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# Optimal Maintenance Decision Based on Remaining Useful Lifetime Prediction for the Equipment Subject to Imperfect Maintenance

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**ABSTRACT** Focusing on the fact that the existing research on optimal maintenance decision for remaining useful lifetime (RUL) prediction and imperfect maintenance has low accuracy of RUL prediction and rationality of decision results, an optimal maintenance decision method based on RUL prediction for the equipment subject to imperfect maintenance is proposed in this paper. Firstly, the nonlinear Wiener process is used to characterize the degradation law of the equipment. Secondly, the imperfect maintenance model that meets the upper limit of the maintenance number is established based on the nonhomogeneous Poisson process. Then, based on the concept of the first hitting time, the probability density function (PDF) of the RUL is derived. Finally, based on the RUL prediction results, the optimal maintenance decision model for the equipment subject imperfect maintenance is constructed. Through the example verification and cost parameter sensitivity analysis, the proposed method can effectively improve the accuracy of the RUL prediction and the scientific of maintenance decision results, which has engineering application value.

**INDEX TERMS** Maintenance decision, imperfect maintenance, remaining useful lifetime prediction, nonlinear Wiener process, nonhomogeneous Poisson process.

## I. INTRODUCTION

Since the industrial revolution, production has improved dramatically in terms of technology and automation levels, and traditional manufacturing enterprises are facing more challenges than ever. As a result, maintenance capacity is playing an increasingly important role in enterprise competitiveness [1]–[4]. To further improve enterprises' maintenance capacity and ensure the economic efficiency and safety of production processes, prognostics and health management (PHM) have gradually emerged, and numerous researches have been achieved for this technique [5]–[9].

In essence, PHM obtains the characteristic information of the equipment using advanced sensor technology and then predicts the variation trend of its health state; thus, it formulates and implements an optimal maintenance

decision [10]. In a general sense, PHM has two core elements, namely, prediction of the equipment RUL and condition-based maintenance (CBM) of the equipment based on the RUL prediction. Based on maintenance results, CBM can be categorized into three types: perfect maintenance (PM), imperfect maintenance (IM), and minimal maintenance (MM) [11]–[13]. PM refers to maintenance actions that can restore the equipment to a brand-new condition. However, equipment degradation is often irreversible. As a result, PM is unattainable in practice during production processes. MM refers to maintenance actions that have basically no impact on the technical condition of the equipment. Due to economic considerations, the application of MM is also relatively limited in practice during production processes. IM refers to a maintenance approach that produces a result between those of PM and MM. IM improves the technical condition of the equipment somewhat but does not come at an immense cost. Consequently, IM has been extensively applied

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in industrial production processes [14]. To date, studies of optimal maintenance decisions for the equipment subject to IM based on RUL predictions have been relatively rarely reported. Ge *et al.* [15] fitted the degradation path of the equipment to a Gamma process and predicted the RUL of the equipment. On this basis, they constructed an optimal maintenance decision model that accounts for IM and determined an optimal inspection cycle length and an optimal number of IM actions. Owing to its strict monotony, it is difficult to apply a Gamma process to the prevalent nonmonotonic degradation behavior in production activities. As a consequence, Ge *et al.*'s model has a limited scope of application. In view of the deficiencies in Gamma models, Guo *et al.* [16] noted that after IM to improve the equipment status, residual degradation remained. They proposed to introduce residual degradation into a Wiener process model. On this basis, they derived a probability density function (PDF) for the equipment RUL that accounts for residual degradation. Furthermore, they constructed an optimal maintenance decision model based on RUL predictions and determined an optimal preventive maintenance (PvM) threshold for the equipment. On the other hand, Zhang *et al.* [17] noted that IM affects the equipment degradation rate. They constructed an optimal maintenance decision model by introducing a drift coefficient improvement factor to a Wiener degradation model and determined an optimal maintenance decision for airborne gyroscopes. The above two methods take into consideration only the effect of IM on the extent and rate of degradation of the equipment. Thus, Pei *et al.* [18] proposed an optimal maintenance decision model for the equipment subject to IM that comprehensively accounts for the extent and rate of degradation and determined an optimal inspection interval and an optimal PvM threshold for the equipment. However, in degradation modeling, the above RUL prediction-based optimal maintenance decisions for the equipment subject to IM all assume that the degradation process is linear and overlook the nonlinear degradation prevalent in real-world environments. This reduces RUL prediction accuracy and affects the scientific basis of strategies. To further improve RUL prediction accuracy for the equipment subject to IM, Hu *et al.* [19] constructed a degradation model for the equipment subject to IM actions based on a nonlinear Wiener diffusion process and analyzed the effect of IM on both the extent and rate of degradation. On this basis, they expanded the scope of application of the method and improved prediction accuracy. However, Hu *et al.*'s method can be used to predict the RUL of the equipment only within a specific degradation stage but is unable to cover the full life cycle of the equipment. Additionally, the maintenance decision did not investigate in this study. By integrating nonlinear Wiener processes at various degradation stages based on the properties of the inverse Gaussian distribution, Zheng *et al.* [20] predicted the RUL in full life cycle of the equipment subject to IM. However, Zheng *et al.*'s method requires a given PvM threshold, which further introduces human errors and is unfavorable to the improvement of prediction accuracy. Additionally, Zheng *et al.* did not

analyze system maintenance decisions. In view of the above problems, assuming that the PvM threshold is unknown, Wang *et al.* [21] modeled the cumulative effect of IM actions using a nonlinear Wiener process and a homogeneous Poisson process and thus improved RUL prediction accuracy. Similarly, this method did not study the maintenance decision of the equipment. Moreover, there is also a potential unrealistic condition in Wang *et al.*'s method — the number of IM activities can be infinite. However, due to the irreversibility of the equipment degradation process and the economic requirements of production activities, there is an upper limit for the total number of IM events during the equipment's life cycle. As a result, Wang *et al.*'s method is also unable to accurately predict the RUL of the equipment subject to IM.

In view of the problems in the available studies on optimal maintenance decisions based on RUL predictions for the equipment subject to IM, in this study, based on the RUL prediction method proposed by Wang *et al.* [21], IM actions are described using a nonhomogeneous Poisson process. Additionally, by introducing an upper limit constraint for the number of IM activities, the degradation pattern of the equipment subject to IM is depicted accurately, and RUL prediction accuracy for the equipment subject to IM is effectively improved. On this basis, an optimal maintenance decision model that accounts for RUL predictions is constructed using the renewal reward theorem. By jointly optimizing the equipment inspection cycle length and the PvM threshold, the average maintenance cost in the whole life of the equipment is minimized.

## II. DEGRADATION MODELING OF THE EQUIPMENT SUBJECT TO IM

### A. DEGRADATION MODEL

In this study, the randomness of the equipment degradation process is described using a nonlinear Wiener process as follows:

$$X^D(t) = X^D(0) + \alpha\psi(t, \beta) + \sigma_B B(t) \quad (1)$$

where  $X^D(t)$  is the extent of performance degradation of the equipment at the time point  $t$ ,  $X^D(0)$  is the initial degradation state (it is generally assumed that  $X^D(0) = 0$ ),  $\alpha$  is the drift coefficient ( $\alpha \sim N(\mu_\alpha, \sigma_\alpha^2)$ ),  $\psi(t, \beta)$  is a continuous function of  $t$  (generally,  $\psi(t, \beta) = t^\beta$ ),  $\sigma_B$  is the diffusion coefficient, and  $B(t)$  is a standard Brownian motion.

### B. IM MODEL

In this study, a compound nonhomogeneous Poisson process is used to describe the IM process to which the equipment is subject to reflect the randomness of IM. It is assumed that these IM actions meet the following conditions:

(1) The equipment is subject to IM. After the IM, the equipment status will be between "as good as new" and "as bad as old"; i.e., the extent of recovery of the performance index ( $X^{E_k}$ ) meets the following condition:  $0 < X^{E_k} < X^D(T_k)$ , where  $X^D(T_k)$  is the extent of performance degradation of the

equipment before the IM and the corresponding  $T_k$  is the time at which the IM is performed.

(2) The duration of maintenance to which the equipment is subject each time is far shorter than its life cycle and thus negligible.

(3) The recovery extent of the performance index of the equipment ( $X^{E_i}$ ) resulting from IM actions each time is independent of those resulting from IM actions at other times and satisfies a common random distribution:  $f_{X^E}(X^E | \varphi)$ , where  $\varphi$  is the distribution parameter.

(4) The number of occasions of IM ( $N(t_k - t_{k-1})$ ) to which the equipment is subject within any arbitrary interval  $(t_{k-1}, t_k]$  within its life cycle  $(0, T]$  satisfies a nonhomogeneous Poisson distribution with  $\lambda(t | \eta)$  as its intensify function. Thus, we have

$$P = \frac{(N(t_k - t_{k-1}) = n_k) (m(t_k) - m(t_{k-1}))^{n_k}}{n_k!} \exp(m(t_{k-1}) - m(t_k)) \quad (2)$$

where  $m(t) = \int_0^t \lambda(\delta | \eta) d\delta$ ,  $n_k$  is the number of occasions of IM within the interval  $(t_{k-1}, t_k]$  ( $t_0 = 0, t_{k-1} < t_k$ ),  $\lambda(t | \eta)$  is a function of time  $t$  ( $\lambda(t | \eta) \geq 0$ ), and  $\eta$  is a parameter.

In summary, IM actions to which the equipment is subject can be represented by the following:

$$X^M(t) = \sum_{i=0}^{N(t)} X^{E_i} \quad (3)$$

where  $N(t) = N(t - t_0)$  and  $E_0 = 0$ .

### C. DEGRADATION MODEL FOR THE EQUIPMENT SUBJECT TO IM

Based on the random system-degradation process, the effect of IM on system performance degradation is taken into consideration. Thus, a comprehensive system-degradation model is constructed as follows:

$$X(t) = X^D(t) - X^M(t) \quad (4)$$

where the negative sign means that IM can help to improve the degraded state of the equipment.

### III. PARAMETER ESTIMATIONS FOR THE DEGRADATION MODEL WITH IM

It is assumed that the degradation data of  $N$  equipment are available. Let  $X(t_{i,j})$  be the extent of performance degradation of the  $i$ th ( $i = 1, 2, \dots, N$ ) equipment at the  $j$ th ( $j = 1, 2, \dots, m_i$ ) time point. Thus,  $X_i = [X(t_{i,1}), X(t_{i,2}), \dots, X(t_{i,m_i})]$  is all the degradation data for the  $i$ th equipment. Let  $T_{i,k}$  and  $X^{E_{i,k}}$  be the time at which the  $i$ th equipment is subject to the  $k$ th ( $k = 1, 2, \dots, d_i$ ) IM and the corresponding extent of recovery of the performance index of the equipment, respectively. Thus,  $d_i$  is the total number of occasions of IM to which the  $i$ th equipment is subject.

### A. ESTIMATION OF THE PARAMETER $\eta$

To ensure the safety and economic efficiency of the full lifecycle operation of the equipment, the number of IM for the equipment is not infinite in engineering practice but instead has an upper limit (denoted by  $a$ ). In this study, it is assumed on this basis that the intensity of IM to which the equipment is subject within the interval  $(t, t + s]$  is directly proportional to the remaining number of occasions of IM (their ratio is denoted by  $b$ ). Thus, based on the above analysis, we have

$$\begin{cases} \lim_{t \rightarrow +\infty} m(t) = a, & t > 0 \\ \lim_{t \rightarrow 0^+} m(t) = 0, & t > 0 \\ \frac{m(t+s) - m(t)}{(t+s) - t} = b(a - m(t)), & t > 0, s > 0 \end{cases} \quad (5)$$

Let  $s \rightarrow 0^+$ . Thus, we have

$$\begin{cases} m'(t) = b(a - m(t)) \\ m(0) = 0, t > 0 \\ m(+\infty) = a \end{cases} \quad (6)$$

By solving the differential equation in Equation (6), we have

$$\begin{cases} m(t) = a(1 - \exp(-bt)) \\ \lambda(t) = ab \exp(-bt) \end{cases} \quad (7)$$

In this study, the parameter  $\eta$  ( $\eta = (a, b)$ ) of the intensity function of the nonhomogeneous Poisson distribution is estimated using the maximum likelihood method.

Based on Equation (2), the likelihood function of the intensity parameters  $a$  and  $b$  can be determined:

$$\begin{aligned} L(a, b) &= \prod_{i=1}^N \prod_{k=1}^{d_i} \frac{(a(\exp(-bT_{i,k-1}) - \exp(-bT_{i,k})))^{n_k}}{n_k!} \\ &\cdot \exp(-a(\exp(-bT_{i,k-1}) - \exp(-bT_{i,k}))) \end{aligned} \quad (8)$$

It is easy to conclude that the number of occasions of IM within an arbitrary interval  $(T_{i,k-1} - T_{i,k}]$  is constant and equals 1, i.e.,  $n_k = 1$ . Thus, Equation (8) is equivalent to

$$\begin{aligned} L(a, b) &= \prod_{i=1}^N \prod_{k=1}^{d_i} a(\exp(-bT_{i,k-1}) - \exp(-bT_{i,k})) \\ &\cdot \exp(-a(\exp(-bT_{i,k-1}) - \exp(-bT_{i,k}))) \end{aligned} \quad (9)$$

Taking the logarithm of Equation (9), we have

$$\begin{aligned} \ln(L(a, b)) &= \sum_{i=1}^N \sum_{k=1}^{d_i} \ln(a(\exp(-bT_{i,k-1}) - \exp(-bT_{i,k}))) \\ &- a \sum_{i=1}^N (1 - \exp(bT_{i,d_i})) \end{aligned} \quad (10)$$

Calculating the partial derivatives of Equation (10) with respect to  $a$  and  $b$ , respectively, we have

$$\frac{\partial \ln(L(a, b))}{\partial a} = \sum_{i=1}^N \frac{d_i}{a} - \sum_{i=1}^N (1 - \exp(bT_{i,d_i})) \quad (11)$$

$$\frac{\partial \ln(L(a, b))}{\partial b} = \sum_{i=1}^N \sum_{k=1}^{d_i} \frac{T_{i,k} \exp(-bT_{i,k}) - T_{i,k-1} \exp(-bT_{i,k-1})}{\exp(-bT_{i,k-1}) - \exp(-bT_{i,k})} - a \sum_{i=1}^N T_{i,d_i} \exp(-bT_{i,d_i}) \quad (12)$$

Let Equations (11) and (12) equal 0, respectively. Thus, we have

$$\hat{a} = \sum_{i=1}^N d_i / \sum_{i=1}^N (1 - \exp(\hat{b}T_{i,d_i})) \quad (13)$$

$$\times \sum_{i=1}^N \sum_{k=1}^{d_i} \frac{T_{i,k} \exp(-\hat{b}T_{i,k}) - T_{i,k-1} \exp(-\hat{b}T_{i,k-1})}{\exp(-\hat{b}T_{i,k-1}) - \exp(-\hat{b}T_{i,k})} = \hat{a} \sum_{i=1}^N T_{i,d_i} \exp(-\hat{b}T_{i,d_i}) \quad (14)$$

By simultaneously solving Equations (13) and (14), estimated values of the parameters  $a$  and  $b$  ( $\hat{a}$ ,  $\hat{b}$ , respectively) are obtained.

### B. ESTIMATION OF THE PARAMETER $\varphi$

The parameter  $\varphi$  is estimated using the maximum-likelihood method.

It is straightforward that the likelihood function corresponding to the parameter  $\varphi$  is

$$L(\varphi | X^E) = \prod_{i=1}^N \prod_{k=1}^{d_i} f_{X^E}(X^{E_{i,k}} | \varphi) \quad (15)$$

By maximizing Equation (15), the estimated value of the parameter  $\varphi$  ( $\hat{\varphi}$ ) can be obtained.

### C. ESTIMATION OF THE PARAMETER $\mu_\alpha, \sigma_\alpha, \beta, \sigma_B$

Clearly, the parameters  $\mu_\alpha, \sigma_\alpha, \beta, \sigma_B$  are components of the random system degradation model. In this study, these parameters are estimated using the maximum-likelihood method.

Based on Equation (4), we have

$$X^D(0) + \alpha t^\beta + \sigma_B B(t) = X(t) + \sum_{i=0}^{N(t)} X^{E_i} \quad (16)$$

If we let

$$Y(t) = X(t) + \sum_{i=0}^{N(t)} X^{E_i}, X^{E_0} = 0 \quad (17)$$

then we have

$$Y(0) = 0 \quad (18)$$

Equation (16) is further transformed into

$$Y(t) = Y(0) + \alpha t^\beta + \sigma_B B(t) \quad (19)$$

$Y(t)$  represents the equivalent extent of performance degradation of the equipment. A comparison of Equations (19) and (1) finds that  $Y(t)$  follows a nonlinear Wiener process. Thus, based on the fundamental properties of a nonlinear Wiener process [21], a logarithmic likelihood function of  $\mu_\alpha, \sigma_\alpha, \beta, \sigma_B$  with respect to the extent of performance degradation ( $Y$ ) can be obtained as shown in Equation (20).

$$\ln L(Y) = -\frac{\ln(2\pi)}{2} \sum_{i=1}^N m_i - \frac{1}{2} \sum_{i=1}^N \ln(|\Sigma_i|) - \frac{1}{2} \sum_{i=1}^N (\Delta Y_i - \mu_\alpha \Delta \psi_i)' \Sigma_i^{-1} (\Delta Y_i - \mu_\alpha \Delta \psi_i) \quad (20)$$

where  $Y = [\Delta Y_1, \Delta Y_2, \dots, \Delta Y_N]$ ,  $\Delta Y_i = [\Delta Y_{i,1}, \Delta Y_{i,2}, \dots, \Delta Y_{i,m_i}]'$ ,  $\Delta Y_{i,j} = Y(t_{i,j}) - Y(t_{i,j-1})$ ,  $t_{i,0} = 0$ ,  $\Sigma_i = \sigma_\alpha^2 \Delta \psi_i \Delta \psi_i' + \sigma_B^2 \Omega_i$ ,  $\Omega_i = \text{diag}(\Delta t_{1,i}, \Delta t_{2,i}, \dots, \Delta t_{j,i})$ ,  $\Delta \psi_i = [\Delta \psi_{i,1}, \Delta \psi_{i,2}, \dots, \Delta \psi_{i,m_i}]'$ , and  $\Delta \psi_{i,j} = \psi(t_{i,j}, \beta) - \psi(t_{i,j-1}, \beta)$ ,  $\Delta t_{i,j} = t_{i,j} - t_{i,j-1}$ .

To facilitate description, let

$$\bar{\sigma}_B^2 = \frac{\sigma_B^2}{\sigma_\alpha^2} \quad (21)$$

$$\bar{\Sigma}_i = \frac{\Sigma_i}{\sigma_\alpha^2} \quad (22)$$

Thus, Equation (20) is equivalent to

$$\ln L(Y) = -\frac{\ln(2\pi)}{2} \sum_{i=1}^N m_i - \frac{1}{2} \ln \sigma_\alpha^2 \sum_{i=1}^N m_i - \frac{1}{2\sigma_\alpha^2} \sum_{i=1}^N (\Delta Y_i - \mu_\alpha \Delta \psi_i)' \bar{\Sigma}_i^{-1} (\Delta Y_i - \mu_\alpha \Delta \psi_i) - \frac{1}{2} \sum_{i=1}^N \ln(|\bar{\Sigma}_i|) \quad (23)$$

By calculating the partial derivatives of the likelihood function  $\ln L(Y)$  with respect to  $\mu_\alpha$  and  $\sigma_\alpha^2$ , respectively, we have

$$\frac{\partial \ln L(Y)}{\partial \mu_\alpha} = \frac{1}{\sigma_\alpha^2} \left( \sum_{i=1}^N \Delta \psi_i' \bar{\Sigma}_i^{-1} \Delta Y_i - \mu_\alpha \sum_{i=1}^N \Delta \psi_i' \bar{\Sigma}_i^{-1} \Delta \psi_i \right) \quad (24)$$

$$\begin{aligned} & \frac{\partial \ln L(Y)}{\partial \sigma_\alpha^2} \\ &= -\frac{1}{2\sigma_\alpha^2} \sum_{i=1}^N m_i + \frac{1}{2(\sigma_\alpha^2)^2} \\ & \quad \times \sum_{i=1}^N (\Delta Y_i - \mu_\alpha \Delta \psi_i)' \bar{\Sigma}_i^{-1} (\Delta Y_i - \mu_\alpha \Delta \psi_i) \end{aligned} \quad (25)$$

By setting the partial derivatives to zero, estimated values of  $\mu_\alpha$  and  $\sigma_\alpha^2$  can be obtained:

$$\hat{\mu}_\alpha = \frac{\sum_{i=1}^N \Delta \psi_i' \bar{\Sigma}_i^{-1} \Delta Y_i}{\sum_{i=1}^N \Delta \psi_i' \bar{\Sigma}_i^{-1} \Delta \psi_i} \quad (26)$$

$$\hat{\sigma}_\alpha^2 = \frac{\sum_{i=1}^N (\Delta Y_i - \hat{\mu}_\alpha \Delta \psi_i)' \bar{\Sigma}_i^{-1} (\Delta Y_i - \hat{\mu}_\alpha \Delta \psi_i)}{\sum_{i=1}^N m_i} \quad (27)$$

Because  $\bar{\Sigma}_i$  contains hidden parameters  $\bar{\sigma}_B^2$  and  $\beta$ , it is impossible to directly determine  $\hat{\mu}_\alpha$  and  $\hat{\sigma}_\alpha$ . To estimate these parameters, Equations (26) and (27) are substituted into Equation (23). Thus, a logarithmic likelihood function of  $Y$  with respect to  $\bar{\sigma}_B^2$  and  $\beta$  is obtained:

$$\ln L(Y) = -\frac{1 + \ln(2\pi) + \ln \hat{\sigma}_\alpha^2}{2} \sum_{i=1}^N m_i - \frac{1}{2} \sum_{i=1}^N \ln(|\bar{\Sigma}_i|) \quad (28)$$

By maximizing Equation (28), estimated values of  $\bar{\sigma}_B^2$  and  $\beta$  can be obtained. Then, by substituting the estimated values of  $\bar{\sigma}_B^2$  and  $\beta$  into Equations (21), (24), and (25), respectively,  $\hat{\sigma}_B$ ,  $\hat{\mu}_\alpha$  and  $\hat{\sigma}_\alpha$  can be determined.

#### IV. RUL PREDICTIONS FOR THE EQUIPMENT SUBJECT TO IM

The life of the equipment refers to the time at which the extent of its performance degradation reaches the failure threshold for the first time (first hitting time). If only the performance degradation of the equipment is taken into account while ignoring IM, then its life  $T$  can be represented by the following:

$$T = \inf \left\{ t : X^D(t) \geq \omega \mid X^D(0) < \omega \right\} \quad (29)$$

where  $\omega$  is the failure threshold.

A previous study [22] demonstrated that a  $T$  corresponding to a nonlinear Wiener process approximately follows an inverse Gaussian distribution and gave the PDF of  $T$  under a fixed failure threshold:

$$\begin{aligned} & f_{T|\omega}(t|\omega) \\ & \approx \frac{1}{\sqrt{2\pi t^2 (\psi(t, \beta)^2 \sigma_\alpha^2 + \sigma_B^2 t)}} \end{aligned}$$

$$\begin{aligned} & \cdot \exp \left( -\frac{(\omega - \psi(t, \beta)\mu_\alpha)^2}{2(\psi(t, \beta)^2 \sigma_\alpha^2 + \sigma_B^2 t)} \right) \\ & \cdot \left( \omega - \xi(t)\mu_\alpha - \frac{\omega - \psi(t, \beta)\mu_\alpha}{\psi(t, \beta)^2 \sigma_\alpha^2 + \sigma_B^2 t} \psi(t, \beta)\sigma_\alpha^2 \right) \end{aligned} \quad (30)$$

where  $\xi(t) = \psi(t, \beta) - t (d\psi(t, \beta)/dt)$ .

Further analysis finds that the RUL of the equipment at the time point  $t_k$  ( $l_k$ ) meets the following condition:  $T = t_k + l_k$ . On this basis, the RUL of the equipment at the time point  $t_k$  can be defined as follows:

$$L = \inf \left\{ l_k : X^D(t_k + l_k) \geq \omega \mid X^D(0) < \omega \right\} \quad (31)$$

If we let  $X^U(l_k) = X^D(t_k + l_k) - X^D(t_k)$ , then we have

$$X^U(l_k) = \alpha v(l_k) + \sigma_B B(l_k) \quad (32)$$

where  $v(l_k) = \psi(t_k + l_k, \beta) - \psi(t_k, \beta)$ . Without loss of generality,  $X^U(0)$  can be set to 0.

It is clear that  $X^U(l_k)$  is equivalent to the nonlinear Wiener process shown in Equation (1). Thus, we have

$$L = \inf \left\{ l_k : X^U(l_k) \geq \omega - d_k \mid X^U(0) < \omega - d_k \right\} \quad (33)$$

On this basis, a conditional PDF of the equipment under a fixed failure threshold can be obtained:

$$\begin{aligned} & f_{T|\omega}(t|\omega) \\ & \approx \frac{1}{\sqrt{2\pi l_k^2 (v(l_k)^2 \sigma_\alpha^2 + \sigma_B^2 l_k)}} \\ & \cdot \exp \left( -\frac{(\omega - d_k - v(l_k)\mu_\alpha)^2}{2(v(l_k)^2 \sigma_\alpha^2 + \sigma_B^2 l_k)} \right) \\ & \cdot \left( \omega - d_k - \Lambda(l_k)\mu_\alpha - \frac{\omega - d_k - v(l_k)\mu_\alpha}{v(l_k)^2 \sigma_\alpha^2 + \sigma_B^2 l_k} v(l_k)\sigma_\alpha^2 \right) \end{aligned} \quad (34)$$

where  $\Lambda(l_k) = v(l_k) - l_k (dv(l_k)/dl_k)$ .

Considering the effect of IM on the equipment degradation state, and assuming  $X^W(l_k) = X(t_k + l_k) - X(t_k)$ , we have

$$\begin{aligned} & X^W(l_k) \\ &= X^D(t_k + l_k) - X^D(t_k) - (X^M(t_k + l_k) - X^M(t_k)) \\ &= \alpha v(l_k) + \sigma_B B(l_k) - \left( \sum_{i=0}^{N(t_k+l_k)} X^{E_i} - \sum_{i=0}^{N(t_k)} X^{E_i} \right) \\ & X^U(l_k) - \left( \sum_{i=0}^{N(t_k+l_k)} X^{E_i} - \sum_{i=0}^{N(t_k)} X^{E_i} \right) = \end{aligned} \quad (35)$$

I.e.,

$$X^U(l_k) = X^W(l_k) + \sum_{i=0}^{N(t_k)} X^{E_i} \quad (36)$$

where  $\sum_{i=0}^{N(l_k)} X^{E_i} = \sum_{i=0}^{N(t_k+l_k)} X^{E_i} - \sum_{i=0}^{N(t_k)} X^{E_i}$ .

On this basis, the RUL of the equipment that accounts for the effect of IM can be defined as follows:

$$L = \inf \left\{ l_k : X^U(l_k) \geq \omega + \sum_{i=0}^{N(l_k)} X^{E_i} - X_k \mid X^U(0) < \omega + \sum_{i=0}^{N(l_k)} X^{E_i} - X_k \right\} \quad (37)$$

A comparison of Equations (37) and (33) finds that the effect of IM on the RUL of the equipment is equivalent to changing the originally fixed failure threshold  $\omega$  to a variable failure threshold  $\omega^* = \omega + \sum_{i=0}^{N(l_k)} X^{E_i}$ . Based on the previous analysis, IM  $X^M(t)$  is a compound nonhomogeneous Poisson process. Thus, based on the law of total probability, we have

$$\begin{aligned} f_{L_k}(l_k) &= E_{\omega^*} (f_{L_k|\omega^*}(l_k|\omega^*)) \\ &= \sum_{i=0}^{\infty} f_{\Omega} \left( \frac{1}{\sqrt{2\pi l_k^2 (v(l_k)^2 \sigma_{\alpha}^2 + \sigma_B^2 l_k)}} \right) \\ &\quad \cdot \left( \omega + R^i - X_k - \Lambda(l_k) \mu_{\alpha} - \frac{\omega + R^i - X_k - v(l_k) \mu_{\alpha}}{v(l_k)^2 \sigma_{\alpha}^2 + \sigma_B^2 l_k} v(l_k) \sigma_{\alpha}^2 \right) \\ &\quad \cdot \exp \left( - \frac{(\omega + R^i - X_k - v(l_k) \mu_{\alpha})^2}{2 (v(l_k)^2 \sigma_{\alpha}^2 + \sigma_B^2 l_k)} \right) \\ &\quad \cdot f_{R^i}^i (X^{R^i} | \tau) dR^i \frac{(a (\exp (b(l_k + t_k)) - \exp (bt_k)))^i}{i!} \\ &\quad \cdot \exp (-a (a (\exp (b(l_k + t_k)) - \exp (bt_k)))) \quad (38) \end{aligned}$$

where  $X^{R^i} = \sum_{j=0}^i X^{E_j}$ ,  $f_{X^{R^i}}^i (X^{R^i} | \tau)$  is the PDF of  $X^{R^i}$  ( $\tau$  is a distribution parameter) and  $\Omega$  is the value range of  $X^{R^i}$ .

In engineering applications, the average predicted RUL is generally used as the predicted RUL, which is shown as follows:

$$l_k^p = E(l_k) = \int_0^{+\infty} f_{L_k}(l_k) l_k dl_k \quad (39)$$

### V. A MAINTENANCE DECISION MODEL FOR THE EQUIPMENT SUBJECT TO IM

To determine optimal maintenance decisions for the equipment subject to IM, in this study a maintenance decision model is constructed based on the renewal reward theorem [23]. The average life-cycle maintenance cost ( $C(T)$ ) for the equipment is treated as an objective function, and the PvM threshold for the equipment (i.e., the RUL threshold ( $l_{pr}$ ) in this study) and the inspection cycle length ( $\Delta\tau_{CM}$ ) are treated as strategy variables. By minimizing the  $C(T)$ , an optimal maintenance decision is obtained. The maintenance decision model is represented by the following:

$$\min C(\Delta\tau_{CM}, l_{pr}) = \lim_{T \rightarrow \infty} \frac{C(T)}{T} = \frac{E(C(T))}{E(T)} \quad (40)$$

where  $\Delta\tau_{CM}$  is the inspection cycle length,  $l_{pr}$  is the RUL threshold for the equipment, and  $T$  is the life of the equipment,  $C(T)$  is the total maintenance cost within the life cycle of the equipment, and  $E(\cdot)$  signifies calculation of the expected value.

Additionally, the basic assumptions for the maintenance decision model are provided as follows. 1) The inspection method is perfect and can accurately detect the degradation states of the equipment without the need to shut the equipment down. Let  $C_{CM}$  be the inspection cost for each time. 2) Compared to the run time of the equipment, the duration of IM (equivalent to the downtime of the equipment) is negligible. Let  $C_{im}$  be the maintenance cost for each time. 3) A preventive replacement (PvR) is performed when the equipment operates fault-free until  $k\Delta\tau_{CM}$  and its RUL  $l_{k\Delta\tau_{CM}} < l_{pr}$ . Let  $C_{pr}$  be the replacement cost. 4) If the equipment breaks down between two consecutive inspections, a failure replacement (FR) needs to be performed immediately. Let  $C_{fr}$  be the sum cost of the replacement and the additional loss resulting from the sudden malfunction. In normal circumstances,  $C_{CM} < C_{im} < C_{pr} < C_{fr}$ .

Based on the above analysis, the  $C(T)$  for the equipment includes  $C_{CM}$ ,  $C_{im}$ ,  $C_{pr}$ , and  $C_{fr}$ , i.e.,

$$E(C(T)) = C_{CM} E(N_{CM}) + C_{im} E(N_{im}) + C_{pr} P_{pr}(\Delta\tau_{CM}, l_{pr}) + C_{fr} P_{fr}(\Delta\tau_{CM}, l_{pr}) \quad (41)$$

where  $P_{pr}(\Delta\tau_{CM}, l_{pr})$  is the probability that the equipment needs a PvR and  $P_{fr}(\Delta\tau_{CM}, l_{pr})$  is the probability that the equipment needs an FR.

The equipment is generally subject to inspections and IM several times within one life cycle and enters the next new life cycle upon being subject to a PvR or FR, as shown in Figure 1.

As demonstrated in Figure 1(a), the equipment is subject to an FR between two consecutive inspections, i.e., the life of the equipment is between  $(k-1)\Delta\tau_{CM}$  and  $k\Delta\tau_{CM}$ , which is equivalent to the RUL of the equipment at the time point  $(k-1)\Delta\tau_{CM}$  being less than  $\Delta\tau_{CM}$ . Thus, the probability that the equipment is subject to an FR within the interval  $((k-1)\Delta\tau_{CM}, k\Delta\tau_{CM})$  is as follows:

$$\begin{aligned} P_{fr,k}(\Delta\tau_{CM}, l_{pr}) &= P\{T < k\Delta\tau_{CM} | T > (k-1)\Delta\tau_{CM}\} \\ &= \frac{P\{(k-1)\Delta\tau_{CM} < T < k\Delta\tau_{CM}\}}{P\{T > (k-1)\Delta\tau_{CM}\}} \\ &= \frac{P\{0 < T - (k-1)\Delta\tau_{CM} < \Delta\tau_{CM}\}}{P\{T - (k-1)\Delta\tau_{CM} > 0\}} \\ &= P\{l_{(k-1)\Delta\tau_{CM}} < \Delta\tau_{CM}\} \quad (42) \end{aligned}$$

On this basis, we have

$$\begin{aligned} P_{fr}(\Delta\tau_{CM}, l_{pr}) &= \sum_{k=1}^{+\infty} P_{fr,k}(\Delta\tau_{CM}, l_{pr}) \\ &= \sum_{k=1}^{+\infty} P\{l_t < \Delta\tau_{CM} | t = (k-1)\Delta\tau_{CM}\} \end{aligned}$$

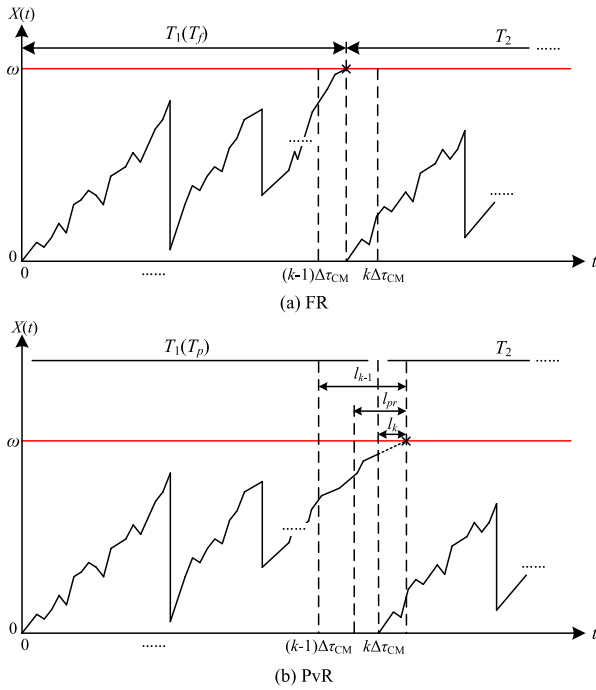


FIGURE 1. The lifecycle maintenance process of the equipment.

$$\begin{aligned}
 &= \sum_{k=1}^{+\infty} F_{l_t}(\Delta\tau_{CM} | t = (k-1)\Delta\tau_{CM}) \\
 &= \sum_{k=1}^{+\infty} \int_0^{\Delta\tau_{CM}} f_{l_t}(l_t | t = (k-1)\Delta\tau_{CM}) dl_t \quad (43)
 \end{aligned}$$

Further analysis finds that a PvR and an FR within the life cycle of the equipment are complementary events. Thus, based on the relevant properties of complementary events in probability theory, the probability that the equipment is subject to a PvR is as follows:

$$P_{pr}(\Delta\tau_{CM}, l_{pr}) = 1 - P_{fr}(\Delta\tau_{CM}, l_{pr}) \quad (44)$$

A PvR is performed simultaneously when the equipment is subject to the  $k$ th inspection. Because  $k \in \mathbf{N}$ , the equipment is subject to a PvR. There are two scenarios, namely,  $k = 0$  and  $k \geq 1$ . When  $k = 0$ , the equipment is subject to a PvR at the initial time point. This is equivalent to a PvR being performed before the operation of the equipment begins. When  $k \geq 1$ , a PvR is needed when the operation of the equipment lasts until the time point  $k\Delta\tau_{CM}$ . Additionally, if a PvR is not performed, the equipment will soon break down.

As demonstrated in Figure 1(b), in the scenario when  $k \geq 1$ , if the equipment is subject to a PvR at the time point  $k\Delta\tau_{CM}$ , its corresponding RUL is less than the  $l_{pr}$  and its RUL at the time point  $(k-1)\Delta\tau_{CM}$  is greater than the  $l_{pr}$ . On this basis, the probability that the equipment is subject to a PvR is determined as follows:

$$\begin{aligned}
 &P_{pr,k}(\Delta\tau_{CM}, l_{pr} | k \geq 1) \\
 &= P\{0 \leq l_{k\Delta\tau} < l_{pr} | l_{(k-1)\Delta\tau} > l_{pr}\}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{P\{0 \leq l_{k\Delta\tau} < l_{pr} \cap l_{(k-1)\Delta\tau} > l_{pr}\}}{P\{l_{k-1} > l_{pr}\}} \\
 &= \frac{P\{0 \leq l_{k\Delta\tau} < l_{pr} \cap l_{k\Delta\tau} > l_{pr} - \Delta\tau\}}{P\{l_{k\Delta\tau} > l_{pr} - \Delta\tau\}} \quad (45)
 \end{aligned}$$

Based on Equation (45), the relative magnitudes of the  $l_{pr}$  and  $\Delta\tau_{CM}$  will directly affect the value of  $P_{pr,k}(\Delta\tau_{CM}, l_{pr})$ . Thus, Equation (45) is transformed into the following:

$$P_{pr,k}(\Delta\tau_{CM}, l_{pr}) = \begin{cases} \frac{P\{l_{pr} - \Delta\tau_{CM} < l_{k\Delta\tau_{CM}} < l_{pr}\}}{P\{l_{k\Delta\tau_{CM}} > l_{pr} - \Delta\tau_{CM}\}}, & l_{pr} > \Delta\tau_{CM} \\ \frac{P\{0 \leq l_{k\Delta\tau_{CM}} < l_{pr}\}}{P\{l_{k\Delta\tau} > l_{pr} - \Delta\tau_{CM}\}}, & l_{pr} \leq \Delta\tau_{CM} \end{cases} \quad (46)$$

By finding the sum of Equation (46), the probability that the equipment is subject to a PvR when  $k \geq 1$  can be determined:

$$\begin{aligned}
 &P_{pr}(\Delta\tau_{CM}, l_{pr} | k \geq 1) \\
 &= \sum_{k=1}^{+\infty} P_{pr,k}(\Delta\tau_{CM}, l_{pr}) \\
 &= \sum_{k=1}^{+\infty} \frac{P\{l_{pr} - \Delta\tau \leq l_t < l_{pr} | t = k\Delta\tau_{CM}\}}{P\{l_t > l_{pr} - \Delta\tau | t = k\Delta\tau_{CM}\}} \\
 &= \sum_{k=1}^{+\infty} \frac{F_{l_t}(l_{pr} | t = k\Delta\tau_{CM}) - F_{l_t}(l_{pr} - \Delta\tau | t = k\Delta\tau_{CM})}{1 - F_{l_t}(l_{pr} - \Delta\tau_{CM} | t = k\Delta\tau_{CM})} \\
 &= \sum_{k=1}^{+\infty} \frac{\int_{\vartheta}^{l_{pr}} f_{l_t}(l_t | t = k\Delta\tau_{CM}) dl_t}{\int_{\vartheta}^{+\infty} f_{l_t}(l_t | t = k\Delta\tau_{CM}) dl_t} \quad (47)
 \end{aligned}$$

where

$$\vartheta = \begin{cases} 0, & l_{pr} \leq \Delta\tau_{CM} \\ l_{pr} - \Delta\tau_{CM}, & l_{pr} > \Delta\tau_{CM} \end{cases} \quad (48)$$

Based on the basic properties of complementary events, the probability that the equipment is subject to a PvR at the initial time point when  $k = 0$  is determined:

$$\begin{aligned}
 &P_{pr,0}(\Delta\tau_{CM}, l_{pr} | t = 0) \\
 &= 1 - P_{pr}(\Delta\tau_{CM}, l_{pr} | t \geq 1) - P_{fr}(\Delta\tau_{CM}, l_{pr}) \quad (49)
 \end{aligned}$$

Additionally, it is easy to conclude

$$\begin{aligned}
 &P_{pr}(\Delta\tau_{CM}, l_{pr}) \\
 &= P_{pr}(\Delta\tau_{CM}, l_{pr} | k \geq 1) + P_{pr,0}(\Delta\tau_{CM}, l_{pr} | t = 0) \quad (50)
 \end{aligned}$$

In this study, to calculate the expected life  $E(T)$  of the equipment,  $E(T)$  is decomposed into two parts, namely, the life corresponding to the FR ( $T_f$ ) and the life corresponding to the PvR ( $T_p$ ).

Let  $T_{f,k}$  be the life of the equipment corresponding to a breakdown that occurs within  $((k-1)\Delta\tau_{CM}, k\Delta\tau_{CM})$ . Thus, based on Figure 1(a), we have

$$\begin{aligned}
 &F_{T_{f,k}}(T) \\
 &= P\{T_{f,k} < T | T_{f,k} > (k-1)\Delta\tau_{CM}, (k-1)\Delta\tau_{CM} < T < k\Delta\tau_{CM}\} \\
 &= \frac{P\{(k-1)\Delta\tau_{CM} < T_{f,k} < T\}}{P\{T_{f,k} > (k-1)\Delta\tau_{CM}\}}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{P\{T_{f,k} > (k-1)\Delta\tau_{CM}\} - P\{T_{f,k} \geq T\}}{P\{T_{f,k} > (k-1)\Delta\tau_{CM}\}} \\
 &= 1 - \frac{P\{T_{f,k} \geq T\}}{P\{T_{f,k} > (k-1)\Delta\tau_{CM}\}} \tag{51}
 \end{aligned}$$

It is clear that

$$\begin{aligned}
 P\{T_{f,k} \geq T\} &= 1 - P\{T_{f,k} < T\} \\
 &= 1 - F(T) = 1 - F_0(l_0) \tag{52}
 \end{aligned}$$

where  $F(T)$  is the cumulative distribution function (CDF) of the life of the equipment ( $T$ ),  $l_0$  is the RUL of the equipment at the initial time point, which is equivalent to  $T$ , and  $F_0(l_0)$  is the CDF corresponding to  $l_0$ .

By substituting Equation (52) into Equation (51), we have

$$F_{T_{f,k}}(l_0) = \frac{F_0(l_0) - F_0((k-1)\Delta\tau_{CM})}{1 - F_0((k-1)\Delta\tau_{CM})} \tag{53}$$

Further analysis finds the following:

$$F_0(l_0) = \int_0^{l_0} f_{l_k|t=0}(l_k | t = 0) dl_k \tag{54}$$

Based on the above analysis as well as Equation (39), we have

$$\begin{aligned}
 &E(T_{f,k}) \\
 &= \int_{(k-1)\Delta\tau_{CM}}^{k\Delta\tau_{CM}} l_0 f_{T_{f,k}}(l_0) dl_0 = \int_{(k-1)\Delta\tau_{CM}}^{k\Delta\tau_{CM}} l_0 dF_{T_{f,k}}(l_0) \\
 &= l_0 F_{T_{f,k}}(l_0) \Big|_{(k-1)\Delta\tau_{CM}}^{k\Delta\tau_{CM}} - \int_{(k-1)\Delta\tau_{CM}}^{k\Delta\tau_{CM}} F_{T_{f,k}}(l_0) dl_0 \\
 &= \frac{k\Delta\tau_{CM} (F_0(k\Delta\tau_{CM}) - F_0((k-1)\Delta\tau_{CM}))}{1 - F_0((k-1)\Delta\tau_{CM})} \\
 &\quad - \frac{1}{1 - F_0((k-1)\Delta\tau_{CM})} \int_{(k-1)\Delta\tau_{CM}}^{k\Delta\tau_{CM}} F_0(l_0) dl_0 \\
 &= \frac{k\Delta\tau_{CM} \int_{(k-1)\Delta\tau_{CM}}^{k\Delta\tau_{CM}} f_{l_k|t=0}(l_k | t = 0) dl_k}{1 - F_0((k-1)\Delta\tau_{CM})} \\
 &\quad - \frac{1}{1 - F_0((k-1)\Delta\tau_{CM})} \\
 &\quad \times \int_{(k-1)\Delta\tau_{CM}}^{k\Delta\tau_{CM}} \int_0^{l_0} f_{l_k|t=0}(l_k | t = 0) dl_k dl_0 \tag{55}
 \end{aligned}$$

Let  $T_{p,k}$  be the life of the equipment when it is subject to a PvR at the time point  $k\Delta\tau_{CM}$ . Thus, based on Figure 1(b), we have

$$E(T_{p,k}) = k\Delta\tau_{CM} P_{pr,k}(\Delta\tau_{CM}, l_{pr}) \tag{56}$$

It is clear that  $T_{p,0} = 0$ . Thus, based on Equations (47), (48), (55), and (56), we have

$$\begin{aligned}
 &E(T) \\
 &= \sum_1^{+\infty} E(T_{f,k}) + \sum_0^{+\infty} E(T_{p,k}) \\
 &= \sum_1^{+\infty} E(T_{f,k}) + \sum_1^{+\infty} E(T_{p,k})
 \end{aligned}$$

$$\begin{aligned}
 &= \sum_{k=1}^{+\infty} \left( \frac{k\Delta\tau_{CM} \int_{(k-1)\Delta\tau_{CM}}^{k\Delta\tau_{CM}} f_{l_k|t=0}(l_k | t = 0) dl_k}{1 - F_0((k-1)\Delta\tau_{CM})} \right. \\
 &\quad - \frac{1}{1 - F_0((k-1)\Delta\tau_{CM})} \\
 &\quad \times \int_{(k-1)\Delta\tau_{CM}}^{k\Delta\tau_{CM}} \int_0^{l_0} f_{l_k|t=0}(l_k | t = 0) dl_k dl_0 \\
 &\quad \left. + \frac{k\Delta\tau_{CM} \int_{\vartheta}^{l_{pr}} f_{l_i}(l_i | t = k\Delta\tau_{CM}) dl_i}{\int_{\vartheta}^{+\infty} f_{l_i}(l_i | t = k\Delta\tau_{CM}) dl_i} \right) \tag{57}
 \end{aligned}$$

Based on the above analysis, equations for calculating  $E(N_{CM})$  and  $E(N_{im})$ , respectively, are obtained:

$$\begin{aligned}
 &E(N_{CM}) \\
 &= \frac{E(T)}{\Delta\tau_{CM}} \tag{58}
 \end{aligned}$$

$$\begin{aligned}
 &E(N_{im}) \\
 &= \sum_{k=1}^{+\infty} (m(E(T_{f,k})) - m((k-1)\Delta\tau_{CM})) P_{fr,k}(\Delta\tau_{CM}, l_{pr}) \\
 &\quad + \sum_{k=1}^{+\infty} (m(k\Delta\tau_{CM}) - m((k-1)\Delta\tau_{CM})) \\
 &\quad \cdot P_{pr,k}(\Delta\tau_{CM}, l_{pr} | k \geq 1) + 0 \times P_{pr,0}(\Delta\tau_{CM}, l_{pr} | k = 0) \\
 &= \sum_{k=1}^{+\infty} (m(E(T_{f,k})) - m((k-1)\Delta\tau_{CM})) \\
 &\quad \cdot \int_0^{\Delta\tau_{CM}} f_{l_i}(l_i | t = (k-1)\Delta\tau_{CM}) dl_i \\
 &\quad + \sum_{k=1}^{+\infty} (m(k\Delta\tau_{CM}) - m((k-1)\Delta\tau_{CM})) \\
 &\quad \cdot \frac{\int_{\vartheta}^{l_{pr}} f_{l_i}(l_i | t = k\Delta\tau_{CM}) dl_i}{\int_{\vartheta}^{+\infty} f_{l_i}(l_i | t = k\Delta\tau_{CM}) dl_i} \tag{59}
 \end{aligned}$$

By substituting Equations (41)-(59) into the maintenance decision model shown in Equation (40), the  $l_{pr}$  and  $\Delta\tau_{CM}$  for the equipment subject to IM can be determined. On this basis, the average  $C(T)$  for the equipment can be minimized.

## VI. PRACTICAL CASE ANALYSIS

Air propelling devices (e.g., fans and air blowers) are key components of the cooling the equipment of large the equipment and play an important role in ensuring the safety and stability of their operation. In engineering applications, vibration data for air-propelling devices are often examined to determine their performance state. Specifically, mechanical wear and dust adherence will cause gradual performance degradation in an air-propelling device and increase its vibration intensity. When its vibration level exceeds a certain threshold, an air-propelling device will fail and must be replaced. In industrial production processes, to further reduce production costs and prolong the useful lives of air-propelling devices, generally, the devices are subject to dynamic balance adjustments (including dust removal and lubrication) to improve their performance. However, these operations are



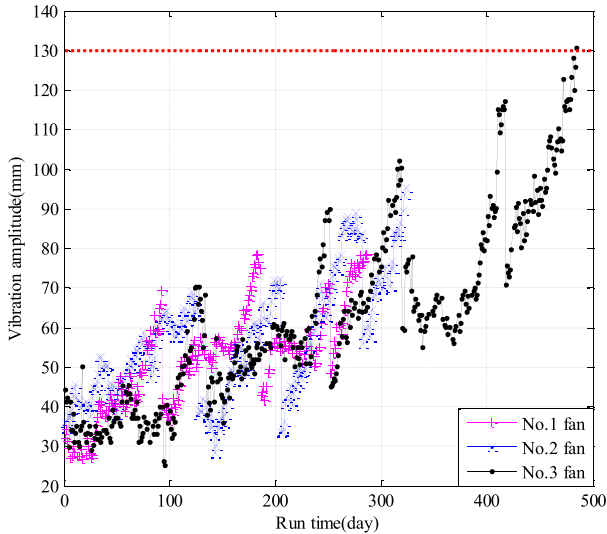


FIGURE 2. Performance degradation data for fans.

able to improve the degradation level of an air propelling device only to a certain extent but are unable to restore it to a brand-new condition. Thus, dynamic balance adjustments performed on air-propelling devices are IM. In this study, performance degradation data acquired for fans of a certain model during a blast-furnace steelmaking process are used for analysis, as shown in Figure 2.

In engineering applications, the fans are often considered to have failed and must be replaced when its vibration amplitude exceeds 130 mm. Thus, 130 mm is the failure threshold for fan performance degradation. As demonstrated in Figure 2, the No. 3 fan failed, whereas the fans No. 1 and No. 2 did not fail. Thus, in this study, the No. 3 fan is selected as the target equipment for analysis to examine the correctness of the maintenance decision method proposed in this study.

In this study, based on the performance degradation data for fans No. 1 and No. 2, the parameters of the degradation model for the equipment subject to IM are estimated in Table 1. To facilitate analysis, the optimal maintenance decision method based on RUL predictions for the equipment subject to IM proposed in this study is denoted by M0. The RUL prediction method proposed in [21] is introduced to the maintenance decision model proposed in this study, and the resulting maintenance decision method is denoted by M1. Further analysis finds that the main difference between M0 and M1 lies in that a nonhomogeneous Poisson process and a homogeneous Poisson process are used to depict IM actions performed on the equipment, respectively.

TABLE 1. Estimated values of the parameters.

M0	$\mu_a$	$\sigma_a$	$\beta$	$\sigma_B$	$a$	$b$
	0.6125	0.6386	0.9454	2.2093	5.1962	0.0021
M1	$\mu_a$	$\sigma_a$	$\beta$	$\sigma_B$	$\lambda$	—
	0.6125	0.6386	0.9454	2.2093	0.0112	—

TABLE 2. K-S hypothesis test results.

Distribution pattern	p	Result
Gamma distribution	0.8282	Not rejected
Normal distribution	0.7768	Not rejected
Weibull distribution	0.7105	Not rejected
Rayleigh distribution	0.2019	Not rejected
Exponential distribution	0.0480	Rejected

To estimate the distribution parameter  $\varphi$  of the extent of recovery ( $E$ ) of the performance index of a fan after IM, it is necessary to first determine its distribution pattern. In this study, the extent of recovery of the performance index of each of the No.1 fan and No.2 fan after IM is subjected to a hypothesis test using the Kolmogorov-Smirnov (K-S) test method (significance level: 5%). Table 2 summarizes the results.

As demonstrated in Table 2, it is more reasonable to use the Gamma distribution to depict the extent of recovery of the performance index of the equipment after IM. Thus, in this study, it is assumed that the extent of recovery of the performance index ( $X^{E_i,k}$ ) satisfies the Gamma distribution. Thus, the value of  $\varphi$  is estimated using the maximum-likelihood method to be (30.02, 0.9628).

$X^{E_i,k}$  is independently identically distributed. Thus, based on the additivity of the Gamma distribution,  $X^{R_i}$  satisfies a Gamma distribution with a parameter  $\tau = (30.02i, 0.9628)$ . The corresponding PDF is as follows:

$$f_{R^i}^i(X^{R^i} | \tau) = \frac{1}{0.9628^{30.02i} \Gamma(30.02i)} (X^{R^i})^{30.02i-1} e^{-\frac{X^{R^i}}{0.9628}}, \quad X^{R^i} > 0 \tag{60}$$

By substituting the estimated values of the parameters in Table 1 and Equation (60) into Equations (38) and (39), the RUL prediction for the target equipment can be obtained, which is shown in Figure 3.

As demonstrated in Figure 3, the PDF of the RUL corresponding to M0 at various time points completely covers the actual RUL of the target equipment, and the corresponding predicted RUL is also closer to the actual RUL. This suggests that M0 is more accurate than M1 in predicting RULs. To further verify that the proposed method is more accurate in predicting RULs, an  $\alpha - \lambda$  index [24] is introduced as an evaluation criterion. The  $\alpha - \lambda$  index is defined as follows:

The  $\alpha - \lambda$  index is primarily used to measure the closeness between the predicted and actual values of the RUL. In this index, a confidence region near  $\pm(\alpha)(100)\%$  of the actual RUL is given (in this study,  $\alpha$  is set to 0.2 based on [24]). If the predicted RUL falls in this region, then it is considered that the predicted RUL meets the accuracy requirement; otherwise, it does not.  $\lambda$  is used to describe a normalized time series and defined as  $\lambda = t_k/T$ .

Thus, the  $\alpha - \lambda$  index values corresponding to M0 and M1 are given, as shown in Figure 4.

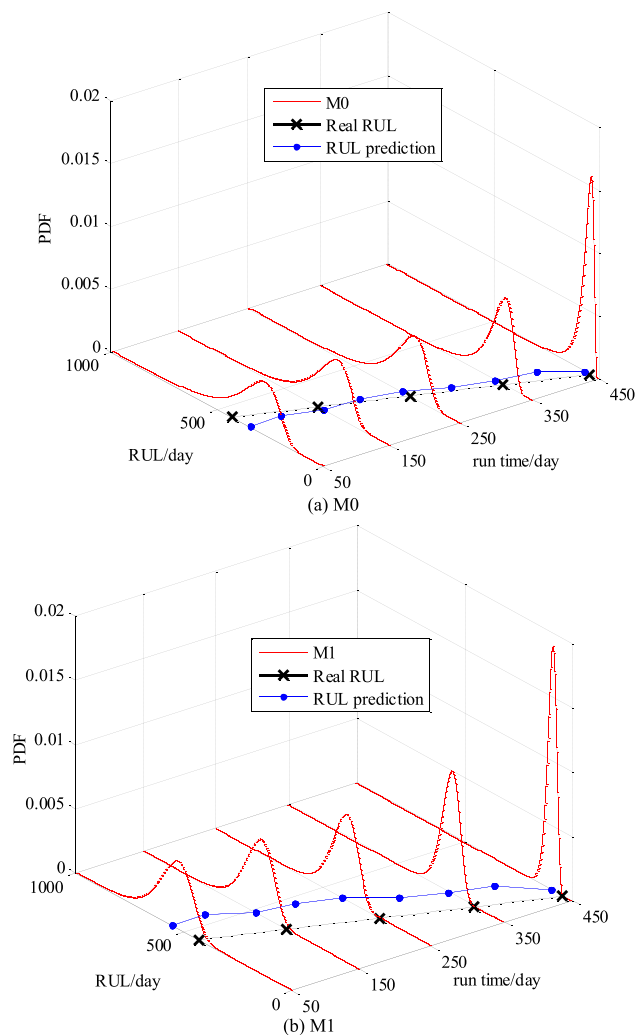


FIGURE 3. RUL prediction.

TABLE 3. Maintenance cost parameters.

Parameter	$C_{CM}$	$C_{im}$	$C_{pr}$	$C_{fr}$
Cost (¥)	5	50	500	5000

As demonstrated in Figure 4, most of the RUL curve corresponding to M0 falls in the confidence interval, whereas the RUL curve corresponding to M1 nearly entirely falls outside the confidence interval. This further demonstrates that the method proposed in this study is more accurate in predicting RULs.

Moreover, the  $C(T)$  parameters for the fan are given, as shown in Table 3.

By substituting the predicted RULs obtained using M0 and M1 into the maintenance decision model proposed in this study, an optimal  $\Delta\tau_{CM}$  and  $l_{pr}$  can be determined. Considering that inspections and replacements are performed based on integer days in the actual maintenance process, it is assumed that  $\Delta\tau_{CM}, l_{pr} \in N$ . Based on the above analysis, the optimization model is solved using the genetic pollen algorithm [25]. Thus, a minimum  $C(T)$  (as shown in Figure 5)

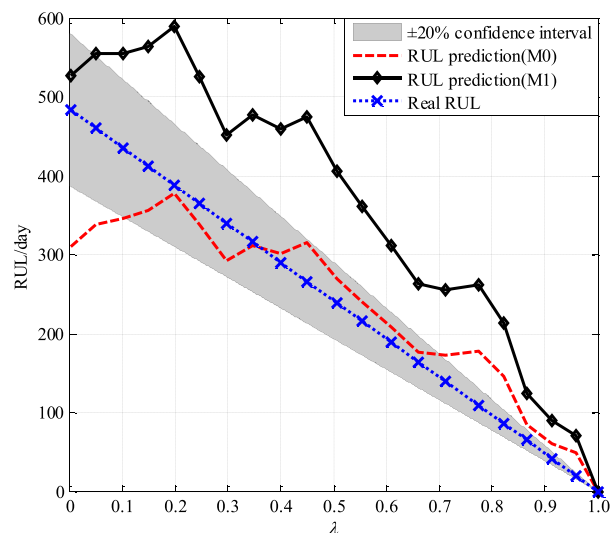


FIGURE 4. Predicted RUL values.

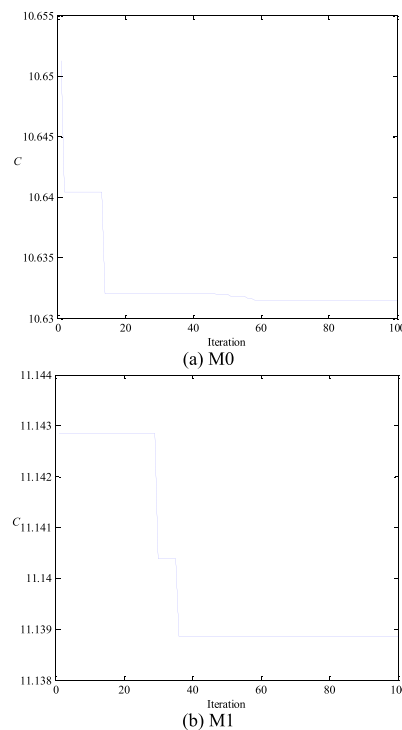


FIGURE 5. Iteration results for the  $C(T)$ .

TABLE 4. Optimal maintenance decisions.

	$\Delta\tau_{CM}$ (day)	$l_{pr}$ (day)	$C$ (¥/day)
M0	1	49	10.6315
M1	1	85	11.1388

and an optimal maintenance decision (as shown in Table 4) are obtained using M0 and M1, respectively.

As demonstrated in Table 4, the optimal  $\Delta\tau_{CM}$  obtained using each of M0 and M1 is one day. However, the  $l_{pr}$  and optimal  $C(T)$  obtained using M0 are both lower than those obtained using M1. This suggests that the maintenance

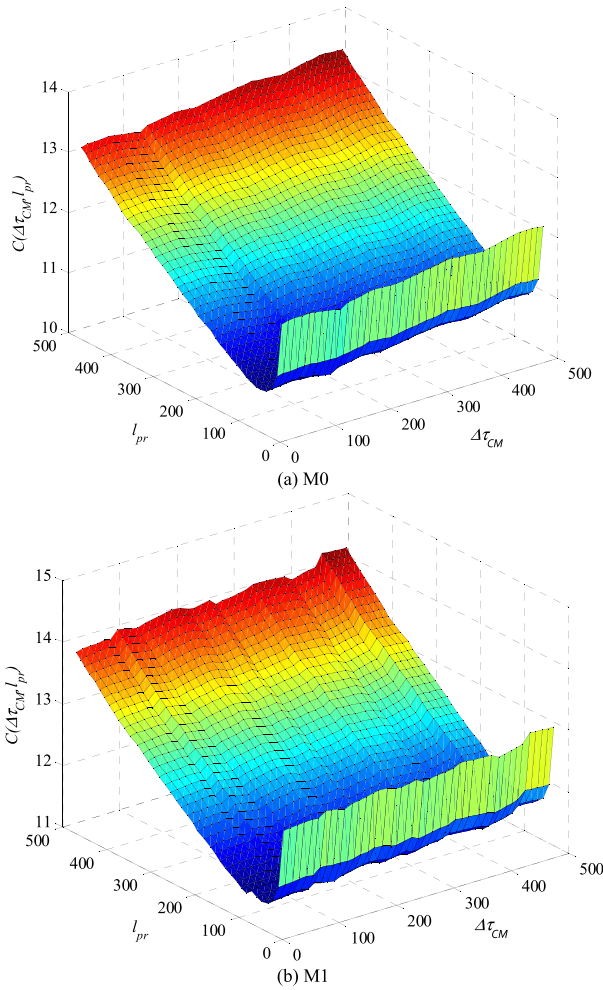


FIGURE 6. Average  $C(T)$  curves for the equipment.

decision obtained using M0 is advantageous over that obtained using M1. Moreover, average  $C(T)$  curves corresponding to M0 and M1, respectively, are given, as shown in Figure 6.

As demonstrated in Figure 6, the average  $C(T)$  for the equipment determined using M0 is, overall, lower than that determined using M1. This further demonstrates that the M0 strategy method is superior to the M1 strategy method. This is mainly because, compared to M1, M0 is more accurate in predicting RULs, and consequently M0 can ensure that PvM is performed in a timelier fashion and thereby reduce the risk of system failure and maintenance cost. This conclusion reflects that RUL prediction accuracy significantly affects the maintenance decision and thereby demonstrates that the proposed method is reasonable.

Furthermore, the sensitivity of each maintenance cost parameter to the optimal system maintenance decision proposed in this study is examined using the control variate method. Because of  $C_{CM} < C_{im} < C_{pr} < C_{fr}$ , let  $C_{CM} \in [1, 50]$ ,  $C_{im} \in [5, 500]$ ,  $C_{pr} \in [50, 5000]$ , and  $C_{fr} \in [500, 8500]$ . Thus, the quantitative relationships between the

cost parameters and the optimal maintenance decision are determined, as shown in Figure 7.

As demonstrated in Figure 7(a), the optimal average  $C(T)$  for the equipment is relatively significantly affected by the  $C_{CM}$ . As the  $C_{CM}$  gradually increases, the average  $C(T)$  increases nearly linearly at an average rate of approximately  $0.1422 \text{ day}^{-1}$ . The  $\Delta\tau_{CM}$  basically remains constant as the  $C_{CM}$  changes and only changes suddenly at a  $C_{CM}$  of approximately ¥30. This suggests that the  $\Delta\tau_{CM}$  is relatively insignificantly affected by the  $C_{CM}$ . When the  $C_{CM}$  per inspection is relatively low, a relatively high inspection frequency can help to improve the operational reliability of the equipment and reduce its failure risk and maintenance cost. When the  $C_{CM}$  per inspection is relatively high, a high inspection frequency will incur a high  $C_{CM}$ , which will account for the majority of the  $C(T)$ . Therefore, it is necessary to reduce the total number of inspections by increasing  $\Delta\tau_{CM}$  to reduce  $C(T)$ . Further analysis finds that there is an approximately linear relationship between the  $l_{pr}$  and  $C_{CM}$ , indicating that the  $C_{CM}$  is relatively highly sensitive to the  $l_{pr}$ . Increasing the  $l_{pr}$ , which is equivalent to shortening the effective run time of the equipment, can reduce the number of inspections and thereby offset the effect of the increase in the  $C_{CM}$ .

As demonstrated in Figure 7(b), the  $C_{im}$  is linearly positively correlated with the optimal average  $C(T)$  for the equipment. Additionally, for every ¥100 increase in the  $C_{im}$ , there is an approximately  $¥0.9404 \text{ day}^{-1}$  increase in the optimal average  $C(T)$  for the equipment. This suggests that the optimal average  $C(T)$  is relatively highly sensitive to the  $C_{im}$ . Throughout the changes in  $C_{im}$ ,  $\Delta\tau_{CM}$  and  $l_{pr}$  change insignificantly. This suggests that  $\Delta\tau_{CM}$  and  $l_{pr}$  are relatively insignificantly affected by the changes in the  $C_{im}$ . This is mainly because when  $\Delta\tau_{CM}$  and  $l_{pr}$  are constant, the number of occasions of IM to which the equipment is subject is also approximately constant. As a result, the  $C(T)$  for the equipment is primarily affected by the  $C_{im}$ .

As demonstrated in Figure 7(c), the  $C_{pr}$  exerts the most significant impact on the optimal average  $C(T)$  for the equipment. As the  $C_{pr}$  increases, the optimal average  $C(T)$  for the equipment increases linearly at a rate of approximately  $1.5138 \times 10^{-3} \text{ day}^{-1}$ . In comparison, changes in the  $C_{pr}$  relatively insignificantly affect  $\Delta\tau_{CM}$  and  $l_{pr}$ . When the value of  $C_{pr}$  is relatively small, both  $\Delta\tau_{CM}$  and  $l_{pr}$  remain constant. When the  $C_{pr}$  is close to the  $C_{fr}$  and gradually increases, both the  $\Delta\tau_{CM}$  ( $> ¥4,100$ ) and  $l_{pr}$  ( $> ¥3,600$ ) begin to increase. This is because when the  $C_{pr}$  is close to the  $C_{fr}$ , the effectiveness of  $C_{pr}$  in reducing the  $C(T)$  weakens, and the effect of  $C_{CM}$  and  $C_{im}$  on the  $C(T)$  for the equipment becomes more prominent. Therefore, it is necessary to reduce the number of inspections and the number of maintenance interventions by increasing  $\Delta\tau_{CM}$  and  $l_{pr}$  to achieve an optimal maintenance decision.

As demonstrated in Figure 7(d), the  $C_{fr}$  relatively significantly affects the optimal average  $C(T)$  for the equipment and its  $l_{pr}$  but almost exerts no impact on the  $\Delta\tau_{CM}$ . This suggests that the  $C_{fr}$  is relatively highly sensitive to the optimal

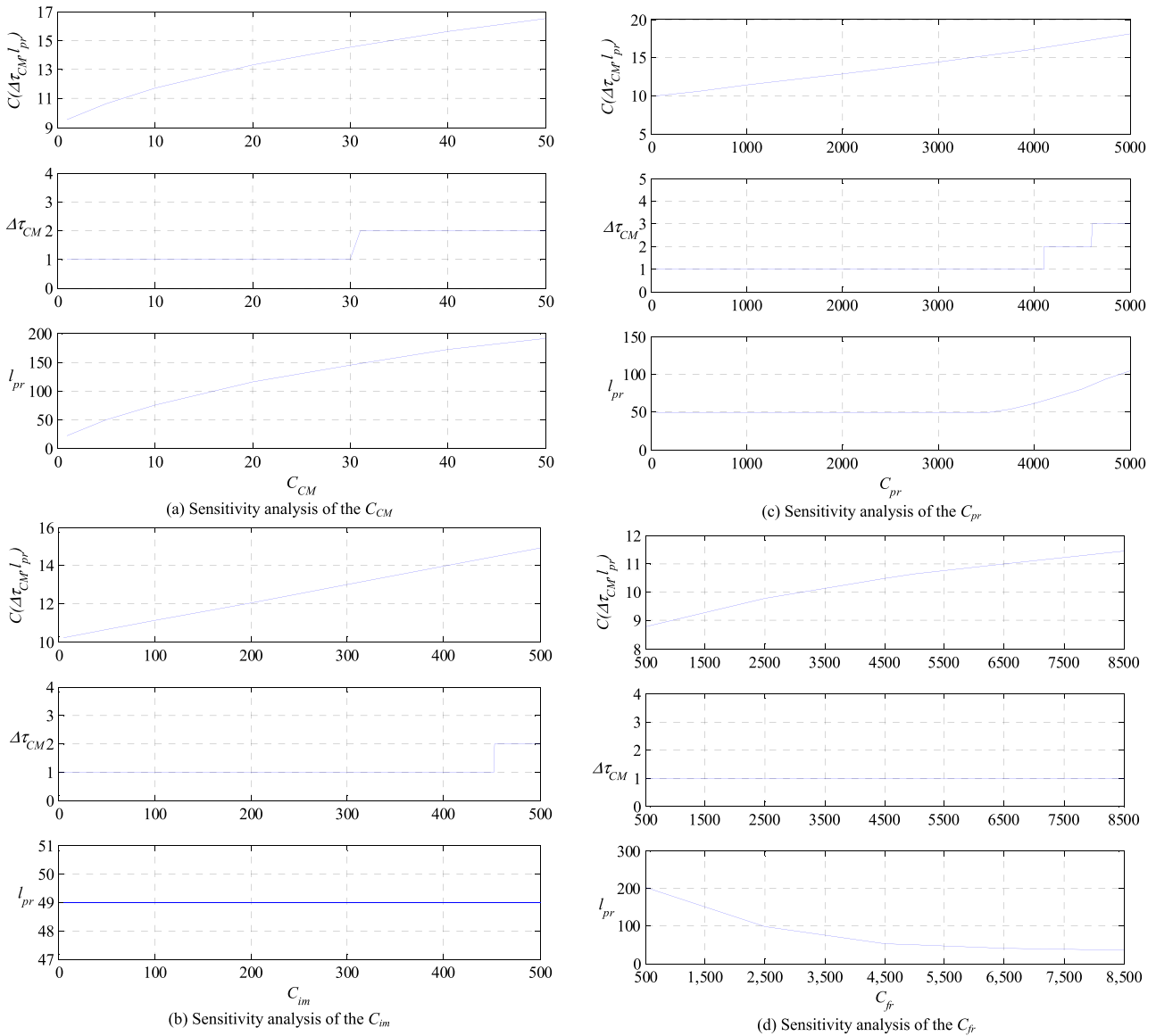


FIGURE 7. Sensitivity analysis of the cost parameters.

average  $C(T)$  for the equipment and its  $l_{pr}$  but relatively weakly sensitive to the  $\Delta\tau_{CM}$ . Further analysis of Figure 7(d) finds the following. When the value of  $C_{fr}$  is relatively small, the  $l_{pr}$  decreases at a relatively high rate as the  $C_{fr}$  increases. When the value of  $C_{fr}$  is relatively large, the  $l_{pr}$  gradually begins to change to a decreasing extent. This is mainly because when the  $C_{fr}$  is close to the  $C_{pr}$ , the effect of  $C_{fr}$  is close to that of  $C_{pr}$ , and the  $C_{im}$  is the primary factor that affects the  $C(T)$ . A relatively high  $l_{pr}$  can help to effectively reduce the expected life of the equipment and result in a relatively small expected number of occasions of IM, thereby reducing the  $C_{im}$  and ensuring an optimal  $C(T)$ . When the value of  $C_{fr}$  is relatively large, the  $C_{fr}$  plays a leading role in the change in the  $C(T)$  for the equipment, and the value of  $l_{pr}$  is also basically constant.

### VII. CONCLUSION

In this study, a compound nonhomogeneous Poisson process is used to construct a comprehensive degradation model for the equipment subject to IM. Additionally, based on the derivation of the RUL distribution, an optimal maintenance decision is obtained. The following conclusions are derived from this study. (1) For the degradation model of the equipment subject to IM, a nonhomogeneous Poisson process is of higher accuracy and applicability than a homogeneous Poisson process. (2) A higher-accuracy RUL prediction can help to determine an optimal maintenance decision for the equipment and further reduce the maintenance cost and improve the effectiveness of maintenance. (3) The maintenance decision method proposed in this study has relatively high modeling generality and RUL prediction accuracy and can be used

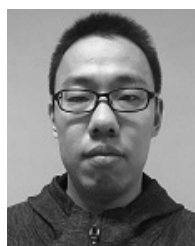
to devise superior strategies and thus has broad engineering application prospects.

## REFERENCES

- [1] M. Noureifath, M.-C. Fitouhi, and M. Machani, "An integrated model for production and preventive maintenance planning in multi-state systems," *IEEE Trans. Rel.*, vol. 59, no. 3, pp. 496–506, Sep. 2010.
- [2] Y. Sun, L. Ma, and J. Mathew, "Optimising preventive maintenance strategy for production lines," in *Asset Condition, Information Systems and Decision Models*. London, U.K.: Springer, 2012.
- [3] A. T. de Almeida, C. A. V. Cavalcante, M. H. Alencar, R. J. P. Ferreira, A. T. de Almeida-Filho, and T. V. Garcez, "Decision making in condition-based maintenance," in *Multicriteria and Multiobjective Models for Risk, Reliability and Maintenance Decision Analysis*. Cham, Switzerland: Springer, 2015.
- [4] N. Li, N. Gebraeel, Y. Lei, L. Bian, and X. Si, "Remaining useful life prediction of machinery under time-varying operating conditions based on a two-factor state-space model," *Rel. Eng. Syst. Saf.*, vol. 186, pp. 88–100, Jun. 2019.
- [5] J. Sheppard, M. Kaufman, and T. Wilmer, "IEEE standards for prognostics and health management," *IEEE Aerosp. Electron. Syst. Mag.*, vol. 24, no. 9, pp. 34–41, Sep. 2009.
- [6] P. V. Varde and M. G. Pecht, "Prognostics and health management," in *Solid State Lighting Reliability*. New York, NY, USA: Springer, 2013.
- [7] H. Q. Song, B. W. Li, T. Sun, and Y. H. Wang, "Design of prognostics and health management system for turbofan engine," *Appl. Mech. Mater.*, vols. 599–601, pp. 2108–2111, Aug. 2014.
- [8] G. P. Michael and K. Myeongsu, *Introduction to PHM: Fundamentals, Machine Learning, and the Internet of Things*. Piscataway, NJ, USA: Wiley, 2019.
- [9] B. Saha, "A Bayesian approach to battery prognostics and health management," in *Advances in Battery Manufacturing, Service, and Management Systems*. Hoboken, NJ, USA: Wiley, 2016.
- [10] N. Li, Y. Lei, T. Yan, N. Li, and T. Han, "A Wiener-process-model-based method for remaining useful life prediction considering unit-to-unit variability," *IEEE Trans. Ind. Electron.*, vol. 66, no. 3, pp. 2092–2101, Mar. 2019.
- [11] H. Pham and H. Wang, "Imperfect maintenance," *Eur. J. Oper. Res.*, vol. 94, no. 3, pp. 425–438, 1996.
- [12] A. K. Jardine, D. Lin, and D. Banjevic, "A review on machinery diagnostics and prognostics implementing condition-based maintenance," *Mech. Syst. Signal Process.*, vol. 20, no. 7, pp. 1483–1510, Oct. 2006.
- [13] A. Grall, C. Bérenguer, and L. Dieulle, "A condition-based maintenance policy for stochastically deteriorating systems," *Rel. Eng. Syst. Saf.*, vol. 76, no. 2, pp. 167–180, May 2002.
- [14] S.-T. Tseng, R. Huei Yeh, and W.-T. Ho, "Imperfect maintenance policies for deteriorating production systems," *Int. J. Prod. Econ.*, vol. 55, no. 2, pp. 191–201, Jul. 1998.
- [15] E. N. Ge, Q. M. Li, and G. Y. Zhang, "Optimization of condition-based maintenance for degradation the equipment under imperfect maintenance," *Acta Aeronautica et Astronautica Sinica*, vol. 34, no. 2, pp. 316–324, 2012.
- [16] C. Guo, W. Wang, B. Guo, and X. Si, "A maintenance optimization model for mission-oriented systems based on Wiener degradation," *Rel. Eng. Syst. Saf.*, vol. 111, pp. 183–194, Mar. 2013.
- [17] M. Zhang, O. Gaudoin, and M. Xie, "Degradation-based maintenance decision using stochastic filtering for systems under imperfect maintenance," *Eur. J. Oper. Res.*, vol. 245, no. 2, pp. 531–541, Sep. 2015.
- [18] H. Pei, C. H. Hu, and X. S. Si, "Remaining life prediction information-based maintenance decision model for equipment under imperfect maintenance," *Acta Automatica Sinica*, vol. 44, no. 4, pp. 145–155, 2018.
- [19] C. Hu, H. Pei, Z. Wang, X. Si, and Z. Zhang, "A new remaining useful life estimation method for equipment subjected to intervention of imperfect maintenance activities," *Chin. J. Aeronaut.*, vol. 31, no. 3, pp. 514–528, Mar. 2018.
- [20] J. F. Zheng, C. H. Hu, and X. S. Si, "Remaining useful life prognostic for the stochastic degradation device subject to imperfect maintenance," *Acta Electronica Sinica*, vol. 45, no. 7, pp. 199–208, 2017.
- [21] Z.-Q. Wang, C.-H. Hu, X.-S. Si, and E. Zio, "Remaining useful life prediction of degrading systems subjected to imperfect maintenance: Application to draught fans," *Mech. Syst. Signal Process.*, vol. 100, pp. 802–813, Feb. 2018.
- [22] X.-S. Si, W. Wang, C.-H. Hu, D.-H. Zhou, and M. G. Pecht, "Remaining useful life estimation based on a nonlinear diffusion degradation process," *IEEE Trans. Rel.*, vol. 61, no. 1, pp. 50–67, Mar. 2012.
- [23] R. Barlow and L. Hunter, "Optimum preventive maintenance policies," *Oper. Res.*, vol. 8, no. 1, pp. 90–100, Feb. 1960.
- [24] A. Saxena, J. Celaya, B. Saha, S. Saha, and K. Goebel, "Evaluating algorithm performance metrics tailored for prognostics," in *Proc. IEEE Aerosp. Conf.*, Mar. 2009.
- [25] Y. J. Gao, F. M. Zhang, Q. Guo, and C. Li, "Research on the searching performance of flower pollination algorithm with three random walks," *J. Intell. Fuzzy Equip.*, vol. 35, no. 1, pp. 333–341, Jul. 2018.

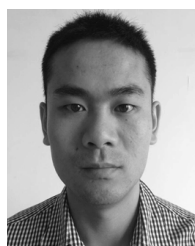


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