IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received December 10, 2019, accepted December 22, 2019, date of publication December 30, 2019, date of current version January 31, 2020.

Digital Object Identifier 10.1109/ACCESS.2019.2963052

An Incremental Feedback Control for Uncertain Mechanical System

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This work was supported in part by the National Natural Science Foundation of China under Grant 11705122, Grant 61902268, and Grant 51575544, in part by the Fundamental Research Funds for the Central Universities, Jinan University, under Grant 12819026, in part by the Hong Kong Research Grants Council under Grant BRE/PolyU 152099/18E and Grant PolyU 15204719/18E, in part by the Fundamental Research Funds for the Central Universities, China University of Geosciences, Wuhan, under Grant CUGGC05, in part by the Natural Science Foundation of The Hong Kong Polytechnic University under Grant G-YW3X, in part by the Sichuan Science and Technology Program under Grant 2019YFSY0045, Grant 2018GZDZX0046, and Grant 2018JY0197, in part by the Open Foundation of Artificial Intelligence Key Laboratory of Sichuan Province under Grant 2018RZJ01, in part by the Nature Science Foundation of Sichuan University of Science and Engineering under Grant 2017RCL52, in part by the Zigong Science and Technology Program of China under Grant 2019YYJC03 and Grant 2019YYJC15, in part by the Key Research and Development Program of Shaanxi Province under Grant 2018ZDXM-GY-036, and in part by the Shaanxi Key Laboratory of Intelligent Processing for Big Energy Date under Grant IPBED7.

ABSTRACT This paper focuses on the tracking problem for fully-actuated mechanical systems with uncertain parameters and external disturbances. Based on the state feedback control of contraction analysis, the robust controller with extra gains is suggested to provide for the tracking of mechanical systems with uncertainties. The proposed control scheme can be redesigned with dual ideas and theoretically prove that the robust control renders uniform roundedness. Further more, the inertia matrices being uniformly bounded above are limited. The simulation is proposed to account for the effectiveness and robustness of the provided method.

INDEX TERMS Increment, contraction analysis, fully-actuated system, robust control.

I. INTRODUCTION

Motivation: In the situation of the booming new economy, mechanical systems are widely used in industrial fields, such as various types of robots [1]–[3], manufacturing [4], ship power [5], [6], and so on. Among them, high-precision tracking as an important factor to measure the performance for mechanical systems is a basic problem in the application of mechanical systems, and decides the market competitiveness of mechanical systems. But the situation is another saying, due to the complexity of the actual engineering, such as changes in the operating objects of the mechanical system and the surrounding environment, motor overheating caused by long-term operation, parameter errors in the manufacturing process of mechanical apparatus, and so on, it is difficult for mechanical systems to measure accurate dynamic models. In response to the problems of high-precision tracking for the inaccurate dynamic models, many scholars have conducted in-depth investigations.

Brief Summary of Prior Literature: For fully-actuated systems, an internal model-based adaptive controller was proposed to solve the robust control problem of fully-actuated passive mechanical systems, where the reference signal and the second derivative of the state can be used for the controller, and the interference signal can be segmented [7]. The control problem for a fully-actuated second-order system was addressed by using state proportional plus derivative feedback [8]. A frame for handling of the path with zero curvature and reducing the complexity of the control law was introduced and the controller design had been achieved

The associate editor coordinating the review of this manuscript and approving it for publication was Salman Ahmed.

by transverse feedback linearization and a parallel transport frame [9]. A robust chattering-free sliding mode asymptotic tracking control design is presented for a fully actuated multirotor [10]. A disturbance observer-based model predictive controller is designed for image-based visual servoing of underwater vehicles subject to field-of view constraint, actuator saturation, and external disturbances [11]. An adaptive second-order fast nonsingular terminal sliding mode control scheme was addressed for the trajectory tracking of fully actuated autonomous underwater vehicles in the presence of dynamic uncertainties and time-varying external disturbances [12]. The control problem for exempt from requiring for accurate dynamic model was addressed by incorporating adaptive sliding-mode and online dynamics estimation schemes [13]. The control problem of tracking a desired trajectory for a fully-actuated marine surface vessel was investigated, where the output constraints were taken into [14]. A model predictive control strategy based on sliding mode observer was proposed for image-based visual servoing fully-actuated underwater vehicles [15]. For under-actuated systems, the control problem for under-actuated ships under stochastic disturbances was addressed by introducing weak and strong nonlinear Lyapunov functions [16]. A methodology for stabilizing the collocated state space of an under-actuated mechanical system has been proposed by employing PDE boundary backstepping control scheme [17]. A leader-follower formation control problem for a group of under-actuated surface vessels with partially known control input functions has been solved [18]. The control performance for uncertain under-actuated mechanical systems has been enhanced by an adaptive fuzzy inference system was combined with a sliding mode controller [19]. The application of model predictive control for high-performance speed control and torsional vibration suppression in the drive system with flexible coupling was demonstrated [20]. To a certain extent, the advantages and disadvantages of fully-actuated and under-actuated are just complementary to each other. Fully-actuated systems are more adept at complex tasks than under-actuated systems, but from the hardware level of electromechanical systems, the overall integration of the system is weak and the cost is high. Many of the machine systems for current interests are under-actuated systems, but fully-actuated systems still have a high value.

The concept of contraction analysis has a short history, which can be traced back to [21]. Contraction is one of the property of incremental stability [22] and can be simply interpreted in Riemannian geometry as: requires the decrease of a distance, defined through a Riemannian metric, along trajectories [23]. In the past decade, applications of contraction analysis include intrinsic observer design [24], consensus problems in complex networks [25], output regulation of non-linear systems [26], design of frequency estimators [27], synchronization of coupled identical dynamical systems [28], stability and robustness analysis of nonlinear system [29]. Recently, the newly developed "control contraction metric" (CCM) concept was used as an unified framework for emerging mechanical control methods [30].

Contribution of This Paper: From previous literature of authors' knowledge, there is a lack of other applications of contraction analysis on mechanical systems. Unlike equilibrium point stability [6], [8], [9], [16], [31], contraction is independent of equilibrium point, especially in the case of multiple equilibrium points or equilibrium point displacements, contraction is more advantageous. The proposed controller will involve real-time optimization to find a minimal-length path with respect to the metric (a geodesic) joining the current state to the desired state. But the existence of a flat metric usually simplifies the control problem [11], [15], [20], [32]. Hence, there is a growing need to extend existing methods or develop new ones for the purpose of applying contraction property on mechanical system. The method proposed in this paper referred to the CCM framework, which is actually a feedback design method, so the excessive introduction to CCM has been omitted, and the relevant literature can be consulted in [33]–[36]. The conclusions about robust control are obtained by performing contraction analysis on uncertain mechanical systems, this is not discussed in [33]–[36]. In this technical note, the main contributions of this paper focus on the following aspects.

- Developed a robust feedback control for fully-actuated mechanical systems with uncertainties and external disturbances, witch has smaller tracking error and faster convergence time than the general Lyapunov method [31];
- Extended a robust feedback control to the CCM framework and provide a new idea for the analysis of nonlinear systems with uncertainties;

However, the present technology is relative to the fully-actuated systems having a triangular structure. In other words, in the case of higher-order [37] or under-actuated systems, new technologies need to be developed. And, research on contraction analysis in finite-time control [38]–[40] is rare, one of the challenges in the future is to extend finite-time control to the CCM framework. Another challenge is the case of matrices have unknown upper and lower bounds,whether the adaptive technologies can be introduced [41]–[43].

Organisation: The rest of this paper is structured as follows: In Section II and III, the model of the fully-actuated mechanical systems with uncertainties and related preliminaries are discussed. In Section IV, firstly, the incremental error dynamics is established. Secondly, proposed the directly incremental feedback control by contraction analysis and the incremental control can be redesigned by Fenchel conjugate. Thirdly, the actual controller by a schematic diagram of incremental controller. lastly, the special case about inertia matrix was discussed. Then the simulation results are described to verify the effectiveness of the proposed distributed control algorithms in Section V. Finally, some concluding remarks are given in Section VI.

II. CONTRACTION ANALYSIS

The contraction analysis is a theory of stability. Unlike the Lyapunov stability theory, it focuses on the incremental stability of the system. The details for contraction analysis refer to [21]. Given a autonomous nonlinear dynamic system and a manifold $\mathcal{M}$

$$\dot{x} = f(x, t), \tag{1}$$

where f is a nonlinear vector field that maps any $(t, x) \in \mathbb{R}^n \times \mathcal{M}$ to a tangent vector $f(t, x) \in T_x\mathcal{M}$. The incremental form of (1) is showed as

$$\dot{\delta}_{x(t)} = \frac{\partial f(x,t)}{\partial x(t)}\delta_{x(t)}, \tag{2}$$

where $\delta_{x(t)}$ is denoted as an infinitesimal displacement at a fixed time.

According to [21], there are the following definition and lemma.

Definition 1: A symmetric and positive definite matrix $M(x, t)$ is called a contraction matrix and a constant β is called a contraction rate if there exist a *Riemann* metric $\delta_x^T M(x, t)\delta_x$ and the strict stabilization constant $\beta \in \mathbb{R}^+$ in system (2) satisfied the inequality

$$\begin{aligned}\frac{d}{dt}(\delta_x^T M\delta_x) &= \delta_x^T\left(\frac{\partial f}{\partial x}^T M + M\frac{\partial f}{\partial x} + \dot{M}\right)\delta_x \\ &\leq -\delta_x^T\beta M\delta_x,\end{aligned} \tag{3}$$

when M is independent of state x, M is called the flat contraction matrix and (3) is similar to *Demidovich* condition.

Lemma 1: Given the system (1), any trajectory, which starts in a ball of constant radius with respect to the metric $M(x, t)$, centered at a given trajectory and contained at all times in a contraction region with respect to $M(x, t)$, remains in that ball and converges exponentially to this trajectory.

III. FULLY-ACTUATED MECHANICAL SYSTEMS WITH UNCERTAINTIES

Considering the nominal Lagrangian formulation of fully-actuated mechanical system dynamics

$$Ru(t) = N\big(q(t)\big)\ddot{q}(t) + C\big(q(t), \dot{q}(t)\big)\dot{q}(t) + G\big(q(t)\big), \tag{4}$$

where $t \in \mathbb{R}$ is the time, $q(t) \in \mathbb{R}^n$ is the joint coordinate, $\dot{q}(t) \in \mathbb{R}^n$ is the joint velocity, $\ddot{q}(t) \in \mathbb{R}^n$ is the joint acceleration, $N\big(q(t)\big) \in \mathbb{R}^{n\times n}$ is an inertia matrix, $C\big(q(t), \dot{q}(t)\big) \in \mathbb{R}^{n\times n}$ is the Corsirio matrix related to centripetal force, $G\big(q(t)\big) \in \mathbb{R}^n$ is the gravitational force, $u(t) \in \mathbb{R}^m$ is the input torque. System (4) is a fully-actuated mechanical system if $m = n$ and R is a fully-rank square matrix.

Next, introducing actual systems, that is, the case of normal system (4) with uncertainties. In the real world, the model of the mechanical system (4) is not precise, for example, payload mass and friction force parameters, which is reflected in the $N\big(q(t)\big)$, $C(q(t), \dot{q}(t))$, $G(q(t))$. In addition, there may exist the external disturbances $\hat{F}(\ddot{q}, \dot{q}, q, t) \in \mathbb{R}^n$ when the external environment changed. This paper considers above uncertainties, the fully-actuated mechanical system with uncertainties is showed as

$$\begin{aligned}R\hat{u}(t) = \hat{N}\big(q(t)\big)\ddot{q}(t) + \hat{C}\big(q(t), \dot{q}(t)\big)\dot{q}(t) \\ + \hat{G}\big(q(t)\big) + \hat{F}(\ddot{q}, \dot{q}, q, t),\end{aligned} \tag{5}$$

where $\hat{N}\big(q(t)\big)$, $\hat{C}\big(q(t), \dot{q}(t)\big)$, $\hat{G}\big(q(t)\big)$ denote $N\big(q(t)\big)$, $C\big(q(t), \dot{q}(t)\big)$, $G\big(q(t)\big)$ affected by uncertainties (payload mass, friction force parameters, etc), and $\hat{u}$ is the practical input torque. Throughout the subsequent analysis we shall assume that the dynamics satisfy the following assumptions.

Assumption 1: $\hat{N}\big(q(t)\big) > 0, \forall q(t) \in \mathbb{R}^n$ and $\big\|\hat{N}\big(q(t)\big)\big\| \leq \xi$, with $\xi > 0$, where $\hat{N}\big(q(t)\big)$ is symmetric, positive definite matrix.

Assumption 2: There exist a positive constant ε which can be estimated to satisfy

$$\frac{1}{\varepsilon} \leq \frac{\big\|\hat{N}\big(q(t)\big)\big\|}{\big\|N\big(q(t)\big)\big\|} \leq \varepsilon, \quad \varepsilon \in \mathbb{R}^+ \geq 1.$$

Remark 1: Assumption 1 comes from [31], which mentioned that for any rigid serial type manipulators with revolute and prismatic joints, the upper bound property of the norm of the inertia matrix is generic (note that the proof about the upper bound property have been proved by [44]). For the upper bound property of the norm of the inertia matrix of the actual model and the nominal model, Assumption 2 further assumes that their ratio is bounded, and the maximum ratio can be measured.

Remark 2: It is easy to see that $\hat{N}\big(q(t)\big)$ is adjacent to $N\big(q(t)\big)$ and ε represents the maximum ranges of proximity. The worst case of uncertainties is that $\hat{N} \approx \varepsilon N$, because the inertia matrix is assumed to a positive definite matrix.

IV. CONTROLLER DESIGN

A. INCREMENTAL ERROR DYNAMICS

Using $q^d(t), \dot{q}^d(t)$ and $\ddot{q}^d(t)$ to denote desired trajectory, desired velocity, and desired acceleration being follow. Assuming $q^d(t), \dot{q}^d(t)$ and $\ddot{q}^d(t)$ are uniformly bounded. Let

$$e(t) = q(t) - q^d(t),$$

and hence $\dot{e}(t) = \dot{q}(t) - \dot{q}^d(t)$, $\ddot{e}(t) = \ddot{q}(t) - \ddot{q}^d(t)$. The system (4) can be rewritten as

$$Ru(t) = N\big(e(t)\big)\ddot{e}(t) + C\big(e(t), \dot{e}(t)\big)\dot{e}(t) + G\big(e(t)\big), \tag{6}$$

and the system (5) can be rewritten as

$$\begin{aligned}R\hat{u}(t) = \hat{N}\big(e(t)\big)\ddot{e}(t) + \hat{C}\big(e(t), \dot{e}(t)\big)\dot{e}(t) \\ + \hat{G}\big(e(t)\big) + \hat{F}(\ddot{e}, \dot{e}, e, t).\end{aligned} \tag{7}$$

For convenience of presentation, ignoring t in the following formula (note that state is related to t). Let $x = [e \quad \dot{e}]^T$, system (6) and (7) can be rewritten as

$$\dot{x} = \underbrace{\begin{bmatrix}\dot{e} \\ N(e)^{-1}\big(-C(e)\dot{e} - G(e)\big)\end{bmatrix}}_{f(x)} + \underbrace{\begin{bmatrix}0 \\ N(e)^{-1}R\end{bmatrix}}_{B(x)}u, \tag{8}$$

and

$$\dot{x} = \underbrace{\begin{bmatrix} \dot{e} \\ \hat{N}(e)^{-1}\big(-\hat{C}(e)\dot{e} - \hat{G}(e) - \hat{F}(\ddot{e}, \dot{e}, e)\big) \end{bmatrix}}_{\hat{f}(x)} + \underbrace{\begin{bmatrix} 0 \\ \hat{N}(e)^{-1}R \end{bmatrix}}_{\hat{B}(x)} \hat{u}. \quad (9)$$

Taking incremental forms of system (8) and system (9), it yields

$$\dot{\delta}_x = A(x, u)\delta_x + B(x)\delta_u, \quad (10)$$

and

$$\dot{\delta}_x = \hat{A}(x, \hat{u})\delta_x + \hat{B}(x)\delta_{\hat{u}}, \quad (11)$$

where

$$A(x, u) = \frac{\partial}{\partial x}\big(f(x) + B(x)u\big),$$
$$\hat{A}(x, \hat{u}) = \frac{\partial}{\partial x}\big(\hat{f}(x) + \hat{B}(x)\hat{u}\big).$$

Combining the increment system (10) and (11), it yields

$$\dot{\delta}_{2x} = \big(A(x, u) + \hat{A}(x, \hat{u})\big)\delta_x + B(x)\delta_u + \hat{B}(x)\delta_{\hat{u}}. \quad (12)$$

Remark 3: Let a manifold $\mathcal{N} = \mathcal{M}_1 \cup \mathcal{M}_2$ be formed by the combination of (8) and (9), where $\mathcal{M}_1 = (8)$ and $\mathcal{M}_2 = (9)$. $x \in \mathcal{M}_2$ can be contracted if $x \in \mathcal{N}$ is contracted, because $\mathcal{M}_2 \subset \mathcal{N}$. Some details of manifolds can be found in [23].

B. DIRECTLY INCREMENTAL FEEDBACK CONTROL

First, we try to design a closed-loop feedback controller such as $\hat{u} = u = k(x, t) + v(t) = k(e, \dot{e}, t) + v(t)$ and a matrix M to satisfy the inequality (3), where $v(t)$ is a external piecewise-continuous signal. It is known from Lemma 1 that system (12) is exponentially convergent and that the controller is robust for the uncertainties in system (12). Rewriting the system (12) as

$$\begin{aligned} \dot{\delta}_{2x} &= \big(A(x, k+v) + \hat{A}(x, k+v)\big)\delta_x + \big(B(x) + \hat{B}(x)\big)\frac{\partial_{k+v}}{\partial x}\delta_x \\ &= \underbrace{\begin{bmatrix} 0 & 2I \\ \frac{\partial f_2(x)}{\partial e} + \frac{\partial \hat{f}_2(x)}{\partial e} & \frac{\partial f_2(x)}{\partial \dot{e}} + \frac{\partial \hat{f}_2(x)}{\partial \dot{e}} \end{bmatrix}}_{\bar{A}} \delta_x \\ &\quad + \underbrace{\begin{bmatrix} 0 \\ \big(N(e)^{-1} + \hat{N}(e)^{-1}\big)R \end{bmatrix}}_{\bar{B}} \frac{\partial_{k+v}}{\partial x}\delta_x. \end{aligned} \quad (13)$$

where $f_2(x)$ denotes $N(e)^{-1}\big(-C(e)\dot{e} - G(e) + R(k+v)\big)$, $\hat{f}_2(x)$ denotes $\hat{N}(e)^{-1}\big(-\hat{C}(e)\dot{e} - \hat{G}(e) - \hat{F}(\ddot{e}, \dot{e}, e) + R(k+v)\big)$.

Considering a flat contraction matrix $M = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$, note that $m_{21} = m_{12}^T$. The rate of change of the square of the increment in system (13) is expressed as

$$\begin{aligned} \frac{d}{dt}\delta_{2x}^T M \delta_{2x} &= \dot{\delta}_{2x}^T M \delta_{2x} + \delta_{2x}^T \dot{M} \delta_{2x} + \delta_{2x}^T M \dot{\delta}_{2x} \\ &= \left(\frac{\bar{A}\delta_{2x} + \bar{B}K\delta_{2x}}{2}\right)^T M \delta_{2x} + 0 + \delta_{2x}^T M \left(\frac{\bar{A}\delta_{2x} + \bar{B}K\delta_{2x}}{2}\right) \\ &= \delta_{2x}^T \left(\frac{\bar{A}^T M + K^T \bar{B}^T M + M\bar{A} + M\bar{B}K}{2}\right)\delta_{2x}, \end{aligned} \quad (14)$$

where $K = \frac{\partial_{k+v}}{\partial x}$, note that $\frac{\partial_{k+v}}{\partial x}\delta_x = \delta_u$. In order to use contraction analysis on (13) via Definition 1 and Lemma 1, to take the matrix part of $\delta_{2x}^T \frac{\beta M}{2}\delta_{2x}$ and $\delta_{2x}^T\left(\frac{\bar{A}^T M + K^T \bar{B}^T M + M\bar{A} + M\bar{B}K}{2}\right)\delta_{2x}$ to get an equation

$$\begin{aligned} &\bar{A}^T M + K^T \bar{B}^T M + M\bar{A} + M\bar{B}K + \beta M \\ &= \begin{bmatrix} \frac{\partial(f_2 + \hat{f}_2)}{\partial e} m_{21} & \frac{\partial(f_2 + \hat{f}_2)}{\partial e} m_{22} \\ 2m_{11} + \frac{\partial(f_2 + \hat{f}_2)}{\partial \dot{e}} m_{21} & 2m_{12} + \frac{\partial(f_2 + \hat{f}_2)}{\partial \dot{e}} m_{22} \end{bmatrix} \\ &\quad + \begin{bmatrix} m_{12}\frac{\partial(f_2 + \hat{f}_2)}{\partial e} & 2m_{11} + m_{12}\frac{\partial(f_2 + \hat{f}_2)}{\partial \dot{e}} \\ m_{22}\frac{\partial(f_2 + \hat{f}_2)}{\partial e} & 2m_{21} + m_{22}\frac{\partial(f_2 + \hat{f}_2)}{\partial \dot{e}} \end{bmatrix} \\ &\quad + \begin{bmatrix} \beta m_{11} & \beta m_{12} \\ \beta m_{21} & \beta m_{22} \end{bmatrix} + K^T \bar{B}^T M + M\bar{B}K. \end{aligned} \quad (15)$$

Theorem 1: Considering the worst case of uncertainties with $\hat{N} \approx \varepsilon N$ in Assumption 1 and Assumption 2, and the system (13) would be contracted if the parameter K designed as

$$K = \begin{bmatrix} \frac{-\frac{\partial f_2(x)}{\partial e} - J}{\big((1+\varepsilon)N(e)\big)^{-1}R} & \frac{\frac{-\partial f_2(x)}{\partial \dot{e}} - P}{\big((1+\varepsilon)N(e)\big)^{-1}R} \end{bmatrix}. \quad (16)$$

■

Proof: Taking (16) into (15) and can get

$$\bar{A}^T M + K^T \bar{B}^T M + M\bar{A} + M\bar{B}K + \beta M = \begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} \\ \bar{S}_{12}^T & \bar{S}_{22} \end{bmatrix}, \quad (17)$$

where the inner elements in formula (17) are

$$\bar{S}_{11} = \left(\frac{\partial \hat{f}_2}{\partial e} - J\right)m_{21} + m_{12}\left(\frac{\partial \hat{f}_2}{\partial e} - J\right) + \beta m_{11},$$

$$\bar{S}_{12} = m_{22}\left(\frac{\partial \hat{f}_2}{\partial e} - J\right) + \left(\frac{\partial \hat{f}_2}{\partial \dot{e}} + \frac{\beta}{2} - P\right)m_{21} + 2m_{11},$$

$$\bar{S}_{22} = 2m_{22}\left(\frac{\partial \hat{f}_2}{\partial \dot{e}} + \frac{\beta}{2} - P\right) + 2m_{12} + 2m_{21}.$$

According to Definition 1 and Lemma 1, the contraction condition in (17) is equivalent via Schur complement to the statement

$$\begin{aligned}
&\bar{S}_{22}-\frac{\bar{S}_{12}^T\bar{S}_{12}}{\bar{S}_{11}}\\
&=2m_{22}\left(\frac{\partial\hat{f}_2}{\partial\dot{e}}+\frac{\beta}{2}-P\right)+2m_{12}+2m_{21}\\
&-\frac{\left(m_{22}\left(\frac{\partial\hat{f}_2}{\partial e}-J\right)+\left(\frac{\partial\hat{f}_2}{\partial\dot{e}}+\frac{\beta}{2}-P\right)m_{21}+2m_{11}\right)^2}{\left(\frac{\partial\hat{f}_2}{\partial e}-J\right)m_{21}+m_{12}\left(\frac{\partial\hat{f}_2}{\partial e}-J\right)+\beta m_{11}}\\
&<0.
\end{aligned}$$

It clearly shows that the appropriate P and J can cause the system (13) to shrink, although the uncertainties are not measured. □

Remark 4: It can be think that the contraction of (17) are to take $m_{21}<-\beta m_{11}/\left(\frac{\partial\hat{f}_2}{\partial e}-J\right)$ to guarantee $\bar{S}_{11}<0$, and to take enough positive definite P to guarantee $\bar{S}_{22}<0$, and to take J to approximate $\bar{S}_{12}\approx 0$. With the establishment of incremental error dynamics, the direct incremental feedback control is proposed. K provides robustness to the system (13), noting that ε can be estimated.

C. DUALITY INCREMENTAL FEEDBACK CONTROL

It is known that a function's dual form have a convex characteristic, this section analyzes the Fenchel conjugate form of metric $\delta_{2x}^T M\delta_{2x}$.

According to the definition of Fenchel conjugate, the conjugate function of original function $F(\delta_{2x})=\delta_{2x}^T M\delta_{2x}$ is

$$\begin{aligned}
F^*(y)&=\sup_{\delta_{2x}\in\mathrm{dom}f}\left(y^T\delta_{2x}-F(\delta_{2x})\right)\\
&=\frac{y^TM^{-1}y}{4},
\end{aligned}\tag{18}$$

where $y=2M\delta_{2x}$ is the tangent of $F(\delta_{2x})$, note that $\dot{y}=2\dot{M}\delta_{2x}+2M\dot{\delta}_{2x}$. Let $W=M^{-1}$, the rate of the changes of the conjugate function $F^*(y)$ can be expressed as

$$\begin{aligned}
&\frac{d}{dt}F^*(y)\\
&=\frac{d}{dt}\left(\frac{y^TWy}{4}\right)\\
&=y^T\dot{W}\frac{y}{4}+\dot{y}^TW\frac{y}{4}+\frac{y^T}{4}W\dot{y}\\
&=y^T\dot{W}\frac{y}{4}+\left(2\dot{M}\delta_{2x}+2W^{-1}\dot{\delta}_{2x}\right)^TW\frac{y}{4}+\frac{y^T}{4}W\\
&\quad\times\left(2\dot{M}\delta_{2x}+2W^{-1}\dot{\delta}_{2x}\right)\\
&=y^T\dot{W}\frac{y}{4}+\left(2\dot{M}\delta_{2x}+2W^{-1}(\bar{A}\delta_x+\bar{B}K\delta_x)\right)^TW\frac{y}{4}\\
&\quad+\frac{y^T}{4}W\left(2\dot{M}\delta_{2x}+2W^{-1}(\bar{A}\delta_x+\bar{B}K\delta_x)\right)\\
&=y^T\dot{W}\frac{y}{4}+\left(2\dot{M}\delta_{2x}+W^{-1}(\bar{A}\delta_{2x}+\bar{B}K\delta_{2x})\right)^TW\frac{y}{4}\\
&\quad+\frac{y^T}{4}W\left(2\dot{M}\delta_{2x}+W^{-1}(\bar{A}\delta_{2x}+\bar{B}K\delta_{2x})\right)\\
&=y^T\dot{W}\frac{y}{4}+\delta_{2x}^T\left(2\dot{M}+(\bar{A}+\bar{B}K)^TW^{-1}\right)W\frac{y}{4}\\
&\quad+\frac{y^T}{4}W\left(2\dot{M}+W^{-1}(\bar{A}+\bar{B}K)\right)\delta_{2x}\\
&=y^T\dot{W}\frac{y}{4}+\delta_{2x}^T\left(\frac{2W^{-1}W}{2}\times(\bar{A}^TW^{-1}+K^T\bar{B}^TW^{-1})\right.\\
&\quad\left.+2W^{-1}W\dot{M}\right)W\frac{y}{4}+\frac{y^T}{4}W\left(2\dot{M}WW^{-1}\right.\\
&\quad\left.+(W^{-1}\bar{A}+W^{-1}\bar{B}K)\times\frac{2WW^{-1}}{2}\right)\delta_{2x}\\
&=y^T\dot{W}\frac{y}{4}+\delta_{2x}^T2W^{-1}\left(\frac{W}{2}\times(\bar{A}^TW^{-1}+K^T\bar{B}^TW^{-1})\right.\\
&\quad\left.+W\dot{M}\right)W\frac{y}{4}+\frac{y^T}{4}W\left(\dot{M}W+(W^{-1}\bar{A}+W^{-1}\bar{B}K)\right.\\
&\quad\left.\times\frac{W}{2}\right)2W^{-1}\delta_{2x}\\
&=y^T\dot{W}\frac{y}{4}+y^T\left(\frac{W}{2}\times(\bar{A}^TW^{-1}+K^T\bar{B}^TW^{-1})+W\dot{M}\right)\\
&\quad\times W\frac{y}{4}+\frac{y^T}{4}W\left(\dot{M}W+(W^{-1}\bar{A}+W^{-1}\bar{B}K)\frac{W}{2}\right)y\\
&=y^T\dot{W}\frac{y}{4}+y^T\left(\frac{W}{2}\times(\bar{A}^T+K^T\bar{B}^T)W^{-1}W+W\dot{M}W\right)\\
&\quad\times\frac{y}{4}+\frac{y^T}{4}\left(W\dot{M}W+WW^{-1}(\bar{A}+\bar{B}K)\times\frac{W}{2}\right)y\\
&=y^T\left(\dot{W}+2W\dot{M}W+\frac{W\bar{A}^T+WK^T\bar{B}^T}{2}\right.\\
&\quad\left.+\frac{\bar{A}W+\bar{B}KW}{2}\right)\frac{y}{4}.
\end{aligned}\tag{19}$$

Note that the derivation from the fourth equation to the fifth equation in (19), there is $2\delta_x=\delta_{2x}$. And the derivation from the eighth equation to the ninth equation in (19), there is $y=2W^{-1}\delta_{2x}$. In order to use contraction analysis on (13) via Definition 1 and Lemma 1, let $W=\begin{bmatrix}w_{11}&w_{12}\\w_{21}&w_{22}\end{bmatrix}$ in (19) be a state-independent matrix and taking the matrix part of $y^T\frac{\beta W}{8}y$ and $y^T\left(\dot{W}+2W\dot{M}W+\frac{W\bar{A}^T+WK^T\bar{B}^T}{2}+\frac{\bar{A}W+\bar{B}KW}{2}\right)\frac{y}{4}$ to get an equation

$$\begin{aligned}
&W\bar{A}^T+\bar{A}W+WK^T\bar{B}^T+\bar{B}KW+\beta W\\
&=\begin{bmatrix}2w_{12} & w_{11}\frac{\partial(f_2+\hat{f}_2)}{\partial e}+w_{12}\frac{\partial(f_2+\hat{f}_2)}{\partial\dot{e}}\\ 2w_{22} & w_{21}\frac{\partial(f_2+\hat{f}_2)}{\partial e}+w_{22}\frac{\partial(f_2+\hat{f}_2)}{\partial\dot{e}}\end{bmatrix}\\
&\quad+\begin{bmatrix}2w_{12} & w_{11}\frac{\partial(f_2+\hat{f}_2)}{\partial e}+w_{12}\frac{\partial(f_2+\hat{f}_2)}{\partial\dot{e}}\\ 2w_{22} & w_{21}\frac{\partial(f_2+\hat{f}_2)}{\partial e}+w_{22}\frac{\partial(f_2+\hat{f}_2)}{\partial\dot{e}}\end{bmatrix}^T\\
&\quad+\begin{bmatrix}\beta w_{11}&\beta w_{12}\\ \beta w_{21}&\beta w_{22}\end{bmatrix}+WK^T\bar{B}^T+\bar{B}KW.
\end{aligned}\tag{20}$$

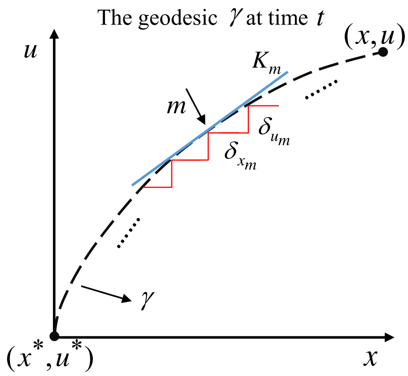

FIGURE 1. The schematic of incremental controller.

Theorem 2: Considering the worst case of uncertainties with $\hat{N} \approx \varepsilon N$ in Assumption 1 and Assumption 2, and the system (13) would be contracted if the parameter K designed as

$$
\begin{aligned}
K &= [K_1 \quad K_2]W^{-1},\\
K_1 &= \frac{-w_{12}\frac{\partial f_2}{\partial \dot{e}} - w_{11}\frac{\partial f_2}{\partial e} - 2w_{22} - \beta w_{12}}{\big((1+\varepsilon)N(e)\big)^{-1}R},\\
K_2 &= \frac{-T - w_{12}\frac{\partial f_2}{\partial e} - w_{22}\frac{\partial f_2}{\partial \dot{e}} - \beta\frac{w_{22}}{2}}{\big((1+\varepsilon)N(e)\big)^{-1}R}.
\end{aligned}
\tag{21}
$$

■

Proof: Taking (21) into (20), the equation (20) is changed to

$$
W\bar{A}^T + \bar{A}W + WK^T\bar{B}^T + \bar{B}KW + \beta W = \begin{bmatrix} \hat{S}_{11} & \hat{S}_{12} \\ \hat{S}_{12}^T & \hat{S}_{22} \end{bmatrix}, \tag{22}
$$

where the inner elements in formula (22) are

$$
\begin{aligned}
\hat{S}_{11} &= 2(w_{12}+w_{21}) + \beta w_{11},\\
\hat{S}_{12} &= w_{11}\frac{\partial \hat{f}_2}{\partial e} + w_{12}\frac{\partial \hat{f}_2}{\partial \dot{e}},\\
\hat{S}_{22} &= 2\Big(-T + w_{12}\frac{\partial \hat{f}_2}{\partial e} + w_{22}\frac{\partial \hat{f}_2}{\partial \dot{e}}\Big).
\end{aligned}
$$

The system (13) is also shrinking if

$$
\begin{aligned}
\hat{S}_{22} - \frac{\hat{S}_{12}^2}{\hat{S}_{11}} &= 2\Big(-T + w_{12}\frac{\partial \hat{f}_2}{\partial e} + w_{22}\frac{\partial \hat{f}_2}{\partial \dot{e}}\Big)\\
&\quad - \frac{\Big(w_{11}\frac{\partial \hat{f}_2}{\partial e} + w_{12}\frac{\partial \hat{f}_2}{\partial \dot{e}}\Big)^2}{2(w_{12}+w_{21}) + \beta w_{11}}\\
&\leq 0.
\end{aligned}
\tag{23}
$$

□

Remark 5: The statement (23) is Schur complement of contraction condition in (22). If $2(w_{12}+w_{21}) + \beta w_{11} < 0$ and T is sufficient positive definite, we can get the conclusion that $\frac{d}{dt}F^* < 0 \Rightarrow F^* \to 0 \Rightarrow y = 2M\delta_{2x} \to 0 \Rightarrow \delta_{2x} = 0$. Therefore, the robustness of the system (13) can be guaranteed.

D. ACTUAL CONTROLLER

Further, the actual controller can be written as

$$
\begin{aligned}
u &= k(x) + v = \int_{\gamma_0}^{\gamma_1} \gamma \delta u,\\
&\approx \sum_{j=1}^{n} \delta u_j + u^\star \approx \sum_{j=1}^{n} K_j \delta_{x_j} + u^\star.
\end{aligned}
\tag{24}
$$

Equation (24) can be explained by Fig. 1 in this paper, where γ denotes the geodesic connecting start point γ_0 (target solution $(u^\star, x^\star)$) and the end point γ_1 (actual solution (u, x)) in system (13) at time t. The above derivation proves the rationality of the incremental controller $\delta_u = \frac{\partial k+v}{\partial x}\delta_x$. It is easy to see that u is the sum of n iterations of δu along γ. Taking the mth incremental controller as an example, the derivative of the solution at m has $K_m = K(x_m) \in \gamma$ and $\delta_{x_m} = \frac{\partial x_m}{\partial t}dt \in \gamma$. So the incremental controller $\delta_{u_m} = K(x_m)\dot{x}_m dt \in \gamma$. After iterating n times at time t, the actual controller is $u = k(x) + v = \sum_{j=1}^{n} K_j \delta_{x_j} + u^\star$.

Remark 6: The geodesic γ is interpreted as the shortest distance measured by the $\delta x^T M \delta x$ between the start point γ_0 (target solution $(x^\star, u^\star)$) and the end point γ_1 (actual solution (x, u)). It's detailed expression is

$$
\begin{aligned}
\gamma &= \inf \int_{\gamma_0}^{\gamma_1} F\Big(\mathbf{c}(s), \frac{\partial \mathbf{c}}{\partial s}\Big) ds,\\
ds &= \begin{cases} \sqrt{\delta x^T G \delta x}, & K \in (16),\\ \sqrt{\dfrac{y^T W y}{4}}, & K \in (21). \end{cases}
\end{aligned}
$$

where $\mathbf{c}(s)$ denotes an arbitrary curve passing points γ_0 and γ_1, and $F(\cdot)$ is a Lagrange function. Another fact is the rate of change of the metric $\delta_{x^T} M \delta_x$ is negative, which leads to the geodesic γ is shortened at the next t. This paper used iteration of the incremental controller to interpret the integral of δu along the geodesic. However, imprecise n can also cause u to be imprecise.

E. SPECIAL CASE: $\bar{B}$ IS INDEPENDENT OF STATE

This special case is usually an independent state of the inertia matrix N, and the physical sense is that the mechanical structure is sufficiently symmetrical.

Theorem 3: If $\bar{B}$ is independent of state, the parameter K of Theorem 1 can be redesigned as

$$
K = \left[\frac{-J}{\big((1+\varepsilon)N(e)\big)^{-1}R} \quad \frac{-\hat{P}-P}{\big((1+\varepsilon)N(e)\big)^{-1}R} \right], \tag{25}
$$

and $u = \int_{\gamma_0}^{\gamma_1} \gamma \delta u$ is simplified to a linear feedback $u = K(x - x^\star) + u^\star$, where $\hat{p} > \frac{\partial f_2(x)}{\partial \dot{e}}$. ■

Proof: Taking (25) into (15), the equation (15) is changed to

$$
\bar{A}^T M + K^T \bar{B}^T M + M\bar{A} + M\bar{B}K + \beta M = \begin{bmatrix} \acute{S}_{11} & \acute{S}_{12} \\ \acute{S}_{12}^T & \acute{S}_{22} \end{bmatrix}, \tag{26}
$$

where the inner elements in formula (26) are

$$\begin{aligned}\acute{S}_{11} &= \beta m_{11} + m_{12}\left(\frac{\partial f_2 + \partial \hat{f}_2}{\partial e} - J\right) \\ &\quad + \left(\frac{\partial f_2 + \partial \hat{f}_2}{\partial e} - J\right) m_{21}, \\ \acute{S}_{12} &= 2m_{11} + \left(\frac{\partial f_2 + \partial \hat{f}_2}{\partial \dot{e}} - P - \hat{P} + \frac{\beta}{2}\right) m_{21} \\ &\quad + m_{22}\left(\frac{\partial f_2 + \partial \hat{f}_2}{\partial e} - J\right), \\ \acute{S}_{22} &= 2m_{22}\left(\frac{\partial f_2 + \partial \hat{f}_2}{\partial \dot{e}} + \frac{\beta}{2} - P - \hat{P}\right) + 2m_{12} + 2m_{21}.\end{aligned}$$

The contraction condition of (26) is equivalent via Schur complement to the statement

$$\acute{S}_{22} - \frac{\acute{S}_{12}^2}{\acute{S}_{11}} < 0. \tag{27}$$

The contraction condition of (27) is similar to (17). The difference is that $\hat{p} > \frac{\partial f_2(x)}{\partial \dot{e}}$ means to keep the stability in Theorem 1. Then, the constant parameter K makes geodesic γ a straight line, so the integral $u = \int_{\gamma_0}^{\gamma_1} \gamma \delta u = K(x - x^\star) + u^\star$. □

Remark 7: The above analysis method is also applicable to (20). Due to similar ideas, detailed descriptions are omitted.

V. ILLUSTRATIVE EXAMPLE

The effectiveness of the algorithm developed in this paper is verified by using an inverted pendulum showed as Fig. 2,

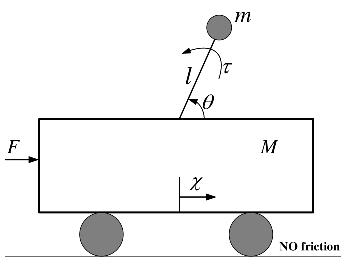

FIGURE 2. Vehicle with an inverted pendulum.

where M (uncertain) denotes the vehicle's mass, F denotes the external force, m (uncertain) denotes the mass of the inverted pendulum and l (uncertain) denotes the length. An external torque τ is the controller applied on the pendulum. The equation of motion of the inverted pendulum can be written in matrix form from using Lagrange's equation as (note that the inverted pendulum comes from [31]):

$$N(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = u,$$

where

$$q = \begin{bmatrix}\chi \\ \theta\end{bmatrix}, \quad \dot{q} = \begin{bmatrix}\dot{\chi} \\ \dot{\theta}\end{bmatrix}, \ \ddot{q} = \begin{bmatrix}\ddot{\chi} \\ \ddot{\theta}\end{bmatrix}, \ u = \begin{bmatrix}F \\ \tau\end{bmatrix},$$

$$\begin{aligned}N(q) &= \begin{bmatrix}M + m & -ml\sin\theta \\ -ml\sin\theta & ml^2\end{bmatrix}, \\ C(q, \dot{q}) &= \begin{bmatrix}0 & -ml\dot{\theta}\cos\theta \\ 0 & 0\end{bmatrix}, \\ G(q) &= \begin{bmatrix}0 \\ mgl\cos\theta\end{bmatrix}.\end{aligned}$$

The desired trajectory $q^\star(t)$, the desired velocity and acceleration $\dot{q}^\star(t)$ are given by

$$\begin{aligned}q^\star(t) &= \begin{bmatrix}\chi^\star \\ \theta^\star\end{bmatrix} = \begin{bmatrix}\sin(t) \\ 1.5 - \cos(t)\end{bmatrix}, \\ \dot{q}^\star(t) &= \begin{bmatrix}\dot{\chi}^\star \\ \dot{\theta}^\star\end{bmatrix} = \begin{bmatrix}\cos(t) \\ \sin(t)\end{bmatrix}.\end{aligned}$$

So the errors can be written as

$$x = \begin{bmatrix}e(t) \\ \dot{e}(t)\end{bmatrix} = \begin{bmatrix}q(t) - q^\star(t) \\ \dot{q}(t) - \dot{q}^\star(t)\end{bmatrix} = \begin{bmatrix}\chi - \chi^\star \\ \theta - \theta^\star \\ \dot{\chi} - \dot{\chi}^\star \\ \dot{\theta} - \dot{\theta}^\star\end{bmatrix}.$$

In order to simulate uncertainties, we take actual parameters $g = 9.8 + \sin(t), l = 1 - (0 \sim 0.2), M = 10 + (0 \sim 2)$, $m = 1 + 0.1$. For simulation, we take standard parameters $g = 9.8, l = 1, M = 10, m = 1$ and control gains $J = 400_{2\times2}, P = 400_{2\times2}, \varepsilon = 4$. The initial condition is chosen as $\chi(0) = 2, \theta(0) = 1, \dot{\chi}(0) = 0.1, \dot{\theta}(0) = 0.1$.

Fig. 3 shows the tracking curves of state q of the proposed directly incremental feedback control (a contraction-based method, denoted by CBR in simulation diagram) and proposed control in [31] (a Lyapunov-based method, denoted by LBR in simulation diagram). Fig. 4 depicts the histories

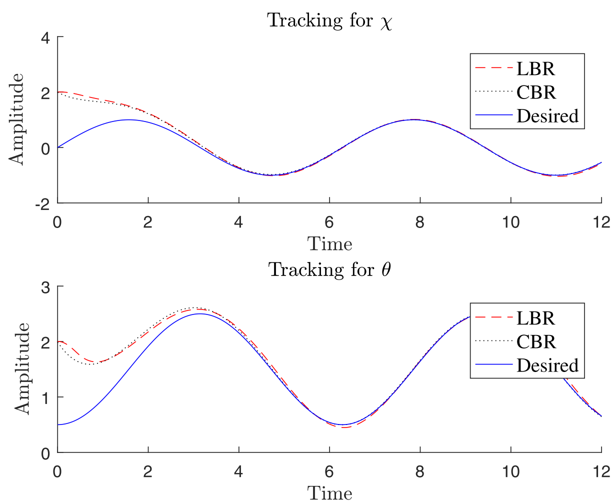

FIGURE 3. The tracking curves.

of q position errors. Although the errors of LBR are already small, CBR is surprisingly smaller than 0.2. It can be concluded that errors can tend to be zero soon if J and P are suitable and with the CBR has a smaller steady state error. Fig. 5 shows the input signal u of CBR and LBR. The disadvantage of CBR is that the initial control signal has a larger

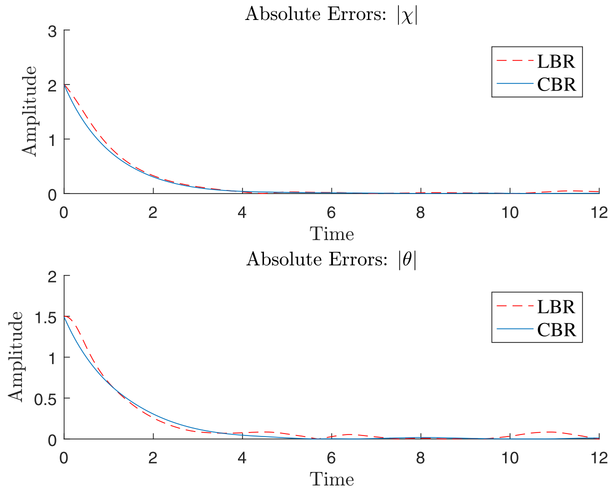

FIGURE 4. **The tracking errors.**

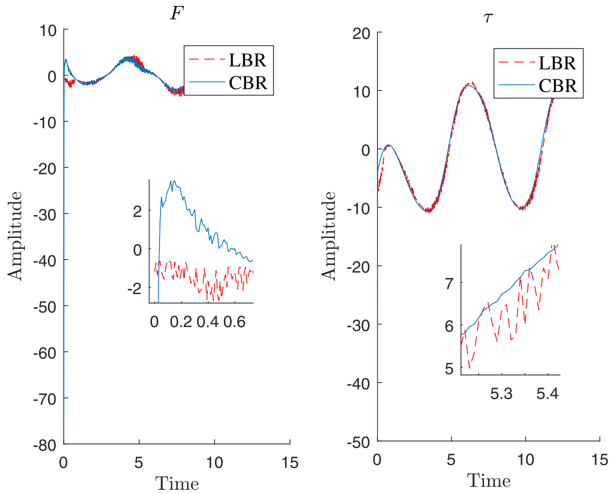

FIGURE 5. **The input *u*.**

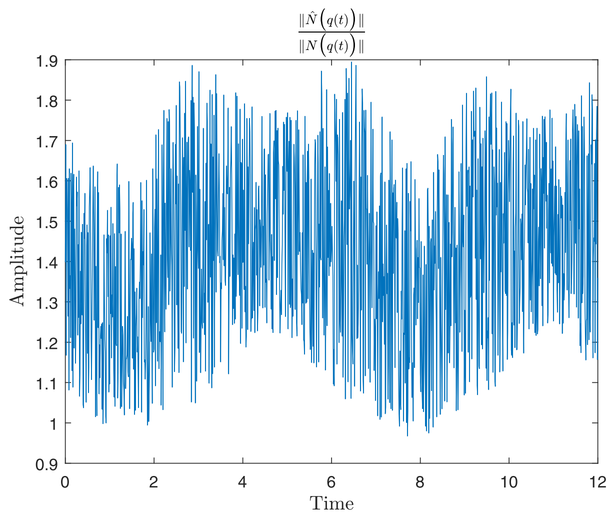

FIGURE 6. $\frac{\|\hat{N}(q(t))\|}{\|N(q(t))\|}$ **in Assumption 2.**

overshoot, but it has a smoother signal and LBR's jitter is obviously. Fig. 6 shows the value of $\frac{\left\|\hat{N}\left(q(t)\right)\right\|}{\left\|N\left(q(t)\right)\right\|}$ in Assumption 2 and ε was set to 5 in this paper. On the other hand, it shows that our assumptions are reasonable. In summary, the proposed method (directly incremental feedback control) has two advantages, one is a smoother control signal, second is a smaller errors, but the disadvantage is a larger input overshoot.

VI. CONCLUSION

The mechanical systems modeled usually have uncertainties, the concept of contraction analysis is introduced to analysis stability and further investigate robust control for that systems. We focus on the tracking problem of fully-actuated mechanical systems with uncertainties of norm bounded conditions and the system contains external disturbances. First, incremental error dynamics are established, followed by a combine process. That is, the result that the upper bound of ε is introduced as a parameter into the uncertain incremental system. Then, assumed a closed-loop feedback controller and derived it based on the contraction theory. It is found that the possibility of stabilizing the system can be achieved by designing reasonable J and P. Moreover, through the dual analysis of $\delta_{2x}^T M \delta_{2x}$, another scheme that can make the system robust is proposed. Finally, the actual controller is refined through graphical interpretation introduced a special case. There is several perspective generalisations of interest to be addressed in next researches among which extending to finite-time control, such as, e.g. interval observer, is worth to remark. Under-actuated mechanical systems are also worth considering in next researches.

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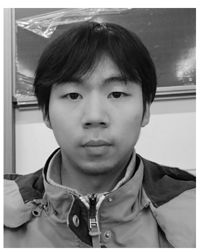

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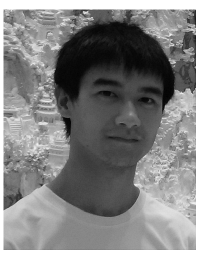

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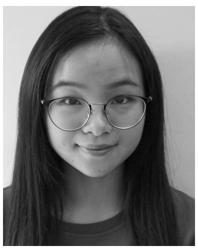

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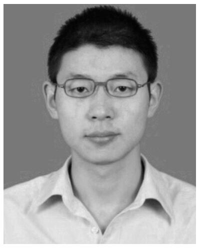

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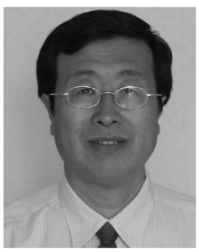

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