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Strong Controllable Siphon Basis-Based Robust Deadlock Control for Manufacturing Systems With Multiple Unreliable Resources

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ABSTRACT In real-world automated manufacturing systems (AMSs), a breakdown of unreliable resources in operation often makes most existing deadlock control policies inapplicable. This work focuses on robust deadlock control problem in AMSs with multi-type and multi-unit unreliable resources. The unreliable resource's failure and repair activities are modeled by Petri nets. We introduce a concept of *strong* controllable siphon basis, which can be seen as an extension of the controllable siphon basis proposed in our previous work. Then by adding a control place with proper depth variable to each strict minimal siphon (SMS) and R-type SMS in a strong controllable siphon basis, we successfully develop a small-scaled robust deadlock controller for AMSs under consideration. Such a robust controller can guarantee that, as long as at least one unit of each unreliable resource type is available, all types of parts can be processed smoothly through any one of their routes even during downtime. Moreover, the number of control places of the proposed controller is no more than that of the activity places in the Petri net model and its size grows polynomially with Petri net model.

INDEX TERMS Automated manufacturing systems (AMSs), robust deadlock controllers, strong controllable siphon basis, Petri nets, unreliable resources.

I. INTRODUCTION

Deadlock-free resource allocation problem in automated manufacturing systems (AMSs) has been an active research area and received increasingly attentions in both industry and academia during the last decades [1]–[4]. In real-world AMSs, resource failures occur unpredictably and may reduce dramatically the resource utilization as it can result in deadlocks which degrade the performance of AMSs greatly. Therefore, it is essential to design an efficient deadlock controller to ensure a smooth production even if resource failures happen.

Three kinds of strategies are proposed to deal with deadlock problems: deadlock detection and recovery [5], deadlock prevention [2], [6]–[13], and deadlock avoidance [14]–[16]. The first one uses a monitoring mechanism for detecting deadlock occurrence and a resolution procedure for appropriately preempting some deadlocked resources. Prevention method is usually achieved by establishing a static resource allocation policy such that the system can never enter a deadlock state. The last one is online control policies that use feedback information on the current resource allocation status and future process resource requirements, keeping the system away from deadlock states.

There have been tremendous works developed on the deadlock control problems for AMSs without unreliable resources. Petri nets are utilized to describe such systems [17]–[20], where deadlocks can be characterized by strict minimal siphons (SMSs) or maximal perfect resource transition circuits (MPCs) [21]–[25]. By adding a control place to each empty SMS or saturated MPC, deadlocks can be prevented from happening, and then various deadlock control policies are developed for AMSs. Further, to obtain a deadlock controller with small structures, the concepts of elementary siphon and controllable siphon basis are introduced in [8] and [12], respectively.

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Note that resource failure is a common problem in a real production system [26]–[29]. However, when an unreliable resource fails, the existing deadlock control policies are always no longer in force and deadlocks or blocked states may rise in the disturbed system. Consequently, a challenge is rising: how to control or avoid deadlocks or blocked states in the operation of AMSs when unreliable resources fail. To date, the existing robust control methods for AMSs with unreliable resources mostly fall into deadlock avoidance. Hsieh modeled the failure of resource as decrease of tokens in Petri net models [30], [31]. Lawley and Sulistyono [32] investigated the AMS with a single unreliable resource, and combined some developed control policies with neighborhood constraints to ensure the system continue producing the part types that do not require the failed resource. Chew and Lawley [33] relaxed the assumption so that the system may have some unreliable resources and every part type at most required one unreliable resource on its route. Their work ensures that when a subset of unreliable resources fails simultaneously, all the type parts that do not require the failed resource can be produced smoothly.

On the other hand, some Petri net-based robust deadlock prevention policies are presented [34]–[42]. Most of them deal with the manufacturing systems with a unique unreliable resource type. In these systems, only one unit of unreliable resources is assumed to fail at a time [36], [38]–[41]. For example, for a class of manufacturing systems with one unreliable resource type, Wang *et al.* [38] concentrated on distributing parts requiring failed resource throughout the buffer space of shared resources that the disturbed parts do not block the production of part types not requiring failed resource. Based on the concept of strong transition covers, Feng *et al.* [39] developed a 1-robust deadlock controller for the same AMSs. Later, Wu *et al.* extended these results to general cases with multi-unit resource failures [41].

This paper concentrates on a robust deadlock prevention policy for AMSs with multi-type and multi-unit unreliable resources. Each resource type is composed of several identical units, and some resources may fail simultaneously even though they belong to different types. The failed resources can return to the system to continue processing parts after their repair. Those AMSs can be modeled by a class of Petri nets, namely, *systems of simple sequential processes with resources* (S^3PRs) in the absence of resource failures. Another Petri net called failure-repair nets are used to model the resource failure and repair activities. Then S^3PR_u, the composition of S^3PR and failure-repair nets, can model the whole behavior of AMSs under consideration. The main contribution of this work is as follows.

(1) We propose two concepts of R-type SMS and strong controllable siphon basis. An SMS is an *R-type* SMS if its resource set contains at least one unreliable resource type. A controllable siphon basis is *strong* if it includes at least one R-type SMS.
(2) By adding a control place with a proper depth control variable and suitable related arcs to each SMS in a strong controllable siphon basis, we develop a robust controller with small size.
(3) Compared with existing works [34], [36]–[41], we believe that the robustness level of our proposed controller is improved largely because it allows that multi-type and multi-unit unreliable resources fail simultaneously.

The rest of the paper is organized as follows. Section II reviews preliminaries used throughout this paper. The concepts of R-type SMS and strong controllable siphon basis are introduced in Section III. Then the design of a robust controller based on strong controllable siphon basis is developed in Section IV. Finally, Section V concludes this paper.

II. PRELIMINARIES

A. PETRI NETS

A Petri net is a 3-tuple $N = (P, T, F)$, where P and T are finite, nonempty and disjoint sets. P is a set of places and T is a set of transitions. $F \subseteq (P \times T) \cup (T \times P)$ is called directed arcs. Given a Petri net $N = (P, T, F)$ and a vertex $x \in P \cup T$, the preset of x is defined as ${}^\bullet x = \{y \in P \cup T | (y, x) \in F\}$, and the post set of x is defined as $x^\bullet = \{y \in P \cup T | (x, y) \in F\}$. The notation can be extended to a set. For example, let $X \subseteq P \cup T$, then ${}^\bullet X = \cup^\bullet_{x \in X} x$ and $X^\bullet = \cup_{x \in X} x^\bullet$. A state machine is a Petri net in which each transition has exactly one input and one output place. A marking or state of N is a mapping M: $P \to \mathbb{Z}^+$, where $\mathbb{Z}^+$ is the non-negative integer set. Given a place $p \in P$ and a marking M, $M(p)$ denotes the number of tokens in p at M, and we use $\Sigma_{p \in P} M(p)p$ to denote vector M. Let $S \subseteq P$ be a set of places, the sum of tokens in all places of S at M is denoted by $M(S)$, i.e., $M(S) = \Sigma_{p \in S} M(p)$. A Petri net N with an initial marking M_0 is called a marked Petri net or net for simplicity, denoted as (N, M_0).

A transition $t \in T$ is enabled at a marking M, denoted by $M[t >$, if $\forall p \in {}^\bullet t$, $M(p) > 0$. An enabled transition t at M can be fired, resulting in a new marking M', denoted by $M[t > M'$, where $M'(p) = M(p) - 1$, $\forall p \in {}^\bullet t \backslash t^\bullet$; $M'(p) = M(p) + 1$, $\forall p \in t^\bullet \backslash {}^\bullet t$; and otherwise $M'(p) = M(p)$, $\forall p \in P - \{{}^\bullet t \backslash t^\bullet, t^\bullet \backslash {}^\bullet t\}$. A sequence of transitions $\alpha = t_1 t_2 \ldots t_k$, $t_i \in T$, $i \in \mathbb{N}_k = \{1, 2, \ldots, k\}$, is feasible from a marking M, if there exist $M_i[t_i > M_{i+1}$, $i \in \mathbb{N}_k$, where $M_1 = M$, and M_i is called a reachable marking from M. Let $R(N, M_0)$ denote the set of all reachable markings of N from M_0. A transition t is live if $\forall M \in R(N, M_0)$, $\exists M' \in R(N, M)$ such that $M'[t >$ holds. A net is live if every transition is live.

A transition t is dead at a marking $M \in R(N, M_0)$ iff $\forall M' \in R(N, M)$, $M'[t >$ does not hold.

The incidence matrix of N is a matrix $[N] : P \times T \to \{-1, 0, 1\}$ such that $[N](p, t) = -1$ if $p \in {}^\bullet t \backslash t^\bullet$; $[N](p, t) = 1$ if $p \in t^\bullet \backslash {}^\bullet t$; and otherwise $[N](p, t) = 0$ if $p \in P - \{{}^\bullet t \backslash t^\bullet, t^\bullet \backslash {}^\bullet t\}$. A P-vector is a column vector I: $P \to \mathbb{Z}$ indexed by P, and a T-vector is a column vector J: $T \to \mathbb{Z}$ indexed by T, where $\mathbb{Z}$ is the integer set. A nonzero P-vector I is a P-invariant if $I \neq 0$ and $I^T[N] = 0^T$.

The support of a P-invariant I is the set of places, $||I|| = \{p \in P | I(p) \neq 0\}$. A P-invariant I is minimal if there does not exist a P-invariant I' such that $||I'|| \subseteq ||I||$. A nonempty subset of places $S \subseteq P$ is a siphon if $^\bullet S \subseteq S^\bullet$, i.e., an input transition is also an output transition of S. If there does not exist a siphon contained in a siphon as a proper subset, it is minimal. A minimal siphon is strict if it does not contain the support of any P-invariant in N. Strict minimal siphon is written as SMS. Let Π denote the set of all SMSs of N throughout the paper.

Two marked Petri nets $(N_i, M_{i0}) = (P_i, T_i, F_i, M_{i0})$, $i \in \mathbb{N}_2$, are compatible if $\forall p \in P_1 \cap P_2, M_{10}(p) = M_{20}(p)$. The composition of two compatible marked Petri nets (N_1, M_{10}) and (N_2, M_{20}) is a marked Petri net $(N_1, M_{10}) \otimes (N_2, M_{20}) = (P, T, F, M_0)$, where $P = P_1 \cup P_2, T = T_1 \cup T_2, F = F_1 \cup F_2$, and $M_0(p) = M_{i0}(p)$ if $p \in P_i, i \in \mathbb{N}_2$.

B. S^{3}PR CLASS

S^{3}PRs are developed in [6] for modeling AMSs with flexible routings and defined in a recursive way.

(1) A Simple Sequential Process with Resources (S^{2}PR) is a Petri net $N = (P \cup P_0 \cup P_R, T, F)$, so that:

- (1.1) $P \neq \emptyset$, $P_0 = \{p_0\}$, $P_R \neq \emptyset$, $P \cap P_0 = \emptyset$, and $(P \cup P_0) \cap P_R = \emptyset$. $p_0, p \in P$ and $r \in P_R$ are called a process idle place, an operation or activity place, and a resource place, respectively;
- (1.2) the subset, N, generated by $P \cup P_0$ and T is a strongly connected state machine;
- (1.3) every circuit of N contains the place p_0.
- (1.4) $\forall p \in P, \forall t \in ^\bullet p, \forall t' \in p^\bullet, ^\bullet t \cap P_R = t'^\bullet \cap P_R = \{r\}$;
- (1.5) The following three statements are verified: a) $\forall r \in P_R, ^{\bullet\bullet}r \cap P = r^{\bullet\bullet} \cap P \neq \emptyset$, b) $\forall r \in P_R, ^\bullet r \cap r^\bullet = \emptyset$; c) $^{\bullet\bullet}(p_0) \cap P_R = (p_0)^{\bullet\bullet} \cap P_R = \emptyset$.

(2) A System of S^{2}PR, called S^{3}PR for short, is defined recursively as follows.

- (2.1) An S^{2}PR is an S^{3}PR.
- (2.2) Let $N_i = (P_i \cup P_{0i} \cup P_{Ri}, T_i, F_i)$, $i \in \mathbb{N}_2$, be two S^{3}PRs, such that $(P_1 \cup P_{01}) \cap (P_2 \cup P_{02}) = \emptyset$, $P_{R1} \cap P_{R2} \neq \emptyset$, $T_1 \cap T_2 = \emptyset$. The net $N = N_1 \otimes N_2 = (P \cup P_0 \cup P_R, T, F)$ resulting of the composition of N_1 and N_2 via $P_{R1} \cap P_{R2}$ is also an S^{3}PR.

Let N be an S^{3}PR. Its acceptable initial marking M_0 must satisfy that 1) $M_0(p_0) \geq 1$ for $p_0 \in P_0$ is a finite number; 2) $M_0(p) = 0, \forall p \in P$; 3) $M_0(r) \geq 1, \forall r \in P_R$, where $M_0(r)$ is the capacity of the resource r.

Let $N = (P \cup P_0 \cup P_R, T, F)$ be an S^{3}PR and a transition $t \in T$, let $^{(o)}t$ and $t^{(o)}$ denote the input and output operation or process idle places of t, respectively, and $^{(r)}t$ and $t^{(r)}$ denote the input and the output resource place of t, respectively. For a given marking $M \in R(N, M_0)$, t is process-enabled at M if $M(^{(o)}t) > 0$, and t is resource-enabled at M if $M(^{(r)}t) > 0$. Only transitions that are process- and resource-enabled at the same time can be fired.

Let $N = (P \cup P_0 \cup P_R, T, F)$ be an S^{3}PR, $\forall r \in P_R$, $H(r) = \{p \in P | ^{(r)\bullet}p = r\}$; $\forall R \subseteq P_R$, $H(R) = \cup_{r \in R} H(r)$. Let S be an SMS of N, $C[S] = H(S \cap P_R) \backslash S$ is called as the complementary set of S.

Let $N = (P \cup P_0 \cup P_R, T, F)$ be an S^{3}PR, and a string of nodes $\alpha = x_1 x_2 x_3 \ldots x_q$ is called a path of N iff $x_{i+1} \in x_i^\bullet$ for $i \in N_{q-1}$, where $q - 1$ is the length of α. An elementary path is a path whose nodes are all different (except, perhaps, x_1 and x_q). $\forall x$ and y be two nodes in $P \cup T$. If there exists a simple path in N from x to y with length greater than 1, which does not contain any place in $P_0 \cup P_R$, we say that x is previous to y in N. This fact is denoted as $x < y$. The fact that x is not previous to y in N is denoted as $x \nless y$. Let $W \subseteq (P \cup T)$ be a set of nodes of N. Then we say that x is previous to W in N, denoted as $x < W$, if and only if $\forall y \in W, x < y$. The fact that x is not previous to W in N, denoted as $x \nless W$, if and only if $\forall y \in W, x \nless y$.

C. CONTROLLABLE SIPHON BASIS

Let $\Xi \subseteq \prod$ be a subset of SMSs in a marked S^{3}PR (N, M_0). For each $S \in \prod \backslash \Xi$, if there exists $\pi_S \subseteq \Xi$ such that $C[S] \subseteq C[\pi_S]$ and $M_0(S) > |\pi_S|$, where $C[\pi_S] = \cup_{S' \in \pi_S} C[S']$ and $|\pi_S|$ denote the number of SMSs in π_S, then Ξ is called a *controllable siphon basis* of (N, M_0), and π_S as the *relevant siphon set* of S.

Definition 1 [12]: Let $(N, M_0) = (P \cup P_0 \cup P_R, T, F, M_0)$ be a marked S^{3}PR, and $\Xi \subseteq \prod$. The net $(C_\Xi, M_{\Xi 0}) = (P_\Xi, T_\Xi, F_\Xi, M_{\Xi 0})$ is called as the *Petri net controller* of (N, M_0) with respect to Ξ if and only if

1) $P_\Xi = \{p_S, S \in \Xi | p_S$ is a control place corresponding to $S\}$ is a set of control places;
2) $F_\Xi = F_{\Xi 1} \cup F_{\Xi 2} \cup F_{\Xi 3}$ such that
$F_{\Xi 1} = \cup_{S \in \Xi} \{(p_S, t) | t \in P_0^\bullet, t < C[S]\}$,
$F_{\Xi 2} = \{(t, p_S) | t \in C[S]^\bullet, t \nless C[S]\}$,
$F_{\Xi 3} = \{(t, p_S) | t \nless C[S]$, and $\exists t_1 \in (^{(o)}t)^\bullet, \ni t_1 < C[S]\}$;
3) $T_\Xi = \cup_{S \in \Xi} (p_S^\bullet \cup ^\bullet p_S)$;
4) $M_{\Xi 0}$ is defined as follows: 4.1) $M_{\Xi 0}(p) = M_0(p), \forall p \in P \cup P_0 \cup P_R$; 4.2) $M_{\Xi 0}(p_S) = M_0(S) - \xi_S$, where $\xi_S \in [1, M_0(S) - 1]$ is an integer called as a *control depth variable*.

According to Definition 1, the controlled Petri net with respect to Ξ is defined as follows.

$$\begin{aligned}(N_C, M_{C0}) &= (N, M_0) \otimes (C_\Xi, M_{\Xi 0}) \\ &= (P \cup P_0 \cup P_R \cup P_\Xi, T, F \cup F_\Xi, M_{C0})\end{aligned}$$

where $M_{C0}(p) = M_0(p), \forall p \in P \cup P_0 \cup P_R$; $M_{C0}(p) = M_{\Xi 0}(p), \forall p \in P_\Xi$.

Lemma 1 [12]: Let $\{S_1, S_2, \ldots, S_n\}$ be a subset of SMSs of a marked S^{3}PR (N, M_0). Add to each S_i a control place and related arcs with the control depth variable ξ_i and denote by (N_C, M_{C0}) the controlled Petri net. For each SMS S different from all S_i, S is not empty at any reachable marking of (N_C, M_{C0}) if $C[S] \subseteq (\cup_{i=1}^n C[S_i])$ and $\Sigma_{i=1}^n \xi_i \geq \Sigma_{i=1}^n M_0(S_i) - M_0(S) + 1$.

Theorem 1 [12]: Let $\Xi \subseteq \prod$ be a controllable siphon basis of a marked S^{3}PR (N, M_0). Add to each $S_i \in \Xi$ a control place and related arcs with the control depth variable ξ_i. Then the obtained controlled Petri net (N_C, M_{C0}) is live.

III. R-TYPE SMS AND STRONG CONTROLLABLE SIPHON BASIS

A. PETRI NETS MODELING FAILURE OF UNRELIABLE RESOURCES

*Definition 2:*Let Â be an AMS with multiple unreliable resources R_u, and an S^{3}PR $(N, M_0) = (P \cup P_0 \cup P_R, T, F, M_0)$ modeling Â without any unreliable ones. The *failure-repair net* is as follows:

$$(N[R_u], M[R_u]) = \cup_{r_u \in R_u} \cup_{p_i \in H(r_u)} (\{p_i, q_i\}, \{\alpha_i, \beta_i\}, F_i, M_{pi})$$

where $p_i \in P \cap H(r_u)$, q_i is a *repair place* corresponding to p_i, α_i and β_i are transitions modeling the resource failure and repair, called as *failure transition* and *repair transition*, respectively. $F_i = \{(p_i, \alpha_i), (\alpha_i, q_i), (q_i, \beta_i), (\beta_i, p_i)\}$, $M[R_u](p_i) = M[R_u](q_i) = 0$.

We compose (N, M_0) and $(N[R_u], M[R_u])$ via shared places in $P \cap H(R_u)$ and obtain the model, S^{3}PR$_u$, for *an S^{3}PR with unreliable resources* as follows:

$$\begin{aligned}(N_u, M_{u0}) &= (N, M_0) \otimes (N[R_u], M[R_u]) \\ &= (P \cup P_0 \cup P_R \cup P_u, T \cup T_u, F \cup F_u, M_{u0})\end{aligned}$$

where $P_u = \{q_i | r_u \in R_u, p_i \in P \cap H(r_u)$ and q_i is the repair place corresponding to $p_i\}$,

$T_u = \{\alpha_i, \beta_i | \alpha_i$ and β_i are failure transition and repair transition$\}$, $F_u = \cup_{r_u \in R_u} \cup_{p_i \in H(r_u)} F_i$, $M_{u0}(p) = M_0(p)$, $p \in P_0 \cup P_R$; otherwise, $M_{u0}(p) = 0$.

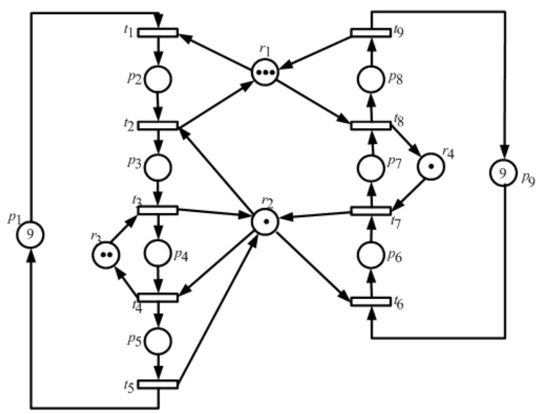

FIGURE 1. An S^3PR (N, M_0).

Example 1: Let us consider an AMS cell that can process two types of parts J_1 and J_2 by four resource types r_1-r_4 with capacities 3, 1, 2 and 1. The unreliable resources are r_1 and r_3. The models without unreliable resources, (N, M_0), and with unreliable resources, (N_u, M_{u0}), are shown in Fig. 1 and Fig. 2, respectively. In Fig. 2 the dashed parts model the failure and repair activities of r_1 and r_3. There are three SMSs $S_1 = \{r_1, r_2, r_4, p_3, p_5, p_8\}$, $S_2 = \{r_2, r_3, p_5, p_6\}$ and $S_3 = \{r_1, r_2, r_3, r_4, p_5, p_8\}$ in (N, M_0). $\Xi = \{S_1, S_2\}$ is a controllable siphon basis of (N, M_0) because

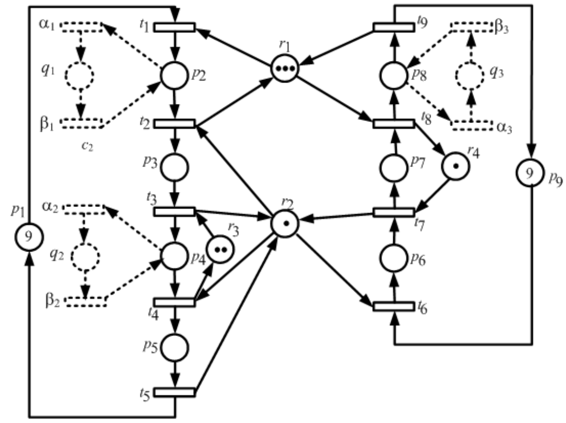

FIGURE 2. An S^3PR_u (N_u, M_{u0}).

$C[S_3] = C[S_1] \cup C[S_2] = \{p_2, p_3, p_4, p_6, p_7\}$ and $M_0(S_3) > 2$. Its corresponding controlled net (N_C, M_{C0}) (shown in Fig. 3) by Definition 1 is live by Theorem 1. The state $M_c = (p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, r_1, r_2, r_3, r_4, c_1, c_2) = (7, 1, 0, 1, 0, 1, 1, 2, 5, 0, 0, 1, 0, 1, 0)$ is live at (N_C, M_{C0}), that is, its corresponding operation process can end smoothly. However, M_c turns to be a blocking state if two units of r_1 fail in p_8 and one of r_3 in p_4 at M_c. This state (denoted by M) can be described in Fig. 4.

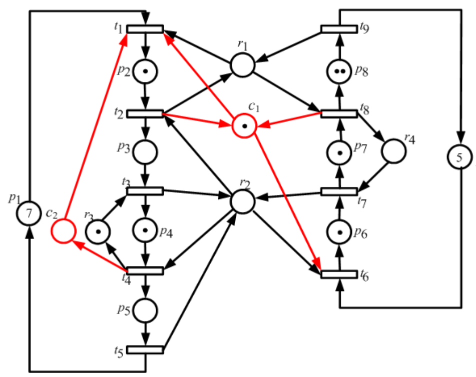

FIGURE 3. A reachable marking M_C.

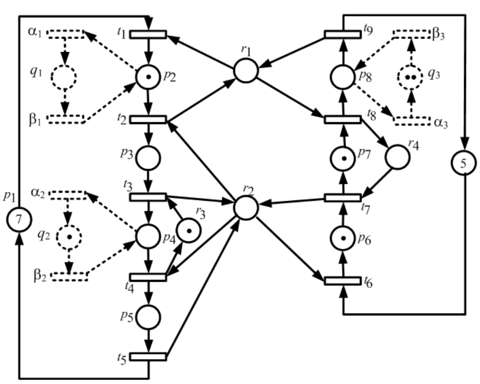

FIGURE 4. A deadlock marking M.

It is easy to observe that S_1 is empty at M before the three unreliable resources are repaired. The reason is that there exists a new resource circular waiting of shared resources r_1, r_2 and r_4. Therefore, the whole manufacturing process is stagnating.

Based on the above analysis, the control policy by Definition 1 is disabled for AMSs with unreliable resources, and it is essential to design a novel control policy to prevent all SMSs from being empty when some unreliable resources fail in the system.

Our desired control objective is to guarantee that any kind of parts can be processed continuously through any one of their processing routes, even if some unreliable resource units fail. This can be defined as follows.

Definition 3: Let (N_u, M_{u0}) be an S^3PR_u with an unreliable resource set R_u, and C_u be a controller for (N_u, M_{u0}). C_u is said to be a robust controller for (N_u, M_{u0}) if the controlled system $(N_u, M_{u0}) \otimes C_u$ satisfies

1) The controlled system is live if no unreliable resource fails;
2) Any kind of parts can be processed smoothly through any of its process routes when k units of each unreliable resource $r_u \in R_u$ fail, where $k \leq M_{u0}(r_u) - 1$.
3) Each failed resource can return to the system after its repair without affecting the continuous operation of any part type.

Note: In our work we assume that at least one unit of each unreliable resource type can work normally at any time. Without loss of generality, we further assume that unreliable resources only fail in the processing time because an unreliable unit can be moved to repair directly if it fails when it is idle.

B. R-TYPE SMS

As shown in Example 1, $S_1 = \{r_1, r_2, r_4, p_3, p_5, p_8\}$ is not empty at Mc but empty at M. The reason is that q_3 occupies two units of r_1, i.e., $M(q_3) = 2$, which implies that q_3 should belong to the complementary set of S_1. However, ${}^\bullet q_3$i s an uncontrollable transition. Therefore, we set ${}^{(o)\bullet}q_3$, p_8, into the complementary set of S_1 and develop a new type SMS in (N_u, M_{u0}). These SMSs are not changed and only their complementary sets are different because their resource sets contain unreliable resources.

Definition 4: Let S be an SMS of (N, M_0), we call S as *an R-type SMS* of (N_u, M_{u0}), denoted by S_u, if S contains unreliable resources. Further, let R_{Su} denote the set of unreliable resources of S_u. $C[S_u] = C[S] \cup (S \cap H(R_{Su}))$ is called as the *complementary set of* S_u.

Due to Definition 4, there is little difference between an SMS S of N and its corresponding R-type SMS S_u of N_u except for their complementary sets. That is, $S = S_u$, but $C[S] \neq C[S_u]$ if $S \cap H(R_{Su}) \neq \emptyset$. For example, in Fig. 2 we have $S_{u1} = S_1 = \{r_1, r_2, r_4, p_3, p_5, p_8\}$, $S_{u2} = S_2 = \{r_2, r_3, p_5, p_6\}$ and $S_{u3} = S_3 = \{r_1, r_2, r_3, r_4, p_5, p_8\}$. However, $C[S_{u1}] = \{p_2, p_6, p_7, p_8\} = C[S_1] \cup \{p_8\}$, $C[S_{u3}] = \{p_2, p_3, p_4, p_6, p_7, p_8\} = C[S_3] \cup \{p_8\}$ and $C[S_{u2}] = \{p_3, p_4\} = C[S_2]$ because $\{r_1, r_3\} \cap S_2 = \emptyset$.

Suppose S_u is an R-type SMS and S is its corresponding SMS, and $C[S] \neq C[S_u]$. $\forall p \in C[S_u] \backslash C[S]$, $q \in P_u$ is the corresponding repair place of p by Definition 2. Let $Q = \{q \in P_u | (p, q) \in (N[R_u], M[R_u]), p \in C[S_u] \backslash C[S]\}$. $\forall M \in R(N_u, M_{u0}), M(S) + M(C[S]) + M(Q) = M_{u0}(S)$. If S_u is not R-type SMS, $C[S] = C[S_u]$ and $Q = \emptyset$, we still have $M(S) + M(C[S]) + M(Q) = M_{u0}(S)$.

Remark: The equality $M(S) + M(C[S]) + M(Q) = M_{u0}(S)$ always holds no matter whether S is an R-type SMS. Without loss of generality, in the follow-up discussion we still denote each SMS as S no matter whether S contains unreliable resources in (N_u, M_{u0}). Let Π_u denote the set of all SMSs of N_u throughout the paper.

Lemma 2: Let (N_u, M_{u0}) be an S^3PR_u and $t \in T$ a dead transition at $M \in R(N_u, M_{u0})$. Then there exists an SMS S such that $M(S) = 0$.

Proof: If there is no unreliable resource failing at M, then there exists an SMS S such that $M(S) = 0$ [6].

On the other hand, ${}^\bullet t \cap P_R \neq \emptyset$ holds because t is dead at M. If $M({}^{(r)}t) \neq 0$, there exists $M' \in R(N_u, M)$ such that $M'[t >$, which contradicts that t is dead at M. Thus $M({}^{(r)}t) = 0$. Let ${}^{(r)}t = r_1$ and $\Im_1 = \{t \in T | {}^{(o)}t \in H(r_1), M({}^{(o)}t) \neq 0\}$. $\forall t_1 \in \Im_1$, ${}^{(r)}t_1 \neq \emptyset$ and $M({}^{(r)}t_1) = 0$. Otherwise, we have a reachable marking $M'' \in R(N_u, M)$ such that $M''(r_1) \neq 0$, which contradicts that t is dead at M. Let ${}^{(r)}t_1 = r_2$. It is natural that ${}^\bullet r_1 = r_2^\bullet$. Repeating the aforementioned procedure, we can get some resources $r_1, r_2, \ldots, r_k$ such that they develop a resource circular wait. Let $R_1 = \{r_1, r_2, \ldots r_k\}$. $\forall r \in R_1$, if $\forall p \in H(r)$, $M(p) \neq 0$, then $M({}^{(r)}(p^\bullet)) = 0$ because t is dead at M. Set $r' =^{(r)} (p^\bullet)$ and add r' into R_1 if $r' \notin R_1$. Let $\Im' = \{t \in T | {}^{(o)}t \in H(r'), M({}^{(o)}t) \neq 0\}$. Do the same procedure for $\Im'$, we obtain $R_2 = \{r_1, r_2, \ldots r_l\}$ such that for each $r \in R_2$, $\exists p \in H(r)$, $M(p) = 0$. The fact that$t$ is dead at M ensures $R_2 \neq \emptyset$. Let $S' = R_2 \cup \{p \in H(R_2), M(p) = 0\}$, we have $M(S') = 0$ because $M(R_2) = 0$ due to the chosen procedure of R_2.

We will prove S' is a siphon as follows. Thus, $\forall p \in S'$, $\exists r \in R_2, \ni p \in H(r)$ such that ${}^\bullet p \in r^\bullet$. On the other hand, $\forall r \in S', \forall t \in {}^\bullet r$, there exists $r' \in S'$ such that ${}^\bullet r \cap r'^\bullet \neq \emptyset$ due to the chosen procedure of R_2. As a result, S' is a siphon. According to the definition of SMS, we can derive an SMS S from S' such that $S \cap P_R = R_2$ and $M(S) = 0$. Furthermore, if there exists at least one unreliable resource in R_2, then S is an R-type SMS, denoted by S_u, and $M(S_u) = 0$. ■

C. STRONG CONTROLLABLE SIPHON BASIS

To avoid adding control place to each SMS in (N_u, M_{u0}) and reduce the structural size of controllers, we will introduce the following concept.

Definition 5: Let $\Xi_u \subseteq \prod_u$ be a subset of SMSs in (N_u, M_{u0}). For each $S \in \prod_u \backslash \Xi_u$, if there exists $\pi_S \subseteq \Xi_u$ such that $C[S] \subseteq C[\pi_S]$ and $M_{u0}(S) > |\pi_S|$, where $C[\pi_S] = \cup_{S' in \pi_S} C[S']$ and $|\pi_S|$ denote the number of SMSs in π_S, then

Ξ_u is called a ***strong controllable siphon basis*** of (N_u, M_{u0}), and π_S as the *relevant siphon set* of S.

It is easy to verify that a strong controllable siphon is similar to a controllable siphon basis, and the difference lies in the complementary sets between SMSs and R-type SMSs. For example, there are three R-type SMSs S_{u1}, S_{u2} and S_{u3}, in (N_u, M_{u0}) of Example 1. $\Xi_u = \{S_{u1}, S_{u2}\}$ is a strong controllable siphon basis because $C[S_{u3}] = C[S_{u1}] \cup C[S_{u2}] and M_0(S_{u3}) > |\Xi_u|$.

Let $\Xi \subseteq \Pi$ in (N, M_0). $\forall S \in \Xi$, if S is R-type SMS, denote as S_u. Set $\Xi_u := (\Xi \backslash \{S\}) \cup \{S_u\}$, and we can draw a conclusion as follows.

Lemma 3: $\forall S \in \Pi_u \backslash \Xi_u$, the relevant siphon set of S in Ξ_u is the same as that of S in Ξ.

Proof: Assume that $\pi_S \subseteq \Xi$ is the relevant siphon set of S in Ξ. If S is not an R-type SMS, then $C[S] = C[S_u]$. The following inequalities $C[S] \subseteq C[\pi_S]$ and $M_{u0}(S) > |\pi_S|$ hold even if there exists an R-type SMS in π_S. As a result, π_S is still the relevant siphon set of S in Ξ_u. On the other hand, if S is an R-type SMS (denoted by S_u), $C[S] \neq C[S_u]$, $\forall p \in C[S_u] \backslash C[S]$, $p \in S$ and further, $\exists r \in S \cap P_u$ such that $p \in H(r)$ because S is an SMS. Due to the strictness of S, $H(r) \cap C[S] \neq \emptyset$. $\forall p' \in H(r) \cap C[S]$, $\exists S' \in \pi_S$ such that $p' \in C[S']$ because π_S is the relevant siphon basis of S in Ξ. Then $r \in S'$ because (N, M_0) is an S^{3}PR that each operation only can be processed by one type resource. Since r is an unreliable resource, S' is an R-type SMS (denoted by S'_u) and $p \in C[S'_u]$. As a result, $C[S_u] \subseteq C[\pi_S]$. Since the initial markings of unreliable resources are constant, $M_{u0}(S_u) > |\pi_S|$ and the conclusion is proved. ■

IV. STRONG CONTROLLABLE SIPHON BASIS-BASED ROBUST CONTROL POLICY

A. A PETRI NET CONTROLLER BASED ON STRONG CONTROLLABLE SIPHON BASIS

Based on the concept of strong controllable siphon basis, we will present the design of Petri net controllers as follows.

Definition 6: Let $(N_u, M_{u0}) = (P \cup P_0 \cup P_R \cup P_u, T \cup T_u, F \cup F_u, M_{u0})$ be a marked S^{3}PR$_u$, and Ξ_u be a strong controllable siphon basis of (N_u, M_{u0}). *A Petri net controller based on* Ξ_u, $(C_{\Xi u}, M_{\Xi u})$, of (N_u, M_{u0}) is defined as follows.

1) $P_{\Xi u} = \{c_S, S \in \Xi_u | c_S$ is a control place corresponding to $S\}$ is a set of control places;
2) $F_{\Xi u} = \cup_{i=1}^{5} F_{\Xi ui}$ such that
   $F_{\Xi u1} = \cup_{S \in \Xi_u} \{(c_S, t) | t \in {}^{\bullet}\mathrm{X} \wedge t < C[S]\}$, where $\mathrm{X} = \cup_{S \in \Xi_u} C[S]$;
   $F_{\Xi u2} = \cup_{S \in \Xi_u} \{(c_S, \beta) | \beta^{\bullet} \cap C[S] = \emptyset \wedge \beta^{\bullet} < C[S] \wedge \beta^{\bullet} \not< \mathrm{X}\}$, where $\beta \in T_u$;
   $F_{\Xi u3} = \cup_{S \in \Xi_u} \{(t, c_S) | t \in C[S]^{\bullet}, t \not< C[S]\}$;
   $F_{\Xi u4} = \cup_{S \in \Xi_u} \{(\alpha, c_S) | {}^{\bullet}\alpha \cap C[S] = \emptyset \wedge {}^{\bullet}\alpha < C[S] \wedge {}^{\bullet}\alpha \not< \mathrm{X}\}$, where $\alpha \in T_u$;
   $F_{\Xi u5} = \cup_{S \in \Xi_u} \{(t, c_S) | t \not< C[S], \exists t_1 \in ({}^{(o)}t)^{\bullet}, \ni t_1 < C[S]\}$;
3) $T_{\Xi u} = F_{\Xi u}^{\bullet} \cup^{\bullet} F_{\Xi u}$;
4) $M_{\Xi u}(c_S) = M_{u0}(S) - \xi_S$, where $\xi_S \in [1, M_{u0}(S) - 1]$ is an integer called as a *control depth variable*.

Denote the controlled Petri net $(N_{Cu}, M_{Cu0}) = (N_u, M_{u0}) \otimes (C_{\Xi u}, M_{\Xi u}) = (P \cup P_0 \cup P_R \cup P_u \cup P_{\Xi u}, T \cup T_u, F \cup F_u \cup F_{\Xi u}, M_{Cu0})$, where $M_{Cu0}(p) = M_{u0}(p)$, $\forall p \in P \cup P_0 \cup P_R \cup P_u$; $M_{Cu0}(p) = M_{\Xi u}(p)$, $\forall p \in P_{\Xi u}$.

Remark: Of course, if $S \in \Xi_u$ is not an R-type SMS, the adding method of control place to S is similar to that in Definition 1. If S is an R-type SMS, denoted by S_u, the aim of controlling S_u is to guarantee that $C[S] \cup Q$ cannot occupy all resources in S_u. Since ${}^{\bullet}Q$ is a set of uncontrollable transitions, we only can pull the control arc to the controllable transitions in ${}^{\bullet}(S_u \cap H(R_u))$. That is, we replace $C[S] \cup Q$ by $C[S_u] = C[S] \cup (S_u \cap H(R_u))$.

Lemma 4: Let (N_u, M_{u0}) be an S^{3}PR$_u$, Ξ_u be its strong controllable siphon basis and (N_{Cu}, M_{Cu0}) the controlled Petri net based on Ξ_u. Then $\forall S \in \Xi_u$, S is marked at any reachable marking of (N_{Cu}, M_{Cu0}).

Proof: By Definition 4, $\forall M \in R(N_{Cu}, M_{Cu0})$, $M(S) + M(C[S]) + M(Q) = M_{u0}(S)$. However, $M(C[S]) + M(Q) \leq M_{u0}(S) - \xi_{Su}$ due to Definition 6. Thus, $M(S) \geq \xi_{Su} \geq 1$ and S is marked at any reachable marking of (N_{Cu}, M_{Cu0}). ■

Lemma 5: Let $S \in \Pi_u \backslash \Xi_u$ be an SMS of (N_u, M_{u0}) and $\pi_S = \{S_1, S_2, \ldots, S_k\}$ be the relevant siphon set of S. A control place is added to S_i by Definition 6 and $\xi_i (i \in \mathbb{N}_k)$ is the control depth variable. Then S is marked at any reachable marking of (N_{Cu}, M_{Cu0}) if the following inequality holds.

$$\Sigma_{i=1}^{k} \xi_i > \Sigma_{i=1}^{k} M_{u0}(S_i) - M_{u0}(S)$$

Proof: $\forall M \in R(N_{Cu}, M_{Cu0})$, $M(C[S_i]) + M(Q_i) \leq M_{u0}(S_i) - \xi_i$ due to Definition 6. $C[S] \subseteq \cup_{i=1}^{k} C[S_i]$ because π_S is the relevant siphon set of S.

Then $M(C[S]) + M(Q) \leq \sum_{i=1}^{k} (M(C[S_i]) + M(Q_i))$. Thus, $M(C[S]) + M(Q) \leq \Sigma_{i=1}^{k} (M_{u0}(S_i) - \xi_i)$ and $M(C[S]) + M(Q) \leq M_{u0}(S) - 1$ because $\Sigma_{i=1}^{k} \xi_i > \Sigma_{i=1}^{k} M_{u0}(S_i) - M_{u0}(S)$. Therefore, $M(S) \geq 1$ and S is always marked at any reachable marking of (N_{Cu}, M_{Cu0}). ■

It is natural that the smaller ξ_S, the more reachable markings of (N_{Cu}, M_{Cu0}) can reach and the better its performance. Combined with the above constraints, the solution of ξ_S is determined by the following integer linear programming (ILP):

$$\text{Min } \Sigma_{S \in \Xi_u} \xi_S$$
$$\text{s.t. } 1 \leq \xi_S \leq M_{u0}(S) - 1 \quad (1)$$
$$\Sigma_{S \in \pi_{S'}} \xi_S \geq \Sigma_{S \in \pi_{S'}} M_{u0}(S) - M_{u0}(S') + 1 \quad (2)$$

$\forall S' \in \Pi_u \backslash \Xi_u$ and $\pi_{S'}$ is the relevant siphon set of S'.

Lemma 6: ILP can be solvable.

Proof: Let $\xi_S = M_{u0}(S) - 1$. It is natural that (1) holds. For $S' \in \Pi_u \backslash \Xi_u$, we can choose $\pi_{S'} \subseteq \Xi_u$ to be the relevant siphon set for S' and $M_{u0}(S') > |\pi_{S'}|$ because Ξ_u is a strong controllable siphon basis of N_u. Thus, we have the following equality:

$$\Sigma_{S \in \pi_{S'}} \xi_S = \Sigma_{S \in \pi_{S'}} M_{u0}(S) - |\pi_{S'}|.$$

Thus (2) holds because of $M_{u0}(S') > |\pi_{S'}|$. As a result, $\{M_{u0}(S) - 1\}$ is a solution of the ILP, and the ILP can be solvable. ■

Theorem 2: Let Ξ_u be a strong controllable siphon basis of (N_u, M_{u0}). $(C_{\Xi u}, M_{\Xi u})$ is the Petri net controller based on Ξ_u, and its control depth variables are obtained by solving the ILP. Then $(C_{\Xi u}, M_{\Xi u})$ is a robust controller to (N_u, M_{u0}).

Proof: Let $(N_{Cu}, M_{Cu0}) = (N_u, M_{u0}) \otimes (C_{\Xi u}, M_{\Xi u})$. There are three cases to be discussed as follows.

(1) There is no unreliable resource failed in the process operation of (N_u, M_{u0}).

$\forall S \in \Pi_u$ and $M \in R(N_{Cu}, M_{Cu0})$, $M(S) \neq 0$ according to Lemma 4 and Lemma 5. Since (N_u, M_{u0}) is still an RCN-net, it proves that the liveness of (N_{Cu}, M_{Cu0}) holds by repeating the proof procedure of Theorem 3 in [12].

(2) Some unreliable resources failure happens, and in the worst case, the maximum failure number of each unreliable resource r_u is $M_{u0}(r_u) - 1$.

Assume, on the contrary, that (N_{Cu}, M_{Cu0}) is not live. Then there exists at least one transition t and $M \in R(N_{Cu}, M_{Cu0})$ such that t is dead at M. Due to Definiton 2 and Defintion 6, $t \notin T_u$. The reason is as follows. $\forall t \in T_u$, ${}^\bullet t \cap P_R = \emptyset$. If ${}^\bullet t \cap P_{\Xi u} = \emptyset$, and t is not dead at any reachable marking of (N_{Cu}, M_{Cu0}). If ${}^\bullet t \cap P_{\Xi u} \neq \emptyset$, there is no circuilar wait because $t^\bullet \cap P_{\Xi u} = \emptyset$. So we only consider $t \in T$.

If ${}^\bullet t \cap P_{\Xi u} = \emptyset$ for $t \in T$, then ${}^{(r)}t \cap P_R \neq \emptyset$. Due to Lemma 2, there exists an SMS S such that $M(S) = 0$ because t is dead at M. If $S \in \Xi_u$, $M(S) \neq 0$ by Lemma 4; if $S \notin \Xi_u$, $M(S) \neq 0$ by Lemma 5, a contradiction.

If ${}^\bullet t \cap P_{\Xi u} \neq \emptyset$ for $t \in T$, there exists $c_u \in P_{\Xi u}$ such that ${}^{(c)}t = c_u$ and $M(c_u) = 0$. According to Definition 6, there exists an SMS S such that $t < C[S]$. If $S \in \Xi_u$, $M(S) \neq 0$ by Lemma 4; if $S \notin \Xi_u$, $M(S) \neq 0$ by Lemma 5. Thus there exists a sequence of transitions $t_1, t_2, \ldots, t_k$ such that $M[t_1 t_2 \ldots t_k > M'$ and $M'(c_u) \neq 0$. Thus, t is not dead at M, a contradiction.

From the above, (N_{Cu}, M_{Cu0}) is live.

(3) The failed resources are repaired and return to the system. Now this condition is similar to (1) and (N_{Cu}, M_{Cu0}) is live.

As a result, $(C_{\Xi u}, M_{\Xi u})$ is a robust controller to (N_u, M_{u0}) by Definition 3. ■

B. ROBUST DEADLOCK CONTROL POLICY FOR AN AMS WITH UNRELIABLE RESOURCES

In the above subsection, we know how to design a robust deadlock controller by Definition 6. A new robust deadlock control policy is synthesized based on strong controllable siphon basis for AMSs with unreliable resources as follows.

Procedure RDCP (Designing a robust deadlock control policy)

Given a marked S^3PR (N, M_0) with a set of unreliable resources $R_u = \{r_1, r_2, \ldots, r_l\}$ and the set of SMSs Π of N.

Step 1: Using **Algorithm 1** and **Algorithm 2** in [12], compute a controllable siphon basis Ξ of N.

Step 2: $\forall S \in \Pi$, if $S \cap R_u \neq \emptyset$, denote S as S_u. Set $\Pi_u := (\Pi \backslash \{S\}) \cup \{S_u\}$ and $\Xi_u := (\Xi \backslash \{S\}) \cup \{S_u\}$ if $S \in \Xi$.

Step 3: Add a control place c_s and related arcs to each $S \in \Xi_u$ and design a Petri net controller $(C_{\Xi u}, M_{\Xi u})$ by Definition 6. Its control depth variable ξ_S is determined by solving the ILP in Step 5.

Step 4: $\forall S' \in \Pi_u \backslash \Xi_u$, let $\pi_{S'}$ be its relevant siphon set output by **Algorithm 2** in [12], the following inequality is

$$\Sigma_{S \in \pi_{S'}} \xi_S \geq \Sigma_{S \in \pi_{S'}} M_{u0}(S) - M_{u0}(S') + 1$$

Step 5: Construct the ILP of Lemma 6 and solve a set of values of ξ_S.

Step 6: Output $(C_{\Xi u}, M_{\Xi u})$.

Theorem 3: $(C_{\Xi u}, M_{\Xi u})$ is a robust Petri net controller by **Procedure RDCP**.

Proof: First, Ξ_u output by Step 2 is a strong controllable siphon basis of Π_u and π_S output by Step 4 is the relevant siphon set of S in Ξ_u due to Lemma 3. According to Theorem 2, $(C_{\Xi u}, M_{\Xi u})$ output by Step 6 is robust. ■

C. AN ILLUSTRATION

Example 2: For (N_u, M_{u0}) shown in Fig. 2, $\{S_{u1}, S_{u2}\}$ is a strong controllable siphon basis, and we add two control places c_1 and c_2 to (N_u, M_{u0}) by Definition 6. Let $X = C[S_{u1}] \cup C[S_{u2}] = \{p_2, p_3, p_4, p_6, p_7, p_8\}$. We draw a directed arc from c_1 to t_1 and t_6 because each of them belongs to ${}^\bullet X$ and is previous to $C[S_{u1}] = \{p_2, p_6, p_7, p_8\}$, and a directed arc from t_2 and t_9 to c_1 because they belong to $C[S_{u1}]^\bullet$ and are not previous to $C[S_{u1}]$. For S_{u2}, we have $C[S_{u2}] = \{p_3, p_4\}$ and ${}^\bullet\alpha_1(\beta_1{}^\bullet) \notin C[S_{u2}] \wedge {}^\bullet\alpha_1(\beta_1{}^\bullet) < C[S_{u2}] \wedge {}^\bullet\alpha_1(\beta_1{}^\bullet) \nless X$. Therefore, a directed arc from c_2 to t_1 and β_1 and another arc from α_1 and t_4 to c_2 are drawn. Furthermore, $\xi_{Su1} + \xi_{Su2} \geq 2$ due to the ILP, and we have $\xi_{Su1} = \xi_{Su2} = 1$. Thus, the controlled Petri net (N_{Cu}, M_{Cu0}) is illustrated in Fig. 5 by Definition 6. It is easy to compute that there are 670 reachable markings in (N_{Cu}, M_{Cu0}).

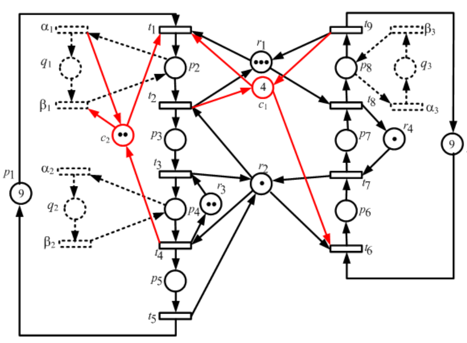

FIGURE 5. A robust controlled net (N_{Cu}, M_{Cu0}) for S^3PR_u in Fig. 2.

V. CONCLUSION

This work deals with the deadlock control problem in AMSs with multi-unit and multi-type unreliable resources and synthesizes their deadlock prevention controllers with small structure. Our control objective is to guarantee that all types of parts can process smoothly their tasks through any of their routes even if some multiple unreliable resources fail simultaneously.

We use a class of Petri nets called S^3PR_u to model AMSs with multiple unreliable resources. To assess the whole performance of unreliable resources, we develop the concept of R-type SMS. An SMS is an R-type SMS if its resource set contains unreliable resources. Then a strong controllable siphon basis is defined. A controllable siphon basis is strong if it includes at least one R-type SMS. By adding a control place with a proper depth control variable and suitable related arcs to each (R-type) SMS in the strong controllable siphon basis, we develop a robust controller with small size for AMSs under consideration. The size of the established controller grows polynomially with Petri net models. Compared with [36]–[41], the studied AMSs contain multiple types of unreliable resources while others contain only one unreliable resource type.

The future research includes improving permissiveness of the proposed method and extending the method to more general models. These are parts of our research agenda.

APPENDIX

The notations that are frequently used in our work are listed as follows.

NOMENCLATURE

Symbols	*Meanings*
P	a set of operation places
P_R	a set of resource places
T	a set of transitions
(N, M_0)	an S^3PR with the initial marking
Π	the set of SMSs in (N, M_0)
Ξ	a controllable siphon basis
(N_C, M_{C0})	a controlled Petri net
(N_u, M_{u0})	an S^3PR_u, a marked S^3PR with unreliable resources
R_u	a set of unreliable resources
Π_u	the set of SMSs in (N_u, M_{u0})
Ξ_u	a strong controllable siphon basis
$(C_{\Xi u}, M_{\Xi u})$	a robust Petri net controller

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