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Multi-Objective Robust Control for Vehicle Active Suspension Systems via Parameterized Controller

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ABSTRACT A parameterized controller design approach is proposed to solve the problem of multi-objective control for vehicle active suspension systems by using symbolic computation. The considered model is a quarter-vehicle model of the active suspension system. The multi-objective robust control performances include the sprung mass acceleration, suspension deflection, and tire deflection. Based on dissipative Hamiltonian systems and Lyapunov function, a multi-objective H_∞ controller design approach is developed, which can avoid solving Hamilton-Jacobi-Issacs equations. Then, an algorithm of solving semi-positive definite polynomial with tuning parameters is proposed by using symbolic computation. Furthermore, a method of parameter optimization is proposed. Simulations and comparation show that the control performance is significantly improved comparing with passive controlled systems and existing other control systems for active suspension systems.

INDEX TERMS Multi-objective control, active suspension system, parameterized controller, symbolic computation, dissipative Hamiltonian systems.

I. INTRODUCTION

Vehicle suspension system plays a vital role in both the driving safety and comfort, where the ride comfort and the road-handling capacity are important in modern vehicles. In order to meet these vehicle performance requirements, three types of vehicle suspension systems, including passive [1], [2], semi-active [3], [4], and active suspensions [5], [6], are currently being studied and applied in practice in the past decades. For active suspension control design, more energy should be added and dissipated by actuators between the wheel-axle and sprung mass than passive and semi-active suspension control. However, the control performance in active suspension system is better than passive system and semi-active suspension system. The active suspension controller for vehicle system obtains more and more attention [7]–[10].

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The aim of this paper is to improve the ride performance for the vehicle active suspension system. Generally, acceleration of sprung mass, suspension stroke, and tire deflection are the multi-objective performances, which should maintain an acceptable level as handling measures. There are various approaches can improve the performance of active suspension. The H_∞ control of active suspension has obtained much attention in recent years. In particular, [11], [12] pointed out that H_∞ control method for vehicle active suspension can be a particularly effective way to manage the tradeoff between conflicting performances. An approach of H_∞ robust control and method of linear matrix inequality (LMI) optimization for non-stationary running condition and parameter uncertainty has been proposed in [13]. For the same problems, [13], [14] proposed an input delay method with sampling measurements. References [15], [16] studied the H_∞ control for half-vehicle active suspensions with time delay. These controllers obtained in [13]–[16] were based on LMI. However, this inequality method has high complexity. A saturated adaptive robust control strategy has been proposed in [17] for system uncertainties and actuator saturation. Based on accurate models, an adaptive sliding mode fault-tolerant controller was proposed for the active suspension systems in [18]. Reference [19] proposed a sliding-mode control method for vehicle active suspension system by using Lyapunov stability theory. However, the multi-objective control problem was not considered in [17]-[19]. Vehicle performance control is complex and multi-objective. An adaptive law for multi-objective suspension control based on the frequency domain was estimated from the road profile [20]. Reference [21] established the corresponding dynamic systems and transformed them into the stochastic system for the multi-objective H_∞ controller design. By using the magnitude of suspension deflection function and road disturbance effect estimate, a nonlinear control for dual object active suspension systems was proposed in [22]. These approaches in [20]–[22] resolved multi-objective control problems for active suspension systems. However, they are the control strategy, which proposed only one fixed controller for active suspension systems. A parameterization design method for all stabilizing controllers was derived for a given plant [23]. However, too many constraints were required to obtain the controller [23].

For a system, designing a controller to satisfy some desired performance objectives is a critical thing. To this end, it is a sophisticated and efficient method to find a parameterized controller to solve multi-objective control problems. Therefore, the parameterized controller can provide a wonderful solution in control theory [24]–[27]. Youla firstly proposed the idea of parameterizing of stabilizing controllers in a linear feedback system in [24]. In the system, the state and the external disturbance are measurable, [25] proposed a family of nonlinear controllers via state-feedback. Yung extended the formulas of state-space and a family of H_∞ controllers for the n-dimensional system in [26]. Reference [27] proposed a family of reliable controllers via solving the Hamilton-Jacobi-Issacs (HJI) inequality (or equations). The process of designing these controllers can be finished by solving a class of HJI inequalities (or equations), which is very difficult [24]–[27]. In order to obtain the parameterized controller, [28] studied the general Hamiltonian system and proposed a set of parameterized H_∞ controllers, which avoided solving HJI inequalities (or equations) by applying good structure and clear physical expression of a general Hamiltonian system. However, so many hard constraints in controller design could not even be achieved. A family of robust simultaneous controllers with tuning parameters for a set of dissipative Hamiltonian systems has been proposed [29], [30].

In this paper, we propose a multi-objective H_∞ parameterized controller design approach by Lyapunov function and symbolic computation for vehicle active suspension systems. The main features of the proposed parameterized controller are as follows:

1) The controller can satisfy the multi-objective control performance for active suspension systems.

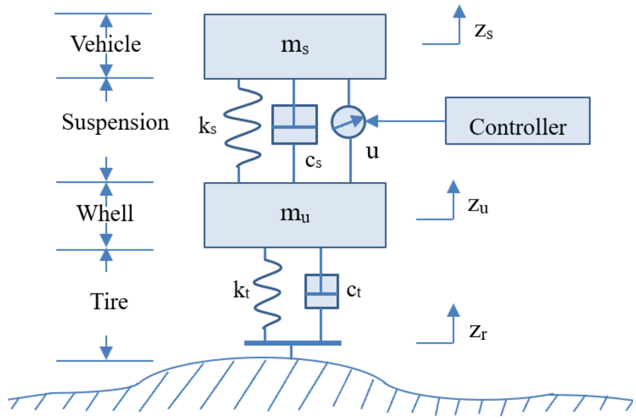

FIGURE 1. Quarter-vehicle model.

2) The controller can optimize the robustness by adjusting parameters' values through tuning parameters.
3) The controller design method can realize easily to avoid solving HJI inequalities (or equations).

The remainder of this paper is organized as follows. In section 2, the problem to be addressed is presented. Section 3 presents some preliminaries and results of controller parameterization, a controller with tuning parameters, an algorithm for solving parameters and a method for optimization parameters, respectively. Then, a family of parameterized controllers is proposed for a quarter-vehicle model with the active suspension system in Section 4. Some examples and simulations for illustrating the effectiveness and feasibility of the multi-objective robust controller are given in Section 5. Finally, Section 6 concludes this paper.

II. PROBLEM FORMULATION

The quarter-vehicle model used in this paper is as shown in Fig. 1. Sprung mass m_s is the car body and may vary for loading different passengers and cargo; unsprung mass m_u is the wheel assembly mass; k_s and c_s represent the coefficient of stiffness and damping, respectively; k_t and c_t stand for the tires' elasticity coefficient and damping coefficient, respectively; z_s and z_u represent the displacements of m_s and m_u, respectively; z_r denotes the road displacement of external input; $u(t)$ stands for the input control power of suspension system. Then, the dynamic equations for the model as follows:

$$\begin{aligned} m_s\ddot{z}_s(t) &= -k_s[z_s(t)-z_u(t)] - c_s[\dot{z}_s(t)-\dot{z}_u(t)] + u(t) \\ m_u\ddot{z}_u(t) &= k_s[z_s(t)-z_u(t)]\, c_s[\dot{z}_s(t)-\dot{z}_u(t)] \\ &\quad -k_t[z_u(t)-z_r(t)] - c_t[\dot{z}_u(t)-\dot{z}_r(t)] - u(t) \end{aligned} \tag{1}$$

We define the following state variable for establishing the state-space form:

$x_1(t) = z_s(t) - z_u(t)$, suspension deflection,
$x_2(t) = z_u(t) - z_r(t)$, tire deflection,

$x_3(t) = \dot{z}_s(t)$, sprung mass velocity,
$x_4(t) = \dot{z}_u(t)$, unsprung mass velocity,

After choosing the disturbance input $\omega(t) = \dot{z}_r(t)$ and the variables as:

$$x(t) = [x_1\ x_2\ x_3\ x_4]^T \tag{2}$$

The dynamic equations (1) can be expressed as the following state-space form:

$$\dot{x}(t) = Ax(t) + B_1 u(t) + B_2 \omega(t) \tag{3}$$

where $A = \begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ -\frac{k_s}{m_s} & 0 & -\frac{c_s}{m_s} & \frac{c_s}{m_s} \\ \frac{k_s}{m_u} & -\frac{k_t}{m_u} & \frac{c_s}{m_u} & -\frac{c_s+c_t}{m_u} \end{bmatrix}$,

$B_1 = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_s} \\ -\frac{1}{m_u} \end{bmatrix}$, $B_2 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ \frac{c_t}{m_u} \end{bmatrix}$.

The performances of ride comfort, road holding ability, and suspension deflection are three main performance criteria in any vehicle suspension design, which are exactly the multi-objective of robust control in this paper.

1) As known, the ride comfort and the vehicle body's vertical acceleration have a direct relationship. Consequently, the sprung mass acceleration (SMA), SMA $= |\ddot{x}_3(t)|$, should be as small as possible.
2) The structural properties of the vehicle active suspension system impose restrictions the suspension deflection within rattle space as the limit x_{rmax}, $x_1(t) < x_{rmax}$. The definition of relative suspension deflection RSD $=\frac{|x_1|}{x_{rmax}}$. Therefore, the performance requirement is RSD < 1.
3) External disturbances caused by road bumps are harmful to safe driving. How to guarantee the uninterrupted contact between wheels and road for vehicle handling is an extremely important issue. The relative tire force (RTF) is RTF $=\frac{|k_t \cdot x_2 + c_t \cdot \dot{x}_2|}{(m_s+m_u)g}$. Therefore, another performance requirement is that RTF < 1.

III. PARAMETERIZED CONTROLLER DESIGN

A. PRELIMINARIES

Consider the dissipative Hamiltonian system [31]

$$\begin{cases} \dot{x} = [J(x) - R(x)]\nabla H(x) + g_1(x)u(t) + g_2(x)\omega(t) \\ y = g_2^T(x)\nabla H(x) \\ z = h(x)g_1^T(x)\nabla H(x) \end{cases} \tag{4}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ and $\omega \in \mathbb{R}^s$ are the state vector, the controller with parameters and the disturbances, respectively. $J(x)$ and $R(x)$ are the skew-symmetric matrix and the positive semi-definite matrix. $g_1(x)$ and $g_2(x)$ are sufficiently smooth functions with proper order. $y \in \mathbb{R}^p$ and $z \in \mathbb{R}^q$ are the output and the penalty, respectively. $h(x)$ is the weight matrix for a specific penalty. The Hamiltonian function $H(x)$ must be a positive semi-definite polynomial and has a local minimum at the equilibrium x_0 of system, and $\nabla H(x) = (\partial H/\partial x)(x)$.

Assumption 1: The Hessian matrix $Hess(H(x_0)) > 0$ and $H(x) \in C^2$.

Remark 1: From Assumption 1 for the system (4), we can find $Hess(H(x_0)) > 0$ guarantees that $H(x)$ is strict convex on some neighborhood of the equilibrium x_0 and $H(x) \in C^2$ guarantees the existence of $Hess(H(x))$.

In this paper, the aim of proposing a parameterized controller approach for systems (4) can be described as: for a given H_∞ disturbance attenuation level $\gamma > 0$, we can obtain a parameterized controller of the form $u = \alpha(x)$ $(\alpha(x_0) = 0)$ such that the L_2 gain is not more than γ for the closed-loop system (from ω to z), and the closed-loop system is locally asymptotically stable when disturbance $\omega = 0$.

Definition 1: [32] If $y(t) = 0$ and $\omega(t) = 0$, $\forall t \geq 0$, then $\nabla H(x(t)) = 0$, $\forall t \geq 0$, system (4) is called zero-energy-gradient (ZEG) observable to y; If $y(t) = 0$ and $\omega(t) = 0$, $\forall t \geq 0$, then $\lim_{t\to\infty} \nabla H(x(t)) = 0$, system (4) is called ZEG detectable with respect to y.

Lemma 1: [31] Consider a nonlinear system

$$\begin{cases} \dot{x} = f(x) + g(x)\omega, f(x_0) = 0 \\ z = h(x) \end{cases} \tag{5}$$

where $x \in \mathbb{R}^n$, $\omega \in \mathbb{R}^s$ and $z \in \mathbb{R}^q$ are the state vector, the disturbances, and the penalty, respectively. If there exists a function $V(x) \geq 0$ $(V(x_0) = 0)$ such that HJI inequality

$$\left(\frac{\partial V}{\partial x}\right)^T f(x) + \frac{1}{2\gamma^2}\left(\frac{\partial V}{\partial x}\right)^T g(x)g^T(x)\frac{\partial V}{\partial x} + \frac{1}{2}h^T(x)h(x) \leq 0 \tag{6}$$

holds, it's indicated that the L_2 gain is not more than $\gamma (\gamma > 0)$ for the closed-loop system (5) (from ω to z), i.e.,

$$\int_0^\infty \|z\|^2 dt \leq \int_0^\infty \gamma^2 \|\omega\|^2 dt \tag{7}$$

B. H_∞ CONTROLLER DESIGN

An H_∞ parameterized controller for the system (4) will be designed in this section. For simplification, we denote $J = J(x)$, $R = R(x)$, $\nabla H = \nabla H(x)$, $u = u(t)$, $g_1 = g_1(x)$, $g_2 = g_2(x)$, $h = h(x)$ in this section.

Theorem 1: Assumption 1 holds and system (4) is generalized ZEG detectable (when $\omega = 0$) and

$$R + \frac{1}{2\gamma^2}\left(g_1 g_1^T - g_2 g_2^T\right) \geq 0 \tag{8}$$

$$\nabla H^T g_1 \alpha(x, a) \leq 0 \tag{9}$$

hold simultaneously (the matrix $\nabla H^T g_1 \alpha(x, a)$ is negative semi-definite and $R + \frac{1}{2\gamma^2}\left(g_1 g_1^T - g_2 g_2^T\right)$ is positive semi-definite). $\alpha(x, a)$ is the item with tuning parameters and a are the tuning parameters. Then, H_∞ control of the system (4) can be realized by the following controller

$$u = \alpha(x, a) + \beta(x) \tag{10}$$

where $\beta(x) = -\frac{1}{2}\left[h^T h + \frac{1}{\gamma^2} I_m\right] g_1^T \nabla H$, I_m is an $m \times m$ identity matrix.

Proof: Consider the candidate Lyapunov function $V(x) = H(x) - c \geq 0$ $(c = H(x_0))$. From Lemma 1 and controller (10), we have

$$\begin{aligned}
&\left(\frac{\partial V}{\partial x}\right)^T f(x) + \frac{1}{2\gamma^2}\left(\frac{\partial V}{\partial x}\right)^T gg^T\left(\frac{\partial V}{\partial x}\right) + \frac{1}{2}z^T z \\
&= -\nabla H^T R \nabla H + \nabla H^T g_1 u + \frac{1}{2\gamma^2}\nabla H^T g_2 g_2^T \nabla H + \frac{1}{2}z^T z \\
&= -\nabla H^T\left(R + \frac{1}{2}g_1 h^T h g_1^T + \frac{1}{2\gamma^2}g_1 g_1^T\right)\nabla H \\
&\quad + \nabla H^T g_1 \alpha(x,a) \\
&\quad + \frac{1}{2\gamma^2}\nabla H^T g_2 g_2^T \nabla H + \frac{1}{2}\nabla H^T g_1 h^T h g_1^T \nabla H \\
&= -\nabla H^T\left(R + \frac{1}{2\gamma^2}\left(g_1 g_1^T - g_2 g_2^T\right)\right)\nabla H \\
&\quad + \nabla H^T g_1 \alpha(x,a) \\
&\leq 0
\end{aligned} \tag{11}$$

which implies the L_2 gain of the closed-loop system (4) controlled by the controller (10) (from ω to z) is bounded by given prescribed disturbance attenuation value γ. Next, we prove that the closed-loop system is asymptotically stable at x_0, when $\omega = 0$. From system (4), controller (10) and $\omega = 0$, it follows that

$$\begin{aligned}
\dot{V}(x) &= \left(\frac{\partial V}{\partial x}\right)^T [J - R]\left(\frac{\partial V}{\partial x}\right) + \left(\frac{\partial V}{\partial x}\right)^T g_1 u \\
&= -\nabla H^T R \nabla H + \nabla H^T g_1 \\
&\quad \times \left[-\frac{1}{2}\left(h^T h + \frac{1}{\gamma^2} I_m\right) g_1^T \nabla H + \alpha(x,a)\right] \\
&= -\nabla H^T\left(R + \frac{1}{2\gamma^2}\left(g_1 g_1^T - g_2 g_2^T\right)\right)\nabla H \\
&\quad - \frac{1}{2}\nabla H^T g_1 h^T h g_1^T \nabla H \\
&\quad - \frac{1}{2\gamma^2}\nabla H^T g_2 g_2^T \nabla H + \nabla H^T g_1 \alpha(x,a) \\
&= -\nabla H^T\left(R + \frac{1}{2\gamma^2}\left(g_1 g_1^T - g_2 g_2^T\right)\right)\nabla H \\
&\quad - \frac{1}{2}\left\|h g_1^T \nabla H\right\|^2 \\
&\quad - \frac{1}{2\gamma^2}\left\|g_2^T \nabla H\right\|^2 + \nabla H^T g_1 \alpha(x,a) \leq 0
\end{aligned} \tag{12}$$

Hence, the closed-loop system converges to the largest invariant set, which is contained in

$$S := \left\{x : \dot{V}(x) = 0\right\} \subset \left\{\begin{array}{l} x : y = g_2^T \nabla H = 0, \\ z = h g_1^T \nabla H = 0, \forall t \geq 0 \end{array}\right\} \tag{13}$$

From Assumption 1 and the generalized ZEG detectable system (4), we obtain

$$\lim_{t\to\infty} \nabla H(x(t)) = \nabla H\left(\lim_{t\to\infty} x(t)\right) = 0 \tag{14}$$

On the other hand, $\nabla H(x_0) = 0$ holds because of the equilibrium x_0 is a local minimum of $H(x)$. So, there exists a neighborhood of x_0, in which $\nabla H\left(\lim_{t\to\infty} x(t)\right) = 0$ implies $\lim_{t\to\infty} x(t) = x_0$. Therefore, it holds that the trajectory of $\dot{V}(x) = 0$ is strict convex at the equilibrium point x_0. The closed-loop system (4) is locally asymptotically stable at x_0 under controller (10) from the LaSalle invariant principle. This completes the proof.

Remark 2:

1) Compared with the controller proposed in [28], the controller (10) has a much simpler form and is easier to realize.
2) For the proposed controller, the parameters' range of polynomial vector $\alpha(x,a)$ can be obtained by solving the condition (9).
3) The proposed controller can be applied to general nonlinear systems with the dissipative Hamiltonian realization method [33], [34].

C. SOLVING TUNING PARAMETERS

From condition (8), we can obtain a special given disturbance attenuation value γ^*. Let $\gamma \geq \gamma^*$ such that condition (8) holds. Then we propose an algorithm to find the ranges of parameters in the controller (10) via solving the parameters of $\alpha(x,a)$ in condition (9) by using cylindrical algebraic decompositions (CAD), which is one method of symbolic computation [35]. The solving turning parameters (STP) algorithm is proceeded as follows [29].

STP Algorithm

INPUT: System's parameters $Sys = \{x_i, n, r, \nabla H, \alpha(x,a)\}$. $//x_i$ is the state variable, n is the number of the state variable, $\alpha(x,a)$ is the constructed polynomial with parameters and r is the such polynomial's highest order, a is the tuning parameters.

OUTPUT: Tuning parameters' range U. $//U$ is an empty set, initially.

The flow diagram of the proposed STP algorithm for condition (9) is outlined in Fig. 2.

Remark 3:

1) Normally, $r = 1$ is the beginning of the STP Algorithm, and if the result is not obtained, $r = r + 1$ until the output solution set U can be obtained.
2) The obtained solution set U is the partial solution, not a complete solution by using CAD.

D. PARAMETER OPTIMIZATION

The parameters optimization will be discussed in the tuning parameters ranges as follows. For system (5), add the controller and rewrite it,

$$\dot{X} = F(X,\mu) + \alpha(X,\mu) \tag{15}$$

According to Lyapunov Theorem, there must have a positive definite symmetric matrix $P = P^T$ in a stabilized system by using the controller (10). The fastest stabilization parameter search problem of the controlled system (15) is

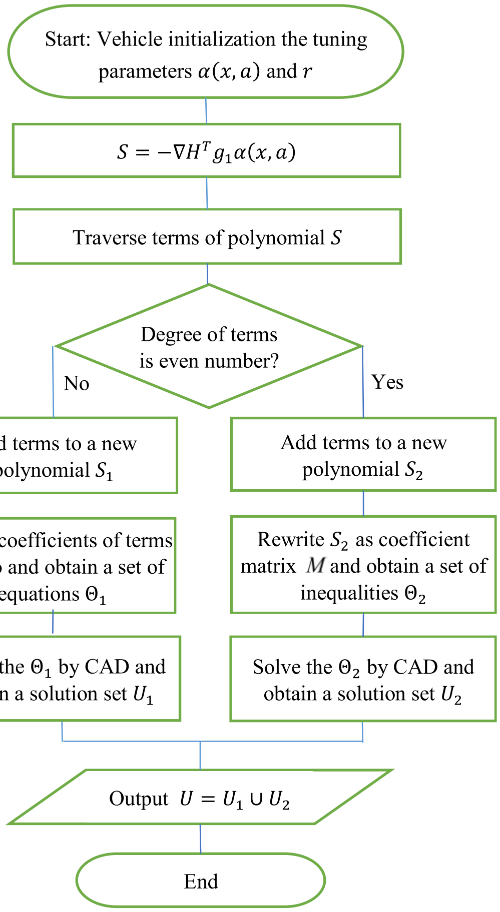

FIGURE 2. Flow diagram of the proposed STP algorithm.

transformed into the characteristic root of P extreme value problem. The design steps for the controller parameter optimization method are given below.

Step1. According to the Jacobi matrix at the equilibrium point of the system, solve the positive define symmetric matrix P. Suppose J_E is the equilibrium of the system, which can be written at the equilibrium as follows:

$$\dot{X} = J_E X \tag{16}$$

Then it is certainly that existing a positive defined symmetric matrix P and a unit matrix Q, which makes the follows hold,

$$V(X) = X^T P X, \quad \dot{V}(X) = -X^T Q X \tag{17}$$

where $V(X)$ is the Lyapunov function of the controlled system, $\dot{V}(X) = \frac{dV(X)}{dt}$. Then according to the linearization of the system, obtain the following equation,

$$\begin{aligned} \dot{V}(X) &= \frac{dV(X)}{dt} = \frac{d\left(X^T P X\right)}{dt} \\ &= \dot{X}^T P X + X^T P \dot{X} \\ X^T\left(J_E^T P + P J_E\right)\dot{X} &= -X^T Q X \end{aligned} \tag{18}$$

That is

$$J_E^T P + P J_E = -Q \tag{19}$$

where J_E and Q can be obtained from the system (15). The positive definite symmetric matrices P can be obtained from formula (19).

Step2. Solve the characteristic equation $H(\lambda) = det(\lambda I - P)$. If the time of adjustment for system states is minimized, it is equivalent to the fastest decay rate of the system. According to Lyapunov theorem, the decay rate of the system can be expressed as follows,

$$\eta = \lambda\left(QP^{-1}\right) \tag{20}$$

Because Q is a unit matrix, considering $\lambda\left(P^{-1}\right) = \frac{1}{\lambda(P)}$, the decay rate of the controlled system to equilibrium point depends on the characteristic root of P^{-1}. So, the problem can be solved by solving the characteristic root extremum to matrix P.

Step3. Solve the extremum problem of $H(\lambda) = 0$. Because J_E and P include the controller parameters k, let $H(\lambda) = 0$ be the implicit function of $\lambda = \bar{H}(k)$. Solve the following equations for obtaining the fastest convergent control parameters,

$$\begin{cases} H(\lambda) = 0 \\ \dfrac{\partial H(\lambda)}{\partial k} = 0 \end{cases} \tag{21}$$

So, we can obtain the optimized tuning parameters for the proposed controller by solving equations (21).

IV. DESIGN EXAMPLE

In this section, the above formulated problem will be solved by the Hamiltonian function method. We rewrite the system (3) as a dissipative Hamiltonian system, firstly.

$$\begin{cases} \dot{x} = (J(x) - R(x))\nabla H(x) + g_1(x)u(t) + g_2(x)\omega(t) \\ z = h(x)g_2^T(x)\nabla H(x) \end{cases} \tag{22}$$

where

$$J(x) = \begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & \dfrac{k_t}{m_u} \\ -1 & 0 & 0 & \dfrac{c_s}{m_s} \\ 1 & -\dfrac{k_t}{m_u} & -\dfrac{c_s}{m_s} & 0 \end{bmatrix},$$

$$R(x) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \dfrac{k_t}{m_u} - 1 \\ \dfrac{k_s}{m_s} - 1 & 0 & \dfrac{c_s}{m_s} & 0 \\ 1 - \dfrac{k_s}{m_u} & 0 & -\dfrac{c_s}{m_s} - \dfrac{c_s}{m_u} & \dfrac{c_s + c_t}{m_u} \end{bmatrix},$$

$$H(x) = \frac{1}{2}\left(x_1^2 + x_2^2 + x_3^2 + x_4^2\right),$$

$$g_1(x) = \left[0 \;\; 0 \;\; \frac{1}{m_s} \;\; -\frac{1}{m_u}\right]^T, \; g_2(x) = \left[0 \;\; -1 \;\; 0 \;\; \frac{c_t}{m_u}\right]^T, \; h(x)$$

is the weight matrix for a specific penalty.

From system (22), it is easy to get $\nabla H(x)|_{x=0} = 0$, $Hess(H(x))|_{x=0} = Diag\{1,1,1,1\} > 0$. So, assumption 1 holds.

Then, we check that condition (8) holds for all x and given γ. From system (22), we have

$$R(x) + \frac{1}{2\gamma^2}\left(g_1(x)g_1^T(x) - g_2(x)g_2^T(x)\right)$$
$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2\gamma^2} & 0 & \frac{k_t}{m_u}-1 \\ \frac{k_s}{m_s}-1 & 0 & \frac{c_s}{m_s}-\frac{1}{2\gamma^2 m_s^2} & -\frac{1}{2\gamma^2 m_s m_u} \\ 1-\frac{k_s}{m_u} & 0 & -\frac{c_s}{m_s}-\frac{c_s}{m_u}-\frac{1}{2\gamma^2 m_s m_u} & \frac{c_s}{m_u}+\frac{1}{2\gamma^2 m_u^2} \end{bmatrix}$$

We assume the specific given disturbance attenuation value $\gamma^* = 0.01$.

$$\gamma \geq 0.01 \tag{23}$$

ensure condition (8) holds.

Then, if system (22) satisfies robustness performance in H_∞ control, condition (9) must hold. The Hamiltonian function $H(x)$ can be used as candidate Lyapunov function $V(x)$, i.e., $V(x) = H(x)$. It follows from controller (10) that

$$\begin{aligned} u(x) &= \alpha(x,a) + \beta(x) \\ &= \alpha(x,a) - \frac{1}{2}\left(h(x)^T h(x) + \frac{1}{\gamma^2}I_m\right) g_1^T(x)\nabla H(x) \end{aligned} \tag{24}$$

From system (22), $n = 4$. Let $r = 1$, we have

$$\alpha(x,a) = a_i x_i = a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 \tag{25}$$

where a_i $(i = 1,2,3,4)$ are the tuning parameters. From system (22), we obtain that $\nabla H(x) = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T$.

Let $\frac{1}{m_s} = b_1$, $\frac{1}{m_u} = b_2$ and $S = -\nabla H(x)^T g_1 \alpha(x,a)$, we have

$$\begin{aligned} S = &-a_1b_1x_1x_3 + a_1b_2x_1x_4 - a_2b_1x_2x_3 + a_2b_2x_2x_4 \\ &-a_3b_1x_3^2 - (a_4b_1 - a_3b_2)x_3x_4 + a_4b_2x_4^2 \end{aligned} \tag{26}$$

Rewrite coefficient of S as a quadratic form M:

$$M = \begin{bmatrix} 0 & 0 & -a_1b_1 & -a_1b_2 \\ 0 & 0 & -a_2b_1 & -a_2b_2 \\ -a_1b_1 & -a_2b_1 & -2a_3b_1 & -a_4b_1+a_3b_2 \\ -a_1b_2 & -a_2b_2 & -a_4b_1+a_3b_2 & 2a_4b_2 \end{bmatrix} \tag{27}$$

As we all know, all principal diagonals of M must be positive semi-definite. So, we can obtain inequalities B from M. From B, we can easy to obtain that $a_1 = 0, a_2 = 0, a_3 \leq 0$ and $a_4 \geq 0$. Substitute a_1 and a_2 into inequalities B for simplifying computation, we obtain simplified inequality $-\left(2a_3a_4b_1b_2 + a_4^2b_2^2 + a_3^2b_1^2\right) \geq 0$. Solving the inequality by using CAD, we obtain $a_3 = -\frac{b_2}{b_1}a_4$. Let $= a_4$. So, we have

$$U = \left\{a_1 = 0, a_2 = 0, a_3 = -\frac{b_2}{b_1}k, a_4 = k, k \leq 0\right\} \tag{28}$$

TABLE 1. Quarter-vehicle model parameters.

Parameter	Value	Units
m_s	320	kg
m_u	40	kg
k_s	18000	N/m
k_t	200000	N/m
c_s	1000	Ns/m
c_t	0	Ns/m
z_{max}	0.08	m

Substitute U into the controller (10), we obtain a family of controllers with parameter

$$u = -\frac{1}{2}\left(h(x)^T h(x) + \frac{1}{\gamma^2}I_m\right)\left(\frac{x_3}{m_s} - \frac{x_4}{m_u}\right) - \frac{b_2}{b_1}kx_3 + kx_4 \tag{29}$$

According to the parameter optimization steps above, we can get the tuning parameter optimization $k = 1525$ for the system (22).

V. SIMULATIONS TO VEHICLE ACTIVE SUSPENSION CONTROL

We will verify our method by some examples and simulations. First, the peak value of the state is the direct reflection of the state change of the system under external disturbance and action of the controller. That is the direct embodiment of the controller's effect. Then, the root-mean-square (RMS) values are used to describe the vibrations of the vehicle body in the vehicle suspension system, especially as the ride comfort of passengers. Hence, we take the peak value and RMS to describe the quantitative indexes.

The quarter-vehicle model parameters have the following values [36] and list in Table 1.

To verify the robustness of the proposed controller, the fluctuations of the road profile are treated as external disturbances. The road disturbance can be seen as discrete events, which are with relatively high intensity and short duration. The road surface is presented in [36].

$$z_r(t) = \begin{cases} \frac{H}{2}\left(1 - \cos\left(\frac{2\pi V_0}{L}t\right)\right), & if\ 0 \leq t \leq \frac{L}{V_0} \\ 0, & if\ t \geq \frac{L}{V_0} \end{cases} \tag{30}$$

where H and L are the height and the length of the bump, respectively. Assume $H = 0.1$m, $L = 10$m, and the vehicle forward velocity is $V_0 = 45$km/h.

A. COMPARISON WITH DIFFERENT PARAMETERS OF CONTROLLERS

The initial parameters of the system (22) are $\gamma = 1$, $h(x) = 0.5$ and the parameters of the controller are $k = 100$, $k = 1525$ (optimization) and $k = 3000$, respectively. We obtain the corresponding controllers (31)-(33),

$$u = -800.00195x_3 + 100.0156x_4 \tag{31}$$
$$u = -12200.00195x_3 + 1525.0156x_4 \tag{32}$$
$$u = -24000.001953x_3 + 3000.0156x_4 \tag{33}$$

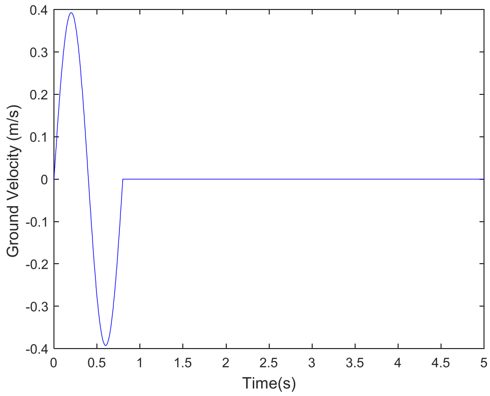

FIGURE 3. Road disturbance velocity.

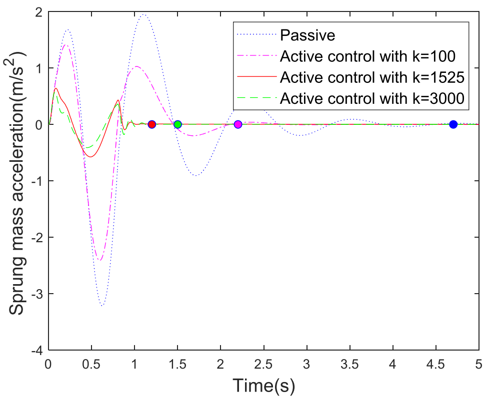

FIGURE 4. Sprung mass acceleration.

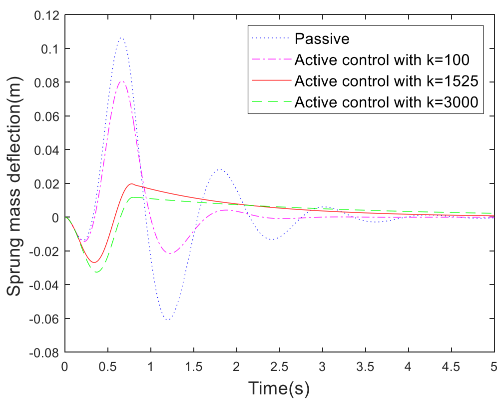

FIGURE 5. Sprung mass deflection.

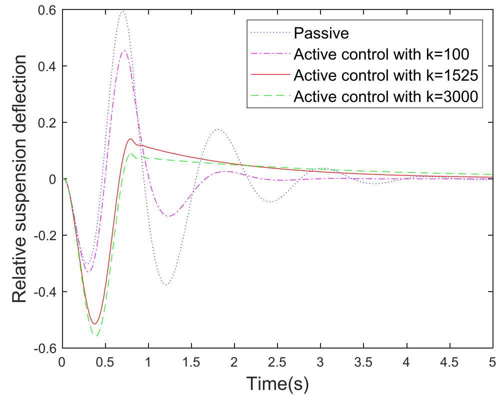

FIGURE 6. Relative suspension deflection.

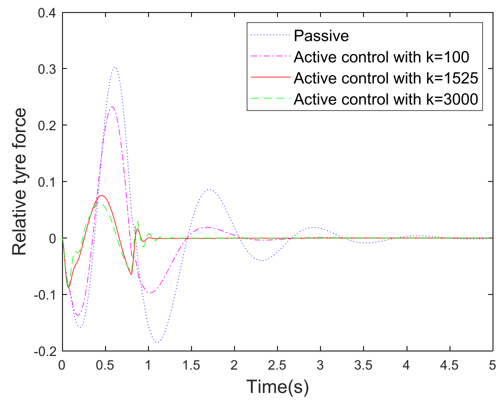

FIGURE 7. Relative tire force.

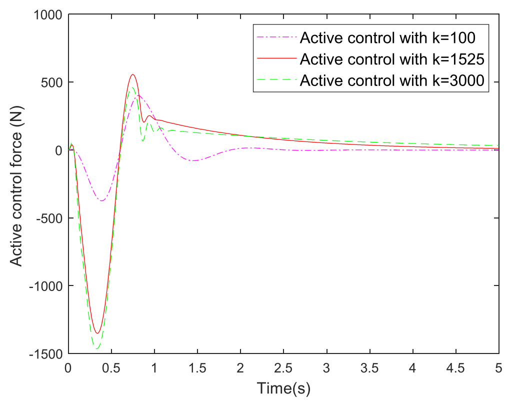

FIGURE 8. Active control force.

In this case, the sprung mass is assumed to be constant with $m_s = 320$kg and the vehicle speed is $V_0 = 45$ km/h. Fig. 3 shows the road disturbance velocity. Fig. 4 and Fig. 5 show the bump response of the SMA and sprung mass deflection (SMD) for passive control (dot line blue) and the controller of parameter values $k = 100$ (dot-dash line purple), $k = 1525$ (solid line red) and $k = 3000$ (dash line green) in the active suspension system. The bump response of the RSD and RTF are shown in Fig. 6 and Fig. 7. Then, Fig. 8 Shows the active control force for the controller of different controller parameter values, respectively.

From Fig. 4-Fig. 8, we can clearly see that good bump response quantities for SMA, SMD, RSD, and RTF can be guaranteed by using H_∞ parameterized control. The time back to the equilibrium position of the vehicle system with active control is less than passive control and the robustness performance of the vehicle system can be further optimized through adjusting the values of controller parameter from $k = 100$ to $k = 3000$, where $k = 1525$ is the better parameter.

B. COMPARISON ANALYSIS WITH DIFFERENT PARAMETERS OF VEHICLE

The sprung mass may vary due to changes in passenger or cargo load and the vehicle speed is the most variable factor. The different values of the sprung mass and vehicle speed

TABLE 2. Comparison of peak and rms value for bump response.

Measure		Peak SMA	RMS SMA	Peak RSD	RMS RTF
$m_s = 320$ $V_0 = 30$	Passive	2.2864	1.0082	0.2977	0.0955
	Proposed	0.4341	0.1700	0.4095	0.0208
$m_s = 480$ $V_0 = 30$	Passive	2.2010	1.0003	0.4354	0.0960
	Proposed	0.3009	0.1199	0.4776	0.0149
$m_s = 320$ $V_0 = 90$	Passive	6.1758	1.7593	0.5627	0.1588
	Proposed	3.1882	0.8499	0.5312	0.1199
$m_s = 480$ $V_0 = 90$	Passive	3.8721	1.2167	0.5184	0.1109
	Proposed	2.1824	0.5816	0.5850	0.0844

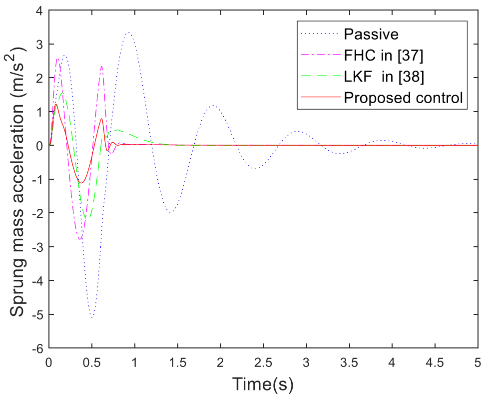

FIGURE 9. Sprung mass acceleration.

are considered as $m_s = 320$kg, $m_s = 480$kg and $V_0 = 30$ km/h, $V_0 = 90$ km/h. The controller (32) is considered in simulations.

To show clearly the comparison results between the different sprung masses and different speeds of the vehicle, the maximum peak values of the bump responses for SMA, RSD, and RTF are listed in Table 2.

From Table 2, it can be seen clearly that the effect of the controller is related to vehicle speed and mass. In the same circumstances, the proposed control method has better suspension's adaptability to the road than the passive suspension control method. With the increase of weight and constant low speed, the passive control has little change, while the proposed controller obtains better control effect. The suspension's adaptability to road will be reduced with the increase of speed. However, the proposed controller can still effectively control the vehicle at high speed $V_0 = 90$ km/h to meet the three control performances.

C. COMPARISON ANALYSIS WITH EXISTING METHODS

In order to further verify the performance of the proposed control method, two existing results in [37] and [38] are presented for the objective of the comparison. The controller (32) is considered in simulations.

In this case, the sprung mass is assumed to be constant with $m_s = 400$ kg and the vehicle speed is $V_0 = 45$ km/h.

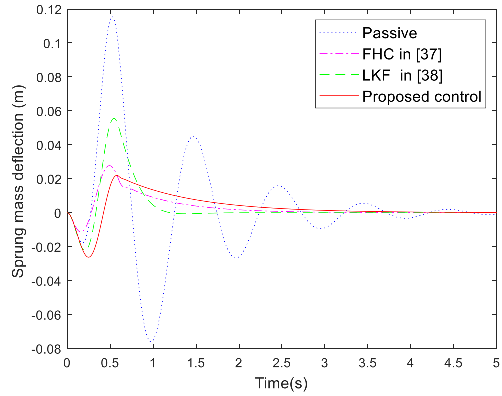

FIGURE 10. Suspension deflection.

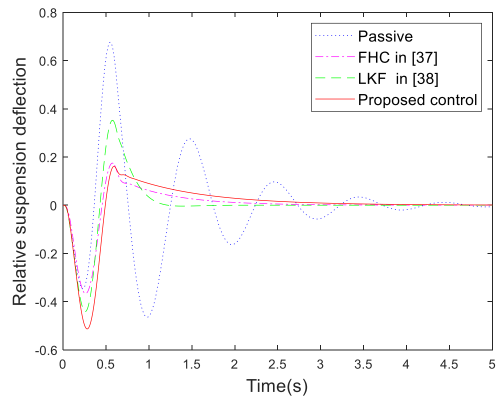

FIGURE 11. Relative suspension deflection.

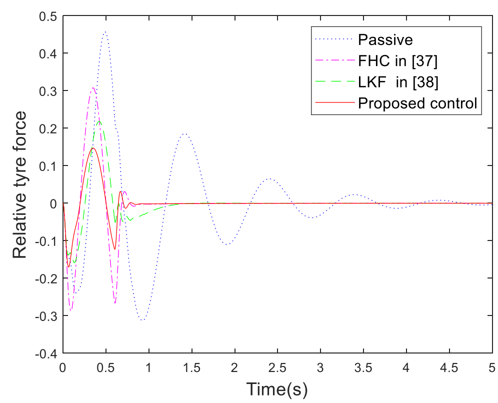

FIGURE 12. Relative tire force.

Simulation results are shown in Fig. 9- Fig. 13. For different control strategies, the SMA, SMD, RSD and RTF and active control force are shown in Fig. 9- Fig. 13, respectively.

As shown in Fig. 9- Fig. 13, the proposed parameterized control can obtain better regulation performance of body acceleration, and simultaneously satisfy the constraint requirements than FHC in [37] and LFK in [38].

Table 3 shows the performance results for the proposed method and existing methods.

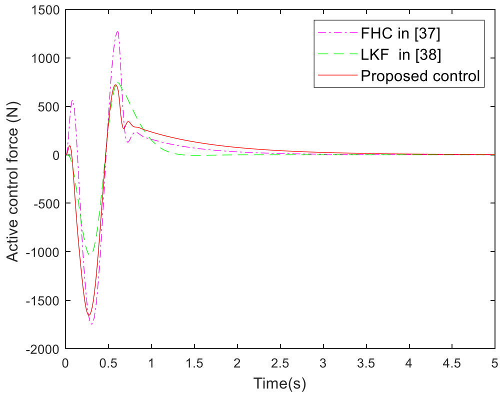

FIGURE 13. Active control force.

TABLE 3. Performance of peak and rms values.

	Passive	FHC in [37]	LKF in [38]	Proposed
Peak SMA	5.0928	2.7814	2.2114	1.2191
RMS SMA	1.8224	0.8450	0.6361	0.3606
Peak RSD	0.6760	0.3667	0.4412	0.5133
RMS RTF	0.1676	0.0944	0.0625	0.0498

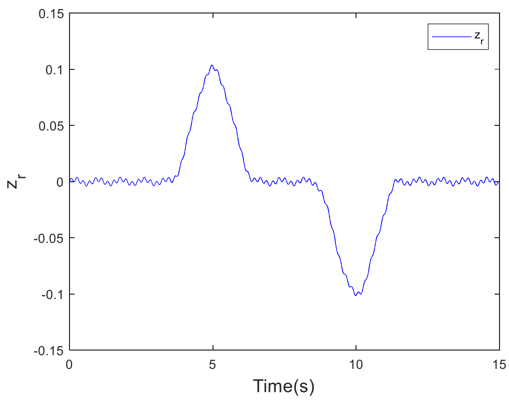

FIGURE 14. Road disturbance velocity.

From Table 3, it can be observed with the parameterized controller is better than the passive method and the other two existing methods in multi-objective robust control.

D. DIFFERENT FLUCTUATION OF THE ROAD PROFILE

In order to further verify the robustness of the proposed controller for different fluctuations of the road profile. We take the road profile from [39], which is described by the following equation,

$$z_r(t) = \begin{cases} -0.0592t_1^3 + 0.1332t_1^2 + b(t), & 3.5 \le t < 5 \\ 0.0592t_2^3 + 0.1332t_2^2 + b(t), & 5 \le t < 6.5 \\ 0.0592t_3^3 - 0.1332t_3^2 + b(t), & 8.5 \le t < 10 \\ -0.0592t_4^3 - 0.1332t_4^2 + b(t), & 10 \le t < 11.5 \\ b(t), & else \end{cases} \tag{34}$$

where $b(t) = 0.002sin(2\pi t) + 0.002sin(7.5\pi t)$ as the disturbance. Assume the vehicle forward velocity is $V_0 =$ 45km/h and the quarter-vehicle model parameters are listed in Table 1. The controller (32) is considered in simulations for system (22).

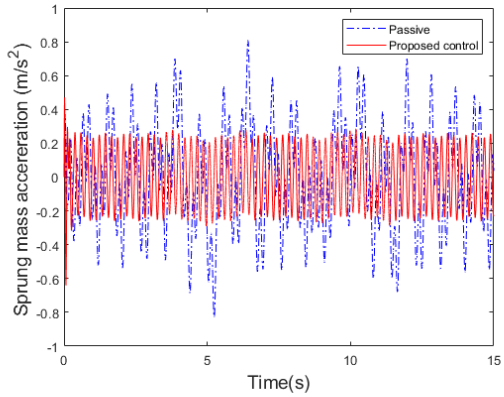

FIGURE 15. Sprung mass acceleration.

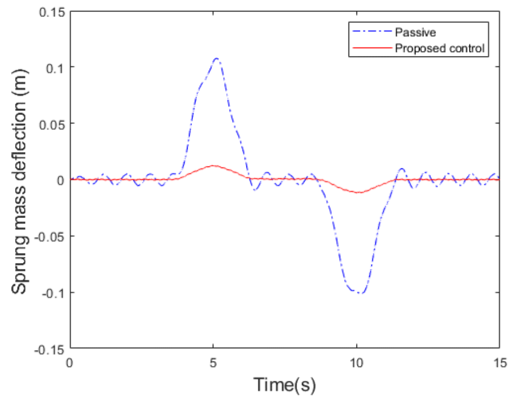

FIGURE 16. Sprung mass deflection.

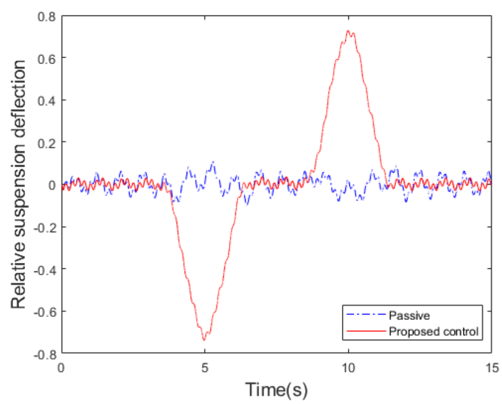

FIGURE 17. Relative suspension deflection.

Fig. 14 shows the road disturbance velocity. Fig. 15 and Fig. 16 show the bump response of the SMA and sprung mass deflection (SMD) for passive control (dot line blue) and the controller of parameter values $k = 1525$ (solid line in red) in the active suspension system. The bump response of the RSD and RTF are shown in Fig. 17 and Fig. 18.

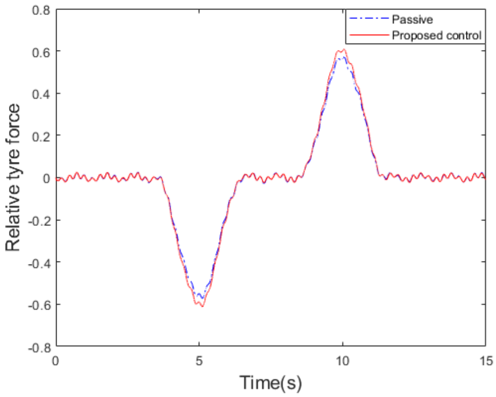

FIGURE 18. Relative tire force.

From Fig. 15-Fig. 18, we can clearly see that good bump response quantities for SMA, SMD, RSD, and RTF can be guaranteed by using H_∞ parameterized control under road disturbance in Fig. 14.

VI. CONCLUSION

The sprung mass may vary due to changes of passenger load, cargo, and vehicle speed. In which, the vehicle speed is the most variable factor. A novel controller with tuning parameters is proposed to achieve the multi-objective control performances of providing the ride safe and ride comfort. The proposed parameterization method uses the Hamiltonian function method and avoids solving HJI inequalities, thus the obtained controllers with parameters are easier as compared to some existing ones. The parameters' range can be obtained by the STP Algorithm and optimal parameters for the controller can be obtained by the parameter optimization method, respectively. Simulations results with two different road profiles validate the multi-objective robust control performances for active suspension vehicles with different sprung masses and different speeds of the vehicle. A comparison with passive system and existing methods shows that the proposed parameterized controller scheme performs better than them in terms of peak and RMS values.

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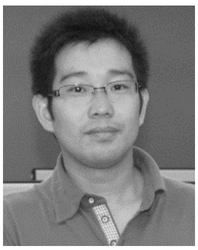

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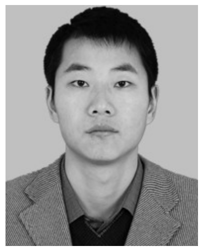

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