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Identification of Underwater Targets Based on Sparse Representation

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ABSTRACT We consider using sparse representations to identify underwater targets, since underwater acoustic signal have sparse characteristics. We consider the identification problem as one of the identifying among multiple linear regression models and believe that the new theory from sparse signal representation provides the key to solving this problem. Based on a sparse representation computed by $\ell^1 - minimization$, we propose a general classification algorithm for (hydroacoustic signal-based) targets identification. This new framework provides new insights into identifying two key issues in underwater targets: feature extraction and robustness of signal loss and noise interference. For feature extraction, we point out that feature extraction is no longer critical if the sparseness of the underwater acoustic signal is properly utilized. The critical is whether the number of features is large enough and whether the sparse representation is correctly computed. This framework can handle errors due to signal loss and noise interference uniformly by exploiting the fact that these errors are often sparse with respect to the standard (hydroacoustic signal) basis. Extensive experiments have been conducted based on a public underwater acoustic signal sampling set to verify the efficacy of the proposed algorithm and corroborate the above claims.

INDEX TERMS Identification of underwater targets, sparse representation, $\ell^1 - norm$, compressed sensing.

I. INTRODUCTION

Progress of underwater sensors and other equipment enable intelligent identification technology of underwater targets to be widely applied in many fields, primarily involving military invasion monitoring [1], exploitation of seabed resources [2], positioning and protection of fish [3] and so on. Due to the complicated underwater environment (*e.g.*, Acoustic medium constraints and heterogeneity), the diversity of target identification and the difficulty in obtaining targeted underwater signal (*e.g.*, Data of sensitive equipment such as military vessels) [4], it is more difficult to intelligently recognize underwater objects than other objects. At present, the main research method for underwater targets identification is the statistical identification method based on the theory of hydroacoustic signal and information processing [5].The low-dimensional observations of sparse representation must contain most of the useful information of the original signal. Most signal in nature are sparse to some extent, and hydroacoustic signal are no exception. If the sparse basis of the hydroacoustic signal can be constructed, the compressed sensing can be applied to underwater signal processing, which can reduce the cost of signal processing, improve the compression efficiency, enhance the anti-noise performance of the identification system as well as the robustness.

In the field of statistical signal, the sparse linear expression for computing a super complete dictionary with basic elements or signal atoms has recently attracted many scholars' attentions [6]–[9]. and the researches maily focus on the following aspects: when the basic elements or signal are sparse enough, the sparse representation can be effectively calculated by convex optimization [6], although commonly it may be very difficult. In order to solve such problems, the coefficients in linear combination are dealt with in paper [6] and [10], instead of solving the problem of number of non-zero coefficients(*i.e.*, $\ell^0 - norm$).

The initial purpose of sparse representation algorithm is not to identify or classify, but to make the representation

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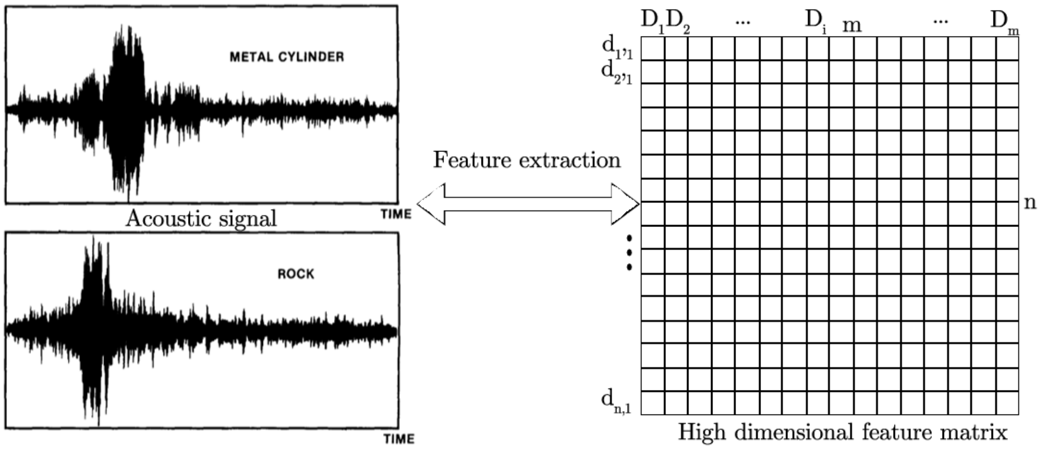


FIGURE 1. Hydroacoustic signal are converted into high-dimensional feature matrices by feature extraction, which increases the processing time of classifier.

and compression of signal have a lower sampling rate than Shannon-Nyquist. Therefore, the performance of the algorithm needs to be measured according to the sparsity of the representation and the fidelity of the original signal. The basic elements in a data model-based dictionary do not have any specific semantics. They are usually selected from standard bases (*e.g.*, Fourier, wavelet, discrete cosine transform and Gabor) or generated from random matrices [11]. However, the sparse representation is naturally different: in all basic quantum sets, it chooses the most accurate input signal and abandons the inaccurate sparse representation.

The function of feature extraction. In fact, the feature extraction technology converts the original hydroacoustic signal into a high dimensional vector, thus causes the problem of ''dimension disaster'', also it can destroy the data structure of the hydroacoustic signal, ignores the associative differences and similarities of hydroacoustic signal of different modal (dimension) (See **Section III-B3**), and dimensions disaster is accompanied by small samples [12], both will reduce the application performance of intelligent identification of underwater objects [13].A large amount of studies focus on various feature transformations based on data, which project high-dimensional test samples into low-dimensional feature space: spectral feature analysis, wavelet analysis, chaotic fractal analysis, and Mel Frequency Cepstrum Coefficient (MFCC) based on speech signal processing, *etc.* [14], [15]. In order to solve the problem of ''dimension disaster'', an innovative feature weight estimation method was proposed in [16], called dynamic representation and neighbor sparse reconstruction-based Relief (DRNSR-Relief).DRNSR-Relief decomposes a nonlinear problem into a set of locally linear ones through local hyperplane with ℓ^1 regularization and then estimates feature weights in a large margin framework, experimental results indicate that DRNSR-Relief is very promising. However, under the framework of sparse representation and classification proposed by us, the compressed sensing means that the selection of feature space is no longer important, and even the randomly selected features can contain enough information for sparse expression, so as to correctly classify hydroacoustic signal [17]. The key of Sparse Representation Classification(SRC) is that the feature space is large enough and the sparse representation can be correctly calculated [18]. In order to improve performance, it is necessary to increase the number of samples, resulting in increased storage space and correspondingly algorithm complexity. As shown in Fig. 1, the spectrum feature diagram generated by an underwater object shows that the hydroacoustic signal extracted by spectrum feature is absolutely a high-dimensional vector.

The robustness of hydroacoustic signal identification. An algorithm of feature extraction may use original acoustic signal. An underwater acoustic channel is characterized by low bandwidth, long propagation delay, high errors probability, Doppler effect, multi-path effect and spatiotemporal variation. Therefore, there are some errors in the hydroacoustic signal, such as signal loss or noise interference, and it is difficult to extract features from high-standard underwater acoustic signal. At the same time, the transmission, storage and processing of hydroacoustic signal also face many challenges. The errors caused by the above problems are unpredictable, and lead to the decline of the identification accuracy [19]. However, a little amount of error signal does not affect the sparse expression of the whole signal, the errors of hydroacoustic signal is regarded as a special kind of sparse representation, which will cause errors classification [19], [20].

In this paper, we mainly propose the effective strategies to solve the issues of underwater targets identification, and propose a robust Sparse Representation Classification(SRC) method to accurate the sparse representation and enhance classification abilities.The main contributions are shown as:

1) We exploit the discriminative nature of sparse representation to perform classification. Instead of using a dictionary based on data model. A overcomplete dictionary D whose base elements are the training

samples themselves(See **Section III-B**). If each class has enough training samples, overcomplete dictionary D can be trained to represent the test samples of the same class linearly. This representation is naturally sparse and uses only a small part of the data in the entire training sample. We believe that the key to using sparse representation is how to use test samples to train an appropriate overcomplete dictionary, so as to obtain the precise sparse linear representation, and finally use sparse representation to automatically classify various classes in the training set.

2) Our classification algorithm based on sparse representation differs significantly from the traditional classification method based on feature extraction (*e.g.*, SVM and Random Forest(RF)).In this paper, instead of using sparsity to identify a related model or related features and then using these features to classify all test samples, the sparse representation of each individual test sample is used directly for classification, and the training sample that gives the most concise representation (See **Section IV-A**) is adaptively selected. Our method strikes a balance among all possible classes, and adaptively selects the minimum number of training samples required to represent each test sample.

3) Signal loss and noise interference (*i.e.*, The errors of hydroacoustic signal) pose a significant obstacle to robust real-world underwater targets identification, which have been taken into account in the sparse representation. When errors have a sparse representation, it can be handled uniformly within our framework: the basis in which the errors are sparse can be treated as a special class of training samples. In experiments with sparse representations of test samples based on extended dictionary for signal loss and noise interference(See **Section IV-B**), the theories of sparse representations and compressed sensing characterize when such source-and-error separation is possible, thus the identification algorithm that determines how much signal loss and noise interference can tolerate these errors.

The structure of the article. **Section II** introduces forward-looking research by many experts and scholars in the field of sparse representation recognition, especially dictionary learning, sparse expression in the field of picture and sound wave recognition and detection. In **Section III**, sparse representation is conducted to given hydroacoustic signal under the framework of compressed sensing theory. In other words, we discuss using $\ell^1 - norm$ optimization methods to make the hydroacoustic signal expression as sparse as possible and how it can be used to classify and validate any given test samples. **Section IV** shows two important problems in studying target identification in water by using sparse identification classification framework: feature extraction and robustness of SRC of acoustic signal. In **Section V**, we verify the previous conjecture with the existing data set of hydroacoustic signal. In our experiment, the experimental results of SRC, SVM and Random Forest(RF) are compared and discussed.

II. RELATED RESEARCH

In this section, we describe the related work on dictionary learning and sparse representation in acoustic signal recognition and detection.

A. DICTIONARY LEARNING

Initially, the dictionary was obtained by using the basis function(*e.g.*, discrete cosine domain), and then developed to obtain the dictionary by iterative algorithm based on training samples(*e.g.*, $\ell^0 - norm$). However, the disadvantages of these methods are that the training time is too long and it is not easy to operate. Research on dictionary training algorithm based on hydroacoustic signal features is an important direction to improve the recognition rate of sparse representation. See [21], the author proposed a discriminative Fisher embedding dictionary learning (DFEDL) algorithm that simultaneously establishes Fisher embedding models on learned atoms and coefficients.See [22], Michal Aharon *et al*. proposed a novel algorithm for adapting dictionaries in order to achieve sparse signal representations, the K-SVD algorithm generalizing the K-means clustering process. Experimental results show that the K-SVD algorithm has achieved the expected results.

B. PREPARATION FOR SPARSE REPRESENTATION

In order to apply the sparse theory to hydroacoustic signal processing, there is an important problem to be solved. Since the time domain and frequency domain data of the sound signal are usually not sparse, some preprocessing must be performed before extracting the acoustic features with significant sparsity. It is essential that these sparse acoustic signal must have sufficient original information.

C. SPARSE REPRESENTATION IN ACOUSTIC SIGNAL RECOGNITION

In recent years, sparse theory has been widely applied in acoustic signal acquisition, compression, modeling, classification, matching and identification, and has become an important branch in the field of acoustic signal processing.

Some sparse algorithm theories have been applied to music and speech signal processing. Then, a dictionary shall be learned by using all the processed frame data, and the dictionary and the initial sparse coding shall be used to train a fast decoder of sparse decomposition, so as to obtain the sparse code of acoustic signal under the dictionary. Mikael Henaff *et al.* proposed a fast encoder based on predictive sparse decomposition (PSD), which can quickly obtain the sparse representation of signal by known dictionary [23]. Music data shall be divided into several frames, and each frame shall be pre-processed. Hui-Hung Wang *et al.* proposed a dictionary learning method, called Paired Discriminative K-SVD (PD-KSVD), to learn discriminative features for visual recognition [24]. A.Lima *et al.* proposed the sparse kernel function principal component analysis (SKPCA) method and applied it to speech identification, achieving the desired goal [25], the research shows that, in some cases, the noise is sparse, and the transcendental sparse feature can be used to weaken the noise and enhance speech. Plumbley *et al.* used sparse representation to solve signal processing tasks from audio encoding, audio enhancement and audio transcription to blind source separation [26].

D. APPLICATION OF ACOUSTIC DETECTION

In addition to using sonar to identify underwater objects, many scholars have gradually detected objects using P-wave or lamb wave recognition techniques. At present, P wave morphology detection is widely used in the classification and identification of ECG diseases.Abed Al Raoof Bsoul *et al.* proposed a P-wave detection method based on multi-feature neural network input [27]. Eduardo de Azevedo Botter *et al.*used neural networks to determine whether the heart beat was P-wave [28], thereby improving the correct detection rate of P-wave. Lamb wave detection has a wide range of applications in the field of structural monitoring [29]. Due to the propagation characteristics of Lamb waves in thin-plate structures, active Lamb wave damage identification techniques can be used to detect objects [30].

III. SPARSE REPRESENTATION CLASSIFICATION BASED ON HYDROACOUSTIC SIGNAL

The basic problem of underwater targets identification is to use d signal samples of different feature categories to train the classifier and finally correctly identify the class of the new test samples. Given n_i training samples from the *ith* class as column vector $D_i = \left[d_{i,1}, d_{i,2}, \ldots, d_{i,n_i}\right] \in \mathcal{R}^{m\times n_i}$. Suppose that the value of each unit in the experimental data of underwater sound is the atomic vector $d \in \mathcal{R}^m$ of sound signal of underwater object D_i, and the column vector of D_i is the sound signal of class *ith*.

A. COMPRESSED SENSING THEORY

Donoho and Huo [31] and Candes [32] *et al.* proposed the compressed sensing (CS) theory, the essence of which is that sparse signal can contain enough information of signal processing through a small number of random linear projections, that is, accurate signal representation can be obtained. Similarly, the sparse representation of hydroacoustic signal inevitably includes all feature information of signal. It is theoretically feasible to use sparse representation as the feature of hydroacoustic signal, and it can reduce data processing tasks.Along with the evolvement of data collection technology, the dimension of data is now getting higher and higher.For example, Dubit sound, high fidelity sound or high redundancy underwater sound,*etc*.In general, when the data dimension increases, the cost in storing, transmitting and analyzing data will inevitably rise. What is more, the increase in data dimension is actually much faster than the advance in communication, storage and computation power.

If a signal is sparse, or a transform domain is sparse, an observation matrix unrelated to the transform basis can be used to ‘‘compress’’ the sparse high-dimensional signal into low-dimensional signal, which directly perceives the compressed data information and breaks the constraints of Shannon-Nyquist theorem. Alternatively, given only the compressed data and sensing matrix, the proposed method, row space pursuit (RSP), recovers the authentic row space that gives correct clustering results under certain conditions [33].

Convert analog signal into digital signal to obtain a lot of sampling, resulting large data size. We obtain the corresponding coefficient by transforming coefficient, then in the process of signal encoding, make values of zero or close to zero, extract only a small amount of coefficient. But these coefficients can represent information of underwater objects at maximum limit [34]. The sparse matrix could store a small amount of values(sparse values) and their positions, and the sparse representation of hydroacoustic signal is obtained.

Under compressed sensing theory, intelligent identification of underwater targets means that the accurate choice of feature space is no longer important, but the dimension of the feature space is enough big and sparse representation calculated precisely, namely the right information for the most part keep the complete original signal information. Signal can be restored by using repeatedly a small amount of information in the subsequent processing, which can avoid the inefficiently resulted by feature extraction and identification methods. Therefore, sparse representation and compressed sensing are ideal methods for effectively solving the identification of underwater targets.

B. TEST SAMPLES ARE USED TO TRAIN AN ACCURATE SPARSE LINEAR COMBINATIONS

Matrix structure D_i has come up with all sorts of classifiers, a joint subspace recovery and enhanced locality based robust flexible label consistent dictionary learning method proposed by Zhao Zhang *et al*.This discriminative dictionary learning algorithm mainly improves the data representation and classification abilities by enhancing the robust property to sparse errors and encoding the locality, reconstruction error and label consistency more accurately. For the robustness to noise and sparse errors in hydroacoustic signas, this algorithm helps to improve recognition rate of error signal [35]. It is especially sensitive to noise and interference subspace when identifying underwater targets. In a typical environment, we first establish the linear subspace of the training sample of a single class, and this subspace will be the only priori sample for underwater targets identification.

Assume that there are sufficient training sample $D_i = \left[d_{i,1}, d_{i,2}, \ldots, d_{i,n_i}\right] \in \mathcal{R}^{m\times n_i}$ in the *ith* class, any test sample y in the same class will be approximately located on the linear subspace of the training samples related to class *ith*:

$$y = \alpha_{i,1}d_{i,1} + \alpha_{i,2}d_{i,2} + \ldots + \alpha_{i,n_i}d_{i,n_i} \tag{1}$$

where the scalar $a_{i,j} \in \mathcal{R}, j = 1, 2, \ldots, n.$.

Since the test samples of the *ith* class are unknown at the time of initialization, we define a new matrix D for the entire training set, $D = [D_1, D_2, \ldots, D_i, \ldots, D_m]$. D can connect n single atom vector d to m training samples as shown

TABLE 1. Partial matrix *D* is obtained according to SRC algorithm, results are predicted according to the matrix, where "1" represents metal cylinder and "0" represents rocks.

Matrix D		Predictions
D_1	D_2	
1.9599	0.1756	1
0.2635	2.4739	0
0.5724	2.5093	0
0.9112	2.0188	0
0.2806	2.8526	0
0.1137	3.0911	0
0.4173	2.8249	0
0.0345	3.0066	0
0.2441	3.4052	0
0.0487	3.1908	0
0.0309	3.2455	0
0.1977	3.4356	0
1.0283	2.5461	0
0.1051	3.1677	0
0.4225	3.5149	0
0.1122	3.3977	0
0.2719	2.9873	0
0.0976	3.3982	0
0.116	3.4097	0
0.5053	3.7453	0
0.0906	3.3923	0

in Fig. 1. Table 1 shows the data samples of the matrix D obtained by SRC algorithm in this experimental database, and the prediction and identification of metal cylinder and rock in two types of underwater objects are carried out.

$$D = [D_1, D_2, \ldots, D_i, \ldots, D_m] = \left[d_{1,1}, \ldots, d_{i,j}, \ldots, d_{i,n_i}\right] \tag{2}$$

Then the linear expression in equation (1) can be rewritten as:

$$y = Da_0 \in \mathcal{R} \tag{3}$$

where, $a_0 = \left[0, \ldots, 0, \alpha_{i,1}, \alpha_{i,2}, \ldots, \alpha_{i,n_i}, 0, \ldots, 0\right]^T \in \mathcal{R}^n$ is a coefficient vector. All data items are 0 except those related to the class *ith*.

Obviously, if $m > n$, according to the knowledge of linear algebra, the solution of $y = Da$ can be uniquely determined. However, according to the sparse condition (*i.e.* $m \ll n$), the solution of the coefficient vector a is usually unique. We can find out the solution with the fewest non-zero elements from all feasible solutions, which satisfies the sparsity. The following mathematical model can be obtained:

$$\left(l^0\right) \quad \hat{a}_0 = argmin\|a\|_0 \; subject\ to\ Da = y \tag{4}$$

where, $\|.\|$ represents $l^0 - norm$, the non-zero number in the coefficient vector. In fact, If D is a non-orthogonal matrix, for $y = Da$, when $a < m/2$, there is a non-zero solution, and a is only sparse solution: $\hat{a}_0 = a_0$ [36]. However, it is hard to find the sparse solution of underdetermined linear equations, or even to approximate [37]–[39]. That is to say, there is no more significant and effective method than finding solutions of $y = Da$ to find the sparse solution.

1) FIND THE MOST SPARSE SOLUTION BY USING $\ell^1 - minimization$

According to the development of sparse representation and compressed sensing theory, since the ℓ^0- or $\ell^1 - norm$ constraint applied in most existing dictionary learning criteria makes the training phase time consuming, to obtain the representation coefficients, discriminative dictionary from the given samples is calculated via minimizing a sparse approximation term [40]–[44]. It is revealed that if the required solution a_0 is sparse enough, then $\ell^1 - minimization$ problem (4) is equivalent to solve the problem $\ell^1 - minimization$:

$$\left(l^1\right) \quad \hat{a}_1 = argmin\|a\|_1 \; subject\ to\ Da = y \tag{5}$$

This problem can be solved in polynomial time by standard linear programming method [42]. When the solution is known to be very sparse, a more efficient approach can be used. For example, homotopy algorithms recovers t solutions with non-zero in time $O\left(t^3 + n\right)$ and is linear in the size of training samples [43].

2) HANDLE SMALL AND DENSE NOISE

So far, we have assumed that equation (3) is completely valid, but in real underwater acoustic signal, there is noise, which may lead to the failure to completely represent test samples as sparse superposition of training samples. By rewriting formula (3) with (6), possible noise problems can be fully considered.

$$y = Da + z \tag{6}$$

where, $z \in \mathcal{R}^m$ is a bounded normal form $\|z\|_2 < \varepsilon$. By solving the stable minimization problem, the sparse solution a can still be approximated to:

$$\left(\ell_s^1\right) \hat{a}_1 = argmin\|a\|_1 \quad subject\ to\ \|Da - y\|_2 \leq \varepsilon \tag{7}$$

The above optimization problem can be solved by second-order cone programming [42].The sparse solution D [43]is guaranteed by ℓ_s^1 to be approximately obtained in the set of random matrices. There are other solutions, for example, Yulin Sun *et al*, propose a structured Robust Adaptive Dictionary Pair Learning(RA-DPL)framework for the discriminative sparse representation learning. To achieve powerful representation ability of the available samples, the setting of RA-DPL seamlessly integrates the robust projective dictionary pair learning, locality-adaptive sparse representation and discriminative coding coefficients learning into a unified learning framework [45].

3) SIMILARITY CORRELATION OF ATOMIC VECTORS *D* OF A SINGLE TRAINING SAMPLE

Coherence refers to the similarity correlation between different atomic vectors d_{i,n_i} in the training sample D. Firstly, it is defined that the sets of different atomic vectors in the training sample are normalized, and then the inner product is taken to take the maximum absolute value of them, and the similarity correlation of training sample is defined as:

$$\mu(D) = \max_{i \neq j} |\langle d_i, d_j \rangle| \quad (8)$$

where d_i, d_j represents two different atoms in a single training sample D. To some extent, size of $\mu(D)$ can reflect the similarity correlation among atoms in the training samples. When the value of $\mu(D)$ is large, it means that the similarity between atoms is strong, otherwise the similarity is weak.

C. SPARSE CLASSIFICATION

Given a test sample y, first calculate its sparse representation $\hat{a}_1$ by (5) or (7), and then calculate the correlation $\mu(D)$ between different atoms in a single training sample D by (8). Ideally, all non-zero items in $\hat{a}_1$ would be associated with columns D_i in a single object the *ith* class, and test samples y could easily be assigned to that class. However, noise interference and modeling errors can cause non-zero terms in multiple classes to be correlated, leading to classification errors. In order to make better use of the sparse linear features, We use the coefficient a to reproduce y, which is the process of classifying y.

For each class i, set $\delta_i : \mathcal{R}^n \to \mathcal{R}^n$ an eigenfunction that selects the coefficients associated with the class *ith*. For $a \in R^n$, $\delta_i(a) \in R^n$ is a new vector, its only non-zero atomic value is the item associated a with the class i.Using coefficients a associated with the class *ith*, the given test sample y can be approximated to $\hat{y}_i = D\delta_i(\hat{a}_1)$, and we can achieve the purpose of classifing y by assigning y to the class with the smallest residuals between y and $\hat{y}_i$.

$$\min_i r_i(y) = \|y - D\delta_i(\hat{a}_1)\|_2 \quad (9)$$

The following algorithm provides a complete identification process of sparse classification, we minimize $\ell^1 - norm$ by implementing linear programming based on the original dual algorithm [44], [46].

Algorithm 1 Sparse Representation-Based Classification (SRC)

1) Input: a matrix of training samples $D_i = [d_{i,1}, d_{i,2}, \ldots, d_{i,n_i}] \in \mathcal{R}^{m \times n_i}$ for ith classes, a test sample $y \in \mathcal{R}^m$,(and an optional error tolerance $\varepsilon > 0$.)
2) Normalize the columns of D to have unit $\ell^1 - norm$.
3) Solve the $\ell^1 - minimization$ problem:
   $\hat{a}_1 = \arg\min_a \|a\|_1 subject \quad to\ Da = y$
   (Or alternatively, slove:
   $\hat{a}_1 = \arg\min_a \|a\|_1 subject \quad to\ \|y - Da\|_2 \le \varepsilon)$
4) Compute the residuals $r_i(y) = \|y - D\delta_i(\hat{a}_1)\|_2$ for $i = 1, 2, \ldots, n_i$
5) Output: $identity(y) = \arg\min_i r_i(y)$.

Experiment 1 (ℓ^1 − Minimization): To illustrate how algorithm 1 works, we selected half of the 208 data sets in the sonar database as the training set and the rest for testing.

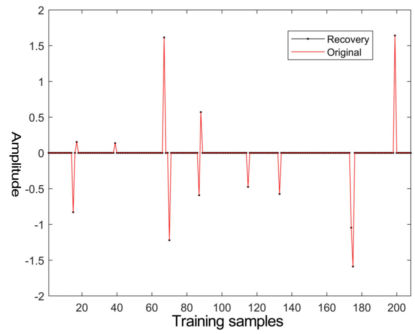


FIGURE 2. The training samples calculated by algorithm 1 are sparse, and the residual error is $r = 4.6099e - 16$.

In this experiment, we combine 60 underwater acoustic samples of objects in water into matrix columns D. Therefore, the size of the matrix is 60×104, $y = Da$ is underdetermined. Fig. 2 shows the sparse coefficient distribution restored by algorithm 1 for test underwater acoustic data. The figure also shows the features and raw data corresponding to the two maximum sparsity. Both maximum coefficients are associated with training samples from the first class of metal objects. With the 60 samples as the feature, algorithm 1 achieves the identification rate of 92.1% in the sonar data set. (See **Section V** for details and performance of other identification algorithms (SVM and RF), as well as a comparison between them.In addition, many researchers also generally use regression analysis or Nearest Neighbor for classification [47]. In order to further improve this experiment, we will further elaborate in future experiments.)

D. VALIDATION BASED ON SPARSE REPRESENTATION CLASSIFICATION

Before classifying a given test sample, we must first determine whether the test signal is a valid sample from a class in the sonar data set. The ability to reject invalid test samples or "outliers" after detection is crucial to identification systems that work in the real world. For example, the underwater intelligent identification system cannot recognize non-hydroacoustic signal and signal that are not in the experimental signal set.

The SVM classification method is to find the distance from the test sample to the hyperplane and then determine whether to accept or reject the test sample, while the decision method of RF is to adopt a special bagging, which uses the decision tree as the model in bagging [48]. Both traditional classifiers classify and identify the test sample base on feature extraction, this paper selects more general MFCC[1] to extract features of hydroacoustic signal.

In the experiment of sparse representation, according to a certain class of hydroacoustic signal data, its characteristic

[1]MFCC:Mel Frequency Cepstrum Coefficient considers human auditory characteristics, first maps the linear spectrum to the Mel nonlinear spectrum based on auditory perception, and then transfers it to the Cepstrum $mel(f) = 2595 \times \log_{10}\left(1 + {}^{f}/_{700}\right)$ where f is the frequency of the acoustical signal.

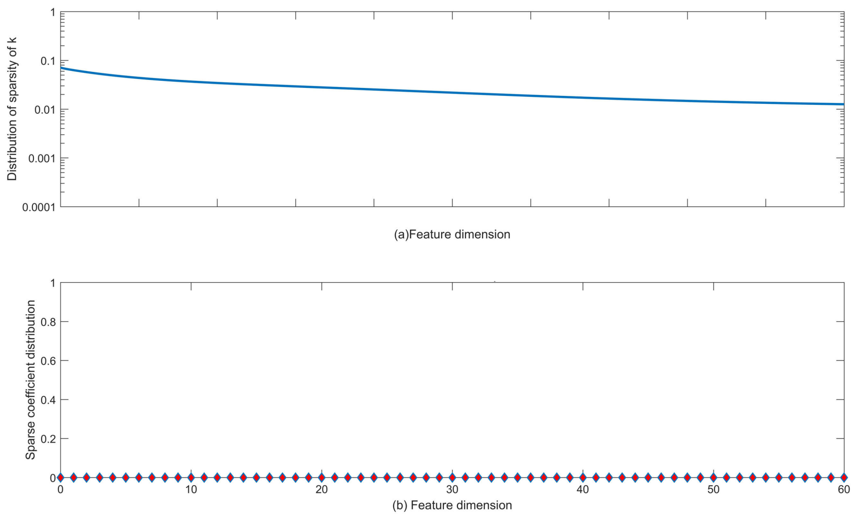


FIGURE 3. **(a) Sparsity tends to 0.01 and exceeds the normal sparse value range. (b) Distribution map of corresponding sparse coefficient $\hat{a}_1$, where "0" indicates any underwater object that does not belong to sparse classification.**

coefficient $\hat{a}_1$ is applied to the sparse linear expression of all relevant data, that is to say, the coefficient $\hat{a}_1$ can be verified by using the joint distribution of its class. This has better verification results than the SVM method of finding the distance from the point to the hyperplane or the special bagging method of RF. The optimization process can be expressed in formula (10):

$$\arg\min_{\{d_i\}\{a^j\}} \sum_{j=1}^{m} \left\| y^j - \sum_{i=1}^{n} a^j{}_i d_i \right\|_2^2 + \varepsilon \sum_{j=1}^{m} \sum_{i=1}^{n} \left| a^j{}_i \right| \tag{10}$$

where, a^j is the sparse representation vector of the j test sample. k is number of non-zero terms of sparse vector a_0, which is also called sparsity [2]

Experiment 2 (Sparsity of the Sparse Coefficient): we randomly selected a hydroacoustic signal (which does not belong to any category of hydroacoustic database) and sparsely represented it. As shown in Fig.3, Fig.3(a) represents the distribution diagram of the obtained sparsity k, while Fig.3(b) represents the corresponding sparsity coefficient $\hat{a}_1$. Compared with the sparsity of effective test samples in Fig.2, $\hat{a}_1$ here are not concentrated on any class, but evenly distributed. Therefore, the distribution of sparsity coefficient $\hat{a}_1$ contains important information about test samples. Effective test samples should have sparse representation, and its non-zero number k should be concentrated around a vector, while the sparse coefficient of invalid hydroacoustic signal is scattered over multiple vectors.

Different from SVM and RF, the SRC avoids the direct use of spatial distance-based classification method which ignores the relevance of the internal features of water objects. The SRC method in our study does not rely on statistics for verification and identification, but separates the information needed for classification and identification: identify residual and verify sparsity. Identify residual refers to the approximate degree between the sparse representation of acoustic signal and test samples, and the sparsity refers to the sparse performance of test samples belonging to this category. We will prove that the sparse expression classification is better than SVM [49] and RF method. Based on the experiment of this database, it can be verified that the classification identification rate is increased by about 10%.

[2]Sparsity: The maximum number of non-zero in matrix D. A zero value indicates that it is less than a certain threshold and is marked as 0, otherwise it is a non-zero value. Given a finite long water acoustic signal y, if the signal $y \in \mathcal{R}$ has at most k non-zero elements, *i.e.* $\|y\|_0 \leq k$, the signal y is said to be a k sparse signal.

IV. TWO TYPES OF PROBLEMS OF SPARSE REPRESENTATION OF HYDROACOUSTIC SIGNAL

In this section, we study the two key influential factors of common classifiers on underwater targets identification:

- selection of feature extraction
- robustness to hydroacoustic signal loss, error and noise interference

Specifically, SRC for robust underwater targets identification is a factor that must be considered in practical scenarios, Underwater acoustic signal are lost or interfered, which usually severely reduces the recognition rate of SRC. To improve the effectiveness of representation coding, some researchers have adopted an error detection machine (EDM) with multiple error detectors (ED) in SRC, is proposed to detect and remove destroyed features on a testing samples [50].

A. FUNCTION OF FEATURE EXTRACTION

For a long time, in order to improve the accuracy of underwater targets identification, many experts and scholars from different angles and application analyzed and studied original signal of target radiated noise, extracted of a series of effective characteristic parameters, including the following categories: auditory feature extraction methods (Loudness and MFCC *etc.*), Visual feature extraction method, the multidimensional feature fusion method and high dimensional feature dimension reduction method,*etc.* [15]. Although a large number of features have been accumulated in the research on classification and identification of objects in water, most of them may be redundant and irrelevant to classification tasks in specific applications. Although useful features can be picked out, this can be a difficult and time-consuming task, especially when the feature of acoustic data is unclear. It is not conducive to discover learning rules to retain irrelevant features and discard relevant features. In addition, irrelevant or redundant features increase the amount of data and slow down the training process of classifier. In this section, we will re-examine the role of feature extraction in the new sparse representation for underwater targets identification.

The advange of feature extraction is that it extends the proposed sparse representation framework and reduces data dimensions and computational costs. For the original acoustic signal data, the corresponding linear system $y = Da$ is very large.For example, if a training sample D is a matrix of 60×104, the dimension n is about 60. Although algorithm 1 can use scalable methods such as linear programming, its direct application to such high-dimensional acoustic signal data is still beyond the capability of conventional computers.

Since linear operations are involved in most feature extraction algorithms (or roughly so), the projection from the original hydroacoustic signal space to the feature space can be expressed as a matrix $R \in \mathcal{R}^{m\times k}$, where $k \ll n$. Apply R to both sides of (3):

$$\tilde{y} = Ry = RDa_0 \in \mathcal{R}^m \tag{11}$$

In fact, the dimension k of the feature space is usually chosen to be much smaller than n_i. In this case, the system $\tilde{y} = RDa \in \mathcal{R}^k$ is underdetermined in the unknown $a \in \mathcal{R}^n$. In order to get the sparse solutions a_0, we solved the problem by reducing $\ell^1 - minimization$:

$$\left(\ell_r^1\right)\ \hat{a}_1 = argmin\|a\|_1 \quad subject\ to\ \|RDa - \tilde{y}\|_2 \leq \varepsilon \tag{12}$$

For a given fault tolerance rate $\varepsilon > 0$, in algorithm 1, the training sample matrix D is replaced by the matrix $RD \in \mathcal{R}^{m\times k}$ of k dimensional features, and the test sample y is replaced by its features $\tilde{y}$

Existing hydroacoustic signal identification experiments show that increasing the dimension k of feature space can improve the identification rate of underwater targets. As long as the distribution of features RD_i is not degenerate [51]. Degeneracy is not a problem for the $\ell^1 - minimization$ which should be in or near the range of $\tilde{y}$, it does not depend on $\Sigma_i = D_i^T R^T RD_i$, it's not singular in classical discriminant analysis. The stable version $\ell^1 - minimization$ (7) or (12) is called Lasso [10] in statistical literature. When the solution is sparse, it effectively standardizes the highly underdetermined linear regression.

For SRC, we want to understand how the choice of feature extraction affects the correct sparse solution (12). In **Section V**, our experimental results will validate $\ell^1 - minimization$, in particular, the performance of stable version (12). As long as the sparse solution a_0 can be correctly obtained, algorithm 1 will always give the same classification result, regardless of which feature is actually used. In other words, the identification performance of algorithm 1 with different features rapidly converges, and the selection of "optimal" feature transformation is no longer the key point. This will be confirmed by the experimental results in **Section V**.

B. ROBUSTNESS OF DESTROYED HYDROACOUSTIC SIGNAL

Hydroacoustic signal loss and noise interference may result in the challage for intelligent identification in underwater environment. This is due to the unpredictability of errors caused by the destruction of hydroacoustic signal: A damaged signal may be an important part, which has a significant impact on the intelligent identification of underwater objects [52]. However, the damaged signal is usually only a small part of the signal, which rarely appears on the entire underwater acoustic signal. When the hydroacoustic signal has such sparse representation, the sparse representation can be used for unified processing. In this case, the linear expression (3) can be rewritten as:

$$y = y_0 + e_0 = Da_0 + e_0 \tag{13}$$

where $e_0 \in \mathcal{R}^m$ is the vector of error, a few of items from the vector e_0 is non-zero. e_0 indicates the signal is damaged or lost. Test sound signal are different, damage may be different [53]. These errors may be uncontrollable, so they cannot be ignored or treated as traditional methods for small noise, as shown in **Section III-B2**.

The basic principle of coding theory [54] points out that redundant data is crucial for detecting and correcting serious errors. When there is redundancy in underwater acoustic signal, the data amount of hydroacoustic signal is much larger than that used to identify objects. In this case, even if a small number of hydroacoustic signal are damaged or lost, the remaining signal can still be used for identification. At the same time, the selection of feature extraction of hydroacoustic signal mentioned in the previous section will help to alleviate the side effects caused by the loss of signal. From this point of view, no method is more useful than the information from the redundant hydroacoustic signal. Therefore, when the signal is destroyed or lost, redundant data with high quality can ensure a good performance of the sparse expression algorithm.

The redundant signal is useless without efficient computation. Next is how to deal with the redundant signal by delivering a linear sparse codes auto-extractor and a multi-class classifier by simultaneously minimizing the sparse reconstruction, discriminative sparse-code, code approximation and classification errors. The auto-extractor is characterized with a projection that bridges Underwater acoustic signal with sparse codes by learning special features from input signals for characterizing sparse codes [55]. It is assumed that the damaged acoustic signal is a small part of the overall acoustic signal, as the sparse vector a_0, the error vector e_0 also has sparse characteristics. Since $y_0 = Da_0$, we can rewrite (13) as:

$$y = [D \quad I] \begin{bmatrix} a_0 \\ e_0 \end{bmatrix} = B\omega_0 \tag{14}$$

Set $B = [D \quad I] \in \mathcal{R}^{m\times(n+m)}$, $y = B\omega_0$ is underdetermined, and ω_0 has no unique solution. However, from the sparsity of a_0 and e_0 discussed above, $\omega_0 = [a_0 \quad e_0]^T$ has at most $n_i + \rho m$ non-zero solution, ρ is the dimension of e_0. Therefore, ω_0 is the most sparse solution of sparse expression $y = B\omega$. In fact, if B is a non-orthogonal matrix, as long as $y = B\tilde{\omega}$, for some nonzero value $\tilde{\omega}$ less than $^m/_2$, $\tilde{\omega}$ is the only sparse solution. Therefore, if the loss e_0 is less than $^{m-n_i}/_2$, the sparse solution $\tilde{\omega}$ is the classifier, $\omega_0 = [a_0 \quad e_0]^T$.

More generally, it can be assumed that the loss error e_0 has representation based on $D_e \in \mathcal{R}^{m\times\rho}$, that is, for some sparse vectors $b_0 \in \mathcal{R}^m$, $e_0 = D_e b_0$ is satisfied. Assuming that $D_e = I \in \mathcal{R}^{m\times m}$, the acoustic signal is sparse relative to the original one, simply add D_e to D to redefine the matrix B to find the sparse solution ω_0:

$$y = B\omega \quad with \quad B = [D \quad D_e] \in \mathcal{R}^{m\times(n+\rho)} \tag{15}$$

In this way, the same formula can be used to deal with more common damaged signal.

As mentioned above, we solve the sparse problem by solving the expanded $\ell^1 - minimization$:

$$\left(\ell_e^1\right) \hat{\omega}_1 = argmin\|\omega\|_1 \quad subject\ to\ B\omega = y \tag{16}$$

So, in algorithm 1, we replace D with an extended matrix $B = [D \quad I]$, replace a with $\omega = [a \quad e]^T$.

According to **Experiments 2**, only when the noise interference and loss of e_0 are not taken into account in the sparse identification and classification system, equation (7) can be available. However, when we use $B = [D \quad I]$ to illustrate the total error, $\ell^1 - minimization$ (16) of $B\omega = y$ with precise limits can still achieve well in classification under medium noise conditions, as shown in Fig.4.

After the sparse solution $\hat{\omega}_1 = \left[\hat{a}_1 \quad \hat{e}_1\right]^T$ is calculated, set $y_r = y - \hat{e}_1$ to represent the complete underwater object signal after the signal is destroyed or lost. We modified the residual $r_i(y)$ in algorithm 1, and calculated according to the compensated acoustic signal y_r:

$$r_i(y) = \left\|y - D\delta_i\left(\hat{a}_1\right)\right\|_2 = \left\|y - \hat{e}_1 - D\delta_i\left(\hat{a}_1\right)\right\|_2 \tag{17}$$

V. EXPERIMENTAL VERIFICATION ON INTELLIGENT UNDERWATER IDENTIFICATION

In this section, we conduct experiments based on publicly available underwater hydroacoustic databases. The experiments can not only prove the effectiveness of the proposed classification algorithm, but also verify the model mentioned in the previous chapters. We first investigate the role of feature extraction in our framework, compare the performance of different feature spaces and feature dimensions with several popular classifiers. Then we demonstrate the robustness of the proposed algorithm in case of the hydroacoustic signal is destroyed or lost. Finally, we verify that SRC can effectively classify test samples and study the robustness of classification recognition in underwater acoustic signal loss and noise interference environments.

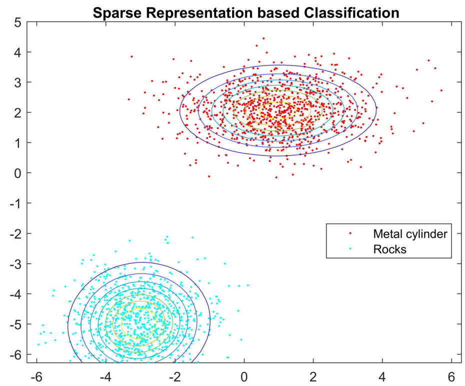


FIGURE 4. Classification of two types of hydroacoustic signal.

A. FEATURE EXTRACTION AND CLASSIFICATION

We use MFCC (Mel frequency cepher coefficient) to verify our classification algorithm. Mel frequency is proposed based on the hearing characteristics of the human ear, which has a nonlinear correspondence with Hz frequency. By using relation between them, the Hz spectrum features are calculated. We use this feature extraction algorithm, compare SVM, RF and SRC classifier in low-dimensional space, testing the optimization problem with errors $\varepsilon = 0.05$ in the process (12). SRC, SVM and RF are running on Matlab 2017a. All algorithms are implemented on a Core i3 2.40GHz with 4G RAM desktop, which will be verified within a tolerable time.

B. HYDROACOUSTIC DATABASE

The data set was contributed into the benchmark collection by Terry Sejnowski, who is at the Salk Institute and the University of California at San Deigo. The data set was developed in collaboration with R. Paul Gorman of Allied-Signal Aerospace Technology Center. The file "sonar.mines" contains 111 patterns obtained by bouncing sonar signal off a metal cylinder at various angles and under various conditions. The file "sonar.rocks" contains 97 patterns obtained from rocks under similar conditions. The transmitted sonar signal is a frequency-modulated chirp, rising in frequency. The data set contains signal obtained from a variety of different aspect angles, spanning 90 degrees for the cylinder and 180 degrees for the rock. As shown in Fig.5.

Each pattern is a set of 60 numbers in the range 0.0 to 1.0. Each number represents the energy within a particular frequency band, integrated over a certain period of time.

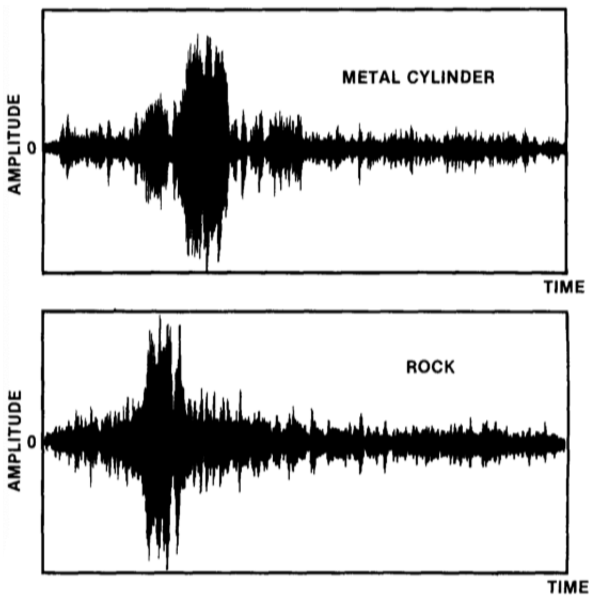


FIGURE 5. Two types of underwater hydroacoustic signal graphs of sonar data set.

The integration aperture for higher frequencies occur later in time, since these frequencies are transmitted later during the chirp.

The label associated with each record contains the letter "R" if the object is a rock and "M" if it is a mine (metal cylinder). The numbers in the labels are in increasing order of aspect angle, but they do not encode the angle directly.

104 hydroacoustic signal in this database were selected as training samples, including 52 metal cylinder and 52 rock, and the two types of samples were randomly distributed. The remaining 104 mixed samples were used as test sample sets.

According to the database of hydroacoustic signal, Fig.6 shows the identification rate[3] of this experiment, the best performance of SRC and SVM in each feature dimension always exceeds the best performance of RF. That is to say, the best identification rate of SRC is 92.1%, and that of SVM is 90.8%, the rate using RF is up to 81%, but it is observed that the performance of SVM and RF changes with the choice of feature space, the two classifiers depends greatly on the performance of "optimal" feature of low feature space dimension, as the "optimal" feature dimensions increase, classifier algorithm is not in convergence.

Here is a brief introduction of RF classifier, which comprehensively analyzes the training results of the base classifier and generates several training samples. Then, each training sample constructs a decision tree. When the node splits, some features are randomly selected to maximize the index (such as information gain), the optimal solution is found in the middle of the extracted features, applied to the node, split again [48], [56]. Table 2 is identification rate of classification of hydroacoustic database by using RF.

[3]Identification rate:Number of test sets C and number of correctly identified test samples c $\eta(\%) = c/C$

TABLE 2. Recognition rate obtained by random forest (RF).

Trees	Recognition rate(%)
1	60.00
3	72.50
5	81.00
7	76.00
10	80.50

TABLE 3. Recognition rate of SRC, SVM and RF in different signal loss percentages.

Methods \ Percent lost(%)	10	20	30
SRC(%)	88.50	83.20	78.40
SVM(%)	83.10	75.40	68.60
RF(%)	76.30	72.60	60.20

C. IDENTIFICATION OF LOSS OF HYDROACOUSTIC SIGNAL

In the second experiment, we artificially intercept some signal from the hydroacoustic database to simulate the loss of signal caused by damage during underwater transmission, as shown in Fig.7. The training and test samples in this experiment are the same as **Section V-B**. For these three types of classifiers, training sample is m, $m = 104$ is larger than the dimension of $n = 60$, so as to ensure that the linear equation (11) have a definite solution. However, the sparse approximate solution a can still be solved by solving problem (12) $\varepsilon - relaxed\ \ell^1 - minimization$ (here $\varepsilon = 4.9390e-16$). The results in table 3 show that the proposed SRC algorithm achieves better identification rate than SVM and RF. These experiments demonstrate the scalability of the algorithm when dealing with more than 104 dimensional features.

D. IDENTIFICATION OF HYDROACOUSTIC SIGNAL AFTER NOISE INTERFERENCE

SRC has better robustness in case of noise interference. Since SRC algorithm actually makes use of the linear correlation among samples of the same category, it can be assumed that these samples of the same category exist in the same feature subspace and can be represented by linear combination with each other. The identification method based on sparse representation constructs a dictionary based on global features, which solves all feature representations of the sample to be detected, but ignore local features between the same samples. However, when the number of training samples of each class is small, sparse decomposition is carried out for the tested hydroacoustic signal, and the sparse representation coefficients a obtained may correspond to multiple categories, resulting in inaccurate classification results.

In this experiment, we test a robust version of SRC. It uses database in **Section V-B** to solve the problem of $\ell^1 - minimization$ (16). In order to simulate the error of hydroacoustic signal, we artificially and randomly modified

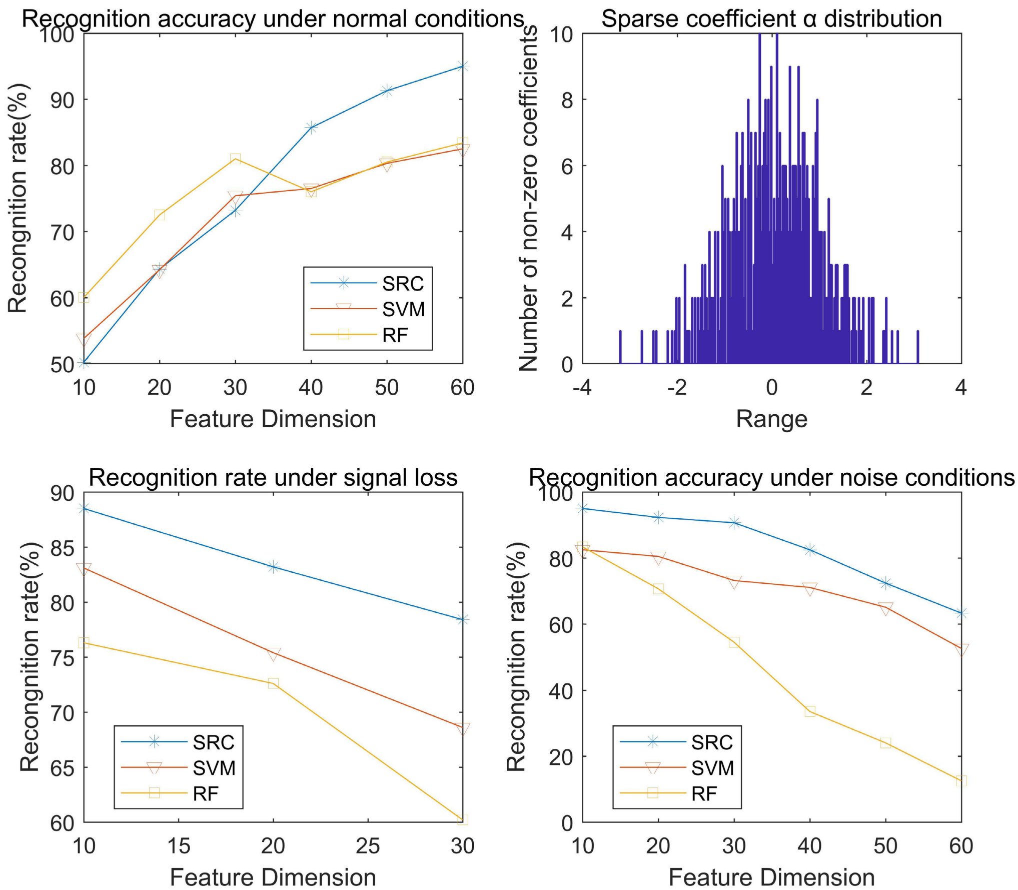
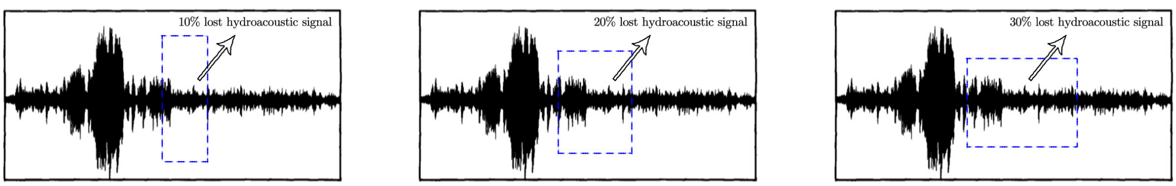


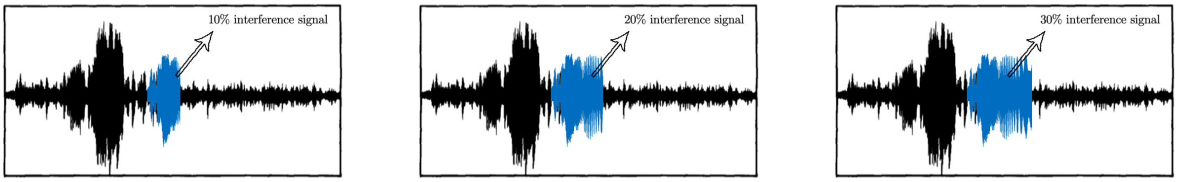
FIGURE 6. Classification results calculated by SRC, SVM and RF classifiers. Where, (a) represents the classification calculated under normal circumstances; (b) is the distribution range of sparse coefficient; (c) represents the classification results of three classifiers under the loss of hydroacoustic signal; (d) represents the classification results generated by noise signal line.



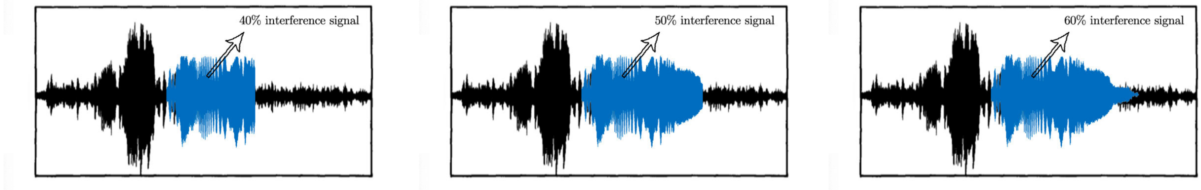
(a) 10 % loss rates of hydroacoustic signal (b) 20 % loss rates of hydroacoustic signal (c) 30 % loss rates of hydroacoustic signal

FIGURE 7. In order to visualize the missing signal of the underwater acoustic data set, we use the dashed box to indicate the missing part of the signal, which accounts for 10%, 20%, 30% of the underwater acoustic signal, respectively.

the signal, and the algorithm could not determine the modification of the signal, so we set the error rate by the proportion of 10% to 60%. In this extreme case, identification rate of SRC, SVM and RF classification algorithms are tested. Similarly, the training samples and test samples in this experiment are the same as **Section V-B**. Fig.8 describes the identification performance of the three classification algorithms. It can be seen that although the identification rate of SRC algorithm is much lower than the correct hydroacoustic signal, it still has a good performance compared with SVM and RF, See table 4, from error rate of between 10% to 60%, SRC is almost able to distinguish two types of underwater objects. When the error rate reaches 60%, SRC can achieve the identification rate of 35.2% while the other two classification algorithms does not exceed 10%.At the same time, although the recognition rate of support vector machine classifiers has also decreased significantly after noise interference, its recognition rate is still better than that caused by RF. Because SVM classifier has two very important parameters: C and gamma. Where C is the penalty coefficient, (*i.e.*, the tolerance of error).By optimizing penalty parameter C and selecting the best parameter, we can reduce the samples noise of underwater acoustic



(a) 10 % interference rate of tested hydroacoustic signal (b) 20 % interference rate of tested hydroacoustic signal (c) 30 % interference rate of tested hydroacoustic signal



(d) 40 % interference rate of tested hydroacoustic signal (e) 50 % interference rate of tested hydroacoustic signal (f) 60 % interference rate of tested hydroacoustic signal

FIGURE 8. In order to visualize the underwater acoustic data set under noise interference, we use the blue acoustic signal to indicate the interference of the tested hydroacoustic signal. The interference signal interferes with the proportion of the entire sonar signal by 10%, 20%, 30%, 40%, 50%, 60% respectively.

TABLE 4. Identification rate of SRC, SVM and RF in different signal interference percentages.

Percent interference(%) / Methods	10	20	30	40	50	60
SRC(%)	95	91.3	85.7	73.2	64.3	35.2
SVM(%)	88.5	75.2	65.1	40.3	20.4	8.1
RF (%)	73.2	63.4	56.9	33.5	13.6	2.5

signal.However, when the loss ratio of underwater acoustic signal is too large, the optimization of penalty parameter C is easy to lead to over fitting, on the contrary, it reduces the recognition rate [57].

E. CONCLUSION AND DISCUSSION

In this paper, We have investigated the underwater target recognition problem with signal sparsity characteristics, and exploit the discriminative properties of sparse representation to classify. Our approach takes into account all possible support (either within a class or across multiple classes) and adaptively selects the minimum number of training samples required to represent each test sample.

We verify theoretically and experimentally that the use of sparsity is crucial for high-performance classification of high-dimensional data. With proper sparsity, feature selection becomes less important than the number of features used. A robust SRC algorithm and its extended version (considering errors and robustness) have good performance in underwater targets identification experiments, especially when the number of features is large. At the same time, it can also be seen from experiments that SVM and RF classification are highly dependent on the selection of feature extraction. When there is serious noise interference or signal loss(often occurs in underwater environment), the performance of SVM and RF algorithm is unstable. There are still some shortcomings in this experiment.It is a future interesting question for robust SRC algorithms, due to the unpredictable nature of the error incurred by signal loss and noise interference: it may affect any part of the underwater acoustic signal and may be arbitrarily large in magnitude. The experiment cannot simulate the real situation, and different experiment database may have inconsistent identification rate, causes conservative result. Next, we will deal with this kind of problems for further research.

APPENDIX

The main SRC, SVM and RF codes are presented in the appendix. All code can be run and get experimental results

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