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Bus Scheduling of Overlapping Routes With Multi-Vehicle Types Based on Passenger OD Data

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ABSTRACT To mitigate the problems caused by bus route overlap in the transit network, this paper proposes a new scheduling method with both large and small vehicle types based on passenger OD (Origin-Destination) data. The minimum of total cost of passenger travel time and bus company operation is taken as the optimization objective, departure intervals and vehicle types are taken as the optimization variables. The impact of route overlap on passenger travel time is analyzed. A heuristic algorithm is developed to solve the optimization model to produce the departure time and vehicle type for each bus trip. Finally, three real bus routes in Harbin city are taken as an example to validate the proposed model using peak-hour data. Compared with the model without considering route overlap, the proposed model can reduce total passenger travel time and cost by 5.2% and 8.8% respectively.

INDEX TERMS Bus route overlap, scheduling method, vehicle types, optimization model.

I. INTRODUCTION

A. BACKGROUND

Giving priority to the public transit development is an effective way to improve the attraction of transit mode and ease traffic congestion. In densely populated urban areas, bus system is characterized by abundant routes and high-frequency services, which is of positive significance for increasing its coverage and capacity [1], [2]. However, bus route overlapping, i.e. multiple routes share common stops, is a widespread phenomenon in crowed urban areas, particularly on arterial roads.

The overlap of high-frequency bus routes would bring about two problems: (i) Passengers traveling in overlapping areas have multiple choices for traveling and they tend to take the earliest arrived bus to reduce waiting time. When subsequent buses arrive in a short period of time, the number of passengers at the stop would be much smaller than the expected value. This will lead to a large discrepancy of dwell times and passenger load rates among buses, and result in bus bunching. (ii) Buses from different routes enter the stop at the same time and interfere with each other, triggering traffic blockage. Several buses enter the stop simultaneously in the case of multiple high-frequency bus routes sharing common stops, which not only increases the bus dwell times, but also blocks the movements of general vehicles [3], [4].

From the above description we can find that the overlap of bus routes will have a significant impact on the bus operations. The number of passengers served by bus i is affected by the time interval between its arrival time at a stop and the departure time of its preceding bus $i-1$. Bus i and bus $i-1$ may belong to different routes. Most existing studies adopt the scheduling strategy with single vehicle type and uniform departure headway, which is difficult to adapt to the large fluctuations of passenger demand resulted by route overlap. This study tries to solve the bus operation problems caused by route overlap via allocating different vehicle types to bus trips on each route. The proposed scheduling method can not only avoid long passenger waiting times and large discrepancy of passenger load rates due to bus bunching, but also can reduce the cost of passenger travels and bus company operations [5], [6].

Some scholars have studied the bus scheduling with multiple vehicle types or considering the influence of route overlap. However, no study has been found to cope with the

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problems caused by route overlap through the use of multiple bus vehicle types.

1) BUS SCHEDULING METHODS WITH MULTI-VEHICLE TYPES

Researches in this group mainly focused on the optimization of departure headways for a bus route. Ceder proposed a methodology to schedule vehicles of different types. The approach used the so called deficit-function method allowing deadheading trip insertion and shifting of departure times within given tolerances to reduce the fleet size [7]. Hassold and Ceder suggested a new methodology for the multi-type vehicle scheduling problem. The methodology was based on a minimum-cost network flow model utilizing sets of Pareto-optimal timetables for individual bus line. The method developed enabled to stipulate the use of a particular vehicle type for a trip or to allow for a substitution either by a larger vehicle or a combination of smaller vehicles with the same or higher total capacity [8]. In recent years, researchers begin to explore a more flexible timetable optimization method based on the multi-vehicle types. Sun et al. took the total cost of passenger travel time and bus company operation into consideration and three different individual route scheduling models for hybrid vehicle, large vehicle and small vehicle were built respectively. The results indicated that the hybrid vehicle scheduling model was superior to the other two in saving passenger travel time and operational cost [9].

2) BUS SCHEDULING METHODS UNDER THE IMPACT OF ROUTE OVERLAP

To avoid the large fluctuations of passenger demand caused by multiple buses arriving at stops at the same time, the real-time coordinated scheduling methods were usually used. Ceder et al. created bus timetables with maximal synchronization of buses that arrive at the same stop, and came up with the feasible solution using a heuristic algorithm [10], [11]. Guihaire and Hao developed a cooperative scheduling method based on the punishment on the deviation from ideal passenger waiting time [12]. Lin *et al.* offered the responsible agency a reliable way to determine the optimal green extension or red truncation duration in responsible to multiple bus priority requests from different routes for headway-based bus operation [13]. Hernández *et al.* developed an optimization model capable of executing a control scheme based on holding strategy for a corridor with multiple bus lines [14]. Given the known bus travel time, Rios-Solis set ideal arrival interval to mitigate the congestion at stops in the overlapping area [15]. Ibarra-Rojas and Muñoz proposed a cooperative method for overlapping bus routes by calculating the weighted sum of the deviation from the scheduled arrival interval [16]. Schmöcker *et al.* presented a passenger queue balancing model with the considerations of bus bunching, passenger boarding behavior and bus overtaking outside stops. Allowing bus overtaking was found to benefit the operation of multi-route transit network [17]. Sun and Schmöcker studied bus bunching and passenger choice behavior at the stop served by multiple bus routes. The results showed that when two buses arrive immediately choosing to board the one after was beneficial to the operation of transit network; in addition, bus overtaking could help prevent the bus ahead from being over crowded [18]. Antoine *et al.* studied substitution strategy in the context of multiple bus lines under either time-independent or time varying settings. They modeled the agency's substitution decisions and retired bus repositioning decisions as a stochastic dynamic program so as to obtain the optimal policy that minimizing the system-wide costs [19].

Although previous researches have obtained rich achievements, some issues do exist in these related studies:

(i) For the multi-vehicle type scheduling methods, the scheduling plan was mainly optimized for a bus route, and the influence of route overlap was not considered. Thus, the proposed plan is not suitable to the overlapped bus routes.

(ii) For the scheduling methods considering the impact of route overlap, the coordinated scheduling plan for multiple routes was established based on real-time traffic information. These studies belong to the dynamic bus scheduling group. However, no research was found on the subject of static bus scheduling with multi-vehicle types.

B. OBJECTIVES AND CONTRIBUTIONS

This study proposes a static bus scheduling method with multi-vehicle types considering the influence of route overlap. The combination of large bus vehicles and small bus vehicles for each bus route is adopted by the optimization model. Minimizing total cost of passenger travel time and bus operation is taken as the optimization objective, and departure interval and vehicle type for each bus trip are set as the optimization variables.

The contributions of this study include two aspects:

(i) Impacts of route overlap on bus operations are analyzed and quantitative methods are proposed. Bus passengers are classified into two types in terms of the influence of route overlap. Travel time costs of two-type passengers and bus company operating cost under the scheduling strategy are analyzed.

(ii) Three real bus routes with common stops in Harbin city of China are employed for the case study. The proposed scheduling model is compared with that without considering route overlap. The influences of two scheduling models on passenger travel time and bus operating cost are analyzed.

II. MODEL DEVELOPMENT

We assume that in the study network the passenger OD of each route is given. Route $l(1 \leq l \leq L)$ has I stops and overlaps with J routes in the overlapping section. Without loss of generality, the number of stops outside the overlapping section is denoted by I_1. The number of stops within the overlapping section is denoted by $\mu_{\max}$ and it equals (I- I_1). The stops on route l and its overlapping routes are shown in Figure 1.

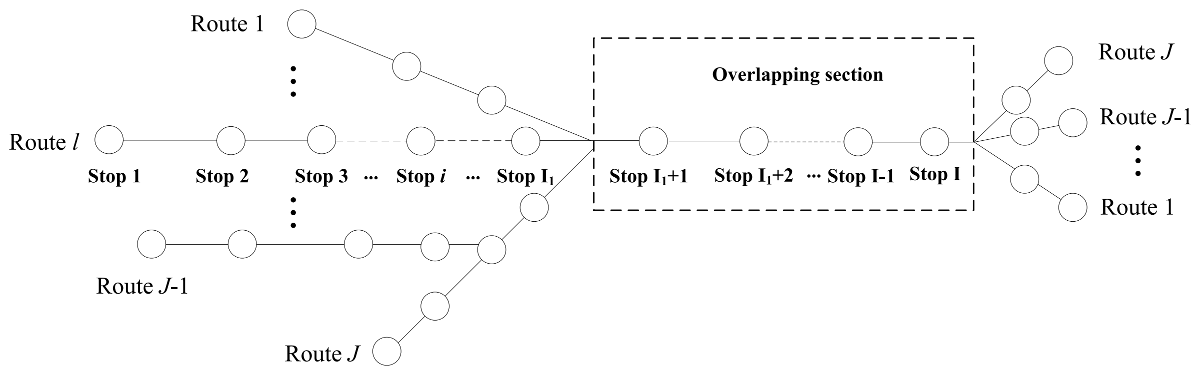

FIGURE 1. The stops on bus route *l* and the overlapping routes.

A. PASSENGER TRAVEL TIME COST

Passenger travel time cost includes in-vehicle travel time cost and waiting time cost at the stops. Passengers can be classified into two types considering the influence of overlapping routes. For the first-type passengers, at least one of their origins or destinations is not in the overlapping area. Hence, they can only select one bus route for traveling. The second-type passengers travel in the overlapping areas, and both their origins and destinations are in overlapping areas. Hence, they have multiple choices of routes for traveling.

1) PASSENGER IN-VEHICLE TRAVEL TIME COST

a: IN-VEHICLE TRAVEL TIME COST OF THE FIRST-TYPE PASSENGERS

Similarly, bus k on route l is taken as an example. The in-vehicle travel time of the first-type passengers that take bus k is the sum of the product of inner-stop travel time and the number of passengers traveling between two adjacent stops. The travel time from stop i to stop i+1 of bus k on route l is determined by inner-stop travel time, acceleration time, deceleration time, and dwell time. Inner-stop travel time equals inner-stop distance divided by average running speed on the road; the acceleration and deceleration when bus entering or leaving a stop are set as a constant, with the same absolute value; operating parameters (speed, acceleration, etc.) do not change with vehicle type. Therefore, the acceleration time, deceleration time, and inner-stop travel time can be calculated by (1) to (3):

$$t_{l,k,i}^{a} = \frac{2S_a}{v} \tag{1}$$

$$t_{l,k,i}^{d} = \frac{2S_d}{v} \tag{2}$$

$$t_{l,k}^{i,i+1} = \frac{S_l^{i,i+1} - S_a - S_d}{v} \tag{3}$$

where $t_{l,k,i}^{a}$ and $t_{l,k,i}^{d}$ represent the acceleration time and deceleration time for bus k of route l at stop i respectively, s; S_a and S_d denote the acceleration and deceleration distances, m; v is the average travel speed on the road, m/s; $t_{l,k}^{i,i+1}$ is inner-stop travel time from stop i to stop i+1 for bus k of route l, s; $S_l^{i,i+1}$ is the distance from stop i to stop i+1 on route l, m.

The dwell time at stop i is expressed by (4), which is the sum of door-opening time, door-closing time and the maximum value between passenger boarding time and alighting time. Passenger boarding time and alighting time are related to the crowding degree of bus carriage, that is, the more crowded the bus, the more time it will take for passengers to get on and off. This study uses the double logarithmic relationship between passenger boarding / alighting time and crowding degree, the number of boarding/alighting passengers proposed by [20] to estimate bus dwell time, as shown in (5) and (6).

$$t_{l,k,i}^{s} = t_o + t_c + \max(t_{l,k,i}^{b}, t_{l,k,i}^{g}) \tag{4}$$

$$\ln t_{l,k,i}^{b} = 0.965 + 0.926 \ln P_{l,k,i}^{b} + 0.085 \ln \eta_{l,k,i} \quad \eta_{l,k,i} > 0 \tag{5}$$

$$\ln t_{l,k,i}^{g} = 0.635 + 0.848 \ln P_{l,k,i}^{g} + 0.092 \ln \eta_{l,k,i} \quad \eta_{l,k,i} > 0 \tag{6}$$

where $t_{l,k,i}^{s}$ is the dwell time of bus k of route l at stop i, s; t_o and t_c denote door-opening time and door-closing time respectively, s; $t_{l,k,i}^{b}$ and $t_{l,k,i}^{g}$ are passenger boarding time and alighting time of bus k on route l at stop i, s; $P_{l,k,i}^{b}$ and $P_{l,k,i}^{g}$ are numbers of boarding and alighting passengers of bus k on route l at stop i; $\eta_{l,k,i}$ is the crowding degree of bus k of route l upon arriving at stop i.

$\eta_{l,k,i}$ is defined by the ratio of the number of standees in bus k upon arriving at stop i to the number of maximum allowed standees, which is shown by (7). (11) to (18) are used to calculate the number of boarding passengers ($P_{l,k,i}^{b}$) and alighting passengers ($P_{l,k,i}^{g}$). The passenger arrival rate at stop i is denoted by R_i and it equals $\sum_{x=1}^{I-i} \lambda_{i,i+x}$. $\lambda_{i,i+x}$ is the arrival rate of passengers that get on the bus at stop i and get off at stop $i + x$, where $x = 1, 2, \ldots, I$-i.

$$\eta_{l,k,i} = \frac{P_{l,k,i}^{st}}{A_{l,k} \times \zeta} \tag{7}$$

$$b_k + s_k = 1 \tag{8}$$

$$b_k s_k = 0 \tag{9}$$
$$A_{l,k} = b_k A^b_{l,k} + s_k A^s_{l,k} \tag{10}$$
$$P^{st}_{l,k,i} = P_{l,k,i} - N_{l,k} \tag{11}$$
$$N_{l,k} = b_k N^b_{l,k} + s_k N^s_{l,k} \tag{12}$$
$$P_{l,k,i} = \sum_{i=1}^{i-1} P^b_{l,k,i} - \sum_{i=2}^{i-1} P^g_{l,k,i} \tag{13}$$
$$P^w_{l,k,i} = \sum_{x=1}^{I-i} \lambda_{i,i+x} \cdot H^{k-1,k}_{l,} \tag{14}$$
$$cap^r_{l,k,i} = cap_{l,k} - P_{l,k,i} + P^g_{l,k,i} \tag{15}$$
$$cap_{l,k} = b_k cap^b_{l,k} + s_k cap^s_{l,k} \tag{16}$$
$$P^b_{l,k,i} = \begin{cases} P^w_{l,k,i} & P^w_{l,k,i} \leq cap^r_{l,k,i} \\ cap^r_{l,k,i} & P^w_{l,k,i} > cap^r_{l,k,i} \end{cases} \tag{17}$$
$$P^g_{l,k,i} = \sum_{e=1}^{i-1} \lambda_{e,i} H^{k-1,k}_l \tag{18}$$

where $P^{st}_{l,k,i}$ is the number of standees upon bus k of route l arriving at stop i; $A_{l,k}$ is the total area in the bus carriage for passengers to stand, m ^{2}; ζ is the number of passengers that can stand in unit area, passengers/ m^2; b_k and s_k are the independent variables to indicate which vehicle type is selected to execute the bus trip, which ensure that only one bus, no matter what the vehicle type is, would complete the bus trip; $A_{l,k} = A^b_{l,k}$ when $b_k = 1 s_k = 0$, which means large vehicle would complete the k-th trip; $A_{l,k} = A^s_{l,k}$ when $s_k = 1 b_k = 0$. $P_{l,k,i}$ is the number of passengers in the carriage upon bus k of route l arriving at stop i; $N_{l,k}$ is the number of seats in bus k; when $b_k = 1 s_k = 0$, $N_{l,k}$ equals the number of seats in large vehicle ($N^b_{l,k}$); when $s_k = 1 b_k = 0$, $N_{l,k}$ equals the number of seats in small vehicle ($N^s_{l,k}$); $P^w_{l,k,i}$ is the number of passengers waiting for route l when bus k arriving at stop i; $cap^r_{l,k,i}$ is the residual capacity of bus k on route l when it arriving at stop i; $cap_{l,k}$ is the capacity of bus k, passenger/veh; $\lambda_{e,i}$ is the passenger arrival rate from stop y to stop i, $e = 1, 2, \ldots, i-1$, passenger/s; $H^{k-1,k}_l$ is the headway between bus $k-1$ and bus k of route l, s.

The departure time of bus k at stop i on route l is determined by its departure time at the previous stop, inner-stop travel time, acceleration time, deceleration time and dwell time, which can be described by (19).

$$T^d_{l,k,i} = T^d_{l,k,i-1} + t^{i-1.i}_{l,k} + t^a_{l,k,i} + t^d_{l,k,i} + t^s_{l,k,i} \tag{19}$$

where $T^d_{l,k,i-1}$ and $T^d_{l,k,i}$ denote departure times of bus k at stop $i-1$ and stop i respectively.

When $i = 1$, the departure time of bus k of route l at stop1 equals to the sum of the arrival time of the first bus, the dwell time of the first bus at stop 1 and the cumulative departure intervals of k buses. (20) represents the departure time of bus k of route l at stop 1, and the headway between bus $k-1$ and bus k of route l is described by (21).

$$T^d_{l,k,1} = T^0_l + t^s_{l,k,1} + \sum_{f=1}^{k} \Delta t_f \tag{20}$$
$$H^{k-1,k}_l = T^d_{l,k,i} - T^d_{l,k-1,i} \tag{21}$$

where T^0_l is the arrival time of the first bus (bus 1) of route l; Δt_f is another independent variable to determine the departure interval between bus f of route l and bus $f-1$, min.

In summary, the in-vehicle travel time of each passenger and the passenger number of bus k of route l from stop i to stop $i+1$ are expressed by (22) and (23).

$$T^{i,i+1}_{l,k} = t^a_{l,k,i} + t^d_{l,k,i} + t^{i,i+1}_{l,k} + t^s_{l,k,i} \tag{22}$$
$$P^{i,i+1}_{l,k} = P_{l,k,i} + P^b_{l,k,i} - P^g_{l,k,i} \tag{23}$$

The in-vehicle travel time cost of the first-type passengers in bus k of route l is denoted by $V_{l,k}$ (I) and it is calculated by (24).

$$V_{l,k}\,(\text{I}) = \frac{c_1}{3600} \cdot \sum_{i=1}^{I_1-1} P^{i,i+1}_{l,k} T^{i,i+1}_{l,k} \tag{24}$$

where c_1 is the unit passenger in-vehicle travel time cost, RMB/h.

b: IN-VEHICLE TRAVEL TIME COST OF THE SECOND-TYPE PASSENGERS

The second-type passengers only travel in the overlapping area and they can select multiple routes for traveling. These passengers prefer to take the earliest arrived bus. The stops in the overlapping area are indicated by μ ($\mu = 1, 2, \ldots, \mu_{max}$). The set of all buses arriving at stop μ between bus $k-1$ and bus k of route l is denoted as $Y_\mu = \{y_1, y_2, y_3, y_4 \ldots\}$, and y_1 is the first arrival bus at stop μ from overlapping routes after the departure of bus $k-1$.

The headway between bus y_1 and bus $k-1$ of route l at stop μ is determined by their departure times. Similarly, the headway between bus y_1 and bus y_2 at stop μ equals to the difference of their departure times.

$$H^{y_1}_{k-1} = T^d_{y_1,\mu} - T^d_{l,k-1,\mu} \tag{25}$$
$$H^{y_1}_{y_2} = T^d_{y_2,\mu} - T^d_{y_1,\mu} \tag{26}$$

Due to the capacity constraint, the number of passengers that take bus y_1 is the minimum value of the number of passengers traveling between stop μ and μ_1(μ_1 is the maximum stop ID that bus y_1 served in the overlapping area, $\mu_1 \leq \mu_{\max}$), and the residual capacity of bus y_1, as shown in (27). Passengers traveling between stop μ and μ_1 include those arrive between the arrival of bus $k-1$ and bus y_1 and passengers left because of the capacity constraint of bus $k-1$.Among all the passengers left at stop μ of bus $k-1$, the number of passengers between two adjacent stops equals the product of total number of passengers left and the ratio of the passenger arrival rate of each travel OD to the total passenger arrival rate in the overlapping area.

$$P_{y_1} = \min(\sum_{x=1}^{\mu_1-\mu} \lambda_{\mu,\mu+x} \cdot H_{k-1}^{y_1} + \sum_{x=1}^{\mu_1-\mu} P_{k-1,\mu,\mu+x}^r, cap_{y_1}^r) \quad (27)$$

(28) and (29) formulate the number of passengers that fail to get on bus $k-1$ of route l at stop μ, and the number of passengers that travel from stop μ to stop $\mu+x$.

$$P_{l,k-1,\mu}^r = \max(P_{l,k-1,\mu}^w - cap_{l,k-1,\mu}^r, 0) \quad (28)$$

$$P_{l,k-1,\mu,\mu+x}^r = P_{l,k-1,\mu}^r \lambda_{\mu,\mu+x} \Big/ \left(\sum_{x=1}^{I-\mu} \lambda_{\mu,\mu+x} \right) \quad (29)$$

Similarly, the numbers of passengers taking bus $y_2, y_3, y_4, \ldots$ can be obtained.

The in-vehicle travel time T_{y_1} for passengers taking bus y_1 is the sum of in-vehicle travel time for passengers traveling between stop μ and μ_1.

If bus y_1 satisfy the capacity constraint at stop μ:

$$T_{y_1} = t_{y_1}^{\mu,\mu+x} \left(\sum_{x=1}^{\mu_1-\mu} \lambda_{\mu,\mu+x} \cdot H_{k-1}^{y_1} + \sum_{x=1}^{\mu_1-\mu} P_{k,\mu,\mu+x}^r \right) \quad (30)$$

Else, T_{y_1} can be got according to the ratio of residual capacity to actual passenger demand.

$$T_{y_1} = t_{y_1}^{\mu,\mu+x} \left(\sum_{x=1}^{\mu_1-\mu} \lambda_{\mu,\mu+x} \cdot H_{k-1}^{y_1} + \sum_{x=1}^{\mu_1-\mu} P_{k,\mu,\mu+x}^r \right) \times cap_{y_1}^r \Big/ \left(\sum_{x=1}^{\mu_1-\mu} \lambda_{\mu,\mu+x} \cdot H_{k-1}^{y_1} + \sum_{x=1}^{\mu_1-\mu} P_{k,\mu,\mu+x}^r \right) \quad (31)$$

$$t_{y_1}^{\mu,\mu+x} = t_{y_1,\mu}^a + t_{y_1}^{\mu,\mu+1} + t_{y_1,\mu+1}^d + t_{y_1,\mu+1}^s + t_{y_1,\mu+1}^a + t_{y_1}^{\mu+1,\mu+2} + t_{y_1,\mu+2}^d + t_{y_1,\mu+2}^s + t_{y_1,\mu+2}^a \cdots + t_{y_1,\mu+x}^d \quad (32)$$

where $t_{y_1}^{\mu,\mu+x}$ is the in-vehicle travel time for passengers taking bus y_1 from stop μ to stop $\mu+x$, s; $t_{y_1,\mu}^a$ is acceleration time for bus y_1 departing from stop μ, s; $t_{y_1,\mu+1}^d$ is deceleration time for bus y_1 entering stop $\mu+1$, s; $t_{y_1,\mu+1}^s$ is the dwell time of bus y_1 at stop $\mu+1$, s.

Therefore, the in-vehicle travel time cost for the second-type passengers of bus k of route l is calculated by (33):

$$V_{l,k}(\text{II}) = \frac{c_1}{3600} \cdot \sum_{\mu=1}^{\mu_{\max}-1} T_{Y_\mu} \quad (33)$$

2) PASSENGER WAITING TIME COST

a: THE FIRST-TYPE PASSENGER WAITING TIME COST

The first-type passenger waiting time of bus k of route l is determined by waiting time of passengers taking bus k and those have to take bus $k+1$ because of capacity constraint. The waiting time of passengers taking bus k is the half of the headway of two consecutive buses, i.e. $H_l^{k-1,k}/2$; the waiting time of passengers left is the sum of headway $H_l^{k-1,k}$ and half of headway $H_l^{k,k+1}$.

(34) shows the number of the first-type passengers who fail to get on bus k of route l at stop i.

$$n_{l,k,i} = \begin{cases} \text{R}_i H_l^{k-1,k} - cap_{l,k,i}^r & \text{R}_i H_l^{k-1,k} > cap_{l,k,i}^r \\ 0 & \text{R}_i H_l^{k-1,k} \le cap_{l,k,i}^r \end{cases} \quad (34)$$

The first-type passenger waiting time cost at stop i for the k-th bus trip of route l is formulated by (35).

$$w_{l,k,i} = \left(\text{R}_i H_l^{k-1,k} - n_{l,k,i}\right) \cdot H_l^{k-1,k}/2 + n_{l,k,i} \cdot \left(H_l^{k-1,k} + H_l^{k,k+1}/2\right) \quad (35)$$

The waiting time cost of the first-type passengers in bus k of route l is:

$$F_{l,k}(\text{I}) = \frac{c_2}{3600} \cdot \sum_{i=1}^{I_1-1} w_{l,k,i} \quad (36)$$

where c_2 is unit waiting time cost, RMB/h.

b: THE SECOND-TYPE PASSENGER WAITING TIME COST

The waiting time of the second-type passengers who take bus y_1 is determined by that of passengers arriving between the arrival of bus $k-1$ and bus y_1 and passengers left because of capacity constraint of bus $k-1$.

If the residual capacity of bus y_1 satisfies passenger demand, the waiting time of passengers taking bus y_1 is shown as follows.

$$w_{y_1,\mu} = \sum_{x=1}^{\mu_1-\mu} \lambda_{\mu,\mu+x} \cdot H_{k-1}^{y_1} \cdot H_{k-1}^{y_1}/2 + \sum_{x=1}^{\mu_1-\mu} P_{l,k-1,\mu,\mu+x}^r \cdot \left(H_{k-2}^{k-1} + H_{k-1}^{y_1}/2\right) \quad (37)$$

Else $P_{y_1} = cap_{y_1}^r$, the waiting time of passengers taking bus y_1 is calculated in terms of the ratio of residual capacity to actual passenger demand.

$$w_{y_1,\mu} = \left[\sum_{x=1}^{\mu_1-\mu} \lambda_{\mu,\mu+x} \cdot H_{k-1}^{y_1} \cdot H_{k-1}^{y_1}/2 + \sum_{x=1}^{\mu_1-\mu} P_{l,k-1,\mu,\mu+x}^r \cdot \left(H_{k-2}^{k-1} + H_{k-1}^{y_1}/2\right) \right] \times cap_{y_1}^r \Big/ \left(\sum_{x=1}^{\mu_1-\mu} \lambda_{\mu,\mu+x} \cdot H_{k-1}^{y_1} + \sum_{x=1}^{\mu_1-\mu} P_{l,k-1,\mu,\mu+x}^r \right) \quad (38)$$

Similarly, the waiting time of the passengers taking bus y_2, y_3, $y_4, \ldots$ is calculated, and that for the second-type passengers arriving between the arrivals of bus $k-1$ and k of route l at stop μ can be obtained. Thus, the waiting time cost of the second-type passengers of bus k of route l is as follows:

$$F_{l,k}\left(\text{II}\right)=\frac{c_2}{3600}\cdot\sum_{\mu=1}^{\mu_{\max}-1}w_{Y_\mu} \tag{39}$$

B. BUS COMPANY OPERATING COST

Bus dispatching with different vehicle types would generate different operating costs for the bus company. The bus company operating cost includes fixed cost and variable cost. Fixed cost refers to expenses that do not change with the workload in the short term, such as hourly salaries, business management cost, route maintenance and repair cost, etc. The variable cost refers to expenses that change with workload, such as fuel cost, wage surcharges issued by vehicle-kilometer. Therefore, the operating costs of the k-th large and small vehicles traveling from the original stop to the terminal stop are as follows:

$$O_{l,k}^b=G+c_3D_l \tag{40}$$
$$O_{l,k}^s=G+c_4D_l \tag{41}$$

where $O_{l,k}^b$ and $O_{l,k}^s$ are operating costs for the k-th large vehicle and small vehicle of route l respectively, RMB; G is the fixed operating cost for a bus running from the original stop to the terminal stop, RMB; D_l is the length of bus route l, km; c_3 and c_4 are unit operation costs for large vehicle and small vehicle, RMB/veh/km.

C. OBJECTIVE FUNCTION

The objective of the proposed scheduling model is minimizing the total cost of passenger travel time and bus company operation. The independent variables are departure intervals and vehicle types. The objective function is displayed by (42). (43) to (47) are model constraints.

$$\min Z=\sum_l\sum_k\left[b_k\left(M_{l,k}^b+O_{l,k}^b\right)+s_k\left(M_{l,k}^s+O_{l,k}^s\right)\right] \tag{42}$$
$$s.\,t.\ M_{l,k}^b=V_{l,k}^b+F_{l,k}^b \tag{43}$$
$$M_{l,k}^s=V_{l,k}^s+F_{l,k}^s \tag{44}$$
$$\Delta t_k=b_k\Delta t_k^b+s_k\Delta t_k^s \tag{45}$$
$$T-\Delta t_{\min}\leq\sum_k^K\Delta t_k\leq T \tag{46}$$
$$\Delta t_k\in\{\Delta t_{\min},\Delta t_{\min}+1,\ldots,\Delta t_{\max}\} \tag{47}$$

where $M_{l,k}^b$ and $M_{l,k}^s$ are passenger travel time costs for the k-th large vehicle and small vehicle of route l, including passenger in-vehicle travel time cost and stop waiting time cost. $V_{l,k}^b$ and $F_{l,k}^b$ are passenger in-vehicle travel time cost and waiting time cost for the k-th large vehicle of route l. $V_{l,k}^s$ and $F_{l,k}^s$ are in a similar way for small vehicle. Independent variable Δt_k is the departure interval between bus $k-1$ and bus k of route l. Δt_k^b and Δt_k^s denote the departure interval of large vehicle and small vehicle respectively. K is the total number of bus trips of route l during study period and (46) is used to restrict the number of bus trips by controlling the sum of intervals not exceeds the length of analyzed time period. (47) guarantees the departure interval should be an integer which lies among the minimum interval $\Delta t_{\min}$ and maximum interval $\Delta t_{\max}$.

To verify the model proposed, a multi-vehicle scheduling model without considering the influence of overlapping routes (the contrast model) is established, that is, the total cost of each bus route in the studied transit network is calculated separately, regardless of their interaction. Finally, the two scheduling models will be compared and analyzed. The form of the optimization objective function and constraint conditions of the contrast model are the same as that of the proposed model. The main difference between the two models is that in the contrast model there is only one type of passengers when calculating the passenger costs.

III. SOLUTION ALGORITHM

The proposed optimization model in this paper is a nonlinear model with multiple independent variables. The optimal solution of such model can be obtained by the solution method of integer programming. Enumeration method is one way to obtain the absolute optimal decisions of integer programming. However, there will be heavy computing burden in the case of large amount of data. Therefore, heuristic algorithms are designed to reduce the computation burden. The steps of the algorithm for multi-vehicle-type scheduling model under the influence of overlapping routes are displayed below:

Step 1: set initial value $k=0$, $T_0=0$, and k indicates the bus trip of route l during the study period; T_0 is the initial departure time.

Step 2: $k=k+1$, $\Delta t_k=0$.

Step 3: $l=0$, $l=l+1$.

Step 4: judge whether $T-\Delta t_{\min}\leq\sum_k\Delta t_k\leq T$? If not, back to step 2; else, turn to step 3.

Step 5: let $\Delta t_k^b=\Delta t_{\max}$, $\Delta t_k^s=\Delta t_{\max}$. Determine whether two types of vehicle scheduling plan satisfying the crowdedness conditions δ or not respectively, and $\delta=P_{l,k,i}/cap_{l,k}$. If it is true, the costs of the k-th large vehicle $C_{l,k}^b$ and small vehicle $C_{l,k}^s$ of route l are calculated; else, let $\Delta t_k=\Delta t_{\max}-1$, until $\Delta t_k=\Delta t_{\min}$.

Step 6: determine whether $l\leq L$? If yes, the k-th large vehicle cost $C_{l,k}^b$ and small vehicle cost $C_{l,k}^s$ of the next bus route are calculated under $\Delta t_k\in[\Delta t_{\min},\Delta t_{\min}+1,\ldots,\Delta t_{\max}]$; else, output minimum cost set of the k-th large vehicles of all routes and that for small vehicles, $\min\left\{C_{1,k}^b,C_{2,k}^b,\ldots,C_{L,k}^b\right\}$ and $\min\{C_{1,k}^s,C_{2,k}^s,\ldots,C_{L,k}^s\}$.

Step 7: combine the elements in the minimum cost sets of large vehicle and small vehicle and calculate the value of *flag* ($flag=\sum C/\sum\Delta t$); select optimal scheduling plan when the value of *flag* is minimum.

Step 8: turn to step 2; solve the optimal scheduling plans of bus $k+1$ of all routes.

The multi-vehicle-type scheduling model without considering the influence of overlapping routes is similar to that

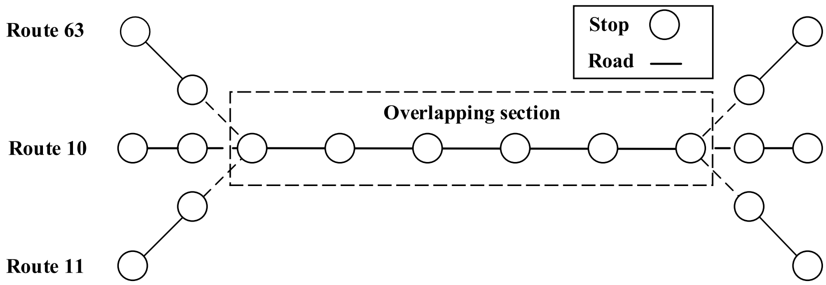

FIGURE 2. The overlapping relationship of the three bus routes in Harbin city.

under the influence of overlapping routes, and the solution algorithm is relatively simple. Steps are as follows:

Step 1: set initial value $k = 0$, $T_0 = 0$, and k indicates the bus trip of route l during the study period; T_0 is the initial departure time.

Step 2: $k = k + 1$, $\Delta t_k = 0$.

Step 3: determine whether $T - \Delta t_{\min} \leq \sum_k \Delta t_k \leq T$? If yes, turn to step 4; else, end the procedure and output results.

Step 4: let $\Delta t_k^b = \Delta t_{\max}$, $\Delta t_k^s = \Delta t_{\max}$. Determine whether two types of vehicle satisfying the crowdedness conditions δ or not respectively. If not, let $\Delta t_k = \Delta t_{\max} - 1$ until $\Delta t_k = \Delta t_{\min}$; else, calculate the costs of the k-th large vehicle $C_{\Delta t_k^b}$ and that of small vehicle $C_{\Delta t_k^s}$, as well as the unit time cost, $flag_{\Delta t_k} = C_{\Delta t_k} / \Delta t_k$.

Step 5: select the departure interval of large vehicle scheduling model with the minimum unit time cost, and that of small vehicle.

Step 6: compare the minimum unit time cost of two types of vehicles, and optimal departure interval and type for this trip corresponding to the smallest one between them.

Step 7: turn to step 2.

Step 8: repeat the steps above; find the optimal scheduling plan for each route.

IV. CASE STUDY

The evening peak hours (17:00-19:00) of bus routes 63, 10 and 11 in Harbin city, China are selected for the case study. These three routes are overlapping with each other, and the overlapping relationship of three routes in Harbin city is shown in Figure 2. At terminal stops, their departure headways are uniform with 5 minutes and the full fare for each route is 1 RMB. The length of route 63 is 10.1 km and the total number of bus stops is 21. Route 10 is 15 km long and has 23 stops. The figures for route 11 are 12.2 km and 23 respectively. These three routes have 6 common stops, which are located at an urban arterial, passing through business districts and large residential areas with large passenger volume.

Parameters of multi-vehicle-type scheduling models are obtained by field survey and data access. For example, the passenger OD was obtained based on smart card data. The bus travel times on the route were obtained based on the historical GPS data. The values of parameters are shown in Table1.

TABLE 1. Parameter values in the multi-vehicle-type scheduling model.

Parameter	Value	Unit	Parameter	Value	Unit
ζ	7	passengers /m^2	c_1	5.4	RMB/h
t_o	1/30	min	c_2	5.4	RMB/h
t_c	1/30	min	c_3	10	RMB (veh·km)
S_a	50	m	c_4	15	RMB (veh·km)
S_d	50	m	$N_{l,k}^b$	40	seat
v	5.6	m/s	$N_{l,k}^s$	20	seat
$\Delta t_{\min}$	3	min	$A_{l,k}^b$	12	m^2
$\Delta t_{\max}$	15	min	$A_{l,k}^s$	6	m^2
G	30	RMB (veh·h)	$cap_{l,k}^b$	120	passengers /veh
T	2	h	$cap_{l,k}^s$	60	passengers /veh
δ	3	-	-	-	-

Scheduling plans of route 63, 10 and 11 output by the multi-vehicle type scheduling model under the influence of overlapping routes in the evening peak hours (17:00-19:00) are listed in Table 2, which are recorded as plan 1, including departure timetable and vehicle type. Table 3 shows scheduling plans of the multi-vehicle-type scheduling model without considering the influence of overlapping routes for three bus routes, and they are noted as plan 2.

Table 4 shows the result of time consumption comparison between plan 1 and plan 2, including passenger in-vehicle travel time, passenger waiting time and passenger total travel time. For route 63, plan 1 can reduce 4.3% of passenger in-vehicle travel time, 10.8% of passenger waiting time and 5.3% of passenger total travel time compared with plan 2. Similar results can be witnessed for route 10 and 11, when route 10 execute scheduling strategy considering the impact of overlapping routes, passengers can save 7.6% of in-vehicle travel time, 9.1% of waiting time and 7.8% of total travel time. The figures for route 11 are 16.1%, 10.1% and 15.3% respectively. On the other hand, 9.8% decrease in passenger in-vehicle travel time, 6.5% decrease in passenger waiting

TABLE 2. Departure timetable and vehicle type of routes 63, 10 and 11 under plan 1.

Bus trip	Route 63		Route 10		Route 11	
ID	Departure time	Vehicle type	Departure time	Vehicle type	Departure time	Vehicle type
1	17:05	large	17:07	small	17:07	large
2	17:15	small	17:18	large	17:13	small
3	17:23	small	17:25	small	17:25	large
4	17:31	small	17:37	small	17:36	large
5	17:38	small	17:46	large	17:44	small
6	17:50	small	17:52	small	17:51	small
7	17:59	large	18:00	small	17:54	small
8	18:02	small	18:08	small	18:00	small
9	18:10	small	18:19	large	18:15	large
10	18:22	small	18:24	small	18:28	large
11	18:35	small	18:32	small	18:38	small
12	18:50	large	18:38	small	18:45	small
13			18:51	large	18:55	large
14			19:00	small		

TABLE 3. Departure timetable and vehicle type of routes 63, 10 and 11 under plan 2.

Bus trip	Route 63		Route 10		Route 11	
ID	Departure time	Vehicle type	Departure time	Vehicle type	Departure time	Vehicle type
1	17:08	large	17:07	large	17:07	large
2	17:13	small	17:14	large	17:13	small
3	17:20	small	17:20	large	17:22	large
4	17:27	small	17:25	small	17:30	large
5	17:38	large	17:28	large	17:40	large
6	17:42	small	17:37	large	17:43	small
7	17:56	large	17:46	small	17:50	small
8	18:02	small	17:56	small	18:00	small
9	18:10	small	18:10	large	18:08	small
10	18:25	large	18:22	large	18:21	large
11	18:38	small	18:29	small	18:30	large
12	18:50	small	18:36	small	18:35	large
13	18:58	small	18:46	small	18:50	large
14			18:54	large		

TABLE 4. Passenger travel time comparisons between plan 1 and plan 2.

Bus	Plan 1			Plan 2			Percentage (%)		
route	PIVTT	PWT	PTTT	PIVTT	PWT	PTTT			
63	9236	1656	10892	9635	1835	11470	-4.3	-10.8	-5.3
10	8825	1642	10467	9495	1792	11287	-7.6	-9.1	-7.8
11	8441	1280	9721	9801	1409	11210	-16.1	-10.1	-15.3
Total	26352	4728	31080	28931	5036	33967	-9.8	-6.5	-9.3

Note: PIVTT-passenger in-vehicle travel time, min; PWT-passenger waiting time, min; PTTT-passenger total travel time, min.

TABLE 5. Cost comparisons between plan 1 and plan 2.

Bus	Plan 1			Plan 2			Percentage (%)		
route	PTTC	BCOC	TC	PTTC	BCOC	TC			
63	980	1722	2702	1032	1903	2935	-5.3	-10.5	-8.6
10	942	2034	2976	1015	2234	3249	-7.7	-9.8	-9.2
11	847	2033	2880	1008	2103	3112	-19.0	-3.4	-8.1
Total	2821	5789	9360	3003	6240	10268	-6.5	-7.8	-9.7

Note: PTTC-passenger travel time cost (the sum of passenger in-vehicle travel time cost and passenger waiting time cost), RMB; BCOC-bus company operation cost, RMB; TC-total cost, RMB.

time and 9.3% decrease in passengers total travel time can be seen for the sum of three bus routes.

Table 5 shows the cost comparison results between plan 1 and plan 2, including passenger travel time cost, bus company operation cost and total cost. Compared with plan 2, plan1 can reduce 5.3% of passenger travel time cost, saving bus company operation cost and total cost by 10.5% and 8.6% for route 63. Meanwhile, plan 1 saves passenger travel time cost by 7.7%, bus company operation cost by 9.8% and total cost by 92% for route 10, with 19.0%, 3.4% and 8.1% for route 11. In total, 6.5% of passenger travel time cost, 7.8% of bus company operation cost and 9.7% of total cost can be saved under the scheduling strategy proposed in this paper.

From the above analysis we can find that the proposed model outperforms the model without considering the impact of overlapping routes both in time consumption and cost. This is mainly because that the proposed model can alleviate the uneven crowding degree among bus carriages and long passenger waiting time caused by bus bunching under routine scheduling plans, which does not consider the interaction among multiple routes and the influence on passenger demand. Thus, plan 1 is designed according to actual passenger demand, which is beneficial for the full utilization of public transit resources and improving operational efficiency.

V. CONCLUSION

This paper aims at the bus scheduling problem considering the influence of overlapping routes in the existing transit system. A multi-vehicle-type scheduling model to minimize the overall cost of passenger travel time and bus company operation is established, and a heuristic algorithm is designed to solve the proposed model. Three real bus routes are selected to validate the model. The following conclusions can be drawn from this study.

(i) The bus route overlap has a direct impact on bus dwell times and passenger waiting times. The impact depends on the arrival sequence of buses from overlapping routes, the headway between two consecutive buses and the number of stops in the overlapping section.

(ii) Compared with the multi-vehicle-type scheduling model without considering the influence of overlapping routes, the proposed model can reduce passenger in-vehicle travel time, waiting time and total travel time by 9.8%, 6.5% and 9.3% respectively. In addition, the proposed model can reduce passenger travel time cost, bus company operation cost and the total cost by 6.5%, 7.8% and 9.7% respectively.

(iii) The results of case study demonstrate that the proposed model outperforms other models that don't consider route overlap. Thus, it is meaningful to employ different vehicle types to solve the problems caused by route overlap.

The proposed model in this study produces the static bus scheduling plan for overlapping routes. In future the dynamic bus scheduling method under the impact of route overlap should be developed.

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