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Nonlinear Merging Consensus for Multi-Agent Systems on Directed and Weighted Signed Graph

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ABSTRACT This paper settles the nonlinear merging consensus for multi-agent systems on a directed and weighted signed network. A novel nonlinear merging control protocol is proposed to drive the states of all agents to arrive at the same state. To be consistent with the reality, the interactions among agents can be either cooperative or competitive and the information flow is considered to be directed. Particularly, the protocol can still ensure the states of all agents to reach an agreement, even though the corresponding directed network is not strongly connected. Besides, to guarantee the efficiency of multi-agent systems, the convergence rate of agents can be sped up by adjusting the parameter of the protocol. Finally, the theoretical analysis and simulation results are provided to verify the effectiveness of this protocol. The current results are instructive for us to comprehend and design an efficient consensus protocol within multi-agent systems.

INDEX TERMS Nonlinear merging consensus, multi-agent systems, signed graph, weighted and directed network.

I. INTRODUCTION

Recently, more and more attention has been drawn to complex networks [1], [2]. It has attracted a great deal of concern for multi-agent systems as a kind of typical complex systems. All kinds of animal communities such as ant swarming [3]–[10], fish school [11]–[17], *etc*, which exist commonly in nature, can reach agreement on velocity and distance without any external force. Inspired by phenomena mentioned above, cooperative control for multi-agent systems, which manipulates multiple dynamic entities to share tasks to arrive at a common state, is discussed by experts from various fields such as sensor networks [18]–[24], vehicle formation [25]–[27] and unmanned aerial vehicles [28]–[32].

As a hot topic of cooperative control, the consensus issue which aims to make the states of all agents reach agreement has received growing attention certainly [33]–[36]. In 1987, Reynolds presented Boid model that met the requirements of three laws: separation, aggregation and velocity match [37]. Then, Vicsek *et al.* researched the Vicsek particle swarm model which updated the direction of motion of each particle based on the nearest neighbor in [38]. In 2004, Olfati-Saber and Murray investigated the continuous-time consensus problems of first-order multi-agent systems, the undirected graph was used to symbolize the communication among agents, and the second smallest eigenvalue of Laplace matrix of graph was considered as an important performance index of the convergence rate of a system [39]. After that, Ren *et al.* further extended the continuous-time consensus on the basis of algebraic graph theory, and proved the necessary and sufficient conditions, in which the system can reach an agreement in [40].

Over the past few years, consensus problems were explored in a growing amount of complex dynamic systems. The distributed consensus of second-order multi-agent systems was also discussed in [41], [42], and higher order consensus protocol for multi-agent systems was further considered in [43]. In addition, the switching communication topologies were taken into account to discuss the consensus problems of multi-agent systems [44], [45].

However, most of existing works of multi-agent systems are based on a common sense that interactions among agents

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are cooperative, which result in the nonnegative weight of the communication topology. In fact, competitive or opposite relationship are widely spread among complex network in reality. For example, there are trust or distrust relationship among users in social networks, activation or inhibition relationship between neurons in neural networks, cooperative or competitive relationship among agents in multi-agent systems. Inspired by this nature, it is essential for multi-agent systems to consider the weighted and signed network, in which a positive weight indicates the cooperative relationship between agents while a negative weight represents the competitive relationship between agents. Although Feng *et al.* considered communication topology whose weights were either positive or negative, there was still a weak point that the direction of weighted links was not discussed in [46]. To this end, the directed and weighted signed network is considered in this paper.

Recently, many researches on consensus problems were based on the assumption that the communication topology was connected or strongly connected. As an example, the distributed quantized consensus problem on connected networks was studied in [47], a consensus protocol was proposed to coordinate the states of all agents to arrive at the same state under a strongly connected network in [48]. While under some realistic scenarios, it is less likely to meet such strong requirements. Accordingly, it is challenging and innovative to present the merging consensus protocol to guarantee all the agents to reach agreement even though the directed communication topology is not strongly connected and there exists the competition between interactions among agents.

The contributions of this paper are summarized in the following three aspects,

1) The interactions among agents exhibit the cooperation or competition relationship; Besides, the information flow of communication topology is directed, which are in line with the reality.
2) Convergence rate which is considered to be a significant index can be greatly sped up as long as one adjusts the parameter of the protocol.
3) Compared with other protocols mentioned in Section IV, the key features of our protocol can be summarized as follows: a) if the directed communication topology whose weights are all positive is strongly connected, then this protocol is capable of making the states of all agents reach agreement more quickly than other protocols; b) if the directed communication topology whose weights are either positive or negative is not strongly connected, then our protocol can still guarantee the states of all agents to converge to the same state, while most of the existing protocols can not complete this goal.

The rest of the paper is structured as follows. The problem description including some necessary notations and lemmas is presented in Section II. A nonlinear merging control protocol and its corresponding proof are discussed in Section III. In Section IV, we carry out the numerical simulations to verify the validity of the theoretical analysis and the effectiveness of protocol. Finally, the concluding remarks are summed up in Section V.

II. PROBLEM DESCRIPTIONS

A. NOTATIONS

Firstly, there are four notations should be introduced. $I_n = \{1, 2, \ldots, n\}$ is on behalf of the set of n agents from 1 to n. x is a vector which equals to $(x_1, x_2, \ldots, x_n)^T$. λ denotes the real part of the eigenvalue. $sign(x)$ is indicated by the following piecewise function of any $x \in R$

$$sign(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases} \quad (1)$$

B. THE DIRECTED SIGNED GRAPH

A directed signed graph $G = (V, \varepsilon, A)$ includes a set of vertex of $V = \{v_1, v_2, \ldots, v_n\}$, a set of edge $\varepsilon \supseteq \{(v_j, v_i) : v_i, v_j \in V\}$, and a weighted adjacency matrix $A = [w_{ij}] \in R^{n\times n}$. Here, w_{ij} can be either positive or negative, and $w_{ij} \neq 0 \Leftrightarrow (v_j, v_i) \in \varepsilon \nLeftrightarrow (v_i, v_j) \in \varepsilon$. There is no self-loop allowed, *i.e.*, $w_{ii} = 0, i = 1, 2, \ldots, n$. $N_i = \{v_j : (v_j, v_i) \in \varepsilon\}$ represents the neighbors set of vertex v_i. Let $A_1 = [w_{ij_1}] \in R^{n\times n}$, $A_2 = [w_{ij_2}] \in R^{n\times n}$, where

$$w_{ij_1} = \begin{cases} w_{ij}, & w_{ij} > 0 \\ 0, & w_{ij} \leq 0 \end{cases}$$

and

$$w_{ij_2} = \begin{cases} 0, & w_{ij} \geq 0 \\ w_{ij}, & w_{ij} < 0 \end{cases}$$

$N_{i_1} = \{v_j : (v_j, v_i) \in \varepsilon, w_{ij} > 0\}$, symbols the cooperative neighbors set of vertex v_i. And the competitive neighbors set of v_i is denoted by $N_{i_2} = \{v_j : (v_j, v_i) \in \varepsilon, w_{ij} < 0\}$. And then $N_i = N_{i_1} \cup N_{i_2}$.

Definition 1: If there exists a directed path between any pair of vertices v_i and v_j in G, then the network G is strong connected.

Definition 2: The Laplacian matrix $L = (l_{ij})_{n\times n}$ of a signed graph G is defined as follows:

$$l_{ij} = \begin{cases} \sum_{k\in N_i} |w_{ik}|, & j = i \\ -w_{ij}, & j \neq i \end{cases} \quad (2)$$

C. LEMMAS

The lemmas introduced in this section will be used in the proof of the protocol.

Lemma 1 [49]: Square matrix $A = (w_{ij}) \in C^{n\times n}$, then its eigenvalues labeled $\lambda_1, \lambda_2, \ldots, \lambda_n$, drop into a complex plane $\cup G_i(A) = \cup\{z\,|z - w_{ii}| \leq P_i\}$, $i = 1, 2, \ldots, n$, $P_i = \sum_{j=1, j\neq i}^{n} |w_{ij}|$.

Lemma 2 [50] : If there exists a function $f(x)$ meets $F(x) \leq f(x) \leq G(x)$ under the assumption that $F(x)$ and $G(x)$ are continuous at x_0, and $\lim_{x\to x_0} F(x) = \lim_{x\to x_0} G(x) = a$, then $\lim_{x\to x_0} f(x) = a$.

III. NONLINEAR MERGING CONSENSUS

In this paper, nonlinear merging consensus problem on directed and weighted signed networks is presented. The dynamics of agent i are described as:

$$\dot{x}_i = u_i \qquad i \in I_n \tag{3}$$

The distributed protocol is investigated:

$$u_i = \sum_{j\in N_i} w_{ij} sign(x_j - rsign(w_{ij})x_i)|x_j - rsign(w_{ij})x_i|^{\frac{m}{n}} \tag{4}$$

where $0 < m < n < 1$. $m, n \in Z$ which are relatively prime. $r > 1$.

The protocol (4) can guarantee the states of all agents to reach a common value zero, although the interactions among agents exist competition and the communication topology is not strong connected. *i.e.*, $x_i(t)$ satisfies:

$$\lim_{t\to\infty} x_i(t) = 0 \qquad i \in I_n \tag{5}$$

on the basis of the following **Assumption 1**.

Assumption 1: A directed graph used to symbol the communication topology, which is not strong connected but can be divided into several strong connected subgraphs with more than one vertex.

Theorem 1: Based on **Assumption 1**, the states of agents of system (3) can converge to the same state zero, when m and n are odd numbers.

Proof: Depending on the sign of w_{ij}, system (3) can be converted into:

$$\begin{aligned}\dot{x}_i &= \sum_{j\in N_{i_1}} w_{ij_1} sign(x_j - rx_i)|x_j - rx_i|^{\frac{m}{n}} \\ &\quad + \sum_{j\in N_{i_2}} w_{ij_2} sign(x_j + rx_i)|x_j + rx_i|^{\frac{m}{n}} \\ &= \dot{x}_{i_1} + \dot{x}_{i_2}\end{aligned} \tag{6}$$

where

$$\begin{aligned}\dot{x}_{i_1} &= \sum_{j\in N_{i_1}} w_{ij_1} sign(x_j - rx_i)|x_j - rx_i|^{\frac{m}{n}} \\ \dot{x}_{i_2} &= \sum_{j\in N_{i_2}} w_{ij_2} sign(x_j + rx_i)|x_j + rx_i|^{\frac{m}{n}} \\ \dot{x}_{i_1} &= \sum_{\substack{j\in N_{i_1} \\ x_{j_{11}} - rx_{i_{11}} \leq -1}} w_{ij_1}(x_{j_{11}} - rx_{i_{11}})^{\frac{m}{n}} \\ &\quad + \sum_{\substack{j\in N_{i_1} \\ -1 < x_{j_{12}} - rx_{i_{12}} \leq 0}} w_{ij_1}(x_{j_{12}} - rx_{i_{12}})^{\frac{m}{n}} \\ &\quad + \sum_{\substack{j\in N_{i_1} \\ 0 < x_{j_{13}} - rx_{i_{13}} \leq 1}} w_{ij_1}(x_{j_{13}} - rx_{i_{13}})^{\frac{m}{n}} \\ &\quad + \sum_{\substack{j\in N_{i_1} \\ x_{j_{14}} - rx_{i_{14}} > 1}} w_{ij_1}(x_{j_{14}} - rx_{i_{14}})^{\frac{m}{n}} \\ &= \dot{x}_{i_{11}} + \dot{x}_{i_{12}} + \dot{x}_{i_{13}} + \dot{x}_{i_{14}}\end{aligned} \tag{7}$$

In order to prove $\lim_{t\to\infty} x_{i_1}(t) = 0$, $\lim_{\substack{t\to\infty \\ j=1,2,3,4}} x_{i_{1j}}(t) = 0$ should be demonstrated. Firstly, $\lim_{t\to\infty} x_{i_{11}}(t) = 0$ will be deduced in the following content.

$$\begin{pmatrix} \sum_{j\in N_{i_1}} w_{1j_1}(x_{j_{11}} - rx_{1_{11}}) \\ \sum_{j\in N_{i_1}} w_{2j_1}(x_{j_{11}} - rx_{2_{11}}) \\ \vdots \\ \sum_{j\in N_{i_1}} w_{nj_1}(x_{j_{11}} - rx_{n_{11}}) \end{pmatrix} = [-L_1 - (r-1)D_1] \begin{pmatrix} x_{1_{11}} \\ x_{2_{11}} \\ \vdots \\ x_{n_{11}} \end{pmatrix} \tag{8}$$

where L_1, D_1 are the Laplacian matrix and degree matrix, respectively.

According to Eq. (8) and **Lemma 1**, Ineq. (9) and Ineq. (10) can be obtained:

$$\lambda_{1i} < 0 \tag{9}$$

where λ_{1i} denotes the real part of the ith eigenvalue of matrix $[-L_1 - (r-1)D_1]$.

$$x_{i_{11}}(t) > 0 \tag{10}$$

The sign-preserving theorem is utilized to demonstrate:

$$\lim_{t\to\infty} x_{i_{11}}(t) > 0 \tag{11}$$

$$\begin{aligned}\dot{x}_{i_{11}} &= \sum_{\substack{j\in N_{i_1} \\ x_{j_{11}} - rx_{i_{11}} \leq -1}} w_{ij_1}(x_{j_{11}} - rx_{i_{11}})^{\frac{m}{n}} \\ &\leq - \sum_{\substack{j\in N_{i_1} \\ x_{j_{11}} - rx_{i_{11}} \leq -1}} w_{ij_1}\end{aligned} \tag{12}$$

Taking the integral on both sides of Ineq. (12), Ineq. (13) can be obtained.

$$x_{i_{11}}(t) \leq x_{i_{11}}(0) - \sum_{\substack{j\in N_{i_1} \\ x_{j_{11}} - rx_{i_{11}} \leq -1}} w_{ij_1} \times t \tag{13}$$

By the theorem of limitation, we can get:

$$\lim_{t\to\infty} x_{i_{11}}(t) \leq 0 \tag{14}$$

By Ineq. (11), Ineq. (14) and **Lemma 2**, Eq. (15) can be acquired.

$$\lim_{t\to\infty} x_{i_{11}}(t) = 0 \tag{15}$$

The proof of $\lim_{t\to\infty} x_{i_{12}}(t) = 0$ which is similar to $\lim_{t\to\infty} x_{i_{11}}(t) = 0$ is omitted here. And then, the following content is the deduced process of $\lim_{t\to\infty} x_{i_{13}}(t) = 0$.

Based on Eq. (8) and Ineq. (9), we have:

$$x_{i_{13}}(t) < 0 \tag{16}$$

The sign-preserving theorem of limitation is used to prove:

$$\lim_{t\to\infty} x_{i_{13}}(t) < 0 \tag{17}$$

$$\begin{aligned}
\dot{x}_{i_{13}} &= \sum_{\substack{j\in N_{i_1}\\ 0<x_{j_{13}}-rx_{i_{13}}\le 1}} w_{ij_1}(x_{j_{13}} - rx_{i_{13}})^{\frac{m}{n}}\\
&\ge \sum_{\substack{j\in N_{i_1}\\ 0<x_{j_{13}}-rx_{i_{13}}\le 1}} w_{ij_1}(x_{j_{13}} - rx_{i_{13}})\\
\dot{x}_{13} &\ge [-L_1 - (r-1)D_1]x_{13}\\
\dot{x}_{i_{13}} &\ge \lambda_{1i} \times x_{i_{13}}
\end{aligned} \tag{18}$$

Taking the integral on both sides of Ineq. (18), Ineq. (19) can be got:

$$x_{i_{13}}(t) \ge e^{\lambda_{1i}t} \times x_{i_{13}}(0) \tag{19}$$

By the sign-preserving of the limitation, we have:

$$\lim_{t\to\infty} x_{i_{13}}(t) \ge 0 \tag{20}$$

By Ineq. (17), Ineq. (20) and **Lemma 2**, Eq. (21) can be obtained:

$$\lim_{t\to\infty} x_{i_{13}}(t) = 0 \tag{21}$$

The deduced process of $\lim_{t\to\infty} x_{i_{14}}(t) = 0$ which is similar to $\lim_{t\to\infty} x_{i_{11}}(t) = 0$ is omitted here.

In summary, we have:

$$\lim_{t\to\infty} x_{i_1}(t) = 0 \tag{22}$$

In the same way, $\lim_{t\to\infty} x_{i_2}(t) = 0$ can be deduced.

Accordingly, the following equality can be got:

$$\lim_{t\to\infty} x_i(t) = \lim_{t\to\infty} x_{i_1}(t) + \lim_{t\to\infty} x_{i_2}(t) = 0 \tag{23}$$

And then, **Theorem 1** can be proved.

Theorem 2: On the basis of **Assumption 1**, the states of agents of system (3) can reach a common value zero, when m is an odd number and n is an even number.

Proof: According to the sign of w_{ij}, system (3) can be transformed into:

$$\begin{aligned}
\dot{x}_i &= \sum_{j\in N_{i_1}} w_{ij_1} sign(x_j - rx_i)|x_j - rx_i|^{\frac{m}{n}}\\
&\quad + \sum_{j\in N_{i_2}} w_{ij_2} sign(x_j + rx_i)|x_j + rx_i|^{\frac{m}{n}}\\
&= \dot{x}_{i_1} + \dot{x}_{i_2}
\end{aligned} \tag{24}$$

where

$$\dot{x}_{i_1} = \sum_{j\in N_{i_1}} w_{ij_1} sign(x_j - rx_i)|x_j - rx_i|^{\frac{m}{n}}$$

$$\dot{x}_{i_2} = \sum_{j\in N_{i_2}} w_{ij_2} sign(x_j + rx_i)|x_j + rx_i|^{\frac{m}{n}}$$

$$\begin{aligned}
\dot{x}_{i_1} &= -\sum_{\substack{j\in N_{i_1}\\ x_{j_{11}}-rx_{i_{11}}\le -1}} w_{ij_1}(x_{j_{11}} - rx_{i_{11}})^{\frac{m}{n}}\\
&\quad - \sum_{\substack{j\in N_{i_1}\\ -1<x_{j_{12}}-rx_{i_{12}}\le 0}} w_{ij_1}(x_{j_{12}} - rx_{i_{12}})^{\frac{m}{n}}\\
&\quad + \sum_{\substack{j\in N_{i_1}\\ 0<x_{j_{13}}-rx_{i_{13}}\le 1}} w_{ij_1}(x_{j_{13}} - rx_{i_{13}})^{\frac{m}{n}}\\
&\quad + \sum_{\substack{j\in N_{i_1}\\ x_{j_{14}}-rx_{i_{14}}>1}} w_{ij_1}(x_{j_{14}} - rx_{i_{14}})^{\frac{m}{n}}\\
&= \dot{x}_{i_{11}} + \dot{x}_{i_{12}} + \dot{x}_{i_{13}} + \dot{x}_{i_{14}}
\end{aligned} \tag{25}$$

$\lim_{\substack{t\to\infty\\ j=1,2,3,4}} x_{i_{1j}}(t) = 0$ should be proved so that we can deduce $\lim_{t\to\infty} x_{i_1}(t) = 0$. At first, the following content is to demonstrate $\lim_{t\to\infty} x_{i_{11}}(t) = 0$.

$$\begin{aligned}
\dot{x}_{i_{11}} &= -\sum_{\substack{j\in N_{i_1}\\ x_{j_{11}}-rx_{i_{11}}\le -1}} w_{ij_1}(x_{j_{11}} - rx_{i_{11}})^{\frac{m}{n}}\\
&\le -\sum_{\substack{j\in N_{i_1}\\ x_{j_{11}}-rx_{i_{11}}}} w_{ij_1}
\end{aligned} \tag{26}$$

Taking the integral on both sides of Ineq. (26), Ineq. (27) can be obtained:

$$x_{i_{11}}(t) \le x_{i_{11}}(0) - \sum_{\substack{j\in N_{i_1}\\ x_{j_{11}}-rx_{i_{11}}\le -1}} w_{ij_1} \times t \tag{27}$$

According to the sign-preserving theorem of limitation, we can get:

$$\lim_{t\to\infty} x_{i_{11}}(t) \le 0 \tag{28}$$

In virtue of Eq. (8) and **Lemma 1**, Ineq. (29) can be acquired.

$$x_{i_{11}}(t) > 0 \tag{29}$$

By the sign-preserving theorem of limitation, we have:

$$\lim_{t\to\infty} x_{i_{11}}(t) > 0 \tag{30}$$

Because of Ineq. (28), Ineq. (30) and **Lemma 2**, Eq. (31) can be got:

$$\lim_{t\to\infty} x_{i_{11}}(t) = 0 \tag{31}$$

And then, $\lim_{t\to\infty} x_{i_{12}}(t) = 0$ will be deduced in the following steps.

On account of Eq. (8) and Ineq. (9), we obtain:

$$x_{i_{12}}(t) > 0 \tag{32}$$

By the sign-preserving theorem of limitation, InEq. (33) can be acquired:

$$\lim_{t\to\infty} x_{i_{12}}(t) > 0 \tag{33}$$

$$
\begin{aligned}
\dot{x}_{i_{12}} &= - \sum_{\substack{j \in N_{i_1} \\ -1 < x_{j_{12}} - r x_{i_{12}} \leq 0}} w_{ij_1} (x_{j_{12}} - r x_{i_{12}})^{\frac{m}{n}} \\
&\leq \sum_{\substack{j \in N_{i_1} \\ -1 < x_{j_{12}} - r x_{i_{12}} \leq 0}} w_{ij_1} (x_{j_{12}} - r x_{i_{12}}) \\
\dot{x}_{12} &\leq [-L_1 - (r-1) D_1] x_{12} \\
\dot{x}_{12} &\leq \lambda_{1i} \times x_{i_{12}}
\end{aligned} \tag{34}
$$

Taking the integral on both sides of Ineq. (34), we have:

$$x_{i_{12}}(t) \leq e^{\lambda_{1i} t} \times x_{i_{12}}(0) \tag{35}$$

Based on the sign-preserving theorem of the limitation, Ineq. (36) can be obtained:

$$\lim_{t \to \infty} x_{i_{12}}(t) \leq 0 \tag{36}$$

On the basis of Ineq. (33) and Ineq. (36), Eq. (37) can be got:

$$\lim_{t \to \infty} x_{i_{12}}(t) = 0 \tag{37}$$

The deduced processes of $\lim_{t \to \infty} x_{i_{13}}(t) = 0$ and $\lim_{t \to \infty} x_{i_{14}}(t) = 0$ are similar to $\lim_{t \to \infty} x_{i_{12}}(t) = 0$ and $\lim_{t \to \infty} x_{i_{11}}(t) = 0$ respectively, so we omit them here. Thus the following equation can be demonstrated:

$$\lim_{t \to \infty} x_{i_1}(t) = 0 \tag{38}$$

With the same method, $\lim_{t \to \infty} x_{i_2}(t) = 0$ can be deduced. And then,

$$\lim_{t \to \infty} x_i(t) = \lim_{t \to \infty} x_{i_1}(t) + \lim_{t \to \infty} x_{i_2}(t) = 0 \tag{39}$$

So *Theorem 2* can be proved.

Theorem 3: According to **Assumption 1**, the states of agents of system (3) can arrive at the same state zero, when m is an odd number and n is an even number.

The proof of **Theorem 3** which is similar to **Theorem 2** is omitted here.

Consequently, the states of agents of system (3) can reach a common state, even though the communication topology is not strong connected but can be divided into several strong connected subgraphs.

IV. SIMULATION RESULTS

The simulation results of three examples are described in this section. Example 1 is given to illustrate that protocol (4) can make the states of all agents in the multi-agent system converge to the same state more quickly than other protocol in [51] when the communication topology is strong connected. Example 2 is considered to demonstrate that protocol (4) can guarantee the agents in the multi-agent system reach agreement even though the communication topology is not strong connected, while other protocol in [52] can not do. Example 3 is presented to verify that the convergence rate of agents can be sped up by adjusting the parameter r of protocol (4).

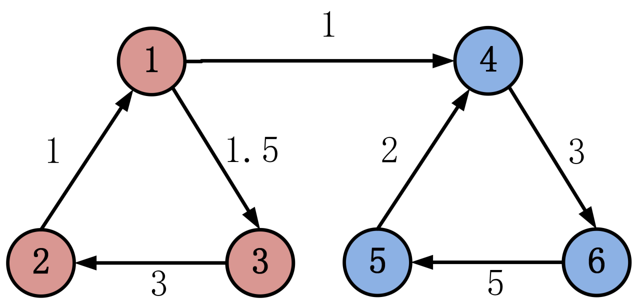

FIGURE 1. The communication topology is strong connected, and the interactions among agents are all cooperative.

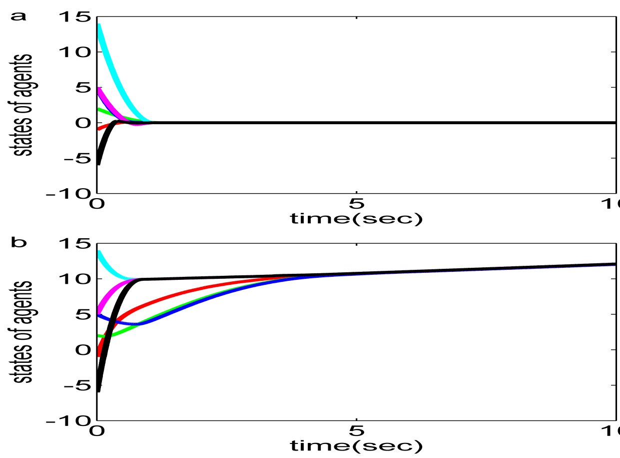

FIGURE 2. Fig. 2(*a*) and Fig. 2(*b*) exhibit the trajectories of agents with communication topology shown in Fig. 1 by applying protocol (4) and the protocol in [51], respectively. We can observe that our protocol can make all the agents in the multi-agent system converge to the same state more quickly than other protocol in [51].

A. EXAMPLE 1

The communication topology is shown in Fig. 1. It is obvious that the communication topology is strong connected and the information flow is directed.

The parameters of protocol (4) are chosen as $m = 1$, $n = 2$, $r = 5$, $x(0) = [-1, 2, 5, 14, 5, -6]^T$. The parameters of protocol in [51] are taken as $\alpha = 0.5$, $\beta = 0.7$, $a = b = 1$. Fig. 2(*a*) and Fig. 2(*b*) respectively exhibit the trajectories of six agents by applying protocol (4) and the protocol in [51]. We can observe that our protocol can make the states of all agents in the multi-agent system converge to the same state more quickly than other protocol in [51].

B. EXAMPLE 2

The communication topology is shown in Fig. 3. It is evident that the communication topology is not strong connected, the information flow is directed and the relationships among agents are either cooperative or competitive.

The parameters of protocol (4) are taken as $m = 1$, $n = 2$, $r = 5$, $x(0) = [-1, 2, 5, 14, 5, -6]^T$. The parameters of protocol in [52] are taken as $\alpha = 0.5$. Fig. 4(*a*) and Fig. 4(*b*) respectively describe the trajectories of agents by applying

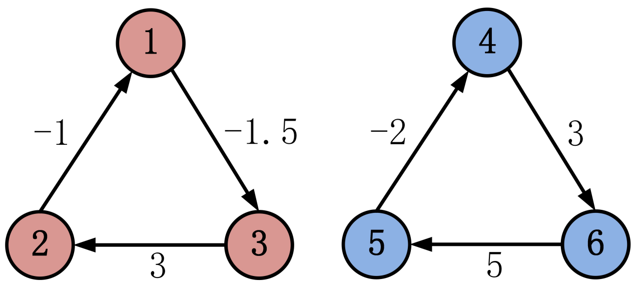

FIGURE 3. **The communication topology is not strong connected, and the interactions among agents are cooperative or competitive.**

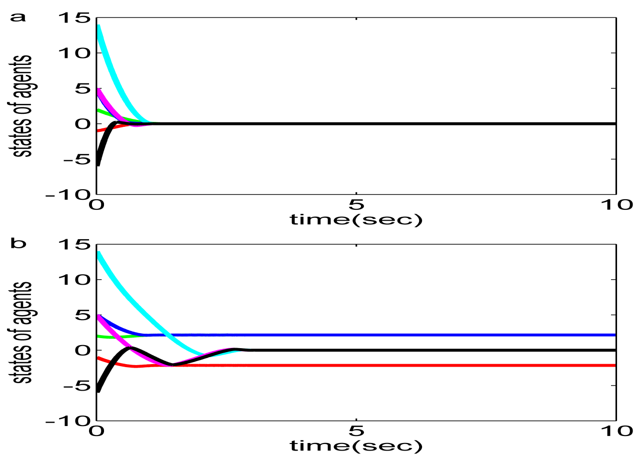

FIGURE 4. **Fig. 4(a) and Fig. 4(b) display the trajectories of agents with communication topology shown in Fig. 3 by applying protocol (4) and the protocol in [52]. It is easy to see that our protocol can guarantee all the agents in the multi-agent system reach agreement, while other protocol in [52] can not do.**

TABLE 1. **The agreement reaching time of protocol (4) with $r = 1.50$, $r = 2.50$, $r = 5.40$, $r = 9.05$.**

r	The corresponding agreement reaching time
$r = 1.50$	4.0766
$r = 2.50$	2.3948
$r = 5.40$	1.4920
$r = 9.05$	0.8610

protocol (4) and the protocol in [52]. It is easy to see that our protocol can guarantee all the agents reach agreement even though the communication topology is not strong connected, while other protocol in [52] can not do.

C. EXAMPLE 3

Firstly, the communication topology is shown in Fig. 3. And this example is given to illustrate that the convergence rate of agents can be sped up by adjusting the parameter r of protocol (4).

We choose the parameters as $m = 1$, $n = 2$, $x(0) = [-1, 2, 5, 14, 5, -6]^T$. The parameters r are taken as $r = 1.50$, $r = 2.50$, $r = 5.40$, $r = 9.05$, respectively. And the corresponding trajectories of the six agents are displayed in Fig. 5(a), Fig. 5(b), Fig. 5(c) and Fig. 5(d) when $r = 1.50$, $r = 2.50$, $r = 5.40$, $r = 9.05$. The agreement reaching time which is illustrated in Table. 1 is about 4.0766 with $r = 1.50$, 2.3948 with $r = 2.50$, 1.4920 with $r = 5.40$, 0.8610 with $r = 9.05$. It is evident that the convergence rate of agents gets faster by increasing the parameter r.

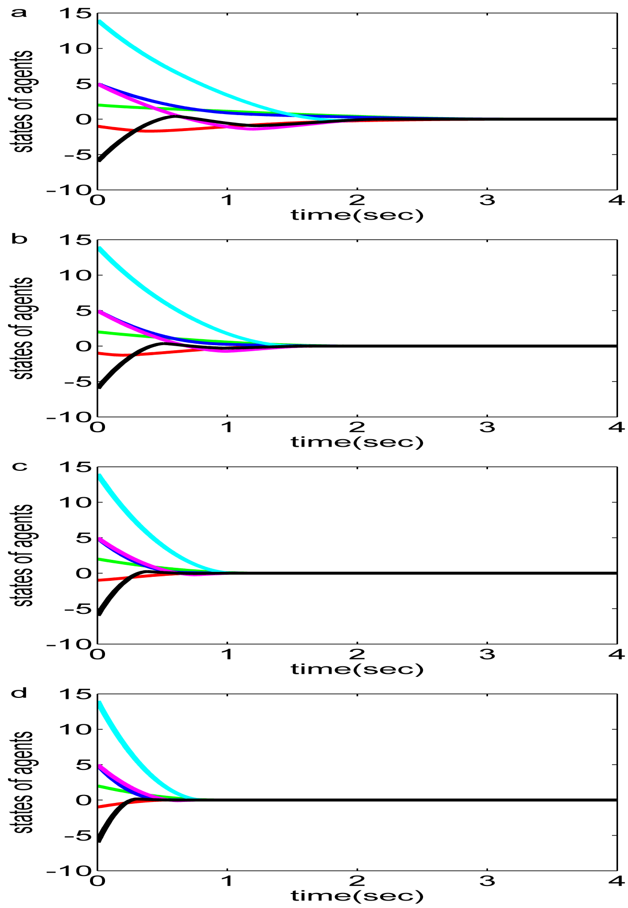

FIGURE 5. **Fig. 5(a), Fig. 5(b), Fig. 5(c) and Fig. 5(d) exhibit the trajectories of agents with communication topology shown in Fig. 3 by applying protocol (4) with $r = 1.50$, $r = 2.50$, $r = 5.40$ and $r = 9.05$, respectively. We can observe that the convergence rate of agents can be sped up by increasing the parameter r.**

V. CONCLUSION

In this paper, the nonlinear merging consensus for multi-agent systems on a directed and weighted signed network is addressed. A merging control protocol is designed to guarantee the system to reach agreement. Firstly, the relationships among agents can be either cooperative or competitive, meanwhile, the information flow is considered to be directed, which is coincident with the real scenarios. Secondly, the consensus can be ensured even though the communication topology is not strongly connected, which is more general under the realistic circumstances. Finally, the convergence rate of agents can become faster by increasing the parameter r of the protocol.

However, the settling time of protocol is not deduced to be an exact mathematical expression, but estimated in the way of numerical simulations. Therefore, we will make further extensions on the proof of an accurate mathematical expression of the settling time of protocol in the future.

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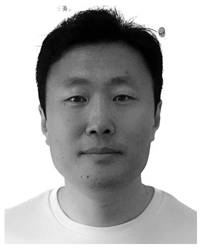

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