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New Results on Finite-Time Stability and Stabilization of Switched Positive Linear Time-Delay Systems

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ABSTRACT In this paper, the problems of finite-time stability and stabilization for switched positive linear time-delay systems under mode-dependent average dwell time (MDADT) are investigated. By proposing a novel multiple piecewise copositive Lyapunov-Krasovskii functional, new results on the sufficient conditions of finite-time stability are obtained. In order to solve the limitation of the multiple piecewise copositive Lyapunov-Krasovskii functional on designing controllers, a novel linear combinatorial copositive Lyapunov-Krasovskii functional is constructed, this provides a possibility for the numerical construction of the controller. Then by using state feedback controller, the finite-time stabilization is achieved. Finally, some simulation results are given to show the advantages of our methods.

INDEX TERMS Switched positive systems, finite-time stability, linear programming, mode-dependent average dwell time, time delay.

I. INTRODUCTION

Switched systems consist of several subsystems described by differential or difference equations and a switching signal orchestrating the switching among these subsystems. Due to the importance of switched systems in theoretical development and practical application, switched systems have attracted much attention from scholars [1]–[6], [7]. When the initial condition is non-negative, the state is always limited to non-negative. Such system is called a positive system. In recent decades, switched positive systems have attracted the wide interest of scholars because switching signals determine the switching rules among multiple positive subsystems. And the study of such systems does meet the practical needs of different fields, such as ecology, industrial engineering, communications and so on [8]–[10]. [11] and [12] have studied the stability problem for continuous-time switched

positive systems by proposing a weak common copositive Lyapunov function approach and a discretized copositive Lyapunov function, respectively. [13] has dealt with the stabilization problem of positive switched linear system with disturbance in both continuous-time case and discrete-time case by introducing a class of weak common linear copositive Lyapunov functions. A state feedback controller has been designed for singular positive Markov jump systems with partly known transition rates in [14]. In the above literature, linear copositive Lyapunov function and linear programming (LP) are considered to be an effective combination of tools to solve the problems of switching positive systems.

In practical applications, time delay phenomenon is widespread in the dynamic systems. Although many achievements have been made in time delay systems, due to the coupling relationship between the complexity of time delay and the particularity of switched positive systems, there are few research achievements on the stability of switched positive linear time-delay systems (SPLTSs) [15]–[18]. In [19],

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by developing a novel multiple discontinuous co-positive Lyapunov-Krasovskii functional approach, the conditions of stability are established for switched positive linear time-delay systems by linear programming approach under mode-dependent average dwell time switching. And exponential stability, L_1 -gain performance and controller design problems have been investigated for a class of switched positive systems with time-varying delays in [20]. [21] has discussed the problem of finite-time L_1 control for a class of switched positive linear systems with time-varying delays. And [22] showed that the proposed weak excitation condition for the delay-free case was also sufficient for the asymptotic stability of the switched positive linear system under unbounded time-varying delays. In [23], the problem of exponential L_1 output tracking control has been addressed for switched positive linear systems with time-varying delays under average dwell time (ADT) switching. [9] has designed an multi-mode observer for a class of switched positive linear time-delay systems. The finite-time control has been discussed for a class of discrete impulsive switched positive time-delay systems under asynchronous switching in [24]. [25] has investigated the problem of static output-feedback L_1 finite-time control for switched positive systems with time-varying delays. And then [26] has considered the problems of exponential stability and L_1 -gain analysis for positive time-delay Markovian jump systems with switching transition rates. Then H_∞ control has been investigated for positive time-delay systems with semi-Markov process and the control scheme has been applied for communication network model in [27].

Stability is a hot topic in the field of switched positive systems. However, most of the literatures study the traditional Lyapunov stability. Different from the Lyapunov stability, finite-time stability focuses on the transient behavior of a system response. More specifically, if once the time interval is fixed, the state of the system does not exceed a certain bound during this time interval, the system is said to be finite-time stable. In normal conditions, Lyapunov asymptotical stability is enough for practical applications. But there are some cases where large values of the state are not acceptable. For example, in chemical process, temperature, pressure or some other quantities require a fall back to the specified range within a fixed-time interval. Thus, introducing such a stability concept is really essential because of its wide applications in practice. Especially in the control of nonlinear dynamics, it needs to keep the states in a certain bound during a finite-time interval to avoid the saturations or the excitations, etc. Therefore, in this context, the study of finite-time stability is crucial. Some related results have been reported in references [4], [28]–[31]. In [10] the concept of finite-time stability was extended to switched positive linear systems and two sufficient conditions were given by using LP technique. Following [10], some results on finite time stability of switching positive linear systems were reported in references [21], [24], [25], [32]–[34]. But as mentioned earlier, time delay usually deteriorates the behavior of the system and even leads to the instability of the system, few literatures dealt with the

finite-time stability for SPLTSSs. And in the above literatures, only [21], [24], [25] dealt with the related issues of finite-time stability for SPLTSSs. Since Zhao et al. first proposed a multi-discontinuous Lyapunov function method in [35], few scholars have used this method to design controllers. Although a time-delay controller is designed for switched positive systems in [36], this study is only limited to the analysis of systems without time delay. For SPLTSSs, how to apply this method presented in [35] to the design the controllers based on linear programming technique is an issue worthy of further study. To the best knowledge of the authors, there has no research reported.

Motivated by the above illustrations, this article considers the problems of finite-time stability and stabilization for SPLTSSs under MDADT switching. First, sufficient conditions are developed for the finite-time stability of SPLTSSs under MDADT switching by using LP approach. By introducing a novel multiple piecewise copositive Lyapunov function, where each copositive Lyapunov function is allowed to have multiple segments during the run time of each subsystem being activated, a smaller lower bound on MDADT can be achieved. This means that more parameters can be adjusted for the MDADT. Then, finite-time stabilization conditions are presented for SPLTSSs under a state-feedback controller. In this paper, the controller design is set as a linear programming problem. By designing a numerical construction of controllers, the controllers can be solved explicitly. For switched positive linear time-delay systems, how to design a numerical construction of controller by using multiple piecewise copositive Lyapunov-Krasovskii functional is an interesting and worthwhile issue. By proposing a novel linear combinatorial copositive Lyapunov-Krasovskii functional, a numerical construction approach of controller is proposed explicitly. And the construction idea of this linear combinatorial copositive Lyapunov-Krasovskii functional method can also be extended to the construction of linear combinatorial copositive Lyapunov function, so as to solve the problems such as the numerical construction forms of controllers or observers. In addition, compared with the existing iterative method in [21], [37]–[40] based on linear matrix inequalities, the numerical construction method of controller presented in this paper has lower complexity and is easier to calculate.

This paper is organized as follows. Section II gives some necessary concepts and definitions of SPLTSSs. In Section III, new results on finite-time stability and stabilization are developed for SPLTSSs with MDADT switching. Two numerical examples are given in Section IV to show the efficiency of our proposed methods. Finally, the paper is concluded in Section V.

Notations: For a real matrix A , A^T denotes its transpose, $A > 0$ ($A \geq 0$) means that all elements of matrix A are positive (i.e. $a_{ij} > 0$) (non-negative, i.e. $a_{ij} \geq 0$). \mathbb{R} , \mathbb{R}^n and $\mathbb{R}^{n \times n}$ denote the field of real numbers, n -dimensional Euclidean space, and the space of $n \times n$ matrix with real entries, respectively. \mathbb{R}_+^n stands for the non-negative orthant in \mathbb{R}^n .

Denote by \mathbb{N} and \mathbb{K} the sets of non-negative numbers and natural numbers, respectively. I denotes identity matrix with an appropriate dimension. Let $1_n = (1, \dots, 1)^T$ with n entries and $1_n^{(i)} = (0, \dots, 0, 1, 0, \dots, 0)^T$ with the i th entry being 1. For $t \in [t_i, t_{i+1})$, $\mathfrak{R}_{\sigma(t_i)} = \{0, 1, \dots, G_{\sigma(t_i)} - 1\}$, $G_{\sigma(t_i)}$ denotes the number of segments of each Lyapunov function during the operation time on each activated subsystem.

II. SYSTEM DESCRIPTION AND PRELIMINARIES

Consider the following switched linear time-delay system:

$$\begin{aligned} x(t+1) &= A_{\sigma(t)}x(t) + A_{d\sigma(t)}x(t-h) + B_{\sigma(t)}u(t), \\ x(\theta) &= \phi(\theta) \in \mathbb{R}_+^n, \forall \theta \in [-h, -h+1, \dots, 0], \end{aligned} \quad (1)$$

where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^r$ are the system state and control input, respectively, $\phi(\theta)$ is the initial condition, h denotes the time delay. $A_{\sigma(t)} \in \mathbb{R}^{n \times n}$, $A_{d\sigma(t)} \in \mathbb{R}^{n \times n}$ and $B_{\sigma(t)} \in \mathbb{R}^{n \times r}$ are system matrices. $\sigma(t)$ denotes switching signal of system and takes values in a finite set $S = \{1, 2, \dots, J\}$. As commonly assumed in the literature, $\sigma(t)$ is continuous from the right everywhere for a switching sequence $0 = t_0 \leq t_1 \leq t_2 \leq \dots$. The $\sigma(t_i)$ th subsystem is said to be activated when $t \in [t_i, t_{i+1})$, $i \in \mathbb{N}$. The system state does not jump at the switching instant.

First, some definitions and lemmas are given as follows.

Definition 1: [15] Given any initial conditions $\phi(\theta) \in \mathbb{R}_+^n$, $\theta \in [-h, -h+1, \dots, 0]$ and any switching signal $\sigma(t)$, if for any non-negative initial condition, the corresponding trajectory $x(t) \in \mathbb{R}_+^n$ and all $u(t) \geq 0$ at all non-negative integers t , then system (1) is said to be a switched positive linear time-delay system.

Definition 2: [32] The time delay system (1) is a switched positive linear system under any switching signals if and only if $A_p \geq 0, \forall p \in S, A_{dp} \geq 0, \forall p \in S$ and $B_p \geq 0, \forall p \in S$.

Definition 3: [21] Given positive constants c_1, c_2 with $c_1 < c_2, T_f \in \mathbb{K}$, and two vectors $\delta > \varepsilon > 0$ and a switching signal $\sigma(t)$. If it holds that

$$\max_{t \in [-h, -h+1, \dots, 0]} \phi(t)^T \delta \leq c_1 \Rightarrow x(t)^T \varepsilon < c_2, \quad t = 1, 2, \dots, T_f, \quad (2)$$

where $\phi(t)$ is a vector-valued initial function defined on $[-h, -h+1, \dots, 0]$, then the switched positive linear time-delay system (1) with $u(t) \equiv 0$ is finite-time stable with respect to $(c_1, c_2, \delta, \varepsilon, T_f, h, \sigma(t))$.

Definition 4: [5] For any time interval $[t_1, t_2](t_2 > t_1 > t_0)$, $N_{\sigma p}(t_2, t_1)$ represents the total switching times when subsystem p is activated, and $T_p(t_2, t_1)$ represents the total running time of subsystem $p, p \in S$. If for any given constants $N_{0p} > 0$ and $\tau_{ap} > 0$, we have

$$N_{\sigma p}(t_2, t_1) \leq N_{0p} + T_p(t_2, t_1)/\tau_{ap}, \quad (3)$$

then τ_{ap} and N_{0p} are called MDADT and the chatter bound. Without loss of generality, as shown in most references, we let $N_{0p} = 0$ and $t_0 = 0$.

III. MAIN RESULTS

In this paper, we aim to establish finite-time stability conditions for system (1). Based on the multiple piecewise copositive Lyapunov-Krasovskii functional (MPCLKF) approach proposed in [36], we first analyze the problem of finite-time stability and then we establish the stabilization conditions for system (1).

A. FINITE-TIME STABILITY

This section will focus on the problem of finite-time stability for switched positive linear time-delay system (1) with $u(t) \equiv 0$.

Theorem 1: Consider system (1) with $u(t) \equiv 0$, for a given time constant $T_f, h > 0, \lambda_p > 0, 0 < \eta_p \leq 1, \bar{\mu}_p > 1$ satisfying $\bar{\mu}_p \eta_p^{G_p-1} > 1$, and two vectors $\delta > \varepsilon > 0$, if there exist positive vectors $v_p^i, \mu_p^i, \varrho_p^i, i \in \mathfrak{R}_p, p \in S$ and positive constants $\xi_1, \xi_2, \xi_3, \xi_4$, such that $i \in \mathfrak{R}_p, \forall (p, m) \in (S \times S), p \neq m$, the following inequalities hold,

$$A_{dp}^T v_p^i - e^{-\lambda_p h} I \mu_p^i - e^{-\lambda_p h} I \varrho_p^i < 0, \quad (4a)$$

$$A_p^T v_p^i - e^{-\lambda_p} I v_p^i + I \mu_p^i + h I \varrho_p^i < 0, \quad (4b)$$

$$v_p^i \leq \eta_p v_p^{i-1}, \quad i \neq 0, \quad (4c)$$

$$v_p^0 \leq \bar{\mu}_p v_m^{G_m-1}, \quad (4d)$$

$$\mu_p^i \leq \eta_p \mu_p^{i-1}, \quad i \neq 0, \quad (4e)$$

$$\mu_p^0 \leq \bar{\mu}_p \mu_m^{G_m-1}, \quad (4f)$$

$$\varrho_p^i \leq \eta_p \varrho_p^{i-1}, \quad i \neq 0, \quad (4g)$$

$$\varrho_p^0 \leq \bar{\mu}_p \varrho_m^{G_m-1}, \quad (4h)$$

$$c_1 \eta_{\sigma(0)}^{G_{\sigma(0)}-1} (\xi_2 + \xi_3 + \xi_4(h-1)) < c_2 \xi_1 e^{\lambda_p K_f}, \quad (4i)$$

$$\xi_1 \varepsilon < v_p^i < \xi_2 \delta, \quad \mu_p^i < \xi_3 \delta, \quad \varrho_p^i < \xi_4 \delta, \quad (4j)$$

then under MDADT

$$\begin{aligned} \tau_{ap} > \tau_{ap}^* &= T_f \ln(\bar{\mu}_p \eta_p^{G_p-1}) / [\ln(c_2 \xi_1 e^{\lambda_p K_f}) \\ &\quad - \ln(c_1 \eta_{\sigma(0)}^{G_{\sigma(0)}-1}) - \ln(\xi_2 + \xi_3 + \xi_4(h-1))], \end{aligned} \quad (5)$$

the system is finite-time stable with respect to $(c_1, c_2, \delta, \varepsilon, T_f, h, \sigma(t))$.

Proof: For given $T_f > 0$ and $t_0 = 0$, suppose we denote the switching sequence as $0 \leq t_1 \leq \dots \leq t_l \leq t_{l+1} \leq \dots \leq t_{N_{\sigma}(T_f, t_0)}$ on time interval $[t_0, T_f]$ with $\sum_{p \in S} N_{\sigma p}(T_f, t_0) = N_{\sigma}(T_f, t_0)$. Choose the following MPCLKF for SPLTS (1):

$$V_{\sigma(t)}(t, x_t) = V_{\sigma(t)}^1(t, x_t) + V_{\sigma(t)}^2(t, x_t) + V_{\sigma(t)}^3(t, x_t), \quad (6)$$

where

$$V_{\sigma(t)}^1(t, x_t) = x^T(t) v_p^i,$$

$$V_{\sigma(t)}^2(t, x_t) = \sum_{j=t-h}^{t-1} e^{\lambda_p(j-t+1)} x^T(j) \mu_p^i,$$

$$V_{\sigma(t)}^3(t, x_t) = \sum_{q=-h+1}^0 \sum_{j=t+q-1}^{t-1} e^{\lambda_p(j-t+1)} x^T(j) \varrho_p^i.$$

By defining $\Delta V(t, x_t) = V(t, x_{t+1}) - e^{-\lambda_p} V(t, x_t)$, for any $\sigma(t) = p \in S$,

$$\begin{aligned} \Delta V_p(t, x_t) &\leq x^T(t)(A_p^T v_p^i - e^{-\lambda_p} v_p^i + I \mu_p^i + h I \varrho_p^i) \\ &\quad + x^T(t-h)(A_{dp}^T v_p^i - e^{-\lambda_p h} I \mu_p^i \\ &\quad - e^{-\lambda_p h} I \varrho_p^i). \end{aligned} \tag{7}$$

From (4a) - (4b), it arrives at

$$V_p(t, x_{t+1}) \leq e^{-\lambda_p} V_p(t, x(t)). \tag{8}$$

From (8), for any $t \in L_{\sigma(t_l)}^i$, we get that

$$V_{\sigma(t_l)}^i(t) \leq e^{-\lambda_{\sigma(t_l)}(t-(t_l+J_{\sigma(t_l)}^i))} V_{\sigma(t_l)}^i(t_l + J_{\sigma(t_l)}^i). \tag{9}$$

Then it follows from (4c), (4e), (4g) and (9) that

$$\begin{aligned} V_{\sigma(t_l)}^{G_{\sigma(t_l)}-1}(t_{l+1} - 1) &\leq e^{-\lambda_{\sigma(t_l)}(t_{l+1}-1-(t_l+J_{\sigma(t_l)}^{G_{\sigma(t_l)}-1}))} \\ &\quad \times V_{\sigma(t_l)}^{G_{\sigma(t_l)}-1}(t_l + J_{\sigma(t_l)}^{G_{\sigma(t_l)}-1}) \\ &\leq e^{-\lambda_{\sigma(t_l)}(t_{l+1}-1-(t_l+J_{\sigma(t_l)}^{G_{\sigma(t_l)}-1}))} \eta_{\sigma(t_l)} \\ &\quad \times V_{\sigma(t_l)}^{G_{\sigma(t_l)}-2}(t_l + J_{\sigma(t_l)}^{G_{\sigma(t_l)}-1}) \\ &\leq \eta_{\sigma(t_l)} e^{-\lambda_{\sigma(t_l)}(t_{l+1}-1-(t_l+J_{\sigma(t_l)}^{G_{\sigma(t_l)}-2}))} \\ &\quad \times V_{\sigma(t_l)}^{G_{\sigma(t_l)}-2}(t_l + J_{\sigma(t_l)}^{G_{\sigma(t_l)}-2}) \\ &\quad \dots \\ &\leq \eta_{\sigma(t_l)}^{G_{\sigma(t_l)}-1} e^{-\lambda_{\sigma(t_l)}(t_{l+1}-1-(t_l+J_{\sigma(t_l)}^0))} \\ &\quad \times V_{\sigma(t_l)}^0(t_l + J_{\sigma(t_l)}^0). \end{aligned} \tag{10}$$

Through continuous integrations, for $T \in [0, T_f)$, (10) can be rewritten as

$$V_{\delta(T)}(T) \leq \prod_{p \in S} (\bar{\mu}_p \eta_p^{G_p-1})^{N_{\sigma p}} \eta_{\sigma(0)}^{G_{\sigma(0)}-1} e^{-\lambda_p T} V_{\sigma(0)}^0(x(0)). \tag{11}$$

Noting that $\bar{\mu}_p \eta_p^{G_p-1} > 1$, by Definition 4, one can obtain that

$$\begin{aligned} V_{\delta(T)}(T) &\leq \exp\{\prod_{p \in S} N_{0p} \ln(\bar{\mu}_p \eta_p^{G_p-1})\} \\ &\quad \times \exp\{\max_{p \in S} [\frac{\ln(\bar{\mu}_p \eta_p^{G_p-1})}{\tau_{ap}} - \lambda_p] \times T_f\} \\ &\quad \times \eta_{\sigma(0)}^{G_{\sigma(0)}-1} V_{\sigma(0)}^0(x(0)). \end{aligned} \tag{12}$$

Given two vectors $\delta \succ \varepsilon > 0$, from (4j) and (6), it follows that

$$V_{\sigma(T)}(T) \geq \xi_1 x^T(T) \varepsilon. \tag{13}$$

$$\begin{aligned} V_{\sigma(0)}^0(0) &\leq \xi_2 x^T(0) \delta + \xi_3 \sum_{j=-h}^{-1} \sup_{-h \leq \theta \leq 0} \{x^T(0 + \theta) \delta\} \\ &\quad + \xi_4 \sum_{j=-h}^{-1} (j - k + h) \\ &\quad \times \sup_{-h \leq \theta \leq 0} \{x^T(0 + \theta) \delta\}. \end{aligned} \tag{14}$$

Combine (12) with (13)-(14), by Definition 3, it leads to

$$\begin{aligned} x^T(T) \varepsilon &\leq \frac{1}{\xi_1} \eta_{\sigma(0)}^{G_{\sigma(0)}-1} e^{\{\max_{p \in S} [\frac{\ln(\bar{\mu}_p \eta_p^{G_p-1})}{\tau_{ap}} - \lambda_p] T_f\}} (\xi_2 + \xi_3 \\ &\quad + \xi_4(h-1)) \times \sup_{-h \leq \theta \leq 0} \{x^T(0 + \theta) \delta\} \\ &\leq \frac{c_1}{\xi_1} \eta_{\sigma(0)}^{G_{\sigma(0)}-1} e^{\max_{p \in S} [\frac{\ln(\bar{\mu}_p \eta_p^{G_p-1})}{\tau_{ap}} T_f - \lambda_p T_f]} \\ &\quad \times e^{\ln(\xi_2 + \xi_3 + \xi_4(h-1))}. \end{aligned} \tag{15}$$

Substituting (5) into (15), we obtain

$$x^T(T) \varepsilon < c_2. \tag{16}$$

According to Definition 3, we can conclude that the switched positive system (1) is finite-time stable with respect to $(c_1, c_2, \delta, \varepsilon, T_f, h, \sigma(t))$. This proof is completed. \square

Remark 1: In Theorem 1, $i \in \mathfrak{R}_p$ is introduced. Just illustrated in Section Notation, for $p \in S$, $\mathfrak{R}_p = \{0, 1, \dots, G_p - 1\}$, G_p denotes the number of segments of each Lyapunov function during the operation time on each activated subsystem. Due to the piecewise continuity of each Lyapunov function required in MPCLKF during the operation time on each activated subsystem, thus i may not be limited to 1 in Theorem 1. Different from the classical multiple copositive Lyapunov function, by using MPCLKF approach, two additional degrees of freedom can be gained when the MDADT is obtained, namely η_p and G_p . Thus, we can adjust these parameters arbitrarily according to actual engineering requirements so that we can get lower bounds of small dwell time for each subsystem.

B. FINITE-TIME STABILIZATION

In this section, we consider the synchronous switching between the controller and the subsystem, and do not consider the delay between the controller and the subsystem, so the control occurs in the case of no delay.

In the sequel, we design a feedback controller for system (1) as follows:

$$u_p(t) = K_p x(t), \tag{17}$$

where $K_p \in \mathbb{R}^{r \times n}$ is the controller gain designed in the form of $\sum_{i=1}^r 1_r^{(i)} z_i^T / (1_r^T B_p^T v_p^i)$ where v_p and z_i , $i = \{1, 2, \dots, r\}$ are variables to be determined.

The following theorem provide sufficient conditions for the stabilization of system (1).

Theorem 2: Consider system (1), for a given time constant $T_f, h > 0, \lambda_p > 0, 0 < \eta_p \leq 1, \bar{\mu}_p > 1$ satisfying $\bar{\mu}_p \eta_p^{G_p-1} > 1$, and two vectors $\delta \succ \varepsilon > 0$, if there exist positive vectors $v_p, \mu_p^i, \varrho_p^i, i \in \mathfrak{R}_p, p \in S$, vectors $z_i \in \mathbb{R}^n, (i = 1, 2, \dots, r)$ and $z \in \mathbb{R}^n$, and positive constants $\xi_1, \xi_2, \xi_3, \xi_4$ such that $\forall i \in \mathfrak{R}_p, \forall (p, m) \in (S \times S), p \neq q$, inequalities (4a) - (4e) and the following inequalities hold,

$$A_p^T v_p + z - e^{-\lambda_p} I v_p + I \mu_p^i + h I \varrho_p^i < 0, \tag{18a}$$

$$A_{dp}^T v_p - e^{-\lambda_p h} I \mu_p^i - e^{-\lambda_p h} I \varrho_p^i < 0, \tag{18b}$$

$$z_i \leq z, \tag{18c}$$

$$1_r^T B_p^T v_p A_p + B_p \sum_{i=1}^r 1_r^{(i)} z_i^T \geq 0, \quad (18d)$$

$$\xi_1 \varepsilon < v_p < \xi_2 \delta, \mu_p^i < \xi_3 \delta, \varrho_p^i < \xi_4 \delta, \quad (18e)$$

then under the state feedback controller (17), the closed-loop system (1) is finite-time stable with respect to $(c_1, c_2, \delta, \varepsilon, T_f, h, \sigma(t))$ for the switching signal $\sigma(t)$ with MDADT (5).

Proof: Similar to the proof the Theorem 1, for given $T_f > 0$ and $t_0 = 0$, we denote the switching sequence as $0 \leq t_1 \leq \dots \leq t_l \leq t_{l+1} \leq \dots \leq t_{N_\sigma(T_f, t_0)}$ on time interval $[t_0, T_f]$ with $\sum_{p \in S} N_{\sigma p}(T_f, t_0) = N_{\sigma}(T_f, t_0)$.

Choose the following MPCLKF for SPLTS (1):

$$V_{\sigma(t)}(t, x_t) = V_{\sigma(t)}^1(t, x_t) + V_{\sigma(t)}^2(t, x_t) + V_{\sigma(t)}^3(t, x_t), \quad (19)$$

where

$$V_{\sigma(t)}^1(t, x_t) = x^T(t) v_p,$$

$$V_{\sigma(t)}^2(t, x_t) = \sum_{j=t-h}^{t-1} e^{\lambda_p(j-t+1)} x^T(j) \mu_p^i,$$

$$V_{\sigma(t)}^3(t, x_t) = \sum_{q=-h+1}^0 \sum_{j=t+q-1}^{t-1} e^{\lambda_p(j-t+1)} x^T(j) \varrho_p^i.$$

By defining $\Delta V(t, x_t) = V(t, x_{t+1}) - e^{-\lambda_p} V(t, x_t)$, for any $\sigma(t) = p \in S$, from (19), we can obtain

$$\Delta V_p^1(t, x_t) = x^T(t) (A_{\sigma(t)}^T v_p + K_{\sigma(t)}^T B_{\sigma(t)}^T v_p - e^{-\lambda_p} I v_p) + x(t-h)^T A_{\sigma(t)}^T v_p \quad (20)$$

$$\Delta V_p^2(t, x_t) = x^T(t) \mu_p^i - e^{-\lambda_p h} x^T(t-h) \mu_p^i \quad (21)$$

$$\Delta V_p^3(t, x_t) \leq h x^T(t) \varrho_p^i - e^{-\lambda_p h} x^T(t-h) \varrho_p^i \quad (22)$$

From (20) - (22), we can further obtain that

$$\Delta V(t, x_t) \leq x^T(t) (A_p^T v_p + z_i B_p^T v_p - e^{-\lambda_p} I v_p + I \mu_p^i + h I \varrho_p^i) + x^T(t-h) (A_{dp}^T v_p - e^{-\lambda_p h} I \mu_p^i - e^{-\lambda_p h} I \varrho_p^i) \quad (23)$$

In view of $1_r^T B_p^T v_p > 0$, from (18a)- (18c), it leads to $\Delta V(t, x_t) \leq 0$. Thus, we can acquire that

$$V(t, x_{t+1}) \leq e^{-\lambda_p} V(t, x_t) \quad (24)$$

In the sequel, similar to the proof in Theorem 1, we can conclude that under the state feedback controller (17), the closed-loop system (1) is finite-time stable with respect to $(c_1, c_2, \delta, \varepsilon, T_f, h, \sigma(t))$ for the switching signal $\sigma(t)$ with MDADT (5). This proof is completed. \square

Remark 2: Different from the construction of the multiple piecewise copositive Lyapunov-Krasovskii functional in Theorem 1, a novel linear combinatorial copositive Lyapunov-Krasovskii functional is proposed in Theorem 2. For example, for the part with no time delay, the traditional continuous copositive Lyapunov function is used, however, for the part with time delay, the multiple piecewise copositive Lyapunov-Krasovskii functional is used. Note that for the switched

positive systems with time delay, the multiple piecewise Lyapunov-Krasovskii functional approach proposed in Theorem 1 is not suitable for designing the numerical structure of the controller. Thus, by constructing a novel linear combinatorial copositive Lyapunov-Krasovskii functional, this provides a possibility for the numerical construction of the controller. And the limitation of multiple piecewise copositive Lyapunov-Krasovskii functional is removed and the conservatism is reduced. Similarly, in order to solve the limitation that multiple piecewise copositive Lyapunov function method cannot be used to design controller, the construction idea of this linear combinatorial copositive Lyapunov-Krasovskii functional method can also be extended to the construction of linear combinatorial copositive Lyapunov function, so as to solve the problems such as the numerical construction forms of controllers or observers. In addition, compared with the existing iterative method in [21], [37]–[40] based on linear matrix inequalities, the numerical construction method of controller presented in this paper has lower complexity and is easier to calculate.

If we use the classical continuous Lyapunov-Krasovskii functional, we can obtain the following corollary 1 and corollary 2.

Corollary 1: Consider system (1) with $u(t) \equiv 0$, for a given time constant $T_f, h > 0, \lambda_p > 0, 0 < \eta_p \leq 1, \bar{\mu}_p > 1$ satisfying $\bar{\mu}_p \eta_p^{G_p-1} > 1$, and two vectors $\delta > \varepsilon > 0$, if there exist positive vectors $v_p, \mu_p, \varrho_p, p \in S$ and positive constants $\xi_1, \xi_2, \xi_3, \xi_4$, such that $\forall (p, m) \in (S \times S), p \neq m$, the following inequalities hold,

$$A_{dp}^T v_p - e^{-\lambda_p h} I \mu_p - e^{-\lambda_p h} I \varrho_p < 0, \quad (25a)$$

$$A_p^T v_p - e^{-\lambda_p} I v_p + I \mu_p + h \varrho_p < 0, \quad (25b)$$

$$v_p \leq \bar{\mu}_p v_m, \quad (25c)$$

$$\mu_p \leq \bar{\mu}_p \mu_m, \quad (25d)$$

$$\varrho_p \leq \bar{\mu}_p \varrho_m, \quad (25e)$$

$$c_1 \eta_{\sigma(0)}^{G(0)-1} (\xi_2 + \xi_3 + \xi_4 (h-1)) < c_2 \xi_1 e^{\lambda_p K_f}, \quad (25f)$$

$$\xi_1 \varepsilon < v_p < \xi_2 \delta, \mu_p < \xi_3 \delta, \varrho_p < \xi_4 \delta, \quad (25g)$$

then under MDADT

$$\tau_{ap} > \tau_{ap}^* = T_f \ln(\bar{\mu}_p) / [\ln(c_2 \xi_1 e^{\lambda_p T_f}) - \ln(\xi_2 + \xi_3 + \xi_4 (h-1))] \quad (26)$$

, the system is finite-time stable with respect to $(c_1, c_2, \delta, \varepsilon, T_f, h, \sigma(t))$.

Corollary 2: Consider system (1), for a given time constant $T_f, h > 0, \lambda_p > 0, 0 < \eta_p \leq 1, \bar{\mu}_p > 1$ satisfying $\bar{\mu}_p \eta_p^{G_p-1} > 1$, and two vectors $\delta > \varepsilon > 0$, if there exist positive vectors $v_p, \mu_p, \varrho_p, p \in S$, vectors $z_i \in \mathbb{R}^n, (i = 1, 2, \dots, r)$ and $z \in \mathbb{R}^n$, and positive constants $\xi_1, \xi_2, \xi_3, \xi_4$ such that $\forall (p, m) \in (S \times S), p \neq q$, inequalities (25a) - (25c) and the following inequalities hold,

$$A_p^T v_p + z - e^{-\lambda_p} I v_p + I \mu_p + h I \varrho_p < 0, \quad (27a)$$

$$A_{dp}^T v_p - e^{-\lambda_p h} I \mu_p - e^{-\lambda_p h} I \varrho_p < 0, \quad (27b)$$

$$z_i \leq z, \quad (27c)$$

$$1_r^T B_p^T v_p A_p + B_p \sum_{i=1}^r 1_r^{(i)} z_i^T \geq 0, \quad (27d)$$

$$\xi_1 \varepsilon < v_p < \xi_2 \delta, \mu_p < \xi_3 \delta, \varrho_p < \xi_4 \delta, \quad (27e)$$

then under the state feedback controller (17), the closed-loop system (1) is finite-time stable with respect to $(c_1, c_2, \delta, \varepsilon, T_f, h, \sigma(t))$ for the switching signal $\sigma(t)$ with MDADT (26).

Remark 3: Compared with the existing references, our results obtained in Corollaries 1 and 2 have two main advantages. First, compared with [21], (26) obtained in Corollary 1 is mode-dependent, this means each subsystem has its own dwell time, which provides more flexibility. Next, we propose the numerical solution of controller gain explicitly. Compared with the iterative solution controller method presented in [21], [37], our method is less conservative. When $G_p = 1$ in Theorem 1, MPCLKF transforms into MCLKF, thus Theorem 1 transforms into Corollary 1.

By replacping MDADT with ADT in Theorems 1-2, we can obtain corollary 3 and corollary 4.

Corollary 3: Consider system (1) with $u(t) \equiv 0$, for a given time constant $T_f, h > 0, \lambda > 0, 0 < \eta \leq 1, \bar{\mu} > 1$ satisfying $\bar{\mu}\eta^{G-1} > 1$, and two vectors $\delta > \varepsilon > 0$, if there exist positive vectors $v_p^i, \mu_p^i, \varrho_p^i, i \in \mathfrak{R}_p, p \in S$ and positive constants $\xi_1, \xi_2, \xi_3, \xi_4$, such that $\forall i \in \mathfrak{R}_p, \forall (p, m) \in (S \times S), p \neq q$, the following inequalities hold,

$$A_{dp}^T v_p^i - e^{-\lambda h} I \mu_p^i - e^{-\lambda h} I \varrho_p^i < 0, \quad (28a)$$

$$A_p^T v_p^i - e^{-\lambda} I v_p^i + I \mu_p^i + h \varrho_p^i < 0, \quad (28b)$$

$$v_p^i \leq \eta v_p^{i-1}, i \neq 0, \quad (28c)$$

$$v_p^0 \leq \bar{\mu} v_m^{G_m-1}, \quad (28d)$$

$$\mu_p^i \leq \eta \mu_p^{i-1}, i \neq 0, \quad (28e)$$

$$\mu_p^0 \leq \bar{\mu} \mu_m^{G_m-1}, \quad (28f)$$

$$\varrho_p^i \leq \eta \varrho_p^{i-1}, i \neq 0, \quad (28g)$$

$$\varrho_p^0 \leq \bar{\mu} \varrho_m^{G_m-1}, \quad (28h)$$

$$c_1 \eta_{\sigma(0)}^{G(0)-1} (\xi_2 + \xi_3 + \xi_4 (h-1)) < c_2 \xi_1 e^{\lambda K_f}, \quad (28i)$$

$$\xi_1 \varepsilon < v_p < \xi_2 \delta, \mu_p^i < \xi_3 \delta, \varrho_p^i < \xi_4 \delta, \quad (28j)$$

then under ADT

$$\tau_a > \tau_a^* = T_f \ln(\bar{\mu} \eta^{G-1}) / [\ln(c_2 \xi_1 e^{\lambda T_f}) - \ln(c_1 \eta^{G-1}) - \ln(\xi_2 + \xi_3 + \xi_4 (h-1))] \quad (29)$$

, the system is finite-time stable with respect to $(c_1, c_2, \delta, \varepsilon, T_f, h, \sigma(t))$.

Remark 4: Compared with the lower bound of ADT obtained in [10], [21], [32], [39], our results have two additional degrees of freedom to adjust for the lower bound of ADT. For example, in addition to the conventional parameters, we can also get a smaller lower bound by increasing G or decreasing η . This undoubtedly brings more flexibility and convenience for practical engineering application.

Corollary 4: Consider system (1), for a given time constant $T_f, h > 0, \lambda > 0, 0 < \eta \leq 1, \bar{\mu} > 1$ satisfying $\bar{\mu}\eta^{G-1} > 1$, and two vectors $\delta > \varepsilon > 0$, if there exist positive vectors $v_p, \mu_p^i, \varrho_p^i, i \in \mathfrak{R}_p, p \in S$, vectors $z_i \in \mathbb{R}^n, (i = 1, 2, \dots, r)$ and $z \in \mathbb{R}^n$, and positive constants $\xi_1, \xi_2, \xi_3, \xi_4$ such that $\forall i \in \mathfrak{R}_p, \forall (p, m) \in (S \times S)$, inequalities (28a) - (28e) and the following inequalities hold,

$$A_p^T v_p + z - e^{-\lambda} I v_p + I \mu_p^i + h I \varrho_p^i < 0, \quad (30a)$$

$$A_{dp}^T v_p - e^{-\lambda h} I \mu_p^i - e^{-\lambda h} I \varrho_p^i < 0, \quad (30b)$$

$$z_i \leq z, \quad (30c)$$

$$1_r^T B_p^T v_p A_p + B_p \sum_{i=1}^r 1_r^{(i)} z_i^T \geq 0, \quad (30d)$$

$$\xi_1 \varepsilon < v_p < \xi_2 \delta, \mu_p^i < \xi_3 \delta, \varrho_p^i < \xi_4 \delta, \quad (30e)$$

then under the state feedback controller (17), the closed-loop system (1) is finite-time stable with respect to $(c_1, c_2, \delta, \varepsilon, T_f, h, \sigma(t))$ for the switching signal $\sigma(t)$ with ADT (29).

IV. ILLUSTRATIVE EXAMPLES

In this section, two examples are given to illustrate the effectiveness of theoretical findings.

A. EXAMPLE 1

Consider system (1) with $u(t) = 0$, two subsystems are given as follows:

$$A_1 = \begin{bmatrix} 0.2 & 0.4 \\ 0.3 & 0.2 \end{bmatrix}, A_2 = \begin{bmatrix} 0.1 & 0.2 \\ 0.3 & 0.41 \end{bmatrix},$$

$$A_{d1} = \begin{bmatrix} 0.01 & 0.02 \\ 0.01 & 0.05 \end{bmatrix}, A_{d2} = \begin{bmatrix} 0.01 & 0.03 \\ 0.02 & 0.01 \end{bmatrix}.$$

If we choose $h = 0.01, c_1 = 0.1, c_2 = 1.5, T_f = 10, \delta = [4; 5], \varepsilon = [0.01; 0.01], \lambda_1 = 0.6, \lambda_2 = 0.6, \eta_1 = 0.8, \eta_2 = 0.7, \bar{\mu}_1 = 10, \bar{\mu}_2 = 10, G_1 = G_2 = 2$, by solving Theorem 1, we can obtain the solutions of the following unknown variables:

$$v_1^0 = \begin{bmatrix} 312.7015 \\ 361.2003 \end{bmatrix}, v_1^1 = \begin{bmatrix} 70.0625 \\ 80.9072 \end{bmatrix},$$

$$v_2^0 = \begin{bmatrix} 266.4822 \\ 393.3153 \end{bmatrix}, v_2^1 = \begin{bmatrix} 60.8959 \\ 89.9051 \end{bmatrix}, \mu_1^0 = \begin{bmatrix} 0.0723 \\ 0.1563 \end{bmatrix},$$

$$\mu_1^1 = \begin{bmatrix} 0.0352 \\ 0.0453 \end{bmatrix}, \mu_2^0 = \begin{bmatrix} 0.3287 \\ 0.2918 \end{bmatrix}, \mu_2^1 = \begin{bmatrix} 0.0622 \\ 0.0554 \end{bmatrix}, \varrho_1^0 =$$

$$\begin{bmatrix} 57.0148 \\ 67.6233 \end{bmatrix}, \varrho_1^1 = \begin{bmatrix} 12.5979 \\ 14.4039 \end{bmatrix}, \varrho_2^0 = \begin{bmatrix} 92.7318 \\ 84.1902 \end{bmatrix}, \varrho_2^1 =$$

$$\begin{bmatrix} 23.7105 \\ 18.8503 \end{bmatrix}, \xi_1 = 0.3776, \xi_2 = 95.6516, \xi_3 = 36.6024, \xi_4 =$$

47.5843. Then we can figure out $\tau_{a1}^* = 5.9194$ and $\tau_{a2}^* = 5.3365$. FIGURE. 1 shows the state response of the system. FIGURE. 2 is the simulation of $x(t)^T \varepsilon$. FIGURE. 3 shows the system mode. From FIGURES. 1-3, it can be shown that

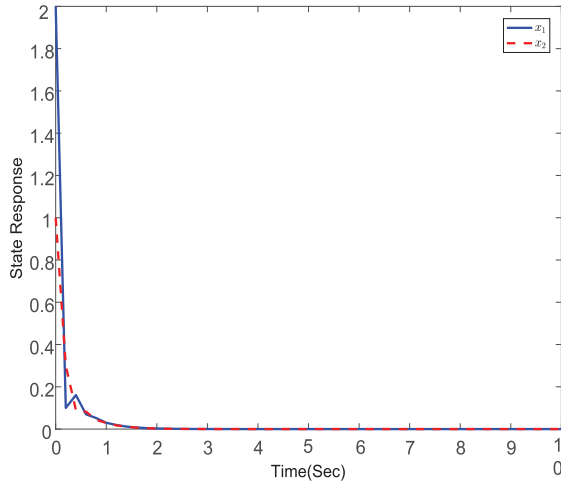


FIGURE 1. Simulation of States.

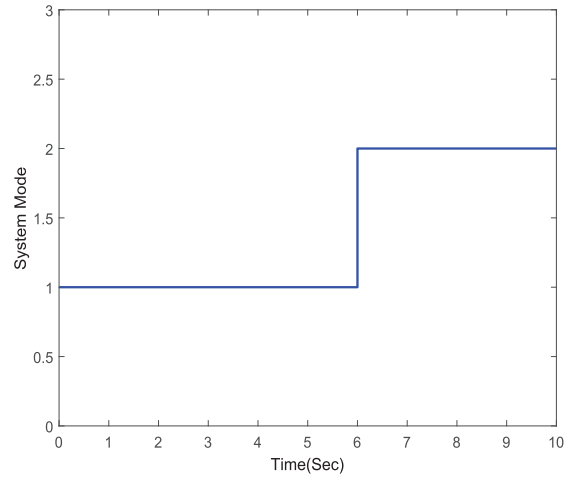


FIGURE 3. Simulation of switching signal.

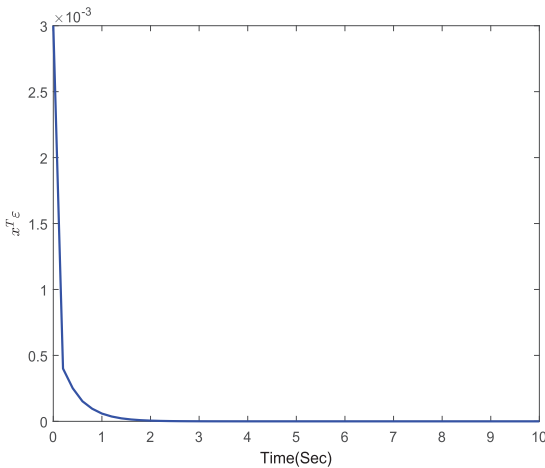


FIGURE 2. Simulation of $x^T \epsilon$.

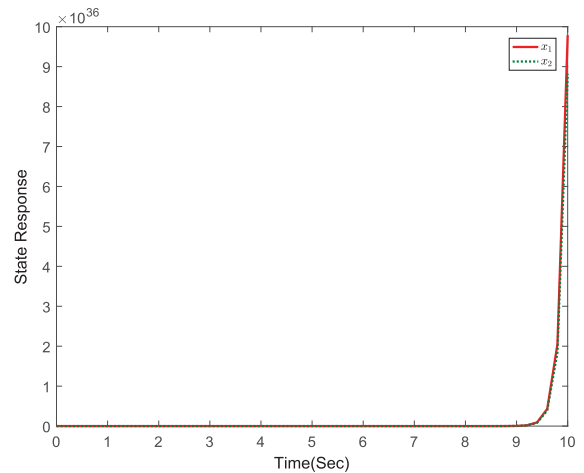


FIGURE 4. Simulation of states (open-loop).

the system can achieve finite-time stability with respect to $(0.1, 1.5, \delta, \epsilon, 10, 0.01, \sigma(t))$.

Remark 5: In [21], a finite-time L_1 control scheme is designed for SPLTSs under ADT switching signals by using the classical linear copositive Lyapunov-Krasovskii functional approach. Based on this approach proposed in [21], we can obtain the lower bound under MDADT switching signals. That is $\tau_{a1} = \tau_{a2} = 23.3248$. It shows that the MPCLKF approach proposed in Theorem 1 can provide more parameters to adjust for obtaining a smaller lower bound of MDADT.

B. EXAMPLE 2

Consider system (1), two subsystems are given as follows:

$$A_1 = \begin{bmatrix} 2.7 & 2.4 \\ 2.6 & 2.5 \end{bmatrix}, A_2 = \begin{bmatrix} 2.6 & 2.4 \\ 2.3 & 2.2 \end{bmatrix},$$

$$A_{d1} = \begin{bmatrix} 0.1 & 0.2 \\ 0.3 & 0.4 \end{bmatrix}, A_{d2} = \begin{bmatrix} 0.1 & 0.2 \\ 0.4 & 0.3 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 3.4 & 3.5 \\ 3.6 & 3.7 \end{bmatrix}, B_2 = \begin{bmatrix} 4.8 & 4.9 \\ 4.8 & 4.6 \end{bmatrix}.$$

If we choose $h = 0.2, c_1 = 0.1, c_2 = 1.5, T_f = 10, \delta = [4; 5], \epsilon = [0.01; 0.01], \lambda_1 = 1.1, \lambda_2 = 1.1, \eta_1 = 0.8, \eta_2 = 0.9, \bar{\mu}_1 = 10, \bar{\mu}_2 = 10, G_1 = G_2 = 2$, by solving Theorem 2, we can obtain the solutions of the following unknown variables:

$$v_1 = \begin{bmatrix} 123.3909 \\ 64.3834 \end{bmatrix}, v_2 = \begin{bmatrix} 113.9465 \\ 87.0304 \end{bmatrix},$$

$$\mu_1^0 = \begin{bmatrix} 0.9164 \\ 0.3087 \end{bmatrix}, \mu_1^1 = \begin{bmatrix} 0.5837 \\ 0.2224 \end{bmatrix}, \mu_2^0 = \begin{bmatrix} 0.1470 \\ 0.3192 \end{bmatrix},$$

$$\mu_2^1 = \begin{bmatrix} 0.1022 \\ 0.2021 \end{bmatrix}, \varrho_1^0 = \begin{bmatrix} 48.4947 \\ 70.0900 \end{bmatrix},$$

$$\varrho_1^1 = \begin{bmatrix} 41.1832 \\ 62.9055 \end{bmatrix}, \varrho_2^0 = \begin{bmatrix} 59.4136 \\ 68.4959 \end{bmatrix}, \varrho_2^1 = \begin{bmatrix} 47.4473 \\ 54.7731 \end{bmatrix},$$

$$z_1 = \begin{bmatrix} -470.5879 \\ -450.1145 \end{bmatrix}, z_2 = \begin{bmatrix} -470.5891 \\ -450.1172 \end{bmatrix},$$

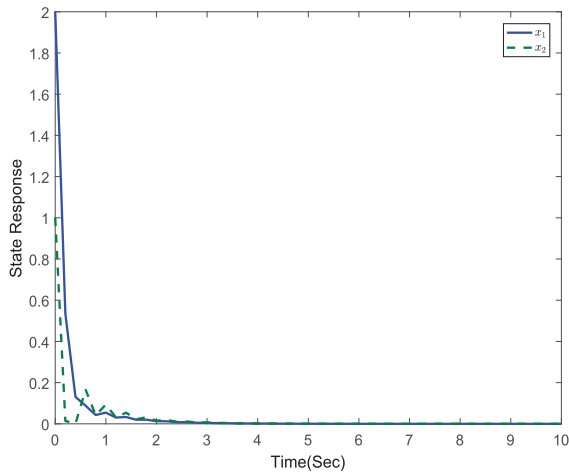


FIGURE 5. Simulation of states (closed-loop).

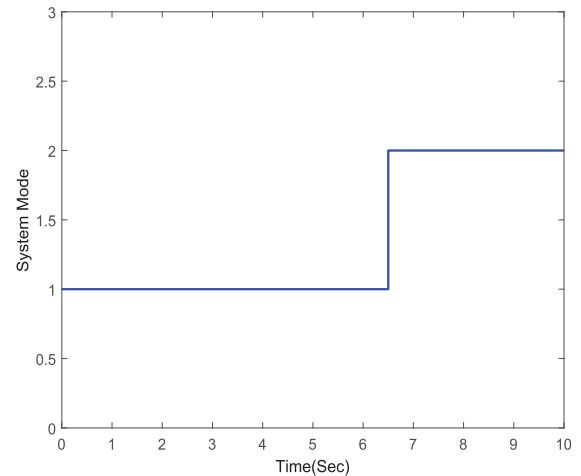


FIGURE 7. Simulation of switching signal.

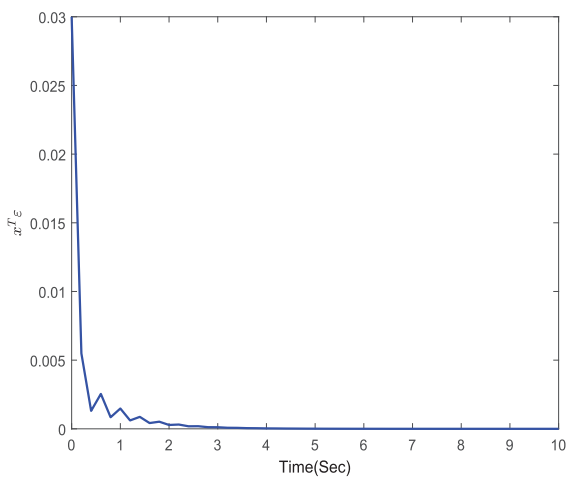


FIGURE 6. Simulation of $x^T \epsilon$.

$$z = \begin{bmatrix} -470.5487 \\ -450.0498 \end{bmatrix}, \xi_1 = 0.0021, \xi_2 = 65.8343, \\ \xi_3 = 44.1017, \xi_4 = 54.2835.$$

Then we can figure out

$$K_1 = \begin{bmatrix} -0.3561 & -0.3406 \\ -0.3561 & -0.3406 \end{bmatrix}, \\ K_2 = \begin{bmatrix} -0.2447 & -0.2340 \\ -0.2447 & -0.2340 \end{bmatrix}, \tau_{a1}^* = 6.3683 \text{ and} \\ \tau_{a2} = 5.8280. \text{ FIGURE. 4 and FIGURE. 5 show the}$$

state responses of the open-loop system and the closed-loop system. FIGURE. 6 is the simulation of $x(t)^T \epsilon$. FIGURE. 7 shows the system mode. From FIGURES. 4-7, it can be shown that even for unstable subsystems, the system can achieve finite-time stabilization with respect to $(0.1, 1.5, \delta, \epsilon, 10, 0.2, \sigma(t))$ under designed state-feedback controllers.

Remark 6: Similar to the discussions in Remark 5, based on this approach proposed in [21], we can obtain the lower bound under MDADT switching signals. That is $\tau_{a1} = \tau_{a2} = 22.0912$. By contrast, the MPCLKF approach proposed in

Theorems 1 and 2 can provide more freedom to obtain smaller lower bound on MDADT. Further, this MPCLKF approach proposed in Theorems 1 and 2 can be applied for the system with ADT switching signals. It means that our results can decrease the conservatism.

V. CONCLUSION

This paper has investigated the finite-time stability and stabilization for switched positive linear time-delay systems with MDADT switching signal. By introducing multiple piecewise copositive Lyapunov-Krasovskii functional, some sufficient conditions of finite-time stability have been developed for switched positive linear time-delay systems by using LP method. And the finite-time stabilization can be achieved by designing a switched state-feedback controller. Since the multiple piecewise copositive Lyapunov-Krasovskii functional method cannot be applied for the numerical structure design of the controller, by putting forward a novel linear combinatorial copositive Lyapunov-Krasovskii functional method, it provides a possibility for the numerical construction of the controller. Thus, the limitation of the multiple piecewise copositive Lyapunov-Krasovskii functional method has been canceled and the conservatism has been reduced. Similarly, in order to solve the limitation that multiple piecewise copositive Lyapunov function method cannot be used to design controller, the construction idea of this linear combinatorial copositive Lyapunov-Krasovskii functional method can also be extended to the construction of linear combinatorial copositive Lyapunov function, so as to solve the problems such as the numerical construction forms of controllers or observers. Further research includes the consideration of adaptive control for nonlinear switched positive systems ([41]–[45]) and stability of probabilistic Boolean networks [46].

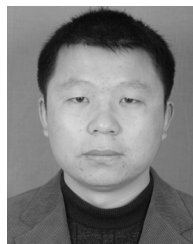
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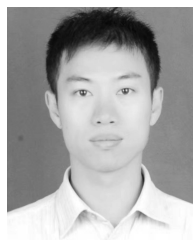
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