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Event-Triggered H-Infinity Control for Continuous-Time Singular Networked Cascade Control Systems With State Delay

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ABSTRACT This paper is concerned with the problems of modeling, stabilization, and H_∞ control for continuous-time singular networked cascade control systems (SNCCSs) with state delay and event-triggered control. An event-triggered scheme is firstly introduced to this system for utilizing the network bandwidth resources efficiently. Considering the effects of both time-varying network-induced delay and event-triggered control, a singular networked cascade control system (SNCCS) model is established. By constructing a suitable Lyapunov-Krasovskii functional, sufficient condition of admissibility for this system is proposed, and the co-design method of event-triggered parameter, primary state feedback controller and secondary state feedback controller are also derived. Furthermore, H_∞ control is concerned for SNCCS via linear matrix inequality (LMI) technique. Finally, a simulation example considering a heating furnace with the structure of SNCCS and event-triggered control is given to illustrate the effectiveness of the proposed method, where it can be seen this method is superior to the existing one with periodic control.

INDEX TERMS Cascade control, singular systems, networked control systems, event-triggered control, stabilization, H_∞ control.

I. INTRODUCTION

Since cascade control is firstly proposed in [1], it has become a very effective strategy to improve the performance of closed-loop control systems. Cascade control systems are usually composed of two control loops: a secondary loop embedded into a primary one. The secondary loop can regulate the key variables rapidly to guarantee the process stability while the primary one is designed to fulfill some performance index [2]. In practice, cascade control has been used in many industrial process control systems, such as power systems [3], [4], chemical systems [5], and so on. From theoretical angle, cascade control has been considered in all kinds of dynamics systems, such as nonlinear systems [2], [6], networked control systems (NCSs) [7]–[9], neural network systems [10], singular networked control systems [11], [12] and so on.

Nowadays, motivated by the fact that SNCCS is widespread in industrial process, many researchers and scholars have paid attention to SNCCS [11], [12]. It has all the characteristics of singular system, NCSs and cascade control systems. As is known, singular system is more general and practical in describing the actual dynamical systems than differential system, since besides the dynamic behavior expressed by differential equation, it also has the capacity to represent the algebraic constraints between physical variables [13]. However, the problem for singular systems is much more complicated than that for differential systems. Simultaneously, SNCCS also has the advantages of both NCSs and cascade control systems, such as reduced system cost, facility of maintenance, shortening the regulating time and controlling timely.

In addition, while bringing the above advantages, the insertion of network and cascade control make the problems of modeling, analysis, and synthesis of the SNCCS much more complicated. In SNCCS, there exist three types of devices with different functions: two sensors, two controller, and an actuator. The network-induced delay may be time-varying and exist everywhere, either in the primary loop or in the secondary one, and the effect of this delay will occur when the

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sampling signals are transmitted through the network. So it need to be considered in this system.

On the other hand, most sampling signals are concerned with periodic control where all signals are transmitted through network. This control mode may result in some problems, such as more data packet loss, network transmission congestion and so on. Recently, to save the limited bandwidth, event-based/event-triggered control has been proposed. It has many potential advantages for the systems connected by network, such as better resource utilization, less transmission traffic. What's more, based on this mode, sampling signals will be transmitted only when some sampling information exceeds a special threshold. In short, only part of the sampling signals need to be transmitted through the network, so it can be viewed as an alternative sampling scheme in terms of the network bandwidth utilization.

In the past decade, event-triggered control has been considered in all kinds of systems and much effort has been made to apply it, such as linear systems [14]–[16], nonlinear systems [17]–[20], NCSs [21]–[29], singular NCSs [30], singular systems [31]–[33], multi-agent systems [34]–[36], and the references therein. Thereinto, in [23], event-triggered H_∞ stabilization is investigated for discrete-time NCCSs with disturbances. In [30], the problem of event-triggered H_∞ control is considered for singular NCSs with both state and input subject to quantizations. In [31], the problem of controller design for linear singular systems with event-triggered control is investigated. Until now, there are few works on the problem of SNCCS with event-triggered control, which is one of the main motivations for this paper.

For SNCCS with state delay and event-triggered control, because of the existence of algebraic subsystems, event-triggered control, and cascade control, the entire structure of SNCCS becomes very complex and many issues should be considered simultaneously, such as regularity, impulse, stability, network-induced delay, state delay, cooperative work of event-triggered scheme and two controllers. Although Lyapunov-Krasovskii method is a popular and efficient one to analyze the problem for dynamical systems [37], [38], the problems of modeling, selection of Lyapunov-Krasovskii functional, and the calculation and processing of its derivative along the motion of SNCCS may be challenging for scholars.

Up to now, the issue of event-triggered control in SNCCS has not yet been fully considered, which inspired us to investigate this challenging issue. Motivated by this consideration, to investigate the effect in practice, we discuss the problems of modeling, stabilization, and H_∞ control for SNCCSs with state delay and disturbance based on event-triggered control. Mainly, the co-design method of event-triggered parameter, primary controller and secondary one will be proposed based on Lyapunov-Krasovskii method which guarantees that the corresponding closed-loop SNCCS is admissible. The main contributions of this paper are summarized as follows.

1) SNCCS exists widely in practical industrial process, in this paper, event-triggered control is introduced to SNCCS for the first time, which can reduce the unnecessary waste of the network bandwidth resources;

2) A new model that takes differential subsystems, differential-algebraic subsystems, network-induced delay, state delay, cascade control and event-triggered control is first constructed. The model is more general and practical in describing the actual dynamical behavior, and the system has the benefits of both cascade control and event-triggered control;

3) The co-design method of event-triggered parameter, primary state feedback controller and secondary one is proposed, which can be easily applied to the corresponding industrial process control systems;

4) H_∞ control for this system with event-triggered control is also considered. The obtained results can be further extended to the industrial process control systems established as SNCCS models.

The organization of this paper is as follows. In Section II, the model of SNCCS is established. Section III gives the sufficient condition of admissibility for SNCCS without disturbance, and the corresponding co-design method of event-triggered parameter, primary and secondary state feedback controllers is also developed. In Section IV, H_∞ control for SNCCS with event-triggered control and disturbance is considered. In Section V, a simulation example considering a heating furnace with SNCCS structure and event-triggered control is given to show the effectiveness of the proposed method, and Section VI is the conclusion of this paper.

Notation: $\|\cdot\|$ is the Euclidean norm of vector or matrix, $X > 0$ (respectively, $X < 0$) for $X \in R^{n\times n}$ means that is a real symmetric positive-definite (respectively, negative-definite) matrix.

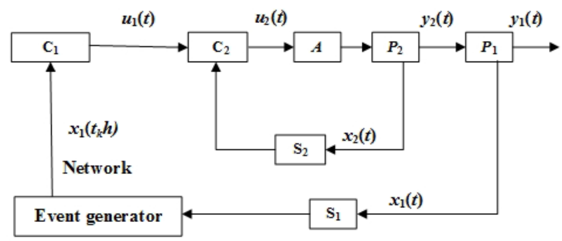

FIGURE 1. Configuration diagram of SNCCSs with event-triggered control.

II. MODELING OF SNCCS WITH EVENT-TRIGGERED CONTROL

In Section II, a configuration diagram of SNCCS similar to the one in [23] is shown in Figure 1 where two plants P_1 and P_2 are concerned.

In Figure 1, S_1 is the primary sensor, C_1 is the primary controller, event generator and C_1 are connected by network, while the secondary sensor S_2 is connected with P_2 and the secondary controller C_2. A is a actuator that is installed in the field connected with P_2. From Figure 1, the output of P_2 is used as the input of P_1, the states of $P_1(P_2)$ are sent to $C_1(C_2)$ by $S_1(S_2)$.

P_2 is continuous-linear time-invariant and described by the following:

$$\begin{cases} E\dot{x}_2(t) = A_2x_2(t) + A_3x_2(t-\tau) + B_2u_2(t) + B_3w(t) \\ y_2(t) = C_2x_2(t) + C_4w(t) \end{cases} \quad (1)$$

where $E \in R^{n\times n}$ may be singular and rank$E = r \le n$, and $x_2(t) \in R^n, u_2(t) \in R^m, w(t) \in R^p$ are the state vector, control input of P_2 and disturbance, respectively, as well as $y_2(t)$ is the output of P_2, τ is a constant and denotes the state delay, A_2, A_3, B_2, B_3, C_2 and C_4 are constant matrices with appropriate dimensions.

P_1 is also continuous-linear time-invariant, which can be written as follows:

$$\begin{cases} \dot{x}_1(t) = A_1x_1(t) + B_1y_2(t) \\ y_1(t) = C_1x_1(t) + C_3w(t) \end{cases} \quad (2)$$

where $x_1(t) \in R^{\tilde{n}}$ is the state vector of P_1, $w(t)$ is assumed to satisfy $L_2[0, \infty)$ which means $w(t)$ has limited energy, $y_2(t)$ is described as the control input of P_1, and $y_1(t)$ is the output of P_1, A_1, B_1, C_1 and C_3 are constant matrices with appropriate dimensions.

Next, an event generator is installed in the outer loop of SNCCS. That is, a scheme is used to determine whether the signal is sent to C_1. The following algorithm is applied to describe this scheme:

$$\begin{aligned} &[x_1(i_kh) - x_1(t_kh)]^T\Omega[x_1(i_kh) - x_1(t_kh)] \\ &\quad \le \sigma x_1^T(i_kh)\Omega x_1(i_kh) \end{aligned} \quad (3)$$

where Ω is a positive-definite matrix, $\sigma \in [0, 1)$, $x_1(i_kh)$ is the current sampling signal at the instant i_kh where $i_kh = t_kh+lh, l = 1, 2, \cdots$, and $x_1(t_kh)$ is the last transmitted signal at the triggering instant t_kh.

Remark 1: The sampling signal $x_1(i_kh)$, just not satisfied the inequality (3), can be sent out through the network. Furthermore, it can be seen there is no Zeno phenomenon in SNCCS.

In this paper, we assume the SNCCSs are completely observable, and the phenomenons of data packet loss and missequence are not concerned. The primary loop of SNCCS is connected by network, where the time-varying network-induced delay may be larger than one sampling period. That is, the network-induced delay mainly exists in the outer loop, and the omitted delay in the inner one is relatively much less than that in the outer loop.

Next, the following controller is adopted in the outer loop, that is

$$u_1(t) = K_1x_1(t) \quad (4)$$

where K_1 is the gain matrix, and $u_1(t)$ is the output of C_1. Considering the existence of network-induced delay, (4) can be rewritten as follows:

$$u_1(t) = K_1x_1(t_kh), t \in [t_kh + \tau_k, t_{k+1}h + \tau_{k+1}) \quad (5)$$

where τ_k is the network-induced delay.

The state feedback controller is also adopted in the inner loop, which is described by

$$\begin{aligned} u_2(t) &= K_1x_1(t_kh) + K_2x_2(t), \\ t &\in [t_kh + \tau_k, t_{k+1}h + \tau_{k+1}) \end{aligned} \quad (6)$$

where K_2 is the gain matrix of C_2.

Define a function $\tau(t)$ that satisfies

$$t - \tau(t) \in [t_kh, t_{k+1}h)$$

where $t \in [t_kh + \tau_k, t_{k+1}h + \tau_{k+1})$, then according to the above formula, we can choose $\tau(t)$ as:

$$0 \le \tau_k \le \tau(t) \le \tau_M$$

where τ_M denotes the upper delay bounds. It means for $t \in [t_kh+\tau_k, t_{k+1}h+\tau_{k+1})$, there exists the corresponding $\tau(t) \in [0, \tau_M]$ to make $t - \tau(t) \in [t_kh, t_{k+1}h)$ hold.

Define $e_k(t) = x_1(t_kh) - x_1(t - \tau(t))$ and combine (3), an event-triggered scheme is applied in SNCCS, that is

$$e_k^T(t)\Omega e_k(t) \le \sigma x_1^T(t - \tau(t))\Omega x_1(t - \tau(t)) \quad (7)$$

where $\sigma \in [0, 1)$, and Ω is a positive-definite matrix.

Using (1), (2), (6), (7), a new model for the closed-loop SNCCS with event-triggered control can be obtained:

$$\begin{cases} \dot{x}_1(t) = A_1x_1(t) + B_1C_2x_2(t) + B_1C_4w(t) \\ E\dot{x}_2(t) = A_2x_2(t) + A_3x_2(t-\tau) + B_2K_2x_2(t) + \\ \qquad B_2K_1x_1(t - \tau(t)) + B_2K_1e_k(t) + B_3w(t) \\ y_1(t) = C_1x_1(t) + C_3w(t) \\ y_2(t) = C_2x_2(t) + C_4w(t) \\ t \in [t_kh + \tau_k, t_{k+1}h + \tau_{k+1}]. \end{cases} \quad (8)$$

III. EVENT-TRIGGERED STABILIZATION FOR SNCCS WITHOUT DISTURBANCE

Now, the following lemmas are introduced which are essential for deriving the main results of this paper.

Lemma 1 [13]: If the pair (E, A) is regular, then for any h times piecewise continuously differentiable input function $f(t)$, the system

$$E\dot{x}(t) = Ax(t) + f(t)$$

is regular, that is, the above system has a unique solution. The above system is impulse free if (E, A) is impulse free.

Remark 2: Based on Lemma 1, the singular subsystem of (8) is regular if the pair $(E, A_2 + B_2K_2)$ is regular, and it is impulse free if $(E, A_2 + B_2K_2)$ is impulse free.

Lemma 2 [38]: For any real matrix E, P, and symmetric positive-definite matrix Z with appropriate dimensions, the following inequality holds

$$P^TE^TZ^{-1}EP \ge \rho E^TEP + \rho P^TE^TE - \rho^2E^TZ^TE.$$

In addition, if $P^TE^T = EP$, we have

$$P^TE^TZ^{-1}EP \ge \rho E^TP^TE^T + \rho EPE - \rho^2E^TZ^TE,$$

where ρ is any chosen constant.

Theorem 1: For the system (8) without disturbance, and given a parameter $\sigma \in [0, 1)$ and matrices K_1, K_2, the system (8) is admissible if there exist a nonsingular matrix P, symmetric positive-definite matrices $R > 0, Z_1 > 0, Z_2 > 0, Q > 0, \Omega > 0$ and matrices $X, Y, T_i, X_{ij}, i = 1, 2, \ldots, 5, j = 1, 2, \ldots, 5$ with appropriate dimensions, such that (9)-(12) hold

$$E^T P = P^T E \geq 0, \quad (9)$$

$$\begin{bmatrix} X & Y \\ Y^T & E^T Z_2 E \end{bmatrix} \geq 0, \quad (10)$$

$$\begin{bmatrix} X_{11} & X_{12} & X_{13} & X_{14} & X_{15} & T_2 \\ * & X_{22} & X_{23} & X_{24} & X_{25} & T_1 \\ * & * & X_{33} & X_{34} & X_{35} & T_4 \\ * & * & * & X_{44} & X_{45} & T_3 \\ * & * & * & * & X_{55} & T_5 \\ * & * & * & * & * & Z_1 \end{bmatrix} > 0, \quad (11)$$

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & \Phi_{15} & \Phi_{16} & \Phi_{17} \\ * & \Phi_{22} & \Phi_{23} & \Phi_{24} & \Phi_{25} & \Phi_{26} & \Phi_{27} \\ * & * & \Phi_{33} & \Phi_{34} & \Phi_{35} & \Phi_{36} & \Phi_{37} \\ * & * & * & \Phi_{44} & \Phi_{45} & \Phi_{46} & \Phi_{47} \\ * & * & * & * & \Phi_{55} & \Phi_{56} & \Phi_{57} \\ * & * & * & * & * & \Phi_{66} & \Phi_{67} \\ * & * & * & * & * & * & \Phi_{77} \end{bmatrix} < 0, \quad (12)$$

where

$$\begin{aligned}
\Phi_{11} &= P^T(A_2 + B_2K_2) + (A_2 + B_2K_2)^T P + Y \\
&\quad + Y^T + \tau X + Q + \tau_M X_{11}, \\
\Phi_{12} &= C_2^T B_1^T R + T_2 + \tau_M X_{12}, \\
\Phi_{22} &= R^T A_1 + A_1^T R + T_1 + T_1^T + \tau_M X_{22}, \\
\Phi_{13} &= P^T A_3 - Y + \tau_M X_{13}, \quad \Phi_{23} = T_4^T + \tau_M X_{23}, \\
\Phi_{33} &= -Q + \tau_M X_{33}, \\
\Phi_{14} &= P^T B_2 K_1 - T_2 + \tau_M X_{14}, \\
\Phi_{24} &= T_3^T - T_1 + \tau_M X_{24}, \quad \Phi_{34} = -T_4 + \tau_M X_{34}, \\
\Phi_{44} &= -T_3 - T_3^T + \sigma\Omega + \tau_M X_{44}, \\
\Phi_{15} &= P^T B_2 K_1 + \tau_M X_{15}, \quad \Phi_{25} = T_5^T + \tau_M X_{25}, \\
\Phi_{35} &= \tau_M X_{35}, \quad \Phi_{45} = -T_5^T + \tau_M X_{45}, \\
\Phi_{55} &= -\Omega + \tau_M X_{55}, \quad \Phi_{16} = \tau_M C_2^T B_1^T Z_1, \\
\Phi_{26} &= \tau_M A_1^T Z_1, \quad \Phi_{36} = 0, \Phi_{46} = 0, \Phi_{56} = 0, \\
\Phi_{66} &= -\tau_M Z_1, \quad \Phi_{17} = \tau(A_2 + B_2K_2)^T Z_2, \Phi_{27} = 0, \\
\Phi_{37} &= \tau A_3^T Z_2, \Phi_{47} = \tau(B_2K_1)^T Z_2, \\
\Phi_{57} &= \tau(B_2K_1)^T Z_2, \quad \Phi_{67} = 0, \Phi_{77} = -\tau Z_2.
\end{aligned}$$

Proof: To show the system (8) with $w(t) = 0$ is admissible, we first show (8) with $w(t) = 0$ regular and impulse free. Without loss of generality, we denote $A_2+B_2K_2$ by A_c and assume E, A_c in (8) have the following forms

$$E = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \quad A_c = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}.$$

From (9),(10) and (12), we can see

$$Y = \begin{bmatrix} Y_{11} & 0 \\ Y_{21} & 0 \end{bmatrix}, \quad P = \begin{bmatrix} P_{11} & 0 \\ P_{21} & P_{22} \end{bmatrix}.$$

From (12), we can obtain $P^T A_c + A_c^T P + Y + Y^T < 0$. That means A_{22} is nonsingular and the pair $(E, A_2 + B_2K_2)$ is regular and impulse free. According to [13], it also implies that the system (8) with $w(t) = 0$ is regular and impulse free.

Next, we will show the system (8) with $w(t) = 0$ is stable. Choose a Lyapunov-Krasovskii functional as follows:

$$V(t) = \sum_{i=1}^{5} V_i(t) \quad (13)$$

where

$$\begin{aligned}
V_1(t) &= x_1^T(t) R x_1(t), \\
V_2(t) &= \int_{-\tau_M}^{0} \int_{t+\beta}^{t} \dot{x}_1^T(\alpha) Z_1 \dot{x}_1(\alpha) d\alpha d\beta, \\
V_3(t) &= x_2^T(t) E^T P x_2(t), \\
V_4(t) &= \int_{t-\tau}^{t} \int_{\beta}^{t} \dot{x}_2^T(\alpha) E^T Z_2 E \dot{x}_2(\alpha) d\alpha d\beta, \\
V_5(t) &= \int_{t-\tau}^{t} x_2^T(s) Q x_2(s) ds,
\end{aligned}$$

where R, Z_1, P, Z_2, Q are defined in Theorem 1 and to be determined.

Taking the derivative of $V(t)$, we can obtain

$$\dot{V}(t) = \sum_{i=1}^{5} \dot{V}_i(t) \quad (14)$$

where

$$\begin{aligned}
\dot{V}_1(t) &= 2x_1^T(t) R [A_1 x_1(t) + B_1 C_2 x_2(t)], \\
\dot{V}_2(t) &= \tau_M \dot{x}_1^T(t) Z_1 \dot{x}_1(t) - \int_{t-\tau_M}^{t} \dot{x}_1^T(\alpha) Z_1 \dot{x}_1(\alpha) d\alpha, \\
\dot{V}_3(t) &= 2x_2^T(t) P^T [A_2 x_2(t) + A_3 x_2(t-\tau) \\
&\quad + B_2K_2 x_2(t) + B_2K_1 x_1(t-\tau(t)) + B_2K_1 e_k(t)], \\
\dot{V}_4(t) &= \tau \dot{x}_2^T(t) E^T Z_2 E \dot{x}_2(t) \\
&\quad - \int_{t-\tau}^{t} \dot{x}_2^T(\alpha) E^T Z_2 E \dot{x}_2(\alpha) d\alpha, \\
\dot{V}_5(t) &= x_2^T(t) Q x_2(t) - x_2^T(t-\tau) Q x_2(t-\tau).
\end{aligned}$$

Using (8), the following equation holds

$$\begin{aligned}
E\dot{x}_2(t) &= (A_2 + B_2K_2) x_2(t) + A_3[x_2(t) \\
&\quad - \int_{t-\tau}^{t} \dot{x}_2(\alpha) d\alpha] + B_2K_1 x_1(t - \tau(t) + B_2K_1 e_k(t).
\end{aligned}$$

Combining Moon [39] inequality and (14), we can obtain

$$\begin{aligned}
\dot{V}_3(t) &\leq 2x_2^T(t) P^T [(A_2 + B_2K_2) x_2(t) + A_3[x_2(t) \\
&\quad - \int_{t-\tau}^{t} \dot{x}_2(\alpha) d\alpha] + B_2K_1 x_1(t - \tau(t) + B_2K_1 e_k(t)] \\
&\leq 2x_2^T(t) P^T [(A_2 + A_3 + B_2K_2) x_2(t) \\
&\quad - A_3 \int_{t-\tau}^{t} \dot{x}_2(\alpha) d\alpha + B_2K_1 x_1(t - \tau(t) + B_2K_1 e_k(t)] \\
&\leq 2x_2^T(t) P^T [(A_2 + A_3 + B_2K_2) x_2(t) + B_2K_1 e_k(t) \\
&\quad + B_2K_1 x_1(t - \tau(t)] + \tau x_2^T(t) X x_2(t)
\end{aligned}$$

$$
\begin{aligned}
&+\int_{t-\tau}^{t} \dot{x}_2^T(\alpha)E^T Z_2 E\dot{x}_2(\alpha)d\alpha \\
&+2x_2^T(t)(Y-P^TA_3)\int_{t-\tau}^{t}\dot{x}_2(\alpha)d\alpha \\
\leq\ & 2x_2^T(t)[P^T(A_2+B_2K_2)x_2(t)+Yx_2(t) \\
&+P^TB_2K_1x_1(t-\tau(t)+P^TB_2K_1e_k(t)] \\
&+\tau x_2^T(t)Xx_2(t)+\int_{t-\tau}^{t}\dot{x}_2^T(\alpha)E^TZ_2E\dot{x}_2(\alpha)d\alpha \\
&+2x_2^T(t)(P^TA_3-Y)x_2(t-\tau) \\
\leq\ & x_2^T(t)[P^T(A_2+B_2K_2)+(A_2+B_2K_2)^TP+Y \\
&+Y^T+\tau X]x_2(t)+2x_2^T(t)P^TB_2K_1x_1(t-\tau(t)] \\
&+2x_2^T(t)P^TB_2K_1e_k(t)+\int_{t-\tau}^{t}\dot{x}_2^T(\alpha)E^TZ_2E\dot{x}_2(\alpha)d\alpha \\
&+2x_2^T(t)(P^TA_3-Y)x_2(t-\tau)
\end{aligned}
$$

where X, Y, Z_2 satisfy $\begin{bmatrix} X & Y \\ Y^T & E^TZ_2E \end{bmatrix} \geq 0$.

Therefore, for any matrices $T_i, i = 1, 2, \ldots, 5$, and $\hat{X}$, defined in Theorem 1, we have the derivative of $V(t)$ along the trajectory of the system (8) as:

$$
\begin{aligned}
\dot{V}(t) =\ & \sum_{i=1}^{5}\dot{V}_i(t)+2[x_1(t)T_1+x_2(t)T_2+x_1(t-\tau(t))T_3 \\
&+x_2(t-\tau)T_4+e_k(t)T_5] \\
&\times[x_1(t)-x_1(t-\tau(t))-\int_{t-\tau(t)}^{t}\dot{x}_1(\alpha)d\alpha] \\
&+\tau(t)\xi^T(t)\hat{X}\xi(t)-\int_{t-\tau(t)}^{t}\xi^T(t)\hat{X}\xi(t)d\alpha \\
\leq\ & \xi^T(t)\Phi\xi(t)-\int_{t-\tau(t)}^{t}\xi_2^T(t,\alpha)\Pi\xi_2(t,\alpha)d\alpha \qquad (15)
\end{aligned}
$$

where $\xi(t) = [x_2^T(t)\ x_1^T(t)\ x_2^T(t-\tau)\ x_1^T(t-\tau(t))\ e_k^T(t)]^T$, $\xi_2(t,\alpha) = [x_2^T(t)\ x_1^T(t)\ x_2^T(t-\tau)\ x_1^T(t-\tau(t))\ e_k^T(t)\ \dot{x}_1^T(\alpha)]^T$.

If $\Phi < 0$, $\Pi > 0$, we can obtain $\dot{V}(t) < 0$ for all nonzero $\xi(t)$, $\xi_2(t,\alpha)$. Therefore,

$$
\Pi = \begin{bmatrix} \hat{X} & T \\ T^T & Z_1 \end{bmatrix} > 0, \qquad (16)
$$

therein

$$
\hat{X} = \begin{bmatrix} X_{11} & X_{12} & X_{13} & X_{14} & X_{15} \\ * & X_{22} & X_{23} & X_{24} & X_{25} \\ * & * & X_{33} & X_{34} & X_{35} \\ * & * & * & X_{44} & X_{45} \\ * & * & * & * & X_{55} \end{bmatrix},
$$

$$
T = \begin{bmatrix} T_2^T & T_1^T & T_4^T & T_3^T & T_5^T \end{bmatrix}^T.
$$

In addition

$$
\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & \Phi_{15} \\ * & \Phi_{22} & \Phi_{23} & \Phi_{24} & \Phi_{25} \\ * & * & \Phi_{33} & \Phi_{34} & \Phi_{35} \\ * & * & * & \Phi_{44} & \Phi_{45} \\ * & * & * & * & \Phi_{55} \end{bmatrix} < 0, \qquad (17)
$$

and

$$
\begin{aligned}
\Phi_{11} &= P^T(A_2+B_2K_2)+(A_2+B_2K_2)^TP+Y \\
&\quad +Y^T+\tau X+Q+\tau_M C_2^TB_1^TZ_1B_1C_2 \\
&\quad +\tau(A_2+B_2K_2)^TZ_2(A_2+B_2K_2)+\tau_M X_{11}, \\
\Phi_{12} &= C_2^TB_1^TR+T_2+\tau_M C_2^TB_1^TZ_1A_1+\tau_M X_{12}, \\
\Phi_{22} &= R^TA_1+A_1^TR+T_1+T_1^T+\tau_M A_1^TZ_1A_1+\tau_M X_{22}, \\
\Phi_{13} &= P^TA_3-Y+\tau(A_2+B_2K_2)^TZ_2A_3+\tau_M X_{13}, \\
\Phi_{23} &= T_4^T+\tau_M X_{23}, \quad \Phi_{33} = -Q+\tau A_3^TZ_2A_3+\tau_M X_{33}, \\
\Phi_{14} &= P^TB_2K_1-T_2+\tau(A_2+B_2K_2)^TZ_2B_2K_1 \\
&\quad +\tau_M X_{14}, \\
\Phi_{24} &= T_3^T-T_1+\tau_M X_{24}, \\
\Phi_{34} &= -T_4+\tau A_3^TZ_2B_2K_1+\tau_M X_{34}, \\
\Phi_{44} &= -T_3-T_3^T+\sigma\Omega+\tau(B_2K_1)^TZ_2B_2K_1+\tau_M X_{44}, \\
\Phi_{15} &= P^TB_2K_1+\tau(A_2+B_2K_2)^TZ_2B_2K_1+\tau_M X_{15}, \\
\Phi_{25} &= T_5^T+\tau_M X_{25}, \\
\Phi_{35} &= \tau A_3^TZ_2B_2K_1+\tau_M X_{35}, \\
\Phi_{45} &= -T_5^T+\tau(B_2K_1)^TZ_2B_2K_1+\tau_M X_{45}, \\
\Phi_{55} &= -\Omega+\tau(B_2K_1)^TZ_2B_2K_1+\tau_M X_{55}.
\end{aligned}
$$

According to Schur complement, $\Phi < 0$ is equivalent to (12).

If the inequalities (9)-(12) hold, then $\dot{V}(t) < 0$ for all nonzero $\xi(t)$, $\xi_2(t,\alpha)$. Therefore, (9)-(12) guarantee that the system (8) with $w(t) = 0$ is stable for $\tau(t)$. As a result, the corresponding system (8) without disturbance is admissible. This completes the proof.

Next, we will propose the co-design method of event-triggered parameter Ω, primary controller gain K_1 and the secondary one K_2 for the system (8) with $w(t) = 0$ and event-triggered scheme (7).

Theorem 2: For the system (8) without disturbance, and given parameters $\sigma \in [0, 1)$, ρ, if there exist a nonsingular matrix P, symmetric positive-definite matrices $\tilde{R} > 0, \tilde{Z}_1 > 0, \tilde{Z}_2 > 0, \tilde{Q} > 0, \tilde{\Omega} > 0$ and matrices $W_1, W_2, \tilde{X}, \tilde{Y}, \tilde{T}_i, \tilde{X}_{ij}, i = 1, 2, \ldots, 5, j = 1, 2, \ldots, 5$ with appropriate dimensions, such that (18)-(21) hold

$$
\tilde{P}^TE^T = E\tilde{P} \geq 0, \qquad (18)
$$

$$
\begin{bmatrix} \tilde{X} & \tilde{Y} \\ \tilde{Y}^T & \rho E^T\tilde{P}^TE^T+\rho E\tilde{P}E-\rho^2E^T\tilde{Z}_2^TE \end{bmatrix} \geq 0, \qquad (19)
$$

$$
\begin{bmatrix} \tilde{X}_{11} & \tilde{X}_{12} & \tilde{X}_{13} & \tilde{X}_{14} & \tilde{X}_{15} & \tilde{T}_2 \\ * & \tilde{X}_{22} & \tilde{X}_{23} & \tilde{X}_{24} & \tilde{X}_{25} & \tilde{T}_1 \\ * & * & \tilde{X}_{33} & \tilde{X}_{34} & \tilde{X}_{35} & \tilde{T}_4 \\ * & * & * & \tilde{X}_{44} & \tilde{X}_{45} & \tilde{T}_3 \\ * & * & * & * & \tilde{X}_{55} & \tilde{T}_5 \\ * & * & * & * & * & \tilde{R}^T+\tilde{R}-\tilde{Z}_1 \end{bmatrix} > 0, \qquad (20)
$$

$$\begin{bmatrix} \tilde{\Phi}_{11} & \tilde{\Phi}_{12} & \tilde{\Phi}_{13} & \tilde{\Phi}_{14} & \tilde{\Phi}_{15} & \tilde{\Phi}_{16} & \tilde{\Phi}_{17} \\ * & \tilde{\Phi}_{22} & \tilde{\Phi}_{23} & \tilde{\Phi}_{24} & \tilde{\Phi}_{25} & \tilde{\Phi}_{26} & \tilde{\Phi}_{27} \\ * & * & \tilde{\Phi}_{33} & \tilde{\Phi}_{34} & \tilde{\Phi}_{35} & \tilde{\Phi}_{36} & \tilde{\Phi}_{37} \\ * & * & * & \tilde{\Phi}_{44} & \tilde{\Phi}_{45} & \tilde{\Phi}_{46} & \tilde{\Phi}_{47} \\ * & * & * & * & \tilde{\Phi}_{55} & \tilde{\Phi}_{56} & \tilde{\Phi}_{57} \\ * & * & * & * & * & \tilde{\Phi}_{66} & \tilde{\Phi}_{67} \\ * & * & * & * & * & * & \tilde{\Phi}_{77} \end{bmatrix} < 0, \quad (21)$$

where

$$\begin{aligned}
&\tilde{\Phi}_{11} = A_2\tilde{P} + B_2W_2 + \tilde{P}^TA_2^T + W_2^TB_2^T + \tilde{Y} + \\
&\qquad \tilde{Y}^T + \tau\tilde{X} + \tilde{Q} + \tau_M\tilde{X}_{11}, \\
&\tilde{\Phi}_{12} = \tilde{P}^TC_2^TB_1^T + \tilde{T}_2 + \tau_M\tilde{X}_{12}, \\
&\tilde{\Phi}_{22} = A_1\tilde{R} + \tilde{R}^TA_1^T + \tilde{T}_1 + \tilde{T}_1^T + \tau_M\tilde{X}_{22}, \\
&\tilde{\Phi}_{13} = A_3\tilde{P} - \tilde{Y} + \tau_M\tilde{X}_{13}, \Phi_{23} = \tilde{T}_4^T + \tau_M\tilde{X}_{23}, \\
&\tilde{\Phi}_{33} = -\tilde{Q} + \tau_M\tilde{X}_{33}, \\
&\tilde{\Phi}_{14} = B_2W_1 - \tilde{T}_2 + \tau_M\tilde{X}_{14}, \\
&\tilde{\Phi}_{24} = \tilde{T}_3^T - \tilde{T}_1 + \tau_M\tilde{X}_{24}, \Phi_{34} = -\tilde{T}_4 + \tau_M\tilde{X}_{34}, \\
&\tilde{\Phi}_{44} = -\tilde{T}_3 - \tilde{T}_3^T + \sigma\tilde{\Omega} + \tau_M\tilde{X}_{44}, \\
&\tilde{\Phi}_{15} = B_2W_1 + \tau_M\tilde{X}_{15}, \Phi_{25} = \tilde{T}_5^T + \tau_M\tilde{X}_{25}, \\
&\tilde{\Phi}_{35} = \tau_M\tilde{X}_{35}, \Phi_{45} = -\tilde{T}_5^T + \tau_M\tilde{X}_{45}, \\
&\tilde{\Phi}_{55} = -\tilde{\Omega} + \tau_M\tilde{X}_{55}, \\
&\tilde{\Phi}_{16} = \tau_M\tilde{P}^TC_2^TB_1^T, \tilde{\Phi}_{26} = \tau_M\tilde{R}^TA_1^T, \tilde{\Phi}_{36} = 0, \\
&\tilde{\Phi}_{46} = 0, \tilde{\Phi}_{56} = 0, \tilde{\Phi}_{66} = -\tau_M\tilde{Z}_1, \\
&\tilde{\Phi}_{17} = \tau\tilde{P}^TA_2^T + \tau\tilde{W}_2^TB_2^T, \tilde{\Phi}_{27} = 0, \tilde{\Phi}_{37} = \tau\tilde{P}^TA_3^T, \\
&\tilde{\Phi}_{47} = \tau W_1^TB_2^T, \\
&\tilde{\Phi}_{57} = \tau W_1^TB_2^T, \tilde{\Phi}_{67} = 0, \tilde{\Phi}_{77} = -\tau\tilde{Z}_2,
\end{aligned}$$

then, the system (8) without disturbance is admissible. Simultaneously, the corresponding event-triggered parameter is given by $\Omega = \tilde{R}^{-T}\tilde{\Omega}\tilde{R}^{-1}$, the primary controller gain can be obtained as $K_1 = W_1\tilde{R}^{-1}$, and the secondary controller gain can be obtained as $K_2 = W_2\tilde{P}^{-1}$.

Proof: By theorem 1, if there exist a nonsingular matrix P, positive-definite matrices $R > 0, Z_1 > 0, Z_2 > 0, Q > 0, \Omega > 0$ and matrices$X, Y, T_i, X_{ij}, i = 1, 2, \ldots, 5, j = 1, 2, \ldots, 5$ with appropriate dimensions satisfying (9)-(12), the system (8) without disturbance is admissible.

Defining $E = diag\{P^{-T}, R^{-T}, P^{-T}, R^{-T}, R^{-T}, Z_1^{-T}, Z_2^{-T}, \}$, and pre- and post-multiplying (12) by E and its transposition, respectively, and letting $\tilde{P} = P^{-1}$, $\tilde{R} = R^{-1}, W_1 = K_1R^{-1}, W_2 = K_2P^{-1}, \tilde{Z}_1^{-1} = Z_1^{-1}, \tilde{Z}_2^{-1} = Z_2^{-1}, \tilde{X} = P^{-T}XP^{-1}, \tilde{Y} = P^{-T}YP^{-1}, \tilde{Q} = P^{-T}QP^{-1}, \tilde{\Omega} = R^{-T}\Omega R^{-1}, \tilde{T}_1 = R^{-T}T_1R^{-1}, \tilde{T}_2 = P^{-T}T_2R^{-1}, \tilde{T}_3 = R^{-T}T_3R^{-1}, \tilde{T}_4 = P^{-T}T_4R^{-1}, \tilde{T}_5 = R^{-T}T_5R^{-1}$, etc., we can obtain

$$\begin{bmatrix} \tilde{\Phi}_{11} & \tilde{\Phi}_{12} & \tilde{\Phi}_{13} & \tilde{\Phi}_{14} & \tilde{\Phi}_{15} & \tilde{\Phi}_{16} & \tilde{\Phi}_{17} \\ * & \tilde{\Phi}_{22} & \tilde{\Phi}_{23} & \tilde{\Phi}_{24} & \tilde{\Phi}_{25} & \tilde{\Phi}_{26} & \tilde{\Phi}_{27} \\ * & * & \tilde{\Phi}_{33} & \tilde{\Phi}_{34} & \tilde{\Phi}_{35} & \tilde{\Phi}_{36} & \tilde{\Phi}_{37} \\ * & * & * & \tilde{\Phi}_{44} & \tilde{\Phi}_{45} & \tilde{\Phi}_{46} & \tilde{\Phi}_{47} \\ * & * & * & * & \tilde{\Phi}_{55} & \tilde{\Phi}_{56} & \tilde{\Phi}_{57} \\ * & * & * & * & * & \tilde{\Phi}_{66} & \tilde{\Phi}_{67} \\ * & * & * & * & * & * & \tilde{\Phi}_{77} \end{bmatrix} < 0 \quad (22)$$

where

$$\begin{aligned}
\tilde{\Phi}_{11} &= A_2\tilde{P} + B_2W_2 + \tilde{P}^TA_2^T + W_2^TB_2^T + \tilde{Y} \\
&\quad + \tilde{Y}^T + \tau\tilde{X} + \tilde{Q} + \tau_M\tilde{X}_{11}, \\
\tilde{\Phi}_{12} &= \tilde{P}^TC_2^TB_1^T + \tilde{T}_2 + \tau_M\tilde{X}_{12}, \\
\tilde{\Phi}_{22} &= A_1\tilde{R} + \tilde{R}^TA_1^T + \tilde{T}_1 + \tilde{T}_1^T + \tau_M\tilde{X}_{22}, \\
\tilde{\Phi}_{13} &= A_3\tilde{P} - \tilde{Y} + \tau_M\tilde{X}_{13}, \quad \Phi_{23} = \tilde{T}_4^T + \tau_M\tilde{X}_{23}, \\
\tilde{\Phi}_{33} &= -\tilde{Q} + \tau_M\tilde{X}_{33}, \\
\tilde{\Phi}_{14} &= B_2W_1 - \tilde{T}_2 + \tau_M\tilde{X}_{14}, \\
\tilde{\Phi}_{24} &= \tilde{T}_3^T - \tilde{T}_1 + \tau_M\tilde{X}_{24}, \quad \Phi_{34} = -\tilde{T}_4 + \tau_M\tilde{X}_{34}, \\
\tilde{\Phi}_{44} &= -\tilde{T}_3 - \tilde{T}_3^T + \sigma\tilde{\Omega} + \tau_M\tilde{X}_{44}, \\
\tilde{\Phi}_{15} &= B_2W_1 + \tau_M\tilde{X}_{15}, \quad \Phi_{25} = \tilde{T}_5^T + \tau_M\tilde{X}_{25}, \\
\tilde{\Phi}_{35} &= \tau_M\tilde{X}_{35}, \quad \Phi_{45} = -\tilde{T}_5^T + \tau_M\tilde{X}_{45}, \\
\tilde{\Phi}_{55} &= -\tilde{\Omega} + \tau_M\tilde{X}_{55}, \\
\tilde{\Phi}_{16} &= \tau_M\tilde{P}^TC_2^TB_1^T, \quad \tilde{\Phi}_{26} = \tau_M\tilde{R}^TA_1^T, \quad \tilde{\Phi}_{36} = 0, \\
\tilde{\Phi}_{46} &= 0, \quad \tilde{\Phi}_{56} = 0, \ \tilde{\Phi}_{66} = -\tau_M\tilde{Z}_1, \\
\tilde{\Phi}_{17} &= \tau\tilde{P}^TA_2^T + \tau\tilde{W}_2^TB_2^T, \quad \tilde{\Phi}_{27} = 0, \ \tilde{\Phi}_{37} = \tau\tilde{P}^TA_3^T, \\
\tilde{\Phi}_{47} &= \tau W_1^TB_2^T, \\
\tilde{\Phi}_{57} &= \tau W_1^TB_2^T, \quad \tilde{\Phi}_{67} = 0, \quad \tilde{\Phi}_{77} = -\tau\tilde{Z}_2.
\end{aligned}$$

Pre- and post-multiplying (11) by $diag\{P^{-T}, R^{-T}, P^{-T}, R^{-T}, R^{-T}, R^{-T}\}$ and its transposition, respectively, we can obtain

$$\begin{bmatrix} \tilde{X}_{11} & \tilde{X}_{12} & \tilde{X}_{13} & \tilde{X}_{14} & \tilde{X}_{15} & \tilde{T}_2 \\ * & \tilde{X}_{22} & \tilde{X}_{23} & \tilde{X}_{24} & \tilde{X}_{25} & \tilde{T}_1 \\ * & * & \tilde{X}_{33} & \tilde{X}_{34} & \tilde{X}_{35} & \tilde{T}_4 \\ * & * & * & \tilde{X}_{44} & \tilde{X}_{45} & \tilde{T}_3 \\ * & * & * & * & \tilde{X}_{55} & \tilde{T}_5 \\ * & * & * & * & * & \tilde{R}^T\tilde{Z}_1^{-1}\tilde{R} \end{bmatrix} > 0. \quad (23)$$

The inequality (23) is not a strictly LMI because of the existence of nonlinear terms. According to $(\tilde{R} - \tilde{Z}_1)^T\tilde{Z}_1^{-1}(\tilde{R} - \tilde{Z}_1) \geq 0$, the following inequality can be obtained

$$\tilde{R}^T\tilde{Z}_1^{-1}\tilde{R} \geq \tilde{R} + \tilde{R}^T - \tilde{Z}_1. \quad (24)$$

Then, if (20) holds, we can obtain (23).

Pre- and post-multiplying (10) by $diag\{P^{-T}, P^{-T}\}$ and its transposition, respectively, we can obtain

$$\begin{bmatrix} \tilde{X} & \tilde{Y} \\ \tilde{Y}^T & \tilde{P}^TE^T\tilde{Z}_2^{-1}E\tilde{P} \end{bmatrix} \geq 0. \quad (25)$$

According to (18) and Lemma 2, we can obtain

$$\tilde{P}^TE^T\tilde{Z}_2^{-1}E\tilde{P} \geq \rho E^T\tilde{P}^TE^T + \rho E\tilde{P}E - \rho^2E^T\tilde{Z}_2E. \quad (26)$$

Then, if (19) holds, we can obtain (25).

As a result, if (18)-(21) hold, the system (8) without disturbance is admissible. The corresponding event-triggered parameter, primary and secondary controller gains can be obtained as: $\Omega = \tilde{R}^{-T}\tilde{\Omega}\tilde{R}^{-1}$, $K_1 = W_1\tilde{R}^{-1}$, $K_2 = W_2\tilde{P}^{-1}$. This completes the proof.

IV. EVENT-TRIGGERED H_∞ CONTROL FOR SNCCS WITH DISTURBANCE

In this section, according to Theorem 1 and Theorem 2, we will concentrate on the problem of event-triggered H_∞ control for SNCCS (8) with state delay and disturbance $w(t)$, which guarantees the system (8) admissible with H_∞ performance index γ. That is, the following conditions are satisfied.
1) The SNCCS (8) is admissible with $w(t) = 0$.
2) Under zero-initial condition, the following inequality $\|y_1((t)\|_2 \le \gamma\|w(t)\|_2$ holds for all nonzero $w(t) \in L_2[0, \infty)$ and the prescribed scalar $\gamma > 0$.

Next, the corresponding co-design method will be proposed as follows.

Theorem 3: For the system (8), given parameters $\sigma \in [0, 1)$, $\gamma > 0$, and matrices K_1, K_2, the system (8) is admissible with H_∞ performance index γ if there exist a nonsingular matrix P, symmetric positive-definite matrices $R > 0, Z_1 > 0, Z_2 > 0, Q > 0, \Omega > 0$ and matrices $X, Y, T_i, X_{ij}, i = 1, 2, \ldots, 6, j = 1, 2, \ldots, 6$ with appropriate dimensions, such that (27)-(30) hold

$$E^TP = P^TE \ge 0, \tag{27}$$

$$\begin{bmatrix} X & Y \\ Y^T & E^TZ_2E \end{bmatrix} \ge 0, \tag{28}$$

$$\tilde{\Pi} = \begin{bmatrix} X_{11} & X_{12} & X_{13} & X_{14} & X_{15} & X_{16} & T_2 \\ * & X_{22} & X_{23} & X_{24} & X_{25} & X_{26} & T_1 \\ * & * & X_{33} & X_{34} & X_{35} & X_{36} & T_4 \\ * & * & * & X_{44} & X_{45} & X_{46} & T_3 \\ * & * & * & * & X_{55} & X_{56} & T_5 \\ * & * & * & * & * & X_{66} & T_6 \\ * & * & * & * & * & * & Z_1 \end{bmatrix} > 0, \tag{29}$$

$$\bar{\Phi} = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \cdots & \Phi_{19} \\ * & \Phi_{22} & \cdots & \Phi_{29} \\ * & * & \vdots & \vdots \\ * & * & * & \Phi_{99} \end{bmatrix} < 0, \tag{30}$$

where

$$\begin{aligned}
\Phi_{11} &= P^T(A_2 + B_2K_2) + (A_2 + B_2K_2)^TP + Y \\
&\quad + Y^T + \tau X + Q + \tau_M X_{11}, \\
\Phi_{12} &= C_2^TB_1^TR + T_2 + \tau_M X_{12}, \\
\Phi_{22} &= R^TA_1 + A_1^TR + T_1 + T_1^T + \tau_M X_{22}, \\
\Phi_{13} &= P^TA_3 - Y + \tau_M X_{13}, \quad \Phi_{23} = T_4^T + \tau_M X_{23}, \\
\Phi_{33} &= -Q + \tau_M X_{33}, \\
\Phi_{14} &= P^TB_2K_1 - T_2 + \tau_M X_{14}, \\
\Phi_{24} &= T_3^T - T_1 + \tau_M X_{24}, \quad \Phi_{34} = -T_4 + \tau_M X_{34}, \\
\Phi_{44} &= -T_3 - T_3^T + \sigma\Omega + \tau_M X_{44}, \\
\Phi_{15} &= P^TB_2K_1 + \tau_M X_{15}, \quad \Phi_{25} = T_5^T + \tau_M X_{25}, \\
\Phi_{35} &= \tau_M X_{35}, \quad \Phi_{45} = -T_5^T + \tau_M X_{45}, \\
\Phi_{55} &= -\Omega + \tau_M X_{55}, \\
\Phi_{16} &= P^TB_3 + \tau_M X_{16}, \\
\Phi_{26} &= RB_1C_4 + T_6^T + \tau_M X_{26}, \quad \Phi_{36} = \tau_M X_{36}, \\
\Phi_{46} &= -T_6^T + \tau_M X_{46}, \quad \Phi_{56} = \tau_M X_{56}, \\
\Phi_{66} &= -\gamma^2 I + \tau_M X_{66}, \quad \Phi_{17} = \tau_M C_2^TB_1^TZ_1, \\
\Phi_{27} &= \tau_M A_1^TZ_1, \quad \Phi_{37} = 0, \ \Phi_{47} = 0, \ \Phi_{57} = 0, \\
\Phi_{67} &= \tau_M C_4^TB_1^TZ_1, \quad \Phi_{77} = -\tau_M Z_1, \\
\Phi_{18} &= \tau(A_2 + B_2K_2)^TZ_2, \quad \Phi_{28} = 0, \ \Phi_{38} = \tau A_3^TZ_2, \\
\Phi_{48} &= \tau(B_2K_1)^TZ_2, \quad \Phi_{58} = \tau(B_2K_1)^TZ_2, \\
\Phi_{68} &= \tau B_3^TZ_2, \quad \Phi_{78} = 0, \ \Phi_{88} = -\tau Z_2, \\
\Phi_{19} &= 0, \quad \Phi_{29} = C_1^T, \ \Phi_{39} = 0, \ \Phi_{49} = 0, \ \Phi_{59} = 0, \\
\Phi_{69} &= C_3^T, \quad \Phi_{79} = 0, \ \Phi_{89} = 0, \ \Phi_{99} = -I.
\end{aligned}$$

Proof: According to Theorem 1, the system (8) with $w(t) = 0$ is admissible if (9)-(12) hold. Noting that for any matrix $\hat{X}$ with appropriate dimension, the following equation holds

$$\tau(t)\xi^T(t)\hat{X}\xi(t) - \int_{t-\tau(t)}^{t} \xi^T(t)\hat{X}\xi(t)d\alpha = 0 \tag{31}$$

where $\xi(t) = [x_2^T(t) \ \ x_1^T(t) \ \ x_2^T(t-\tau) \ \ x_1^T(t-\tau(t)) \ \ e_k^T(t) \ \ w^T(t)]^T$.

Choosing a Lyapunov-Krasovskii functional as (14), taking the derivative of $V(t)$ along the solution of (8), and for any appropriately dimensioned matrices $T_i, i = 1, 2, \ldots, 6$, we can obtain

$$\begin{aligned}
\dot{V}(t) &= \sum_{i=1}^{5} \dot{V}_i(t) + 2[x_1(t)T_1 + x_2(t)T_2 + x_1(t-\tau(t))T_3 \\
&\quad + x_2(t-\tau)T_4 + e_k(t)T_5 + w^T(t)T_6] \\
&\quad \times [x_1(t) - x_1(t-\tau(t)) - \int_{t-\tau(t)}^{t} \dot{x}_1(\alpha)d\alpha] \\
&\quad + \tau(t)\xi^T(t)\hat{X}\xi(t) - \int_{t-\tau(t)}^{t} \xi^T(t)\hat{X}\xi(t)d\alpha \\
&\le \xi^T(t)\Omega\xi(t) - \int_{t-\tau(t)}^{t} \xi_2^T(t,\alpha)\Pi\xi_2(t,\alpha)d\alpha
\end{aligned} \tag{32}$$

where $\xi_2(t,\alpha) = [x_2^T(t) \ \ x_1^T(t) \ \ x_2^T(t-\tau) \ \ x_1^T(t-\tau(t)) \ \ e_k^T(t) \ \ w^T(t) \ \ \dot{x}_1^T(\alpha)]^T$.

Define a functional as:

$$J = \int_0^\infty [y_1^T(t)y_1(t) - \gamma^2 w^T(k)w(t)]dt \tag{33}$$

for the system (8) with disturbance $w(t)$.

Assuming $y_1^T(t)y_1(t) - \gamma^2 w^T(t)w(t) + \dot{V}(t) = \xi_t^T\Upsilon\xi_t$, we can obtain $J \le \int_0^\infty \xi_t^T\Upsilon\xi_t dt$ and $J \le 0$ for all nonzero ξ_t if $\Upsilon < 0$.

Combining (31), (32), we can obtain

$$
\begin{aligned}
J &= \int_0^{\infty} [y_1^T(t)y_1(t) - \gamma^2 w^T(t)w(t) + \dot{V}(t)]dt \\
&\quad - \int_0^{\infty} \dot{V}(t)dt \\
&= \int_0^{\infty} [\xi^T(t)\tilde{\Phi}\xi(t) - \int_{t-\tau(t)}^{t} \xi_2^T(t,\alpha)\tilde{\Pi}\xi_2(t,\alpha)d\alpha]dt \\
&\quad - V(\infty) + V(0)
\end{aligned} \tag{34}
$$

In (34), $\tilde{\Pi} > 0$ is described as (29), and $\tilde{\Phi} < 0$ can be written as:

$$
\tilde{\Phi} = \begin{bmatrix}
\Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & \Phi_{15} & \Phi_{16} \\
* & \Phi_{22} & \Phi_{23} & \Phi_{24} & \Phi_{25} & \Phi_{26} \\
* & * & \Phi_{33} & \Phi_{34} & \Phi_{35} & \Phi_{36} \\
* & * & * & \Phi_{44} & \Phi_{45} & \Phi_{46} \\
* & * & * & * & \Phi_{55} & \Phi_{56} \\
* & * & * & * & * & \Phi_{66}
\end{bmatrix} < 0,
$$

therein

$$
\begin{aligned}
\Phi_{11} &= P^T(A_2 + B_2K_2) + (A_2 + B_2K_2)^T P + Y \\
&\quad + Y^T + \tau X + Q + \tau_M C_2^T B_1^T Z_1 B_1 C_2 \\
&\quad + \tau(A_2 + B_2K_2)^T Z_2(A_2 + B_2K_2) + \tau_M X_{11}, \\
\Phi_{12} &= C_2^T B_1^T R + T_2 + \tau_M C_2^T B_1^T Z_1 A_1 + \tau_M X_{12}, \\
\Phi_{22} &= R^T A_1 + A_1^T R + T_1 + T_1^T + \tau_M A_1^T Z_1 A_1 \\
&\quad + \tau_M X_{22} + C_1^T C_1, \\
\Phi_{13} &= P^T A_3 - Y + \tau(A_2 + B_2K_2)^T Z_2 A_3 + \tau_M X_{13}, \\
\Phi_{23} &= T_4^T + \tau_M X_{23}, \\
\Phi_{33} &= -Q + \tau A_3^T Z_2 A_3 + \tau_M X_{33}, \\
\Phi_{14} &= P^T B_2 K_1 - T_2 + \tau(A_2 + B_2K_2)^T Z_2 B_2 K_1 \\
&\quad + \tau_M X_{14}, \\
\Phi_{24} &= T_3^T - T_1 + \tau_M X_{24}, \\
\Phi_{34} &= -T_4 + \tau A_3^T Z_2 B_2 K_1 + \tau_M X_{34}, \\
\Phi_{44} &= -T_3 - T_3^T + \sigma\Omega + \tau(B_2K_1)^T Z_2 B_2 K_1 + \tau_M X_{44}, \\
\Phi_{15} &= P^T B_2 K_1 + \tau(A_2 + B_2K_2)^T Z_2 B_2 K_1 + \tau_M X_{15}, \\
\Phi_{25} &= T_5^T + \tau_M X_{25}, \\
\Phi_{35} &= \tau A_3^T Z_2 B_2 K_1 + \tau_M X_{35}, \\
\Phi_{45} &= -T_5^T + \tau(B_2K_1)^T Z_2 B_2 K_1 + \tau_M X_{45}, \\
\Phi_{55} &= -\Omega + \tau(B_2K_1)^T Z_2 B_2 K_1 + \tau_M X_{55}, \\
\Phi_{16} &= P^T B_3 + \tau_M C_2^T B_1^T Z_1 B_1 C_4 \\
&\quad + \tau(A_2 + B_2K_2)^T Z_2 B_3, \\
\Phi_{26} &= R B_1 C_4 + \tau_M A_1^T Z_1 B_1 C_4 + T_6^T + C_1^T C_3, \\
\Phi_{36} &= \tau A_3^T Z_2 B_3, \quad \Phi_{46} = \tau K_1^T B_2^T Z_2 B_3 - T_6^T, \\
\Phi_{56} &= \tau K_1^T B_2^T Z_2 B_3, \\
\Phi_{66} &= \tau_M C_4^T B_1^T Z_1 B_1 C_4 + C_3^T C_3 - \gamma^2 I + \tau B_3^T Z_2 B_3.
\end{aligned}
$$

Using Schur complement, $\tilde{\Phi} < 0$ is equivalent to (30).

It follows if the inequalities $\tilde{\Phi} < 0$, $\tilde{\Pi} > 0$ hold, then $\int_0^{\infty}[y_1^T(t)y_1(t) - \gamma^2 w^T(t)w(t) + \dot{V}(t)]dt < 0$ for all nonzero $\xi(t)$, $\xi_2(t,\alpha)$.

Since $\tilde{\Phi} < 0$, $\tilde{\Pi} > 0$, $V(0) = 0$ under zero initial condition and $\lim_{t\to\infty} V(t) \geq 0$, the following LMI $J < 0$ holds, this gives the desired result

$$
\|y_1(t)\|_2 \leq \gamma \|w(t)\|_2. \tag{35}
$$

That implies the system (8) has H_∞ performance index γ. Similar to the proof of Theorem 1, we can obtain (27)-(30). Moreover, (27)-(30) also imply that the system (8) with $w(t) = 0$ is admissible. This completes the proof.

Next, the co-design problems of the event-triggered parameter Ω, the primary controller gain K_1, and the secondary one K_2 for the system (8) will be considered.

Theorem 4: For given parameters $\sigma \in [0, 1)$, $\rho, \gamma > 0$, if there exist a nonsingular matrix $\hat{P}$, symmetric positive-definite matrices $\hat{R} > 0, \hat{Z}_1 > 0, \hat{Z}_2 > 0, \hat{Q} > 0, \hat{\Omega} > 0$ and matrices $W_1, W_2, \hat{X}, \hat{Y}, \hat{T}_i, \hat{X}_{ij}, i = 1, 2, \ldots, 6, j = 1, 2, \ldots, 6$ with appropriate dimensions, such that (36)-(39) hold

$$
\hat{P}^T E^T = E\hat{P} \geq 0, \tag{36}
$$

$$
\begin{bmatrix}
\hat{X} & \hat{Y} \\
\hat{Y}^T & \rho E^T \hat{P}^T E^T + \rho E \hat{P} E - \rho^2 E^T \hat{Z}_2 E
\end{bmatrix} \geq 0, \tag{37}
$$

$$
\hat{\Pi} = \begin{bmatrix}
\hat{X}_{11} & \hat{X}_{12} & \hat{X}_{13} & \hat{X}_{14} & \hat{X}_{15} & \hat{X}_{16} & \hat{T}_2 \\
* & \hat{X}_{22} & \hat{X}_{23} & \hat{X}_{24} & \hat{X}_{25} & \hat{X}_{26} & \hat{T}_1 \\
* & * & \hat{X}_{33} & \hat{X}_{34} & \hat{X}_{35} & \hat{X}_{36} & \hat{T}_4 \\
* & * & * & \hat{X}_{44} & \hat{X}_{45} & \hat{X}_{46} & \hat{T}_3 \\
* & * & * & * & \hat{X}_{55} & \hat{X}_{56} & \hat{T}_5 \\
* & * & * & * & * & \hat{X}_{66} & \hat{T}_6 \\
* & * & * & * & * & * & \Psi
\end{bmatrix} > 0, \tag{38}
$$

$$
\hat{\Phi} = \begin{bmatrix}
\hat{\Phi}_{11} & \hat{\Phi}_{12} & \cdots & \hat{\Phi}_{19} \\
* & \hat{\Phi}_{22} & \cdots & \hat{\Phi}_{29} \\
* & * & \vdots & \vdots \\
* & * & * & \hat{\Phi}_{99}
\end{bmatrix} < 0, \tag{39}
$$

where

$$
\begin{aligned}
\Psi &= \hat{R}^T + \hat{R} - \hat{Z}_1, \\
\hat{\Phi}_{11} &= A_2\hat{P} + B_2W_2 + \hat{P}^T A_2^T + W_2^T B_2^T + \hat{Y} \\
&\quad + \hat{Y}^T + \tau\hat{X} + \hat{Q} + \tau_M \hat{X}_{11}, \\
\hat{\Phi}_{12} &= \hat{P}^T C_2^T B_1^T + \hat{T}_2 + \tau_M \hat{X}_{12}, \\
\hat{\Phi}_{22} &= A_1\hat{R} + \hat{R}^T A_1^T + \hat{T}_1 + \hat{T}_1^T + \tau_M \hat{X}_{22}, \\
\hat{\Phi}_{13} &= A_3\hat{P} - \hat{Y} + \tau_M \hat{X}_{13}, \quad \hat{\Phi}_{23} = \hat{T}_4^T + \tau_M \hat{X}_{23}, \\
\hat{\Phi}_{33} &= -\hat{Q} + \tau_M \hat{X}_{33}, \\
\hat{\Phi}_{14} &= B_2W_1 - \hat{T}_2 + \tau_M \hat{X}_{14}, \\
\hat{\Phi}_{24} &= \hat{T}_3^T - \hat{T}_1 + \tau_M \hat{X}_{24}, \quad \hat{\Phi}_{34} = -\hat{T}_4 + \tau_M \hat{X}_{34}, \\
\hat{\Phi}_{44} &= -\hat{T}_3 - \hat{T}_3^T + \sigma\hat{\Omega} + \tau_M \hat{X}_{44}, \\
\hat{\Phi}_{15} &= B_2W_1 + \tau_M \hat{X}_{15}, \quad \hat{\Phi}_{25} = \hat{T}_5^T + \tau_M \hat{X}_{25}, \\
\hat{\Phi}_{35} &= \tau_M \hat{X}_{35}, \quad \hat{\Phi}_{45} = -\hat{T}_5^T + \tau_M \hat{X}_{45}, \\
\hat{\Phi}_{55} &= -\hat{\Omega} + \tau_M \hat{X}_{55}, \\
\hat{\Phi}_{16} &= B_3 + \tau_M \hat{X}_{16}, \quad \hat{\Phi}_{26} = B_1C_4 + \hat{T}_6^T + \tau_M \hat{X}_{26}, \\
\hat{\Phi}_{36} &= \tau_M \hat{X}_{36}, \quad \hat{\Phi}_{46} = -\hat{T}_6^T + \tau_M \hat{X}_{46}, \hat{\Phi}_{56} = \tau_M \hat{X}_{56},
\end{aligned}
$$

$$\hat{\Phi}_{66} = -\gamma^2 I + \tau_M \hat{X}_{66}, \quad \hat{\Phi}_{17} = \tau_M \hat{P}^T C_2^T B_1^T,$$
$$\hat{\Phi}_{27} = \tau_M \hat{R}^T A_1^T, \quad \hat{\Phi}_{37} = 0, \hat{\Phi}_{47} = 0, \hat{\Phi}_{57} = 0,$$
$$\hat{\Phi}_{67} = \tau_M C_4^T B_1^T, \quad \hat{\Phi}_{77} = -\tau_M \hat{Z}_1,$$
$$\hat{\Phi}_{18} = \tau(\hat{P}^T A_2^T + W_2^T B_2^T), \quad \hat{\Phi}_{28} = 0, \hat{\Phi}_{38} = \tau \hat{P}^T A_3^T,$$
$$\hat{\Phi}_{48} = \tau W_1^T B_2^T, \hat{\Phi}_{58} = \tau W_1^T B_2^T, \hat{\Phi}_{68} = \tau B_3^T,$$
$$\hat{\Phi}_{78} = 0, \quad \hat{\Phi}_{88} = -\tau \hat{Z}_2, \quad \hat{\Phi}_{19} = 0, \hat{\Phi}_{29} = \hat{R}^T C_1^T,$$
$$\hat{\Phi}_{39} = 0, \quad \hat{\Phi}_{49} = 0, \hat{\Phi}_{59} = 0, \hat{\Phi}_{69} = C_3^T, \hat{\Phi}_{79} = 0,$$
$$\hat{\Phi}_{89} = 0, \quad \hat{\Phi}_{99} = -I.$$

then the system (8) is admissible with H_∞ performance index γ. Simultaneously, the desired event-triggered parameter is $\Omega = \hat{R}^{-T}\hat{\Omega}\hat{R}^{-1}$, the primary controller gain is $K_1 = W_1\hat{R}^{-1}$, and the secondary one is $K_2 = W_2\hat{P}^{-1}$.

Proof: Similar to Theorem 2, if there exist a nonsingular matrix P, positive-definite matrices $R > 0, Z_1 > 0, Z_2 > 0, Q > 0, \Omega > 0$ and matrices $X, Y, T_i, X_{ij}, i = 1, 2, \ldots, 6, j = 1, 2, \ldots, 6$ satisfying (27)-(30), the system (8) is admissible with $\gamma > 0$.

Letting $E = diag\{P^{-T}, R^{-T}, P^{-T}, R^{-T}, R^{-T}, I, Z_1^{-T}, Z_2^{-T}, I\}$, pre- and post-multiplying (30) by E and its transpose, respectively, and applying Lemma 1, Theorem 3, and defining $\hat{P} = P^{-1}, \hat{R} = R^{-1}, W_1 = K_1R^{-1}, W_2 = K_2P^{-1}, \hat{Z}_1 = Z_1^{-1}, \hat{Z}_2 = Z_2^{-1}, \hat{X} = P^{-T}XP^{-1}, \hat{Y} = P^{-T}YP^{-1}, \hat{Q} = P^{-T}QP^{-1}, \hat{\Omega} = R^{-T}\Omega R^{-1}, \hat{T}_1 = R^{-T}T_1R^{-1}, \hat{T}_2 = P^{-T}T_2R^{-1}, \hat{T}_3 = R^{-T}T_3R^{-1}, \hat{T}_4 = P^{-T}T_4R^{-1}, \hat{T}_5 = R^{-T}T_5R^{-1}, \hat{T}_6 = T_6R^{-1}$, etc., $\bar{\Phi} < 0$ is equivalent to (39).

Letting $F = diag\{P^{-T}, R^{-T}, P^{-T}, R^{-T}, R^{-T}, I, R^{-T}\}$, and pre- and post-multiplying (29) by F and its transpose, respectively, we can obtain (38). Similar to Theorem 2, we can also obtain (36), (37). As a result, if (36)-(39) hold, the system (8) is admissible with $\gamma > 0$. The corresponding event-triggered parameter, the primary and secondary controller gains can be obtained as: $\Omega = \hat{R}^{-T}\hat{\Omega}\hat{R}^{-1}$, $K_1 = W_1\hat{R}^{-1}, K_2 = W_2\hat{P}^{-1}$. This completes the proof.

Remark 3: Compared with those systems, such as NCS, singular system, singular NCS, with event-triggered control, it is noticed that the methods in those systems are not feasible for SNCCS, and the existing results are difficult to be extended to SNCCS.

Furthermore, we can see that this system is more general than differential system, and has the advantages of cascade control, event-triggered control and networked control. According to the obtained result, it can be easily applied to the practical industrial control systems.

V. SIMULATION EXAMPLE

In this section, a simulation example of SNCCS with event-triggered scheme is provided to illustrate the effectiveness of the obtained results.

Considering the power transmission system as described in [11] and [12], the heating furnace could be an important part of the steam generation process. Proper control of the heated material temperature is extremely important to ensure

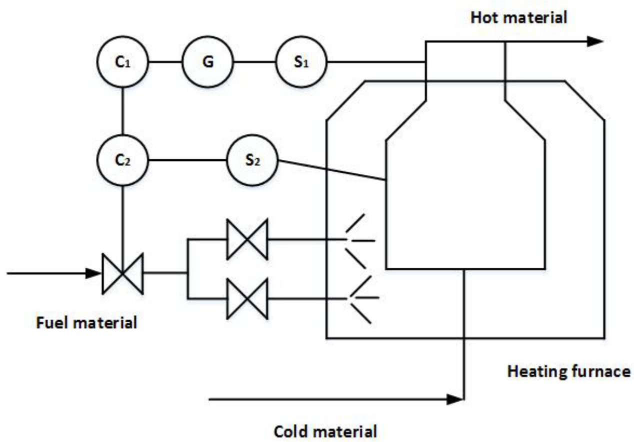

FIGURE 2. **Schematic diagram of SNCCS for heating furnace with event generator.**

the security and effectiveness of the overall system. Cascade control is an effective mean for the temperature control of the heated material.

In the process, the primary sensor S_1 and secondary one S_2 are associated with the furnace, and transform the information of the primary plant and the secondary one, respectively. The disturbance exists mainly in the inner loop which can be suppressed rapidly. The information of the primary/secondary plants includes the temperature, disturbance and so on, of the furnace mouth/hearth. The secondary controller C_2 can control the valve of the furnace, thus the quantity of the fuel material can be modulated. G denotes the event generator. The event generator G, S_1, the primary controller C_1, and C_2 are connected by network where the network-induced delay phenomena may appear in the transmission.

Figure 2 is the schematic diagram of SNCCS for heating furnace with event generator and can be transformed to Figure 1. Assume that the state and disturbance of the secondary plant have algebraic relation, the representation of the secondary plant can be described by (1) with the following parameters:

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1.3 & 1 \\ 0.2 & 0 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} 0.2 & 0.1 \\ 0.2 & 1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.2 \\ 1 \end{bmatrix}, B_3 = \begin{bmatrix} -0.4 \\ 0.1 \end{bmatrix},$$
$$C_4 = 0.1, C_2 = \begin{bmatrix} -0.3 & 0.1 \end{bmatrix}.$$

The representation of the primary plant is described by (2) with

$$A_1 = \begin{bmatrix} -1 & 0 \\ -1 & -2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}, C_1 = \begin{bmatrix} 0 & 0.1 \end{bmatrix}$$
$$C_3 = 0.2.$$

Next, assuming the sampling period $h = 0.1s$, we will co-design the event-triggered parameter Ω, the primary and secondary controller gains K_1, K_2 for (1), (2) without/with $w(t)$ according to Theorem 2/Theorem 4 proposed above by LMI Toolbox.

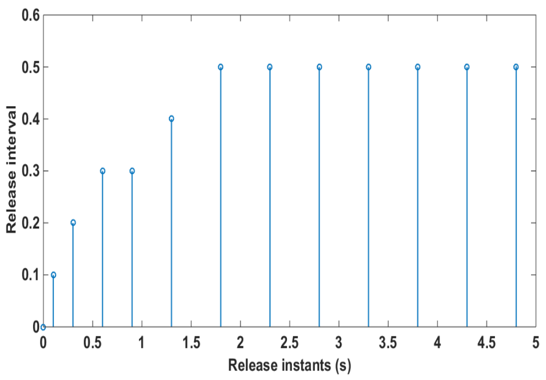

FIGURE 3. The release instants and interval with K_1, K_2 and Ω in (18-21).

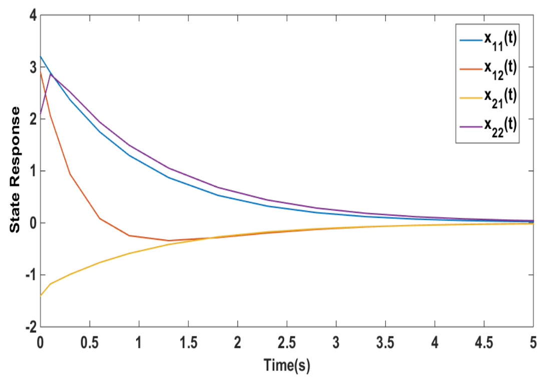

FIGURE 4. The state response of system with feedback gains in (18-21).

A. FOR SNCCS WITHOUT DISTURBANCE

According to Theorem 2, the state feedback stabilization controllers and the event-triggered parameter for system (1), (2) without disturbance will be co-designed with $\sigma = 0.3$, $\tau = 0.1$, $\rho = 1$, and $\tau_M = 0.2$. Some matrices can be obtained as follows:

$$\tilde{P} = \begin{bmatrix} 170.2611 & 0 \\ -408.0562 & 42.4675 \end{bmatrix},$$
$$\tilde{R} = \begin{bmatrix} 299.9353 & -73.4348 \\ -73.4348 & 215.7686 \end{bmatrix},$$
$$\tilde{\Omega} = \begin{bmatrix} 213.7686 & -8.0909 \\ -8.0909 & 209.1496 \end{bmatrix}, \quad \tilde{Y} = \begin{bmatrix} -70.1697 & 0 \\ -147.5125 & 0 \end{bmatrix},$$
$$\tilde{Z}_1 = \begin{bmatrix} 386.8310 & -106.7157 \\ -106.7157 & 256.8347 \end{bmatrix},$$
$$\tilde{Z}_2 = 10^3 \times \begin{bmatrix} 0.1766 & -0.0204 \\ -0.0204 & 2.0193 \end{bmatrix},$$
$$W_1 = \begin{bmatrix} 0.4926 & 0.1535 \end{bmatrix},$$
$$W_2 = \begin{bmatrix} 172.7256 & -382.9190 \end{bmatrix},$$

For simplicity, the other obtained matrices are omitted. Thus, the event-triggered parameter is obtained as:

$$\Omega = \tilde{R}^{-T}\tilde{\Omega}\tilde{R}^{-1} = \begin{bmatrix} 0.0031 & 0.0021 \\ 0.0021 & 0.0056 \end{bmatrix},$$

and the primary and secondary controller gains are obtained as:

$$K_1 = W_1\tilde{R}^{-1} = \begin{bmatrix} 0.0020 & 0.0014 \end{bmatrix},$$
$$K_2 = W_2\tilde{P}^{-1} = \begin{bmatrix} -20.5955 & -9.0168 \end{bmatrix}.$$

If the initial conditions are:

$$x_1(0) = \begin{bmatrix} 3.2 & 2.9 \end{bmatrix}^T, \quad x_2(0) = \begin{bmatrix} -1.4 & 2.1 \end{bmatrix}^T,$$

the release instants and release interval are plotted as Figure 3, and Figure 4 is the state response of this system. The simulation results for $t \in [0, 5]s$ show that the maximum release interval is 0.5s, only 13 signals need to be sent out to the controller, and the last transmitted signal is at the instant of $t = 4.8$s, it can be computed that the event generator leads to an average release period of 0.3692s. That is, only 26% of the sampling signals need to be sent to the primary controller.

As can be seen clearly, the SNCCS without disturbance is admissible, and the states of this system converge to zeroes. At the same time, the average release period 0.3692s is much larger than the fixed sampling period 0.1s, which means that only less signals need to be transmitted in the network.

B. FOR SNCCS WITH DISTURBANCE

It is assumed that the disturbance can be written as follows:

$$w(t) = \begin{cases} sin(10t) & 0 < t \leq 3 \\ 0 & otherwise \end{cases}$$

The primary and secondary controllers, and the event-triggered parameter for (1), (2) with disturbance will be co-designed according to Theorem 4 proposed above.

Setting $\sigma = 0.3$, $\gamma = 3$, $\tau = 0.1$, $\rho = 1$, and $\tau_M = 0.2$, some of the obtained matrices can be described as follows:

$$\hat{P} = \begin{bmatrix} 8.1769 & 0 \\ -20.3035 & 2.5457 \end{bmatrix},$$
$$\hat{R} = \begin{bmatrix} 16.3671 & -3.3604 \\ -3.3604 & 11.9928 \end{bmatrix},$$
$$\hat{\Omega} = \begin{bmatrix} 10.6987 & -0.3873 \\ -0.3873 & 10.4804 \end{bmatrix}, \quad \hat{Y} = \begin{bmatrix} -3.7463 & 0 \\ -4.2249 & 0 \end{bmatrix},$$
$$\hat{Z}_1 = \begin{bmatrix} 21.5729 & -4.5399 \\ -4.5399 & 15.2025 \end{bmatrix},$$
$$\hat{Z}_2 = \begin{bmatrix} 11.8141 & -1.1028 \\ -1.1028 & 104.3935 \end{bmatrix},$$
$$W_1 = \begin{bmatrix} 0.0273 & -0.0026 \end{bmatrix},$$
$$W_2 = \begin{bmatrix} 6.6206 & -21.9869 \end{bmatrix}.$$

Therefore, the event-triggered parameter is obtained as:

$$\Omega = \hat{R}^{-T}\hat{\Omega}\hat{R}^{-1} = \begin{bmatrix} 0.0475 & 0.0271 \\ 0.0271 & 0.0843 \end{bmatrix},$$

the primary and secondary controller gains can be obtained as:

$$K_1 = W_1\hat{R}^{-1} = \begin{bmatrix} 0.0017 & 0.0003 \end{bmatrix},$$
$$K_2 = W_2\hat{P}^{-1} = \begin{bmatrix} -20.6356 & -8.6367 \end{bmatrix}.$$

If the initial conditions are:

$$x_1(0) = \begin{bmatrix} 3.2 & 2.9 \end{bmatrix}^T, \quad x_2(0) = \begin{bmatrix} -1.4 & -2.2 \end{bmatrix}^T,$$

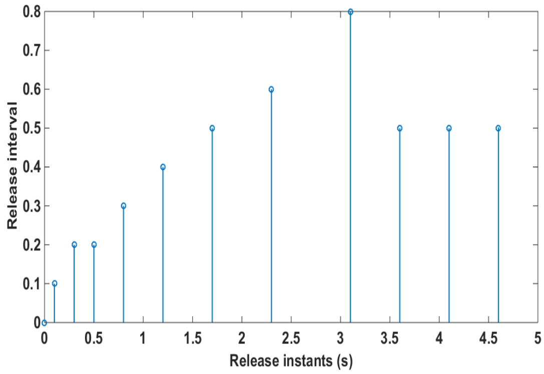

FIGURE 5. The release instants and interval with K_1, K_2 and Ω in (36-39).

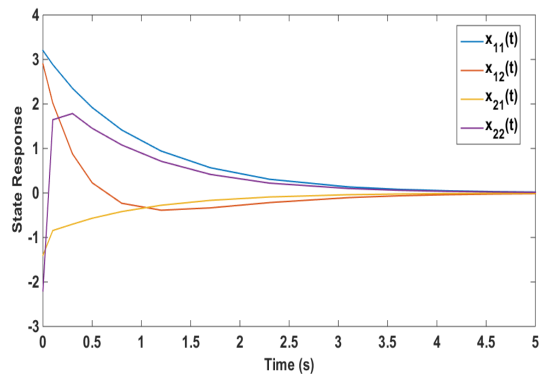

FIGURE 6. The state response of system with feedback gains in (36-39).

Figure 5 plots the release instants and release interval. The simulation results for $t \in [0, 5]s$ show that only 12 signals need to be sent out to the controller, the maximum release interval is 0.8s, and the last transmitted signal is at the instant of $t = 4.6$s, it can be computed that the event generator leads to an average release period of 0.3833s. That is, only 24% of the sampling signals will be sent to the controller.

On the other hand, the response of this system can be presented as Figure 6. It can be seen clearly that the system with event generator and disturbance is admissible with H_∞ performance index $\gamma = 3$, and the states of SNCCS converge to zeroes. Simultaneously, the average release period 0.3833s is much larger than the sampling period 0.1s, which means the effectiveness and improvement of the obtained results.

VI. CONCLUSION

In this paper, the co-design problem of the event-triggered parameter, the primary controller and the secondary one is mainly considered for SNCCS with event-triggered control which is firstly introduced into this system. First, a new model is constructed which assures whether the sampling signal need to be sent to the primary controller. Based on Lyapunov stability theory, sufficient condition of admissible for this system is presented, and the co-design method of the primary and secondary controllers, and the event-triggered parameter is also proposed. Then, the problem of H_∞ control for SNCCS is also concerned. Finally, a simulation example considering a heating furnace temperature control system with cascade control and event-triggered control is given to illustrate the effectiveness and applicability of the proposed method. Future work will address the control issues of SNCCS with two or more event triggers.

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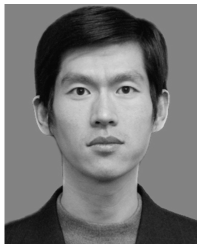

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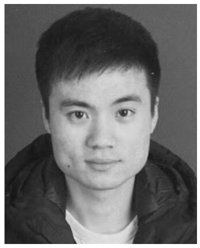

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