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A Day-to-Day Stochastic Traffic Flow Assignment Model Based on Mixed Regulation

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ABSTRACT In many cases, the final path selection of travellers' is not the shortest path, due to the limited computing power and high cost of path search. To solve the problem, this paper proposes a day-to-day (DTD) stochastic traffic flow assignment model that regulates the traffic flow based on the travel time (travel cost) and residual congestion of optional paths. The regulation mechanism is called the mixed regulation. Then, the authored proved the existence, uniqueness and stability of the model solution. The proposed model was verified through simulation on a Nguyen-Dupuis road network. The results show that traffic flows and travel times of all paths reached the equilibrium state, thanks to the DTD mixed regulation for 20 $\sim$ 30 days. From the traffic flows and congestion degrees of different sections, it can be seen that our model with mixed regulation diverts the traffic flow to the sections with a low congestion degree, and encourages travellers to drive through the sections with a low traffic flow. In addition, the congestion degrees of the four most congested sections decreased by 5.8%, 4%, 7% and 1.2%, respectively, and the entire road network exhibited a slight downward trend in mean congestion degree. These results prove that our model can uniformize the traffic flow, improve the operation efficiency and alleviate the congestion of the road network. These findings shed new light on the control, guidance and planning of traffic flow in road networks.

INDEX TERMS Path selection, traffic assignment, residual congestion, stability, Nguyen-Dupuis road network.

I. INTRODUCTION

In the road network, the traffic flow is often unequilibrated under external or internal factors [1]. If there are multiple paths ahead, the traffic flow will be assigned to different paths, depending on the path selection of travellers. To disclose the effect of travellers' path selection, it is necessary to explore how the traffic flow gradually evolves from disequilibrium state to equilibrium state.

In urban road network, the path selection is a complex noncooperative game [2], [3]. To maximize their personal interests, numerous travellers constantly adjust their travel paths according to historical selections or traffic information. The adjustments based on historical selections exhibit as the behavioural inertia, while those based on traffic information reflects the travellers' learning ability. Through these adjustments, the unequilibrated traffic flow gradually evolves to an equilibrium. The dynamic evolution of traffic flow should be fully examined to gain insights into the operation of traffic network, and promote urban traffic planning and management.

The early models on traffic flow evolution generally assume the travellers can grasp the traffic information well and minimize their travel costs. However, their results are impractical because real-world travellers have bounded rationality (BR) [4], [5]. To solve the problem, many scholars started to investigate traffic flow evolution under the BR. One of the most representative results is the BR behaviour principle put forward by Simon.

Meanwhile, the traffic flow evolution has been modelled based on the impact of changing external environment on travellers' path selections. For example, Peeta [6] simulated

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the dynamic traffic flow in a day, adjusted the weight of travellers' path selection rules in the light of real-time road conditions, and established a hybrid model of intra-day and day-to-day (DTD) path selections by travellers. The hybrid model can predict the travellers' path selections according to real-time information. Considering the hysteresis of traffic information, Guo and Huang [7] introduced the concept of path selection cost, and constructed a dynamic evolution model of traffic flow in the presence of automatic traffic information system (ATIS).

Assuming that travellers' path selections are based on expected total travel time and local traffic information (rather than global traffic information), Shengxue *et al.* [8] set up a dynamic evolution model of traffic flow through dynamic projection, which includes a formula to update path flow and a formula to estimate the shortest travel time. Liu and Guan [9] held that travellers with BR select paths based on the difference between the travel time via the current path and that via the shortest path, and created a travel time update formula and a path selection model under complete and incomplete information.

Drawing on behavioural game theory, He and Peeta [10] constructed a marginal cost function to describe the shadow price incurred by path change, defined the numerical difference between marginal profit and marginal cost as marginal utility, and then developed a DTD traffic flow evolution model under marginal utility. Iryo [11] believed that the existing DTD models only cover travellers' behaviours under incomplete information, and designed a DTD model for dynamic traffic flow evolution that collects travellers' information under complete and incomplete information.

Cantarella and Watling [12] prepared a deterministic process model of simple discrete time and continuous time based on exponential smoothing, and relied on the model to study how travellers empirically predict the current service level and select paths based on the prediction. Lou et al. [13] established a DTD dynamic traffic flow evolution model for three types of travellers: risk-averse travellers, risk-taking travellers without the ATIS, and risk-taking travellers with the ATIS. Considering the impact of congestion charge on travellers' path selections, Li and Guan [14] assumed that the charge is positively correlated with the travel time in congested section, and introduced the congestion charge policy to the DTD dynamic evolution model of traffic flow.

To date, most of the existing traffic flow evolution models, which consider the BR and heterogeneity of travellers, take account of the difference between travellers in time value [15]–[17]. In real-world road network, the travel cost is a random variable. Gao *et al.* [18] pointed out that the final path selection of travellers' is not the shortest path, due to the limited computing power and high cost of path search. In other words, the travellers are unable to estimate the theoretical minimum travel time through path selection. The inability directly causes many differences in travel behaviour.

As a matter of experience, the travellers are more sensitive to travel time than any other factor in path selection. Unable to estimate the theoretical minimum travel time, the travellers often focus on the most congested sections. With the aid of the ATIS, the congestion information is directly accessible to all travellers with a smartphone. Therefore, it is assumed here that travellers can obtain the congestion degree of each section and select the preferred path, i.e., the path selections completely depend on the congestion conditions.

Of course, the congestion-based path selections reflect the BR. On the one hand, the path selections might be irrational, due to the limited computing power and high cost of path search. On the other hand, some travellers may detour through less congested sections, because they pursue the freedom and comfort of driving.

To sum up, unbounded rational travellers make path selections purely based on travel cost, i.e. the time difference between different paths, while bounded rational travellers with complete information constantly adjust their selections through DTD learning of the real-time traffic information until the traffic flow reaches a new equilibrium.

Based on non-Walrasian equilibria [19]–[21] and path congestion degree, this paper proposes a DTD stochastic traffic flow assignment model that regulates the traffic flow based on the travel time (travel cost) and residual congestion of optional paths. The regulation mechanism is called the mixed regulation. This simple and stable model can reasonably describe how some travellers choose the paths based on congestion information, and lead to the equilibrium of the traffic flow.

II. PRELIMINARIES

A. CONGESTION DEGREE AND RESIDUAL CONGESTION

The congestion degree of a path is the weighted sum of the congestion degrees of multiple key sections. The key sections are the most crowded segments of the path, i.e. the sections that attract the most travellers. The residual congestion refers to the residual capacity of a path, i.e. the difference between the saturation state or congestion state and the current state.

The congestion degree of section a can be computed by $f_a/K_a(f_a \leq K_a)$, where f_a is the traffic flow of section a, and K_a is the traffic capacity of section a. Then, the residual congestion v_a on the n-th day of section a can be defined as:

$$v_a(f_a(n)) = \varpi - \frac{f_a(n)}{K_a} \tag{1}$$

where, ϖ is the maximum traffic flow. If the section is saturated, $\varpi = 1$; if the section is oversaturated, $\varpi \geq 1$.

B. HYPOTHESES

The following two hypotheses were put forward on travellers' path selections:

Hypothesis 1: The travellers only consider the most congested sections in path selection. The residual congestion v_r of the path and the residual congestion v_a of a section of the path should satisfy:

$$v_r(h_r(n)) = \min_{a \in A}\{\delta_{ar} v_a(f_a(n))\}, \quad \delta_{ar} v_a(f_a(n) > 0 \tag{2}$$

where, A is the set of sections within the path ; $h_r(n)$ is the flow of path r on the n-th day; δ_{ar} is the coefficient of association between the path and the section. If $a \in r$, then $\delta_{ar} = 1$; otherwise, $\delta_{ar} = 0$.

Hypothesis 2: In path selection, the residual congestion v_r of the path equals the weighted sum of the residual congestions v_a of multiple key sections. The residual congestions of all sections in the path can be sorted in ascending order as:

$$\begin{aligned} \delta_{a_1 r} v_{a_1}(f_{a_1}(n)) &\leq \delta_{a_2 r} v_{a_2}(f_{a_2}(n)) \leq \\ &\cdots \leq \delta_{a_l r} v_{a_l}(f_{a_l}(n)) \\ &\leq \delta_{a_{l+1} r} v_{a_{l+1}}(f_a(n)) \leq \cdots \\ &\leq \delta_{a_k r} v_{a_k}(f_a(n)) \\ \delta_{a_k r} v_a(f_a(n) &> 0, \quad a_k \in B \end{aligned} \tag{3}$$

where, B is the set of sections within path r. Then, the top-L sections in the ranking are selected to compute the residual congestion of the path:

$$\begin{aligned} v_r(h_r(n)) = \omega_1 &\delta_{a_1 r} v_{a_1}(f_{a_1}(n)) \\ &+ \omega_2 \delta_{a_2 r} v_{a_2}(f_{a_2}(n)) + \ldots \\ &+ \omega_l \delta_{a_l r} v_{a_l}(f_{a_l}(n)) \sum_{i=1}^{l} \omega_i = 1, \\ &0 \leq \omega_i \leq 1 \end{aligned} \tag{4}$$

where, ω_i is defined as the correlation coefficient between section congestion and path congestion, indicating the importance of section congestion affecting path selection. Generally speaking, the more congested the section, the greater the value.

III. MODEL CONSTRUCTION

A. DTD STOCHASTIC TRAFFIC FLOW ASSIGNMENT MODEL BASED ON RESIDUAL CONGESTION

The expected residual congestion V_r on the n-th day is defined as the weighted sum of the expected residual congestion and the residual congestion of the path on the (n-1)-th day:

$$V_r(n) = \eta V_r(n-1) + (1-\eta) v_r(h_r(n-1)) \tag{5}$$

where, $\eta \in [0, 1)$ is a parameter related to travel characteristics, and the dependence of travellers' path selections on the residual congestion of the path on previous day. The higher the value of η, the less dependence. On the contrary, the smaller the value of η, the greater dependence. According to the stochastic user equilibrium model, suppose that the random variables ε_r and ε_k are independent and obey the same Gumbel distribution, the probability p_r that path r is selected on the n-th day can be computed by:

$$\begin{aligned} p_r(n) &= P(V_r(n) + \varepsilon_r \geq \bigcup_{k \in R_w} (V_k(n) + \varepsilon_k)) \\ &= \frac{1}{1 + \sum_{k \neq r} e^{-\varphi(V_k(n) - V_r(n))}}, \quad \forall k,\ r \in R_w \end{aligned} \tag{6}$$

where, $\varphi > 0$ is the travellers' sensitivity to the expected residual congestion. The higher the value of φ, the more sensitive the congestion degree of the path, and the less random the traveler to choose the path. On this basis, the DTD stochastic traffic flow assignment model based on residual congestion can be established as:

$$\begin{cases} p_r(n) = \dfrac{1}{1 + \sum_{k \neq r} e^{-\varphi(V_k(n) - V_r(n))}} \\ h_r(n) = d_w p_r(n) \\ V_r(n) = \eta V_r(n-1) + (1-\eta) v_r(h_r(n-1)) \end{cases} \tag{7}$$

where d_w is the distance between a pair of origin (O) and destination (D) among the set of OD pairs w; ε_r is the random error in path r; $\sum_{r \in R_w} p_r(n) = 1$.

B. DTD STOCHASTIC TRAFFIC FLOW ASSIGNMENT MODEL BASED ON MIXED REGULATION

As mentioned before, the mixed regulation refers to regulating the traffic flow based on the travel time and residual congestion of optional paths. According to the theory on stochastic traffic flow assignment, the travel time c_r of a path and the travel time c_a of a section of the path satisfies:

$$c_r(h_r(n)) = \sum_{a \in A} \delta_{ar} c_a(f_a(n)) \tag{8}$$

Similarly, the traffic flow $h_r(n)$ of a path and the traffic flow f_a of a section of the path satisfies:

$$f_a(n) = \sum_{w \in W} \sum_{r \in R_w} \delta_{ar} h_r(n) \tag{9}$$

The expected travel time C_r of the path on the n-th day equals the weighted sum of the expected travel time and the travel time of the path on the (n-1)-th day:

$$C_r(n) = \kappa C_r(n-1) + (1-\kappa) c_r(h_r(n-1)) \tag{10}$$

where, $\kappa \in [0, 1)$ is the dependence of travellers' path selections on the travel time of the path on the previous day.

The total travel cost s_r of the path on the n-th day is defined as the weighted sum of the travel time c_r and residual congestion v_r of the path:

$$s_r(h_r(n)) = \lambda c_r(h_r(n)) - (1-\lambda) v_r(h_r(n)) \tag{11}$$

where, $\lambda \in [0, 1]$ is the travellers' sensitivity to the travel time and residual congestion of the path.

The travel time c_r and residual congestion v_r of the path have different dimensions. Hence, the two variables can be normalized by:

$$\begin{aligned} \tilde{c}_r(h_r(n)) &= \frac{c_r(h_r(n)) - \underset{r \in R_w}{min}\{c_r(h_r(n))\}}{\underset{r \in R_w}{max}\{c_r(h_r(n))\} - \underset{r \in R_w}{min}\{c_r(h_r(n))\}} \\ \tilde{v}_r(h_r(n)) &= \frac{v_r(h_r(n)) - \underset{r \in R_w}{min}\{v_r(h_r(n))\}}{\underset{r \in R_w}{max}\{v_r(h_r(n))\} - \underset{r \in R_w}{min}\{v_r(h_r(n))\}} \end{aligned} \tag{12}$$

Then, formula (11) can be rewritten as:

$$s_r(h_r(n)) = \lambda \tilde{c}_r(h_r(n)) - (1-\lambda) \tilde{v}_r(h_r(n)) \tag{13}$$

The expected total travel cost S_r of the path on the n-th day equals the weighted sum of the expected travel time c_r and the expected residual congestion v_r of the path:

$$S_r(n) = \lambda \tilde{C}_r(n) - (1-\lambda)\tilde{V}_r(n) \tag{14}$$

According to the stochastic user equilibrium model, the probability p_r that path r is selected on the n-th day can be computed by:

$$\begin{aligned} p_r(n) &= P(S_r(n) + \varepsilon_r \leq \bigcup_{k\in R_w} (S_k(n) + \varepsilon_k)) \\ &= \frac{1}{1+\sum_{k\neq r} e^{-\theta\{[\lambda\tilde{C}_k(n)-(1-\lambda)\tilde{V}_k(n)]-[\lambda\tilde{C}_r(n)-(1-\lambda)\tilde{V}_r(n)]\}}}, \\ &\quad \forall k, r \in R_w \end{aligned} \tag{15}$$

where, $\varphi > 0$ is the travellers' sensitivity to the expected total travel cost. On this basis, the DTD stochastic traffic flow assignment model based on mixed regulation can be established as:

$$\begin{cases} p_r(n) = \dfrac{1}{1+\sum_{k\neq r} e^{-\theta\{\lambda\tilde{C}_k(n)-(1-\lambda)\tilde{V}_k(n)-[\lambda\tilde{C}_r(n)-(1-\lambda)\tilde{V}_r(n)]\}}} \\ h_r(n) = d_w p_r(n) \\ \tilde{C}_r(n) = \kappa \tilde{C}_r(n-1) + (1-\kappa)\tilde{c}_r(h_r(n-1)) \\ \tilde{V}_r(n) = \eta \tilde{V}_r(n-1) + (1-\eta)\tilde{v}_r(h_r(n-1)) \end{cases} \tag{16}$$

IV. STABILITY ANALYSIS

A. EXISTENCE OF A UNIQUE SOLUTION

Theorem 1: If the travel time of a path is a continuous and strictly monotonically increasing function of the traffic flow of the path, and if the residual congestion of the path is a continuous and strictly monotonically decreasing function of the traffic flow of the path, there exists a unique solution for the DTD stochastic traffic flow assignment model based on mixed regulation under fixed travel demand.

Proof: The first step is to prove the existence of solution(s). Since the travel demand is bounded, the feasible set of traffic flows of the path is a non-empty bounded closed convex set. By the definitions of travel time and residual congestion, the feasible set of travel time and the feasible set of residual congestion of the path are also non-empty bounded closed convex sets. Then, model (16) is a continuous mapping from non-empty bounded closed convex set to itself. According to Brouwer fixed-point theorem, model (16) must have at least one solution.

The next step is to prove the uniqueness of solution. Let $\tilde{C}_r(n) = \tilde{C}_r(n-1) = C_r^*$ be the expected travel time at a stable place, $\tilde{V}_r(n) = \tilde{V}_r(n-1) = V_r^*$ be the expected residual congestion, and $h_r(n) = h_r(n-1) = h_r^*$ be the path flow. Then, we have:

$$C_r^* = \tilde{c}_r(h_r^*) \tag{17}$$

$$V_r^* = \tilde{v}_r(h_r^*) \tag{18}$$

$$h_r^* = \frac{d_w}{1+\sum_{k\neq r} e^{-\theta\{\lambda C_k^*-(1-\lambda)V_k^*-[\lambda C_r^*-(1-\lambda)V_r^*]\}}} \tag{19}$$

The above fixed-point problem can be converted into the variational inequality below:

$$\begin{aligned} \sum\nolimits_{w\in W} \sum\nolimits_{r\in R_w} [\lambda \tilde{c}_r(h_r^*) - (1-\lambda)\tilde{v}_r(h_r^*) \\ + \frac{1}{\theta} lnh_r^*](h_r - h_r^*) \geq 0 \end{aligned} \tag{20}$$

Assuming that the model (16) has two different solutions, namely, $(C_r^{1*}, V_r^{1*}, h_r^{1*})$ and $(C_r^{2*}, V_r^{2*}, h_r^{2*})$, the following can be derived from inequality (20):

$$\begin{aligned} \sum\nolimits_{w\in W} \sum\nolimits_{r\in R_w} [\lambda \tilde{c}_r(h_r^{1*}) - (1-\lambda)\tilde{v}_r(h_r^{1*}) \\ + \frac{1}{\theta} lnh_r^{1*}](h_r^{2*} - h_r^{1*}) \geq 0 \end{aligned} \tag{21}$$

$$\begin{aligned} \sum\nolimits_{w\in W} \sum\nolimits_{r\in R_w} [\lambda \tilde{c}_r(h_r^{2*}) - (1-\lambda)\tilde{v}_r(h_r^{2*}) \\ + \frac{1}{\theta} lnh_r^{2*}](h_r^{1*} - h_r^{2*}) \geq 0 \end{aligned} \tag{22}$$

Adding up inequalities (21) and (22), we have:

$$\begin{aligned} \sum\nolimits_{w\in W} \sum\nolimits_{r\in R_w} [\lambda(\tilde{c}_r(h_r^{1*}) - \tilde{c}_r(h_r^{2*})) \\ - (1-\lambda)(\tilde{v}_r(h_r^{1*}) - \tilde{v}_r(h_r^{2*})) \\ + \frac{1}{\theta}(lnh_r^{1*} - lnh_r^{2*})](h_r^{1*} - h_r^{2*}) \leq 0 \end{aligned} \tag{23}$$

Since the travel time function of the path is strictly monotonically increasing, $\tilde{c}_r(h_r)$ must be strictly monotonically increasing with respect to h_r. Likewise, $\tilde{v}_r(h_r)$ is strictly monotonically decreasing with respect to h_r. In addition, lnh_r is strictly monotonically increasing with respect to h_r:

$$\begin{aligned} \sum\nolimits_{w\in W} \sum\nolimits_{r\in R_w} [\lambda(\tilde{c}_r(h_r^{1*}) - \tilde{c}_r(h_r^{2*})) \\ - (1-\lambda)(\tilde{v}_r(h_r^{1*}) - \tilde{v}_r(h_r^{2*})) \\ + \frac{1}{\theta}(lnh_r^{1*} - lnh_r^{2*})](h_r^{1*} - h_r^{2*}) > 0 \end{aligned} \tag{24}$$

Inequality (24) contradicts inequality (23), indicating that model (16) has a unique solution.

B. STABILITY OF SOLUTION

Suppose there are m paths between each OD pair in w. Since model (16) is stable, $\tilde{C}_r(n) = C_r^*, \tilde{V}_r(n) = V_r^*, h_r(n) = h_r^*$, and $\forall r = 1, \cdots, m$. Let

$$c_r' = \left.\frac{\partial \tilde{c}_r(n)}{\partial h_r(n)}\right|_{(C_r^*, V_r^*, h_r^*)} \tag{25}$$

Theorem 2: If model (16) has a unique solution and satisfies $\left|\kappa + (1-\kappa)d_w(c_r' p_{rC_r}' - c_1' p_{1C_r}')\right| < 1$ and $\left|\eta - (1-\eta)d_w(p_{rV_r}' - p_{1V_r}')\right| < 1$ for each r, the only solution of the model must be asymptotically stable.

Proof: According to the nonlinear dynamic stability theory is introduced, the equilibrium point of a system is asymptotically stable at the equilibrium point of the nonlinear discrete system, if the moduli of all the eigenvalues of Jacobian matrix are less than 1. Here, the unique solution of model

(16) can be regarded as an equilibrium point. The Jacobian matrix J of model (16) can be expressed as:

$$J=\begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} \tag{28}$$

where,

$$J_{11}=\begin{pmatrix} \frac{\partial \tilde{C}_1(n)}{\partial \tilde{C}_1(n-1)} & \cdots & \frac{\partial \tilde{C}_1(n)}{\partial \tilde{C}_m(n-1)} \\ \vdots & \ddots & \vdots \\ \frac{\partial \tilde{C}_m(n)}{\partial \tilde{C}_1(n-1)} & \cdots & \frac{\partial \tilde{C}_m(n)}{\partial \tilde{C}_m(n-1)} \end{pmatrix} = \begin{pmatrix} \kappa+d_w(1-\kappa)c_1'p_{1C_1}' & \cdots & d_w(1-\kappa)c_1'p_{1C_m}' \\ \vdots & \ddots & \vdots \\ d_w(1-\kappa)c_m'p_{mC_1}' & \cdots & \kappa+d_w(1-\kappa)c_m'p_{mC_m}' \end{pmatrix} \tag{29}$$

$$J_{12}=\begin{pmatrix} \frac{\partial \tilde{C}_1(n)}{\partial \tilde{V}_1(n-1)} & \cdots & \frac{\partial \tilde{C}_1(n)}{\partial \tilde{V}_m(n-1)} \\ \vdots & \ddots & \vdots \\ \frac{\partial \tilde{C}_m(n)}{\partial \tilde{V}_1(n-1)} & \cdots & \frac{\partial \tilde{C}_m(n)}{\partial \tilde{V}_m(n-1)} \end{pmatrix} = \begin{pmatrix} d_w(1-\kappa)c_1'p_{1V_1}' & \cdots & d_w(1-\kappa)c_1'p_{1V_m}' \\ \vdots & \ddots & \vdots \\ d_w(1-\kappa)c_m'p_{mV_1}' & \cdots & d_w(1-\kappa)c_m'p_{mV_m}' \end{pmatrix} \tag{30}$$

$$J_{21}=\begin{pmatrix} \frac{\partial \tilde{V}_1(n)}{\partial \tilde{C}_1(n-1)} & \cdots & \frac{\partial \tilde{V}_1(n)}{\partial \tilde{C}_m(n-1)} \\ \vdots & \ddots & \vdots \\ \frac{\partial \tilde{V}_m(n)}{\partial \tilde{C}_1(n-1)} & \cdots & \frac{\partial \tilde{V}_m(n)}{\partial \tilde{C}_m(n-1)} \end{pmatrix} = \begin{pmatrix} -d_w(1-\eta)p_{1C_1}' & \cdots & -d_w(1-\eta)p_{1C_m}' \\ \vdots & \ddots & \vdots \\ -d_w(1-\eta)p_{mC_1}' & \cdots & -d_w(1-\eta)p_{mC_m}' \end{pmatrix} \tag{31}$$

$$J_{22}=\begin{pmatrix} \frac{\partial \tilde{V}_1(n)}{\partial \tilde{V}_1(n-1)} & \cdots & \frac{\partial \tilde{V}_1(n)}{\partial \tilde{V}_m(n-1)} \\ \vdots & \ddots & \vdots \\ \frac{\partial \tilde{V}_m(n)}{\partial \tilde{V}_1(n-1)} & \cdots & \frac{\partial \tilde{V}_m(n)}{\partial \tilde{V}_m(n-1)} \end{pmatrix} = \begin{pmatrix} \eta-d_w(1-\eta)p_{1V_1}' & \cdots & -d_w(1-\eta)p_{1V_m}' \\ \vdots & \ddots & \vdots \\ -d_w(1-\eta)p_{mV_1}' & \cdots & \eta-d_w(1-\eta)p_{mV_m}' \end{pmatrix} \tag{32}$$

It can be seen from equation sets (26) and (27), as shown at the bottom of this page, that $p_{rC_1}'+\cdots+p_{rC_m}'=0$ and $p_{rV_1}'+\cdots+p_{rV_m}'=0$. The eigen-polynomial of Jacobian matrix J can be expressed as:

$$\begin{aligned} &\det|J-\lambda E| \\ &\quad=(\kappa-\lambda)[\kappa+d_w(1-\kappa)(c_2'p_{2C_2}'-c_1'p_{1C_2}')-\lambda] \\ &\qquad\cdots[\kappa+d_w(1-\kappa)(c_m'p_{mC_m}'-c_1'p_{1C_m}')-\lambda] \\ &\qquad\cdot(\eta-\lambda)[\eta-d_w(1-\eta)(p_{2V_2}'-p_{1V_2}')-\lambda] \\ &\qquad\cdots[\eta-d_w(1-\eta)(p_{mV_m}'-p_{1V_m}')-\lambda] \end{aligned} \tag{33}$$

$$\begin{cases} p_{rC_r}'=\frac{\partial p_r(n)}{\partial \tilde{C}_r(n)} \\ \quad=-\theta\lambda\left.\frac{\sum_{k\neq r}e^{-\theta\{[\lambda\tilde{C}_k-(1-\lambda)\tilde{V}_k]-[\lambda\tilde{C}_r-(1-\lambda)\tilde{V}_r]\}}}{\left(1+\sum_{k\neq r}e^{-\theta\{[\lambda\tilde{C}_k-(1-\lambda)\tilde{V}_k]-[\lambda\tilde{C}_r-(1-\lambda)\tilde{V}_r]\}}\right)^2}\right|_{(C_r^*,V_r^*,h_r^*)} \\ p_{rC_k}'=\frac{\partial p_r(n)}{\partial \tilde{C}_k(n)} \\ \quad=\theta\lambda\left.\frac{e^{-\theta\left\{\left[\lambda\tilde{C}_k-(1-\lambda)\tilde{V}_k\right]-\left[\lambda\tilde{C}_r-(1-\lambda)\tilde{V}_r\right]\right\}}}{\left(1+\sum_{k\neq r}e^{-\theta\left\{\left[\lambda\tilde{C}_k-(1-\lambda)\tilde{V}_k\right]-\left[\lambda\tilde{C}_r-(1-\lambda)\tilde{V}_r\right]\right\}}\right)^2}\right|_{(C_r^*,V_r^*,h_r^*)}, \quad \forall k\neq r \end{cases} \tag{26}$$

$$\begin{cases} p_{rV_r}'=\frac{\partial p_r(n)}{\partial \tilde{V}_r(n)}=\theta(1-\lambda)\left.\frac{\sum_{k\neq r}e^{-\theta\{[\lambda\tilde{C}_k-(1-\lambda)\tilde{V}_k]-[\lambda\tilde{C}_r-(1-\lambda)\tilde{V}_r]\}}}{\left(1+\sum_{k\neq r}e^{-\theta\{[\lambda\tilde{C}_k-(1-\lambda)\tilde{V}_k]-[\lambda\tilde{C}_r-(1-\lambda)\tilde{V}_r]\}}\right)^2}\right|_{(C_r^*,V_r^*,h_r^*)} \\ p_{rV_k}'=\frac{\partial p_r(n)}{\partial \tilde{V}_k(n)}=-\theta(1-\lambda)\left.\frac{e^{-\theta\{[\lambda\tilde{C}_k-(1-\lambda)\tilde{V}_k]-[\lambda\tilde{C}_r-(1-\lambda)\tilde{V}_r]\}}}{\left(1+\sum_{k\neq r}e^{-\theta\{[\lambda\tilde{C}_k-(1-\lambda)\tilde{V}_k]-[\lambda\tilde{C}_r-(1-\lambda)\tilde{V}_r]\}}\right)^2}\right|_{(C_r^*,V_r^*,h_r^*)}, \quad \forall k\neq r \end{cases} \tag{27}$$

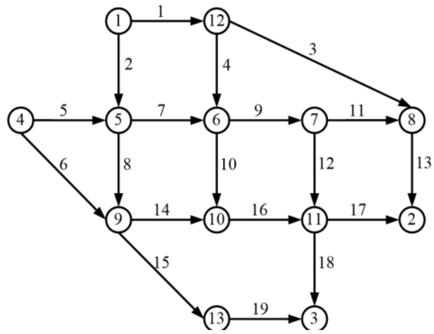

FIGURE 1. The Nguyen-Dupuis road network.

TABLE 1. Parameters of the travel cost function for each section.

Section	1	2	3	4	5	6	7	8	9	10
A_a	8.00	5.00	2.00	6.00	4.00	5.00	3.50	2.00	5.00	8.02
B_a	0.06	0.09	0.03	0.075	0.045	0.06	0.03	0.03	0.075	0.055
K_a	30	30	50	30	30	40	30	50	30	30
Section	11	12	13	14	15	16	17	18	19	
A_a	5.40	8.00	2.00	2.50	3.00	1.02	4.00	1.00	6.00	
B_a	0.06	0.05	0.03	0.085	0.045	0.08	0.09	0.04	0.075	
K_a	30	50	50	40	30	30	40	50	30	

Equation (33) shows that the Jacobian matrix J has 2m eigenvalues, namely, $\lambda_1 = \kappa, \lambda_2 = \kappa + d_w(1-\kappa)(c_2' p_{2C_2}' - c_1' p_{1C_2}'),\ldots, \lambda_m = \kappa + d_w(1-\kappa)(c_m' p_{mC_m}' - c_1' p_{1C_m}')$, $\lambda_{m+1} = \eta, \lambda_{m+2} = \eta - d_w(1-\eta)(p_{2V_2}' - p_{1V_2}'),\ldots, \lambda_{2m} = \eta - d_w(1-\eta)(p_{mV_m}' - p_{1V_m}')$. Since $|\kappa| < 1, |\eta| < 1, \left|\kappa + (1-\kappa)d_w(c_r' p_{rC_r}' - c_1' p_{1C_r}')\right| < 1$ and $\left|\eta - (1-\eta)d_w(p_{rV_r}' - p_{1V_r}')\right| < 1$, the moduli of all the eigenvalues of Jacobian matrix J are less than 1. Therefore, the unique solution of model (16) must be asymptotically stable.

V. ROAD NETWORK TEST

To verify its operation effect, the proposed model was tested in the Nguyen-Dupuis road network [22]–[25]. As shown in Figure 1, the network contains 13 nodes, 19 sections and 4 OD pairs.

Table 1 lists the relationship between each OD pair and each section/path, and the values of relevant parameters. The cost function of each section is $c_a(f_a) = A_a + B_a(\frac{f_a(t)}{K_a})^4$.

During the road network test, the path selections were made according to Hypothesis 2 and formula (4). If a path has five or more sections, its residual congestion was computed as the weighted sum of the three smallest section residual congestions, with the weight coefficients being 0.5, 0.3 and 0.2, respectively; if a path has four sections, its residual congestion was computed as the weighted sum of the two smallest section residual congestions, with the weight coefficients being 0.6 and 0.4, respectively; if a path has fewer than four sections, its residual congestion was computed as the smallest section residual congestion.

In our model, the weight λ of residual congestion is 50%. Meanwhile, the demand function for an OD pair is $D_{12}, D_{13}, D_{42}, D_{43} = (25, 20, 15, 20)$. The adjusted model parameters are listed in Table 2.

TABLE 2. Adjusted model parameters.

Parameter	Value
κ	0.9
η	0.9
θ	0.3
λ	0.5

TABLE 3. The traffic flows, travel times and congestion degrees of the paths in the equilibrium road network.

OD pair	Path	Sections	Traffic flow	Travel time	Congestion degree
(1, 2)	1	1, 10, 19	3.600	22.015	0.679
	2	2, 6, 9, 16, 19	2.803	22.108	0.836
	3	2, 6, 9, 15, 17	2.793	22.130	0.836
	4	2, 6, 14, 11, 17	3.030	22.102	0.785
	5	2, 5, 7, 11, 17	3.077	22.093	0.775
	6	1, 13, 9, 16, 19	3.301	22.063	0.732
	7	1, 13, 9, 15, 17	3.039	22.085	0.785
	8	1, 13, 14, 11, 17	3.358	22.057	0.721
(1, 3)	9	2, 5, 8, 12	3.407	19.050	0.754
	10	2, 6, 9, 15, 18	2.982	19.114	0.836
	11	2, 6, 14, 11, 18	3.234	19.087	0.785
	12	2, 5, 7, 11, 18	3.316	19.078	0.769
	13	1, 13, 9, 15, 18	3.343	19.069	0.764
	14	1, 13, 14, 11, 18	3.718	19.042	0.696
(4, 2)	15	4, 7, 11, 17	3.090	19.045	0.729
	16	3, 6, 9, 16, 19	2.944	19.068	0.759
	17	3, 6, 9, 15, 17,	2.762	19.090	0.800
	18	3, 6, 14, 11, 17	3.053	19.063	0.736
	19	3, 5, 7, 11, 17	3.151	19.054	0.715
(4, 3)	20	4, 8, 12	4.793	16.003	0.393
	21	4, 7, 11, 18	2.994	16.030	0.704
	22	3, 5, 8, 12	3.595	16.011	0.584
	23	3, 6, 9, 15, 18	2.655	16.075	0.780
	24	3, 6, 14, 11, 18	2.953	16.047	0.712
	25	3, 5, 7, 11, 18	3.011	16.038	0.700

After a period of testing, the traffic flows and travel times of all paths reached the equilibrium state. Figures 2 $\sim$ 7 present the dynamic evolution of traffic flow on paths 1 $\sim$ 8. It can be seen that our model achieved the equilibrium of traffic flow through mixed regulation after 20 $\sim$ 30 days.

Table 3 lists the traffic flows, travel times and congestion degrees of the paths between different OD pairs after the road network reached the equilibrium state ($\lambda = 0.5$). Note that

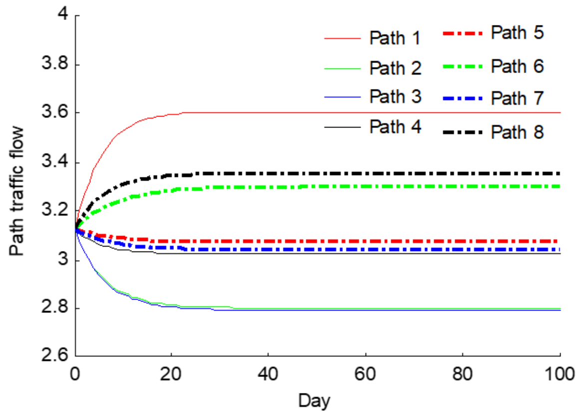

FIGURE 2. Traffic flow evolution on paths 1 ∼ 8 ($\lambda = 0.5$).

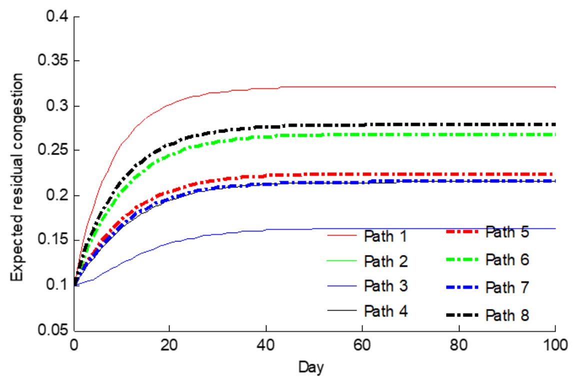

FIGURE 3. Congestion evolution on paths 1 ∼ 8 ($\lambda = 0.5$).

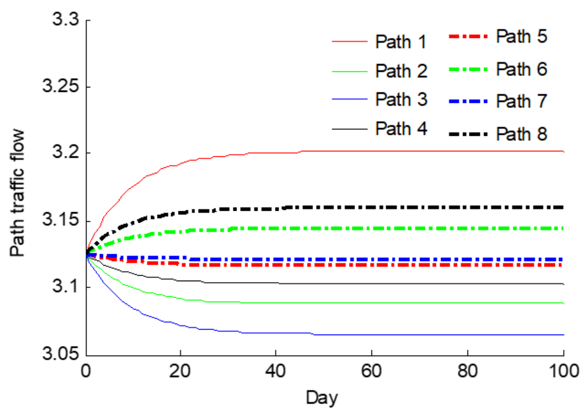

FIGURE 4. Traffic flow evolution on paths 1 ∼ 8 ($\lambda = 1$).

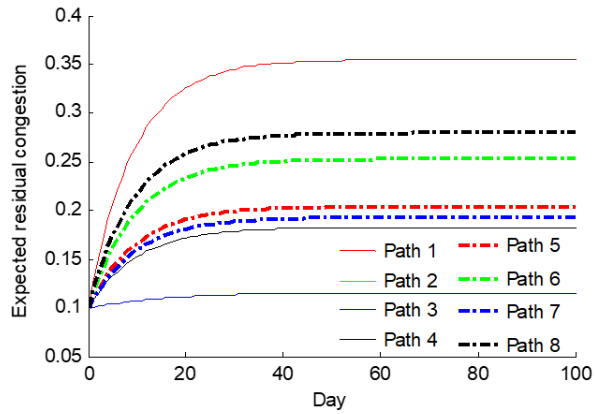

FIGURE 5. Congestion evolution on paths 1 ∼ 8 ($\lambda = 1$).

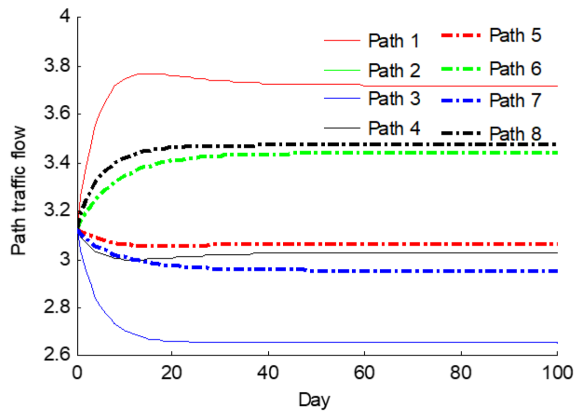

FIGURE 6. Traffic flow evolution on paths 1 ∼ 8 ($\lambda = 0$).

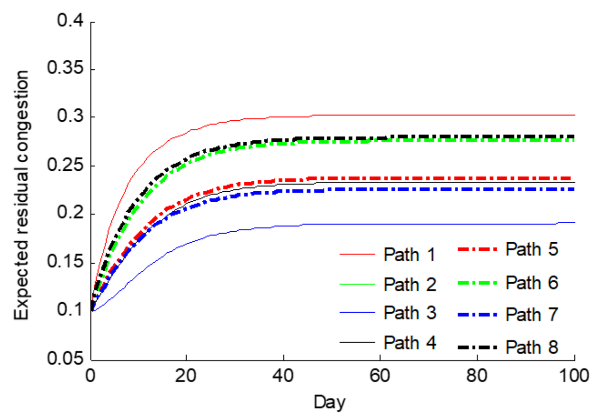

FIGURE 7. Congestion evolution on paths 1 ∼ 8 ($\lambda = 0$).

$\lambda = 1$ is the result of the travel time (travel cost) regulation mechanism while $\lambda = 0$ is the result of the residual congestion regulation mechanism.

From the traffic flow and residual congestion curves of paths $1 \sim 6$, it can be seen that the significance of residual congestion regulation lies in diverting the traffic flow to less congested paths.

To further illustrate the significance of residual congestion regulation, the congestion conditions of each section under travel time (travel cost) regulation were compared with those under mixed regulation (Figures 8 and 9).

From Figures 8 and 9, it is learned that our model with mixed regulation diverts the traffic flow to the sections with a low congestion degree, and encourages travellers to drive through the sections with a low traffic flow. For four most congested sections (9,2,6,11), their congestion degrees were adjusted from 0.9421, 0.8553, 0.7851 and 0.7688 to 0.8874,

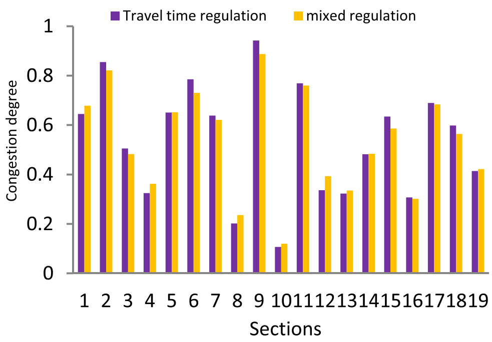

FIGURE 8. Comparison of congestion degree.

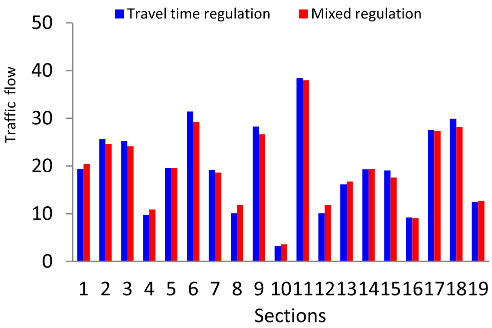

FIGURE 9. Comparison of traffic flow.

0.8214, 0.7302 and 0.7597, respectively, that is, falling by 5.8%, 4%, 7% and 1.2%, respectively. Meanwhile, for the four least congested sections (10, 8, 16, 13), their congestion degrees were adjusted from 0.1067, 0.2019, 0.3074 and 03228 to 0.1200, 0.2359, 0.3016 and 0.3352, respectively. In general, 16 sections witnessed slight declines in congestion degree, while the other three sections saw an increase in congestion degree by 12.4%, 16.8% and 3.8%, respectively. The entire road network exhibited a slight downward trend in mean congestion degree (−0.4%).

VI. CONCLUSION

Because the travel time (travel cost) is highly stochastic, the final path selection of travellers' is not the shortest path, due to the limited computing power and high cost of path search. To solve the problem, this paper proposes a DTD stochastic traffic flow assignment model based on mixed regulation.

The proposed model was verified through simulation on a Nguyen-Dupuis road network. The results show that traffic flows and travel times of all paths reached the equilibrium state, thanks to the DTD mixed regulation for 20 $\sim$ 30 days. From the traffic flows and congestion degrees of different sections, it can be seen that our model with mixed regulation diverts the traffic flow to the sections with a low congestion degree, and encourages travellers to drive through the sections with a low traffic flow. Therefore, in the traditional regulation mechanism, can only keep adjusting the travel paths and learning the congestion information of sections, owing to the limited computing power or incomplete information. In our model, however, the λ value, i.e. the proportion of travellers' relying on residual congestion regulation in path selection, can be calibrated flexibly depending on the actual situation.

Compared with the travel time (travel cost) regulation, the residual congestion regulation can alleviate the congestion in the most congested sections and balance the congestion conditions in the road network. our model can uniformize the traffic flow, improve the operation efficiency and alleviate the congestion of the road network. These findings shed new light on the control, guidance and planning of traffic flow in road networks.

Based on our findings, the future research will further explore the dynamic evolution of traffic flow from three angles: First, the weights of travel time and residual congestion will be further investigated; Second, the DTD stochastic traffic flow assignment model will be calibrated according to the actual state of the road network to reflect the changes in the actual traffic flow; Third, the proposed model will be extended to road networks with elastic travel demands, and the relationship between dynamic evolution and traffic information other than congestion will be discussed.

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