldots}^{m-1} 4g_{0_\psi}(t_j) \\ \quad + \sum_{j=2,4,\ldots}^{m-1} 2g_{0_\psi}(t_j) + g_{0_\psi}(t_m)\Bigg) \\ \quad +\Gamma_{2_\psi}\frac{\Delta h}{3}\Bigg(g_{1_\psi}(t_0) + \sum_{j=1,3,\ldots}^{m-1} 4g_{1_\psi}(t_j) \\ \quad + \sum_{j=2,4,\ldots}^{m-1} 2g_{1_\psi}(t_j) + g_{1_\psi}(t_m)\Bigg) \\ \hat{d}_\psi = \Gamma_{0_\psi}\big(F_{\psi_{11}}^{+}x_5 + F_{\psi_{12}}^{+}x_6 - z_\psi\big) \\ \quad +\Gamma_{1_\psi}\frac{\Delta h}{3}\Bigg(g_{0_\psi}(t_0) + \sum_{j=1,3,\ldots}^{m-1} 4g_{0_\psi}(t_j) \\ \quad + \sum_{j=2,4,\ldots}^{m-1} 2g_{0_\psi}(t_j) + g_{0_\psi}(t_m)\Bigg) \\ \quad +\Gamma_{2_\psi}\frac{\Delta h}{3}\Bigg(g_{1_\psi}(t_0) + \sum_{j=1,3,\ldots}^{m-1} 4g_{1_\psi}(t_j) \\ \quad + \sum_{j=2,4,\ldots}^{m-1} 2g_{1_\psi}(t_j) + g_{1_\psi}(t_m)\Bigg) \end{cases} \tag{36}$$

where $\hat{d}_\psi$ and z_ψ represent disturbance estimation and auxiliary variable for high-order DO, respectively.

D. YAW TRACKING CONTROL DESIGN

After developing a DO for disturbance estimation, it is required to construct a control scheme for tracking by following the similar method of previous sections. The subsystem for yaw model based on tracking error can be written as follows:

$$\begin{aligned} \dot{\xi}_{\psi_5} &= \xi_{\psi_6} + F_{\psi_{11}}d_\psi \\ \dot{\xi}_{\psi_6} &= x_2x_4k_{1_\psi} + l_\psi U_\psi + F_{\psi_{21}}d_\psi - \ddot{x}_{\psi_D} \end{aligned} \tag{37}$$

where x_{ψ_D} is desired angle. Now by designing a sliding mode surface as $\zeta_\psi = \delta_{1_\psi}\xi_{\psi_5} + \delta_{2_\psi}\xi_{\psi_8}$, an SMC control is developed as

$$\begin{aligned} U_\psi = -\frac{1}{l_\psi\delta_{2_\psi}}\big[&\delta_{1_\psi}\xi_{2_\psi} + \delta_{2_\psi}\big(x_2x_4k_{1_\psi} - \ddot{x}_{\psi_D}\big) \\ &+ \eta_\psi\hat{d}_\psi + K_\psi\zeta_\psi + L_\psi sgn(\zeta_\psi)\big] \end{aligned} \tag{38}$$

where $\hat{d}_\psi(t)$ is disturbance estimation, $K_\psi > 0$, $L_\psi > 0$ is switching constant and $\eta_\psi = \big(\delta_{1_\psi}F_{\psi_{11}} + \delta_{2_\psi}F_{\psi_{21}}\big)$.

Now, for the stability of yaw system, defining a Lyapunov candidate function is chosen as follows:

$$\begin{aligned} V_y &= V_\psi(t) + V_{\psi_D} \\ &= \frac{1}{2}\zeta_\psi^2 + \frac{1}{2}\tilde{d}_\psi^2 \end{aligned} \tag{39}$$

where $\tilde{d}_\psi = \hat{d}_\psi - d_\psi$. Now, taking derivative of (39) yields

$$\begin{aligned} \dot{V}_y &= \dot{V}_\psi + \dot{V}_{\psi_D} \\ &= \zeta_\psi\dot{\zeta}_\psi + \tilde{d}_\psi\dot{\tilde{d}}_\psi \end{aligned} \tag{40}$$

With similar derivation of roll and pitch, it can be derived that

$$\dot{V}_y \le -2\bar{\Gamma}_\psi V_{\psi_D} - (2K_\psi - (L_\psi^2 + 1))V_\psi + B_\psi^{*} \tag{41}$$

where $\bar{\Gamma}_\psi > 0$ is a constant. Furthermore, $B_\psi^{*} = \frac{1}{2}\big(B_{1_\psi} + B_{2_\psi}\big) + B_{3_\psi}$ with $B_{1_\psi} = \max\{g_{0_\psi}^2 + g_{1_\psi}^2\}$, $B_{2_\psi} = \max\{\kappa_{1_\psi}^2 + \kappa_{2_\psi}^2 + \varrho_\psi^2\}$ and $B_{3_\psi} \ge \max\frac{1}{2}\{\eta_\psi^2(\tilde{d}_\psi + \Delta_\psi)^2 + 1\}$ are representing bounds. ϱ_ψ and Δ_ψ denotes the upper bound on rate of change of disturbances and an error, respectively. κ_{1_ψ} and κ_{2_ψ} are bounds on the approximation errors e_{1_ψ} and e_{2_ψ}, respectively. These errors are incurred because of the Simpson's approximation technique used for the derivation of $g_{1_\psi}(t)$ and $g_{2_\psi}(t)$. Now, with appropriate choice of $\bar{\Gamma}_\psi \ge \big(\Gamma_{0_\psi} - \Gamma_{1_\psi}^2 - \Gamma_{2_\psi}^2 - \frac{1}{2}\big)$, $K_\psi \ge \frac{1}{2}(L_\psi^2 + 1)$ and L_ψ, $\dot{V}_y \le 0$ can be ensured.

E. DISTURBANCE OBSERVER BASED ALTITUDE CONTROL

From (1), altitude model can be written as follows:

$$\begin{aligned} \dot{x}_7 &= x_8 + F_{h_{11}}d_h \\ \dot{x}_8 &= g - \frac{U_h}{m}(\cos x_1 \cos x_3) + F_{h_{21}}d_h \end{aligned} \tag{42}$$

where d_h is representing disturbances, $F_{h_{11}}$ and $F_{h_{12}}$ are the elements of the matrix F_h associated with high-order disturbances affecting altitude subsystem.

1) DO FOR PARABOLIC DISTURBANCES (SECOND ORDER)

A second order model for disturbances is assumed as $d_h = d_{0_h} + d_{h_1}t + d_{h_2}t^2$, where d_{0_h}, d_{1_h} and d_{2_h} are unknown constants. Now, a DO can be developed as follows:

$$\begin{cases} \dot{z}_h = F_h^{+}f_h(x_h, U_h; t) + \Gamma_{0_h}g_{0_h}(t) \\ \quad +\Gamma_{1_h}g_{1_h}(t) + \Gamma_{2_h}g_{2_h}(t); \\ \hat{d}_h = \Gamma_{0_h}g_{0_h}(t) + \Gamma_{1_h}g_{1_h}(t) + \Gamma_{2_h}g_{2_h}(t) \end{cases} \tag{43}$$

where $F_h^{+} = [F_{h_{11}}^{+} \quad F_{h_{12}}^{+}]$. Γ_{0_h}, Γ_{1_h} and Γ_{2_h} are chosen by using Lemma 2. Furthermore, we have $g_{0_h}(t) = F_{h_{11}}^{+}x_7 + F_{h_{12}}^{+}x_8 - z_h$, $g_{1_h}(t) = \Delta h/3\big(g_{0_h}(t_0) + \sum_{i=1,3,\ldots}^{n-1} 4g_{0_h}(t_i) + \sum_{j=2,4,\ldots}^{m-1} 2g_{0_h}(t_j) + g_{0_h}(t_m)\big)$ and $g_{2_h}(t) = \Delta h/3\big(g_{1_h}(t_0) + \sum_{j=1,3,\ldots}^{m-1} 4g_{1_h}(t_j) + \sum_{j=2,4,\ldots}^{m-1} 2g_{1_h}(t_j) + g_{1_h}(t_m)\big)$. Now, $\dot{z}_h$ and $\hat{d}_h$

can be obtained as follows:

$$\begin{cases} \dot{z}_h = \Gamma_{0_h} F^{+}_{h_{11}} x_7 + \left(F^{+}_{h_{11}} + \Gamma_{0_h} F^{+}_{h_{12}} x_8\right) \\ \quad +F^{+}_{h_{11}}\left(g - \frac{U_h}{m}\left(\cos x_1 \cos x_3\right)\right) \\ \quad -\Gamma_{0_h} z_h + \Gamma_{1_h} \frac{\Delta h}{3}\Bigg(g_{0_h}(t_0) \\ \quad + \sum\limits_{j=1,3,\ldots}^{m-1} 4g_{0_h}(t_j) + \sum\limits_{j=2,4,\ldots}^{m-1} 2g_{0_h}(t_j) \\ \quad +g_{0_h}(t_m)\Bigg) + \Gamma_{2_h}\frac{\Delta h}{3}\Bigg(g_{1_h}(t_0) \\ \quad + \sum\limits_{j=1,3,\ldots}^{m-1} 4g_{1_h}(t_j) + \sum\limits_{j=2,4,\ldots}^{m-1} 2g_{1_h}(t_j) + g_{1_h}(t_m)\Bigg) \\ \hat{d}_h = \Gamma_{0_h}\left(F^{+}_{h_{11}} x_7 + F^{+}_{h_{12}} x_8 - z_h\right) \\ \quad +\Gamma_{1_h}\frac{\Delta h}{3}\Bigg(g_{0_h}(t_0) + \sum\limits_{j=1,3,\ldots}^{m-1} 4g_{0_h}(t_j) \\ \quad + \sum\limits_{j=2,4,\ldots}^{m-1} 2g_{0_h}(t_j) + g_{0_h}(t_m)\Bigg) \\ \quad +\Gamma_{2_h}\frac{\Delta h}{3}\Bigg(g_{1_h}(t_0) + \sum\limits_{j=1,3,\ldots}^{m-1} 4g_{1_h}(t_j)\Bigg) \\ \quad + \sum\limits_{j=2,4,\ldots}^{m-1} 2g_{1_h}(t_j) + g_{1_h}(t_m)\Bigg) \end{cases} \tag{44}$$

where $\hat{d}_h$ and z_h represent disturbance estimation and auxiliary variable for high-order DO, respectively.

2) ALTITUDE TRACKING CONTROL DESIGN

To obtain a tracking control, similar procedure of previous sections is followed. Firstly, a second-order nonlinear model of altitude system is derived based on error as follows:

$$\begin{aligned} \dot{\xi}_{h7} &= \xi_{h8} + F_{h_{11}} d_h \\ \dot{\xi}_{h8} &= g - \frac{U_h}{m}\left(\cos x_1 \cos x_3\right) - \ddot{x}_{hD} + F_{h_{21}} d_h \end{aligned} \tag{45}$$

where x_{hD} is desired height. To start with, a sliding mode surface is assumed as $\zeta_h = \delta_{1_h}\xi_{h7} + \delta_{2_h}\xi_{h8}$ where δ_{1_h} and δ_{2_h} are sliding mode control parameters. Now, following the steps of SMC controller, following control scheme is obtained for tracking of altitude.

$$\begin{aligned} U_h = \frac{m}{\delta_{2_h}\cos x_1 \cos x_3}\Big[&\delta_{1_h}\xi_{h8} + \delta_{2_h}\left(g - \ddot{x}_{hD}\right) \\ &+ \eta_h \hat{d}_h + K_h \zeta_h + L_h sgn(\zeta_h)\Big] \end{aligned} \tag{46}$$

where $\hat{d}_h$ is disturbance estimation, $K_h > 0$, $L_h > 0$ is the switching gain constant and $\eta_h = \left(\delta_{1_h}F_{h_{11}} + \delta_{2_h}F_{h_{21}}\right)$. For obtaining the stability criteria of altitude model, let us define a Lyapunov candidate as follows:

$$V_h = V_h(t) + V_{h_D} = \frac{1}{2}\zeta_h^2 + \frac{1}{2}\tilde{d}_h^2 \tag{47}$$

where $\tilde{d}_h = \hat{d}_h - d_h$. Following similar procedure of proving stability for attitude model, it can be derived that

$$\dot{V}_h \le -2\bar{\Gamma}_h V_{h_D} - (2K_h - (L_h^2 + 1))V_h + B_h^* \tag{48}$$

where $\bar{\Gamma}_h$ is a positive constant, $B_h^* = \frac{1}{2}\left(B_{1_h} + B_{2_h}\right) + B_{h_3}$ with $B_{1_h} = \max\{g_{0_h}^2 + g_{1_h}^2\}$, $B_{2_h} = \max\{\kappa_{1_h}^2 + \kappa_{2_h}^2 + \varrho_h^2\}$ and $B_{h_3} \ge \max\frac{1}{2}\{\eta_h^2(\tilde{d}_h + \Delta_h)^2 + 1\}$ are upper bounds. ϱ_h and Δ_h represents the upper bound on rate of change of disturbances and an error, respectively. κ_{1_h} and κ_{2_h} denotes the bounds on the approximation errors introduced while using Simpson's approximation technique for the derivation of $g_{1_h}(t)$ and $g_{2_h}(t)$, respectively. Now, if the design constants $\bar{\Gamma}_h \ge \left(\Gamma_{0_h} - \Gamma_{1_h}^2 - \Gamma_{2_h}^2 - \frac{1}{2}\right)$ and $K_h \ge \frac{1}{2}(L_h^2 + 1)$ are chosen along with the appropriate design of L_h, then $\dot{V}_y \le 0$.

It should be noted that $K_i\zeta_i$ where $i \in (\phi, \theta, \psi, h)$, is introduced in the controller for minimizing the chattering effect [48]. Since, $K_i\zeta_i$ and $L_i sgn(\zeta_i)$ appear in same channel, hence, by appropriately design K_i can yield domination over discontinuous function, resulting in reducing chattering effect. Now, all of the above analysis and design can be summarized as the following theorem.

Theorem 1: For a model of quadrotor suffering from unknown but bounded high-order disturbances, a disturbance observer based tracking control (DOBTC) scheme can be constructed by incorporating a DO proposed in (10), (29), (36) and (44), controller developed using SMC for roll, pitch, yaw and altitude in (14), (31), (38) and (46), respectively. The developed DOBTC yields asymptotic stability when observer gain parameters are designed according to Lemma 2 and following criteria are ensured.

$$\bar{\Gamma}_\tau \ge \left(\Gamma_{0_\tau} - \Gamma_{1_\tau}^2 - \Gamma_{2_\tau}^2 - \frac{1}{2}\right), K_\tau > \frac{1}{2}(L_\tau^2 + 1) \tag{49}$$

where $B_\tau^* = \frac{1}{2}\left(B_{1_\tau} + B_{2_\tau}\right) + B_{3_\tau}$ with $B_{1_\tau} = \max\{g_{0_\tau}^2 + g_{1_\tau}^2\}$, $B_{2_\tau} = \max\{\kappa_{1_\tau}^2 + \kappa_{2_\tau}^2 + \varrho_\tau^2\}$ and $B_{3_\tau} \ge \max\frac{1}{2}\{\eta_\tau^2(\tilde{d}_\tau + \Delta_\tau)^2 + 1\}$ representing the upper bounds. ϱ_τ and Δ_τ denotes the upper bound on rate of change of disturbances and an error, respectively. κ_{1_τ} and κ_{2_τ} are the upper bounds on approximation errors occurred because of the use of Simpson's technique for the derivation of $g_{1_\tau}(t)$ and $g_{2_\tau}(t)$, respectively. Furthermore, it is required to design L_τ sufficiently large to satisfy sliding mode criterion, and to make sure the states does not escape the sliding manifold.

Proof: The stability of the proposed control scheme can be analyzed using the Lapunov stability criterion. According to the stability criterion, the origin yields asymptotic stability given that the Lyapunov candidate V is positive definite and its time derivative $\dot{V}$ is negative definite. Since, the developed control scheme incorporates tracking control and DO, hence, to analyze the stability of quadrotor, the Lyapunov candidate can be defined as follows:

$$V_Q = V_r + V_p + V_y + V_h \tag{50}$$

where V_r, V_p, V_y and V_h represent Lyapunov functions for roll, pitch, yaw and height, respectively. Now, taking time derivative

$$\dot{V}_Q = \dot{V}_r + \dot{V}_p + \dot{V}_y + \dot{V}_h \tag{51}$$

Substituting (26), (33), (41) and (48)

$$\begin{aligned} \dot{V}_Q \le &-2\bar{\Gamma}_\phi V_{\phi_D} - (2K_\phi - (L_\phi^2 + 1))V_\phi + B_\phi^* \\ &- 2\bar{\Gamma}_\theta V_{\theta_D} - (2K_\theta - (L_\theta^2 + 1))V_\theta + B_\theta^* \end{aligned}$$

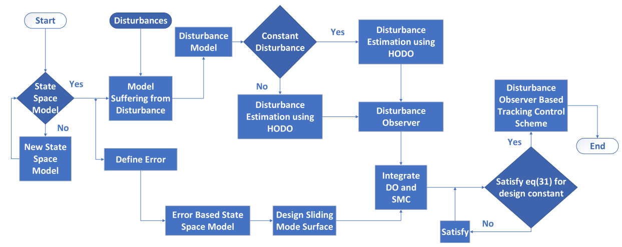

FIGURE 5. Flow chart of disturbance observer based tracking control.

TABLE 1. Quadrotor parameters.

Parameter	symbol	Value
Airframe Inertia of Roll	I_{xx}	4.856×10^{-3} kgm^2
Airframe Inertia of Pitch	I_{yy}	4.856×10^{-3} kgm^2
Airframe Inertia of Yaw	I_{zz}	8.801×10^{-3} kgm^2
Drag force coefficient in Roll direction	A_x	0.25 kg/s
Drag force coefficient in Pitch direction	A_y	0.25 kg/s
Drag force coefficient in Yaw direction	A_z	0.25 kg/s

$$-2\bar{\Gamma}_\psi V_{\psi_D} - (2K_\psi - (L_\psi^2 + 1))V_\psi + B_\psi^*$$
$$-2\bar{\Gamma}_h V_{h_D} - (2K_h - (L_h^2 + 1))V_h + B_h^* \qquad (52)$$

To obtain $\dot{V}_Q < 0$, the Lyapunov stability for attitude and altitude model of quadrotor, the DO gain parameters are required to be design according to the Lemma 2, i.e. $\rho_i(s) = 0$ resulting in convergent dynamics [29]. And also with $\tau \in (\phi, \theta, \psi, h)$,

$$\bar{\Gamma}_\tau \geq \left(\Gamma_{0_\tau} - \Gamma_{1_\tau}^2 - \Gamma_{2_\tau}^2 - \frac{1}{2}\right), \quad K_\tau > \frac{1}{2}(L_\tau^2 + 1) \quad (53)$$

This completes the proof. A complete flow diagram to obtain tracking scheme of a quadrotor with the ability of disturbance rejection using proposed DOBTC is shown in Fig. 5.

IV. SIMULATION RESULTS

For simulation purpose, the parameters of quadrotor are taken as, gravity $g = 9.91$m/s^2, mass $m = 0.468$kg. distance $l = 0.225$m, thrust coefficient $k = 2.980 \times 10^{-6}$, drag coefficient $b = 1.140 \times 10^{-7}$, rotor inertia $I_M = 3.357 \times 10^{-5}$ kgm^2. The airframe inertia and drag coefficients for roll, pitch and yaw are shown in the table 1. For Simpson's approximation, $n = 1000$ during simulations of all subsystems. Furthermore, the matrix F associated with disturbances and its Moore-Penrose pseudo inverse commuted using mathematical program are as follows:

$$F = \begin{bmatrix} 4 \\ 6 \end{bmatrix} \qquad F^+ = \begin{bmatrix} 1.6000 & 0.8000 \end{bmatrix} \qquad (54)$$

A. SIMULATION RESULTS OF ROLL MODEL

The design constants of the developed control scheme and DO for roll angle are shown in the following table 2. It should be noted that the DO is constructed for second-order parabolic disturbances. However, the simulations are conducted for constant, ramp and chirp disturbances also.

TABLE 2. Design constants for tracking control of roll with DO.

Disturbances	L_ϕ (sgn)	L_ϕ (sat)	δ_{1_ϕ}	δ_{2_ϕ}	Γ_{0_ϕ}	Γ_{1_ϕ}	Γ_{2_ϕ}
Constant	5	50	0.9	0.01	200	—	—
Ramp	100	250	50	0.1	500	10	—
Parabolic	12	50	5	0.1	600	20	10
Chirp	100	50	5	0.1	600	20	15

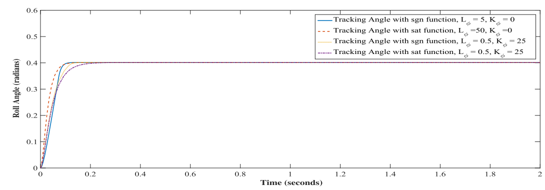

FIGURE 6. Tracking roll angle.

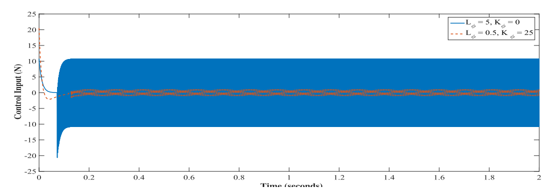

FIGURE 7. Control input using *sgn* function.

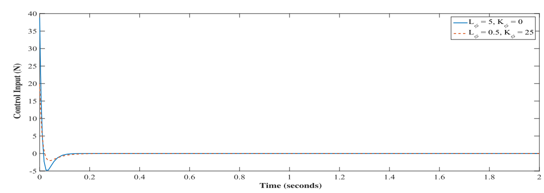

FIGURE 8. Control input using *sat* function.

For constant and ramp disturbances, $d_1 = d_2 = 0$ and $d_2 = 0$ are taken, respectively. While for chirp disturbances, the DO of parabolic disturbance is used but with different tuned gain constants to achieve the estimation. Furthermore, the reference desired angle for roll tracking is assumed to be $x_{\phi D} = 0.4$ *rad*. Since SMC technique is used for achieving tracking performance, DOBTC can be constructed either by using $sgn(.)$ or $sat(.)$ functions. The former function exhibits chattering in the controller while the latter negates it. Furthermore, the developed control scheme contains two control design parameters, i.e. L_ϕ and K_ϕ. Hence, in Fig. 6, various results are presented for tracking performance followed by the control inputs required in Fig. 7 and Fig. 8, respectively. A significant reduction in chattering can be noticed in control input by introducing the design gain constant of $K_\phi = 25$, as shown in Fig. 7. Furthermore, Fig. 8 shows comparison of control inputs for different values of control gain parameters. Hence, from the simulation study it can be said that with appropriate choice of L_ϕ and K_ϕ, chattering can be reduced.

The simulation results for proposed DO to estimate constant, ramp, parabolic and chirp disturbances are presented in Fig. 9, Fig. 10, Fig. 11 and Fig. 12, respectively.

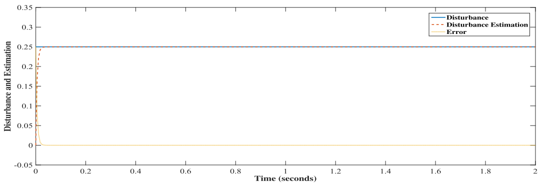

FIGURE 9. Constance disturbance and disturbance estimation.

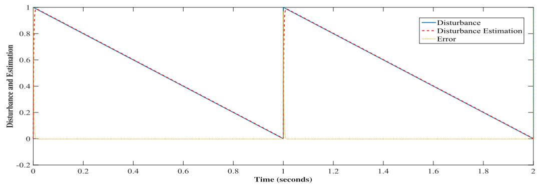

FIGURE 10. Ramp disturbance and disturbance estimation.

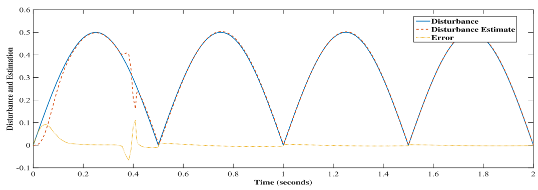

FIGURE 11. Parabolic disturbance and disturbance estimation.

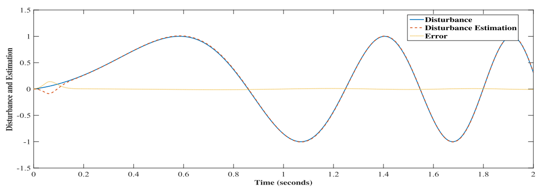

FIGURE 12. Chirp disturbance and disturbance estimation.

TABLE 3. Design constants for tracking control of pitch with DO.

Disturbance	L_θ (sgn)	L_θ (sat)	δ_{1_θ}	δ_{2_θ}	Γ_{0_θ}	Γ_{1_θ}	Γ_{2_θ}
Constant	50	15	10	0.1	500	–	–
Ramp	100	125	30	0.1	700	15	–
Parabolic	15	20	3	0.01	1200	25	20
Chirp	100	50	5	0.1	1200	25	25

B. SIMULATION RESULTS OF PITCH MODEL

The design parameter constants of control and DO for the decoupled model of pitch subsystem of attitude are enlisted in table 3. For pitch, the desired tracking angle is taken as $x_{\theta_D} = 0.5$ *rad*. In Fig. 13, tracking performance of pitch is presented for different values of L_θ and K_θ. Fig. 14 and Fig. 15 shows the control inputs with different designed control parameters by using signum and saturation functions, respectively.

C. SIMULATION RESULTS OF YAW MODEL

The design control parameters for the decoupled model of yaw are presented in table 4. For simulation purpose, the desired yaw angle is considered to be $x_{\psi_D} = 0.3 rad$.

TABLE 4. Design constants for tracking control of yaw with DO.

Disturbances	L_ψ (sgn)	L_ψ (sat)	δ_{1_ψ}	δ_{2_ψ}	Γ_{0_ψ}	Γ_{1_ψ}	Γ_{2_ψ}
Constant	50	15	10	0.1	500	–	–
Ramp	80	100	30	0.1	350	12	–
Parabolic	15	20	3	0.01	850	20	18
Chirp	50	30	5	0.01	850	20	20

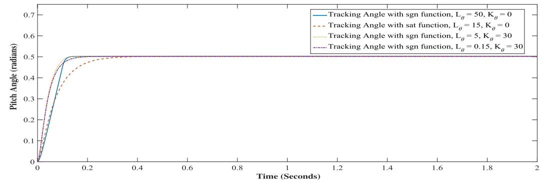

FIGURE 13. Tracking pitch angle.

TABLE 5. Design constants for tracking control of altitude with DO.

Disturbances	L_h (sgn)	L_h (sat)	δ_{1_h}	δ_{2_h}	Γ_{0_h}	Γ_{1_h}	Γ_{2_h}
Constant	45	20	1	0.1	10	–	–
Ramp	100	130	2	0.1	275	15	–
Parabolic	40	55	10	0.1	400	10	15
Chirp	75	30	50	0.01	350	10	15

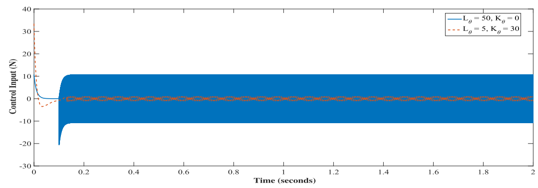

FIGURE 14. Control input using *sgn* function.

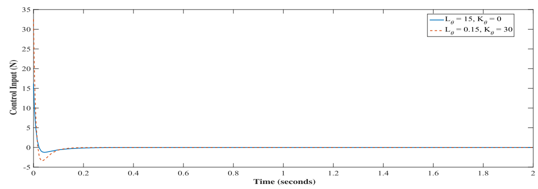

FIGURE 15. Control input using *sat* function.

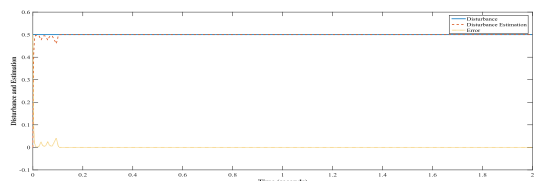

FIGURE 16. Constant disturbance and disturbance estimation.

Fig. 20 shows the tracking performance for yaw model for various L_ψ and K_ψ. The control inputs by using signum and saturation functions are shown in Fig. 21 and Fig. 22, respectively. It is evident from Fig. 21 that the chattering is reduced after invoking K_ψ in the control technique. After in-depth simulation analysis, in Fig. 23, Fig. 24, Fig. 25 and Fig. 26,

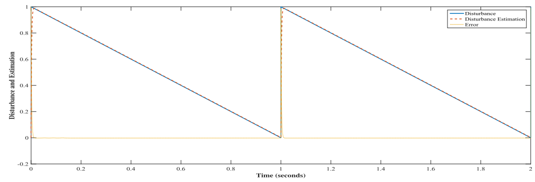

FIGURE 17. Ramp disturbance and disturbance estimation.

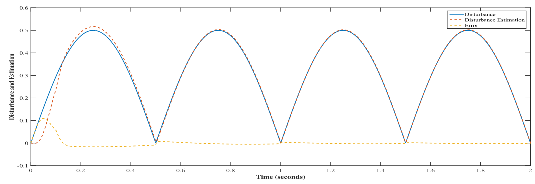

FIGURE 18. Parabolic disturbance and disturbance estimation.

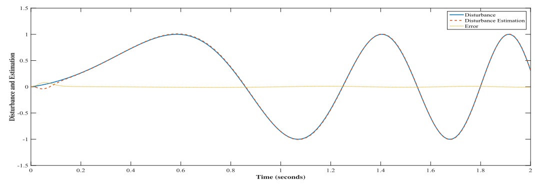

FIGURE 19. Chirp disturbance and disturbance estimation.

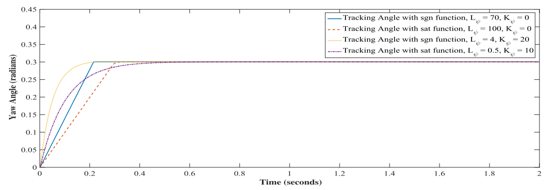

FIGURE 20. Tracking yaw angle.

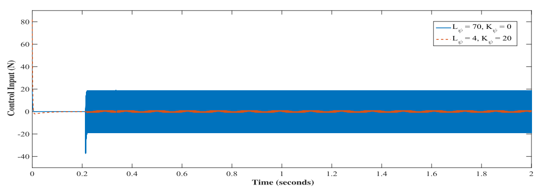

FIGURE 21. Control input using *sgn* function.

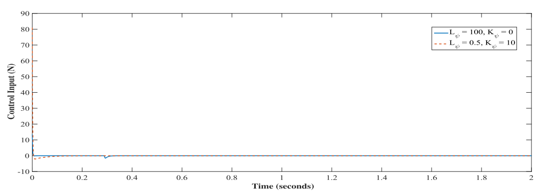

FIGURE 22. Control input using *sat* function.

are presented for DO estimating the disturbances of constant, ramp, parabolic and chirp models, respectively.

D. SIMULATION RESULTS OF ALTITUDE/HEIGHT MODEL

For altitude/height subsystem of the quadrotor, the desired height considered is $25m$. From Fig. 27, it can be seen that

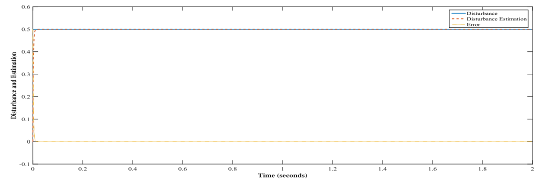

FIGURE 23. Constant disturbance and disturbance estimation.

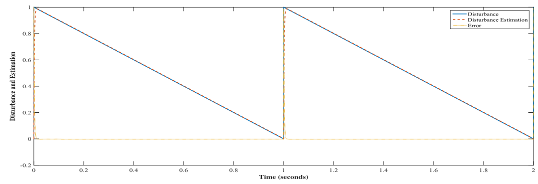

FIGURE 24. Ramp disturbance and disturbance estimation.

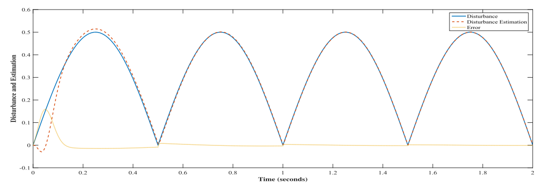

FIGURE 25. Parabolic disturbance and disturbance estimation.

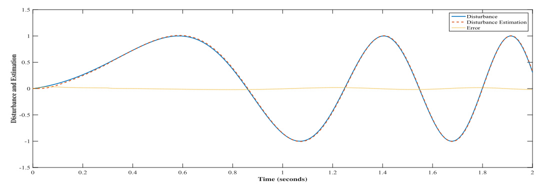

FIGURE 26. Chirp disturbance and disturbance estimation.

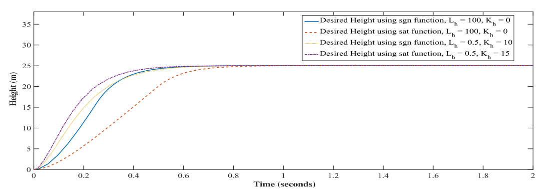

FIGURE 27. Desired height.

the desired height is achieved in the presence of disturbances. Furthermore, it is shown that the desired height is achieved for various design gain parameters, i.e. L_h and K_h. The control inputs required to achieve the desired height are shown in Fig. 28 and Fig. 29. The former control input is obtained when *sgn* function is used in the control scheme while the later is obtained when the *sgn* function is replaced by *sat* function in DOBTC scheme. It can be seen that by introducing K_h, the chattering can be reduced in the control input. The performance of DO can be seen in Fig. 30, Fig. 31, Fig. 32 and Fig. 33 where the disturbance estimation of constant, ramp, parabolic and chirp function is shown, respectively.

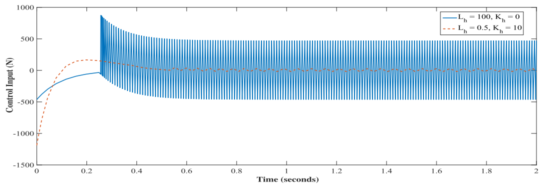

FIGURE 28. Control input using sgn function.

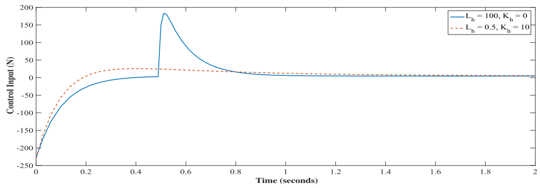

FIGURE 29. Control input using sat function.

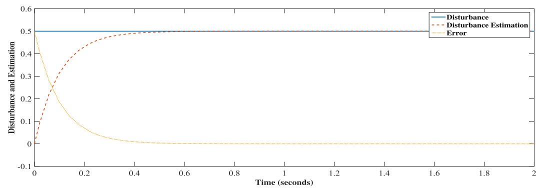

FIGURE 30. Constant disturbance and disturbance estimation.

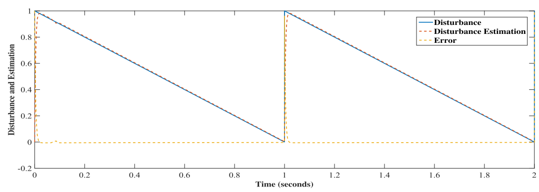

FIGURE 31. Ramp disturbance and disturbance estimation.

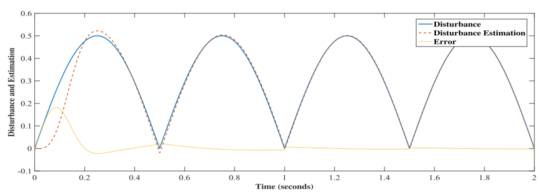

FIGURE 32. Parabolic disturbance and disturbance estimation.

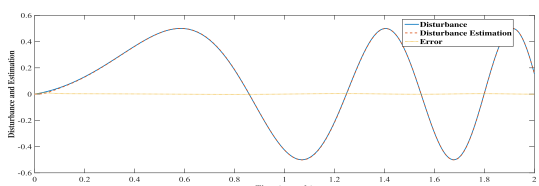

FIGURE 33. Chirp disturbance and disturbance estimation.

The tracking performance can be varied by designing different the switching gain constants regardless of using signum or saturation function used in switching function during the developed DOBTC based on SMC control. Designing switching gain higher yields quick performance of the designed control and the tracking performance is obtained quickly while with the small design value of switching gain results in slower tracking performance. Furthermore, the performance can also be improved by introducing K_τ in the controller. In fig. 6, fig. 13 and fig. 20, the tracking performance of roll, pitch and yaw are presented, respectively. Fig. 27, represents the height achieved using DOBTC. These tracking performances can be varied by adjusting L_τ and K_τ according to the requirements. With the introduction of the latter design gain constant, the chattering can also be reduced in the control input. Moreover, it should be noted down that the attitude model of a quadrotor is independent of altitude model. However, the altitude model is dependent on from attitude model. Hence, with the control of attitude, the altitude tracking performance can be varied.

V. CONCLUSION

In this paper, a DOBTC technique with the ability of tracking and disturbance estimation is presented. The developed DO is an integrator-free DO to avoid the unnecessary saturation and adverse effects on the controller. When patched with the controller, it attenuates both matched and mismatched high-order disturbances as well as tracks the desired references. After conducting the simulation study, a conclusion can be drawn that the DO can estimate disturbances with both constant and variable frequencies. Furthermore, the developed SMC technique achieves tracking performance of the UAV, quadrotor. Hence, with the support of asymptotic stability obtained using Lyapunov criteria, it can be stated that the control scheme is effective and stable.

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