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PI-Type Iterative Learning Consensus Control for Second-Order Hyperbolic Distributed Parameter Models Multi-Agent Systems

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ABSTRACT This paper considers the consensus control problem of multi-agent systems (MAS) with second-order hyperbolic distributed parameter models. Based on the framework of network topologies, a PI-type iterative learning control protocol is proposed by using the nearest neighbor knowledge. Using Gronwall inequality, a sufficient condition for the convergence of the consensus errors with respect to the iteration index is obtained. Finally, the validity of the proposed method is verified by two numerical examples.

INDEX TERMS Multi-agent systems, iterative learning control, Gronwall inequality, hyperbolic distributed parameter system.

I. INTRODUCTION

Iterative learning control (ILC) is an effective technique of tracking control aiming at improving system tracking performance from trial to trial in a repetitive mode [1], [2]. The basic idea is to use information collected from previous executions of the same task repetitively to form the control action for the current operation in order to improve tracking performances from iteration to iteration [3]–[5]. In the last three decades, it has been widely applied in many fields and applications [6]–[8].

One of the major analysis tools have been used in the design of ILC is the contraction mapping method [8]–[10]. A frequency-domain criteria for the convergence of ILC was derived in [9]. In [10], a PI controller combined with a simple ILC algorithm was designed for mechanical ventilator topology with periodic disturbance. The two dimension (2D) analytical model is often utilized to investigate the convergence of ILC. Several robust ILC schemes in time domain were proposed in [11]–[13] by using an equivalent 2D system model. A 2D delay compensation based ILC for batch processes with both input and state delays was investigated in [14]. Composite energy function (CEF) is another useful method for the design of ILC [15]. On the basis of CEF, a boundary ILC law was proposed in [16] to guarantee the learning convergence. A norm-optimal ILC algorithm incorporating variable cycle durations was developed in [17]. The results confirmed that the algorithm is able to prevent the dilatation of the ventricle and adapt to varying cycle lengths.

Distributed parameter systems (DPS) are described by partial differential equations, such as heat transfer and diffusion, vibration and fluid dynamical models, whose states depend on spatial position and time [18], [19]. In recent years, the application of ILC to distributed parameter systems has become a new topic. To attenuate the unknown periodic speed variation for a stretched string system on a transporter, a differential-difference type ILC was augmented in [20]. In [21], an ILC of flow rate was considered in a center pivot irrigator used in dry-land farming. The similar ILC scheme was combined to compensate the unknown periodic motion for axially moving material systems in [22]. The convergence condition and robustness of the P-type ILC for parabolic heat conduction DPS were discussed in [23]. In [24], ILC with forgetting factor was proposed and the conditions for convergence of algorithm were established. In [25], a P-type ILC scheme was proposed for a class of second-order hyperbolic DPSs with uncertainties. The robust boundary ILC for the output tracking and disturbance attenuation of the nonlinear

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hyperbolic system was addressed in [26]. For dynamical systems governed by partial differential inclusions, a numerical solution for finite time tracking problem based on ILC technique was presented in [27].

Multi-agent systems (MAS) are a collection of mutiple computable agents, in which the agents have communication, sensing and execution capabilities [28]. Nowadays, is widely used in satellite attitude control, multi-mobile robots and self-organized underwater fleet. In very recent years, ILC-based consensus control of MAS has attracted much attention. A new distributed adaptive ILC scheme for nonlinear MAS with uncertainties was presented in [29]. In [30], the learning consensus problem for heterogenous high-order nonlinear MAS with output constraints was considered with the help of a novel barrier Lyapunov function. In [31], a PI-type ILC was applied to both linear and nonlinear fractional-order MAS to solve consensus tacking problem. In very recently, the study on the consensus control via ILC for MAS with DPS has also received considerable attention [32], [33]. Based on the framework of network topologies, a consensus-based ILC protocol was proposed for a class of MAS described with DPS in [32]. The consensus control problem of distributed parameter models MAS with time-delay was considered in [33], and the conclusions were also extended to Lipschitz nonlinear case. In [34], [35], a second-order iterative learning consensus control protocol was proposed for MAS with parabolic DPS or hyperbolic DPS.

The study of this paper is motivated by the following facts. Firstly, up to the present, most of the references that address ILC of MASs are focusing on P-type ILC, where the accumulated errors were neglected. Secondly, many ILC control of MAS for DPS, the identical initialization condition was used, where the output of each follower agent was required to start from the same initial value of the leader. In fact, not all the follower agents can obtain the information of the leader, therefore, the identical initialization condition is not practical. Thirdly, all the above literatures that discussed the consensus of MAS were described by ordinary differential equations or first order partial differential equation (PDE). However, in practice, the spatial dynamics of MAS is related to second-order PDE is often exists.

The main contributions are as follows. 1) Extending to the consensus control of MAS for second-order hyperbolic distributed parameter models, and using the nearest neighbor knowledge and the prior information of the control input, a PI-type ILC protocol with initial state learning is proposed and the existing result is generalized. 2) Using Cauchy-Schwarz inequality and Gronwall inequality, the convergence analysis for consensus errors is given in detail. The obtained condition is less conservative than the existing one.

The rest of this paper is organized as follows: Some preliminaries and problem formulation are presented in Section II. The main results are derived in Section III. Section IV present two numerical examples that demonstrates the effectiveness of the method. Finally, some conclusions are drawn in Section V.

Throughout this paper, $\mathbb{R}^n$ denotes an n-dimensional Euclidean space, I_m means an $m \times m$ dimensional identity matrix. For the n dimensional vector $W = (w_1, w_2, \cdots, w_n)^T$, its 2-norm for the n-dimensional vector $w = (w_1,\ w_2,\ \cdots,\ w_n)$ is defined as $\|w\| = \sqrt{\sum_{i=1}^n w_i^2}$ and the spectrum norm of the $n \times n$ -order square matrix A is $\|A\| = \sqrt{\lambda_{max}(A^T A)}$, where λ_{max} represents the maximum eigenvalue. Let $L^2(\Omega)$ be Hilbert space. If $Q_i \in L^2(\Omega)(i = 1,\ 2,\ \cdots, n)$, we define $Q = (Q_1,\ Q_2,\ \cdots,\ Q_n) \in \mathbf{R}^n \bigcap L^2(\Omega)$, then $\|Q\|_{L^2} = \{\int_\Omega (Q(x)^T Q(x))dx\}^{\frac{1}{2}}$. For the function $f(x,t)\ :\ \Omega \times [0,T] \to \mathbf{R}^n$, $f(x,t) \in \mathbf{R}^n \bigcap L^2(\Omega)$, $t \in [0,T]$, we define the norm of $(L_2,\ \lambda)$ as

$$\|f(x,t)\|_{(L_2,\ \lambda)} = \sup_{t\in[0,\ T]} \{\|f(x,t)\|_{L^2} e^{-\lambda t}\},\ \lambda > 0.$$

II. PRELIMINARIES AND PROBLEM FORMULATION

Some basic properties are first introduced, which will be used in the following sections.

Lemma 1 [36]: Let $M \in \mathbb{R}^{n\times m}$, $N \in \mathbb{R}^{n\times l}$, $\zeta \in \mathbb{R}^m$, $\eta \in \mathbb{R}^l$. Then

$$2\zeta^T M^T N\eta \le \varepsilon\zeta^T M^T M\zeta + \frac{1}{\varepsilon}\eta^T N^T N\eta.$$

Lemma 2 (Gronwall Inequality [37]): Let $M \in R$, $u(t)$, $a \ge 0$ and $w(t)$ be continuous functions on $t \in [t_0, \infty)$. If

$$u(t) \le M + \int_0^t \Big[au(s) + bw(s)\Big]ds,$$

then

$$u(t) \le Me^{at} + \int_0^t e^{a(t-s)}bw(s)ds.$$

In order to describe the connection among these agents, a directed graph $\mathbb{G} = (\mathbb{V}, \mathbb{E}, \mathbb{A})$ will be used, where $\mathbb{V} = \{1, 2, \cdots, N\}$ and $\mathbb{E} \subseteq \mathbb{V} \times \mathbb{V}$ are the sets of vertices and edges of the graph $\mathbb{G}$, respectively. In $\mathbb{G}$, we use the ith vertex to represent the ith agent, and use a directed edge from i to j to represent an ordered pair $(i,j) \in \mathbb{E}$, which means that agent j can directly receive information from agent i. The communication graph can be represented by two types of matrices: the adjacency matrix $\mathbb{A} = (a_{i,j}) \in R^{N\times N}$ with $a_{i,j} = 1$ if $(i,j) \in \mathbb{E}$ and $a_{i,j} = 0$, otherwise. The Laplacian matrix $L = [l_{ij}] \in \mathbb{R}^{N\times N}$ is denoted as $L = \mathbb{D} - \mathbb{A}$, where $\mathbb{D} = \text{diag}\{d_1, d_2, \cdots, d_N\}$ is the degree matrix whose diagonal elements are defined by $d_i = \sum_{j=1}^N a_{ij},\ i \in \{1, 2, \cdots, N\}$. It is well known that Laplace matrix L has a simple zero eigenvalue and all the other eigenvalues have positive real parts if and only if $\mathbb{G}$ has a directed spanning tree.

In this paper, we consider a set of N agents, whose dynamics are described by the following second-order hyperbolic distributed parameter systems

$$\begin{cases} \dfrac{\partial^2 q_{i,k}(x,t)}{\partial t^2} = D\Delta q_{i,k}(x,t) + Aq_{i,k}(x,t) + Bu_{i,k}(x,t), \\ y_{i,k}(x,t) = Cq_{i,k}(x,t) + Gu_{i,k}(x,t), \end{cases} \quad (1)$$

where $i \in \{1, 2, \ldots, N\} = \bar{N}$, subscript k denotes the iterative number of the process; x and t respectively denote space and time variables, $(x, t) \in \Omega \times [0, T]$; $\Omega = [a, b]$ is a bounded open subset with smooth boundary $\partial\Omega$; $\Delta = \frac{\partial^2}{\partial x^2}$ is a Laplace operator on Ω. $q_{i,k}(\cdot,\cdot) \in \mathbb{R}^n$, $u_{i,k}(\cdot,\cdot) \in \mathbb{R}^p$, $y_{i,k}(\cdot,\cdot) \in \mathbb{R}^m$ are the state, input and output of the ith agent at the kth iteration, respectively. $d > 0$ denotes the constant time-delay. $A,\ B,\ C,\ D,\ G$ are constant matrices of appropriate dimensions and $D > 0,\ G \neq 0$.

The corresponding boundary condition of system (1) is given as

$$q_{i,k}(a, t) = f_{i,1}(t), \quad q_{i,k}(b, t) = f_{i,2}(t), \tag{2}$$

$$\left\| \frac{\partial q_{i,k+1}(x, 0)}{\partial t} - \frac{\partial q_{i,k}(x, 0)}{\partial t} \right\|_{L^2}^2 \leq l r^k, \tag{3}$$

$$\left.\frac{\partial q_{i,k+1}(x, t)}{\partial x}\right|_{t=0} = \left.\frac{\partial q_{i,k}(x, t)}{\partial x}\right|_{t=0}. \tag{4}$$

where $i \in \bar{N}$, $(x, t) \in \partial\Omega \times [0, T]$, $l > 0$ and $0 < r < 1$.

Definition 1 ([38]): For MAS (1), protocols $u_{i,k}(x, t)$ are said to solve consensus if for any initial and boundary values, the states of agents satisfy

$$\lim_{k\to\infty} \| y_{j,k}(x, t) - y_{i,k}(x, t) \|_{L^2} = 0, \quad i, j \in \bar{N}. \tag{5}$$

Let $y_r(x, t),\ (x, t) \in \Omega \times [0, T]$ be the desired sufficiently smooth trajectory for consensus tracking, which is accessible to a subset of followers only. We also regard $y_r(x, t)$ as a leader. Thus, for all agents, the consensus target of system (1) is equivalent to finding the correct control inputs $u_{i,k}(x, t)$ such that

$$\lim_{k\to\infty} \| y_r(x, t) - y_{i,k}(x, t) \|_{L^2} = 0, \quad i \in \bar{N}. \tag{6}$$

To solve the consensus target (5) (or (6)), we construct the following distributed PI-type ILC protocol at the $(k + 1)$th iteration for each agent:

$$\begin{cases} u_{i,k+1}(x, t) = u_{i,k}(x, t) + \Lambda\eta_{i,k}(x, t) + \Gamma\int_0^t \eta_{i,k}(x, \tau)d\tau, \\ q_{i,k+1}(x, 0) = q_{i,k}(x, 0) + \Upsilon\eta_{i,k}(x, 0), \end{cases} \tag{7}$$

where $u_{i,1}(x, t)$ is given initial input, $q_{i,1}(x, 0)$ is the initial state of the agent for the iteration, Γ, Λ and Υ are learning gain matrices to be determined later. $\eta_{i,k}(x, t)$ is the available information at the $(k + 1)$th iteration for the ith agent which denoted by

$$\eta_{i,k}(x, t) = \sum_{l\in N_i} a_{l,i}[y_{l,k}(x, t) - y_{i,k}(x, t)] + s_i[y_r(x, t) - y_{i,k}(x, t)], \tag{8}$$

where i denotes the agent index, N_i is a collection of neighbor agents of agent i. $s_i = 1$ if the ith agent can access the desired trajectory and $s_i = 0$ otherwise.

Remark 1: Parabolic DPS and hyperbolic DPS are two kinds of the most important PDEs. Studies ILC-based consensus for the MAS with parabolic DPS has been widely investigated [33], [34], while ILC-based consensus for the MAS with hyperbolic DPS are limited. In this paper, ILC technique is applied to the consensus control of MAS described by a class of second-order hyperbolic DPS.

Remark 2: PI-type ILC protocol (7) is borrowed from [31]. When $\Gamma = 0$, it is degenerated to the case of [25], [33], but without initial state learning. That is to say, the proposed control algorithm is an extension of the learning control algorithm developed in [25], [33]. On the other hand, $\Gamma\int_0^t \eta_{i,k}(x, \tau)d\tau$ is a feedback of integral of the local neighbor output error, which can help to eliminate the steady-state error.

Let $e_{i,k}(x, t) = y_r(x, t) - y_{i,k}(x, t)$ be the tracking error. Then, (8) can be rewritten as

$$\eta_{i,k}(x, t) = \sum_{l\in N_i} a_{l,i}[e_{i,k}(x, t) - e_{l,k}(x, t)] + s_i e_{i,k}(x, t). \tag{9}$$

For the kth iteration, we define

$$\begin{aligned} \eta_k(x, t) &= [\eta_{1,k}^T(x, t)\ \eta_{2,k}^T(x, t)\ \cdots\ \eta_{N,k}^T(x, t))]^T, \\ q_k(x, t) &= [q_{1,k}^T(x, t)\ q_{2,k}^T(x, t)\ \cdots\ q_{N,k}^T(x, t)]^T, \\ e_k(x, t) &= [e_{1,k}^T(x, t)\ e_{2,k}^T(x, t)\ \cdots\ e_{N,k}^T(x, t))]^T. \end{aligned}$$

Therefore, using Kronecker product, we can write (1) and (9) in the following compact forms

$$\begin{cases} \frac{\partial^2 q_k(x, t)}{\partial t^2} = (I_N \otimes D)\Delta q_k(x, t) + (I_N \otimes A)q_k(x, t) \\ \qquad + (I_N \otimes B)u_k(x, t), \\ y_k(x, t) = (I_N \otimes C)q_k(x, t) + (I_N \otimes G)u_k(x, t), \end{cases} \tag{10}$$

and

$$\eta_k(x, t) = ((L + S) \otimes I_m)e_k(x, t), \tag{11}$$

respectively. Besides, the compact form of (7) is

$$\begin{cases} u_{k+1}(x, t) = u_k(x, t) + ((L + S) \otimes \Lambda)e_k(x, t) \\ \qquad + ((L + S) \otimes \Gamma)\int_0^t e_k(x, \tau)d\tau, \\ q_{k+1}(x, 0) = q_k(x, 0) + ((L + S) \otimes \Upsilon)e_k(x, 0), \end{cases} \tag{12}$$

where L denotes graph Laplacian, and

$$S = \text{diag}(s_1, s_2, \cdots, s_N), \quad s_i \geq 0,\ i \in \bar{N}.$$

III. MAIN RESULTS

To obtain the main results, we first give following lemmas.

Lemma 3: For $k \geq 1$, denote

$$\begin{aligned} \delta q_{k+1}(x, t) &= q_{k+1}(x, t) - q_k(x, t), \\ \delta u_{k+1}(x, t) &= u_{k+1}(x, t) - u_k(x, t). \end{aligned}$$

Then

$$\delta u_{k+1}(x, t) = ((L + S) \otimes \Lambda)e_k(x, t) + ((L + S) \otimes \Gamma)\int_0^t e_k(x, \tau)d\tau, \tag{13}$$

and

$$e_{k+1}(x, t) = e_k(x, t) - (I_N \otimes C)\delta q_{k+1}(x, t) - (I_N \otimes G)\delta u_{k+1}(x, t). \tag{14}$$

Proof: The proof is simple, which is omitted here.

Lemma 4: For MAS (1), considering PI-type ILC protocol (7), if

$$\|I_{Nm}-(L+S)\otimes C\Upsilon-(L+S)\otimes G\Lambda\|^2<1, \tag{15}$$

then

$$\lim_{k\to\infty}\|e_k(x,0)\|_{L^2}=0. \tag{16}$$

Proof: It follows form (13) and (14), we have

$$\delta u_{k+1}(x,0)=((L+S)\otimes\Lambda)e_k(x,0), \tag{17}$$

and

$$e_{k+1}(x,0)=e_k(x,0)-(I_N\otimes C)\delta q_{k+1}(x,0) -(I_N\otimes G)\delta u_{k+1}(x,0). \tag{18}$$

From (12), one has

$$\delta q_{k+1}(x,0)=((L+S)\otimes\Upsilon)e_k(x,0). \tag{19}$$

So, combining with (17), (18) and (19), we have

$$e_{k+1}(x,0)=\Pi e_k(x,0), \tag{20}$$

where $\Pi=I_{Nm}-(L+S)\otimes C\Upsilon-(L+S)\otimes G\Lambda$. Using condition (15) and the contracting mapping principle, we can deduce

$$\lim_{k\to\infty}\|e_k(x,0)\|_{L^2}=0. \tag{21}$$

The proof is complete.

Lemma 5: Choosing sufficiently large enough λ and denoting that

$$\begin{aligned}
c_1 &=(1+\epsilon_1+\epsilon_2)\,\|\,I_{Nm}-((L+S)\otimes G\Lambda)\|^2,\\
c_2 &=(1+\frac{1}{\epsilon_1}+\epsilon_3)\|I_N\otimes C^TC\|^2,\\
c_3 &=(1+\frac{1}{\epsilon_2}+\frac{1}{\epsilon_3})\|(L+S)\otimes G\Gamma\|^2,\quad \epsilon_1,\ \epsilon_2,\ \epsilon_3>0,\\
M &=2\|(L+S)\otimes\Lambda\|^2+\frac{2\|(L+S)\otimes\Gamma\|^2}{\lambda^2},
\end{aligned}$$

then

$$\begin{aligned}
\|\,e_{k+1}(x,t)\|^2_{(L^2,\lambda)} &\le c_1\|e_k(x,t)\|^2_{(L^2,\lambda)}+c_2\|\delta q_{k+1}(x,t)\|^2_{(L^2,\lambda)}\\
&\quad+\frac{c_3}{\lambda^2}\|e_k(x,\tau)\|^2_{(L^2,\lambda)},
\end{aligned} \tag{22}$$

$$\|\,\delta u_{k+1}(x,t)\|^2_{(L^2,\lambda)}\le M\|e_k(x,\tau)\|^2_{(L^2,\lambda)}. \tag{23}$$

Proof: According to the first equation in (12), we have

$$\delta u_{k+1}(x,t)=((L+S)\otimes\Lambda)e_k(x,t) +((L+S)\otimes\Gamma)\int_0^t e_k(x,\tau)d\tau. \tag{24}$$

Substituting (24) into (14), we get

$$\begin{aligned}
e_{k+1}(x,t)&=(I_{Nm}-((L+S)\otimes G\Lambda))e_k(x,t)\\
&\quad-(I_N\otimes C)\delta q_{k+1}(x,t)\\
&\quad-((L+S)\otimes G\Gamma)\int_0^t e_k(x,\tau)d\tau.
\end{aligned} \tag{25}$$

So, using Lemma 1, we have

$$\begin{aligned}
e^T_{k+1}(x,t)e_{k+1}(x,t) &\le c_1e^T_k(x,t)e_k(x,t)\\
&\quad+c_2\delta q^T_{k+1}(x,t)\delta q_{k+1}(x,t)\\
&\quad+c_3\int_0^t e^T_k(x,\tau)d\tau\int_0^t e_k(x,\tau)d\tau,
\end{aligned} \tag{26}$$

where $c_1, c_2,\ c_3$ are the same as in Lemma 5. This gives that,

$$\begin{aligned}
\|\,e_{k+1}(x,t)\|^2_{L^2} &\le c_1\|e_k(x,t)\|^2_{L^2}+c_2\|\delta q_{k+1}(x,t)\|^2_{L^2}\\
&\quad+c_3\Big\|\int_0^t e_k(x,\tau)d\tau\Big\|^2_{L^2}\\
&\le c_1\|e_k(x,t)\|^2_{L^2}+c_2\|\delta q_{k+1}(x,t)\|^2_{L^2}\\
&\quad+c_3\Big(\int_0^t\|e_k(x,\tau)\|_{L^2}d\tau\Big)^2.
\end{aligned} \tag{27}$$

Multiplying $e^{-2\lambda t}$ on both sides of (27) and taking λ-norm, one has (22). By the same argument, it follows from (24), we have (23). The proof is finished.

Lemma 6: Denoting that

$$\begin{aligned}
c_6 &=\lambda_{max}(I_N\otimes(A^TA)),\\
c_7 &=\lambda_{max}(I_N\otimes(B^TB))\\
c_8 &=\|(L+S)\otimes\Upsilon\|^2,
\end{aligned}$$

and choosing sufficiently large enough $\lambda>1+\sqrt{c_6}$, then

$$\begin{aligned}
\|\delta q_{k+1}(x,t)\|^2_{(L^2,\lambda)} &\le\frac{c_8}{1-\frac{c_6}{4(\lambda-1)^2}}\|e_k(x,0)\|^2_{L^2}\\
&\quad+\frac{\frac{1}{2(\lambda-1)}}{1-\frac{c_6}{4(\lambda-1)^2}}Nlr^k\\
&\quad+\frac{\frac{c_7}{4(\lambda-1)^2}}{1-\frac{c_6}{4(\lambda-1)^2}}\|\delta u_{k+1}(x,t)\|^2_{(L^2,\lambda)}.
\end{aligned} \tag{28}$$

Proof: From (10), we can see

$$\frac{\partial^2\delta q_k(x,t)}{\partial t^2}=(I_N\otimes D)\Delta\delta q_k(x,t)+(I_N\otimes A)\delta q_k(x,t) +(I_N\otimes B)\delta u_k(x,t). \tag{29}$$

Since

$$\frac{d}{dt}\Big\|\frac{\partial\delta q_{k+1}(x,t)}{\partial t}\Big\|^2_{L^2} =2\int_\Omega\Big\{\Big(\frac{\partial\delta q_{k+1}(x,t)}{\partial t}\Big)^T\frac{\partial^2\delta q_{k+1}(x,t)}{\partial t^2}\Big\}dx,$$

we get

$$\begin{aligned}
\frac{d}{dt}\Big\|\frac{\partial\delta q_{k+1}(x,t)}{\partial t}\Big\|^2_{L^2} &=2\int_\Omega\Big(\frac{\partial\delta q_{k+1}(x,t)}{\partial t}\Big)^T(I_N\otimes D)\Delta\delta q_{k+1}(x,t)dx\\
&\quad+2\int_\Omega\Big(\frac{\partial\delta q_{k+1}(x,t)}{\partial t}\Big)^T(I_N\otimes A)\delta q_{k+1}(x,t)dx\\
&\quad+2\int_\Omega\Big(\frac{\partial\delta q_{k+1}(x,t)}{\partial t}\Big)^T(I_N\otimes B)\delta u_{k+1}(x,t)dx\\
&:=I_1+I_2+I_3.
\end{aligned} \tag{30}$$

Integrating by parts and using the boundary condition (2), we have

$$I_1 = 2\Big(\frac{\partial \delta q_{k+1}(x,t)}{\partial t}\Big)^T (I_N \otimes D)\frac{\partial \delta q_{k+1}(x,t)}{\partial x}\Big|_\Omega - 2\int_\Omega \Big\{\Big(\frac{\partial^2 \delta q_{k+1}(x,t)}{\partial t \partial x}\Big)^T \times (I_N \otimes D)\frac{\partial \delta q_{k+1}(x,t)}{\partial x}\Big\}dx \le -c\frac{d}{dt}\Big\|\frac{\partial \delta q_{k+1}(x,t)}{\partial x}\Big\|^2_{L^2}, \quad (31)$$

where $c = \lambda_{min}(I_N \otimes D)$.

Using Lemma 1 to I_2 and I_3, we can obtain

$$I_2 \le \Big\|\frac{\partial \delta q_{k+1}(x,t)}{\partial t}\Big\|^2_{L^2} + c_6\|\delta q_{k+1}(x,t)\|^2_{L^2}, \quad (32)$$

$$I_3 \le \Big\|\frac{\partial \delta q_{k+1}(x,t)}{\partial t}\Big\|^2_{L^2} + c_7\|\delta u_{k+1}(x,t)\|^2_{L^2}, \quad (33)$$

where $c_6 = \lambda_{max}(I_N \otimes (A^T A))$, $c_7 = \lambda_{max}(I_N \otimes (B^T B))$.

Thus, from (30) to (33), it yields

$$\frac{d}{dt}\Big\|\frac{\partial \delta q_{k+1}(x,t)}{\partial t}\Big\|^2_{L^2} + c\frac{d}{dt}\Big\|\frac{\partial \delta q_{k+1}(x,t)}{\partial x}\Big\|^2_{L^2} \le 2\Big\|\frac{\partial \delta q_{k+1}(x,t)}{\partial t}\Big\|^2_{L^2} + c_6\|\delta q_{k+1}(x,t)\|^2_{L^2} + c_7\|\delta u_{k+1}(x,t)\|^2_{L^2}. \quad (34)$$

Integrating both sides of (34) above t and combining with the boundary condition (3), we can get

$$\Big\|\frac{\partial \delta q_{k+1}(x,t)}{\partial t}\Big\|^2_{L^2} \le \Big\|\frac{\partial \delta q_{k+1}(x,t)}{\partial t}\Big\|^2_{L^2} + c\Big\|\frac{\partial \delta q_{k+1}(x,t)}{\partial x}\Big\|^2_{L^2} \le Nlr^k + 2\int_0^t \Big\|\frac{\partial \delta q_{k+1}(x,s)}{\partial s}\Big\|^2_{L^2}ds + c_6\int_0^t \|\delta q_{k+1}(x,s)\|^2_{L^2}ds + c_7\int_0^t \|\delta u_{k+1}(x,s)\|^2_{L^2}ds. \quad (35)$$

Applying Gronwall inequality, we have

$$\Big\|\frac{\partial \delta q_{k+1}(x,t)}{\partial t}\Big\|^2_{L^2} \le Nlr^k e^{2t} + c_6\int_0^t e^{2(t-s)}\|\delta q_{k+1}(x,s)\|^2_{L^2}ds + c_7\int_0^t e^{2(t-s)}\|\delta u_{k+1}(x,s)\|^2_{L^2}ds. \quad (36)$$

As a result, for $\lambda > 1$,

$$\Big\|\frac{\partial \delta q_{k+1}(x,t)}{\partial t}\Big\|^2_{(L^2,\lambda)} \le Nlr^k + \frac{c_6}{2(\lambda-1)}\cdot\|\delta q_{k+1}(x,t)\|^2_{(L^2,\lambda)} + \frac{c_7}{2(\lambda-1)}\cdot\|\delta u_{k+1}(x,t)\|^2_{(L^2,\lambda)}. \quad (37)$$

On the other hand, using the basic inequality, we get

$$\frac{\partial(\|\delta q_{k+1}(x,t)\|^2_{L^2})}{\partial t} = 2\int_\Omega (\delta q_{k+1}(x,t))^T\frac{\partial \delta q_{k+1}(x,t)}{\partial t}dx \le \|\delta q_{k+1}(x,t)\|^2_{L^2} + \Big\|\frac{\partial \delta q_{k+1}(x,t)}{\partial t}\Big\|^2_{L^2}. \quad (38)$$

Integrating both sides of (38) above t and using (19), we have

$$\|\delta q_{k+1}(x,t)\|^2_{L^2} \le c_8\|e_k(x,0)\|^2_{L^2} + \int_0^t \|\delta q_{k+1}(x,s)\|^2_{L^2}ds + \int_0^t \Big\|\frac{\partial \delta q_{k+1}(x,s)}{\partial s}\Big\|^2_{L^2}ds, \quad (39)$$

where $c_8 = \|(L+S)\otimes \Upsilon\|^2$.

Applying Gronwall inequality, it yields,

$$\|\delta q_{k+1}(x,t)\|^2_{L^2} \le c_8\|e_k(x,0)\|^2_{L^2}e^t + \int_0^t e^{(t-s)}\Big\|\frac{\partial \delta q_{k+1}(x,s)}{\partial s}\Big\|^2_{L^2}ds. \quad (40)$$

Therefore, for $\lambda > 1$,

$$\|\delta q_{k+1}(x,t)\|^2_{(L^2,\lambda)} \le c_8\|e_k(x,0)\|^2_{L^2} + \frac{1}{(2\lambda-1)}\Big\|\frac{\partial \delta q_{k+1}(x,t)}{\partial t}\Big\|^2_{(L^2,\lambda)} \le c_8\|e_k(x,0)\|^2_{L^2} + \frac{1}{2(\lambda-1)}\Big\|\frac{\partial \delta q_{k+1}(x,t)}{\partial t}\Big\|^2_{(L^2,\lambda)}. \quad (41)$$

Substituting (37) into (41), for $\lambda > 1 + \sqrt{c_6}$, we have (28). The proof is complete.

Theorem 1: For MAS (1), considering PI-type ILC protocol (7), if

$$\|I_{Nm} - (L+S)\otimes C\Upsilon - (L+S)\otimes G\Lambda\|^2 < 1, \quad (42)$$

and

$$\|I_{Nm} - (L+S)\otimes G\Lambda\|^2 < \frac{1}{1+\epsilon_1+\epsilon_2}, \quad (43)$$

where ϵ_1, ϵ_2 are positive constants, then the consensus of MAS (1) can be achieved asymptotically.

Proof: It follows from Lemma 5 and Lemma 6, we have

$$\|\delta q_{k+1}(x,t)\|^2_{(L^2,\lambda)} \le \frac{c_8}{1-\frac{c_6}{4(\lambda-1)^2}}\|e_k(x,0)\|^2_{L^2} + \frac{\frac{1}{2(\lambda-1)}}{1-\frac{c_6}{4(\lambda-1)^2}}Nlr^k + \frac{\frac{c_7}{4(\lambda-1)^2}}{1-\frac{c_6}{4(\lambda-1)^2}}\cdot M\|e_k(x,t)\|^2_{(L^2,\lambda)}. \quad (44)$$

Substituting (44) into (22), for $\lambda > 1+\sqrt{c_6}$, we get

$$\begin{aligned}\| e_{k+1}(x,t)\|^2_{(L^2,\lambda)} &\leq (c_1+\frac{c_3}{\lambda^2})\|e_k(x,t)\|^2_{(L^2,\lambda)} \\ &+\frac{c_2c_8}{1-\frac{c_6}{4(\lambda-1)^2}}\|e_k(x,0)\|^2_{L^2} \\ &+\frac{\frac{c_2}{2(\lambda-1)}}{1-\frac{c_6}{4(\lambda-1)^2}}Nlr^k \\ &+\frac{\frac{c_2c_7}{4(\lambda-1)^2}}{1-\frac{c_6}{4(\lambda-1)^2}}\cdot M\|e_k(x,t)\|^2_{(L^2,\lambda)}. \end{aligned} \tag{45}$$

Obviously, by Lemma 4, we can see that

$$\lim_{k\to\infty} \| e_k(x,0) \|_{L^2}=0$$

via condition (42). Further, when λ is large enough, from (43), we can deduce that

$$\lim_{k\to\infty} \|e_k(x,t)\|^2_{(L^2,\lambda)}=0. \tag{46}$$

Note that

$$\begin{aligned}\|e_k(x,t)\|^2_{L^2} &\leq \|e_k(x,t)\|^2_{L^2}e^{-2\lambda t}e^{2\lambda T} \\ &\leq \|e_k(x,t)\|^2_{(L^2,\lambda)}e^{2\lambda T}. \end{aligned} \tag{47}$$

Then, it follows from (46), (47), we have

$$\lim_{k\to\infty} \| e_k(x,t) \|_{L^2}=0. \tag{48}$$

This completes the proof.

The following corollaries are obvious.

Corollary 1: For MAS (1) with the initial condition $q_{j,k}(x,0)=0$, considering PI-type ILC protocol,

$$u_{j,k+1}(x,t)=u_{j,k}(x,t)+\Lambda\eta_{j,k}(x,t)+\Gamma\int_0^t \eta_{i,k}(x,\tau)d\tau, \tag{49}$$

if

$$\|I_{Nm}-(L+S)\otimes G\Lambda\|^2 < \frac{1}{1+\epsilon_1+\epsilon_2}, \tag{50}$$

where ϵ_1, ϵ_2 are positive constants, then the consensus of MAS (1) can be achieved asymptotically.

Remark 3: The convergence condition (50) does not include the integration gain Γ. Therefore, PI-type ILC provides an extra freedom for the choices of the parameters in controller (49).

Corollary 2: For MAS (1) with the initial condition $q_{j,k}(x,0)=0$, considering P-type ILC protocol,

$$u_{j,k+1}(x,t)=u_{j,k}(x,t)+\Lambda\eta_{j,k}(x,t), \tag{51}$$

if

$$\|I_{Nm}-(L+S)\otimes G\Lambda\|^2 < \frac{1}{1+\epsilon_1}, \tag{52}$$

where ϵ_1 is a positive constant, then the consensus of MAS (1) can be achieved asymptotically.

Remark 4: In particular, if simply setting $\epsilon_1=1$, the convergence condition (52) becomes

$$\|I_{Nm}-(L+S)\otimes G\Lambda\|^2 < \frac{1}{2}, \tag{53}$$

which is the same result as that in [25]. Obviously, the convergence condition (52) is less conservative than (53) (See, e.g., the Example 2 in Section 4 Numerical Examples).

IV. NUMERICAL EXAMPLES

In this section, two numerical examples are presented to demonstrate the validity of the design method.

Example 1: Consider the second-order hyperbolic distributed parameter systems (1) with $D=I_2$ and

$$A=\begin{bmatrix}1.5 & 0\\ 0 & 0.5\end{bmatrix},\quad B=\begin{bmatrix}0.3 & 0\\ 0.25 & 0.2\end{bmatrix},$$
$$C=\begin{bmatrix}-0.2 & 0\\ 0 & 0.1\end{bmatrix},\quad G=\begin{bmatrix}1 & 0\\ 0 & 0.2\end{bmatrix},$$

and $(x,t)\in[0,1]\times[0,1]$.

Assume that the MAS consist of four agents. The Laplacian matrix is given as:

$$L=\begin{bmatrix}1 & -1 & 0 & 0\\ -1 & 2 & -1 & 0\\ 0 & 0 & 1 & -1\\ 0 & -1 & -1 & 2\end{bmatrix} \tag{54}$$

and $S=\mathrm{diag}\{1,0,1,0\}$. The desired reference trajectory (the trajectory of the virtual leader) is

$$y_r(x,t)=\begin{bmatrix}-4\sin(\pi t)\sin(2\pi x)\\ -4\sin(\pi t)\sin(2\pi x)\end{bmatrix}.$$

We use the ILC protocol (7) and take the learning gain matrices

$$\Lambda=\begin{bmatrix}0.3 & 0\\ 0 & 0.2\end{bmatrix},$$
$$\Gamma=\begin{bmatrix}0 & 0.10\\ 0.20 & 0\end{bmatrix},$$
$$\Upsilon=\begin{bmatrix}0.1 & 0\\ 0 & 0.2\end{bmatrix}.$$

Set $\epsilon_1=\epsilon_2=0.02$. It is not difficult to verify that the conditions in Theorem 1 are satisfied.

The numerical simulation is done with initial state values $q_{i,1}(x,0)=[0.02\,x, 0.01\sin x]^T$, $i=1,2,3,4$. The control input values at the beginning of learning are set to be 0.

Fig. 1 shows the output of four agents without controller. Fig. 2 is the virtual leader and the output of four agents at the number of iterations $k=6$. It can be seen that the output of the four agents is almost the same as the output of the virtual leader. Fig. 3 shows the L_2 norm of the tracking errors for all agents in each iteration. When the number of iterations is from $k=1$ to $k=10$, the tracking errors between them gradually converges to zero in the sense of L_2 norm. Therefore, applying the designed PI-type iterative control algorithm to the above MAS, the consensus of $q_i(x,t)(i=1,2,3)$ can be reached.

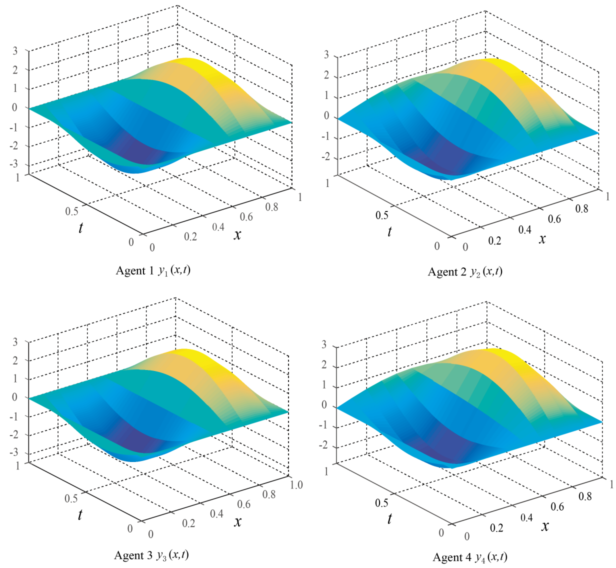

FIGURE 1. The output of four agents without controller.

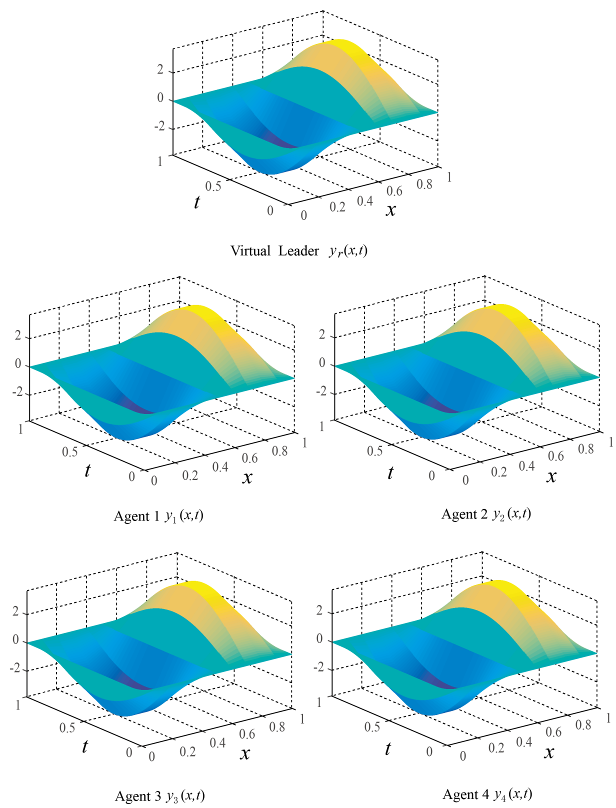

FIGURE 2. The virtual leader and the output of four agents at $k = 6$.

Table 1 shows comparison of tracking errors with different ILC in 10th iteration. The errors value in the 10th iteration is always smaller than 0.002. But the tracking errors under PI-type ILC are smaller than those of P-type ILC, which displays that PI-type ILC surpasses P-type ILC.

Example 2: Consider MAS with the same parameters and conditions in Example 1 but $q_{j,k}(x,0) = 0$. In this case,

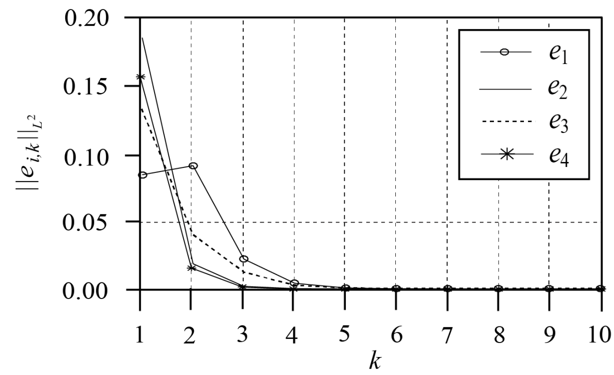

FIGURE 3. The L_2 norm of the tracking errors for all agents in each iteration.

TABLE 1. Tracking errors with different ILC.

Agent	P-type ILC [25]	PI-type ILC
1	0.001263	0.001142
2	0.001865	0.001672
3	0.001749	0.001331
4	0.001982	0.001877

we use the ILC protocol (51) and let the learning gain matrix

$$\Lambda = \begin{bmatrix} 0.6 & 0.1 \\ 0.8 & 2 \end{bmatrix}.$$

It is easy to find that

$$\|I_8 - (L + S) \otimes G\Lambda\|^2 = 0.8468 > \frac{1}{2}, \tag{55}$$

which implies the convergence condition in [25] is not satisfied. However,

$$\|I_8 - (L + S) \otimes G\Lambda\|^2 = 0.8468 < \frac{1}{1 + 0.1}. \tag{56}$$

That is to say, if we set $\epsilon_1 = 0.1$, then the condition in Corollary 2 is valid. So, we can use the ILC protocol (51), which indicates that the convergence condition presented in this paper is less conservative than that in [25].

V. CONCLUSION

In this paper, the consensus control of MAS described by second-order hyperbolic distributed parameter models is studied. By using the knowledge of neighbors and considering the information transfer between any two agents, a PI-type ILC protocol is proposed. The consensus condition is derived, which is less conservative than the existing one in the case of P-type ILC protocol. Two examples are simulated and verified the effectiveness of the proposed algorithms.

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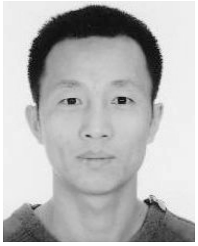

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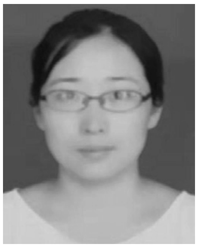

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