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A Vehicle Trajectory Tracking Method With a Time-Varying Model Based on the Model Predictive Control

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ABSTRACT The vehicle trajectory tracking algorithm is one of the key and difficult issues of intelligent driving technologies. In current control algorithms for the vehicle trajectory tracking, there are three main assumptions: the standard working condition for the driving path, the same control model used for the entire control process, and a fixed value for the longitudinal vehicle speed. However, the above determinations in current control algorithms are inconsistent with the actual vehicle driving conditions. To overcome those problems, a vehicle trajectory tracking method with a time-varying model is proposed. The time-varying model is developed by using a two-dimensional vehicle kinematics model. This method considers the influences of the longitudinal speed and road curvature on the vehicle trajectory tracking stability under the low-speed complex driving condition. Thus, the proposed method can improve the trajectory tracking accuracy when the unmanned vehicle is located at the road with a sharp curvature under the low-speed complex driving condition. Moreover, the proposed model can achieve the real time calculation. Meanwhile, the prediction accuracy of the vehicle kinematics model is ensured. The proposed approach with the above characteristics can complete the trajectory tracking for the route composed of arbitrary curves. The results show that the proposed method can effectively improve the trajectory tracking stability of the unmanned vehicle on the roads with different curvatures under complex driving conditions.

INDEX TERMS Model predictive control, stability, time-varying model control, vehicle trajectory tracking method.

I. INTRODUCTION

As the information and intelligent technology develops, unmanned vehicles are rapidly developed and widely used in various fields [1]. Although the unmanned vehicle with a higher driving speed is an important trend during the development of the current unmanned vehicle technologies [1]–[3], the low-speed unmanned vehicles for special purposes can also be used in various applications [4], [5]. At present, the researches and applications of the unmanned vehicles for the low-speed driving conditions includes the autonomous parking [6], road cleaning, express delivery, parcel sorting, and mobile robots, etc.

The steering stability of the unmanned vehicle is constrained by the nonlinear vehicle dynamics. When the vehicles are on the roads with the complex curvatures or low adhesion coefficients, the sideslip or out of control can be easily produced [7]. In addition, most previous works focused on the wide driving conditions of the unmanned vehicles including the double shifting and sinusoidal working conditions, which missed the arbitrary driving trajectories. Therefore, it is necessary to study the trajectory tracking stability of vehicles for driving under nonstandard road (such as arbitrary trajectory) conditions by considering various longitudinal vehicle speeds and roads with different curvatures.

The model predictive control (MPC) algorithm considers the nonlinear characteristics of the vehicle model. It has an absolute advantage in dealing with the optimal problems with multiple constraints. Moreover, it can constraint the controlling and state quantities [8]–[12]. Therefore, the model predictive control algorithm is wildly used in the motion planning, path tracking, obstacle avoidance, emergency control

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and lane change control of the unmanned vehicles [13]–[19]. Although the MPC method provides a good control effectiveness in their relative applications, it usually assumes a simple road topography, which means that it ignores the road curvature and tilt angle. Zhang *et al.* [15] and Hauser [16] proposed an adaptive MPC method. The control time domain of the algorithm is automatically revised. The control domain of this algorithm is determined by the curvature change of the tracking trajectory. Their method can ensure the control effect and reduce the calculation resources of the controller. Moreover, the real-time performance of the system can be improved. Liu *et al.* [20] proposed a model predictive control algorithm with a variable step size. This algorithm can ensure the real-time performance as well as the prediction accuracy of the model prediction. Moreover, it can save the calculation time reasonably. Rafaila and Livint [21] proposed a nonlinear model predictive control (NMPC) method to control the steering for an autonomous ground vehicle. This work studied the feasibility of classical NMPC method. But the mathematical model of the system are very complex. Moreover, the previous nonlinear models request large computational resources, which will affect the real-time performance of the unmanned vehicle control [22]. The study of linear parameter variation (LPV) MPC scheme can be found in [23] and [24]. Generally, the performance of this scheme is highly dependent on the nonlinearity of the model. In fact, as the input trajectory and state deviate from the current working point, the model mismatch will increase. This will produce a large prediction error, which leads to the instability of the closed-loop system.

The biggest advantage of the MPC algorithm with the linear vehicle model (LMPC) is that its calculation processing is simple and its real-time performance is well. Moreover, its calculation efficiency is high but the tracking accuracy is poor. Thus, the LMPC can only be used to predict and control the vehicle motion for a region with a small curvature variation or a straight-line. For the linear model, the prediction time domain is small and the discretization time step is long. However, for the high speed conditions or the road conditions with a large curvature variation, the nonlinear dynamic models were generally used to describe the vehicle motions. The NMPC has a higher tracking accuracy, a poor real-time performance, and a longer prediction time domain than those of the linear models. In addition, its discretization time step is greatly reduced, and the tracking deviation of the unmanned vehicle under an arbitrary trajectory condition is reduced. However, the nonlinear models can lead to the increase of the computational time.

In the above literature, many fixed vehicle models were used in their relative control processing. Some methods assumed that the longitudinal speeds were fixed during the whole movement, but those assumptions were inconsistent with the actual vehicle driving situations. A small number of the path tracking controllers were designed without considering different previewed road information, and the influence of the model error on the trajectory tracking precision was not discussed. The linear driving dynamic models were generally used in the unmanned vehicles. The calculation method of this model is simple and the real-time performance is well, but its tracking accuracy is poor. Although the NMPC has a higher tracking accuracy, it has a poor real-time performance.

To overcome above problems, the effects of the vehicle longitudinal speed and road curvature on the vehicle trajectory tracking stability are analyzed. Based on this analysis, a vehicle trajectory tracking method with a time-varying model depending on a two-dimensional (2DOF) vehicle model is proposed. Both the liner MPC and NMPC algorithms are used in the proposed method. Thus, the proposed method can be used for arbitrary complex trajectory conditions. In this method, the curvature variation of the road is used to determine the control models in the unmanned vehicle travels. The control algorithm adopts the basic idea of segmentation processing. The linear predictive control model is only used in a region with a small curvature variation or a straight-line, by which the vehicle motion is predicted and controlled, whereas the NMPC is used for the road with a large curvature variation. Although the real-time performance is locally poor, the NMPC can reduce the tracking deviation. The total tracking accuracy and real-time performance are improved by the proposed method.

In conclusion, our main contribution is to provide a vehicle trajectory tracking method with a time-varying model. This method is based on a two-dimensional vehicle model. Both the LMPC and NMPC algorithms are used in the proposed method. Thus, this method can be used for arbitrary complex trajectory conditions. In this method, the curvature variation of the road is used to determine the control models in the unmanned vehicle travels. The total tracking accuracy and real-time performance can be improved. Thus, this work can show some guidances for vehicle tracking under arbitrary complex trajectory conditions.

II. PROBLEM DESCRIPTION

The steering stability of the unmanned vehicles is constrained by their nonlinear vehicle dynamics. When the vehicles are on the complex curved roads or the roads with a low adhesion coefficient, the sideslip or out of control can be easily produced. Under the low speed and complex driving conditions, a major problem is that a large tracking deviation of the unmanned vehicles may be produced when the curvature of the road changes sharply. In order to solve this problem, two methods are provided as shown in Fig. 1. The first method is used to change the sharp curvature. The Expected Trajected-1 trajectory is carried on the smoothing processing to obtain the Expected Trajected-2. The smoothed trajectory is tracked to appropriately reduce the tracking deviation. The second method is used to propose a trajectory tracking control algorithm with the time-varying model depending on the MPC method, which is the proposed one in this work. In this method, the vehicle uses the linear model to predict and control vehicle motion in a straight line or a road region with a small curvature variation, which can reduce

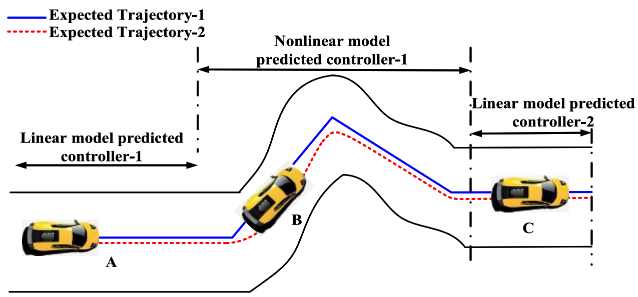

FIGURE 1. The path tracking under arbitrary trajectory conditions.

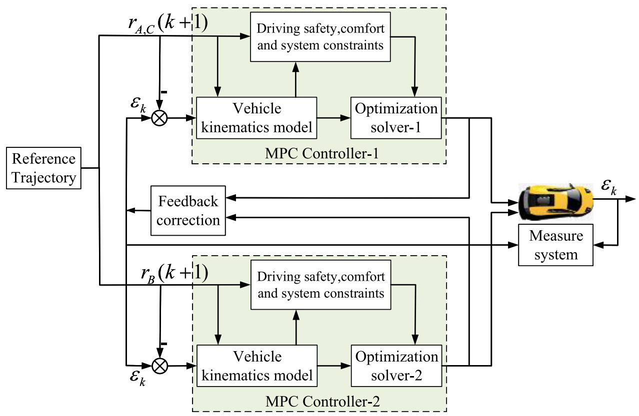

FIGURE 2. A trajectory tracking method with the time-varying model based on the model predictive control.

the prediction time domain and increase the sampling time interval. For the road region with a large curvature variation, the vehicle use the nonlinear models to predict and control the vehicle motion, which can increase the prediction time domain and reduce the sampling time interval. Moreover, since the nonlinear model uses a smaller longitudinal speed, it can reduce the tracking deviation of the arbitrary paths. According the above reasons, the NMPC algorithm has more advantages in the road conditions with large curvature variation. The algorithm can improve the tracking accuracy for the road with a large curvature variation. Moreover, it can improve the real-time performance and ensure the tracking accuracy.

III. A TRAJECTORY TRACKING METHOD WITH THE TIME-VARYING MODEL BASED ON THE MODEL PREDICTIVE CONTROL

Figure 2 show the established trajectory tracking method with the time-varying model depending on the MPC algorithm. A piecewise processing approach is used in the proposed method. The unmanned vehicle uses different control algorithm models according to the trajectory characteristics. The vehicle uses the linear model to predict and control the vehicle motion in the road region with the straight line or a small curvature variation. The nonlinear model with a higher tracking accuracy is used for the road regions with a large curvature variation. The predictive control algorithm can not only improve the tracking accuracy, but also insure the tracking accuracy and improve the real-time performance. More descriptions of the proposed method are given in Section 3.2.

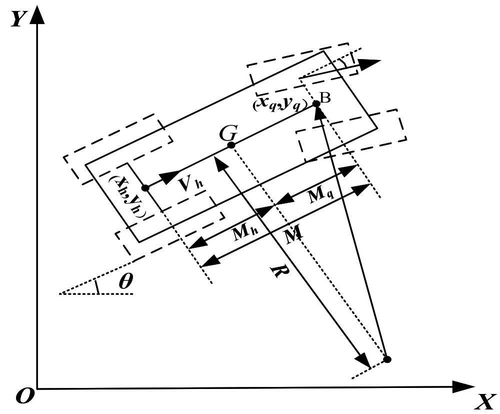

FIGURE 3. A 2 DOF kinematic model of a vehicle.

A. VEHICLE DYNAMIC MODELING

In practice, the dynamic processing of a vehicle on the road surface is very complex. To ensure the real-time performance of the prediction algorithm, the predictive control model should show good vehicle kinematics and dynamic constraints, which can obtain the desired predictive control. For the good road surface and low speed conditions, the dynamic problems in the vehicle stability control can be neglected. A 2DOF kinematic model can be used to describe the kinematic characteristics of the vehicles [25], [26]. Figure 3 depicts a typical 2 DOF vehicle steering kinematic model. Here, (x_h, y_h) and (x_q, y_q) respectively denote the axle coordinates of rear and front axles; M denotes the length of the wheelbase; δ_q describes the front wheel deflection angle; θ denotes the vehicle heading angle; v_q describes the front wheel speed; v_h denotes the rear wheel speed; and R describes the instantaneous vehicle turning radius.

Here, the road curvature radius is defined as the instantaneous vehicle turning radius. The coordinate relationship between the vehicle rear and front axle centers can be expressed as [27]

$$\begin{cases} x_q = x_h + M\cos\theta \\ y_q = y_h + M\sin\theta \end{cases} \tag{1}$$

The relationship between the speed and the coordinates of the central point of the rear axle is given by

$$v_h = \dot{x}_h\cos\theta + \dot{y}_h\sin\theta \tag{2}$$

If there is no sideslip and the sideslip angle of the mass center remains fixed during the steering processing, the kinematic constraints between the rear and front axles are defined as

$$\begin{cases} \dot{x}_h\sin\theta - \dot{y}_h\cos\theta = 0 \\ \dot{x}_q\sin(\theta+\delta_q) - \dot{y}_q\cos(\theta+\delta_q) = 0 \end{cases} \tag{3}$$

Combining Eqs. 2 and 3 yields

$$\begin{cases} \dot{x}_h = v_h \cos\theta \\ \dot{y}_h = v_h \sin\theta \end{cases} \tag{4}$$

In Fig. 3, the relationship among the vehicle steering radius R, rear axle central point velocity v_h, and vehicle yaw angular velocity ω is written as

$$\begin{cases} w = v_h/R \\ \tan\delta_q = M/R \end{cases} \tag{5}$$

By substituting Eqs. 1 and 4 into Eq. 3, the yaw angular velocity ω can be expressed as

$$w = \frac{v_h}{M}\tan\delta_q \tag{6}$$

The relationship between the vehicle heading angle θ and yaw angular velocity ω is

$$\dot{\theta} = w \tag{7}$$

The expression of the vehicle kinematics model is

$$\begin{bmatrix} \dot{x}_h \\ \dot{y}_h \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta \\ \sin\theta \\ \tan\delta_q/M \end{bmatrix} v_h \tag{8}$$

In the processing of the trajectory tracking control, the control quantity and the state quantity are applied to determine the vehicle kinematic model. Thus, the model can be expressed as

$$\dot{\varepsilon} = f(\varepsilon, u) \tag{9}$$

where the state quantity $\varepsilon = \begin{bmatrix} x_h & y_h & \theta \end{bmatrix}^T$ and the control quantity $u = \begin{bmatrix} v_h & \delta_q \end{bmatrix}^T$.

B. MODEL PREDICTIVE CONTROL ALGORITHMS

The MPC algorithm is a multivariable control algorithm. In Fig. 4, the algorithm uses a model describing the dynamic object behavior to realize the prediction of future system dynamic control quantity. The specific algorithm processing is as follows: the state and control variables at the kth time step is defined as the input parameter. The input parameter is defined as the initial condition for obtaining the future system dynamics at the current time. The open-loop optimization issue considering a finite time is calculated online. Moreover, a first component u_k for control sequence is applied for the dynamic behavior model to obtain the system output η_{k+1} at k+1. The input u_k at k is used to control the output η_{k+1} at $k+1$ and makes the output greatly reach the expected value at $k+1$. For the following sampling time, the previous step will be repeated; and the updated measurement value ξ_k will be given as the initial parameter to predict the future system dynamics. The optimization issue is updated and calculated again until the whole control process is completed.

The key difference between the traditional control and the MPC algorithms is to obtain the open-loop optimization sequence by solving the on-line open-loop optimization problems. The traditional control method usually solves a feedback control law offline and acts on the system all the time.

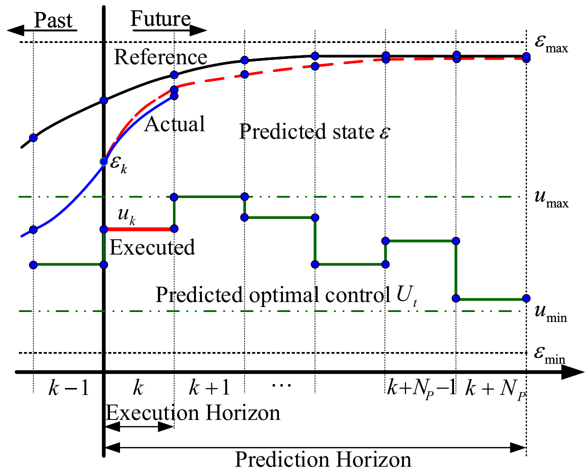

FIGURE 4. A schematic of the MPC.

TABLE 1. MPC controller parameters.

Parameters	Value(unit)	Description
Np	30	Prediction horizon
Nc	25	Control horizon
T_l	0.5s	Sampling time-interval of linear model
T_n	0.05s	Sampling time-interval of non-linear model

The greatest attraction for predictive control is used to deal with the explicit constraints, which is depended on the model-based prediction for future system dynamic behavior. It can add on constraints to the future output, input, or state parameters. Moreover, the constraints are explicitly formulated in non-linear or on-line quadratic programming problems.

The MPC has the strong robustness and good control effect. It can address the nonlinearity, uncertainties, and parallelism during the relative processing. It can also conveniently solve the various constraints for controlled process and state variables.

1) DISCRETIZATION METHOD FOR THE LINEAR PREDICTION MODEL

The 2 DOF vehicle kinematic model in Fig. 3 is a nonlinear system, which is generally linearized in a low-speed driving environment and used for a time-varying LMPC. According to the model in Fig. 3, a desired trajectory of vehicle kinematic model is defined in Fig. 4. The state quantity and the control amount at any time of the desired trajectory can be described by the relationship as follows

$$\dot{\varepsilon}_c = f(\varepsilon_c, u_c) \tag{10}$$

where $\varepsilon_c = \begin{bmatrix} x_c & y_c & \theta_c \end{bmatrix}^T$ and $u_c = \begin{bmatrix} v_c & \delta_c \end{bmatrix}^T$.

The Taylor series expansion is performed at any point (ε_c, u_c) in this *equation*, which is given by

$$\dot{\varepsilon} = f(\varepsilon_c, u_c) + \left.\frac{\partial f}{\partial \varepsilon}\right|_{\substack{\varepsilon=\varepsilon_c \\ u=u_c}} (\varepsilon - \varepsilon_c) + \left.\frac{\partial f}{\partial u}\right|_{\substack{\varepsilon=\varepsilon_c \\ u=u_c}} \times (u - u_c) + R_n(\varepsilon, u) \tag{11}$$

where $R_n(\varepsilon, u)$ is the high order term of Taylor series ($n = 2, 3, 4, 5 \ldots\ldots$), which can be ignored in Eq. 11.

Subtracting Eqs. 9 and 10, Eq.12 can be obtained

$$\dot{\hat{\varepsilon}} = A(t)\hat{\varepsilon} + B(t)\hat{u} \tag{12}$$

where $\hat{\varepsilon} = \begin{bmatrix} x_h - x_c \\ y_h - y_c \\ \theta - \theta_c \end{bmatrix}$, $\hat{u} = u - u_c = \begin{bmatrix} v_h - v_c \\ \delta_q - \delta_c \end{bmatrix}$, $A(t)$ is the Jacobi matrix between f and ε, and $B(t)$ is the Jacobi matrix between f and u. Eq. 12 is a new state equation. But the state equation is continuous and cannot be directly used for model predictive control. Therefore, it needs to be approximated and discretized [28]. In the discrete systems, the current moment is represented by k and the next moment of the current moment is represented by $k+1$. During the discretization process, $A_{k,t}$ and $B_{k,t}$ are given as

$$A_{k,t} = I + TA(t) \tag{13}$$

$$B_{k,t} = TB(t) \tag{14}$$

where I and T are the unit matrix and the sample time interval respectively. By combining Eqs. 12, 13, and 14 yields

$$\hat{\varepsilon}(k+1) = A_{k,t}\hat{\varepsilon}(k) + B_{k,t}\hat{u}(k-1) \tag{15}$$

Equation 15 is a discrete linearization system. This discrete linear system is applied to the vehicle trajectory tracking kinematic model represented by Eq. 8. Thus, $A_{k,t}$ and $B_{k,t}$ are given by

$$A_{k,t} = \begin{bmatrix} 1 & 0 & -v_c \sin\theta_c T \\ 0 & 1 & v_c \cos\theta_c T \\ 0 & 0 & 1 \end{bmatrix} \tag{16}$$

$$B_{k,t} = \begin{bmatrix} \cos\theta_c T & 0 \\ \sin\theta_c T & 0 \\ \tan\delta_c T/M & v_c/\left(M\cos^2\delta_c\right) \end{bmatrix} \tag{17}$$

$$\hat{\varepsilon} = \begin{bmatrix} x_h - x_c \\ y_h - y_c \\ \theta - \theta_c \end{bmatrix} \tag{18}$$

$$\hat{u} = \begin{bmatrix} v_h - v_c \\ \delta_q - \delta_c \end{bmatrix} \tag{19}$$

The control quantity in Eq. 15 will be calculated in each control cycle, which are the rear axle center point speed v_h and the front wheel steering angle δ_q. Moreover, $\hat{u}(k-1)$ and $\hat{\varepsilon}(k)$ in Eq. 15 are expressed as

$$x(k) = \begin{bmatrix} \hat{\varepsilon}(k) \\ \hat{u}(k-1) \end{bmatrix} \tag{20}$$

Equation 15 is converted into Eq. 21, which yields

$$\begin{cases} x(k+1|t) = \hat{A}_{k,t}x(k|t) + \hat{B}_{k,t}\Delta u(k|t) \\ \eta(k|t) = \hat{C}_{k,t}x(k|t) \end{cases} \tag{21}$$

where $\eta(k|t)$ and $x(k+1|t)$ are the output and state quantity in the predict time domain respectively. $\hat{A}_{k,t}$, $\hat{B}_{k,t}$, and $\hat{C}_{k,t}$ are given as

$$\hat{A}_{k,t} = \begin{bmatrix} A_{k,t} & B_{k,t} \\ 0_{m\times n} & I_m \end{bmatrix} \tag{22}$$

$$\hat{B}_{k,t} = \begin{bmatrix} B_{k,t} \\ I_m \end{bmatrix} \tag{23}$$

$$\hat{C}_{k,t} = [I_{5\times 5}] \tag{24}$$

where $m = 2$ and $n = 3$; I_{m} is the two dimensional unit matrix; and $\hat{C}_{k,t}$ is the 5×5 unit matrix. For further simplification, $\hat{A}_{k,t} = \hat{A}_t$, $\hat{B}_{k,t} = \hat{B}_t$, $\hat{C}_{k,t} = \hat{C}_t$, $k = 1, 2, \cdots t+N-1$. If the system prediction and control horizons are N and M respectively, the state quantity and system output in the predicted time domain are calculated by

$$x(t+N|t) = \hat{A}_t^N x(t|t) + \hat{A}_t^{N-1}\hat{B}_t\Delta u(t|t) + \cdots + \hat{A}_t^{N-M-1}\hat{B}_t\Delta u(t+M|t) \tag{25}$$

$$\eta(t+N|t) = \hat{C}_t\hat{A}_t^N x(t|t) + \hat{C}_t\hat{A}_t^{N-1}\hat{B}_t\Delta u(t|t) + \cdots + \hat{C}_t\hat{A}_t^{N-M-1}\hat{B}_t\Delta u(t+M|t) \tag{26}$$

The matrix expression for the future output variables of the system is

$$Y(t) = \psi_t x(t|t) + \Theta_t \Delta U(t) \tag{27}$$

where

$$Y(t) = \begin{bmatrix} \eta(t+1|t) \\ \eta(t+2|t) \\ \cdots \\ \eta(t+M|t) \\ \cdots \\ \eta(t+N|t) \end{bmatrix}, \quad \psi_t = \begin{bmatrix} \hat{C}_t\hat{A}_t \\ \hat{C}_t\hat{A}_t^2 \\ \cdots \\ \hat{C}_t\hat{A}_t^M \\ \cdots \\ \hat{C}_t\hat{A}_t^N \end{bmatrix},$$

$$\Delta U(t) = \begin{bmatrix} \Delta u(t|t) \\ \Delta u(t+1|t) \\ \cdots \\ \Delta u(t+M|t) \end{bmatrix}$$

$$\Theta_t(t) = \begin{bmatrix} \hat{C}_t\hat{B}_t & 0 & 0 & 0 \\ \hat{C}_t\hat{A}_t\hat{B}_t & \hat{C}_t\hat{B}_t & 0 & 0 \\ \cdots & \cdots & \ddots & \cdots \\ \hat{C}_t\hat{A}_t^{M-1}\hat{B}_t & \hat{C}_t\hat{A}_t^{M-2}\hat{B}_t & \cdots & \hat{C}_t\hat{B}_t \\ \hat{C}_t\hat{A}_t^M\hat{B}_t & \hat{C}_t\hat{A}_t^{M-1}\hat{B}_t & \cdots & \hat{C}_t\hat{A}_t\hat{B}_t \\ \vdots & \vdots & \ddots & \vdots \\ \hat{C}_t\hat{A}_t^{N-1}\hat{B}_t & \hat{C}_t\hat{A}_t^{N-2}\hat{B}_t & \cdots & \hat{C}_t\hat{A}_t^{N-M-1}\hat{B}_t \end{bmatrix}$$

The current state quantity $x(t|t)$ and control increment $\Delta U(t)$ in the control time domain are used to obtain the state and the output quantities in the prediction time domain. Then, the prediction function of the prediction model is obtained.

During the driving processing of an unmanned vehicle, the control sequence in the control time domain must be obtained by solving the set objective function; then, the constraints on the control quantity are applied; finally, the optimal system output in the prediction time domain is predicted. Then, the optimization objective function is given as

$$J\left(\mathrm{x}\left(t\right),u\left(t-1\right),\Delta u\left(t\right)\right) = \sum_{k=1}^{M} \parallel \eta\left(t+k|t\right)-\eta_c\left(t+k|t\right)\|_Q^2 + \sum_{k=1}^{N-1}\parallel \Delta u\left(t+k|t\right)\parallel_R^2 \tag{28}$$

where $\eta\left(t+k|t\right)$ denotes the prediction model output, $k = 1, 2, \cdots, N$; $\eta_c\left(t+k|t\right)$ is the reference trajectory output, $k = 1, 2, \cdots, N$; R and Q are the weight matrices; and $\Delta u\left(t+k|t\right)$ is the prediction model control increment, $k = 1, 2, \cdots, M-1$.

The goal of optimization function is to enable the unmanned vehicles to track a given desired trajectory as soon as possible and satisfy the passengers' comfort requirements. The first part of Eq. 28 reflects the system ability for tracking the desired trajectory. Its second part denotes the stability requirements of control quantity. In the actual control system, to satisfy the vehicle actuator and safety requirements, some constraints must be added on to the system control and output. The constrained optimization targets are given as

$$u_{\min}\left(t+k\right) \le u\left(t+k\right) \le u_{\max}\left(t+k\right), k = 1, \cdots, m-1 \tag{29}$$

$$y_{\min}\left(t+k\right) \le y\left(t+k\right) \le y_{\max}\left(t+k\right), k = 1, \cdots, m-1 \tag{30}$$

The control sequences in the control time domain can be obtained. The first control quantity of control sequence is utilized for the actual control system until the new time system is re-predicted [29].

2) DISCRETIZATION METHOD FOR THE NONLINEAR PREDICTION MODEL

The NMPC algorithm is a better choice than the LMPC algorithm according to the analysis in previous sections. The NMPC algorithm has a better control influence within the allowable range of hardware level. As given in Fig. 3, the 2 DOF kinematic model is a nonlinear one, and the general form of discrete model is expressed as

$$\varepsilon\left(t+1\right) = f\left(\varepsilon\left(t\right),\ u\left(t\right)\right) \varepsilon\left(t\right) \in \chi,\ u\left(t\right) \in \Gamma \tag{31}$$

where ε is a three dimensional state variable; $f\left(\cdot,\cdot\right)$ is the state transition function; χ is a state variable constraint; u is a two dimensional control variable; and Γ is a control variable constraint. Moreover, $f\left(0,0\right) = 0$ is the stability point for this nonlinear system. For this purpose, the optimization objective function in any prediction horizon N is given as

$$J_N\left(\varepsilon\left(t\right),U\left(t\right)\right) = \sum_{k=t}^{t+N-1} l\left(\varepsilon\left(t\right),u\left(t\right)\right) + P\left(\varepsilon\left(t+N\right)\right) \tag{32}$$

where $\varepsilon\left(t\right)$ is the state vector track after the input sequence vector $U\left(t\right)$; $U\left(t\right) = \left[u\left(t\right), u\left(t+1\right), \cdots, u\left(t+N-1\right)\right]^T$ denotes the control quantity input sequence in the prediction horizon N; $l\left(\cdot,\cdot\right)$ reflects the tracking ability of the system for the desired trajectory; and $P\left(\cdot\right)$ reflects the terminal state quantity constraint.

According to Eqs. 31 and 32, the NMPC is used to calculate the constrained finite time domain optimization issue in each step. The relative equations are given as

$$\min_{U_t,\varepsilon_{t+1},\cdots\varepsilon_{t+N,t}} J_N\left(\varepsilon_t, U_t\right) \tag{33}$$

$$s.t.\ \varepsilon_{k+1,t} = f\left(\varepsilon_{k,t}, u_{k,t}\right), \quad k = t, \cdots, N-1 \tag{34}$$

$$\varepsilon_{k,t} \in \chi \quad k = t+1, \cdots, t+N-1 \tag{35}$$

$$u_{k,t} \in \Gamma \quad k = t, \cdots, t+N-1 \tag{36}$$

$$\varepsilon_{t,t} \in \varepsilon\left(t\right)_{initial} \tag{37}$$

$$\varepsilon_{N,t} \in \chi_{final} \tag{38}$$

Here, Equation 34 denotes the state constraint; Eqs. 35 and 36 are the state quantity and control quantity constraints, respectively; and Eqs. 37 and 38 are the initial and terminal state constraints, respectively.

When a feasible solution is obtained in the optimization issue, the optimal control in Eq. 39 is determined. Based on the MPC principle, the first part in the control sequence is applied to the controlled object, Eq. 40 can be obtained. They are given as

$$U_t^*\left(t\right) = \left[u_{t,t}^*, \cdots, u_{t+N-1,t}^*\right]^T \tag{39}$$

$$u\left(\varepsilon\left(t\right)\right) = u_{t,t}^* \tag{40}$$

For the following sampling moment, the initial state with the new sampling time from Eqs. 33 to 38 will be recalculated, which results in a new control sequence. Then, the first part will be used in the system to solve it cyclically when the entire control processing is completed.

IV. SIMULATION RESULTS AND ANALYSIS

In order to verify the control stability of the trajectory tracking for the unmanned vehicle from the proposed predictive control algorithm with the time-varying model and ensure the adequacy of the simulation, the control results from the predictive control algorithm with the linear model are analyzed by considering different longitudinal speed conditions and different road conditions with various curvatures. The double-shifting condition and the arbitrary trajectory condition are discussed. The double-shifting condition is a standard working condition; and the arbitrary trajectory condition is composed of segments including the straight and sinusoidal lines.

A. INFLUENCE OF THE VEHICLE SPEED ON THE CONTROL MODEL TRACKING STABILITY

1) DOUBLE-SHIFTING CONDITION

For the double-shifting condition, the vehicle longitudinal speed are 1 and 2 m/s, respectively. Figures 5 and 6 give

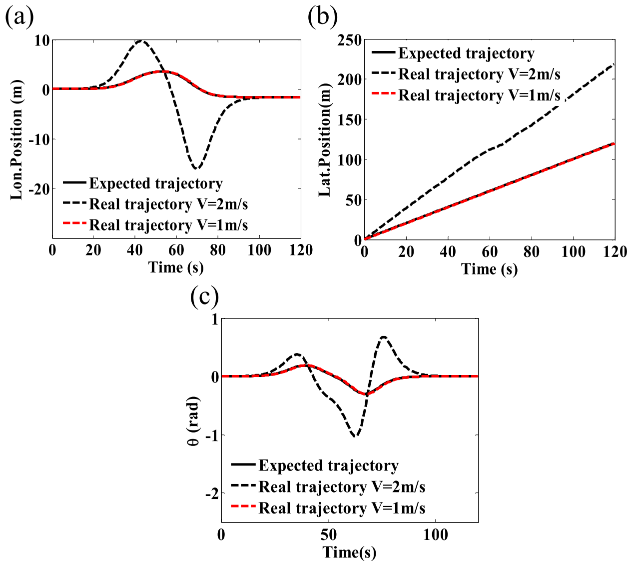

FIGURE 5. Influence of the vehicle speed on the (a) longitudinal coordinates, (b) lateral coordinates, and (c) heading angle for the double-shifting condition for different speed conditions.

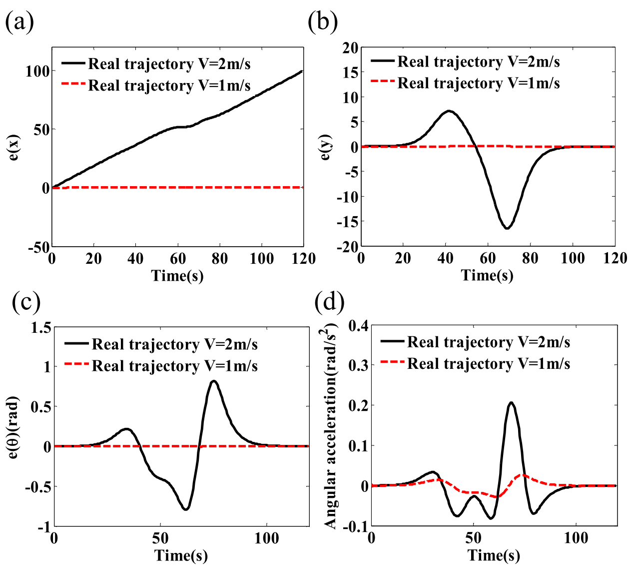

FIGURE 6. Influence of the vehicle speed on the (a) lateral deviation, (b) longitudinal deviation, (c) heading angle deviation, and (d) angular acceleration for the double-shifting condition for different speed conditions.

the corresponding simulation results. When the longitudinal speed is 1 m/s, the longitudinal coordinate, the lateral coordinate, and the heading angle are basically coincident with the desired trajectory. However, when the longitudinal speed is 2 m/s, there is a large deviation. The changes in the heading angle and the angular acceleration in different directions are shown in Fig. 5. When the longitudinal speed is 1 m/s, the maximum absolute deviation along the X direction is 0.0043 m. The absolute maximum deviation along the Y direction is 0.0223 m. The absolute maximum deviation of the heading angle is 0.0047 rad/s. The changes in the angular acceleration are relatively small. The maximum angular

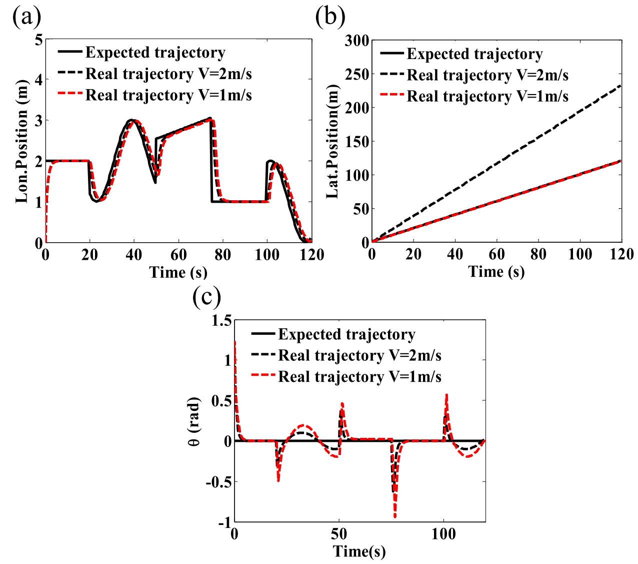

FIGURE 7. Influence of the vehicle speed on the (a) longitudinal coordinates, (b) lateral coordinates, and (c) heading angle for the arbitrary trajectory condition for different speed conditions.

acceleration is 0.0260 rad/s^2. When the longitudinal speed is 2 m/s, the maximum absolute deviation along the X direction is 98.97 m. The absolute maximum deviation along the Y direction is 16.4874 m. The maximum absolute value of heading deviation is 0.08193 rad/s. The absolute value of angular acceleration is 0.2061 rad/s^2. The results show that the predictive control algorithm with the linear model has a good tracking effect on the double-shift line condition when the longitudinal speed is 1 m/s. Moreover, it has a great influence on the tracking stability with the increment of the longitudinal speed.

2) ARBITRARY TRAJECTORY CONDITION

The arbitrary trajectory condition is more complex than the double-shifting working condition. The arbitrary trajectory condition includes a straight running condition and a sinusoidal working condition. The longitudinal speeds of the unmanned vehicle are 1 and 2 m/s, respectively. It shows that both the longitudinal speed and the road curvature can significantly affect the tracking stability. In Fig. 7, the curvature of the desired trajectory has changed drastically during 20-100s. In Fig. 8, the tracking deviations are greatly changed at 20 s, 50 s, 75 s, and 100 s.

B. INFLUENCE OF THE ROAD CURVATURE ON THE CONTROL MODEL TRACKING STABILITY

Because of the extreme changes in the road curvature, the actual driving vehicle will reduce the vehicle speed according to the road surface condition to decrease the tracking deviation. On the other hand, the unmanned vehicle can reduce the road curvature and smooth the desired trajectory to decrease the tracking deviation. In Figs. 5 and 6, the tracking results before and after the trajectory smoothing processing are compared. The Expected trajectory-1 represents

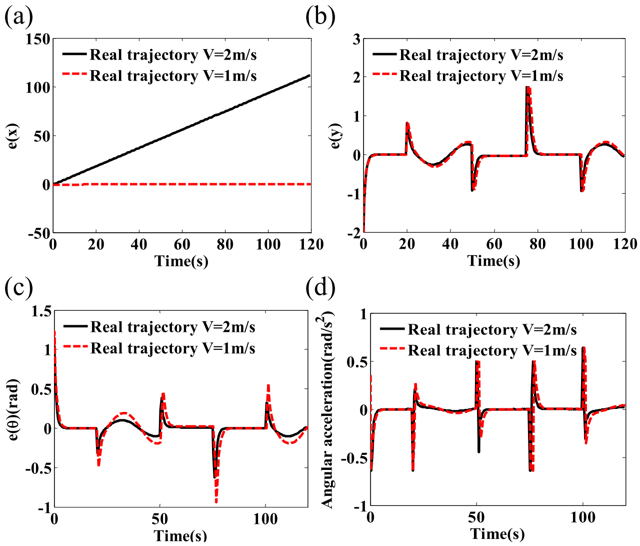

FIGURE 8. **Influence of the vehicle speed on the (a) lateral deviation, (b) longitudinal deviation, (c) heading angle deviation, and (d) angular acceleration for the arbitrary trajectory condition for different speed conditions.**

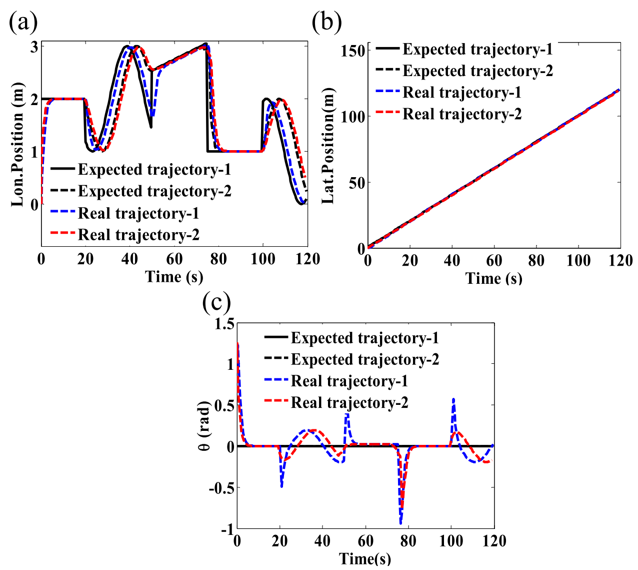

FIGURE 9. **Influence of the road curvature on the (a) longitudinal coordinates, (b) lateral coordinates, and (c) heading angle for the arbitrary trajectory condition for different road curvature conditions.**

the desired trajectory before the smoothing processing. The Expected trajectory-2 represents the desired trajectory after smoothing processing. The Real trajectory-1 represents the tracking before the desired trajectory smoothing processing. Moreover, the Real trajectory-2 represents the tracking result after the desired trajectory smoothing processing. According to the simulation results for different vehicle speed conditions in Section 4.1, the tracking results are better ones when the longitudinal speed is 1 m/s. Therefore, the longitudinal speed is defined as 1 m/s for the Expected trajectory-1 and Expected trajectory-2. In Figs. 9 and 10, the curvature of the desired track is changed. For Expected trajectory-2, the deviations for the longitudinal coordinates, the lateral coordinates, the

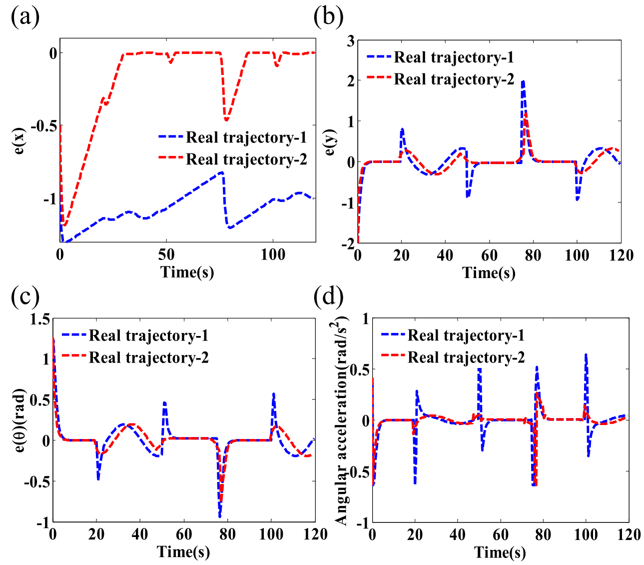

FIGURE 10. **Influence of the road curvature on the (a) lateral deviation, (b) longitudinal deviation, (c) heading angle deviation, and (d) angular acceleration for the arbitrary trajectory condition for different road curvature conditions.**

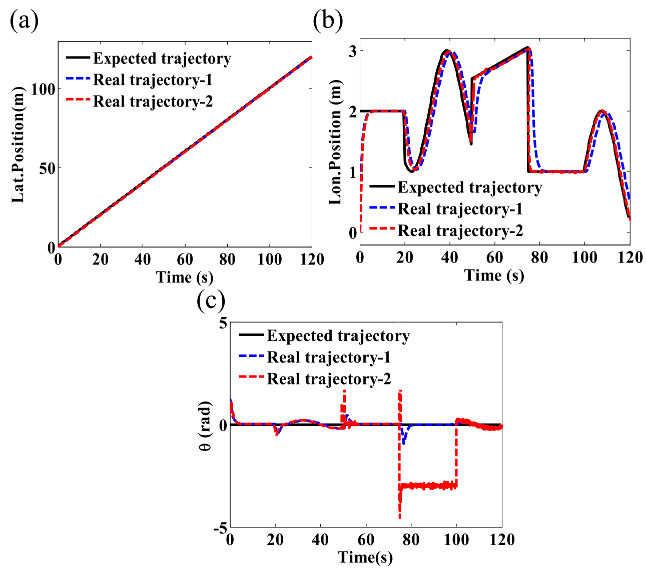

FIGURE 11. **Influence of different algorithms on the (a) longitudinal coordinates, (b) lateral coordinates, and (c) heading angle for the arbitrary trajectory condition for different algorithms.**

heading angle, and the desired trajectory are significantly reduced. The angular acceleration decreases at 75 s; and it decreases significantly at 20 s, 50 s, and 100 s. The above results show that the ride comfort for driverless vehicles is significantly improved.

C. INFLUENCE OF DIFFERENT ALGORITHMS ON THE VEHICLE TRAJECTORY TRACKING STABILITY

1) ARBITRARY TRAJECTORY CONDITION

Similarly, due to the extreme changes in the road curvature, the actual driving vehicle will reduce the vehicle speed according to the road surface condition to decrease the

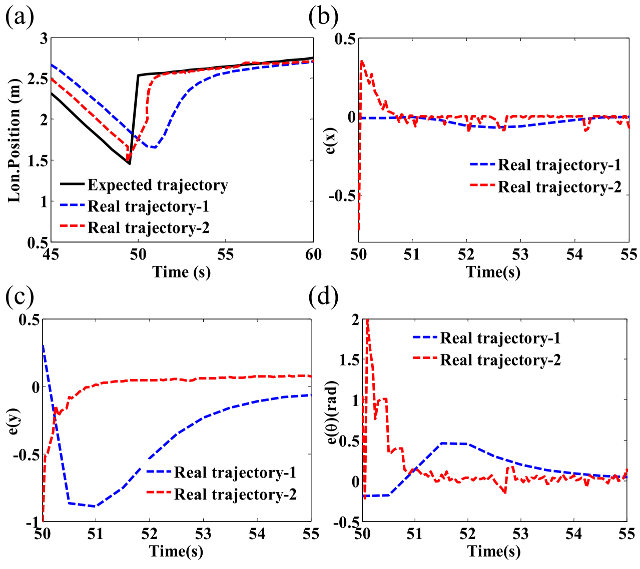

FIGURE 12. Influence of different algorithms on the (a) longitudinal coordinates, (b) lateral deviation, (c) longitudinal deviation, and (d) heading angle deviation for the arbitrary trajectory condition for different algorithms at 50 s.

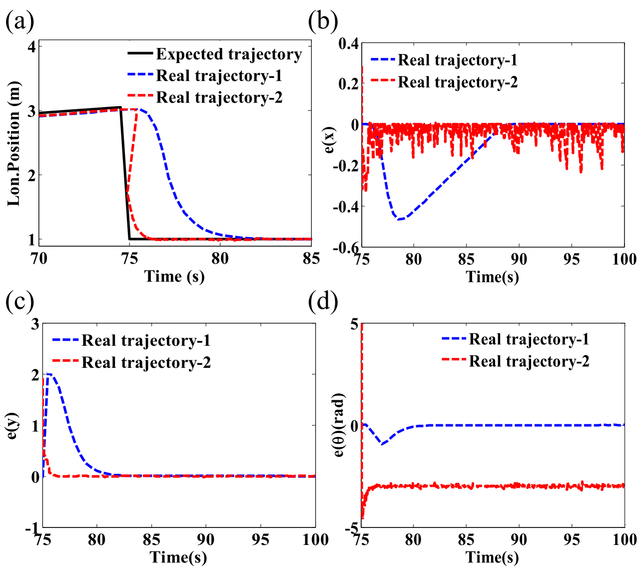

FIGURE 13. Influence of different algorithms on the (a) longitudinal coordinates, (b) lateral deviation, (c) longitudinal deviation, and (d) heading angle deviation for the arbitrary trajectory condition for different algorithms at 75 s.

tracking deviation. The unmanned vehicle can not only reduce the road curvature for decreasing the tracking deviation, but also use the predictive control algorithm with the nonlinear model to obtain a higher local tracking accuracy when the road surface curvature changes sharply. By these ways, the tracking accuracy and the real-time performance can be improved; and it can ensure the tracking accuracy too.

The road curvature is defined as $1/R_r$, where R_r is the road curve radius. The maximum longitudinal vehicle speed set in this paper is 2 m/s. In the Fig.9 (a), the real trajectory 2 (Red dotted line) represents the tracking result used the LMPC

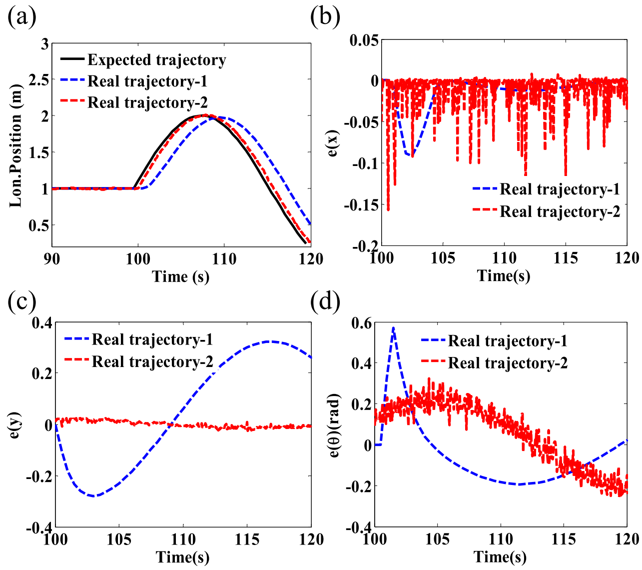

FIGURE 14. Influence of different algorithms on the (a) longitudinal coordinates, (b) lateral deviation, (c) longitudinal deviation, and (d) heading angle deviation for the arbitrary trajectory condition for different algorithms at 100 s.

method when the maximum road curvature of the expected trajectory-2 is 0.67. The results show that the tracking performance is well. But when the maximum road curvature of the expected trajectory-1 is 1.25, the real trajectory 1(blue dotted line) represents the tracking result when the maximum road curvature of the expected trajectory-1 is 1.25, the tracking deviation obviously increases.

So, the value of the road curvature division is set to 1.25 in the time-varying model predictive control method. According to the division value, In the Fig. 11(b), the NMPC algorithm is used at 50 s, 75 s, and 100 s in the arbitrary trajectory here.

In Fig. 11, the Real trajectory-1 represents a tracking trajectory using the LMPC algorithm, and the Real trajectory-2 represents a tracking trajectory using a predictive control algorithm with the time-varying models. In Fig. 11, it seems that the tracking deviation is significantly reduced when the curvature of the road changes drastically.

The tracking results and deviations at each point are shown in Figs.12, 13, and 14. In Figs. 12(a), 13(a), and 14(a), for the expected trajectory with large curvature variations at 50 s, 75 s, and 100 s, the convergence speed of the predictive control algorithm with the time-varying model is faster than that with the linear model. Figures 12(b) to 12(d), 13(b) to 13(d), and 14(b) to 14(d) show the deviations between the tracking trajectory and the expected trajectory in the lateral, longitudinal and heading angles. The tracking error of the predictive control algorithm with the time-varying model is less than that of the linear model.

2) LANE CHANGE TRAJECTORY CONDITION

To show more validations for the reliability of the time-varying model predictive control algorithm, the vehicle lane change trajectory is tracked in this section.

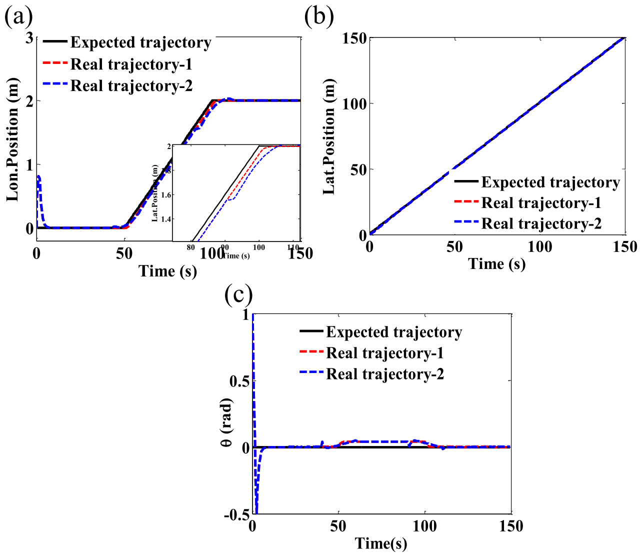

FIGURE 15. Influence of different algorithms on the (a) longitudinal coordinates, (b) lateral coordinates, and (c) heading angle for the lane change trajectory condition for different algorithms.

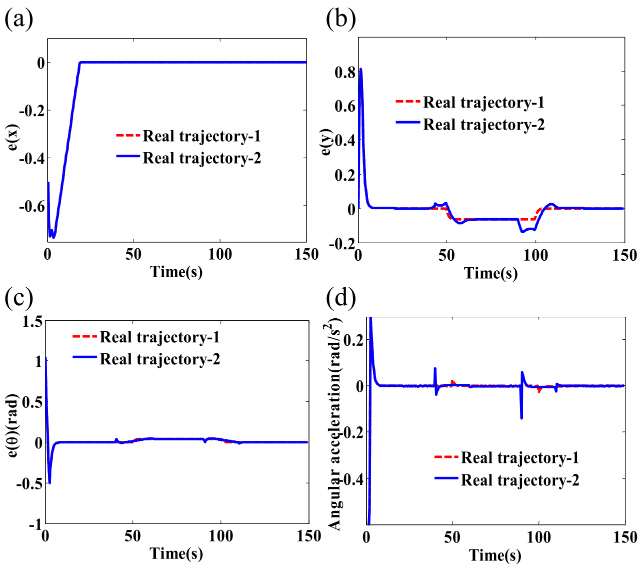

FIGURE 16. Influence of different algorithms on the (a) longitudinal deviation, (b) lateral deviation, (c) heading angle deviation, and (d) angular acceleration for the lane change trajectory condition for different algorithms.

The results are shown in Figs. 15 and 16. In Figs. 15 and 16, the Real trajectory-2 represents a tracking trajectory using the LMPC algorithm, and the Real trajectory-1 represents a tracking trajectory using a predictive control algorithm with the time-varying model. For the Real trajectory-1, the nonlinear model is uesd near the points at 50 s and 100 s. Figures 16(a) to 16(d) show the deviations between the tracking trajectory and the expected trajectory in the lateral, longitudinal, heading angles and angular accelerations. The tracking deviation of the time-varying model predictive control algorithm is significantly reduced near the points at 50 s and 100 s, and the tracking accuracy is improved too. It seems that the algorithm has universal applicability.

TABLE 2. Employed model parameters.

Symbol	Description
x_q, y_q	axle coordinates of the front axles
x_h, y_h	axle coordinates of the rear axles
θ	heading angle of the vehicle
δ_q	front wheel deflection angle
v_q	speed of the front wheel
v_h	speed of the rear wheel
M	length of the wheelbase
R	instantaneous turning radius of the vehicle
ω	yaw angular velocity
x_c, y_c	axle coordinates of the rear axles for the desired trajectory
θ_c	heading angle of the vehicle for the desired trajectory
v_c	longitudinal reference speed
ε_c	state variable of the desired trajectory
u_c	control variable of the desired trajectory
$A(t)$	Jacobi matrix between f and ε
$B(t)$	Jacobi matrix between f and u
$A_{k,t}$	Discrete $A(t)$
$B_{k,t}$	Discrete $B(t)$
T	sample time interval
I	unit matrix
$\hat{\varepsilon}$	state variable increment
$\hat{u}$	control variable increment
$x(k+1\|t)$	state quantity in the predict time domain
$\eta(k\|t)$	output quantity in the predict domain
Q, R	weight matrices

V. CONCLUSION

This work proposes a vehicle trajectory tracking method with the time-varying model based on a two-dimensional vehicle model. The influences of the longitudinal vehicle speed and road curvature on the tracking path and deviation of the unmanned vehicle with a low-speed for the double-shift line condition and the arbitrary trajectory condition are analyzed. Some conclusions can be drawn from this study and are listed as follows:

(1) The predictive control algorithm with the linear model has a good tracking effect on the double-shift line condition when the longitudinal speed is 1 m/s. Moreover, it has a great influence on the tracking stability with the increment of the longitudinal speed. Both the longitudinal speed and road curvature can greatly affect the tracking stability

(2) The actual driving vehicle can reduce the vehicle speed according to the road surface condition to decrease the tracking deviation. The tracking results are better when the longitudinal speed is 1 m/s. For Expected trajectory-2, the deviations for the longitudinal coordinates, the lateral coordinates, the heading angle, and the desired trajectory are significantly reduced. The angular acceleration decreases at 75 s; and it decreases significantly at 20 s, 50 s, and 100 s. The above results show that the ride comfort for driverless vehicles is significantly improved.

(3) The unmanned vehicle can not only reduce the road curvature for decreasing the tracking deviation, but also use

the predictive control algorithm with the nonlinear model to obtain a higher local tracking accuracy when the road surface curvature changes sharply. By these ways, the tracking accuracy and the real-time performance can be improved; and it can also ensure the tracking accuracy. The tracking deviation is significantly reduced when the curvature of the road changes drastically.

Moreover, this work studies a low longitudinal vehicle speed, which can be used for the low-speed driving environments, such as the road cleaning, parcel sorting, parcel delivery, etc. Future work will mainly focus on how to achiever the real intelligent autonomous model selection for the unmanned vehicles according to the road curvature, which is not a method according to the setting path and selection situation.

APPENDIX

See Table 2.

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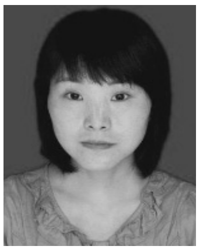

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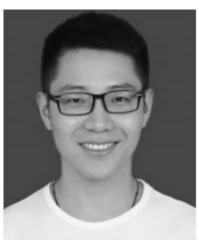

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