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# Design and Implementation of Pseudo-Inverse Thrust Allocation Algorithm for Ship Dynamic Positioning

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**ABSTRACT** The pseudo-inverse method cannot optimize the angle and needs to give the azimuth angle artificially, therefore, when the dynamic positioning system has variable vector propellers and rudders, the pseudo-inverse method cannot be applied to the full automatic control and cannot realize the optimal exactly. In this paper, in view of the above problems, considering the propeller and the environment load conditions, a thrust allocation scheme based on the pseudo inverse algorithm with weight is proposed for minimizing the energy consumption. The thrust allocation scheme distributes the thrust after optimizing the angle. It combines the full-revolving propulsion angle optimization with the thrust optimization to realize the ship thrust allocation reasonably and efficiently and achieves the minimum energy consumption and wear. Finally, the experimental results verify the effectiveness of the proposed method.

**INDEX TERMS** Pseudo-inverse method, dynamic positioning, thrust allocation, optimize.

## I. INTRODUCTION

The main purpose of the ship dynamic positioning system (DPS) is to enable the ship to maintain a fixed position and yaw angle or make the ship automatically navigated according to the set track. The controller of DPS calculates the thrust and torque generated by the ship propeller according to the deviation between the current ship position and the actual position measured by the sensor, then the thrust and torque are assigned to each thruster by the thrust allocation algorithm to realize the ship's dynamic positioning. Therefore, the fast and efficient thrust allocation algorithm is very important for the ship DPS, which is a bridge between the controller and propeller system and contributes to the rapid realization of ship DPS[1]. In order to maintain the ship's position and yaw angle, the ship DPS needs to generate a force in a specific direction and a yaw moment. If the propeller is full-revolving propellers which can rotate 360 degrees, then only two propellers need to be installed to meet the thrust requirements. However, a single propeller can

only produce a limited thrust, and in order to meet the reliability of ship maneuvering, ships are usually equipped with multiple propellers. So the propeller system equipped with multiple propellers constitutes a redundant system, there are multiple combinations of thrust and direction that can meet the force and yaw moment required by ship motion. Then how to select a set of optimal values from these combinations of thrust and direction is the thrust allocation optimization problem of the propeller system [2].

Scholars have proposed various thrust allocation methods for DPS. Liang proposed the sequential quadratic programming (SQP) thrust allocation optimization control algorithm to obtain the economy and effectiveness of energy consumption [3]. Ronghua Xu proposed the direct thrust allocation method, which needs to cycle many times to find the optimal value [4]. Under the constraints of the propulsion position, thrust and prohibited angle, Johansen proposed SQP thrust allocation method to minimize the energy consumption, while also avoid the singular value appear [5], [6]. Eivind Ruth presents an anti-helical thrust allocation method to prevent the energy losses caused by the propeller wear and tear that resulted from the propeller water infiltration

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and ventilated. This method can extend the service life of the propulsion system, reduce the energy and torque fluctuation, and improve the precision of the ship's dynamic positioning or the propeller assisted berthed positioning, however, this thrust allocation method remains to be further studied in actual ships [7]. Na Yang proposed a thrust allocation that combined static and dynamic state, which needs to be further improved when the full-revolving propeller is in the thrust restricted area [8]. Although the thrust allocation using SQP can calculate the thrust and angle rapidly and steadily, and the constraints of the full-revolving propeller forbidden angle is feasible and effective, then the DPS greatly improves the coefficient of security, reliability and positioning capability. This method relies much on the initial value, it can get different optimization results from different initial value, and it is difficult to obtain global convergence, its application in engineering needs further study [9]–[12].

In this paper, a thrust allocation scheme based on pseudo inverse algorithm with weight is proposed for the minimum energy consumption of the propulsion system, which can achieve the best optimization under many restricted condition. This method is divided into two steps, azimuth angle optimization and dynamic thrust allocation. Azimuth angle optimization distributes propulsion angle before thrust allocation in case of blade-blade interaction, and effectively avoids thrust attenuation between adjacent propulsion thrust. Dynamic thrust allocation is to distribute the thrust after optimizing the azimuth angle of the full rotary propeller, considering that the thrust provided by each propeller is limited and the minimum energy consumption is required. It becomes a thrust allocation algorithm for energy optimization after combining propulsion angle optimization with thrust optimization. Finally, simulation results and experimental tests show that the thrust allocation algorithm under azimuth angle optimization requires less energy and causes less error than the algorithm without angle optimization.

The structure of this paper is as follows: first, the thrust allocation system in our laboratory is introduced, and then the optimization of the pseudo inverse algorithm in the thrust allocation is introduced. Finally, the simulation results are given to verify the effectiveness of the proposed thrust allocation method.

## II. THE PROPULSION SYSTEM DESCRIPTION

In general, the ship's dynamic positioning only consider three directions motion: surge, sway and yaw [13]. The propeller position shown in Fig. 1,  $F_x$  is the surge force,  $F_y$  is the sway force, the ship has two full-revolving propellers and two channel propellers, and the full-revolving propeller can provide thrust from 0 to 360 degrees, such as  $F_3$  and  $F_4$ , while the channel propellers can only provide force in sway direction, such as  $F_1$  and  $F_2$ .  $R_1, R_2$  are the distance between the center of ship mass and the channel propeller,  $L_1$  is the distance between the center of ship mass and the full-revolving propeller,  $\phi_3, \phi_4$  are the angle from the vertical axis to the line joining between the center of ship mass and the

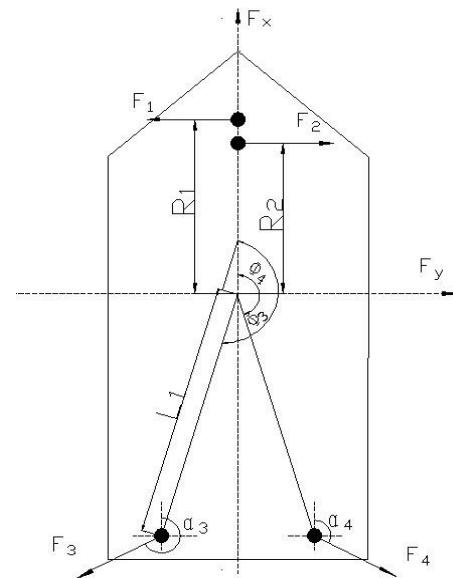


FIGURE 1. The propeller position.

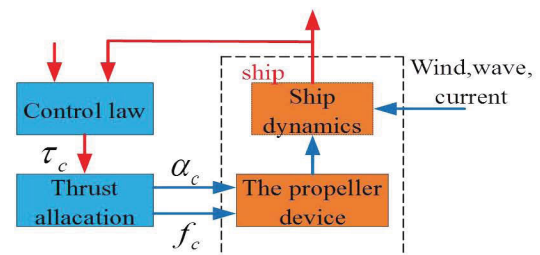


FIGURE 2. Thrust allocation structure graph.

full-revolving propeller,  $\alpha_3, \alpha_4$  are the direction angle from the vertical axis to the full-revolving propeller.

The dynamic positioning controller sends the thrust  $\tau_c$  to the propulsion system,  $f_c, \alpha_c$  are assigned to the corresponding thruster by thrust allocation, as shown in Fig.2.  $\tau_c$  is the thrust commands from the controller,  $f_c$  is the force of each thruster,  $\alpha_c$  is the thruster angle of the full-rotating propeller. In this case, the thrust allocation problem is described as the optimal problem [14], and its performance standards are:

(1) The smaller the value of  $J_1 = \frac{1}{T} \int_0^T \|\tau_c - C(\alpha_c)f_c\|_2 dt$  is the better, which used to reduce the error between the thrust command and the actual distribution thrust,  $T$  is the simulation time.

(2) The smaller the value of  $J_2 = \frac{1}{T} \int_0^T \|f\|_2 dt$  is the better, which used to energy consumption of the thruster.  $f$  is the thrust of the propeller, and  $T$  is the simulation time.

(3)  $J_3 = \frac{1}{T} \int_0^T \|f_i - f_{i+1}\|_2 dt$  is the amplitude difference of force distributed between two thruster,  $J_3$  is as small as possible, it reflects the balance of the force distribution.  $f_i$  is the thrust of one of the thruster while  $f_{i+1}$  is the thrust of the other thruster.

(4)  $\alpha_i(t), F_i(t)$  change as slowly as possible, it adapts to the dynamic response of the propeller and reduce the wear of

the propeller device.  $\alpha_i(t)$  is the thrust angle, and  $F_i(t)$  is the thrust of the propeller.

(5) The thrust of the propeller is less than the maximum thrust that can be provided.

### III. OPTIMIZATION OF PSEUDO INVERSE ALGORITHM IN THRUST ALLOCATION

#### A. PSEUDO INVERSE METHOD

During the optimization procedure, the constraints of thrust allocation, such as thrust amplitude, the propulsion position, the rate of change of angular velocity and thrust prohibited angle were considered. Taking unconstrained optimization problem as an example, the pseudo inverse method is introduced as followed [10]:

Objective function:  $\min f = u^T w u$ .

Constraints:  $\tau - B(\alpha)u = 0$ , where  $\alpha$  is the constraints.

Define a Lagrange function:

$$L(u, \lambda) = u^T W u + \lambda^T (\tau - B(\alpha)u) \quad (1)$$

Take the derivative of  $u$ :

$$\frac{\partial L}{\partial u} = (W + W^T)u - B(\alpha)^T \lambda = 0 \quad (2)$$

$$\frac{\partial^2 L}{\partial u^2} = (W + W^T)^T \quad (3)$$

From the local optimization theorem, we can get:

$$u = (W + W^T)^{-1} B(\alpha)^T \lambda \quad (4)$$

$f$  can achieve the minimum value when  $(W + W^T)^T > 0$ , where  $W + W^T$  is a singular matrix, and  $W_{ij} > 0 (i = 1, \dots, n)$ . From the boundary conditions, we can get:

$$\tau = B(\alpha)u = B(\alpha)(W + W^T)^{-1} B(\alpha)^T \lambda \quad (5)$$

It can obtained from Eq. (5):

$$\lambda = (B(\alpha)(W + W^T)^{-1} B(\alpha)^T)^{-1} \tau \quad (6)$$

Substituting Eq. (6) into Eq. (4), we obtain thrust as follows:

$$u = (W + W^T)^{-1} B(\alpha)^T (B(\alpha)(W + W^T)^{-1} B(\alpha)^T)^{-1} \tau \quad (7)$$

If matrix  $B(\alpha)^{-1}$  exists, we can obtain from Eq.(7):

$$\begin{aligned} u &= (W + W^T)^{-1} B(\alpha)^T (B(\alpha)^T)^{-1} (W + W^T) B(\alpha)^{-1} \tau \\ &= B(\alpha)^{-1} \tau \end{aligned} \quad (8)$$

That is, when  $B(\alpha)$  is non-singular matrix, Eq. (8) can realize the inverse allocation of thrust. However, structure matrix  $B(\alpha)$  is usually singular matrix or non-square matrix, so we use  $W + W^T$  to construct non-singular matrix to realize the inverse allocation of thrust. When  $W \neq I$  ( $I$  as a unit matrix), Eq. (7) is a pseudo-inverse thrust allocation algorithm with weight. While  $W = I$ , Eq. (7) is an equivalent or no weight pseudo-inverse thrust allocation algorithm.

#### B. ALGORITHM OPTIMIZATION IN LABORATORY ENVIRONMENT

In the optimization algorithm, according to the balance principle of force and torque, the thrust and torque of the propeller on the plane can be expressed as [15]:

$$\begin{cases} F_u = p_3 f_3 \cos \alpha_3 + p_4 f_4 \cos \alpha_4. \\ F_v = p_1 f_1 + p_2 f_2 + p_3 f_3 \sin \alpha_3 + p_4 f_4 \sin \alpha_4. \\ N_z = p_1 f_1 l_{1x} + p_2 f_2 l_{2x} + p_3 f_3 (\sin \alpha_3 l_{3x} - \cos \alpha_3 l_{3y}) \\ \quad + p_4 f_4 (\sin \alpha_4 l_{4x} - \cos \alpha_4 l_{4y}). \end{cases} \quad (9)$$

That is:

$$\tau = B(\alpha) P f \quad (10)$$

where  $F_u$  is the surge force,  $F_v$  is the sway force,  $N_z$  is the rotational momentum, and  $\tau = [F_u, F_v, N_z]^T$ , according to the type of thruster and its position, the matrix  $B(\alpha)$  and  $P$  can be expressed as:

$$B(\alpha) = \begin{bmatrix} 0 & 0 & \cos \alpha_3 & \cos \alpha_4 \\ 1 & 1 & \sin \alpha_3 & \sin \alpha_4 \\ l_{1x} & l_{2x} & \sin \alpha_3 l_{3x} - \cos \alpha_3 l_{3y} & -\cos \alpha_4 l_{4y} + \sin \alpha_4 l_{4x} \end{bmatrix},$$

$$P = \begin{bmatrix} p_1 & 0 & 0 & 0 \\ 0 & p_2 & 0 & 0 \\ 0 & 0 & p_3 & 0 \\ 0 & 0 & 0 & p_4 \end{bmatrix},$$

$l_{ix} (1 \leq i \leq 4)$  and  $l_{iy} (1 \leq i \leq 4)$  are the propeller position shown in Table1,  $\alpha_3$  and  $\alpha_4$  are the full-revolving propeller azimuth angle,  $P$  is the diagonal matrix of the thruster[14],

$$p_i = \begin{cases} 0 & \text{failure in the } i\text{th thrust} \\ 1 & \text{effectivity in the } i\text{th thrust} \end{cases} \quad (i = 1, 2, 3, 4),$$

the force vector is  $f = [f_1, f_2, f_3, f_4]^T$ , which represent the force produced by each thruster, suppose the four thrusters are in normal work,  $P$  is unit matrix, Eq. (10) can be written as:

$$\tau = B(\alpha) f \quad (11)$$

When the number of the output variables is greater than the number of equations, the system is called a overdrive system. From Eq. (9), we known that there are three equations for the thrust allocation of DPS, but there are six unknowns  $\alpha_3, \alpha_4, f_1, f_2, f_3, f_4$ . Therefore, the system is a overdrive system, which should supplements the equation that characterize the system characteristic and calculated by the optimization method.

The controller calculates the required thrust according to the deviation value between input and feedback. The thrust command  $\tau_c$  obtained from the DPS controller is sometimes unstable, where a first order low pass filter can be used [10], that is:

$$\dot{\tau} + \Lambda \tau = \Lambda \tau_c \quad (12)$$

Then we can obtain the gently changing thrust command  $\tau$ ,  $\Lambda$  in Eq.(11) is the diagonal matrix with positive elements and  $\Lambda = \text{diag}(\frac{1}{T_1}, \frac{1}{T_2}, \frac{1}{T_3})$ .  $T_1, T_2, T_3$  represent the time constant in the directions of surge, sway and yaw respectively.

It can be seen as the optimization problems for Eq.(9), the minimum  $f$  demanded when satisfy Eq.(9), the objective function of optimization can be written as:

$$\begin{aligned} \min_{f'} u &= \min_{f'} f'^T W f' \\ \text{s.t. } \tau &= B(\alpha) P f' \end{aligned} \quad (13)$$

**C. OPTIMIZATION OF THE AZIMUTH ANGLE OF FULL-REVOLVING PROPELLER**

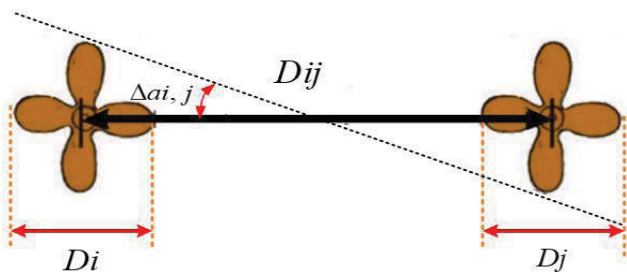
1) CHANGE WEIGHTING FACTOR

The disadvantage of the pseudo inverse algorithm is that it can not optimize the propeller azimuth angle. When two adjacent full-revolving propellers or channel propellers are working together, the wake flow of upstream propeller will change the inflow velocity of the downstream propeller, which will cause the downstream propeller thrust losses. According to this case, we can first optimize the angle by changing the weight matrix of the allocation algorithm, so that the thrust direction of the adjacent propeller is not in a straight line, and the problem of thrust attenuation caused by blade-blade interaction is avoided.

According to the type of propeller used in this article, weight matrix defined as follows:

$$W = \begin{bmatrix} w_{1y} & 0 & 0 & 0 & 0 & 0 \\ 0 & w_{2y} & 0 & 0 & 0 & 0 \\ 0 & 0 & w_{3x} & 0 & 0 & 0 \\ 0 & 0 & 0 & w_{3y} & 0 & 0 \\ 0 & 0 & 0 & 0 & w_{4x} & 0 \\ 0 & 0 & 0 & 0 & 0 & w_{4y} \end{bmatrix} \quad (14)$$

where  $w_{3x}, w_{4x}$  are the proportion factor of the third and fourth full-revolving propeller in surge direction,  $w_{1y}, w_{2y}, w_{3y}, w_{4y}$  are the proportion factor of each full-revolving propeller in sway direction. The higher the proportion factor, the smaller the propeller thrust.



**FIGURE 3.** The relationship between distance and deviation angle of thrusters.

2) INTRODUCING THE PROPELLER DEVIATING ANGLE  $\Delta\alpha_{i,j}$

If the reasonable distance between two adjacent full-revolving propellers  $i$  and  $j$  are not satisfied  $d_{i,j} > 2(D_i + D_j)$ ,  $D_i, D_j$  are the diameter of the  $i$ th and  $j$ th propeller, as shown in Fig. 3. The propeller deviating angle  $\Delta\alpha_{i,j}$  introduced to solve the thrust interaction between two adjacent

full-revolving propellers. The propeller angle is satisfied:

$$\begin{cases} \alpha_i = \alpha - \Delta\alpha_{i,j} \\ \alpha_j = \alpha + \Delta\alpha_{i,j} \end{cases} \quad i \neq j \quad (15)$$

where

$$\alpha = \begin{cases} \frac{\pi}{2}, & F_u = 0, \quad F_v > 0 \\ \frac{3\pi}{2}, & F_u = 0, \quad F_v < 0 \\ \arctan \frac{F_v}{F_u} + \pi, & F_u < 0 \\ \arctan \frac{F_v}{F_u}, & F_u > 0, \quad F_v > 0 \\ \arctan \frac{F_v}{F_u} + 2\pi, & F_u > 0, \quad F_v < 0 \end{cases} \quad (16)$$

And  $\alpha_i, \alpha_j$  is the  $i$ th and  $j$ th propeller angle, the propeller deviating angle  $\Delta\alpha_{i,j}$  is satisfied:

$$\Delta\alpha_{i,j} > \arcsin \frac{D_i + D_j}{2d_{i,j}} \quad (17)$$

It can avoid the propeller failure by introducing the propeller deviation angle when one full-revolving propeller located on another wake flow at ship stern.

**D. OPTIMUM OF THE PROPELLER THRUST**

After the optimization of the full-revolving propeller azimuth angle, the command angle of the full-revolving propeller is [14]:

$$\alpha_c = \alpha \pm \Delta\alpha_{i,j} \quad (18)$$

Consider the finite thrust provided by each thruster, and require minimal energy consumption, that is:

$$\min_f u' = \min_f \frac{1}{2} (f_c + c)^T W_2 (f_c + c) \quad (19)$$

Satisfy condition:

$$\tau_c = C(\alpha_c) f_c \quad (20)$$

where  $f_{c,jmin} \leq f_{c,j} \leq f_{c,jmax}, j$  is 1, 2, 3, 4.  $W_2$  is the weight matrix,  $c$  represent offset, to solve this problem, define Hamilton variables:

$$H = \frac{1}{2} (f_c^T W_2 f_c + c^T W_2 f_c + f_c^T W_2 c + c^T W_2 c) + \xi (C(\alpha_c) f_c - \tau_c) \quad (21)$$

where  $\xi \in R^n$  is an uncertain Lagrangian multiplier,  $H$  takes a partial differential and makes it equal to zero, that is:

$$\begin{aligned} \frac{\partial H}{\partial f_c} &= W_2 f_c + \frac{1}{2} (c^T W_2)^T + \frac{1}{2} W_2 c + (\xi C(\alpha_c))^T = 0 \\ &\Rightarrow W_2 f_c = -W_2 c - C(\alpha_c)^T \xi^T \end{aligned} \quad (22)$$

$$\begin{aligned} \frac{\partial H}{\partial \xi} &= C(\alpha_c) f_c - \tau_c = 0 \Rightarrow C(\alpha_c) W_2^{-1} W_2 f_c = \tau_c \\ &\Rightarrow C(\alpha_c) W_2^{-1} [-W_2 c - C(\alpha_c)^T \xi^T] = \tau_c \end{aligned} \quad (23)$$

Solve Eq. (22) can obtain:

$$\xi^T = -(C(\alpha_c) W_2^{-1} C(\alpha_c)^T)^{-1} (\tau_c + C(\alpha_c) c) \quad (24)$$

Substituting Eq. (23) into Eq. (21):

$$\begin{aligned}
 W_2 f_c &= -W_2 c + C(\alpha_c)^T (C(\alpha_c) W_2^{-1} C(\alpha_c)^T)^{-1} (\tau_c + C(\alpha_c) c) \\
 \Rightarrow f_c &= -c + W_2^{-1} C(\alpha_c)^T (C(\alpha_c) W_2^{-1} C(\alpha_c)^T)^{-1} \\
 &\quad \times (\tau_c + C(\alpha_c) c) \tag{25}
 \end{aligned}$$

Eq. (24) can solve the problem of the thrust allocation, the initial value of  $c$  is zero, if the thrust amplitude allocated by each propellers are within the scope of the provided thrust, then the above process stops. If a thrust is bigger than the maximum, you could optimize again and amend Eq. (24) as follows:

- (1) The  $j$ th column element of the structure matrix  $C(\alpha_c)$  corresponding to the saturation value  $f_{cj}$  in appropriate position is set at 0, while  $C(\alpha_c)$  in  $\tau_c + C(\alpha_c) c$  remain unchanged.
- (2) The  $j$ th element of the vector  $C(\alpha_c)$  is set as negative saturation value, while  $C(\alpha_c)$  in  $\tau_c + C(\alpha_c) c$  remain unchanged.



(a)



(b)

FIGURE 4. Physical map. (a) 11m×6m×1m Pool. (b) 2.8 meters ship model.

## IV. SIMULATION ANALYSES

### A. EXPERIMENTAL PLATFORM

The experiment in this paper relies on the Key Laboratory of marine engineering of Guangdong University of Technology, where has an experiment pool with 11m×6m×1m and a 2.8m ship model of an offshore supply vessel with a scale of 1:26, as shown in Fig. 4. The experiment simulation platform of ship dynamic positioning system was composed of a set of controller, a work station and a mobile operation terminal.



FIGURE 5. Practicality picture of ship control system.



FIGURE 6. Mounting image of attitude sensor and mounting image of wind sensor.

ship DP system is an indoor positioning system based on ultrasonic distance measurement[13], [16]. The main control system uses EC31-RTX as the controller, and the physical map of the control system is shown in Fig.5. The installation drawing of the posture sensor and the wind sensor is shown in Fig.6. The thrust system is mainly composed of two channel propellers on the ship bow and two full-revolving propellers on the ship stern, and its main thruster physical map is shown in Fig.7.

### B. EXPERIMENTAL PROCEDURES

The steps to the optimal combination of the propeller azimuth angle and dynamic thrust allocation are as follows:

Step1:  $\tau_0 = 0, \alpha_0 = 0, f_0 = 0, \Delta\tau_0 = 0, e_0 = 0, k = 0, h$  is the sampling time.

Step2:  $\alpha_k = \arctan \frac{F_{v,k}}{F_{u,k}}, P = E_{4 \times 4}$ .

Step3:  $\alpha_{c,k} = \alpha_k \pm \Delta\alpha_k$ .

Step4:

$$\begin{aligned}
 f'_k &= (W_1 + W_1^T)^{-1} P'^T B'^T (P'^T B'^T)^{-1} (B' P' (W_1 \\
 &\quad + W_1^T)^{-1})^{-1} \tau_k.
 \end{aligned}$$



FIGURE 7. Practicality picture of main thruster drive gear.

Step5:  $Y_k = Y(\alpha_k, f'_k)$ .  
 Step6: If  $\dot{\alpha}_{c,k} < \Delta$ , executes Eq. (8), otherwise adjusts it to  $\dot{\alpha}_{c,k} < \Delta$ .  
 Step7:  $f_{c,k} = -c + W_2^{-1} C^T (C W_2^{-1} C^T)^{-1} (\tau_{c,k} + Cc)$ .  
 Step8: If  $f_{c,j} < f_{j\max}$  or  $f_{c,j} < f_{j\min}$ , the  $j$ th column element of vector  $C$  is zero, the  $j$ th element of vector  $C$  is set as negative saturation value, return to Step8, otherwise proceed to next step.  
 Step9: Low pass filters:  $\dot{\tau}_k = \wedge(\tau_{c,k} - \tau_k)$ ,  $\tau_{k+1} = \tau_k + h\dot{\tau}_k$   
 Step10: Second order filter:  $\dot{e}_k = \Delta\tau_k$ ,  $\Delta\dot{\tau}_k = -2\xi\omega_n\Delta\tau_k - \omega_n^2 e_k + K\tau_{c,k}$ ,  $e_{k+1} = e_k + h\dot{e}_k$ ,  $\Delta\tau_{k+1} = \Delta\tau_k + h\Delta\dot{\tau}_k$ .  
 Step11:  $k = k + 1$ , return to Step2.

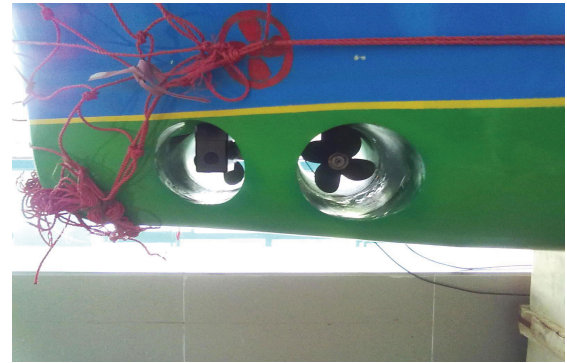
C. SIMULATION RESULTS

This article research as thrust allocation strategy based on the 2.8 meters ship model, the bow propeller using channel propeller, where installed speed sensor, the stern propeller and other parts use full-revolving propeller, where installed speed sensor and full-revolving angle sensor[17]. The Power plants of each propeller use Yaskawa SGD V-2R8A01A ac servo motor with power 0.4kW, the rated speed is 3000r/min, and the propeller blade diameter  $D = 95mm$ . The parameters of each thruster are shown in Table 1:

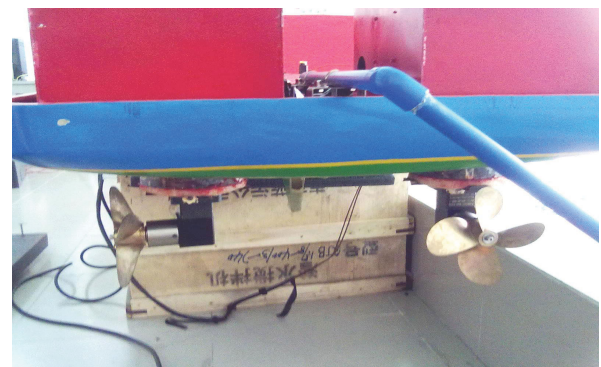
TABLE 1. Comparison of various technologies.

The number of propeller	position- $l_{ix}$ (mm)	position- $l_{iy}$ (mm)	The range of propeller thrust(N)	The maximum speed of propulsion angle (degree/s)
1#	1085	0	(-4, 4)	0
2#	935	0	(-4, 4)	0
3#	-1205	-170	(-10, 10)	30
4#	-1205	170	(-10, 10)	30

The installation of the four propellers shown as Fig.8. Simulation tests are carried out on the 2.8 meters ship model to demonstrate the validity of the proposed thrust allocation algorithm, simulation parameter are set up as follows: the time constants of the low pass first order filter are set to:  $T_1 = T_2 = T_3 = 1.59$ , the parameters of the second order band-pass filter are set to  $\omega_{n,k} = 0.63$ ,  $\xi_k = 1.0$ , input



(a)



(b)

FIGURE 8. Ship thrusters. (a) The funnel thruster in the bow. (b) The azimuth thruster in the poop.

command:  $\tau_c = [10 \sin(w_1 ht), 10 \cos(w_2 ht), 5 \sin(w_3 ht)]^T$ , where  $w_1 = 2\pi \times 0.03$ ,  $w_2 = 2\pi \times 0.03$ ,  $w_3 = 2\pi \times 0.02$ ,  $h = 0.5$  as sampling time,  $t$  is time and  $\dot{\alpha} = 30^\circ/s$ ,

$$W_1 = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.25 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.25 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.25 \end{bmatrix},$$

$$W_2 = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0.25 \end{bmatrix}.$$

Suppose the maximum thrust provided by the channel propeller is 4N, the maximum thrust provided by the full-revolving propeller is 10N, the controller sent the thrust command  $\tau_c = [F_u, F_v, N_z]^T$  to the propulsion system, as shown in Fig.9.  $F_u$  is the surge force,  $F_v$  is the sway force, and  $N_z$  is the torque. In order to simulate the thrust command in all directions, the direction of joint force  $F_u, F_v$  change from  $0^\circ$  to  $360^\circ$ , which can improve the ergodicity of experiments.

The optimized azimuth angle of the full-revolving propeller, as shown in Fig. 10, the interval of each sampling point is 0.5, the angle change is less than 30 degrees, the azimuth angle changing from 0 to 360 degrees does not belong to angle hopping, the azimuth angle  $0^\circ$  is equivalent to  $360^\circ$ .

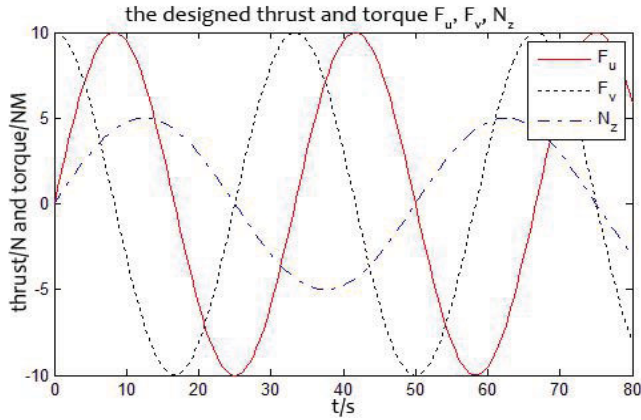


FIGURE 9. Thrust command  $\tau_c$ .

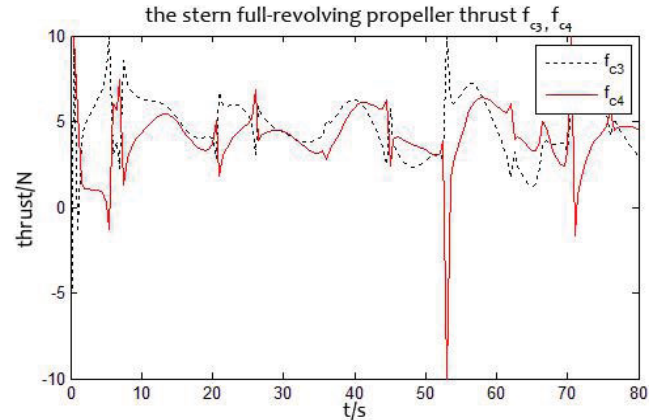


FIGURE 12. Thrusts of azimuth thrusters.

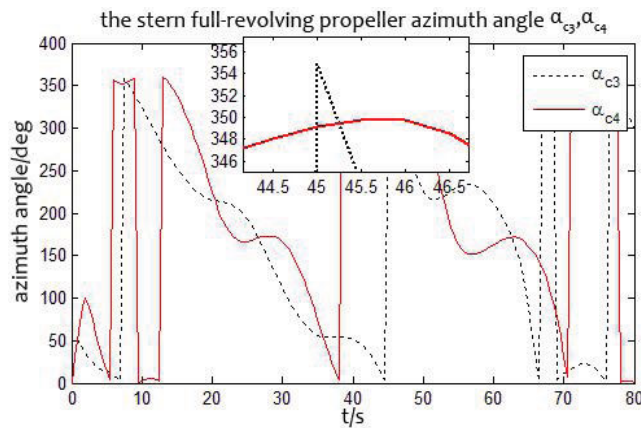


FIGURE 10. Angles of azimuth thrusters.

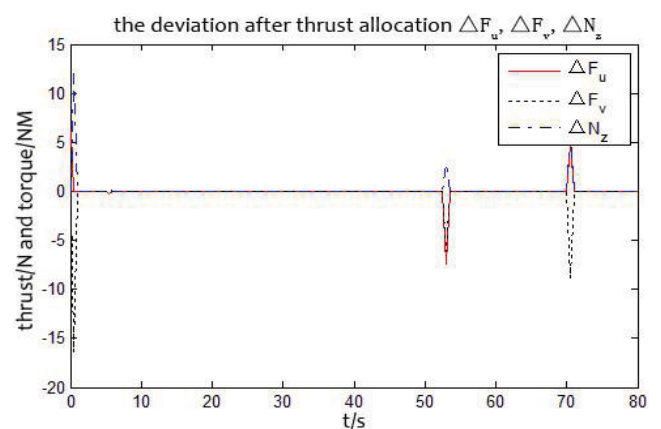


FIGURE 13. Deviation  $\Delta\tau$ .

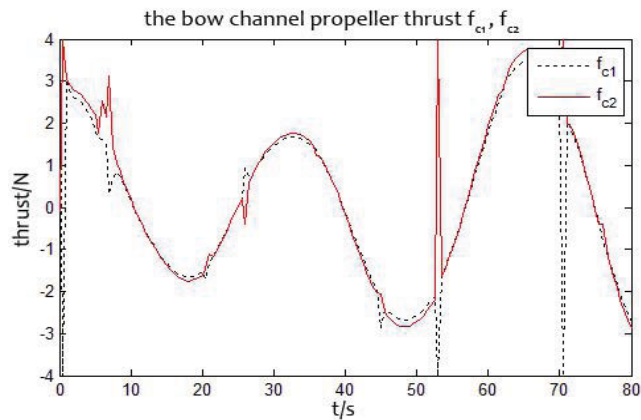


FIGURE 11. Thrusts of funnel thrusters.

The thrust allocated to the bow channel propeller is shown in Fig. 11, where the minus represent the reverse of the bow channel propeller.

The thrust allocated to the stern full-revolving propeller is shown in Fig. 12, where the minus represents the reverse of the stern full-revolving propeller. It can be seen that each propeller works properly, if a propeller fails, its thrust is zero.

The deviation  $\Delta\tau = (\Delta F_u, \Delta F_v, \Delta N_z) = \tau_c - C(\alpha_c)f_c$  is shown in Fig. 13, it can be seen that the deviation is very close

TABLE 2. Comparison of static and dynamic combination and pure static thrust allocation method.

thrust allocation method	$J_1$	$J_2$	$J_3$	$\dot{\alpha}$
azimuth angle optimization	Close to 0	10.115	3.23	$30^\circ/s$
angle-free optimization	Close to 0	14.568	4.76	$30^\circ/s$

to zero, which indicates that the control commands from the controller are almost all applied to the propeller.

The thrust allocation algorithm requires the performance indicators  $J_1$  close to 0,  $J_2, J_3$  as small as possible. At the same thrust command and angle change, the thrust allocation algorithm of angle-free consumes more energy than the azimuth angle optimization. The deviation between them is close to zero, as shown in Table 2. In addition, you can get different results by adjusting the time constant and weight matrix.

It can be seen from Fig. 9-Fig. 13 and Table 2 that the thrust allocation algorithm based on the azimuth angle optimization can not only optimize the azimuth angle of the full-revolving propeller, but also make it change slowly to reduce the wear and tear of the propeller and avoid the blade-blade interaction. Meanwhile the thrust of the propeller is processed to be saturated and limited, the amplitude and azimuth angles of each propeller thrust are optimized in real time. You can obtain different thrust allocation result by adjusting the weight matrix, comparing with the non-optimized algorithm,

the total power is reduced, energy saved and improve the ship's maneuverability. In addition, there is no iteration in the proposed thrust allocation algorithm, and its calculation is simple. Simulation experiments also verify the effectiveness of the algorithm.

## V. CONCLUSION

Based on the pseudo-inverse algorithm and the characteristics of the ship model propulsion system, a thrust allocation algorithm for energy optimization is designed in this paper. Thrust allocation algorithm which combines propulsion angle optimization with thrust optimization can consume less energy and the error is close to zero. Then, according to the actual situation of the ship model, adjust the thrust allocation method under the azimuth angle optimization, and the process of thrust allocation and its simulation experiment are given. Finally, the validity of the optimized thrust allocation algorithm in the ship dynamic positioning system is verified under different laboratory conditions.

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