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Fault-Tolerant Control of Multi-Agent Systems With Saturation and L_2-Disturbances

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ABSTRACT This paper studies the fault-tolerant tracking consistency problem of multi-agent systems with actuator faults and saturation and L_2 disturbance. Firstly, a new index is given to describe the anti-disturbance ability and H_∞ performance of the multi-agent system. In addition, a new method of designing adaptive fault-tolerant controller is given for multi-agent systems which can guarantee that the closed-loop systems H_∞ performance and disturbance tolerance performance are better than the fixed gains method. For this new method, by using the adaptive law, the controller parameters are updated on-line to compensate for the fault effects on multi-agent systems. The results show that the adaptive controller designed can achieve system consistency and better performance in the case of multi-agent system with actuator failure, saturation and L_2 disturbance. An example illustrating the validity of the results is given.

INDEX TERMS Actuator faults and saturation, multi-agent systems, tracking consistency, H_∞ performance, L_2 disturbance.

I. INTRODUCTION

An agent is an entity that exists in a specific environment and can be perceived and changed according to the environment. This concept was first proposed by American professor M. Minsky (see [1]). Multi-agent systems (MAS) are proposed by biologists in the study of group behaviours of social organisms in nature. For example, a flock of birds in flight, a school of swimming fish, or a colony of worker ants. With the increasing demand for engineering applications in recent years, research scholars have been inspired by the self-organization phenomenon of nature, and proposed the concept of MAS. And now with the rapid development of artificial intelligence, sensors and other technologies, the research of MAS has received more and more attention, gaining extensive application in such areas as multi-satellite formation, drone formation and robot group control ([2], [3]).

The research direction of MAS includes consistency, formation control, optimization, distributed task allocation, estimation and intelligent coordination. The consistency issue is the most representative research direction ([4], [5]). Consistency means that the state of each individual in the system tends to be the same by designing a reasonable control strategy. At present, the research on the consistency of multi-agent systems has been deeply studied. For the multi-agent system with general linear constant dynamic structure, Ma and Zhang [6] give the necessary and sufficient conditions for consistency, and clarify the relationship between consistency and system dynamics and communication topology. In [7] Sheng *et al.* have studied the optimal consistency control algorithm for the communication time lag in the system, while Zhu and Jing [8] use their study [7] as a basis to consider the condition with event trigger. Wen *et al.* [9] have studied the consistency of multi-agent systems under intermittent communication and their L_2 gain performance.

The MAS requires each individual to operate normally. If some members' actuators fail and saturate, the control law may not meet the consistency requirements of the multi-agent system. Therefore, during the process of design of the consistency strategy, it is necessary to consider the actuator faults, and design the corresponding fault-tolerant control law to ensure that the MAS can achieve consistency in the event of faults. Research on fault-tolerant control of single-body systems has achieved a lot of results in recent years

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(see [10], [12], [13]), but there is little research on fault-tolerant control for multi-agent systems. In work done by Tichy *et al.* [14], they gave key aspects of fault-tolerant control in multi-agent systems and established different system structures. The work of [15] studied the aggregation problem of nonlinear systems with agent faults under time-varying communication topology. Compared with single system, the fault-tolerant control of multi-agent system is affected by communication topology and neighbours. At present, the research results on fault-tolerant control of multi-agent systems are still in the initial stage.

It is well known that actuator faults and saturation phenomena sometimes occur simultaneously in practice. When actuator saturation and faults occur simultaneously, the consistency of multi-agent systems will be greatly affected. This paper considers both fault and saturation of actuators of multi-agent systems under external disturbances by designing adaptive gain compensation control law to guarantee the closed-loop H_∞ performance of MAS.

II. PRELIMINARIES AND PROBLEM STATEMENT

A. NOTATION AND GRAPH THEORY

1) NOTATION

$\lambda_{max}(L)$ and $\lambda_{min}(L)$ represent the maximum eigenvalue and minimum eigenvalue of matrix L. $A > 0$ denotes A is a positive definite matrix. Let $diag\{A_1, \cdots, A_N\}$ be the block-diagonal matrix with matrices $A_1, \cdots, A_N$ on its principal diagonal. $A \otimes B$ denotes the Kronecker product of matrices A and B.

2) GRAPH THEORY

The theory of graph theory is used to describe the communication topology of multi-agent systems. Figure $G = \{v, \varepsilon\}$ represents the leader and follower multi-agent system communication topology, $v = \{0, 1, \cdots, N\}$ means all individuals, 0 represents the leader, $i = 1, \cdots, N$ stands for follower. Subgraph $G = \{v_i, \varepsilon_i\}$ represents the communication topology of the follower, where $v_i = \{0, 1, \cdots, N\}$ and $\varepsilon_i \subseteq v_i \times v_i$ represent the set of followers and the set of communication links between them respectively. An element $(i, j) \in \varepsilon$ in the set ε indicates that the individual i can obtain the information of the individual j, $N_i = \{j, (i, j) \in \varepsilon\}$ is called the neighbour set of the individual i. The elements in the adjacency matrix $E = [a_{ij}] \in R^{(N+1)\times(N+1)}$ are defined as follows: if $(i, j) \in \varepsilon$ then $a_{ij} = 1$; otherwise, $a_{ij} = 0$. $L = [l_{ij}] \in R^{(N+1)\times(N+1)}$ is defined as $l_{ij} = \sum_{i \neq j} a_{ij}$, if $(i, j) \in \varepsilon_1 \Leftrightarrow (j, i) \in \varepsilon_1$, then graph G is called an undirected graph. In undirected graphs, if there are paths between any two nodes, the undirected graph is connected.

Assumption 1: The leader's information can be obtained by at least one follower, and the leader cannot obtain the follower's information, that is, the graph G contains a spanning tree, and the leader is the root node. On the other hand, it is assumed that G is an undirected connection figure.

B. SYSTEM DESCRIPTION

The dynamics of the followers are governed by

$$\begin{aligned} \dot{x}_i(t) &= Ax_i(t) + B_1\omega_i(t) + B_2\sigma(u_i(t)) \\ z_i(t) &= Cx_i(t) + D\sigma(u_i) \end{aligned} \tag{1}$$

For each leader, assume that its control input is 0, the dynamic model of the leader are governed by

$$\begin{aligned} \dot{x}_0(t) &= Ax_0(t) + B_1\omega_0(t) \\ z_0(t) &= Cx_0(t) \end{aligned} \tag{2}$$

The $x_i(t) \in R^n$, $Z_i(t) \in R^s$, $\omega_i(t) \in R^d$ and $\sigma(u_i) \in R^m$ represents the follower's state vector, regulated output, exogenous disturbance in $L_2[0, \infty]$, state vector and control input with saturation, $x_0 \in R^n$, $Z_0(t) \in R^s$, $\omega_0(t) \in R^d$ represent the leader's state vector, regulated output and exogenous disturbance in $L_2[0, \infty]$. A, B_1, B_2, C, D are matrices with appropriate dimensions.

In order to study the consistency of multi-agent systems, the error between the follower and the leader is defined as $\xi_i(t) = x_i(t) - x_0(t)$, reference output error is $\zeta_i(t) = z_i(t) - z_0(t)$, $i = 1, 2, \cdots, N$ and $\xi_0 = \zeta_0 = 0$. The goal of leader-following multi-agent system consistency is to design the control law for each follower so that the follower state tends to the leader state, *i.e.* $\lim_{t\to\infty} \|x_i(t) - x_o(t)\| = 0, \forall\, i = 1, \cdots, N$. then

$$\begin{aligned} \dot{\xi}_i(t) &= A\xi_i(t) + B_2\sigma(u_i(t)) + B_1(\omega_i(t) - \omega_0(t)) \\ \zeta_i(t) &= C\xi_i(t) + D\sigma(u_i(t)) \end{aligned} \tag{3}$$

Assumption 2: For a linear system, the system's immunity to interference is described by the L_2 gain. However, when there is input saturation in the system, the external disturbance is too large and the closed-loop system state and output divergence, so that the L_2 gain will become meaningless. Therefore, we assume that the energy of the external disturbance of the system is bounded, *i.e.*

$$\Im_\delta := \left\{\omega : R_+ \to R^d : \int_0^\infty \omega^T(t)\,\omega(t)\,dt \le \delta\right\} \tag{4}$$

C. FAULT MODEL

We use the actuator faults model of [13], [16] to cause errors in fault-tolerant control

$$u_{kq}^F(t) = \left(1 - p_k^q\right)\sigma(u_k(t)) \tag{5}$$

where $k \in I[1, m]$, $q \in I[1, L]$, L is the number of total fault modes. p_k^q is an unknown constant which satisfies $0 \le \underline{p}_k^q \le p_k^q \le \bar{p}_k^q \le 1$.

The fault model (5) implies that: (i)$\underline{p}_k^q = \bar{p}_k^q = 0$ means that there is no fault; (ii) $0 \le \underline{p}_k^q \le \bar{p}_k^q \le 1$ signifies the loss-of-effectiveness fault; (iii)$\underline{p}_k^q = \bar{p}_k^q = 1$ represents the outage fault.

Denote

$$u_q^F(t) = \left(1 - p^q\right)\sigma(u(t)) \tag{6}$$

where $q \in I[1,L]$, $u_q^F(t) = \left[u_{1q}^F(t), u_{2q}^F(t), \cdots u_{mq}^F(t)\right]^T$, $u(t) = [u_1(t), u_2(t), \cdots, u_m(t)]$, $p^q = diag\left[p_1^q, p_2^q, \cdots, p_m^q\right]$. For simplicity, the fault model is presented as

$$u^F(t) = (I-p)\,\sigma(u(t)), \quad p \in \left\{p^1 \cdots p^L\right\} \tag{7}$$

where $p = diag\,[p_1, p_2, \cdots, p_m]$

D. CONTROL OBJECTIVE

The purpose of this paper is to design a distributed adaptive fault-tolerant controller with actuator faults and saturation and L_2 disturbance for multi-agent systems to achieve the following performance indicators.

(I) For $\omega \in \Im_\delta$, any closed-loop system trajectory starting from zero will remain in the region $\varepsilon^*(L_1 \otimes P, \delta^*)$

(II) In normal mode, *i.e.*, $p = 0$, for $\xi(0) = 0$

$$\int_0^\infty \zeta^T(t)\,\zeta(t)\,dt \le r_n^2 \int_0^\infty \omega^T(t)\,\omega(t)\,dt + r_n^2 \sum_{i=1}^N \sum_{k=1}^m \frac{\tilde{p}_{ik}^2(0)}{\tau_{ik}} \tag{8}$$

In fault mode, *i.e.*, $p \in \left\{p^1 \cdots p^L\right\}$, for $\xi(0) = 0$

$$\int_0^\infty \zeta^T(t)\,\zeta(t)\,dt \le r_f^2 \int_0^\infty \omega^T(t)\,\omega(t)\,dt + r_f^2 \sum_{i=1}^N \sum_{k=1}^m \frac{\tilde{p}_{ik}^2(0)}{\tau_{ik}} \tag{9}$$

where $\tilde{p}_{ik}(t) = \hat{p}_{ik}(t) - p_{ik}$

E. DEFINITION AND LEMMAS

In order to solve the system fault and saturation problems, the following definitions and lemmas are proposed.

Definition 1: We use C_{clk} to represent the kth row in the matrix $C_{cl} \in R^{m\times n}$, define

$$\partial(C_{cl}) = \left\{x \in R^n : |C_{clk}\xi| \le 1, k \in I[1,L]\right\}$$

where $\partial(C_{cl})$ represents the portion that is not saturated in the state space.

Definition 2: Let $P \in R^{n\times n}$ be a positive definition matrix. Denote

$$\varepsilon(P,\delta) = \left\{\xi \in R^n : \xi^T P \xi \le \delta\right\},$$
$$\varepsilon^*(P,\delta) = \left\{\xi \in R^n : \xi^T P \xi + \sum_{i=1}^N \sum_{k=1}^m \frac{\tilde{p}_{ik}^2(t)}{\tau_{ik}} \le \delta\right\}$$

Assume $\tau_{ik} > 0$ is given, we denote $\delta^* = \delta + max\left\{\sum_{i=1}^N \sum_{k=1}^m \frac{\tilde{p}_{ik}^2(0)}{\tau_{ik}}\right\}$

Definition 3: Consider the following system

$$\begin{aligned} \dot{\xi}(t) &= A_a\left(\hat{p}(t), p\right)\xi(t) + B_a\left(\hat{p}(t), p\right)\omega(t) \\ \zeta(t) &= C_a\left(\hat{p}(t), p\right)\xi(t), \quad \xi(0) = 0 \end{aligned} \tag{10}$$

where $\hat{p}(t)$ is the time-varying parameter vector to be selected. Suppose system (8) has an adaptive H_∞ performance index no greater than r, where $r > 0$ is a given constant, then for any $\varepsilon > 0$, the following inequality holds

$$\int_0^\infty \zeta^T(t)\,\zeta(t)\,dt \le r^2 \int_0^\infty \omega^T(t)\,\omega(t)\,dt + \varepsilon \tag{11}$$

Definition 4: The nonlinear process of actuator saturation is given as follows

$$\sigma(u_k) = \begin{cases} u_k, & |u_k| \le u_k^{max} \\ sign(u_k)\,u_k{}^{max}, & |u_k| > u_k^{max} \end{cases} \tag{12}$$

where $k \in I[1,m]$. Here, for the convenience of the following description, we will use σ to describe both the scalar form and the saturation function of the vector form and assume $u_k{}^{max} = 1$

Lemma 1: By assumption 1, its Laplacian matrix can be written as Frobenius standard form as follows

$$L = \begin{bmatrix} 0 & 0^{1\times N} \\ L_2 & L_1 \end{bmatrix}$$

where $L_2 \in R^{N\times 1}$, $L_1 \in R^{N\times N}$, the communication topology between followers is undirected, and $L_1 > 0$ is symmetric.

Lemma 2: Let $u, v \in R^m$ with $u = [u_1, u_2, \cdots, u_m]^T$ and $v = [v_1, v_2, \cdots, v_m]^T$. Suppose that $|v_k| \le 1$ for all $k \in I[1,m]$, then

$$\sigma(u) \in co\left\{D_d u + D_d^- v : d \in I\left[0, 2^m - 1\right]\right\} \tag{13}$$

where co denotes the convex hull.

$$\sigma(u) = \sum_{d=0}^{2^m-1} \eta_d\left(D_d u + D_d^- v\right)$$

where $\sum_{d=0}^{2^m-1} \eta_d = 1$, $0 \le \eta_d \le 1$, and D_d is a set of $m \times m$ diagonal matrices, and only 1 and 0 are taken on the diagonal of the elements in the set. There are 2^m elements in the D_d, such as $m = 2$,

$$D = \left\{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right\}$$

denote $D_d^- = I - D_d$, it is easy to see that $D_d^- \in D_d$.

Lemma 3 [17]: Let $\xi \in \partial\left(H\left(\hat{p}(t)\right)\right)$, for each $k \in I[1,m]$

$$\lambda_k\left(\xi(t), \hat{p}(t)\right) = \begin{cases} 1, \quad if\ K\left(\hat{p}(t)\right)_k \xi(t) = H\left(\hat{p}(t)\right)_k \xi(t) \\ \dfrac{\sigma\left(K\left(\hat{p}(t)_k \xi(t)\right)\right) - H\left(\hat{p}(t)\right)_k \xi(t)}{\left(K\left(\hat{p}(t)\right)\right)_k - H\left(\hat{p}(t)\right)_p\right)\xi(t)}, \\ \quad otherwise \end{cases}$$

and allow $k = s_1 2^{m-1} + s_2 2^{m-2} + \cdots + s_m$ to satisfy the condition of $s_k \in \{0,1\}$, define

$$\eta_k\left(\xi(t), \hat{p}(t)\right) = \prod_{k=1}^m s_k\left[\left(1 - \lambda_k\left(\xi(t), \hat{p}(t)\right)\right) + (1 - s_k)\,\lambda_k\left(\xi(t), \hat{p}(t)\right)\right]$$

Then η_d's are Lipschitz in x and $\hat{p}$.

Lemma 4 [18]: Suppose there is a determinant as follows $Z, Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{12}^T & Z_{22} \end{bmatrix}$, and $Z_{11}, Z_{22} \in R^{Nn\times Nn}$, so we can get the formula:

$$Z_{22kk} \leq 0, \quad k \in I[1, N]$$

$Z_{11} + Z_{12}\Delta(\delta) + (Z_{12}\Delta(\delta))^T + \Delta(\delta) Z_{22}\Delta(\delta) \geq 0$ $\delta \in \Delta_\delta$; $\begin{bmatrix} Q & E \\ E^T & F \end{bmatrix} + V^TV + Y^TZY < 0$, $p \in \{p^1 \cdots p^L\}$, $p^q \in N_{p^q}$ then inequality

$$M(\delta) = \sum_{k=1}^{N} \delta_k E_k + \left(\sum_{k=1}^{N} \delta_k E_k\right)^T + \sum_{k=1}^{N}\sum_{p=1}^{N} \delta_k\delta_p F_{kp} + \left(V_0 + \sum_{k=1}^{N} \delta_k V_k\right)^T \left(V_0 + \sum_{k=1}^{N} \delta_k V_k\right) + Q < 0$$

holds for all $\delta_k \in \left[\underline{\delta}_{ik}^f \bar{\delta}_{ik}^f\right]$, where $Q = Q^T \in R^{n\times n}$, and $F_{pk} = F_{pk}^T \in R^{n\times n}$, $E_k \in R^{n\times n}$.

$$\Delta(\delta) = diag\left[\delta_1 I_{n\times n} \cdots \delta_N I_{n\times n}\right], E = [E_1, E_2 \cdots E_N]$$

$$F = \begin{bmatrix} F_{11} & \cdots & F_{1N} \\ \cdots & \cdots & \cdots \\ F_{N1} & \cdots & F_{NN} \end{bmatrix}, \quad V = \begin{bmatrix} I_{n\times n} & \\ \cdots & 0 \\ I_{n\times n} & \\ 0 & I_{Nn\times Nn} \end{bmatrix}$$

Remark 1: By definition 3, for $\int_0^\infty \omega^T(t)\,\omega(t)\,dt \geq \beta$, where $\beta > 0$ and $\varepsilon = \beta^2$, we have

$$\int_0^\infty \zeta^T(t)\,\zeta(t)\,dt \leq \left(r^2 + \beta\right) \int_0^\infty \omega^T(t)\,\omega(t)\,dt \tag{14}$$

for $\int_0^\infty \omega^T(t)\,\omega(t)\,dt \leq \beta$, it follows

$$\int_0^\infty \zeta^T(t)\,\zeta(t)\,dt \leq r^2\theta + \theta^2 \tag{15}$$

Remark 2: To satisfy the conditions of the above problem, we believe that each $p \in \{p^1 \cdots p^L\}$ is stable to $(A, B(I-p))$.

III. MAIN RESULTS

The multi-agent system with fault (7) and saturation (12) is described by

$$\begin{aligned} \dot{x}_i(t) &= Ax_i(t) + B_1\omega_i(t) + (I_m - p_i) B_2\sigma(u_i(t)) \\ z_i(t) &= Cx_0(t) + D(I_m - p_i)\,\sigma(u_i(t)) \end{aligned} \tag{16}$$

The controller adopts the following structure

$$u_i(t) = K(\hat{p}_i(t)) \sum_{j=1}^{N} a_{ij}\left(x_i(t) - x_j(t)\right) \tag{17}$$

where $K(\hat{p}_i(t)) = \left(K_0 + K_a(\hat{p}_i(t)) + K_b(\hat{p}_i(t))\right)$, $\hat{p}_i$ is the estimation of p_i, and $K_a(\hat{p}_i) = \sum_{k=1}^{m} K_{ak}\hat{p}_{ik}$, $K_b(\hat{p}_i) = \sum_{k=1}^{m} K_{bk}\hat{p}_{ik}$.

By lemma 2, the saturated feedback controller can be expressed as

$$\sigma(u_i(t)) = \sum_{d=0}^{2^m-1} \eta_{id}\left[D_{id}K(\hat{p}_i(t)) + D_{id}^- H(\hat{p}_i(t))\right] \sum_{j=1}^{N} a_{ij}\left(x_i(t) - x_j(t)\right) \tag{18}$$

for some scalars $0 \leq \eta_{id} \leq 1, d \in I[0,2^m-1]$, such that $\sum_{d=0}^{2^m-1} \eta_{id} = 1$ and the following equality holds

$$\begin{aligned} &(I-p)\,\sigma(u(t)) \\ &= \sum_{d=0}^{2^m-1} \eta_{id}[(I_m - p_i) D_{id}K_0 + D_{id}K_a(p_i) - p_iD_{id}K_a(\hat{p}_i) \\ &\quad + (I_m - \hat{p}_i(t)) D_{id}K_b(\hat{p}_i(t)) + D_{id}K_a(\tilde{p}_i(t)) \\ &\quad + \tilde{p}_iD_{id}K_b(\hat{p}_i(t)) + (I_m - p_i) D_{id}^-H_0 + D_{id}^-H_a(p_i) \\ &\quad - D_{id}^-H_a(\hat{p}_i) + (I_m - \hat{p}_i(t)) D_{id}^-H_b(\hat{p}_i(t)) \\ &\quad + D_{id}^-H_a(\tilde{p}_i(t)) + \tilde{p}_iD_{id}^-H_b(\hat{p}_i(t))] \sum_{j=1}^{N} a_{ij}\left(x_i(t) - x_j(t)\right) \end{aligned}$$

where $\tilde{p}_i(t) = \hat{p}_i(p) - p_i$, $B^k = \left[0 \cdots b^k \cdots 0\right]$ with $B = \left[b^1 \cdots b^m\right]$, and $B = \sum_{k=1}^{m} B^k$. Denote $\Delta_{\hat{p}} = \left\{\hat{p} = (\hat{p}_1 \cdots \hat{p}_m) : \hat{p}_k \in \left\{\min_q \left\{\underline{p}_k^q\right\}, \max_q \left\{\bar{p}_k^q\right\}\right\}\right\}$

By lemma 3, the equality (16) can be expressed as follows:

$$\begin{aligned} \dot{x}_i(t) &= Ax_i(t) + B_2 \sum_{d=0}^{2^m-1} \eta_{id}\left[(I_m - p_i) D_{id}(K_0\right. \\ &\quad + K_a(\hat{p}_i(t)) + K_b(\hat{p}_i(t))) \\ &\quad + (I_m - p_i) D_{id}^-(H_0 + H_a(\hat{p}_i(t)) \\ &\quad \left. + H_b(\hat{p}_i(t))\right] \sum_{j=1}^{N} a_{ij}(x_i(t) - x_j(t)) \\ &\quad + B_1\omega_i(t) \end{aligned} \tag{19}$$

then

$$\begin{aligned} \dot{\xi}_i &= A\xi_i + B_2(I_m - p_i) \sum_{d=0}^{2^m-1} \eta_{id}\left[D_{id}K(\hat{p}_i)\right. \\ &\quad \left. + D_{id}^-H(\hat{p}_i)\right] \sum_{j=1}^{N} l_{ij}\xi_j + B_1(\omega_i - \omega_0) \\ \zeta_i &= C\xi_i + D(I_m - p_i) \sum_{d=0}^{2^m-1} \eta_{id}\left[D_{id}K(\hat{p}_i)\right. \\ &\quad \left. + D_{id}^-H(\hat{p}_i)\right] \sum_{j=1}^{N} l_{ij}\xi_j \end{aligned} \tag{20}$$

define $\xi = \left[\xi_1^T, \xi_2^T, \ldots, \xi_N^T\right]$, $\zeta = \left[\zeta_1^T, \zeta_2^T, \ldots, \zeta_N^T\right]$ and $\tilde{\omega} = diag\left[\omega_1 - \omega_0, \omega_2 - \omega_0, \cdots, \omega_N - \omega_0\right]$, then

$$\begin{aligned} \dot{\xi} &= (I_N \otimes A)\,\xi + \Omega(L_1 \otimes I_N)\,\xi + (I_N \otimes B_1)\,\tilde{\omega} \\ \zeta &= (I_N \otimes C)\,\xi + \Psi(L_1 \otimes I_N)\,\xi \end{aligned} \tag{21}$$

where

$$\Omega = diag\left[B_2(I_m-p_1)\sum_{d=0}^{2^m-1}\eta_{1d}\left[D_{1d}K(\hat{p}_1)+D_{1d}^-H(\hat{p}_1)\right],\right.$$
$$\left.\ldots,B_2(I_m-p_N)\sum_{d=0}^{2^m-1}\eta_{Nk}\left[D_{Nk}K(\hat{p}_N)+D_{Nk}^-H(\hat{p}_N)\right]\right]$$

$$\Psi = diag\left[D(I_m-p_1)\sum_{d=0}^{2^m-1}\eta_{1d}\left[D_{1d}K(\hat{p}_1)+D_{1d}^-H(\hat{p}_1)\right],\right.$$
$$\left.\ldots,D(I_m-p_N)\sum_{d=0}^{2^m-1}\eta_{Nk}\left[D_{Nk}K(\hat{p}_N)+D_{Nk}^-H(\hat{p}_N)\right]\right]$$

Obviously, if equality (21) is asymptotically stable, then the fault-tolerant consistency problem of the original multi-agent system can be solved.

Theorem 1: The control objectives (I) and (II) can be achieved, if there exist $X > 0$, O_0, O_{ak}, O_{bk}, Y_0, Y_{ak}, Y_{bk}, and matrices Z^d, $d \in I[0, 2^m-1]$ with

$$Z = \begin{bmatrix} Z^d{}_{11} & Z^d{}_{12} \\ Z^{dT}_{12} & Z^d{}_{22} \end{bmatrix}$$

and $Z^d{}_{11}, Z^d{}_{22} \in R^{mn\times mn}$, let all $D_{id} \in D$ satisfy the following inequalities, and $\varepsilon(L_1\otimes P,\delta) \subset \partial(H(\hat{p}_i))$, *i.e.*, $|H(\hat{p}_i)_k\xi| \leq 1$, for all $\xi \in \varepsilon^*(L_1\otimes P,\delta)$, $k\in I[1,m]$ and $d\in I[0,2^m-1]$.

$$\begin{gathered} Z^d{}_{22kk} \leq 0 \\ Z^d{}_{11} + Z^d{}_{12}\Delta(\hat{p}_i) + (Z^d{}_{12}\Delta(\hat{p}_i))^T + \Delta(\hat{p}_i)Z^d{}_{22}\Delta(\hat{p}_i) \geq 0 \\ \begin{bmatrix} N_{0d} & N_{1d} \\ N_{1d}{}^T & N_{2d}\end{bmatrix} + \frac{1}{r_n^2}V_d{}^TV_d + Y^TZ^dY < 0, \quad \rho = 0 \\ \begin{bmatrix} N_{0d} & N_{1d} \\ N_{1d}{}^T & N_{2d}\end{bmatrix} + \frac{1}{r_f^2}V_d{}^TV_d + Y^TZ^dY < 0 \end{gathered} \tag{22}$$

where $p \in \{p^1, p^2, \cdots, p^L\}$, $p^q \in N_{p^q}$, $\hat{p}\in\Delta_{\hat{p}}$, and

$$\begin{aligned} N_{0d} &= \frac{1}{\lambda_{max}(L_1)}AX + B_2(I-p_i)D_{id}Y_0 + \left[\frac{1}{\lambda_{max}(L_1)}AX\right]^T \\ &\quad + [B_2(I-p_i)D_{id}Y_0]^T + B_2\sum_{k=1}^m p_{ik}D_{id}Y_{ak} \\ &\quad + (B_2\sum_{k=1}^m p_{ik}D_{id}Y_{ak})^T + B_2(I-p_i)D_{id}^-O_0 \\ &\quad + [B_2(I-p_i)D_{id}^-O_0]^T + B_2\sum_{k=1}^m p_{ik}D_{id}^-O_{ak} \\ &\quad + \left[B_2\sum_{k=1}^m p_{ik}D_{id}^-O_{ak}\right]^T + \frac{1}{\lambda_{max}(L_1)}B_1B_1^T \end{aligned}$$

$$Y = \begin{bmatrix} I_{n\times n} & \\ \cdots & 0 \\ I_{n\times n} & \\ 0 & I_{mn\times mn}\end{bmatrix}$$

$$\begin{aligned} N_{1d} &= [-B_2p_iD_{id}Y_{a1} + B_2D_dY_{b1} - B_2p_iD_{id}^-O_{a1} \\ &\quad + B_2D_d^-O_{b1}, \cdots, -B_2p_iD_{id}Y_{am} + B_2D_{id}Y_{bm} \\ &\quad - B_2p_iD_{id}^-O_{am} + B_2D_{id}^-O_{bm}] \end{aligned}$$

$$\begin{aligned} V_d &= \left[\frac{1}{\lambda_{max}(L_1)}CX\right. \\ &\quad + D(I-p_i)D_{id}Y_0 + D(I-p_i)D_{id}^-O_0 \\ &\quad \left. D(I-p_i)[D_{id}(Y_{ak}+Y_{bk}) + D_{id}^-(O_{ak}+O_{bk})]\right] \end{aligned}$$

$$\begin{aligned} N_{kp} &= -B_2^kD_{id}Y_{bp} - \left[B_2^kD_{id}Y_{bp}\right]^T \\ &\quad - B_2^kD_{id}^-O_{bp} - \left[B_2^kD_{id}^-O_{bp}\right]^T \end{aligned}$$

and determine $\hat{p}_{ik}(t)$ according to the adaptive law

$$\begin{aligned} \dot{\hat{p}}_{ik} &= \Pr oj_{[\min_q\{\underline{p}_{ik}^q\},\max_q\{\overline{p}_{ik}^q\}]}\{T_{ik}\} \\ &= \begin{cases} if & \hat{p}_{ik} = \min_q\left\{\underline{p}_{ik}^q\right\} \text{ and } T_{ik}\leq 0 \\ 0, & or\ \hat{p}_{ik} = \max_q\left\{\overline{p}_{ik}^q\right\} \text{ and } T_{ik}\geq 0 \\ T_{ik}, & otherwise \end{cases} \end{aligned} \tag{23}$$

where

$$\begin{aligned} T_{ik} &= -\tau_{ik}\left[\sum_{j=1}^N a_{ij}(x_i-x_j)\right]^T [PB_2\left(\sum_{d=0}^{2^m-1}\eta_{id}D_{id}\right)K_{ak} \\ &\quad + PB_2^k\left(\sum_{d=0}^{2^m-1}\eta_{id}D_{id}\right)K_b(\hat{p}_i) + PB_2^k\left(\sum_{d=0}^{2^m-1}\eta_{id}D_{id}\right)H_{ak} \\ &\quad + PB_2^k\left(\sum_{d=0}^{2^m-1}\eta_{id}D_{id}\right)H_b(\hat{p}_i)]\left[\sum_{j=1}^N a_{ij}(x_i-x_j)\right] \end{aligned}$$

$P = X^{-1}$, $K_{ak} = Y_{ak}X^{-1}$, $K_{bk} = Y_{bk}X^{-1}$, $H_{aj} = O_{ak}X^{-1}$, $H_{bj} = O_{bk}X^{-1}$, $\tau_{ik} > 0$ ($k \in I[1,m]$), and $\delta > 0$ belong to a gain of the system, and it is adaptively selected according to the actual situation. Then the control gain is given by

$$K(\hat{p}_i) = Y_0X^{-1} + \sum_{k=1}^m \hat{p}_{ik}Y_{ak}X^{-1} + \sum_{k=1}^m \hat{p}_{ik}Y_{bk}X^{-1} \tag{24}$$

Proof: see appendix

Corollary 1: When $r_n > r_f > 0$, the controller gain is given by (23) and (24), condition (22) is established, then the adaptive H_∞ performance index is less than r_n and r_f in normal and fault modes, respectively.

Algorithm 1: Let r_n and r_f to describe the adaptive H_∞ performance of the closed-loop system (21) in normal mode and fault mode, respectively. The perturbation tolerance level of the closed loop system is described by δ. Then, we can minimize the indicators r_n, r_f and maximize the index

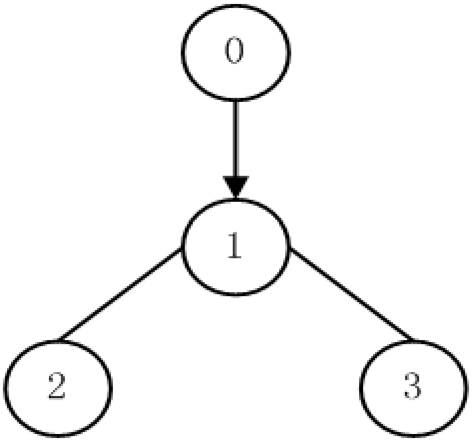

FIGURE 1. Topology of multi-agent systems.

by solving the following optimization process.

$$\begin{aligned} &\min\ \theta = \alpha\theta_n + \beta\theta_f + \gamma\theta_\delta \\ &\text{s.t. } (a)(20) \\ &\qquad (b)\ \varepsilon^*\left(L_1 \otimes P, \delta\right) \subset \partial\left(H\left(\hat{p}_i\right)\right) \end{aligned} \tag{25}$$

where $\theta_\delta = \frac{1}{\delta^*} = \frac{1}{\delta + max\left\{\sum_{i=1}^{N}\sum_{k=1}^{m}\frac{\tilde{p}_{ik}^2}{\tau_{ik}}\right\}}$, $\theta_n = r_n^2$, $\theta_f = r_f^2$, and α, β, γ are weighting coefficients.

However, by definition 2, we have that (b) cannot be shown as LMIs directly. Obviously $\varepsilon^*\left(L_1 \otimes P, \delta\right) \subset \varepsilon\left(L_1 \otimes P, \delta\right)$ which implies that (b) holds if (b1) holds, where

$$(bl)\ \varepsilon\left(L_1 \otimes P, \delta\right) \subset \partial\left(H\left(\hat{p}_i\right)\right) \tag{26}$$

condition (b1) is equivalent to

$$\begin{aligned} &\delta h(\hat{p})_k\left(L_1 \otimes P^{-1}\right) h\left(\hat{p}_i\right)_k^T \leq 1 \\ \Leftrightarrow &\begin{bmatrix} 1 & h(\hat{p}_i)_k\left(\dfrac{L_1 \otimes P^{-1}}{\delta}\right)^{-1} \\ * & \left(\dfrac{L_1 \otimes P^{-1}}{\delta}\right)^{-1} \end{bmatrix} \geq 0 \end{aligned} \tag{27}$$

for all $k \in I[1, m]$, where $h(\hat{p}_i)_k$ be the jth row of $H\left(\hat{p}_i\right)$. We have that (27) is equivalent to the following inequalities.

$$\begin{bmatrix} \dfrac{-1}{\lambda_{\max}\left(L_1\right)} & -O_{os} \\ * & -X \end{bmatrix} + \sum_{i=1}^{N}\sum_{k=1}^{m}\hat{p}_{ik}\begin{bmatrix} 0 & -O_{aks} - O_{bks} \\ * & 0 \end{bmatrix} \leq 0 \tag{28}$$

IV. EXAMPLES

Consider the system of form (1) and (2) with

$$A = \begin{bmatrix} -3 & 4 \\ -4 & -3 \end{bmatrix},\quad B_1 = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix},\quad B_2 = \begin{bmatrix} 0 & 3 \\ 3 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}^T,\quad D = \begin{bmatrix} 0 & 0.6 & 0 \\ 0 & 0 & 1 \end{bmatrix}^T$$

The topology of the multi-agent system is shown in Figure 1

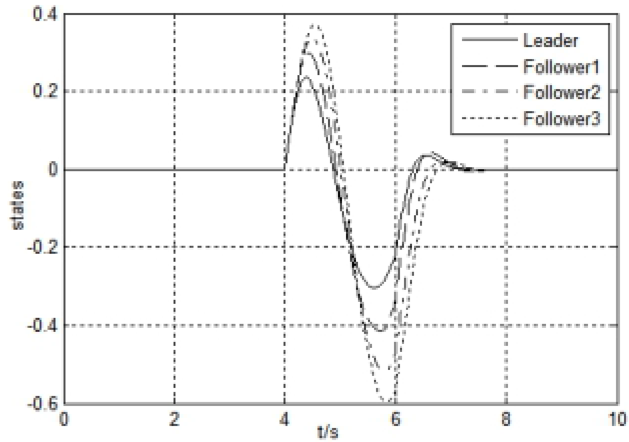

FIGURE 2. State difference curve of multi-agent system under disturbance.

Fault Mode 1: The first actuator is outage and the second actuator may be normal or loss of effectiveness, described by $p_1^2 = 1, 0 \leq p_2^2 \leq a$, where $a = 0.6$ denotes the maximal loss of effectiveness for the second actuator. Fault mode 2: Both of the two actuators are normal.

Let $\alpha = 10$, $\beta = 1$,$\gamma = 10$, the optimization index obtained by the fixed gain controller design method is $\theta_n = 0.6626$, $\theta_f = 1.6657$, $\theta_\delta = 0.2894$, $\theta = 11.1854$. By solving the optimization process (25), the optimization index can be given as $\theta_n = 0.5449$, $\theta_f = 1.2611$, $\theta_\delta = 0.2893$, $\theta = 9.6031$, in order to better reflect the superiority of the adaptive design method, we choose $\alpha = 110$, $\beta = 0.3$, $\gamma = 0.6$, then we get $\theta_n = 0.1735$, $\theta_f = 1.4733$, $\theta_\delta = 3.6237$. This phenomenon indicates that the adaptive controller design method presented in this paper has higher superiority than the fixed gain H_∞ controller design method.

In order to demonstrate the effectiveness of the proposed method, a system simulation is given. First, we consider the simulation of multi-agent system under actuator faults and saturation and disturbance. Agents 1 and 3 use fault 1, agent 2 use fault 2, The disturbance is given as

$$\omega_i(t) = \begin{cases} sin\,(2t), & 4 \leq t \leq 6 \\ 0, & otherwise \end{cases}$$

Considering that the system is in a steady state and then adding a disturbance signal, the state difference curve of the system is shown in Figure 2. The result shows that the system can still return to a stable state when it is disturbed. In order to show that the designed adaptive controller has better control effect than the fixed gain controller, the follower 1 is compared under the control of the adaptive controller and the fixed gain controller respectively. The disturbance is given as

$$\omega_1(t) = \begin{cases} 1, & 4 \leq t \leq 6.5 \\ 0, & otherwise \end{cases}$$

Figure 3 shows the state controller of follower 1 under adaptive controller and fixed gain controller. The results show that the adaptive controller has better control effect.

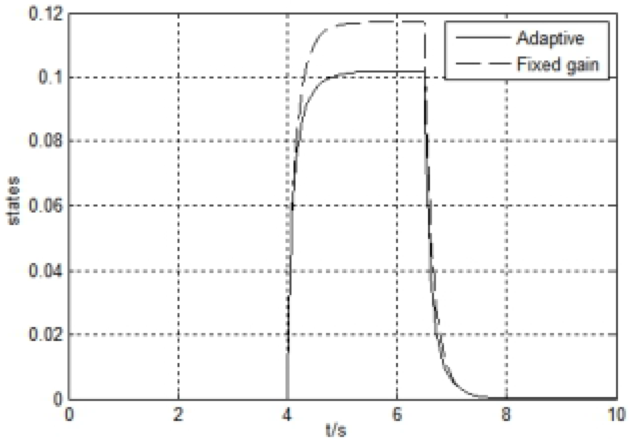

FIGURE 3. State curve of follower 1 under disturbance (The solid line is the adaptive controller and the dotted line is the fixed gain controller.).

V. CONCLUSION

This paper considers the problem of multi-agent system fault-tolerant control with both actuator faults and saturation and external disturbance. The fault-tolerant consistency of multi-agent systems is achieved by designing a control law with adaptive gain. By analyzing the closed-loop asymptotic stability of the tracking error system, the conditions for the consistency of the original multi-agent system are given. The results show that under the condition of actuator faults and saturation and external disturbance, the consistency of multi-agent system can be realized by reasonable selection of control parameters.

APPENDIX

Proof of Theorem 1: Choose the following Lyapunov function

$$V=\xi^T\left(L_1\otimes P\right)\xi+\sum_{i=1}^N\sum_{k=1}^m\frac{\tilde{p}_{ik}^2}{\tau_{ik}}$$

then from the derivative of V along the closed-loop system, it follows

$$\begin{aligned}\dot{V}&+\frac{1}{r_f^2}\zeta^T\zeta-\tilde{\omega}^T\tilde{\omega}\\ &\le M+\xi^T\left(L_1\otimes PB_1B_1^TP\right)\xi+\frac{1}{r_f^2}N^TN\\ &\quad-\left(\tilde{\omega}^T-\xi^T\left(L_1\otimes PB_1\right)\right)\left(\tilde{\omega}-\left(B_1^TP\otimes L_1\right)\xi\right)\end{aligned}$$

where

$$\begin{aligned}M&=\xi^T\left(L_1\otimes\left(PA+A^TP\right)\right)\xi+\left[\sum_{j=1}^N l_{1j}\xi_j\right]^T\left(M_1\right.\\ &\quad\left.+M_1^T\right)\left[\sum_{j=2}^N l_{1j}\xi_j\right]+\cdots+\left[\sum_{j=1}^N l_{Nj}\xi_j\right]^T\left(M_1\right.\\ &\quad\left.+M_1^T\right)\left[\sum_{j=1}^N l_{Nj}\xi_j\right]+2\left[\sum_{j=1}^N l_{ij}\xi_j\right]^T\left[PB_2\sum_{d=0}^{2^m-1}\eta_{id}\left[\right.\right.\\ &\quad\times D_{id}K_a\left(\tilde{p}_i\right)+\tilde{p}_iD_{id}K_b\left(\hat{p}_i\right)+D_{id}^-H_a\left(\tilde{p}_i\right)\\ &\quad\left.\left.+\tilde{p}_iD_{id}^-H_b\left(\hat{p}_i\right)\right]\right]\left[\sum_{j=1}^N l_{ij}\xi_j\right]+2\sum_{i=1}^N\sum_{k=1}^m\frac{\tilde{p}_{ik}\dot{\hat{p}}_{ik}}{\tau_{ik}}\end{aligned}$$

$$\begin{aligned}M_1&=PB_2\sum_{d=0}^{2^m-1}\eta_{id}\left[\left(I_m-p_i\right)D_{id}K_0+D_{id}K_a\left(p_i\right)\right.\\ &\quad-p_iD_{id}K_a\left(\hat{p}_i\right)+\left(I_m-\hat{p}_i\right)D_{id}K_b\left(\hat{p}_i\right)\\ &\quad+\left(I_m-p_i\right)D_{id}^-H_0+D_{id}^-H_a\left(p_i\right)-p_iD_{id}^-H_a\left(\hat{p}_i\right)\\ &\quad\left.+\left(I_m-\hat{p}_i\right)D_{id}^-H_b\left(\hat{p}_i\right)\right]\end{aligned}$$

$$\begin{aligned}N&=\sum_{d=0}^{2^m-1}\eta_{id}\left[I_N\otimes C+\left(L_1\otimes D\right)\left(I_m-\rho_i\right)\right.\\ &\quad\left.\left[D_{id}K_b\left(\hat{p}_i\right)+D_{id}^-H_a\left(\hat{p}_i\right)\right]\right]\xi\end{aligned}$$

let $B=\left[b^1,\cdots b^m\right]$, $B^k=\left[0,\cdots b^k\cdots,0\right]$, we have

$$\dot{V}+\frac{1}{r_f^2}\zeta^T\zeta-\tilde{\omega}^T\tilde{\omega}\le M+\xi^T\left(L_1\otimes PB_1B_1^TP\right)\xi+\frac{1}{r_f^2}N^TN$$

Let $P=X^{-1}$, $K_0=Y_0X^{-1}$, $K_{ak}=Y_{ak}X^{-1}$, $K_{bk}=Y_{bk}X$, $H_0=O_0X^{-1}H_{ak}=O_{ak}X^{-1}$, $H_{bk}=O_{bk}X^{-1}$, choose the adaptive laws as (23), we can know

$$\begin{aligned}&\left[\sum_{j=1}^N l_{ij}\xi_j\right]^T\left[PB_2\sum_{d=0}^{2^m-1}\eta_{id}\left[D_{id}K_a\left(\tilde{p}_i\right)+\tilde{p}_iD_{id}K_b\left(\hat{p}_i\right)\right.\right.\\ &\quad\left.\left.+D_{id}^-H_a\left(\tilde{p}_i\right)+\tilde{p}_iD_{id}^-H_b\left(\hat{p}_i\right)\right]\right]\left[\sum_{j=1}^N l_{ij}\xi_j\right]\\ &\quad+\sum_{i=1}^N\sum_{k=1}^m\frac{\tilde{p}_{ik}\dot{\hat{p}}_{ik}}{\tau_{ik}}\le 0\end{aligned}$$

then

$$\begin{aligned}M&+\xi^T\left(L_1\otimes PB_1B_1^TP\right)\xi+\frac{1}{r_f^2}N^TN\\ &\le\xi^T\left(L_1\otimes\left(PA+A^TP+PB_1B_1^TP\right)\right)\xi\\ &\quad+\left[\sum_{j=1}^N l_{1j}\xi_j\right]^T\left(\Gamma_1+\Gamma_1^T\right)\left[\sum_{j=1}^N l_{1j}\xi_j\right]+\ldots\\ &\quad+\left[\sum_{j=1}^N l_{Nj}\xi_j\right]^T\left(\Gamma_N+\Gamma_N^T\right)\left[\sum_{j=1}^N l_{Nj}\xi_j\right]+\frac{1}{r_f^2}N^TN\end{aligned}$$

where

$$\begin{aligned}\Gamma_i&=PB_2\sum_{d=0}^{2^m-1}\left[\left(I_m-p_i\right)D_{id}K_0+D_{id}K_a\left(p_i\right)\right.\\ &\quad-p_iD_{id}K_a\left(\hat{p}_i\right)+\left(I_m-\hat{p}_i\right)D_{id}K_b\left(\hat{p}_i\right)\\ &\quad+\left(I_m-p_i\right)D_{id}^-H_0+D_{id}^-H_a\left(p_i\right)-p_iD_{id}^-H_a\left(\hat{p}_i\right)\end{aligned}$$

$$+\left(I_m-\hat{p}_i\right)D_{id}^{-}H_b\left(\hat{p}_i\right)\Big]$$

then

$$\dot{V}\leq\xi^T\left(L_1\otimes I_n\right)\left(\sum_{d=0}^{2^m-1}\eta_{id}\Phi+\frac{1}{r_f^2}W^TW\right)\left(L_1\otimes I_n\right)\xi$$

where $\Phi=diag\left(\phi_{1,\ldots}\phi_n\right)$

$$\phi_i=\frac{1}{\lambda_{max}\left(L_1\right)}\left(PA+A^TP+PB_1B_1^TP\right)+\Gamma_i+\Gamma_i^T$$

If for all $p\in\left\{p^1,p^1,\cdots,p^L\right\}$, $p^q\in N_{p^q}$

$$\sum_{d=0}^{2^m-1}\eta_{id}\left[K_{0d}+K_{1q}\left(\hat{p}_{ik}\right)+K_{2q}\left(\hat{p}_{ik}\right)\right]+\frac{1}{r_f^2}W^TW<0$$

then the adaptive rate can be chosen (23) to make $\dot{V}<0$ is established, where $K_{0d}=N_{0d}$, and

$$\begin{aligned}K_{1q}\left(\hat{p}_{ik}\right)=&-B_2p_iD_{id}\sum_{k=1}^{m}\hat{p}_{ik}Y_{ak}+B_2D_d\sum_{k=1}^{m}\hat{p}_{ik}Y_{bk}\\&+\left(-B_2p_iD_{id}\sum_{k=1}^{m}\hat{p}_{ik}Y_{ak}+B_2D_d\sum_{k=1}^{m}\hat{p}_{ik}Y_{bk}\right)^T\\&-B_2p_iD_{id}^{-}\sum_{k=1}^{m}\hat{p}_{ik}O_{ak}+B_2D_d^{-}\sum_{k=1}^{m}\hat{p}_{ik}O_{bk}\\&+\left(-B_2p_iD_{id}^{-}\sum_{k=1}^{m}\hat{p}_{ik}O_{ak}+B_2D_d^{-}\sum_{k=1}^{m}\hat{p}_{ik}O_{bk}\right)^T\\K_{2q}\left(\hat{p}_{ik}\right)=&\sum_{k=1}^{m}\sum_{p=1}^{m}\hat{p}_{ik}\hat{p}_{ip}\Big(-B^kD_{id}Y_{bp}-\left[B^kD_{id}Y_{bp}\right]^T\\&-B^kD_{id}^{-}O_{bp}-\left[B^kD_{id}^{-}O_{bp}\right]^T\Big)\\W=&\sum_{d=0}^{2^m-1}\eta_d\Big[\frac{1}{\lambda_{max}\left(L_1\right)}CX+D\left(I-p_i\right)D_{id}Y_0\\&+D\left(I-p_i\right)D_{id}^{-}O_0+\sum_{k=1}^{m}\hat{p}_{ik}D\left(I-p_i\right)\\&\left[D_{id}(Y_{ak}+Y_{bk})+D_{id}^{-}\left(O_{ak}+O_{bk}\right)\right]\Big]\end{aligned}$$

By lemma 3 and (22), it follows that $\dot{V}<0$, for any $\xi_i\in\partial\left(H\left(\hat{p}_i\right)\right)$, $p\in\left\{p^1,p^1,\cdots,p^L\right\}$ and $\hat{p}_i$ satisfying (23).

Proof Control Objective (I): $\dot{V}\leq M+\xi^T\left(L_1\otimes PB_1\tilde{\omega}\right)+(L_1\otimes\tilde{\omega}^TB_1^TP)\xi$, noting that $\xi^T\left(L_1\otimes PB_1\tilde{\omega}\right)+(L_1\otimes\tilde{\omega}^TB_1^TP)\xi\leq\xi^T\left(L_1\otimes PB_1B_1^TP\right)\xi+\tilde{\omega}^T\tilde{\omega}$, we have

$$\dot{V}\leq M+\xi^T\left(L_1\otimes PB_1B_1^TP\right)\xi+\tilde{\omega}^T\tilde{\omega}$$

By the proof of control objective (II), we have $\dot{V}\leq\tilde{\omega}^T\tilde{\omega}$, which implies that

$$V\left(\xi\left(t\right)\right)\leq\int_0^\infty\tilde{\omega}^T\left(t\right)\tilde{\omega}\left(t\right)dt+\sum_{i=1}^{N}\sum_{k=1}^{m}\frac{\tilde{p}_{ik}^2\left(0\right)}{\tau_{ik}}$$

then, the conclusion can be drawn that trajectories of the closed-loop system that start from the origin will remain inside $\varepsilon^*\left(L_1\otimes P,\delta^*\right)$.

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