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Improving the Reliability of Underwater Gravity Matching Navigation Based on a Priori Recursive Iterative Least Squares Mismatching Correction Method

ZHAOWEI LI¹, WEI ZHENG^{1,2,3,4,5}, AND FAN WU¹

¹Qian Xuesen Laboratory of Space Technology, China Academy of Space Technology, Beijing 100094, China

²School of Geomatics, Liaoning Technical University, Fuxin 123000, China

³School of Surveying and Landing Information Engineering, Henan Polytechnic University, Jiaozuo 454000, China

⁴School of Geomatics and Marine Information, Jiangsu Ocean University, Lianyungang 222005, China

⁵State Key Laboratory of Geodesy and Earth's Dynamics, Institute of Geodesy and Geophysics, Chinese Academy of Sciences, Wuhan 430077, China

Corresponding author: Wei Zheng (zhengwei1@qxslab.cn)

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ABSTRACT This study focuses on the reliability of underwater gravity matching navigation. Firstly, the research started with the post-processing of mismatching. Under the guidance of priori recursive multiple matching and iterative least squares, a new priori recursive iterative least squares mismatching correction (PRILSMC) method was proposed based on statistics and the fitting principle. Secondly, according to factors such as matching algorithm probability, error of INS, and the actual situation, we comprehensively considered the effects of recursive sampling points, priori matching points, etc. The new mismatching judgment and dynamic correction (MJDC) model was constructed based on the PRILSMC method. Finally, under the same conditions, the MJDC model was used to verify a new matching point in three matching regions. The results showed that in the excellent-suitability region, the matching probability increased from about 96% to 100%, i.e., all mismatching points could essentially be eliminated. In the general-suitability region, the matching probability increased from about 64% to 92%, indicating that the probability of mismatching point was greatly reduced, and the reliability of the matching navigation was improved.

INDEX TERMS PRILSMC method, MJDC model, reliability, underwater gravity matching navigation, gravity suitability.

I. INTRODUCTION

Marine gravity matching navigation is an external aiding technique that aims to correct the accumulative errors of inertial navigation system (INS) for underwater vehicles. It does not radiate energy to the outside and does not require the underwater vehicles to approach the water surface to receive signals. It is a real passive navigation technology that is not limited by location, weather, and other external conditions, which is helpful to realize autonomous, continuous, accurate, and long-endurance navigation for underwater vehicles [1]–[8]. The basic principles of gravity matching navigation are as follows: the information of the surrounding gravity

field is collected by using the marine gravimeter/gravity gradiometer when the carrier passes through a region with abundant gravity characteristics; the information is compared to a gravity reference map pre-stored in the navigation systems; and the best position estimation is determined according to several minimization criteria.

The key factors affecting underwater gravity matching navigation are not only the gravity matching algorithms [9]–[11], the high-precision and high-resolution global marine gravity anomaly reference map [12], [13], the high-precision gravity measurement systems [14], and the optimized gravity suitability regions [15]–[17], but also the fact that mismatching has a negative impact on the underwater gravity matching. In the actual situation, the mismatching is inevitable in the gravity matching process based on the gross error theory.

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Therefore, the various algorithms cannot be completely matched. For the underwater vehicles, the carrier itself cannot judge the correctness of the matching position. Furthermore, due to the computational complexity of underwater feature information and the similarity of trajectory distribution in the search area of the gravity field, the matching process may be mismatched, that is, the matching point may be far away from the correct position. The mismatching point not only cannot correct the accumulative error of INS, but also can affect the navigation security and the strike accuracy for underwater vehicles. Therefore, determining how to improve the matching reliability is an important research hotspot in the field of underwater navigation. At present, many scholars focus on the improved positioning accuracy and the optimized gravity suitability regions in order to directly reduce the mismatching probability, and thus improve matching reliability. Zhao et al. introduced the matching criterion based on Hausdorff distance into the terrain contour matching (TERCOM) algorithm, and they proposed a new idea by increasing the rotation change and determining the optimal rotation angle, which effectively improved the accuracy of underwater matching navigation [18]. Zhang et al. studied the pre-translation simplification of the iterative closest contour point (ICCP) algorithm. The results showed that mismatching was effectively reduced, and the positioning accuracy and reliability were improved by the optimization algorithm [19]. Cai and Chen proposed a selection criterion of a gravity matching region based on the analytic hierarchy process. Navigation was carried out by optimizing the suitability region with abundant gravity characteristics, and the positioning accuracy and reliability were directly improved [20]. Zhang and Wang proposed an online mismatching criterion based on the joint probability of multi-reference points in a correlation plane. They established the mean square difference (MSD) probability distribution density function of the point matched by analyzing the statistical distribution of elevation measurement noise and judged the minimum value point in the MSD correlation plane using the threshold [21].

Different from the previous studies, for the post-processing of mismatching, under the guidance of priori recursive multiple matching and iterative least squares, a new priori recursive iterative least squares mismatching correction (PRILSMC) method is proposed based on statistics and the fitting principle. After that, the mismatching judgment and dynamic correction (MJDC) model is constructed using the functional relationship that was obtained by iteratively fitting a series of priori matching points based on the PRILSMC method. The purpose is to judge whether a new matching point is a mismatching point, and if so, to correct it. The rationality of the PRILSMC method is verified by the TERCOM simulation algorithm.

II. CALCULATION PRINCIPLE OF THE PRILSMC METHOD

In this paper, the PRILSMC method is proposed based on the idea of priori recursive multiple matching of sequence of sampling points and iterative least squares, in order to construct

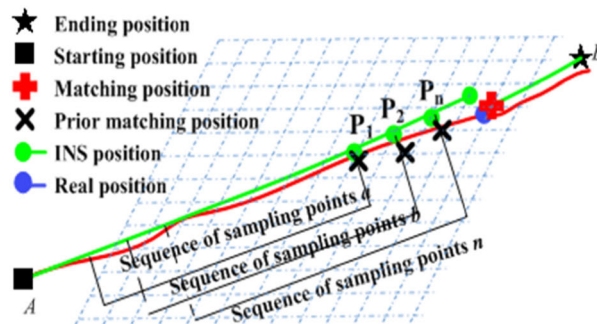


FIGURE 1. Schematic of underwater gravity matching navigation based on the PRILSMC method.

the MJDC model to judge and correct a new matching point. The calculation principle of underwater gravity matching navigation based on the PRILSMC method is shown in Figure 1.

The algorithm flowchart of the PRILSMC method is shown in Figure 2.

Step 1: We calculated the position coordinates of priori matching points. The navigation task covers from position A to position B. Furthermore, each sequence of sampling points has the same length (sequence of sampling points a, sequence of sampling points b, ..., sequence of sampling points n). After entering the matching area, when the sequence of sampling points a is long enough, the first priori matching is performed (only the position coordinates of priori matching point is calculated, but the error of INS is not modified), and the first priori matching point $P_1(x_1, y_1)$ is obtained. Then, we recursively process the sequence of sampling points a, remove N sampling points at the back end (N is the number of recursive sampling points), and add N sampling points at the front end to form a new sequence of sampling points b. Thus, the second priori matching point $P_2(x_2, y_2)$ is obtained by the second priori matching. Similarly, we can get several priori matching points $P_3(x_3, y_3), \dots, P_{n-1}(x_{n-1}, y_{n-1}), P_n(x_n, y_n)$.

Step 2: We constructed the MJDC model. The least squares method is used to find the best fitted function model for a set of data by minimizing the sum of squared errors. We usually calculate the error from each priori matching point to the fitted function model. If the error of some matching point is larger than the threshold, this priori matching point is considered as a priori mismatching point. The advantages of the least squares method are that it is simple and fast. Its shortcoming is that the error of fitted function model is large when some matching point has a large deviation. Therefore, in order to overcome the negative influence of large error matching points on the fitted function model, this paper introduces the idea of iteration, and the MJDC model is constructed by the iterative least squares principle.

This paper introduced the idea of priori recursive matching with N sampling points as a unit. Due to the low maneuverability of underwater vehicles, the shape of navigation

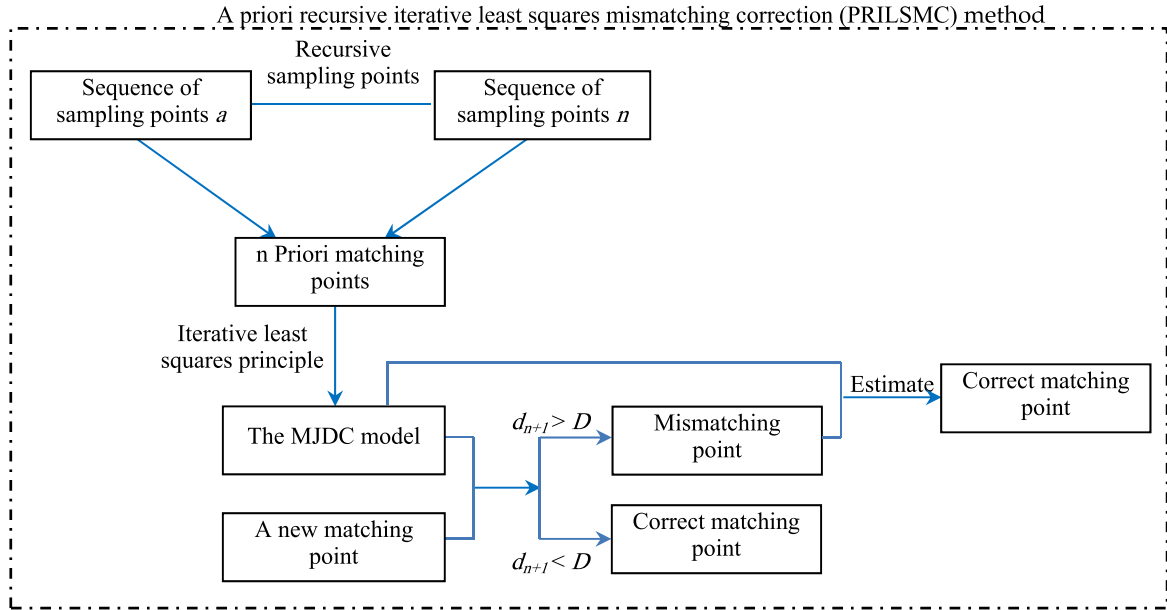


FIGURE 2. Algorithm flowchart of the PRILSMC method.

trajectory is smoother for a short time. Therefore, we assumed that the trajectory is approximately linearly distributed within a short time in the matching area. That is, the curve fitted using all priori matching points $P_1(x_1, y_1), P_2(x_2, y_2), \dots, P_n(x_n, y_n)$ is approximately a straight line. We set the threshold D as a limiting condition and obtain the maximum distance from each priori matching point to the fitted straight line. If the maximum distance is larger than the threshold D , the corresponding priori matching point is considered as a priori mismatching point and should be eliminated. After, we used the residual priori matching points to obtain the straight line again, and judged until all maximum distances are less than the threshold D .

The specific practices were as follows,

① We assumed the fitted function model is $y = a_0 + a_1x$, where a_0 and a_1 can be obtained according to the least squares principle [22].

$$a_0 = \frac{\sum_{i=1}^n y_i - a_1 \sum_{i=1}^n x_i}{n}, \tag{1}$$

$$a_1 = \frac{n \sum_{i=1}^n x_i y_i - \sum_{i=1}^n x_i \sum_{i=1}^n y_i}{n \sum_{i=1}^n x_i^2 - (\sum_{i=1}^n x_i)^2} \tag{2}$$

② According to the distance formula of point to line [23], the distance d_i from priori matching points $P_i(x_i, y_i)$ to the fitted function model $y = a_0 + a_1x$ can be calculated.

$$d_i = \left| \frac{a_1 x_i - y_i + a_0}{\sqrt{a_1^2 + 1}} \right|, \quad i = 1, 2, \dots, n \tag{3}$$

The maximum distance is $d_{\max} = \max(d_i)$; if d_{\max} is larger than the threshold D , this priori matching point is considered to be a priori mismatching point.

③ For the residual $n - 1$ priori matching points, ① and ② are repeated until all maximum distances d_{\max} are less than the threshold D ; then it is considered that the elimination is complete, and the MJDC model is obtained.

Step 3: We judged and corrected a new matching point. Based on the MJDC model, a new matching point is judged whether it is a mismatching point, and if so, the real matching point is estimated. We calculate the distance d_{n+1} from a new matching point $P_{n+1}(x_{n+1}, y_{n+1})$ to the MJDC model $y = a_0 + a_1x$. If d_{n+1} is less than the threshold D , then the matching point is considered to be a correct matching point. On the contrary, if it is a mismatching point, it is eliminated, and we estimate the position coordinates of the real matching point using the accumulative error relationship of residual priori matching points.

III. CONSTRUCTION OF THE MJDC MODEL

As shown in Figure 3, this study data was located in the South China Sea and was taken from the website of the University of California, San Diego (https://topex.ucsd.edu/cgi-bin/get_data.cgi), ranging between longitude $133^\circ - 135^\circ$ E and latitude $39^\circ - 41^\circ$ N. The resolution of the original marine gravity anomaly reference map was $1' \times 1'$, whose accuracy reached 2-8 mGal [24]–[26], which could be interpolated onto a grid with 100×100 m resolution, which provided a good base for the simulation analysis of matching navigation [27]–[30].

Figure 3 shows the 2D/3D marine gravity anomaly reference map with 100×100 m resolution in the study area. As seen in Figure 3, the overall gravity field fluctuated

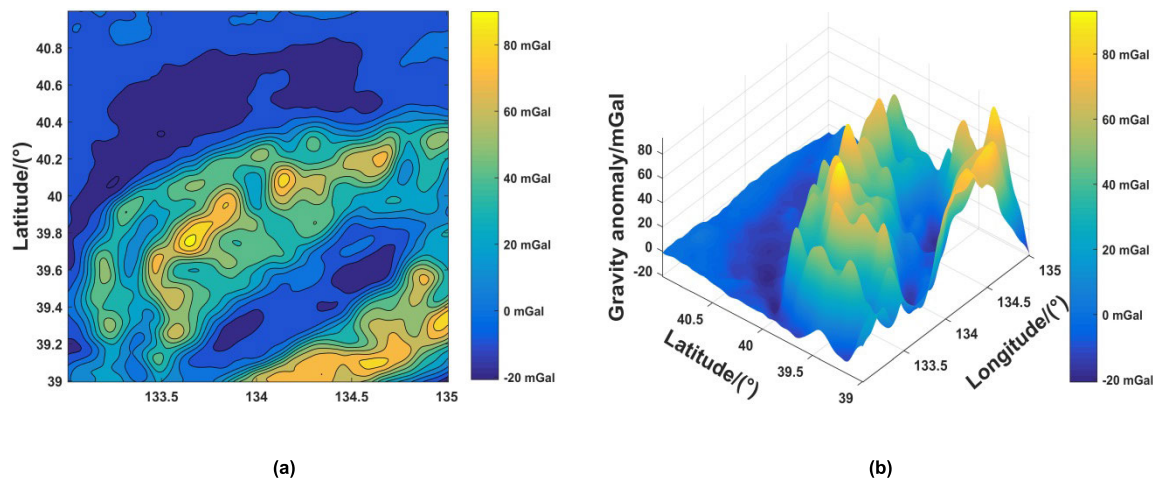


FIGURE 3. Marine gravity anomaly reference map with 100×100 m resolution. (a) 2D; (b) 3D.

drastically in the study area. The gravity field changed more drastically in the central and southern areas, and more gently in the northern and western areas.

The gravity field characteristics not only affected the gravity matching accuracy, but also had a strong correlation with matching probability. The matching effects were different in different gravity characteristic regions even if the same algorithm was used. The region with abundant gravity characteristics could obviously improve the matching accuracy and the matching probability.

Many parameters have been defined to describe the gravity field characteristics, such as standard deviation of gravity anomaly, standard deviation of slope, roughness, differential entropy of gravity anomaly, and fractal dimension, which are used to evaluate the matching effect of a gravity anomaly reference map. However, a single characteristic parameter contains limited gravity information, so the gravity suitability cannot be effectively evaluated on this basis. As a consequence, Li et al. comprehensively analyzed the above mentioned characteristic parameters of a gravity field. Furthermore, they proposed a principal component weighted average normalization method based on principal component analysis criteria and weighted average principle in order to obtain the overall characteristic parameter index that can evaluate the matching results of a gravity anomaly reference map [31]. Based on this index, the suitability of the gravity anomaly reference map was divided in the study area, as shown in Figure 4.

Figure 4 shows the suitability division results of gravity matching in the study area. Among them, the white area was an excellent-suitability region with high positioning accuracy and good matching effect. The orange area was the general-suitability region, and the matching effect was general. The black area was the non-suitability region with poor matching effect, and it was not suitable for matching. The red area was the dangerous region (for example, the water depth was less than 400 m) that was obtained by overlapping the seabed

digital terrain data in the same area. There were many shoals and reefs, and we concluded that during route planning, this area should be avoided for safety reasons. Therefore, as seen in Figure 4, we verified the feasibility of the PRILSMC method in three types of regions (for example, the excellent-suitability region (A), the general-suitability region (B), and the non-suitability region (C)).

Analysis and Discussion of Parameter Settings: Both the number of recursive sampling points and priori matching points had an important effect on the positioning accuracy and matching probability.

Firstly, for example, if the number of the sequence of sampling points is 100, theoretically, the value range of the number of recursive sampling points N is 1–100. When the value is 1, 1 sampling point is processed at a time, and then 99% of sampling points are the same in the adjacent sequences, such as: the sequence of sampling points a , the sequence of sampling points b , ..., the sequence of sampling points n . In this case, these priori matching points were greatly affected by local gravity and had a strong correlation with each other, which may have led to an overall deviation. However, with the increases of recursive sampling points N , the overlap rate of sampling points was reduced in the adjacent sequences. This was beneficial to reduce the negative influence of local gravity and ensure the independence of each priori matching point.

Secondly, the number of priori matching points is important data for constructing the MJDC model. When the number of priori matching points is 0, the algorithm degenerates into the conventional TERCOM algorithm. With the increases of priori matching points, it is helpful to improve the effects of the MJDC model.

Finally, according to the actual situation, the number of recursive sampling points and priori matching points was not as large as possible. The main reasons are as follows: ① With the increase of recursive sampling points and priori matching points, regarding the shape of the navigation trajectory, it is

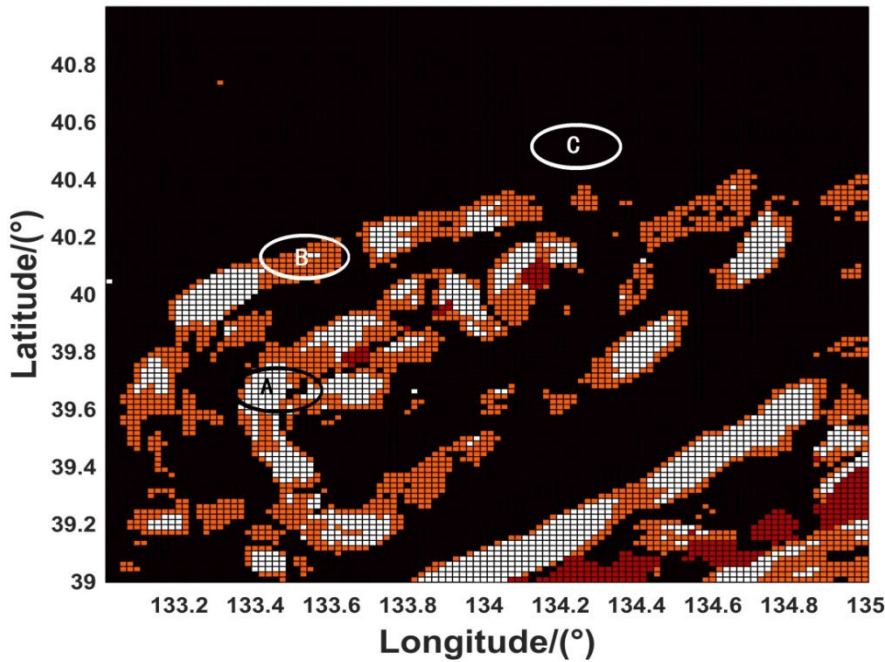


FIGURE 4. Suitability division of gravity matching region.

TABLE 1. Simulation parameters.

Simulation parameters	Numerical value
Gyroscope drift	0.01°/h
Accelerometer zero bias	10 ⁻³ m/s ²
Velocity	10 m/s
Initial position error	0 m
Velocity error	0.02 m/s
Heading error	0.03°
Number of sampling points	100
Sampling period	20 s
Threshold <i>D</i>	1 grid
Recursive sampling points	5
Priori matching points	10

not easy to maintain an approximately straight line if a long time has passed. Furthermore, it is not conducive to the rapid correction of underwater vehicles in case of emergency. ② If the entire priori matching process takes too much time, there needs to be a large matching region, which improves the selection criteria of matching region.

In summary, the number of recursive sampling points and priori matching points need to be fully considered according to the actual situation. In this paper, some numerical simulations were carried out by taking 5 recursive sampling points and 10 priori matching points as examples.

The numerical simulation parameters of the TERCOM algorithm were set as follows Table 1:

The data of the sampling points are the sum of the sampled value in the gravity anomaly reference map and the random

noise with a standard deviation of 1 mGal. In this paper, if the positioning accuracy was within the diagonal length of a grid, this matching point was defined as an effective matching point. Then, the matching probability = (number of effective matching points / number of experiments) × 100%.

A new MJDC model was constructed based on the PRILSMC method. As shown in Figure 5, under the same conditions, 10 priori matching points were obtained separately in the excellent-suitability region, general-suitability region, and non-suitability region. The right graphs of Figure 5a–c show the MJDC model. Table 2 represents the statistical results of the MJDC model in three regions, which depicts the iterative process optimization of the right graph in Figure 5a–c. According to the factors, including matching algorithm probability, error of INS, and actual situation, the threshold is set to 1 grid from the priori matching point to the fitted function model. If it is larger than the threshold, the corresponding priori matching point is a priori mismatching point and should be eliminated. Then, the residual priori matching points are used to obtain the fitted function model again and are judged until all maximum distances are less than the threshold 1, when it is considered that the MJDC model is optimized completely.

As shown in Table 2, in the excellent-suitability region, the fitted function model of the priori matching points ($P_1 \sim P_{10}$) was $y = 0.359x + 606.9$. The maximum distance from each point to the model was less than the threshold 1, so there was no priori mismatching point in the priori matching points ($P_1 \sim P_{10}$); then the MJDC model was $y = 0.359x + 606.9$. In the general-suitability region, the fitted function model of the priori matching points ($P_1 \sim P_{10}$) was

TABLE 2. Statistical results of the MJDC model in three regions.

Regions	Number of priori matching points	Number of iterations	Priori matching points participating in the fitted model	MJDC model	Maximum distance (grid)	Priori mismatching point
Excellent-suitability	10	0	$P_1 \sim P_{10}$	$y = 0.359x + 606.9$	0.411	no
		0	$P_1 \sim P_{10}$	$y = 0.269x + 991.3$	1.445	$P_4 (620, 1157)$
General-suitability	10	1	$P_1 \sim P_3, P_5 \sim P_{10}$	$y = 0.267x + 993.4$	1.174	$P_5 (629, 1160)$
		2	$P_1 \sim P_3, P_6 \sim P_{10}$	$y = 0.266x + 994.2$	1.027	$P_8 (658, 1168)$
		3	$P_1 \sim P_3, P_6, P_7, P_9, P_{10}$	$y = 0.269x + 992.3$	0.732	no
Non-suitability	10	0	$P_1 \sim P_{10}$	$y = 0.319x + 1281.6$	1.867	$P_1 (1385, 1725)$
		1	$P_2 \sim P_{10}$	$y = 0.338x + 1254.4$	1.308	$P_2 (1396, 1727)$
		2	$P_3 \sim P_{10}$	$y = 0.354x + 1230.1$	1.094	$P_8 (1448, 1742)$
		3	$P_3 \sim P_7, P_9, P_{10}$	$y = 0.358x + 1224.8$	0.811	no

$y = 0.269x + 991.3$. The distance from the priori matching point $P_4 (620, 1157)$ to the fitted function model was the largest, which was 1.445 grids. It was larger than the threshold 1, so P_4 was considered as a priori mismatching point. The fitted function model of the residual priori matching points ($P_1 \sim P_3, P_5 \sim P_{10}$) was $y = 0.267x + 993.4$. The distance from the priori matching point $P_5 (629, 1160)$ to the fitted function model was the largest, which was 1.174 grids. It was larger than the threshold 1, so P_5 was considered as a priori mismatching point. The fitted function model of the residual priori matching points ($P_1 \sim P_3, P_6 \sim P_{10}$) was $y = 0.266x + 994.2$. The distance from the priori matching point $P_8 (658, 1168)$ to the fitted function model was the largest, which was 1.027 grids. It was larger than the threshold 1, so P_8 was also considered as a priori mismatching point. The fitted function model of the residual priori matching points ($P_1 \sim P_3, P_6, P_7, P_9, P_{10}$) was $y = 0.269x + 992.3$. The maximum distance from each point to the model was less than the threshold 1, so there was no priori mismatching point in the residual priori matching points ($P_1 \sim P_3, P_6, P_7, P_9, P_{10}$); then the MJDC model was $y = 0.269x + 992.3$. In a similar way, in the non-suitability region, we judged and eliminated some priori mismatching points, $P_1 (1385, 1725)$, $P_2 (1396, 1727)$, and $P_8 (1448, 1742)$, respectively. Then the MJDC model was $y = 0.358x + 1224.8$.

The left graphs of Figure 5a–c show the accuracy of 10 priori matching points. Table 3 depicts the statistical results of 10 priori matching points in three regions from Figure 5a–c. As seen in Table 3, in the excellent-suitability region, the effect of gravity matching was significant with an average positioning accuracy of 67.71 m and a standard deviation of positioning accuracy of 27.34 m. The matching probability was near to 100% (all the 10 priori matching points

were correct), and the number of priori mismatching points was 0. In the general-suitability region, the effect of gravity matching was general, with an average positioning accuracy of 107.96 m and a standard deviation of positioning accuracy of 56.86 m. The matching probability was about 70% (7 of the 10 priori matching points were correct), and the number of priori mismatching points was 3. In the non-suitability region, the effect of gravity matching was poor, with an average positioning accuracy of 255.65 m and a standard deviation of positioning accuracy of 142.52 m. The matching probability was about 20% (2 of the 10 priori matching points were correct), but the number of priori mismatching points was 3 instead of 8, because there were only two correct points among the 10 priori matching points, and the reliability of the priori matching points was too low in the non-suitability region. This led to the low reliability and accuracy of the MJDC model, which was not suitable for the elimination and correction of mismatching points. Thus, the MJDC model cannot be applied in the non-suitability region.

IV. VERIFICATION AND APPLICATION OF THE MJDC MODEL

The correction effect of a new matching point was verified based on the MJDC model. Section III, ‘onstruction of the MJDC model’ described the construction process of the MJDC model, and the MJDC model needed to be rebuilt before each new matching point was judged and corrected. As shown in Figure 6, we separately conducted 50 simulation verifications of new matching points in the excellent-suitability region, general-suitability region, and non-suitability region under the same conditions.

The statistical results of Figure 6 are shown in Table 4. In the excellent-suitability region, the effects of gravity matching and the MJDC model were both good. The average

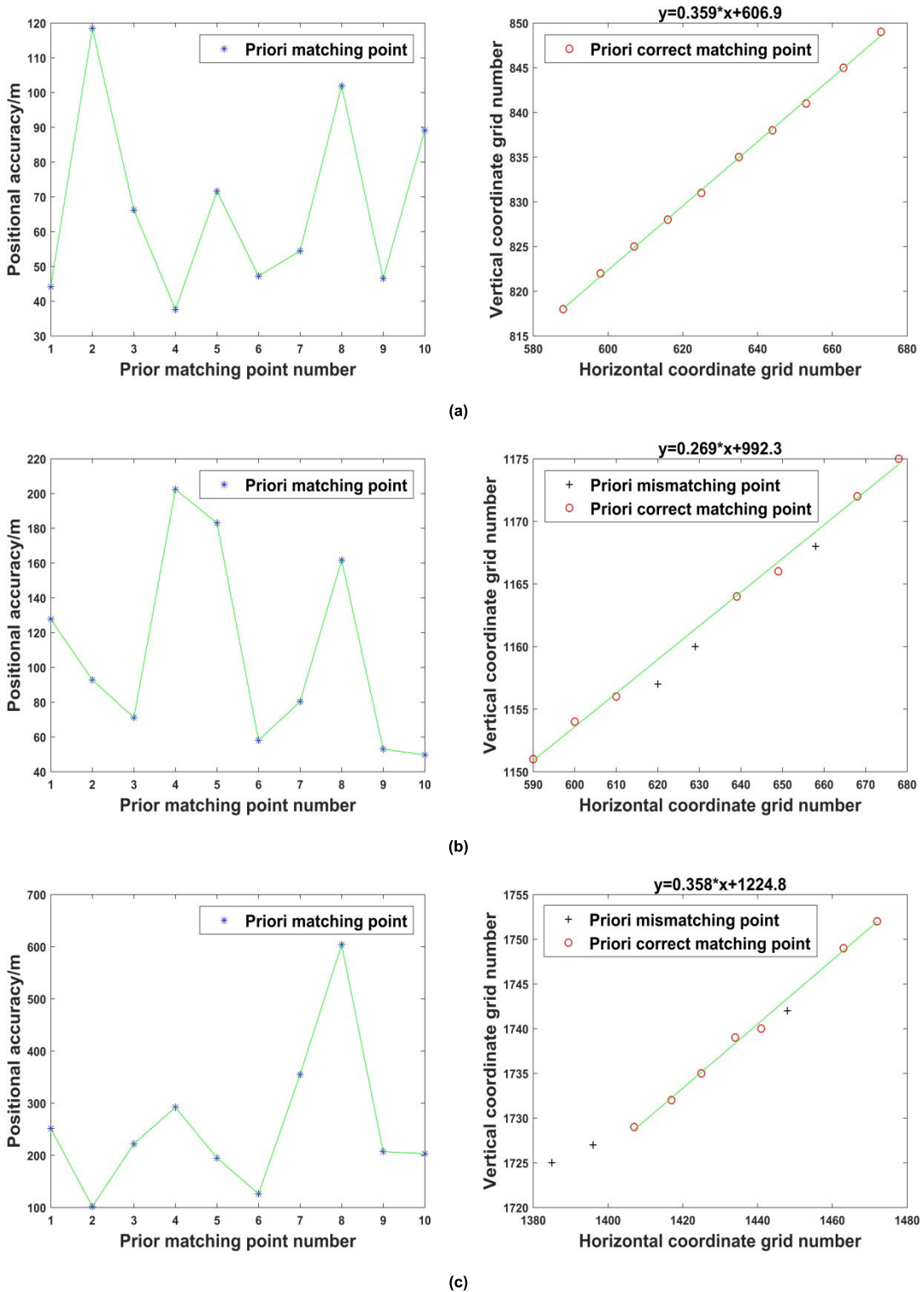


FIGURE 5. Comparison of positioning accuracy and location of 10 priori matching points in three regions. (a) Excellent-suitability; (b) General-suitability; (c) Non-suitability.

positioning accuracy was improved from 68.39 m to 61.65 m, and the matching reliability was high; after being corrected, the number of mismatching points was reduced from 2 to 0 (the matching probability increased from about 96% to 100%,

and all mismatching points could essentially be eliminated). In the general-suitability region, the gravity matching effect was general. However, the MJDC model effect was significant. The average positioning accuracy was improved from

TABLE 3. Statistical results of 10 priori matching points in three regions.

Regions	Average positioning accuracy (m)	Standard deviation of positioning accuracy (m)	Matching probability	Number of priori mismatching points
Excellent-suitability	67.71	27.34	100%	0
General-suitability	107.96	56.86	70%	3
Non-suitability	255.65	142.52	20%	3

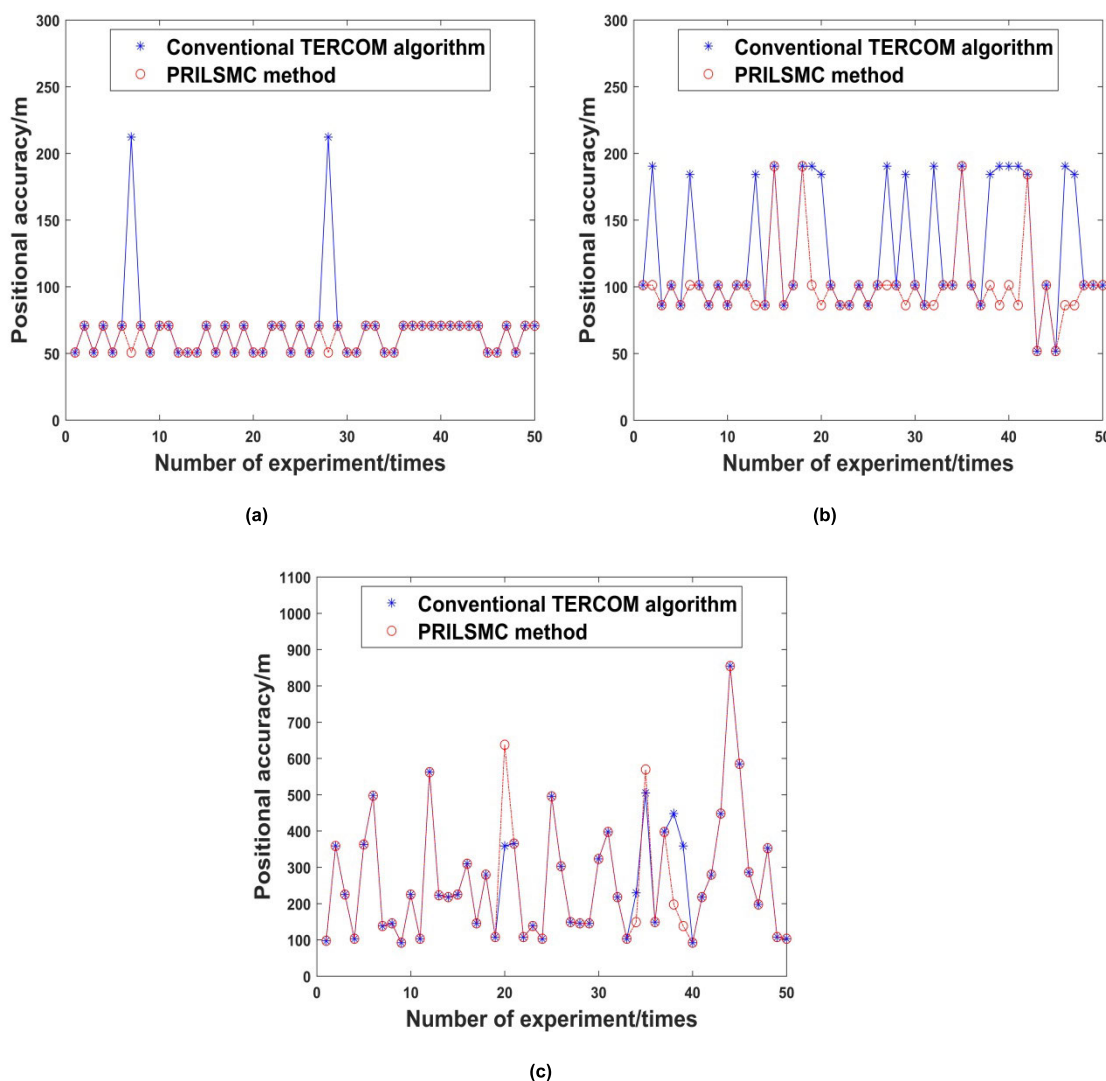


FIGURE 6. Results of the mismatching correction in three regions. (a) Excellent-suitability; (b) General-suitability; (c) Non-suitability.

127.13 m to 99.23 m, and after being corrected, the number of mismatching points was reduced from 18 to 4 (the matching probability increased from about 64% to 92%), indicating that the reliability of the matching navigation was

improved. In the non-suitability region, the effects of gravity matching and the MJDC model were both poor, and this was because the reliability of priori matching points was too low.

TABLE 4. Statistical results from Figure 6.

Regions	Conventional TERCOM algorithm				The PRILSMC method			
	Average positioning accuracy (m)	Standard deviation of positioning accuracy (m)	Matching probability	Number of mismatching points	Average positioning accuracy (m)	Standard deviation of positioning accuracy (m)	Matching probability	Number of mismatching points
Excellent-suitability	68.39	31.26	96%	2	61.65	10.61	100%	0
General-suitability	127.13	47.26	64%	18	99.23	33.58	92%	4
Non-suitability	267.59	162.51	26%	37	263.42	172.03	28%	36

V. CONCLUSION

This paper proposed a new PRILSMC method to reduce the probability of mismatching and improve the reliability of matching navigation.

(1) We developed a PRILSMC method. This study put forward the PRILSMC method based on the idea of priori recursive multiple matching and iterative least squares, in order to identify and correct a new matching point.

(2) We constructed the MJDC model. According to factors including matching algorithm probability, error of INS, and actual situation, the MJDC model was constructed based on the PRILSMC method.

(3) We verified the correction effect to a new matching point. The results indicated the following: ① In the excellent -suitability region, the effect of the MJDC model was good. After being corrected, the number of mismatching points was reduced from 2 to 0. ② In the general-suitability region, the effect was significant. After being corrected, the number of mismatching points was reduced from 18 to 4, indicating that the probability of mismatching points was greatly reduced, which improved the reliability of matching navigation. As a consequence, the PRILSMC method is beneficial to improve the reliability of underwater vehicles.

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Wei Zheng is co-first author

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WEI ZHENG received the Ph.D. degree from the College of Physics, Huazhong University of Science and Technology, in 2007. From 2007 to 2015, he worked as an Associate Research Fellow with the Institute of Geodesy and Geophysics, Chinese Academy of Sciences. Since 2015, he has been working as a Research Fellow with the Qian Xuesen Laboratory of Space Technology, China Academy of Space Technology. His research interests include satellite gravity, underwater integrated navigation, satellite sea surface altimetry, and satellite navigation.



ZHAOWEI LI received the bachelor's degree in geographic information system (GIS) from Northwest University, China, in 2010, and the master's degree in global navigation satellite system (GNSS) from Paris Diderot University–Paris 7, France, in 2015. He participated the joint master's degree programme in remote sensing (RS) technology at the Institut de Physique du Globe de Paris (IPGP), France, from 2013 to 2015. Since 2016, he has been working as an Assistant Research Fellow with the Qian Xuesen Laboratory of Space Technology, China Academy of Space Technology. His research interests include underwater gravity matching navigation, underwater terrain matching navigation, and satellite ocean remote sensing.



FAN WU received the bachelor's degree in optical information science and technology, the master's degree in electronics and communication engineering, and the Ph.D. degree in ocean detection technology from the Ocean University of China, in 2010, 2013, and 2017, respectively. He participated the joint Ph.D. programme, supported by the China Scholarship Council with the University of Rhode Island, from 2014 to 2016. Since 2017, he started a postdoctoral programme with the Qian Xuesen Laboratory of Space Technology, China Academy of Space Technology. His research interests include satellite sea surface altimetry and satellite ocean remote sensing.

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