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Synchronization of Fuzzy Control Design Based on Bessel–Legendre Inequality for Coronary Artery State Time-Delay System

RUIKUN WANG[1], BOHAN LI[1], ZHANSHAN ZHAO[1,2], JIE GUO[1], AND ZHENYU ZHU[1]

[1]School of Computer Science and Technology, Tiangong University, Tianjin 300387, China
[2]State Key Laboratory of Process Automation in Mining and Metallurgy/Beijing Key Laboratory of Process Automation in Mining and Metallurgy Research, Beijing 102628, China

Corresponding author: Zhanshan Zhao (zhzhsh127@163.com)

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ABSTRACT This paper investigates the problem of fuzzy state feedback control for a category of fuzzy-model-based coronary artery system(FMBCAS) with state time-delay. T-S fuzzy model is employed to close in coronary artery system(CAS) which exists unknown nonlinear characteristics. Through the use of linear matrix inequality technique, the synchronization controller for CAS are obtained. By utilizing the delay-partitioning method, we can get a less conservative results. The Bessel-Legendre inequality and the Moon's et al. inequality with convex analysis are used to further reduce the conservatism of the system. Finally, a simulation of CAS proves the effectiveness of our strategy.

INDEX TERMS Chaos synchronization, coronary artery system, Bessel-Legendre inequality, fuzzy control, delay-partitioning method.

I. INTRODUCTION

Chaos synchronization has become an important topic in the field of nonlinearity, it exists widely in biomedical, information and other fields [1]–[5]. Confidential communication, chemical reactions, cardiovascular disease treatment are some examples which use chaos synchronization [6], [7]. With the changes of blood vessel and blood pressure, the coronary artery system will produce non-linear chaotic dynamic behavior. Thus, the coronary artery system is also a special chaotic system. Recently years, many researchers study on the coronary artery system. Xu et al. in 1986 put forward the mathematical equation of the CAS and the main parameter values, mathematically prove that coronary artery spasm will occur chaotic behavior [8].

Researchers have proposed a number of methods for the study of nonlinear systems[9]–[18], such as adaptive control [19], [20], sliding mode control [21], [22], fuzzy control [23]–[25], event-triggered control [26], [27]. In the study of CAS, the researchers applied the control methods of nonlinear system to chaos synchronization.

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Reference [28] designed a chaos suppression controller using sliding mode control scheme for CAS. Reference [29] used high-order sliding mode combined with adaptive control method to achieve CAS synchronization in a limited time. Reference [30] proposed a terminal sliding mode adaptive control for the synchronization of CAS with unknown uncertainty boundary and the disturbance. It is a pity that above literatures did not take the influence of time-delay into account.

In practice of coronary heart disease treatment, the duration of the drug, and the situation of the drug absorption affect the stability of the system. There is a general phenomenon of time-delay in CAS, and usually the time-delay changes over time. Therefore, it has practical significance to research the synchronization problem of uncertain CAS with time-delay. The work [31] achieved the synchronization control of CAS under interval time-varying delay through delay-partitioning method. The paper [32] proposed a novel synchronization control method by designing chaotic observers. Reference [33] used Wirtinger-based integral inequality to process time delay to achieve chaos synchronization. Reference [34] used Jensen inequality to obtain delay-dependent stability conditions for chaotic finance systems. The methods proposed in the above literatures to deal with the time-delay chaotic system has achieved good results in obtaining system stability criteria, but there is still room for further improvement. According to the Lyapunov stability theory, the conservation of the delay-dependent stability criterion depends on constructing an appropriate Lyapunov-Krasovskii functional (LKF) and scaling the derivative of the constructed LKF. At present, the mainstream methods are Wirtinger inequality [35], Jensen inequality [36] and free-matrix-based integral inequality [37]. Recently, a new inequality named Bessel-Legendre inequality was proposed [38], which has a less conservatism. In addition, the Moon's et al. inequality with convex analysis is also an effective way to reduce the conservatism [39].

CAS is more complex than existing mathematical models, and existing mathematical models can't approximate the real system well. Fuzzy control is an effective method for modeling and controlling complex nonlinear systems. Through the fuzzy logic, the language information is constructed on the control system, so it is needn't to establish the precise mathematical model of controlled plant in the design, so that the control strategy is easy to design and apply to practical system. The powerful universal approximation of the Takagi-Sugeno (T-S) fuzzy system makes it a powerful tool for controlling nonlinear systems. The T-S fuzzy model approach uses a combination of fuzzy logic theory and linear system theory to deal with complex nonlinear systems. References [40] and [41] show the advantages of modeling using the T-S model.

Motivated by the above discussions, fuzzy state feedback synchronization control for coronary artery system with state time-varying delay is investigated in this paper. By utilizing the linear matrix inequality technique, a new synchronization controller for CAS are obtained. The contribution of the presented synchronization method are emphasized as follows:(a) By constructing the fuzzy dynamic equation of the CAS, the model further approximates the real CAS, thus achieving better control effect.(b) By utilizing the delay-partitioning technology [42], we can get a less conservative results.(c) Bessel-Legendre inequality and the Moon's et al. inequality with convex analysis are used to further reduce the conservatism of the proposed method.

The rest is organized as follows. The problem description and preliminaries are described in Section 2. Section 3 elaborates the fuzzy synchronization control strategy for CAS with state time-delay. The simulation results are illustrated in Section 4 to verify the validity of the presented methodology. Finally, the conclusions are drawn in Section 5.

II. PROBLEM FORMULATION AND PRELIMINARIES

In this paper, the mathematical equation of CAS is expressed as follows:

$$\dot{x}(t) = Ax(t) + Bx(t-\tau(t)) + \zeta, \tag{1}$$

where $x(t) = [x_1(t), x_2(t)]^T$, $x_1(t)$ and $x_2(t)$ represent inner diameter changes and inner pressure changes of CAS.

Accordingly, we can derive the fuzzy model of coronary artery drive system. In this paper, let us use T-S fuzzy model to describe CAS with r fuzzy rules:

Rule i: IF $\varphi_1(t)$ is δ_{i1} and ... and $\varphi_p(t)$ is δ_{ip}
THEN

$$\begin{cases} \dot{x}_m(t) = A_i x_m(t) + B_i x_m(t-\tau(t)) + \zeta_i, \\ x_m(t) = \psi(t), \quad -\tau_M \le t \le 0, \ i = 1, 2, \ldots, r. \end{cases} \tag{2}$$

where $\varphi(t) = (\varphi_1(t), \varphi_2(t), \ldots, \varphi_p(t))^T$ is the premise variable, $\delta_{ij} (i = 1 \ldots r, j = 1 \ldots p)$ is the fuzzy set, $x_m(t) \in R^n$ is the state vector, $\tau(t) \in [0, \tau_M]$ denotes a time-varying delay and τ_M is a positive constant. $A_i, B_i \in R^{n*n}$ are constant real matrices, ζ_i is periodic perturbation.

The master system model of FMBCAS can be inferred as follows:

$$\begin{cases} \dot{x}_m(t) = \sum_{i=1}^{r} h_i(\varphi(t))\{A_i x_m(t) + B_i x_m(t-\tau(t)) + \zeta_i\}, \\ x_m(t) = \psi(t), \quad -\tau_M \le t \le 0, \ i = 1, 2, \ldots, r. \end{cases} \tag{3}$$

where fuzzy weighting functions

$$h_i(\varphi(t)) = \epsilon_i(\varphi(t)) / \sum_{i=1}^{r} \epsilon_i(\varphi(t)).$$

It is clear that $h_i(\varphi(t)) \ge 0$, $\sum_{i=1}^{r} h_i(\varphi(t)) = 1$.

Similar to the master system, we can describe the slave system using the following rules:

Rule i: IF $\varphi_1(t)$ is δ_{i1} and ... and $\varphi_p(t)$ is δ_{ip}
THEN

$$\begin{cases} \dot{x}_s(t) = A_i x_s(t) + B_i x_s(t-\tau(t)) + \zeta_i + \omega(t) + u(t), \\ x_s(t) = \bar{\psi}(t), \quad -\tau_M \le t \le 0, \ i = 1, 2, \ldots, r. \end{cases} \tag{4}$$

The slave system model with controller can be represented by:

$$\begin{cases} \dot{x}_s(t) = \sum_{i=1}^{r} h_i(\varphi(t))\{A_i x_s(t) + B_i x_s(t-\tau(t)) \\ \qquad +\zeta_i\} + \omega(t) + u(t), \\ x_s(t) = \bar{\psi}(t), \quad -\tau_M \le t \le 0, i = 1, 2, \ldots, r. \end{cases} \tag{5}$$

where $\omega(t)$ is the external disturbance and fuzzy weighting functions

$$h_i(\varphi(t)) = \epsilon_i(\varphi(t)) / \sum_{i=1}^{r} \epsilon_i(\varphi(t)).$$

It is clear that $h_i(\varphi(t)) \ge 0$, $\sum_{i=1}^{r} h_i(\varphi(t)) = 1$.

Defining $e(t) = x_m(t) - x_s(t)$, the state equation of the error system can be expressed as follows:

$$\begin{aligned} \dot{e}(t) &= \dot{x}_m - \dot{x}_s \\ &= \sum_{i=1}^{r} h_i(\varphi(t))[A_i x_m(t) + B_i x_m(t-\tau(t)) + \zeta_i] \\ &\quad - \sum_{i=1}^{r} h_i(\varphi(t))(A_i x_s(t) + B_i x_s(t-\tau(t)) + \zeta_i) \\ &\quad - u(t) - \omega(t) \\ &= \sum_{i=1}^{r} h_i(\varphi(t))(B_i e(t-\tau(t)) + (A_i - K_i) e(t)) - \omega(t) \end{aligned} \tag{6}$$

In this paper, the state feedback controller will be designed to make the slave system synchronized to master system. We consider the following control law:

$$u(t)=\sum_{i=1}^{r}h_i(\varphi(t))K_ie(t) \tag{7}$$

K_i are the control gain matrices.

Remark 1: The TS fuzzy model is a nonlinear system described by a set of If-then fuzzy rules. The TS fuzzy rules divided the nonlinear system into many local linear systems. Each rule represents a subsystem. Comparing with the traditional nonlinear system model, the local linearized dynamic model can be easily analyzed and designed by modern control theory.

Assumption 1: The error system (6) satisfies the following condition

$$\delta^2\int_0^{\infty}\omega^T(t)\omega(t)dt-\int_0^{\infty}e^T(t)e(t)dt\geq 0, \tag{8}$$

where δ represents disturbance attenuation rate.

Lemma 1 [38]: For given symmetric positive definite matrix Z, x is a continuously differentiable function $x\in[p,q]\to\mathbb{R}^n$, we can obtain:

$$\int_p^q\dot{x}^T(v)Z\dot{x}(v)dv\geq\frac{1}{q-p}\Im^T diag(Z,3Z,5Z)\Im$$

where

$$\Im=\begin{bmatrix}x(q)-x(p)\\ x(q)+x(p)-\dfrac{2}{q-p}\displaystyle\int_p^q x(v)dv\\ x(q)-x(p)-\dfrac{6}{q-p}\displaystyle\int_p^q\varrho_{p,q}(v)x(v)dv\end{bmatrix},$$

$$\varrho_{p,q}(v)=2(\frac{v-p}{q-p})-1.$$

Lemma 2 [39]: Suppose exist the symmetric matrices $M\in\mathbb{R}^{n\times n}$, the following inequality holds for the given matrices $N_1,N_2\in\mathbb{R}^{2n\times n}$, and any scalar $\alpha\in(0,1)$:

$$\begin{bmatrix}\frac{1}{\alpha}M & 0\\ * & \frac{1}{1-\alpha}M\end{bmatrix}\geq He(N_1[I\quad 0]+N_2[0\quad I])-\alpha N_1M^{-1}N_1^T-(1-\alpha)N_2M^{-1}N_2^T.$$

Remark 2: In this paper, we use the Bessel-Legendre inequality. It encompasses the Wirtinger inequality. The Bessel-Legendre inequality depends not only on $x(t)$, $x(t-\tau)$, $\int_p^q x(v)dv$, but also on the integral term $\int_p^q\varrho_{p,q}(v)x(v)dv$. Using the Bessel-Legendre inequality, we can obtain a less conservative results.

III. MAIN RESULTS

In this paper, we divide the delay interval $[0,\tau_M]$ into N equivalent subintervals, where $N>0$. $[\tau_{k-1},\tau_k]$ represents the k-th subinterval$(1\leq k\leq N)$. We define $\bar{\tau}=\tau_k-\tau_{k-1}=\tau_M/N$, $0=\tau_0<\tau_1<\cdots<\tau_N=\tau_M$.

Theorem 1: Coronary artery error system (7) is asymptotically stability by using the new designed controller for any time-delay that satisfies $0<\tau(t)<\tau_M$, if there are positive-definite symmetric matrices $P_1,P_2,P_3,Q_k,R_k(k=1,\ldots,N)$, matrices X, $G_i(i=1,\ldots,r)$, $N_{j,k}(j=1,2;k=1,\ldots,N)$ of appropriate dimensions, and the scalars β_1,β_2,δ, so that the following LMI hold:

$$\begin{bmatrix}\Xi & N_j\\ N_j^T & -\hat{R}\end{bmatrix}<0,\quad(j=1,2) \tag{9}$$

where

$$\begin{aligned}
\Xi&=\Phi-\sum_{k=1}^{N}He([N_{2,k}N_{1,k}]\Lambda_k),\\
\Phi&=2e_1^TP_1e_{N+2}+\Theta\\
&\quad+e_{N+2}^T(\sum_{k=1}^{N}\bar{\tau}^2R_k+\tau_M^2P_3)e_{N+2}-\Lambda^T\hat{P}_3\Lambda\\
&\quad+2[\sigma X(-e_{N+2}^T+\sum_{i=1}^{r}h_i(\varphi(t))(A_ie_1^T\\
&\quad+B_ie_{N+3}^T+e_{N+4}^T)-\sum_{i=1}^{r}h_i(\varphi(t))\sigma G_ie_1^T]\\
&\quad+e_1^Te_1-\delta^2e_{N+4}^Te_{N+4}\\
\Theta&=diag\{P_2+Q_1,-Q_1+Q_2,\ldots,\\
&\quad-Q_{N-1}+Q_N,-Q_N-P_2,\overbrace{0,\cdots,0}^{4N+5}\},\\
N_j&=[N_{j,1},\ldots,N_{j,N}],\\
\hat{R}&=diag\{\hat{R}_1,\ldots,\hat{R}_N\},\\
\hat{P}_3&=diag\{P_3,3P_3,5P_3\},\\
\hat{R}_k&=diag\{R_k,3R_k,5R_k\},\\
\sigma&=\beta_1e_1+\beta_2e_{N+2},\\
\Lambda&=\begin{bmatrix}e_1-e_{N+1}\\ e_1+e_{N+1}-2e_{3N+5}\\ e_1-e_{N+1}-6e_{5N+6}\end{bmatrix},\\
\Lambda_k&=\begin{bmatrix}\Lambda_{k,1}\\ \Lambda_{k,2}\end{bmatrix},\\
\Lambda_{k,1}&=\begin{bmatrix}e_k-e_{N+3}\\ e_k+e_{N+3}-2e_{N+4+k}\\ e_k-e_{N+3}-6e_{3N+5+k}\end{bmatrix},\\
\Lambda_{k,2}&=\begin{bmatrix}e_{N+3}-e_{k+1}\\ e_{N+3}+e_{k+1}-2e_{2N+4+k}\\ e_{N+3}-e_{k+1}-6e_{4N+5+k}\end{bmatrix},
\end{aligned} \tag{10}$$

The error system (6) is asymptotically stable, the control gain can be calculated by $K_i=X^{-1}G_i$.

Proof: We select Lyapunov function as

$$V(t)=\sum_{i=1}^{3}V_i(t) \tag{11}$$

where

$$V_1(t) = e^T(t)P_1e(t) \tag{12}$$

$$V_2(t) = \sum_{k=1}^{N}\int_{t-\tau_k}^{t-\tau_{k-1}} e^T(\nu)Q_ke(\nu)d\nu + \int_{t-\tau_M}^{t} e^T(\nu)P_2e(\nu)d\nu \tag{13}$$

$$V_3(t) = \sum_{k=1}^{N}\bar{\tau}\int_{-\tau_k}^{-\tau_{k-1}}\int_{t+\nu}^{t} \dot{e}^T(\varphi)R_k\dot{e}(\varphi)d\varphi d\nu + \tau_M\int_{-\tau_M}^{0}\int_{t+\nu}^{t} \dot{e}^T(\varphi)P_3\dot{e}(\varphi)d\varphi d\nu \tag{14}$$

Let us denote:

$$\begin{aligned}
\xi^T &= [e(t), u_1, \dot{e}(t), e(t-\tau(t)), \omega(t), u_2, \\
&\quad u_3, u_4, u_5, u_6, u_7] \\
u_1 &= [e(t-\tau_1), \ldots, e(t-\tau_N)] \\
u_2 &= [\frac{1}{\tau(t)-\tau_0}\int_{t-\tau(t)}^{t-\tau_0} e^T(\nu)d\nu, \ldots, \\
&\quad \frac{1}{\tau(t)-\tau_{N-1}}\int_{t-\tau(t)}^{t-\tau_{N-1}} e^T(\nu)d\nu] \\
u_3 &= [\frac{1}{\tau_1-\tau(t)}\int_{t-\tau_1}^{t-\tau(t)} e^T(\nu)d\nu, \ldots, \\
&\quad \frac{1}{\tau_N-\tau(t)}\int_{t-\tau_N}^{t-\tau(t)} e^T(\nu)d\nu] \\
u_4 &= \frac{1}{\tau_M}\int_{t-\tau_M}^{t} e^T(\nu)d\nu \\
u_5 &= [\frac{1}{\tau(t)-\tau_0}\int_{t-\tau(t)}^{t-\tau_0} \varrho_{1,1}e^T(\nu)d\nu, \ldots, \\
&\quad \frac{1}{\tau(t)-\tau_{N-1}}\int_{t-\tau(t)}^{t-\tau_{N-1}} \varrho_{N,1}e^T(\nu)d\nu] \\
u_6 &= [\frac{1}{\tau_1-\tau(t)}\int_{t-\tau_1}^{t-\tau(t)} \varrho_{1,2}e^T(\nu)d\nu, \ldots, \\
&\quad \frac{1}{\tau_N-\tau(t)}\int_{t-\tau_N}^{t-\tau(t)} \varrho_{N,2}e^T(\nu)d\nu] \\
u_7 &= \frac{1}{\tau_M}\int_{t-\tau_M}^{t} \varrho_M e^T(\nu)d\nu \\
\varrho_{k,1} &= 2(\frac{\nu-(t-\tau(t))}{\tau(t)-\tau_{k-1}})-1, \quad (k=1,\ldots,N) \\
\varrho_{k,2} &= 2(\frac{\nu-(t-\tau_k)}{\tau_k-\tau(t)})-1, \quad (k=1,\ldots,N) \\
\varrho_M &= 2(\frac{\nu-(t-\tau_M)}{\tau_M})-1
\end{aligned} \tag{15}$$

Taking the derivative of $V(t)$, we can get:

$$\dot{V}(t) = \sum_{i=1}^{3}\dot{V}_i \tag{16}$$

where

$$\begin{aligned}
\dot{V}_1(t) &= 2e^T(t)P_1\dot{e}(t) \\
&= 2\xi^T(t)e_1^TP_1e_{N+2}\xi(t)
\end{aligned} \tag{17}$$

$$\begin{aligned}
\dot{V}_2(t) &= \sum_{k=1}^{N}[e^T(t-\tau_{k-1})Q_ke(t-\tau_{k-1}) \\
&\quad -e^T(t-\tau_k)Q_ke(t-\tau_k)] + e^T(t)P_2e(t) \\
&\quad -e^T(t-\tau_M)P_2e(t-\tau_M) \\
&= \xi^T(t)\Theta\xi(t)
\end{aligned} \tag{18}$$

$$\begin{aligned}
\dot{V}_3(t) &= \sum_{k=1}^{N}[\bar{\tau}^2\dot{e}^T(t)R_k\dot{e}(t) \\
&\quad -\bar{\tau}\int_{t-\tau_k}^{t-\tau_{k-1}} \dot{e}^T(\varphi)R_k\dot{e}(\varphi)d\varphi] + \tau_M^2\dot{e}^T(t)P_3\dot{e}(t) \\
&\quad -\tau_M\int_{t-\tau_M}^{t} \dot{e}^T(\varphi)P_3\dot{e}(\varphi)d\varphi \\
&= \xi^T(t)e_{N+2}^T(\sum_{k=1}^{N}\bar{\tau}^2R_k + \tau_M^2P_3)e_{N+2}\xi(t) \\
&\quad -\sum_{k=1}^{N}\bar{\tau}\int_{t-\tau_k}^{t-\tau_{k-1}} \dot{e}^T(\varphi)R_k\dot{e}(\varphi)d\varphi \\
&\quad -\tau_M\int_{t-\tau_M}^{t} \dot{e}^T(\varphi)P_3\dot{e}(\varphi)d\varphi
\end{aligned} \tag{19}$$

Using Lemma 1 to the integral term in (19), we can get:

$$\begin{aligned}
&-\tau_M\int_{t-\tau_M}^{t} \dot{e}^T(\varphi)P_3\dot{e}(\varphi)d\varphi \\
&\quad \leq -\xi^T(t)\Lambda^T\hat{P}_3\Lambda\xi(t)
\end{aligned} \tag{20}$$

$$\begin{aligned}
&\quad -\sum_{k=1}^{N}\bar{\tau}\int_{t-\tau_k}^{t-\tau_{k-1}} \dot{e}^T(\varphi)R_k\dot{e}(\varphi)d\varphi \\
&\quad = -\sum_{k=1}^{N}(\bar{\tau}\int_{t-\tau(t)}^{t-\tau_{k-1}} \dot{e}^T(\varphi)R_k\dot{e}(\varphi)d\varphi \\
&\qquad +\bar{\tau}\int_{t-\tau_k}^{t-\tau(t)} \dot{e}^T(\varphi)R_k\dot{e}(\varphi)d\varphi) \\
&\quad \leqslant -\sum_{k=1}^{N}(\frac{\bar{\tau}}{\tau(t)-\tau_{k-1}}\xi^T(t)\Lambda_{k,1}^T\hat{R}_k\Lambda_{k,1}\xi(t) \\
&\qquad +\frac{\bar{\tau}}{\tau_k-\tau(t)}\xi^T(t)\Lambda_{k,2}^T\hat{R}_k\Lambda_{k,2}\xi(t)) \\
&\quad = -\sum_{k=1}^{N}\xi^T(t)\Lambda_k^T\begin{bmatrix} \frac{1}{\alpha}\hat{R}_k & 0 \\ * & \frac{1}{1-\alpha}\hat{R}_k \end{bmatrix}\Lambda_k\xi(t)
\end{aligned} \tag{21}$$

where

$$\Lambda_k = \begin{bmatrix} \Lambda_{k,} & 1 \\ \Lambda_{k,} & 2 \end{bmatrix}, \quad \alpha_k = \frac{\tau(t)-\tau_{k-1}}{\bar{\tau}}$$

According to (6), for any scalars β_1, β_2, and any appropriate dimensional matrix X, we can get

$$\begin{aligned}
&2[\beta_1e^T(t) + \beta_2\dot{e}^T(t)]X[-\dot{e}(t) + \sum_{i=1}^{r}h_i(\varphi(t))(A_ie(t) \\
&\qquad + B_ie(t-\tau(t)) + \omega(t) - K_ie(t)] = 0
\end{aligned} \tag{22}$$

which can be written as

$$2\xi^T(t)[\sigma X(-e_{N+2}^T+\sum_{i=1}^{r}h_i(\varphi(t))(A_ie_1^T+B_ie_{N+3}^T+e_{N+4}^T) - \sum_{i=1}^{r}h_i(\varphi(t))\sigma G_ie_1^T]\xi(t)=0 \quad (23)$$

where $G_i = XK_i$, $\sigma = \beta_1 e_1 + \beta_2 e_{N+2}$.

Combining(17)-(23),we can get the following inequality:

$$\begin{aligned}\dot{V}(t) \le\ & \xi^T(t)(2e_1^TP_1e_{N+2}+\Theta \\ & + e_{N+2}^T(\sum_{k=1}^{N}\bar{\tau}^2R_k+\tau_M^2P_3)e_{N+2}-\Lambda^T\hat{P}_3\Lambda \\ & + 2[\sigma X(-e_{N+2}^T+\sum_{i=1}^{r}h_i(\varphi(t))(A_ie_1^T \\ & + B_ie_{N+3}^T+e_{N+4}^T)-\sum_{i=1}^{r}h_i(\varphi(t))\sigma G_ie_1^T] \\ & - \sum_{k=1}^{N}\Lambda_k^T\begin{bmatrix}\frac{1}{\alpha}\hat{R}_k & 0\\ * & \frac{1}{1-\alpha}\hat{R}_k\end{bmatrix}\Lambda_k)\xi(t) \quad (24)\end{aligned}$$

According to Assumption 1, in order to achieve the H_∞ performance, we define

$$J(e(t),\omega(t)) = \int_0^\infty [e^T(t)e(t)-\delta^2\omega^T(t)\omega(t)]dt. \quad (25)$$

Based on the zero initial condition, we can get

$$\begin{aligned}J(e(t),\omega(t)) =\ & \int_0^\infty [e^T(t)e(t)-\delta^2\omega^T(t)\omega(t) \\ & +\dot{V}(t)]dt - V(t)|_{t\to\infty} \\ \le\ & \int_0^\infty [e^T(t)e(t)-\delta^2\omega^T(t)\omega(t)+\dot{V}(t)]dt \\ =\ & \int_0^\infty \xi^T(t)(\Phi+\Pi)\xi(t)dt \quad (26)\end{aligned}$$

where

$$\begin{aligned}\Phi =\ & 2e_1^TP_1e_{N+2}+\Theta \\ & + e_{N+2}^T(\sum_{k=1}^{N}\bar{\tau}^2R_k+\tau_M^2P_3)e_{N+2}-\Lambda^T\hat{P}_3\Lambda \\ & + 2[\sigma X(-e_{N+2}^T+\sum_{i=1}^{r}h_i(\varphi(t))(A_ie_1^T \\ & + B_ie_{N+3}^T+e_{N+4}^T)-\sum_{i=1}^{r}h_i(\varphi(t))\sigma G_ie_1^T] \\ & + e_1^Te_1-\delta^2e_{N+4}^Te_{N+4}\end{aligned}$$

$$\Pi = -\sum_{k=1}^{N}\Lambda_k^T\begin{bmatrix}\frac{1}{\alpha_k}\hat{R}_k & 0\\ * & \frac{1}{1-\alpha_k}\hat{R}_k\end{bmatrix}\Lambda_k$$

From Lemma 2, we can get the following inequality

$$-\sum_{k=1}^{N}\xi^T(t)\Lambda_k^T\begin{bmatrix}\frac{1}{\alpha_k}\hat{R}_k & 0\\ * & \frac{1}{1-\alpha_k}\hat{R}_k\end{bmatrix}\Lambda_k\xi(t) \le -\sum_{k=1}^{N}\xi^T(t)\Lambda_k^T\Upsilon_k\Lambda_k\xi(t) \quad (27)$$

where

$$\Upsilon_k = He([M_{1,k}\quad M_{2,k}])-\alpha_kM_{1,k}\hat{R}_k^{-1}M_{1,k}^T - (1-\alpha_k)M_{2,k}\hat{R}_k^{-1}M_{2,k}^T$$

We define

$$\begin{aligned}\Psi =\ & \Phi-\sum_{k=1}^{N}He(\Lambda_k^T[M_{1,k}M_{2,k}]\Lambda_k) \\ & +\alpha_k\Lambda_k^TM_{1,k}\hat{R}_k^{-1}M_{1,k}^T\Lambda_k \\ & +(1-\alpha_k)\Lambda_k^TM_{2,k}\hat{R}_k^{-1}M_{2,k}^T\Lambda_k. \quad (28)\end{aligned}$$

Denote $\Lambda_k^TM_{1,k} = N_{2,k}$ and $\Lambda_k^TM_{2,k} = N_{1,k}$, we can get

$$\begin{aligned}\Psi =\ & \Xi+\sum_{k=1}^{N}\alpha_kN_{2,k}\hat{R}_k^{-1}N_{2,k}^T \\ & +\sum_{k=1}^{N}(1-\alpha_k)N_{1,k}\hat{R}_k^{-1}N_{1,k}^T \\ =\ & \alpha_k(\Xi+\sum_{k=1}^{N}N_{2,k}\hat{R}_k^{-1}N_{2,k}^T) \\ & +(1-\alpha_k)(\Xi+\sum_{k=1}^{N}N_{1,k}\hat{R}_k^{-1}N_{1,k}^T), \quad (29)\end{aligned}$$

where

$$\Xi = \Phi-\sum_{k=1}^{N}He([N_{2,k}N_{1,k}]\Lambda_k).$$

According to (29), when $\Xi+\sum_{k=1}^{N}N_{2,k}\hat{R}_k^{-1}N_{2,k}^T < 0$ and $\Xi+\sum_{k=1}^{N}N_{1,k}\hat{R}_k^{-1}N_{1,k}^T < 0$, we get $\Psi < 0$. Applying Schur complement, the following inequality can be obtained

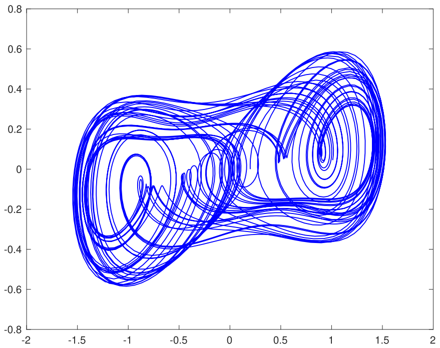
$$\begin{bmatrix}\Xi & N_1\\ N_1^T & -\hat{R}\end{bmatrix} < 0, \quad (30)$$

$$\begin{bmatrix}\Xi & N_2\\ N_2^T & -\hat{R}\end{bmatrix} < 0, \quad (31)$$

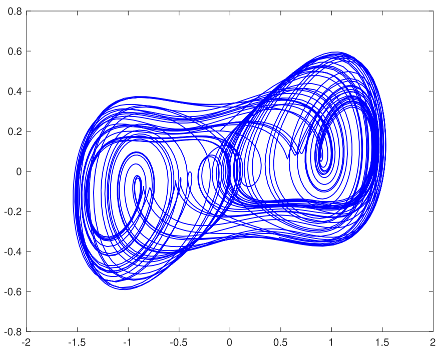
where

$$\begin{aligned}N_1 &= [N_{1,1},\ldots,N_{1,N}],\\ N_2 &= [N_{2,1},\ldots,N_{2,N}],\\ \hat{R} &= diag\{\hat{R}_1,\ldots,\hat{R}_N\}.\end{aligned}$$

When (30) and (31) are satisfied, the error converges to zero with the controller $K_i = X^{-1}G_i$.



(a) The phase diagram of the master system



(b) The phase diagram of the slave system

FIGURE 1. **The phase diagrams of the master and slave system without the control input.**

IV. SIMULATION

In this section, we demonstrate the effectiveness of our proposed controller by the following two examples.

Example 1: Considering FMBCAS (3) and (5) with time-varying delays and disturbances, the system parameters are as follows:

$$A_1 = \begin{bmatrix} -0.15 & 1.7 \\ -49.425 & -0.35 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -0.15 & 1.7 \\ 0.575 & -0.35 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0 & 0 \\ 0.01 & 0.01 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 & 0 \\ 0.01 & 0.01 \end{bmatrix}$$

The membership value of the FMBCAS (3) and (5) is

$$h_1(\varphi(t)) = \frac{\varphi^2(t)}{100},$$
$$h_2(\varphi(t)) = 1 - \frac{\varphi^2(t)}{100}, \tag{32}$$

Initial conditions value are $x_m(0) = (0.2, 0), x_s(0) = (-0.1, 0.2)$. The phase diagrams of the master-slave system without the control strategy are shown in Fig 1. The trajectory of the error system without the control input is shown in Fig 2. It is obvious that the slave system is not synchronized to master system.

Choose $\tau(t) = 0.3+0.1sin(t)$, $\tau_M = 0.6$, $\delta = 0.6$, $\beta_1 = 1$, $\beta_2 = 1, N = 2$ and $\zeta_i = 0.3\cos(t)$, applying Theorem 1,

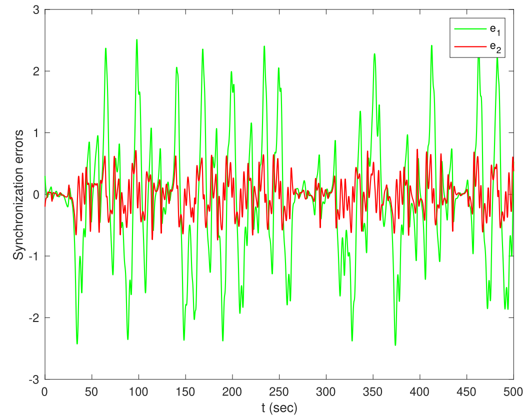
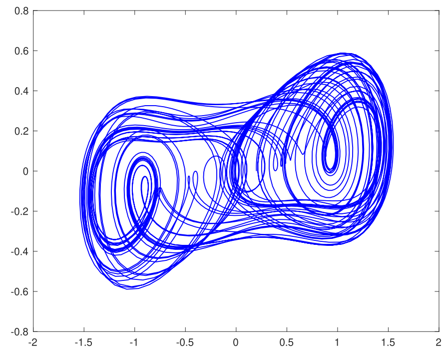
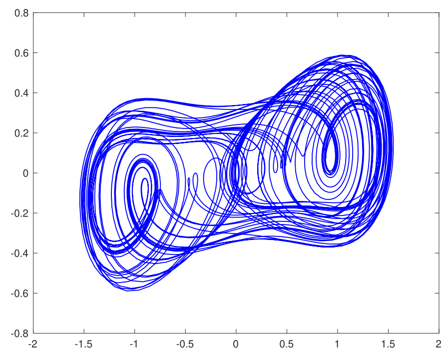


FIGURE 2. **The trajectory of the error system without the control input.**



(a) The phase diagram of the master system



(b) The phase diagram of the slave system

FIGURE 3. **The phase diagrams of the master and slave system with the controller.**

the state feedback control gains are

$$K_1 = \begin{bmatrix} 8.9404 & 1.6967 \\ -49.4119 & 8.6873 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} 8.8745 & 1.6967 \\ 0.5880 & 8.6873 \end{bmatrix},$$

Under the controller K_1 and K_2, the slave system and master system is synchronized. It is shown in Fig 3. The error system with the controller is displayed in Fig 4. From Fig 4, we can find that with the time changes, the error system converges to zero under the controller.

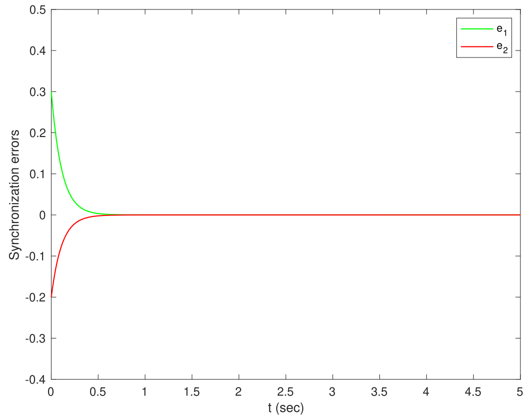
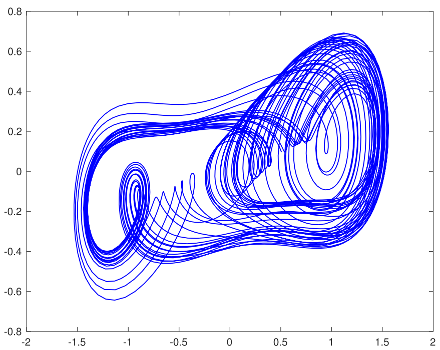
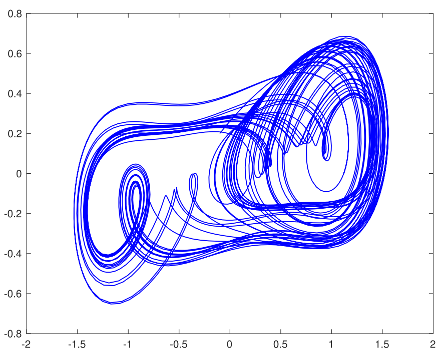


FIGURE 4. **The trajectory of the error system with the controller.**



(a) The phase diagram of the master system



(b) The phase diagram of the slave system

FIGURE 5. **The phase diagrams of the master and slave system without the control input.**

Example 2: Considering FMBCAS (3) and (5) with time-varying delays and disturbances, the system parameters are as follows:

$$A_1 = \begin{bmatrix} -0.2 & 1.5 \\ -49.425 & -0.25 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -0.2 & 1.5 \\ 0.575 & -0.25 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0 & 0 \\ 0.03 & 0.03 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 & 0 \\ 0.03 & 0.03 \end{bmatrix}$$

The membership values, initial conditions and other parameters are the same as Example 1.

The phase diagrams of the master-slave system without the control strategy are shown in Fig 5. The trajectory of the error system without the control input is shown in Fig 6. It is obvious that the slave system is not synchronized to master system. Applying Theorem 1, the state feedback control gains are

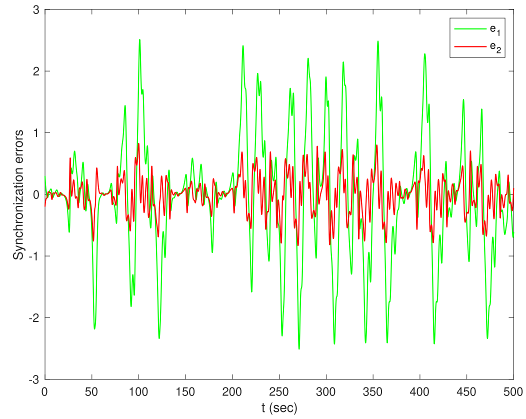
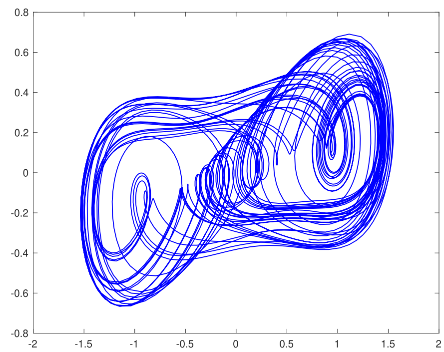
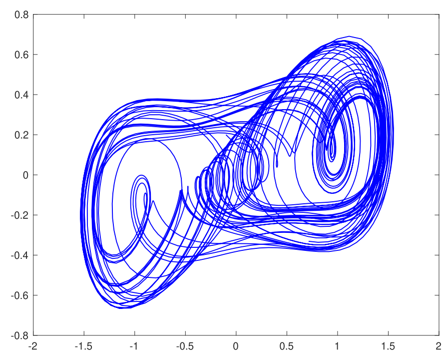


FIGURE 6. **The trajectory of the error system without the control input.**



(a) The phase diagram of the master system



(b) The phase diagram of the slave system

FIGURE 7. **The phase diagrams of the master and slave system with the controller.**

$$K_3 = \begin{bmatrix} 9.0204 & 1.4898 \\ -49.3855 & 8.8119 \end{bmatrix},$$
$$K_4 = \begin{bmatrix} 8.8228 & 1.4898 \\ 0.6139 & 8.8119 \end{bmatrix},$$

Under the controller K_3 and K_4, the slave system and master system is synchronized. It is shown in Fig 7. The error system

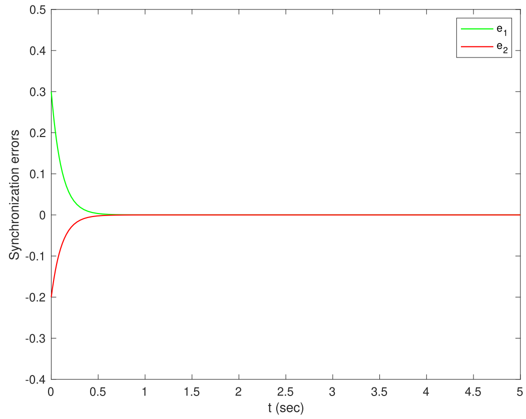


FIGURE 8. The trajectory of the error system with the controller.

with the controller is displayed in Fig 8. From Fig 8, we can find that with the time changes, the error system converges to zero under the controller.

The above results show that our control strategy has been successfully applied in the synchronization of CAS.

V. CONCLUSION

The paper addresses the synchronization for the healthy and diseased CAS with the time-delay based on T-S fuzzy model. By constructing a fuzzy dynamic equation, the model is closer to the real CAS. By choosing the appropriate LKF and combining delay-partitioning method, Bessel-Legendre integral inequality, the Moon's et al. inequality with convex analysis, a new synchronization controller is obtained. The numerical simulations of CAS elaborate the effectiveness of the presented synchronization control approach. Recently, interval type-2 fuzzy control has been studied extensively [43]. In the near future, the research on synchronization of CAS based on interval type-2 fuzzy model will be considered.

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