Received November 30, 2019, accepted December 17, 2019, date of publication December 20, 2019,
date of current version January 2, 2020.

Digital Object Identifier 10.1109/ACCESS.2019.2961142

A New Symbolic-Based Flow Aggregation and Disaggregation Modular Approach for Tree-shaped Networks

WALAA IBRAHIM GABR[1], (Member, IEEE), AND HASSEN TAHER DORRAH[2], (Life Member, IEEE)
[1]Department of Electrical Engineering, Benha Faculty of Engineering, Benha University, Benha 13511, Egypt
[2]Department of Electrical Engineering, Faculty of Engineering, Cairo University, Giza 12613, Egypt

Corresponding author: Hassen Taher Dorrah (dorrahht@aol.com)

This work was supported in part by the Benha University and Cairo University, Egypt.

ABSTRACT This paper presents the representation and modeling of real-life tree-shaped natural and man-made networks. It is shown that the tree-shaped networks could be composed of two entities of different functionalities that can operate separately or jointly. The first entity is the **feet/head aggregation networks**, while the second entity is the **head/feet disaggregation networks**. Each entity is represented with the same symbolic-based modular model expressions. Moreover, it is illustrated that the aggregation entity network can be mapped through a mirroring type process to an analogous disaggregation entity network and vice versa. The suggested technique is demonstrated by an application of the modeling of 20-nodes real-life tree-shaped irrigation network. The paper also addresses simultaneously the analogy between natural plant tree morphology and natural/man-made operational network of both the aggregation or disaggregation types. It is highlighted that such analogy with the natural tree system could help in future schematizing of stages of operational networks expansion in the most efficient way as learnt from nature and in building advanced generations of operational networks. Furthermore, it is pointed out that the new approach has unlimited scope of real-life applications in engineering/technology such as electric generation, water basins, sewage, agriculture drainage, highway transportation ..etc. networks for the **aggregation entity**, and electric distribution, irrigation, oil, gas, potable water, roads transport, ..etc. networks for the **disaggregation entity**. In all respects, the paper has succeeded within the area of tree-shaped networks in **crossing the boundaries** between the **Science of Botany** and **Engineering/technology** (and **vice versa**) and to create new common areas of important shared interests of great benefits to these disciplines and the science world as a whole. Finally, the new notion of crossing boundaries between sciences can also be extended **to and among** many other sciences themselves dealing with tree-shaped systems.

INDEX TERMS Crossing the boundaries between sciences, ecology and hydrology, evolutionary systems, feet/head aggregation networks, head/feet disaggregation networks, life sciences, medicine and biology, multi-step modular modeling, real-life natural and man-made operational networks, engineering and technology, symbolic-based mathematical modeling, the tree of life, the science of botany, tree-shaped morphology, water and energy.

I. INTRODUCTION

The tree network has been extensively applied in computer systems, where the main branch is referred to as the parent and its subsidiary is denoted as the child [1]. The top of the tree-shaped network could be visualized as the ''**network head**'' and the bottom as the ''**network feet**''.

On other hand, natural and man-made real life (physical) tree-shaped operational networks have been given much less concentration in the literature. Examples of such operational networks are the natural flow systems such as river basins or man-made systems such as electricity, oil, gas, potable water and sewage, irrigation and drainage, roads and transport . . . etc. networks [2].

The application of symbolic-based modeling and analysis of natural and man-made systems has been recently more emphasized [3]. The implementation of such symbolic-based mathematical approach has been extended in many classes of systems in areas of engineering, automatic control, life sciences, utilities operational networks, . . . etc. [4]–[6].

The associate editor coordinating the review of this manuscript and approving it for publication was Lin Wang.

In addition, in the symbolic-based approach, the solution has been derived in the form of generic and exact mathematical functions or expressions, that can be implemented using one of the techniques of cyber-physical symbolic-based systems [7]–[9].

Operational networks are now well advanced than before. They are equipped with flexible resources such as of changeable form within the nodes that adapts itself with the ongoing situations. Furthermore, they could be equipped with additional moveable resources that can move from one node to another to achieve the highest efficiency and reliability. In addition to these requirements, there is an urgent need to develop the tree-shaped aggregation and disaggregation models of the operational networks problem to be considered in the investigation.

Similar to the operational network, there are natural trees that have same tree-shape topologies. In the science of botany, a tree is a perennial plant with an elongated stem (or trunk) supporting branches and leaves [10]. Below ground, the roots branches are spread widely as they serve to anchor the tree and extract moisture and nutrients from soil. Both trees roots and stems systems usually follow the same tree-shape topology.

II. PROBLEM MOTIVATION

Nature in fact has inspired us with the two types of tree-shaped configurations as shown in **Figure 1**(source: https://hippos.ml/post/wiringpi-not-root). The ground surface in **Figure 1** divides the natural tree into two separate entities describes as follows:

The first entity is of the **aggregation function** part of the natural tree which is located under the ground surface. This entity can be visualized as composing of the ‘‘**roots feet**’’ at the bottom and the ‘‘**roots head**’’ at the top. We will refer to this entity as the **feet/head aggregation networks**. This entity head ends just touching the ground surface from **its beneath**. The function of this first natural tree entity is for harvesting water and mineral resources at the first tree feet.

The second entity is of the **disaggregation function** part of the tree and is located above the ground surface. This entity can be visualized as composing of the ‘‘**stems head’**’’ at the bottom positioned upside down and the ‘‘**stems feet’**’’ at the top of the figure. We will refer to this entity as the **head’/feet’ disaggregation networks**. This entity head just begins just touching the ground surface **from its above**. The function of this second natural tree entity is to conduct collected water and minerals raised by the first entity to the head and the plant branches (second tree feet).

In fact, there are lots of analogies between the real-life natural and man-made operational networks of the aggregation and the disaggregation system with the tree structure. Moreover, there are many efforts carried out in the literature to look at what nature has optimized its networks and correlation between growth rate and life span. Examples are the research works of the statistical physicist **Geoffrey Brian West** and others [11].

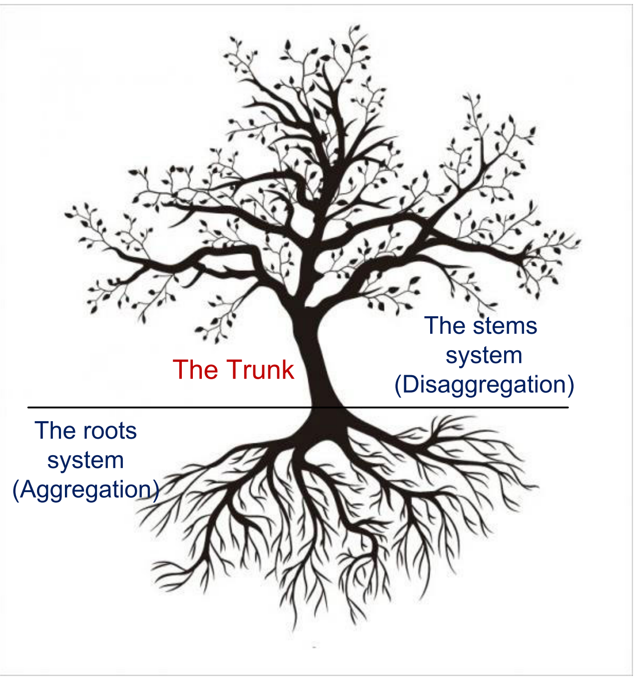


FIGURE 1. A natural plant tree comprising two joint entities at their heads namely the roots entity (feet/head aggregation type) beneath the ground surface and stems entity (head’/feet’ disaggregation type) above the ground surface as created by mother nature.

In this investigation, we will be guided by the natural aggregation and disaggregation of trees entities in developing and understanding both systems for the natural and man-made operational networks. Many lessons can be learnt from our natural plant systems to be applied to our man-made created tree-shaped networks.

In this respect of tree-shaped topology networks, the paper will attempt **to cross the boundaries** between the science of **botany and Engineering/technology** (and **vice versa**) and to create a new common area of important shared interests between all these disciplines.

III. PROBLEM FORMULATION

The concept of the problem of flow aggregation and disaggregation from the mathematical point of view was arisen in the seventies by the water resources analysts. They required these models for distributing their annual synthetic inflows of rivers to consecutive monthly, weekly and daily flows. Their approaches were mainly of the statistical form relying heavily on the conditional probabilities and extreme events analysis [12]–[15]. Nevertheless, the present proposed approach will be mainly deterministic and the models are of the symbolic-based mathematical type.

Aggregation is defined as the act of accumulation or the state of being collected together, while ***disaggregation*** means the act of division or the state of separating of an aggregate body into its component parts. Simply each term is the opposite of the other.

In this work, the modeling and analysis of flow aggregation and disaggregation will be directed mainly to real-life physical tree-shaped operational networks. These networks are categorized by the following:

- **a:** They are operational and subjected to parameters varying supply and demand constraints.
- **b:** They are flexible and could be equipped with movable and changeable resources.
- **c:** Each network part of the tree shapes cannot operate separately but has to interact with other tree-shaped networks at different order levels.
- **d:** All nodes and branches are assumed lossless and there is no travelling times of flow all over the network.

In addition, the main rule governing the operational networks is the flow continuity equations all over the networks. These equations apply at each node of the system and are specified as that system flow output at each node is equal to all system inputs minus demand.

One of the important requirements in designing the nodes and branch capacities of the interconnected operational network is the conformity of the design capacity of various system units based on required inputs, outputs and demands. If a node/branch capacity is under-designed, shortages will take place affecting this unit node/branch and upward.

This means that a single shortage at one node/branch could have multiple shortage effects at many other successive nodes. This is an important aspect that should be taken into consideration when formulating the modeling problem of such tree-shaped operational networks.

IV. NATURAL AND MAN-MADE TREE-SHAPED FLOW NETWORKS

A. TREE-SHAPED FLOW FEET/HEAD AGGREGATION NETWORKS

Physical flows in many real-life operational networks follow the tree-shape pattern with some origin inflow sources and then flows accumulate to end as integrated outflow [16]–[18]. Examples are river basins, agriculture drainage, sewage, highway transports, …etc. networks. Such flow pattern can be illustrated in a typical form as shown in **Figure 2.**

In this pattern, the outflow is denoted as the order one which is also the "**tree head**". The branches are sequentially numbered in a higher order following the stems system of the "**tree feet**" in its **backward flow direction.** This case will be referred to as the **Flow aggregation case**.

For **feet/head aggregation trees** the branch ordering is based on the two following rules when moving in the backward direction:

a: RULE 1

The main nodes at "**tree head**" (order 1) collects flow from all connected branches fed from nodes 21, 31, 41, m1, …etc (lower-order). The amounts of these flows are represented in symbolic form as $x_{21}, x_{31}, x_{41}, x_{m1}, \ldots$etc respectively.

b: RULE 2

The node mn collects flow from all its connected lower orders branches fed from nodes mn1, mn2, .., mnk, …etc. The amounts of these flows are represented in symbolic form as $x_{mn1}, x_{mn2}, \ldots, x_{mnk}, \ldots$,etc respectively.

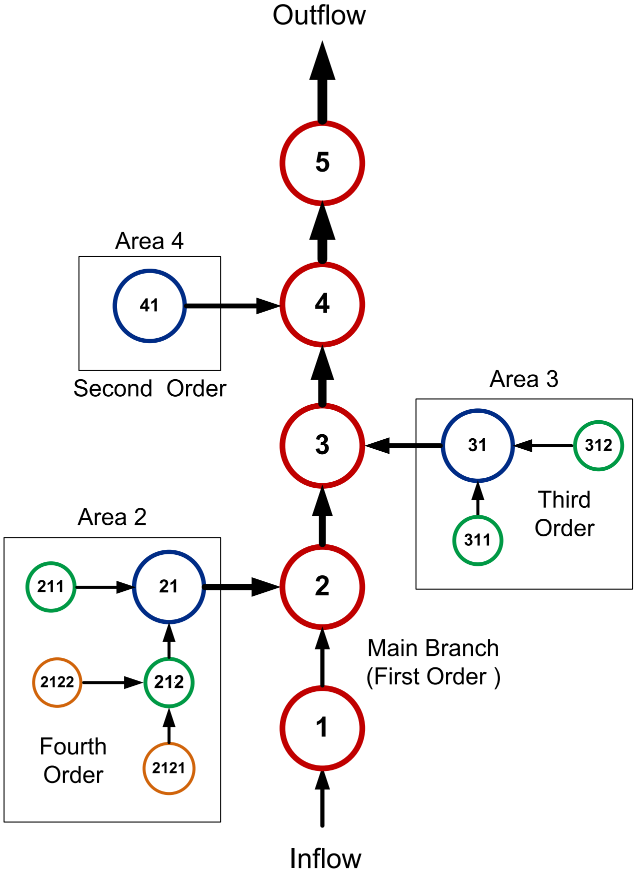


FIGURE 2. A typical example of tree-shaped operational network topology of the feet/head aggregation entity.

Each tree is composed of smaller sub-trees to be referred to as an area (or sub-area) connected to the tree main branch (or sub-branch). The area or sub-area is a division of the **tree feet** system and follows the same tree rules. No inter-connections are permitted between one area or another area. The aggregation of all areas will form the whole tree topology.

B. TREE-SHAPED FLOW HEAD/FEET DISSAGGREGATION NETWORKS

Same tree type could appear in an opposite form, where the system commences with the main inflow. The inflow then branches in a forward tree form to subsequent tree sub-branches. Similarly, the main inflow is denoted as the first order or "**tree head"**". The branches are sequentially numbered in higher order following the "**tree feet"**" system in **forward flow direction**.

Examples are the electric distribution, gas, oil, potable water, roads transport, and irrigation networks. Such flow pattern can be represented in its typical form as shown in **Figure 3**. This case will be referred to as **Flow disaggregation case.**

For **head/feet disaggregating trees,** the branch ordering is based on the two following rules when moving in the forward flow direction:

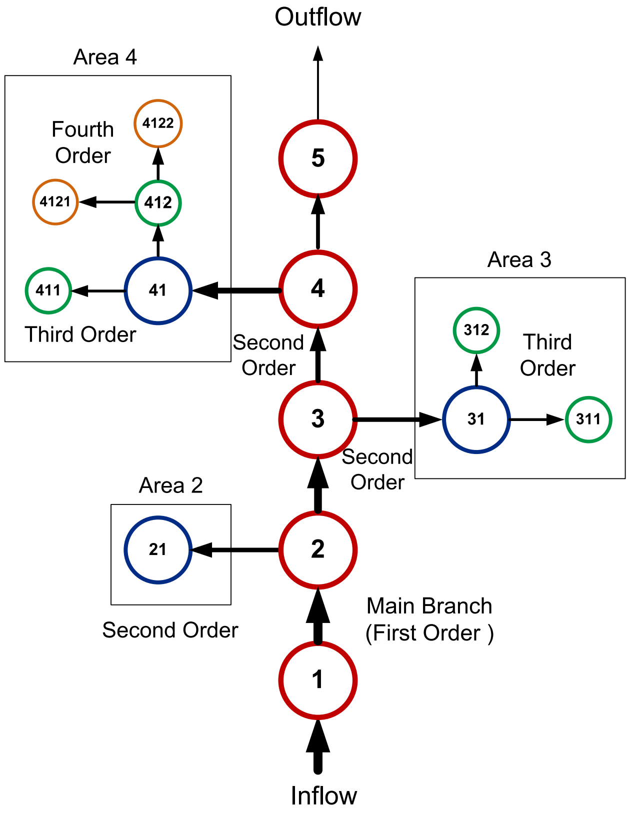


FIGURE 3. A typical example of tree-shaped operational network topology of the head’/feet’ disaggregation entity.

a: RULE 3

The main node “**tree head’**” (order 1) diverts the flow through branches to feed nodes 21, 31, 41, m1, …etc (lower order). The amounts of these flows are represented in symbolic form as $x_{21}, x_{31}, x_{41}, x_{m1}$, ..etc respectively.

b: RULE 4

The node mn diverts flows to all its connected lower order branches to feed nodes mn1, mn2, .., mnk, …etc. The amounts of these flows are represented in symbolic form as $x_{mn1}, x_{mn2}, \ldots, x_{mnk}, \ldots$,etc respectively.

In summary, for both aggregation and disaggregation types of the tree-shaped networks, the terminologies used are: **order 1** is always the “**tree head’**”, **order 2** is the following order after the main stream, **…, orders (n-1)** has the second smallest branches and **orders n** has finally the smallest branches at **“tree feet’”**.

C. ANALOGY OF THE TREE MORPHOLOGY WITH TREE-SHAPED OPERATIONAL NETWORKS

A natural plant tree system usually possesses jointly two separate entities. These are the **feet/head aggregation** of roots entity followed by the **head/feet disaggregation** of stems entity.

The aggregation model is many-to-one variable mappings of the problem parameters, where information is augmented. Therefore, it is essential to work with its mirroring model of the disaggregation model to achieve the problem detailed operational requirements. The developed disaggregation model will be also useful during network operation. It could also be used as an effective tool for testing several operational scenarios necessary for decision making.

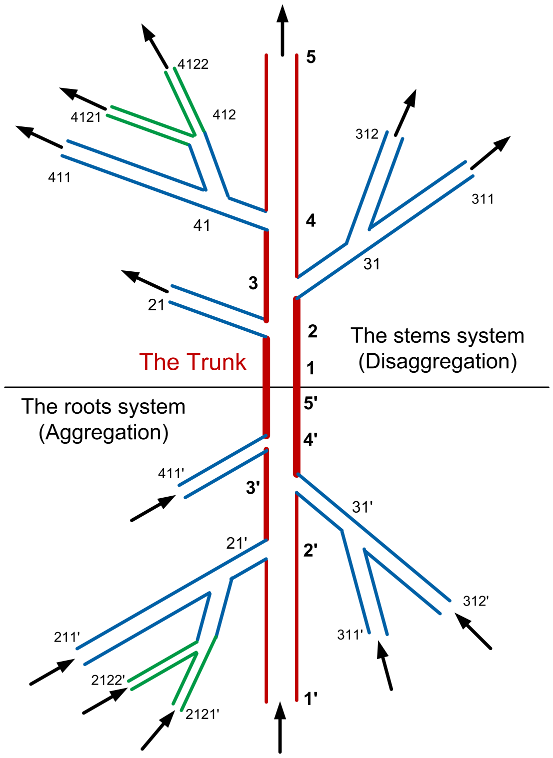


FIGURE 4. Mirroring between the two considered typical examples of aggregation and disaggregation tree-shaped entities illustrating the analogy with natural trees sharing the same symbolic-based modular models.

Extending the above aggregation and disaggregation concept to both examples given in sections IV (A,B), it is easy to assemble an analogous topology to the natural roots and stems systems as shown in **Figure 4**. In this figure, there is a clear mapping through mirroring between the two networks of the aggregation and disaggregation entities and both systems are sharing the same **symbolic-based mathematical models**. Analogous to the plant, the two networks are joined by the two heads of both entities.

V. PROPOSED MULTI-STEP TREE-SHAPED FLOW MODULAR REPRESENTATION

A. ONE-STEP FLOW AGGREGATION MODULAR REPRESENTATION

In order to simplify the modeling process of flows in the overall tree-shaped operational network, the analysis will commence by introducing the one-step module for the aggregation entity as illustrated in **Figure 5**. In this module, the lower level is represented in the form of n several areas where each area comprises several aggregating nodes that can be described as follows:

a: **Area 1** contains nodes 11, 12, …, 1m aggregating flow as $x_1 = x_{11} + x_{12} + \ldots + x_{1m}$.

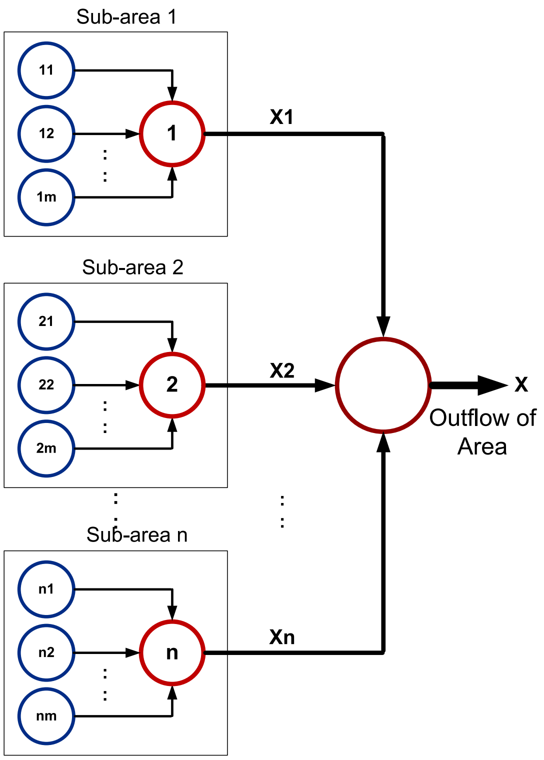


FIGURE 5. A typical tree configuration of one-order step module of the flow aggregation entity showing sub-areas and the overall area.

- **b:** **Area 2** contains nodes 21, 22, …, 2m aggregating flow as $x_2 = x_{21} + x_{22} + \ldots + x_{2m}$.
- **c:** **Area n** (general case) contains nodes n1, n2, …, nm aggregating flow as $x_n = x_{n1} + x_{n2} + \ldots + x_{nm}$.

The outflow of the module can be expressed as:

$$X = x_1 + x_2 + \ldots + x_n = \sum\nolimits_{i=1}^{n} x_i. \tag{1}$$

Or after substitution with x_i components, we can get the overall flow aggregation value for this typical module as follows:

$$X = \sum\nolimits_{i=1}^{n} x_i = \sum\nolimits_{i=1}^{n} \sum\nolimits_{j=1}^{m} x_{ij}. \tag{2}$$

For various modules constituting the overall network, we will have corresponding relationship of (2) for each one-step module.

B. ONE-STEP FLOW DISAGGREGATION MODULAR REPRESENTATION

A typical one-step tree shaped flows disaggregation module is shown in **Figure 6**. In this figure, the inflow to the module is distributed into sub-flows of $x_1, x_2, \ldots, x_n$ for consequent areas respectively in the lower level. Each sub-flow is then distributed at each area to various smaller flows described as follows:

- **a:** **For Area 1**, the sub-flow x_1 is distributed into smaller flows to feed nodes 11, 12, …, 1m such as $x_1 = x_{11} + x_{12} + \ldots + x_{1m}$.

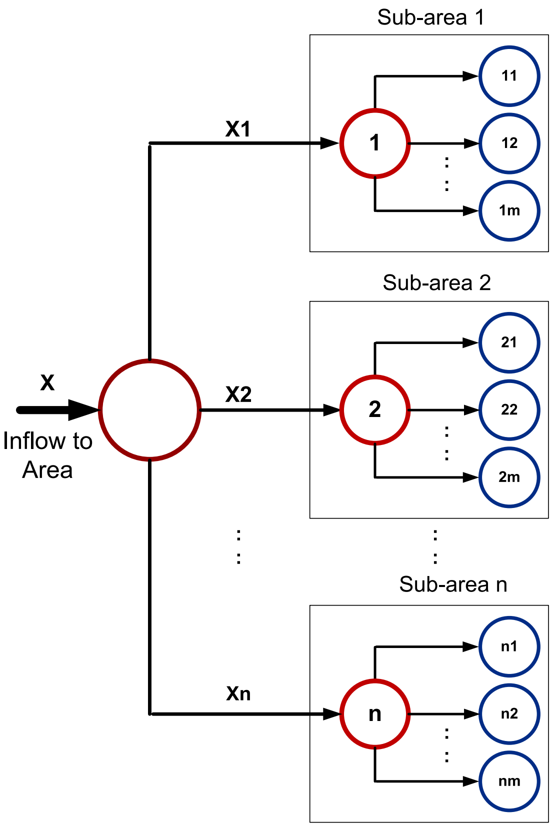


FIGURE 6. A typical tree configuration of one-order step module of the disaggregation entity showing the overall area and sub-areas.

- **b:** **For Area 2**, the sub-flow x_2 is distributed into smaller flows to feed nodes 21, 22, …, 2m such as $x_2 = x_{21} + x_{22} + \ldots + x_{2m}$.
- **c:** **For Area n** (general case), the sub-flow x_n is distributed into smaller flows to feed nodes n1, n2, …, nm, such as $x_n = x_{n1} + x_{n2} + \ldots + x_{nm}$.

The inflow of the module can be expressed by its sub-flows as follows:

$$X = x_1 + x_2 + \ldots + x_n = \sum\nolimits_{i=1}^{n} x_i. \tag{3}$$

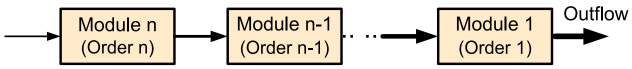
Or after substitution with x_i components, we can get the overall flow disaggregation value for this typical module as follows:

$$X = \sum\nolimits_{i=1}^{n} x_i = \sum\nolimits_{i=1}^{n} \sum\nolimits_{j=1}^{m} x_{ij}. \tag{4}$$

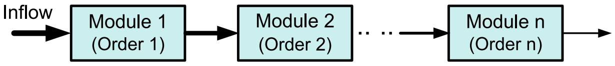
Both equations (3) and (1), and also (4) and (2) are analogous but physically are representing flows in backward order.

C. MULTI-STEP FLOW AGGREGATION AND DISSAGGREGATION MODULAR REPRESENTATION

Based on the modularity principle, the cascaded one-order step flow aggregation or disaggregation modules can then be assembled to model the overall operational network. This is illustrated in **Figure 7(a,b)**, where various modules are arranged sequentially for feet/head aggregating the flows or head/feet disaggregating the flow.



a) Aggregation (Collection) type networks.



b) Disaggregation (Distribution) type networks.

FIGURE 7. Proposed multi-step sequential modular modeling for the tree-shaped aggregation and disaggregation networks entities (The terminologies used for both types are: Order 1 is always the head Order 2 is the following order after the main stream, …, Order (n-1) has the second smallest branches and Order n has finally the smallest branches of the tree feet).

The technique is based on systematic moving step by step from lower-order branches (Module n) at the **tree feet** to the main-order branch or the **tree head** (Module 1) for aggregation networks. Similarly, moving from the main-order branch or the **tree head** (Module 1) to the lower-order branches (Module n) at the **tree feet** for disaggregation networks.

Such multi-step modular representation will lead to reducing considerably the dimensionality of the system as it only takes into consideration existing areas and avoiding any formulations of the matrix or tensor forms. Most real-life systems of irrigation and transportation networks of the tree type could have orders of 7+. Their orders may reach even 10+ for large-scale networks, that could cause severe dimensional complexities for any matrix forms approaches.

In fact, for each operational network both multi-step flow aggregation networks and multi-step flow disaggregation networks are needed for operation and analysis. In addition, the relationship of flow aggregation is not reversable as it is not one-to-one relation with constituent sub-flows. This means knowing the overall augmented outflow cannot reveal the corresponding detailed values of such sub-flows.

Side by side with the aggregation feet/head model, an analogous disaggregation head/feet model can be developed to operate as the mirroring form of the aggregation model. Thus, for both cases of aggregation (or disaggregation) entity models, the other mirroring model of disaggregation (or aggregation) should be built as well for any complete analysis.

VI. EXAMPLES OF MULT-STEP FLOW MODULAR REPRESENTATION

A. FLOW FEET/HEAD AGGREGATION NETWORKS

The multi-step flow **feet/head aggregation** modular representation is now presented using the tree configuration shown in **Figure 2** and following the methodology described in **Figure 7(a).** The technique is based on systematic moving step by step from lower-order branches (**MODULE n**) at the "**tree feet**" to the main-order branch or "**tree head**" (**MODULE 1**).

The equations of the tree-shaped network can then be expressed in symbolic-based mathematical form for this specific problem avoiding any high-dimensionality formulations. The obtained results after simplifications are as follows:

a: MODULE 4

Continuity equation at node 212 (**fourth level**):

$$x_{212} = x_{2121} + x_{2122} \tag{5}$$

b: MODULE 3

Continuity equations at nodes 21 and 31 (**third level**):

$$x_{21} = x_{211} + x_{212} \tag{6}$$

and

$$x_{31} = x_{311} + x_{312} \tag{7}$$

c: MODULE 2

Continuity equations at nodes 2, 3 and 4 given the input flow x_1 (**second level**):

$$x_2 = x_1 + x_{21} \tag{8}$$

$$x_3 = x_2 + x_{31} \tag{9}$$

and

$$x_4 = x_3 + x_{41} \tag{10}$$

d: MODULE 1

Continuity equation at node 5 (**first level**):

$$Outflow = x_5 = x_4. \tag{11}$$

Equations (5) to (11) are applied sequentially to obtain at the end the overall aggregated outflow. The approach is systematic as it decomposes the entity into parts, and each part operates at a specific level or tree order.

These equations can be substituted sequentially to give the following overall flow balanced relationship for the aggregation case:

$$x_5 = x_1 + x_{2121} + x_{2122} + x_{211} + x_{311} + x_{312} + x_{41}. \tag{12}$$

The above balanced system assumes the ideal operation of all nodes/branches and all capacities are selected in a harmonious way. However, in real operations some of the nodes' operation capacities and the supply side could be affected. In such situations, we could be confronted with one of the two unbalanced situations (x_5 is total required supply):

a:*THE SUPPLY SHORTAGE CASE:*

$$x_5 < (x_1 + x_{2121} + x_{2122} + x_{211} + x_{311} + x_{312} + x_{41}). \tag{13}$$

b:*THE SUPPLY SURPLUS CASE:*

$$x_5 > (x_1 + x_{2121} + x_{2122} + x_{211} + x_{311} + x_{312} + x_{41}). \tag{14}$$

Intervention is needed in such cases to solve such situations by locating the sources of bottlenecks and the problem nodes causing such imbalance.

B. FLOW HEAD/FEET DISAGGREGATION NETWORKS

Similar to the multi-step flow **feet/head aggregation** modular representation presented above, the flow **head/feet disaggregation** case is illustrated using the tree configuration shown in **Figure 3** and following the methodology depicted in **Figure 7(b)**. The technique is based on systematic moving step by step from the main-order branch or "**tree head**"' (**MODULE 1**) to the lower-order branches (**MODULE** n) at "**tree feet**"'.

The mathematical equations in symbolic-based form of the tree-shaped network can be expressed for this specific problem at each tree order and also avoiding any high-dimensionality formulations. The results obtained after simplifications can be expressed as follows:

a: MODULE 1

Continuity equation at node 5 (**first level**):

$$Outflow = x_5 \tag{15}$$

b: MODULE 2

Continuity equations at nodes 4, 3 and 2 (**second level**):

$$x_5 = x_4 - x_{41} \tag{16}$$

$$x_4 = x_3 - x_{31} \tag{17}$$

and

$$x_2 = x_1 \tag{18}$$

c: MODULE 3

Continuity equations at nodes 41 and 31 (**third level**):

$$x_{41} = x_{411} + x_{412} \tag{19}$$

and

$$x_{31} = x_{311} + x_{312} \tag{20}$$

d: MODULE 4

Continuity equation at node 412 (**fourth level**):

$$x_{412} = x_{4121} + x_{4122}. \tag{21}$$

Equations (15) to (21) are applied sequentially to obtain at the end the overall disaggregated inflow. The approach is similarly systematic as it also decomposes the problem into parts, and each part operates at a specific level or tree order.

These equations can be substituted sequentially to give after terms arrangements the following overall flow balanced relationship for the disaggregation case:

$$x_1 = x_{21} + x_{311} + x_{312} + x_{411} + x_{4121} + x_{4122} + x_5. \tag{22}$$

The above balanced system assumes the ideal operation of all nodes/branches and all capacities are selected in a harmonious way. However, in real operations some of the nodes' operation capacities and the demand side are affected and we could be confronted with one of the two unbalanced situations (x_1 is total required demand):

a:*THE DEMAND SHORTAGE CASE:*

$$x_1 > (x_{21} + x_{311} + x_{312} + x_{411} + x_{4121} + x_{4122} + x_5). \tag{23}$$

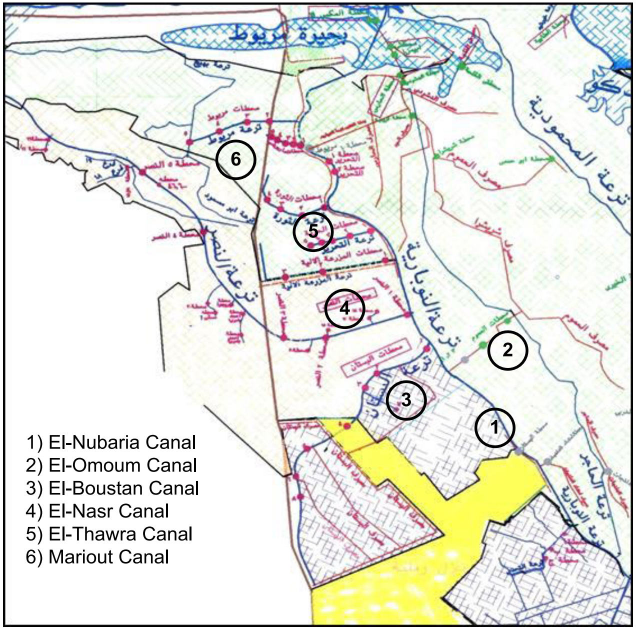


FIGURE 8. A map showing the considered application of the West Nile delta irrigation network (disaggregation type) and its canals location [19].

b:*THE DEMAND SURPLUS CASE:*

$$x_1 < (x_{21} + x_{311} + x_{312} + x_{411} + x_{4121} + x_{4122} + x_5). \tag{24}$$

Intervention is needed in such cases to solve such situations by locating the sources of bottlenecks at the problem nodes causing such imbalance. Diagnosis analysis of such shortcoming can then be applied using the developed modular model of the network.

VII. APPLICATION TO A TREE-SHAPED IRRIGATION OPERATIONAL NETWORK

The application selected for demonstrating the efficacy of the presented modularity approach of the tree-shaped aggregation and disaggregation network is the irrigation network of the West Nile Delta of Egypt as shown in **Figure 8** [19], [20].

From this network only the basic part is considered in this application. The selected part is fed from El-Noubaria canal and includes the following five canals:

- **a:** El-Omoum canal serving a total of 140 thousand acres of agriculture lands.
- **b:** El-Boustan canal serving a total of 125 thousand acres of agriculture lands
- **c:** El-Nasr canal serving a total of 385 thousand acres of agriculture land.
- **d:** El-Thawra canal serving a total of 15 thousand acres of agriculture land.
- **e:** Mariout canal serving a total of 82 thousand acres of agriculture land.

The total area of agriculture lands served within this irrigation network application is 747 thousand acres. The total inflow

TABLE 1. **Main information data of the considered application of West Nile delta (Egypt) irrigation tree-shaped network controlled by electric operating pumping systems [19].**

Node No.	Canal Name	Description of Node	Pump Stations Information Data (Size, type, capacity)	Flow Capacity ($M.m^3/$ $10\ day$)	Capacity of Available Moving Resources
1	El-Noubaria	Input inflow to network		1272.6	
2	El-Noubaria	Diverting node to El-Nasr Canal			
3	El-Noubaria	Diverting node to El-Thawra Canal			
4	El-Noubaria	Diverting node to Mariout Canal			
5	El-Noubaria	Outflow diverted to sea		60.6	
11	El-Omoum	El-Omoum P. S. #2	Large, Electric, 62.5 $m^3/sec.$	166.9	
12	El-Omoum	El-Omoum P. S. #3	Large, Electric, 62.5 $m^3/sec.$	150.0	M12
121	El-Omoum	Shershara P. S.	Medium, Electric, 48 $m^3/sec.$	143.3	
122	El-Omoum	Kalaa P. S. #1	Large, Electric, 13 $m^3/sec.$	23.6	
13	El-Boustan	El-Boustan P. S. #1	Large, Electric, 60 $m^3/sec.$	165.0	M13
131	El-Boustan	El-Boustan P. S. #7	Large, Electric, 30 $m^3/sec.$	97.2	
132	El-Boustan	El-Boustan P. S. #8	Large, Electric, 30 $m^3/sec.$	95.6	
21	El-Nasr	El-Nasr P. S. #1	Large, Electric, 150 $m^3/sec.$	766.7	
22	El-Nasr	El-Nasr P. S. #3	Large, Electric, 135 $m^3/sec.$	633.3	M22
23	El-Nasr	El-Nasr P. S. #4	Large, Electric, 90 $m^3/sec.$	322.2	
24	El-Nasr	El-Nasr P. S. #5	Large, Electric, 90 $m^3/sec.$	322.2	
221	El-Nasr	Pump Station #20	Medium, Electric, 20 $m^3/sec.$	48.2	
231	El-Nasr	Badawy P. S. and others	Medium, Electric, 11 $m^3/sec.$ with others P. S. total capacities 34 $m^3/sec.$	311.1	
31	El-Thawra	El-Thawra P. S. #2	Medium, Electric, 16 $m^3/sec.$	18.9	
41	Mariout	Mariout P. S. #2	Medium, Electric, 33.8 $m^3/sec.$	66.7	

going into the selected system of El-Noubaria Canal is 1272.6 $Million.m^3/10\ day$, while the output outflow of the network is 60.6 $Million.m^3/10\ day$ and is diverted into the Mediterranean Sea.

The selected irrigation network was then represented in the diagram form shown in **Figure 9** following the same rules and numbering presented in the paper. The operational flows were collected for this network as given in **Table 1**, indicating the existence of shortages at some points of the system. All pump stations operating the irrigation network are of the electric operated type of different sizes and capacities.

A thorough investigation of the irrigation network of the disaggregation type (analogous to tree roots system) was carried out by creating its analogous network of the aggregation form (analogous to the tree stems system). Upon moving from roots to main trunk and performing flow balance investigation, several bottlenecks are located. These locations are at nodes 12, 13 and 22 where additional resources of moving pumps are needed respectively.

The capacities of the required moving pumps were calculated from the flow balance equations of the network as: 16.9, 28.8, and 85.2 $Million.m^3/10\ day$ respectively. These flows are equivalent to moveable pump stations capacities of 7.04, 10.81 and 18.16 $m^3/sec.$ respectively.

The symbolic-based mathematical representation of the irrigation network elucidated in **Figure 9** using the suggested multi-step modular approach of the disaggregation head/feet type can be written as:

a: MODULE 1

Continuity equation at node 5 (**first level**):

$$Outflow = x_5 \tag{25}$$

b: MODULE 2

Continuity equation at nodes 5, 4 and 2 (**second level**):

$$x_5 = x_4 - x_{41} \tag{26}$$

$$x_4 = x_3 - x_{31} \tag{27}$$

and

$$x_2 = x_1 - x_{12} - x_{13} \tag{28}$$

c: MODULE 3

Continuity equation at nodes 23, 22, 13 and 12 (**third level**):

$$x_{23} = x_{24} + x_{231} \tag{29}$$

$$x_{22} = x_{23} + x_{221} - M_{22} \tag{30}$$

$$x_{13} = x_{131} + x_{132} - M_{13} \tag{31}$$

and

$$x_{12} = x_{121} + x_{122} - M_{12}. \tag{32}$$

The overall flow balance equation of the application network can be expressed after terms arrangements as follows:

$$\begin{aligned} x_1 &= x_{121} + x_{122} + M_{12} + x_{131} + x_{132} + M_{13} \\ &\quad + x_{221} + x_{231} + M_{22} + x_{24} + x_{31} + x_{41} + x_5. \end{aligned} \tag{33}$$

TABLE 2. Diversity of the tree-shaped morphological parameters of the six sample trees examples elucidated in Figure 12.

Ser.	Morphological Parameter	Sample Tree No.					
		1	2	3	4	5	6
	1. Roots Feet/Head Entity:						
a.	Total projected area (m^2)	0.230	0.377	0.363	0.170	0.267	0.610
b.	Order of roots entity network	4	4	4	4	4	5
c.	Total length of roots entity main head (m)	1.45	1.70	1.35	0.90	1.45	1.75
d.	Total number of connections (nodes)	53	47	57	39	44	148
e.	Bifurcation ratio (BR)	3.528	3.778	3.597	3.202	3.292	3.737
	2. Stems Head/Feet Entity:						
a.	Total projected area (m^2)	0.727	0.747	0.760	0.780	0.360	0.447
b.	Order of stems entity network	4	3	4	5	3	3
c.	Total length of stems entity main head (m)	1.60	2.50	1.90	2.25	2.15	1.8
d.	Total number of connections (nodes)	71	19	35	163	29	44
e.	Bifurcation ratio (BR)	3.738	3.750	2.999	4.005	3.167	3.180
	3. Stems Entity/Roots Entity:						
a.	Projected area of stems entity/Area of roots entity	3.161	1.981	2.094	4.588	1.348	0.733
b.	Order of stems entity/Order of roots entity	1.00	0.75	1.00	1.25	0.75	0.60
c.	Number of stems connections/Number of roots connections	1.339	0.404	0.614	4.179	0.659	0.297
d.	Stems entity main head length/Roots entity main head length	1.103	1.471	1.407	2.500	1.483	1.029
e.	Bifurcation ratio of stems entity/Bifurcation ratio of roots entity	1.059	0.993	0.834	1.251	0.962	0.851

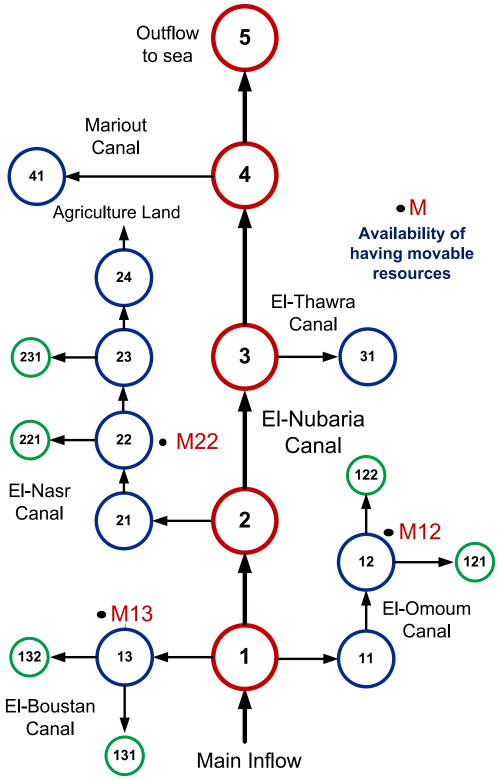


FIGURE 9. A schematic diagram of 20-nodes tree-shape disaggregation application of selected part of West Nile Delta (Egypt) irrigation network.

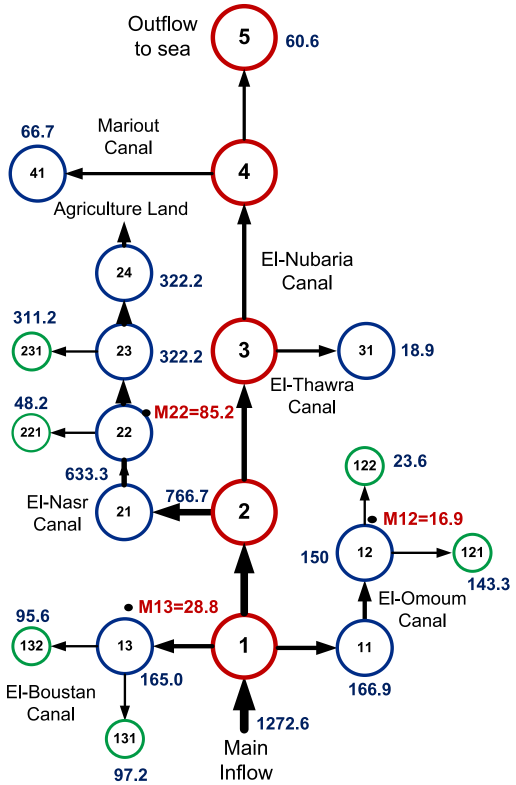


FIGURE 10. Results of balanced flow distribution in the considered 20-nodes irrigation system application all in Millions M^3/10 day.

The solution of the application was carried out sequentially using (25) to (32) and the flow results at each network node are written beside each node as shown in **Figure (10).** The implementation of the multi-step modularity technique is straightforward and can be easily extended without too many complexities even to very high-scale tree-shaped operational networks.

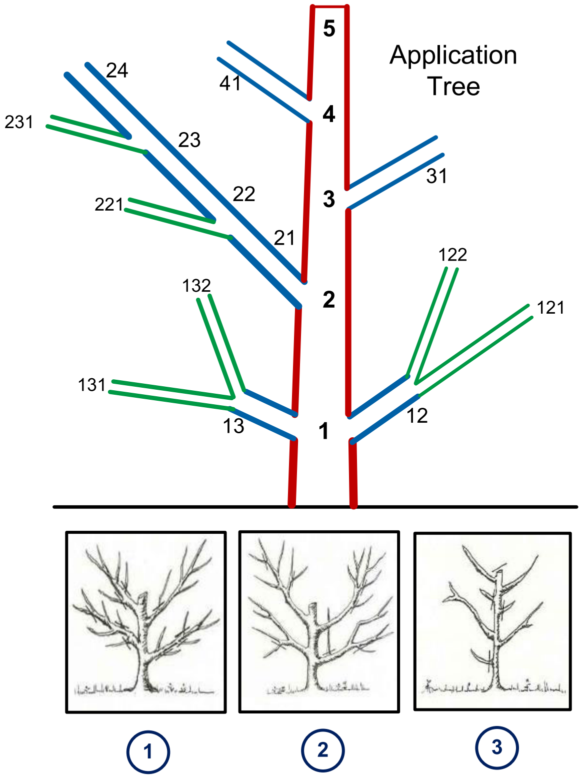


FIGURE 11. Analogy between the considered application of tree-shape irrigation network entity with the natural pruned fruits trees.

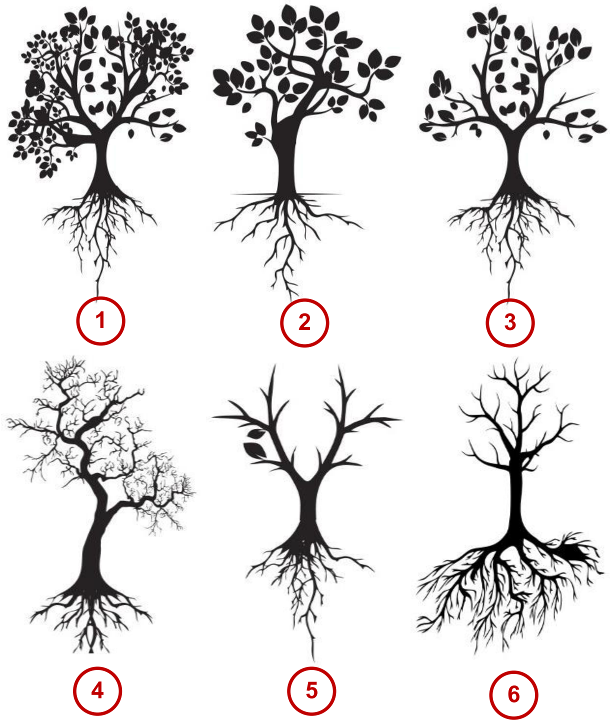


FIGURE 12. Examples of six samples shapes of natural tree roots (feet/head aggregation type) and stems (head/feet disaggregation type) systems.

VIII. ANALOGIES BETWEEN TREE-SHAPED OPERATIONAL NETWORKS AND NATURAL TREES TOPOLOGY

As it appears from the previous sections that the natural and man-made tree-shaped operational networks have many analogies and resemblance with natural tree topology. Considering the operational network in the considered application in section VII, such network topology can be sketches in the form of the tree structure with main trunk and stems as shown in **Figure 11.**

The topology of this considered application network could be visualized as analogous to the category of pruned fruits trees illustrated beneath the application network sketch.(source: http://www.janesdeliciousgarden.com/).

In fact, the realm of trees is vast and greatly extended. Some sample examples are illustrated in **Figure 12** showing six natural trees with different topologies of both the roots and stems systems

(source: https://www.vecteezy.com/vector-art/146886-black-silhouettes-tree-with-roots-vector).

The morphological parameters factors required for comparing different tree-shaped roots, stems or roots versus stems systems are summarized as follows [10], [14]:

a: Total **projected area** of the roots or stems systems. A large area roots system could accompany a small area stems system and vice versa.

b: The **order of the overall system** starting from tail of the network until reaching the network head or vice versa, to be denoted as n.

c: Total **length of the main trunk** or stream for both roots and stems systems. A long length root provides a good anchoring of the plant.

d: **Total number of branches or nodes**. This information gives indication about the complexity of the network.

e: The **bifurcation Ratio (BR)** defined as the slope of the relations between the number of branches in order i (y-axis) versus its order i (x-axis). This slope is usually determined through a simple linear regression analysis.

Applying the above morphological parameters to the six natural sample trees elucidated in **Figure 12**, we obtain the results shown in **Table 2** for the trees roots, stems and stems/roots systems. The table shows the diversity of such tree-shaped morphological parameters of the samples. The bifurcation ratios of these samples' trees were found to be in the range 3 to 4. The analogous bifurcation ratio (BR) of the considered irrigation network application of section VII is found to be 3.00, which is comparable with the corresponding range of the six plant trees samples.

For the natural sample tree #3 of **Figure 12**, the analogous operational network of the roots system (aggregation) and the stems system (disaggregation) are sketched in **Figures 13 and 14** respectively sharing the same symbolic-based modular models. The figures complete the depth needed for understanding the analogous process of tree morphology to tree-shaped operational networks types. It can be pointed out now that the implementation of all the above morphological metrics for tree-shaped operational networks could represent an additional effective tool during tree-shaped operational

TABLE 3. Summary of the overall benefits gained in crossing the boundaries between the Engineering/technology and the science of botany and (other real-life sciences in general) and vice versa.

Ser.	Aspect	Benefits Gained
1	Modeling, analysis and simulation	Applying ample numbers of Engineering and technology mathematical modeling and analysis tools as well as simulation packages to transformed natural tree-shaped stems and roots system from botany sharing the same symbolic-based mathematical models
2	Tree-shaped networks expansion and growth	Performing knowledge exchange between the progress of tree growth in botany versus the expansion operational networks in Engineering and technology, to get benefit from the wisdom of nature in these regards (see **Figure 16**).
3	Tree architectures matching and patterns categorization	Developing an architecture-based shape matching tools between trees in botany versus similar ones in Engineering and technology. This will help in building common **morphological categorization database** of such shapes including various combinations of natural stems/roots patterns in botany versus disaggregation/aggregation operational networks in Engineering and technology.
4	Networks aging and their life cycles	Carrying out knowledge exchange regarding aging progress in both natural trees in botany and operational networks in Engineering and technology. This will provide more in-depth information regarding the commencement of aging process and the progress of life cycle of each system. This will strongly encourage developing new improved generations of trees-shaped networks based on the outcome of these aging analysis.
5	Environmental effects	Performing in-depth information exchange regarding the environmental effects on the stems/roots system in botany versus operational networks in Engineering and technology, and compiling lessons learnt in these regards.
6	Extension to other sciences with tree-shaped architecture representations	Extending the gained experience of the above aspects **to and among other** disciplines and sciences with tree-shaped architecture representation. Such extension can be applied for both **physical or conceptual** systems. Examples of these sciences are all life sciences such as biology, medicine, ecology, hydrology and evolutionary systems with strong emphasis on the **Tree of Life** [10]. In general, the new will open an unlimited scope of fruitful investigations for the mutual benefits of the **science world as a whole**.

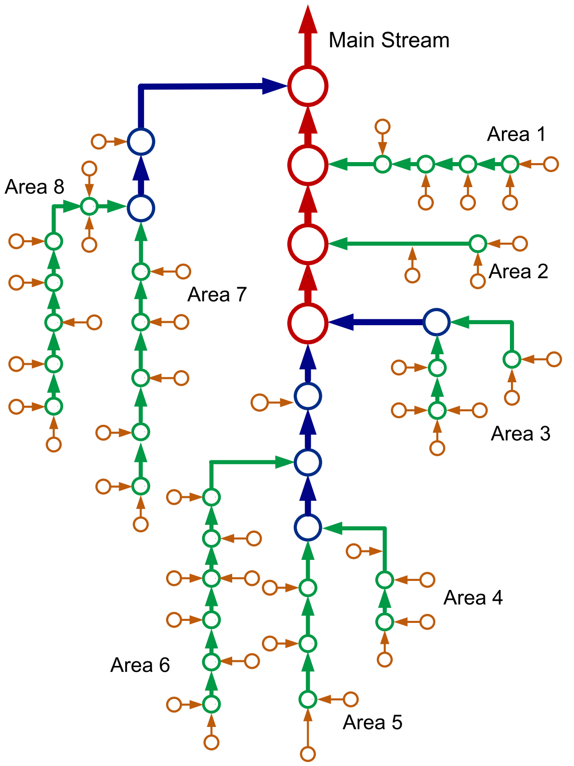


FIGURE 13. Analogous operational network (feet/head aggregation entity) of the roots system of natural sample tree #3 of Figure 12 sharing the same symbolic-based modular model.

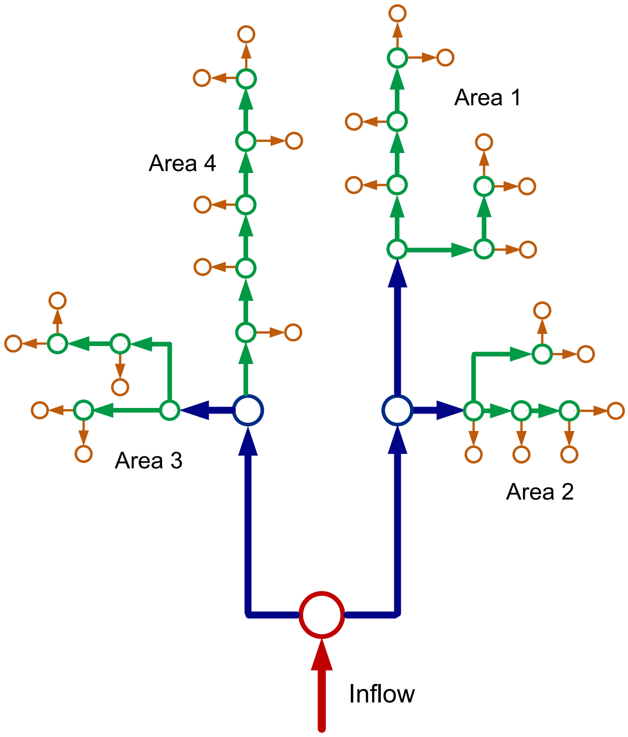


FIGURE 14. Analogous operational network (head/feet disaggregation entity) of the stems system of natural sample tree #3 of Figure 12 sharing the same symbolic-based modular model.

networks design and expansion specially for the comparison between several suggested design scenarios.

Based on the developed tree-shaped architecture similarities (or analogies) given in the paper, the descriptions of some of the benefits gained from **crossing the boundaries** between **Engineering/technology** and **Science of Botany in specific** as illustrated in **Figure 15** and all the other sciences and disciplines **in general** (and vice versa) are presented in **Table 3.**

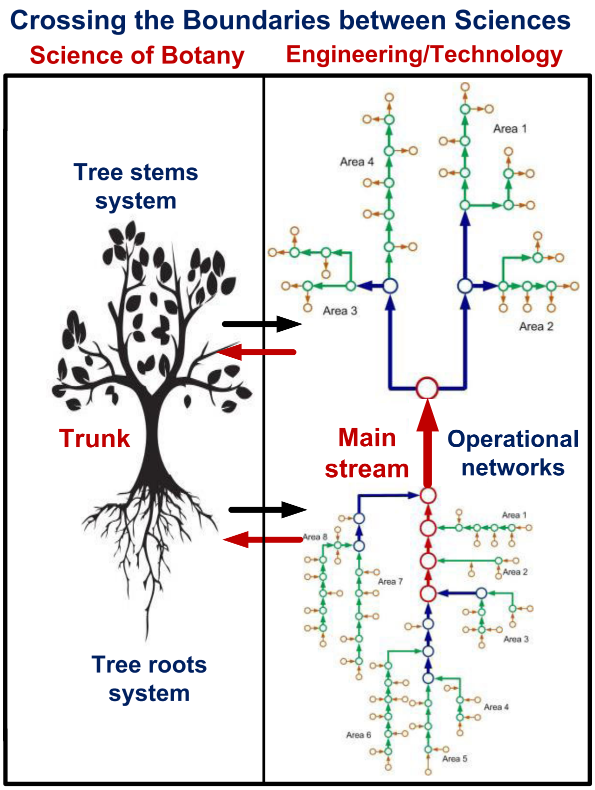


FIGURE 15. An overall diagram illustrating the new notion of crossing the boundaries between the Science of Botany and the Engineering/Technology disciplines for both tree-shaped aggregation and disaggregation entities sharing the same symbolic-based mathematical models.

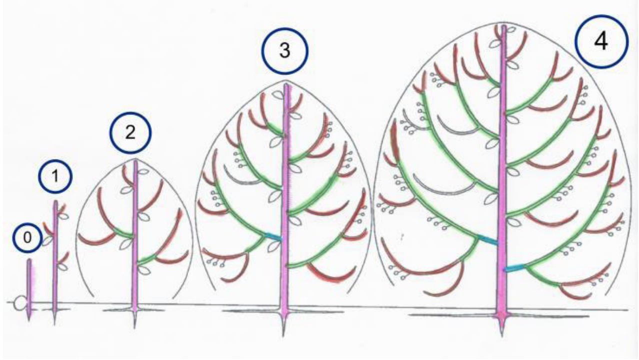


FIGURE 16. Example of the typical architecture model in the science of botany of the growth progress of the natural plant tree showing the head/feet entity [10].

Attached with **Table 3** an example of the architecture model in the science of botany of the growth progress of the natural plant tree elucidated as in **Figure 16**. Some studies of tree shapes and their statistics can be found in [21], [22] that can give some additional inference of how the trees be formed and the difference in their growth.

IX. CONCLUSION

The paper has shown with examples and applications that the tree-shaped networks could be composed of two entities of different functionalities that can operate separately or jointly. The first entity is the **feet/head aggregation networks**, while the second entity is the **head/feet disaggregation networks**. Each entity is represented with the same symbolic-based modular model expressions.

It is illustrated that both networks entities can be interchangeably mapped one to another through a straightforward mirroring process. Both forms provide good mathematical flexibility in the design, analysis and testing process.

A new multi-step modular approach has been presented for the modeling of both the aggregation and disaggregation networks. The proposed approach was solely based on symbolic representation or simply mathematical modular modeling to escape during the tree-shaped analogous processes from falling into the trap of the conversion between different physical units and metrics used in various sciences.

The presented approach is shown to be sequential in its implementation that avoids the use of high-dimensional matrix formulations. The approach was demonstrated by an application of 20-nodes of tree-shape network representing part of the irrigation network of the West Nile Delta of Egypt of the disaggregation type.

The study has illustrated that for real-life operational networks the **feet/head aggregation** entity network (analogous to natural tree roots system) can be mapped through a simple mirroring process to a **head/feet disaggregation** entity network (analogous to natural tree stems system) and vice versa. Both mirrored networks are analogous but with reverse flow direction.

It is highlighted that the study of various entities of natural trees roots (**feet/head aggregation**) and stems (**head/feet disaggregation**) systems shapes versus surrounding environments could provide more in-depth knowledge in our future trends in designing real-life operational networks. This will be highly beneficial in developing new generations of networks imitating the wisdom of nature.

If we recall now that the results of Google search of word "**tree**".[1] could amount to an average exceeding **10^{10}** (**ten billion**), we can easily realize based on the common tree-shaped architecture spread in most real-life sciences that the Engineering/technology disciplines possess now **unlimited golden opportunities** to penetrate through the common boundaries of most of these sciences and vice versa. This will open many joint aspects for very fruitful future investigations in the **science world as a whole**.

The new notion can also be extended **to and among** all other sciences themselves dealing with tree-shaped systems sharing the feature of having tree-shaped **conceptual or physical** representations and the same symbolic-based mathematical models. Some applications of these other general sciences are the group of life sciences such as biology, medicine, ecology, hydrology, and evolutionary systems with emphasis on the celebrated **Tree of Life**.

---

[1] Similar to Google search, the word '**tree**' has been **adopted** in its general sense in our investigation interchangeably between the **natural tree in botany** and any **other tree-shaped system** in various sciences and disciplines.

Last but not least, the answer of the main question of:

> “Why our **nature and universe** as a whole have selected ‘**tree-shape**’ topology as one of their **top preferences**?”

is extremely important and is left to be uncovered.

X. RECOMMENDED TOPICS FOR FUTURE WORK

Some additional topics for future research:

a: Carrying out research studies of how to generate a tree-like operational network and their expansion that accords with the real natural trees.

b: Investigating the functionality of the modular models be investigated to shed light on optimal design or resource distribution.

c: Extending the new notion to similar problems in biology, medicine, ecology, hydrology and evolutionary systems with strong emphasis on the celebrated **Tree of Life**,

d: Building common morphological categorization database of such shapes including various combinations of tree-shapes from various sciences.

ACKNOWLEDGEMENT

This work is dedicated to the memory of:

Ibrahim Mahmoud Gabr and **Taher Hassen Dorrah**.

REFERENCES

[1] C. Pozrikidis, *An Introduction to Grids, Graphs, and Networks*. Oxford, U.K.: Oxford Univ. Press, 2014.

[2] W. I. Gabr, H. T. Dorrah, and S. A. El-Gindy, “Symbolic-based optimal operation of flexible reconfigurable networks (FRNs) using movable/changeable resources,” *Sylwan J.*, vol. 163, no. 9, pp. 196–219, Sep. 2019.

[3] H. T. Dorrah, W. I. Gabr, and M. S. Elsayed, “Generic symbolic parameters varying systems frameworks versus other techniques: Returning back to the roots,” *Alexandria Eng. J.*, vol. 57, no. 4, pp. 3577–3594, Dec. 2018.

[4] H. T. Dorrah, W. I. Gabr, and M. S. Elsayed, “Derivation of symbolic-based embedded feedback control stabilization with experimentation,” *J. Electr. Syst. Inf. Technol.*, vol. 5, no. 3, pp. 427–441, Dec. 2018.

[5] W. I. Gabr, H. T. Dorrah, and M. Z. A. Magiud, “On the consolidity-inhibitors theory of change,” *Sylwan J.*, vol. 163, no. 9, pp. 134–147, Sep. 2019.

[6] W. I. Gabr, H. T. Dorrah, and M. S. Elsayed, “A new symbolic-based continuous (infinite) modal approach for systems control and operation using computational mathematics,” *Ain Shams Eng. J.*, to be published, doi: 10.1016/j.asej.2019.11.001.

[7] M. Merro and R. Lanotte, *A Calculus of Cyber-Physical Systems, Presentation, Department of Computer Science*. Verona, Italy: Univ. of Verona, 2017.

[8] C. Alippi and S. Ozawa, “Computational intelligence in the time of cyber-physical systems and the Internet of Things,” in *Artificial Intelligence in the Age of Neural Networks and Brain Computing*, R. Kozma, C. Alippi, Y. Choe, and F. C. Morabito, Eds. New York, NY, USA: Academic, 2019, ch. 12, pp. 245–263.

[9] S. Munir, A. Stankovic, C. M. Liang, and S. Lin, “Cyber physical system challenges for human-in-the-loop control,” in *Proc. 8th Int. Workshop Feedback Comput.*, San Jose, CA, USA, Jun. 2013.

[10] A. Shipunov, *Introduction to Botany*. Minot, ND, USA: Minot State Univ., 2018.

[11] B. J. Enquist, G. B. West, E. L. Charnov, and J. H. Brown, “Allometric scaling of production and life-history variation in vascular plants,” *Nature*, vol. 401, pp. 907–911, Oct. 1999.

[12] F. Clautiaux, S. Hanafi, R. Macedo, E. Voge, and C. Alves, “Iterative aggregation and disaggregation algorithm for pseudo-polynomial network flow models with side constraints,” *Eur. J. Oper. Res.*, vol. 258, pp. 467–477, Apr. 2017.

[13] D. F. Rogers, R. Plante, R. Wong, and J. R. Evans, “Aggregation and disaggregation techniques and methodology in optimization,” *Oper. Res.*, vol. 39, no. 4, pp. 553–582, Aug. 1991.

[14] L. C. Leung, Y. Van Hui, G. Chen, and W. H. Wong, “Aggregate-disaggregate approach to an airfreight forwarder’s planning under uncertainty: A case study,” *J. Oper. Res. Soc.*, vol. 68, no. 6, pp. 695–710, Jun. 2017.

[15] E. Afreeda and B. Kannan, “Determination of watershed morphological parameters using remote sensing and GIS,” *Int. J. Eng. Sci. Comput. (IJESC)*, vol. 8, no. 3, pp. 16109–16115, Mar. 2018.

[16] G. A. Walters and D. K. Smith, “Evolutionary design algorithm for optimal layout of tree networks,” *Eng. Optim.*, vol. 24, no. 4, pp. 261–281, Apr. 2007.

[17] P. H. Zipkin, “Aggregation and disaggregation in convex network problems,” *Networks*, vol. 12, no. 2, pp. 101–117, 1982.

[18] R. Basbous and B. Nagy, “Strategies to fast evaluation of tree networks,” *Acta Polytechnica Hungarica*, vol. 12, no. 6, pp. 127–148, 2015.

[19] *Mechanical and Electrical Department, Ministry of Water Resources and Irrigation (Egypt)*, Annu. Rep. West Delta Region, Giza, Egypt, 2018.

[20] W. I. Gabr, H. T. Dorrah, and S. A. El-Gendy, “Optimal analysis of flexible reconfigurable networks using movable and changeable components,” in *Proc. 20th Int. Middle East Power Syst. Conf. (MEPCON)*. Cairo, Egypt: Cairo Univ., Dec. 2018, Paper 24, pp. 74–81.

[21] L. M. Bettencourt, J. Lobo, D. Helbing, C. Kühnert, and G. B. West, “Growth, innovation, scaling, and the pace of life in cities,” *Proc. Nat. Acad. Sci. USA*, vol. 104, no. 17, pp. 7301–7306, Apr. 2007.

[22] C. Metzig, O. Ratmann, D. Bezemer, and C. Colijn, “Phylogenies from dynamic networks,” *PLoS Comput. Biol.*, vol. 15, no. 2, Feb. 2019, Art. no. e1006761.

WALAA IBRAHIM GABR received the B.Sc. degree in electrical engineering from the Shoubra Faculty, Benha University, Egypt, in 2000, and the M.Sc. and Ph.D. degrees in automatic control from Cairo University, Giza, Egypt, in 2006 and 2008, respectively. From 2009 to 2010, she worked as a Research and Development (R&D) Senior Consultant with SDA Engineering Canada, Inc., Toronto, ON, Canada, in the area of intelligent systems and their applications. From March 2014 to September 2015, she was a Visiting Professor with the Department of Electrical and Computer Engineering, Case Western Reserve University, Cleveland, OH, USA. Since January 2011, she joined the Benha Faculty of Engineering, Benha University, Benha, where she is currently serving as an Associate Professor of electrical engineering. Her main interests are system engineering, automatic control, intelligent systems, symbolic computations, optimization techniques, probability and statistics, operations research, and smart power grids.

HASSEN TAHER DORRAH (LM’14) received the B.Sc. degree in electrical engineering from Cairo University, Cairo, Egypt, in 1968, and the M.Sc. and Ph.D. degrees in electrical engineering from the University of Calgary, Calgary, AB, Canada, in 1972 and 1975, respectively. From 1975 to 1976, he was a Postdoctoral Fellow with the University of New Brunswick, Fredericton, NB, Canada. In 1977, he joined Cairo University, where he has been working as a Full Professor of electrical engineering, since 1987. From 2007 to 2008, he served as the Head of the Department of Electric Power Engineering. In 1996, he established SDA Engineering Canada, Inc., Toronto, ON, Canada, where he is currently serving as the President. He has been a Registered Professional Engineer in Canada in both Ontario, since 1987 and New Brunswick, since 1976 and Life Member, since 2011. His main interests are system engineering, automatic control, intelligent systems, symbolic computations, smart grids, fuzzy systems, water and energy engineering, and operations research.

• • •