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Trajectory Tracking With Constrained Sensors and Unreliable Communication Networks

CHANG LIU, FEN LIU, HONGXIA RAO, AND YONG XU

Guangdong Provincial Key Laboratory of Intelligent Decision and Cooperative Control, School of Automation, Guangdong University of Technology, Guangzhou 510006, China

Corresponding author: Hongxia Rao (raohxia@163.com)

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ABSTRACT This work investigates the remote vehicle tracking issue over constrained monitoring sensors and unreliable communication networks. A saturation function is used to describe the bounded time varying acceleration of the vehicle. A set of matrices are introduced to model the sensor monitoring conditions called captured states (CSs), and a Markov chain with time varying and partially unknown transition probability (TVPUTP) is proposed to analyze the conditions of the CSs. Then, a CS dependent nonfragile estimator is designed based on the measured unreliable vehicle information, and the estimation error system (EES) is derived. Two theorems are established to ensure that the EES satisfies the finite-horizon (FH) H_∞ performance. Finally, an example is introduced to show the effectiveness of the results.

INDEX TERMS Trajectory tracking, constrained sensors, Markov chain, finite-horizon H_∞ performance.

I. INTRODUCTION

Vehicle intelligence is a main trend in both automotive and transportation fields, and the research of intelligent vehicle mainly focuses on improving the safety, as well as providing excellent human-vehicle interface, and so on [1], [2]. This field attracts a lot of attention from researchers, and many theories have been put forward. In [3], a survey of the frameworks of traffic control and intelligent vehicle highway systems, and some outstanding issues and future challenges were identified. In [4], based on the chicken-game-theory algorithm, the researchers proposed a Cooperative Adaptive Cruise Control systems to control vehicle movements at uncontrolled intersections. In [5], according to the special radio frequency identification, a smart traffic control system was presented for special vehicle control. However, the vehicle states are indispensable for control and safety, therefore how to estimate the vehicle's state (i.e., the tracking problem) becomes critical.

In general circumstance, tracking for a vehicle needs many monitoring sensors when it is running on a level road [6], [7]. Under the practical circumstance, it is impossible and unrealistic that the monitoring sensors monitor the moving vehicle ideally at all times, so the information captured by the monitoring sensors may be incomplete or missing [8], [9]. Some theories were proposed for vehicle tracking based on the obtained information by the sensor [10], [11]. According to the extended Kalman filter, a unique approach for vehicle tracking by simultaneous detection and estimation was designed to efficiently integrate all available information into the tracking [12]. A new technique about nonlinear observer design was proposed by utilizing better bounds on nonlinear functions in the dynamics for this challenge [13]. Considering a vehicle with partial observations and varying viewpoints, the authors proposed a part-based particle filter for vehicle tracking [14]. In some works, the actual measurement cases of the monitoring sensor was modeled by random Bernoulli process [15] and Markov model [16]–[21]. Inspired by these works, how to study a more general Markov model is an interesting and important problem, which has not been fully studied.

For wireless transmission networks, with the increasing of the amount of data transmitted, packet dropouts and time delays are inevitable in the case of limited communication channel capacity [22]–[24]. How to study networked control systems has become an important problem [25]–[28]. In [29], considering the unreliable communication link, a stochastic Bernoulli variable was proposed to describe the packet

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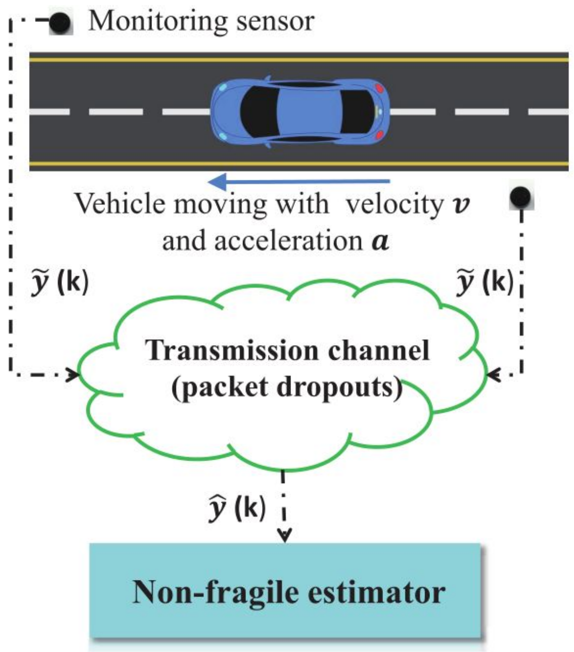

FIGURE 1. The simplified system structure diagram for vehicle tracking.

dropouts. In [30], a finite-state Markov process was used to describe the packet dropouts, and time varying Kalman filter was proposed to analyse the peak covariance stability. In [17], packet dropouts, time delays and sensor nonlinearity caused by unreliable communication link were simultaneously considered. In [31], an adaptive transmission policy was designed for the wireless fading channels with the delay-sensitive and bursty traffic source. Therefore, it is difficult and necessary for researchers to design a reasonable transmission model, which is worth further studying.

This work addresses the issue of vehicle tracking over limited monitoring sensors and unreliable transmission channels. The simplified system structure diagram is shown in FIGURE 1. An estimator is designed based on the available data, and two results are obtained to ensure that the estimation error system (EES) achieves the finite-horizon (FH) H_∞ performance. Finally, a numerical example is introduced to show the effectiveness of the derived results. This paper possesses the following contributions:

- With the purpose of establish a more general model of the vehicle acceleration, a saturation function $\sigma(\cdot)$ is used to describe a bounded vehicle acceleration with a time varying change rate $\rho(k)$.
- The CSs are introduced to model the sensor monitoring condition. A Markov chain with time varying and partially unknown transition probability TVPUTP) is proposed to describe the variation of these CSs, which is a more general model than [34], [35].
- A CS dependent non-fragile estimator is designed on the basis of the unreliable measurements to improve the robustness of the estimator. Besides, the FH H_∞ performance of the EES is analyzed.

The organization of the paper is as follows. A moving vehicle with limited monitoring sensors and unreliable communication networks are described in Section II. In Section III, sufficient conditions of the FH H_∞ performance are given. In Section IV, the CS dependent non-fragile estimator gain design is shown. In Section V, an example is described, and conclusions are given in Section VI.

Notations: The symbols $\mathbf{R}^n$ and $\mathbf{R}^{m\times n}$ stand for n-dimensional vector and $m \times n$ real matrix, respectively. The symbol X^T means the transpose of the matrix X, $diag_n\{\cdot\}$ denotes the diagonal matrix belonging to $\mathbf{R}^{n\times n}$. $P_\iota(k)_{[0,N]}$ denotes the matrix $P_\iota(k)$, $k \in [0, N]$. $\mathbb{E}\{\cdot\}$ means the expectation of a stochastic variable.

II. PROBLEM FORMULATION AND PRELIMINARIES

A. VEHICLE MOVING DESCRIPTION

A vehicle moving with acceleration $\boldsymbol{a}$ on a level road is considered, where the acceleration $\boldsymbol{a}$ satisfies the following equilibrium equations [32]:

$$\begin{cases} 2(\mathcal{F}_{x_1} + \mathcal{F}_{x_2}) = \mathrm{m}\boldsymbol{a} \\ 2(\mathcal{F}_{z_1} + \mathcal{F}_{z_2}) - \mathrm{m}\boldsymbol{g} = 0 \\ -2\mathcal{F}_{z_1}a_1 + 2\mathcal{F}_{z_2}a_2 - 2(\mathcal{F}_{x_1} + \mathcal{F}_{x_2})h = 0 \end{cases} \tag{1}$$

where parameters $\mathcal{F}_{x_1}$ and $\mathcal{F}_{x_2}$ respectively represent the braking force on the front wheels and rear wheels of a four-wheel-drive. The parameters $\mathcal{F}_{z_1}$ and $\mathcal{F}_{z_2}$ are the vertical forces under the front and rear wheels, respectively. The constant vector $\boldsymbol{g}$ is the gravitational acceleration. The constant scalar a_1 is the length between the vehicle's mass center and front axle, a_2 is the length between the vehicle's mass center and the rear axle, and h is the height of the vehicle's mass center.

Considering the all-wheel drive vehicle, and assuming that the friction coefficients at the tires are equal, and all tires reach their maximum traction at the same instant,

$$\begin{aligned} \mathcal{F}_{x_1} &= \pm\mu\mathcal{F}_{z_1} \\ \mathcal{F}_{x_2} &= \pm\mu\mathcal{F}_{z_2} \end{aligned} \tag{2}$$

where μ stands for the friction coefficient.

In term of the equation (1), the maximum acceleration $\boldsymbol{a}_{max}$ is

$$\boldsymbol{a}_{max} = \pm\mu\boldsymbol{g}. \tag{3}$$

When the vehicle runs in a complex environment, the acceleration may change within the bound $\mathcal{C} \triangleq [-\mu\boldsymbol{g}, +\mu\boldsymbol{g}]$, and the change rate of acceleration is $\rho(k) \in [-\rho_1, \rho_2]$, where ρ_1 and ρ_2 are positive numbers and dependent on the mechanical properties of the vehicle. A saturation function $\sigma(\rho(k)\boldsymbol{a}(k)) \in \mathcal{C}$ is introduced to describe the acceleration at the instant k and is defined as

$$\sigma(\rho(k)\boldsymbol{a}(k)) \triangleq \mathrm{sign}(\rho(k)\boldsymbol{a}(k))\min\{\|\rho(k)\boldsymbol{a}(k)\|, \mu\boldsymbol{g}\}.$$

The kinematics model for the moving vehicle on a level road is as follows,

$$\begin{cases} \boldsymbol{s}(k+1) = \boldsymbol{s}(k) + t_0\boldsymbol{v}(k) + \frac{t_0^2}{2}\boldsymbol{a}(k) \\ \boldsymbol{v}(k+1) = \boldsymbol{v}(k) + t_0\boldsymbol{a}(k) + \boldsymbol{\omega}(k) \\ \boldsymbol{a}(k+1) = \sigma(\rho(k)\boldsymbol{a}(k)) \end{cases} \tag{4}$$

where the variables $\boldsymbol{s}(k)$, $\boldsymbol{v}(k)$, $\boldsymbol{a}(k)$ denote the displacement, velocity and acceleration, respectively. The constant scalar t_0 is the sampling interval, the one-dimensional variable $\boldsymbol{\omega}(k)$ is the external noise.

Lemma 1 [33]: A nonlinear function $\psi(\mathrm{v})$ belongs to the sector $[\mathfrak{c}_1, \mathfrak{c}_2]$, if the following inequality:

$$(\psi(\mathrm{v}) - \mathfrak{c}_1\mathrm{v})(\psi(\mathrm{v}) - \mathfrak{c}_2\mathrm{v}) \le 0, \quad \forall \mathrm{v} \in R \tag{5}$$

holds for $\mathfrak{c}_1$ and $\mathfrak{c}_2$.

Based on the inequality (5) and the characteristic of saturation function, we assume that there exist scalars h_1 and h_2 such that $0 \le h_1 < 1 \le h_2$, the saturation function $\sigma(\rho(k)\boldsymbol{a}(k))$ is supposed to consist of a linear part and a nonlinear part as

$$\sigma(\rho(k)\boldsymbol{a}(k)) = h_1\rho(k)\boldsymbol{a}(k) + \psi(\rho(k)\boldsymbol{a}(k)) \tag{6}$$

where $\psi(\rho(k)\boldsymbol{a}(k))$ satisfies a sector condition with $\mathfrak{c}_1 = 0$ and $\mathfrak{c}_2 = h_2 - h_1$, i.e., $\psi(\rho(k)\boldsymbol{a}(k))$ meets the following inequality:

$$\psi(\rho(k)\boldsymbol{a}(k))(\psi(\rho(\mathrm{k})\boldsymbol{a}(\mathrm{k})) - \mathfrak{c}_2\rho(\mathrm{k})\boldsymbol{a}(\mathrm{k})) \le 0. \tag{7}$$

Define a state vector $\mathbf{x}(k) \triangleq [\boldsymbol{s}(k)\ \boldsymbol{v}(k)\ \boldsymbol{a}(k)]^T$ and an objective vector $\mathbf{z}(k)$, the time varying state space model can be obtained as the following formula:

$$\begin{cases} \mathbf{x}(k+1) = A(k)\mathbf{x}(k) + B\psi(\rho(k)H\mathbf{x}(k)) + E\boldsymbol{\omega}(k) \\ \mathbf{z}(k) = L\mathbf{x}(k) \end{cases} \tag{8}$$

where

$$A(k) = \begin{bmatrix} 1 & t_0 & \frac{t_0^2}{2} \\ 0 & 1 & t_0 \\ 0 & 0 & h_1\rho(k) \end{bmatrix}, \quad L = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}^T$$
$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad E = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, H = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}^T. \tag{9}$$

B. LIMITED MONITORING SENSORS

In order to improve the transportation efficiency, road safety, and intelligence, the first important thing is to obtain the moving states of the vehicle. Thus, when the vehicle runs on a road, the monitoring sensors are distributed on both sides of the road, whose measurement output $\mathbf{y}(k)$ is

$$\mathbf{y}(k) = C\mathbf{x}(k) + D\boldsymbol{v}(k) \tag{10}$$

where the variable $\boldsymbol{v}(k) \in \mathbb{R}^{m\times 1}$ is the external noise and the known matrices $C \in \mathbb{R}^{2\times 3}$ and $D \in \mathbb{R}^{2\times m}$.

In fact, sensors are often sparsely distributed, it is impossible and unrealistic for the monitoring sensors to monitor the moving vehicle at all times, in view of this, there are four cases that occur, which are described as follows.

1) The vehicle is missing for the monitoring sensors.
2) The vehicle is captured by the monitoring sensors, but only the displacement information is detected.
3) The vehicle is captured by the monitoring sensors, but only the velocity information is detected.
4) The vehicle is captured by the monitoring sensors, and both information are detected.

A Markov chain $\vartheta(k) \in S \triangleq \{1, 2, 3, 4\}$ with TVPUTP is introduced to describe the above cases, whose transition probability matrix is expressed as

$$\Pi(k) = \left[\ \pi_{\imath\jmath}(k)\ \right]_{4\times 4} \tag{11}$$

where some elements in matrix $\Pi(k)$ are unknown, i.e., denote the $S_K^\imath(k) \triangleq \{\jmath : \pi_{\imath\jmath}(k)$ is known$\}$ and $S_{uK}^\imath(k) \triangleq \{\jmath : \pi_{\imath\jmath}(k)$ is unknown$\}$, for any $\imath \in S$, $\sum_{\jmath=1}^{s} \pi_{\imath\jmath}(k) = \sum_{\jmath\in S_K^\imath(k)} \pi_{\imath\jmath}(k) + \sum_{\jmath\in S_{uK}^\imath(k)} \pi_{\imath\jmath}(k) = 1$. The monitoring measurement output of sensors (10) can be rewritten as

$$\widetilde{\mathbf{y}}(k) = M_{\vartheta(k)}\mathbf{y}(k) \tag{12}$$

where $M_{\vartheta(k)}$ are CS matrices, and

$$M_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad M_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$
$$M_3 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad M_4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{13}$$

Remark 1: The vehicle may be missed or captured by the limited monitoring sensors, which was described by random Bernoulli process [15], Markov model [16], [17]. In the actual situation, the probability is always not all available, and not only the cases occur where the target is presence/absence [16]. Therefore, in this work, we consider the Markov chain $\vartheta(k)$ with TVPUTP, which is a more general description than the partially unknown probability of transition Markov model [34], [35] and other existing ones.

With the wide application of remote state estimation, the issue that communication capacity constraints occur in the process of transmission is inevitable. In this paper, we aim at researching the problems of packet dropouts caused by the communication capacity constraint. In order to describe the packet dropouts, the received information by estimator is described as

$$\widehat{\mathbf{y}}(k) = \alpha_{\vartheta(k)}(k)\widetilde{\mathbf{y}}(k) \tag{14}$$

where $\alpha_{\vartheta(k)}(k) \in \{0, 1\}$, $\alpha_{\vartheta(k)}(k) = 1$ denotes the successful data transmission, otherwise, denotes the packet dropouts, and $\alpha_{\vartheta(k)}(k)$ satisfies the following conditions:

$$\begin{cases} \mathbb{E}\{\alpha_1(k) = 1\} = 0 \\ \mathbb{E}\{\alpha_i(k) = 1\} = \bar{\alpha}_i, \quad i = 2, 3, 4 \\ \mathbb{E}\{(\alpha_i(k) - \bar{\alpha}_i)^2\} = \bar{\alpha}_i(1 - \bar{\alpha}_i) = \varsigma_i^2. \end{cases} \tag{15}$$

Remark 2: The CS matrices $M_{\vartheta(k)}$ have four cases. It needs to be emphasized that the vehicle is missing for the monitoring sensors and doesn't need to transmit the zero data to remote estimator when $\vartheta(k) = 1$, in this case, we define $\alpha_1(k)$ satisfying $\mathbb{E}\{\alpha_1(k) = 1\} = 0$. In the other cases, considering the data of transmissions with limited bandwidth, packet dropout rates are closely related to transfer load, therefore, different transmission modes have different packet dropout rates.

C. NON-FRAGILE ESTIMATOR AND EES

A CS dependent non-fragile estimator is designed to estimate the states of the vehicle based on the received information, which is as follows,

$$\begin{cases} \hat{\mathbf{x}}(k+1) = A(k)\hat{\mathbf{x}}(k) + B\psi(\rho(k)H\hat{\mathbf{x}}(k)) \\ \qquad + \alpha_{\vartheta(k)}(k)\bar{K}_{\vartheta(k)}(k) \\ \qquad \times M_{\vartheta(k)}(\mathbf{y}(k) - C\hat{\mathbf{x}}(k)) \\ \hat{\mathbf{z}}(k) = L\hat{\mathbf{x}}(k) \end{cases} \tag{16}$$

where the vectors $\hat{\mathbf{x}}(k)$ and $\hat{\mathbf{z}}(k)$ are the estimations for $\mathbf{x}(k)$ and $\mathbf{z}(k)$, respectively.

The matrix $\bar{K}_{\vartheta(k)}(k) \triangleq K_{\vartheta(k)}(k) + \Delta K_{\vartheta(k)}(k) \in \mathbb{R}^{3\times 2}$, where $K_{\vartheta(k)}(k)$ is the CS dependent non-fragile gain, $\Delta K_{\vartheta(k)}(k)$ is the time varying matrix, which is assumed to be the following form:

$$\Delta K_{\vartheta(k)}(k) = F_{\vartheta(k)}\Lambda(k)N_{\vartheta(k)}$$

where $F_{\vartheta(k)} \in R^{3\times 2}$ and $N_{\vartheta(k)} \in R^{2\times 2}$ are known matrices. $\Lambda(k) \in R^{2\times 2}$ is unknown time varying matrix, and satisfies the inequality $\Lambda(k)^T\Lambda(k) < I$.

The estimation error is defined as $\mathbf{e}(k) \triangleq \mathbf{x}(k) - \hat{\mathbf{x}}(k)$, the noises are described as $\mathbf{v}(k) \triangleq \left[\mathbf{w}(k)^T \; \boldsymbol{\nu}(k)^T\right]^T$, and assume $\phi(\rho(k)H\hat{\mathbf{e}}(k)) \triangleq \psi(\rho(k)H\mathbf{x}(k)) - \psi(\rho(k)\, H\hat{\mathbf{x}}(k))$. In terms of (8) and (16), the time varying EES (17) can be obtained.

$$\begin{cases} \mathbf{e}(k+1) = \bar{A}(k)\mathbf{e}(k) + B\phi(\rho(k)H\hat{\mathbf{e}}(k)) + \bar{D}\mathbf{v}(k) \\ \bar{\mathbf{z}}(k) = L\mathbf{e}(k) \end{cases} \tag{17}$$

where

$$\begin{aligned} \bar{A}(k) &= A(k) - \bar{\alpha}_{\vartheta(k)}\bar{K}_{\vartheta(k)}(k)M_{\vartheta(k)}C \\ &\quad - (\alpha_{\vartheta(k)}(k) - \bar{\alpha}_{\vartheta(k)})\bar{K}_{\vartheta(k)}(k)M_{\vartheta(k)}C \\ \bar{D}(k) &= [E \quad -\bar{\alpha}_{\vartheta(k)}\bar{K}_{\vartheta(k)}(k)M_{\vartheta(k)}D \\ &\quad - (\alpha_{\vartheta(k)}(k) - \bar{\alpha}_{\vartheta(k)})\bar{K}_{\vartheta(k)}(k)M_{\vartheta(k)}D]. \end{aligned} \tag{18}$$

Definition 1 [36]: Given a scalar $\gamma > 0$ and a matrix $Q > 0$, if the inequality (19) holds for $k \in [0, N]$, then the time varying EES (17) satisfies the FH H_∞ performance.

$$\mathbb{E}\left\{\sum_{k=0}^{N}\|\bar{\mathbf{z}}(k)\|^2\right\} \le \gamma^2\left\{\sum_{k=0}^{N}\|\mathbf{v}(k)\|^2 + \mathbf{e}(0)^T Q\mathbf{e}(0)\right\}. \tag{19}$$

Lemma 2 [37]: For matrices $\Phi = \Phi^T \in R^{n\times n}$, $\Gamma_1 \in R^{n\times n}$, $\Gamma_2 \in R^{n\times n}$, and $\Lambda^T\Lambda \le I$. The inequality

$$\Phi + \Gamma_1\Lambda\Gamma_2 + (\Gamma_1\Lambda\Gamma_2)^T < 0$$

holds if and only if there exists a positive scalar $\epsilon > 0$ such that

$$\Phi + \epsilon\Gamma_1\Gamma_1^T + \epsilon^{-1}\Gamma_2^T\Gamma_2 < 0.$$

III. MAIN RESULTS

In this section, the FH H_∞ performance is analyzed for the EES (17) with TVPUTP.

Theorem 1: Given a scalar $\gamma > 0$, and a matrix $Q > 0$, the EES (17) with constrained sensors and unreliable communication networks meets the FH H_∞ performance with the initial condition $P_\iota(0) < \gamma^2 Q$, if there exist matrices $P_\iota(k)_{[0,N+1]} > 0$, $\tilde{K}_\iota(k)_{[0,N]}$, $\varepsilon_1 > 0$, such that the matrix inequalities (20) hold, for $\forall\iota \in S$, $k \in [0, N]$.

$$\begin{bmatrix} \Phi_{11} & -R_1 & 0 & \Phi_{14} & \Phi_{15} \\ * & -2\varepsilon_1 I & 0 & B^T & 0 \\ * & * & -\gamma^2 I & \Phi_{34} & \Phi_{35} \\ * & * & * & \Phi_{44} & 0 \\ * & * & * & * & \Phi_{55} \end{bmatrix} < 0 \tag{20}$$

where

$$\Phi_{11} = -P_\iota(k) + L^T L, \; \pi_{K,\iota}(k) = \sum_{\jmath\in S_K^\iota(k)} \pi_{\iota\jmath}(k)$$

$$\Phi_{14} = A(k)^T - \bar{\alpha}_\iota C^T M_\iota^T \bar{K}_\iota(k)^T$$

$$\Phi_{15} = \varsigma_\iota C^T M_\iota^T \bar{K}_\iota(k)^T, \quad \Phi_{35} = \varsigma_\iota[0 \;\; \bar{K}_\iota(k)M_\iota D]^T$$

$$\Phi_{34} = [E \;\; -\bar{\alpha}_\iota\bar{K}_\iota(k)M_\iota D]^T, \quad R_1 = -\varepsilon_1\mathfrak{c}_2 H^T\rho(k)$$

$$\Phi_{44} = \Phi_{55} = \begin{cases} -\hat{P}_\jmath^{-1}(k+1), & \jmath \in S_K^\iota(k) \\ -\check{P}_\jmath^{-1}(k+1), & \jmath \in S_{uK}^\iota(k) \end{cases}$$

$$\hat{P}_\jmath(k+1) = \pi_{K,\iota}(k)^{-1}\sum_{\jmath\in S_K^\iota(k)} \pi_{\iota\jmath}(k)P_\jmath(k+1)$$

$$\check{P}_\jmath(k+1) = P_\jmath(k+1), \quad \jmath \in S_{uK}^\iota(k).$$

Proof: The Lyapunov function as in (21) is considered,

$$V(k) = \mathbf{e}(k)^T P_{\vartheta(k)}(k)\mathbf{e}(k) \tag{21}$$

where $P_{\vartheta(k)}(k) > 0, \forall k \in [0, N]$.

Denote $\vartheta(k) \triangleq \iota$, $\vartheta(k+1) \triangleq \jmath$, and the difference of Lyapunov function in the mean sense is defined as

$$\Delta V(k) \triangleq \mathbb{E}\{V(k+1) - V(k)\}. \tag{22}$$

In term of Lemma 1, $\mathcal{C} \triangleq [-\mu g, +\mu g]$ and the inequality (7), the nonlinear function $\phi(\rho(k)H\mathbf{e}(k))$ satisfies the inequality (23) for any scalar $\varepsilon_1 > 0$,

$$2\varepsilon_1\phi(\rho(k)H\mathbf{e}(k))(\phi(\rho(k)H\mathbf{e}(k)) - \mathfrak{c}_2\rho(k)H\mathbf{e}(k)) \le 0 \tag{23}$$

which is equal to

$$\begin{bmatrix} \mathbf{e}(k) \\ \phi(\rho(k)H\mathbf{e}(k)) \end{bmatrix}^T \begin{bmatrix} 0 & R_1 \\ * & 2\varepsilon_1 I \end{bmatrix} \begin{bmatrix} \mathbf{e}(k) \\ \phi(\rho(k)H\mathbf{e}(k)) \end{bmatrix} \le 0. \tag{24}$$

Considering the inequality constraint (24) for the nonlinear part of saturation function and submitting the time varying

EES (17) into (22), we have

$$
\begin{aligned}
&\Delta V(k) \\
&\quad \le \mathbb{E}\Big\{\mathbf{e}(k)^T(A(k)^T-\bar{\alpha}_\iota C^T M_\iota^T \bar{K}_\iota^T(k)) \\
&\qquad \times \bar{P}_\iota(k+1)(A(k)-\bar{\alpha}_\iota \bar{K}_\iota(k)M_\iota C)\mathbf{e}(k) \\
&\qquad +(\alpha_\iota(k)-\bar{\alpha}_\iota)^2\mathbf{e}(k)^T C^T M_\iota^T \bar{K}_\iota(k)^T \\
&\qquad \times \bar{P}_\iota(k+1)\bar{K}_\iota(k)M_\iota C\mathbf{e}(k) \\
&\qquad +\phi^T(\rho(k)H\mathbf{e}(k))B^T\bar{P}_\iota(k+1)B\phi(\rho(k)H\mathbf{e}(k)) \\
&\qquad +\mathbf{v}^T(k)\check{D}(k)^T\bar{P}_\iota(k+1)\check{D}(k)\mathbf{v}(k) \\
&\qquad +(\alpha_\iota(k)-\bar{\alpha}_\iota)^2\mathbf{v}^T(k)\hat{D}(k)^T\bar{P}_\iota(k+1)\hat{D}(k)\mathbf{v}(k) \\
&\qquad +2\mathbf{e}(k)^T(A(k)^T-\bar{\alpha}_\iota C^T M_\iota^T \bar{K}_\iota(k)^T) \\
&\qquad \times \bar{P}_\iota(k+1)B\phi(\rho(k)H\mathbf{e}(k)) \\
&\qquad +2\mathbf{e}(k)^T(A(k)^T-\bar{\alpha}_\iota C^T M_\iota^T \bar{K}_\iota(k)^T \\
&\qquad \times \bar{P}_\iota(k+1)\check{D}(k)\mathbf{v}(k) \\
&\qquad +2(\alpha_\iota(k)-\bar{\alpha}_\iota)^2\mathbf{e}(k)^T C^T M_\iota^T \bar{K}_\iota(k)^T \\
&\qquad \times \bar{P}_\iota(k+1)\hat{D}(k)\mathbf{v}(k) \\
&\qquad +2\mathbf{v}^T(k)\check{D}(k)^T\bar{P}_\iota(k+1)B\phi(\rho(k)H\mathbf{e}(k)) \\
&\qquad -\mathbf{e}(k)^T P_\iota(k)\mathbf{e}(k)\Big\} \\
&\qquad -\begin{bmatrix}\mathbf{e}(k)\\ \phi(\rho(k)H\mathbf{e}(k))\end{bmatrix}^T \begin{bmatrix}0 & R_1\\ * & 2\varepsilon_1 I\end{bmatrix}\begin{bmatrix}\mathbf{e}(k)\\ \phi(\rho(k)H\mathbf{e}(k))\end{bmatrix} \\
&\quad =\pi_{K,\iota}(k)(\xi(k)^T\acute{\Psi}(k)\xi(k)) \\
&\qquad +\sum_{\jmath\in S_{uK}^\iota(k)}\pi_{\iota\jmath}(k)\xi(k)^T\grave{\Psi}(k)\xi(k)
\end{aligned}
\tag{25}
$$

where

$$
\begin{aligned}
\bar{P}_\iota(k+1) &= \sum_{\jmath\in S_K^\iota(k)}\pi_{\iota\jmath}(k)P_\jmath(k+1) \\
&\quad +\sum_{\jmath\in S_{uK}^\iota(k)}\pi_{\iota\jmath}(k)\check{P}_\jmath(k+1) \\
\check{D}(k) &= [E \quad -\bar{\alpha}_\iota\bar{K}_\iota(k)M_\iota D] \\
\hat{D}(k) &= [0 \quad \bar{K}_\iota(k)M_\iota D] \\
\xi(k) &= [\mathbf{e}(k)^T \ \ \phi(\rho(k)H\mathbf{e}(k))^T \ \ \mathbf{v}(k)^T]^T \\
\acute{\Psi}(k) &= \begin{bmatrix}\acute{\Psi}_{11} & \acute{\Psi}_{12} & \acute{\Psi}_{13}\\ * & \acute{\Psi}_{22} & \acute{\Psi}_{23}\\ * & * & \acute{\Psi}_{33}\end{bmatrix} \\
\grave{\Psi}(k) &= \begin{bmatrix}\grave{\Psi}_{11} & \grave{\Psi}_{12} & \grave{\Psi}_{13}\\ * & \grave{\Psi}_{22} & \grave{\Psi}_{23}\\ * & * & \grave{\Psi}_{33}\end{bmatrix}
\end{aligned}
\tag{26}
$$

with

$$
\begin{aligned}
\acute{\Psi}_{11} &= (A(k)^T-\bar{\alpha}_\iota C^T M_\iota^T\bar{K}_\iota^T(k))\hat{P}_\jmath(k+1) \\
&\quad \times (A(k)-\bar{\alpha}_\iota\bar{K}_\iota(k)M_\iota C) \\
&\quad +\varsigma_\iota^2 C^T M_\iota^T\bar{K}_\iota(k)^T\hat{P}_\jmath(k+1) \\
&\quad \times \bar{K}_\iota(k)M_\iota C-P_\iota(k) \\
\acute{\Psi}_{12} &= (A(k)^T-\bar{\alpha}_\iota C^T M_\iota^T\bar{K}_\iota(k)^T)\hat{P}_\jmath(k+1)B-R_1 \\
\acute{\Psi}_{13} &= (A(k)^T-\bar{\alpha}_\iota C^T M_\iota^T\bar{K}_\iota(k)^T)\hat{P}_\jmath(k+1)\check{D}(k) \\
&\quad +\varsigma_\iota^2 C^T M_\iota^T\bar{K}_\iota(k)^T\hat{P}_\jmath(k+1)\hat{D}(k) \\
\acute{\Psi}_{22} &= B^T\hat{P}_\jmath(k+1)B-2\varepsilon_1 I \\
\acute{\Psi}_{23} &= B^T\hat{P}_\jmath(k+1)\check{D}(k) \\
\acute{\Psi}_{33} &= \check{D}(k)^T\hat{P}_\jmath(k+1)\check{D}(k)+\varsigma_\iota^2\hat{D}(k)^T \\
&\quad \times \hat{P}_\jmath(k+1)\hat{D}(k) \\
\grave{\Psi}_{11} &= (A(k)^T-\bar{\alpha}_\iota C^T M_\iota^T\bar{K}_\iota^T(k))\check{P}_\jmath(k+1) \\
&\quad \times (A(k)-\bar{\alpha}_\iota\bar{K}_\iota(k)M_\iota C) \\
&\quad +\varsigma_\iota^2 C^T M_\iota^T\bar{K}_\iota(k)^T\check{P}_\jmath(k+1) \\
&\quad \times \bar{K}_\iota(k)M_\iota C-P_\iota(k) \\
\grave{\Psi}_{12} &= (A(k)^T-\bar{\alpha}_\iota C^T M_\iota^T\bar{K}_\iota(k)^T)\check{P}_\jmath(k+1)B-R_1 \\
\grave{\Psi}_{13} &= (A(k)^T-\bar{\alpha}_\iota C^T M_\iota^T\bar{K}_\iota(k)^T)\check{P}_\jmath(k+1)\check{D}(k) \\
&\quad +\varsigma_\iota^2 C^T M_\iota^T\bar{K}_\iota(k)^T\check{P}_\jmath(k+1)\hat{D}(k) \\
\grave{\Psi}_{22} &= B^T\check{P}_\jmath(k+1)B-2\varepsilon_1 I \\
\grave{\Psi}_{23} &= B^T\check{P}_\jmath(k+1)\check{D}(k) \\
\grave{\Psi}_{33} &= \check{D}(k)^T\check{P}_\jmath(k+1)\check{D}(k)+\varsigma_\iota^2\hat{D}(k)^T \\
&\quad \times \check{P}_\jmath(k+1)\hat{D}(k).
\end{aligned}
$$

Adding the following zero term to (25):

$$
\|\bar{\mathbf{z}}(k)\|^2-\gamma^2\|\mathbf{v}(k)\|^2-(\|\bar{\mathbf{z}}(k)\|^2-\gamma^2\|\mathbf{v}(k)\|^2)=0 \tag{27}
$$

then the inequality (25) can be modified as

$$
\begin{aligned}
\Delta V(k) &\le \pi_{K,\iota}(k)(\xi(k)^T\acute{\Psi}(k)\xi(k)) \\
&\quad +\sum_{\jmath\in S_{uK}^\iota(k)}\pi_{\iota\jmath}(k)\xi(k)^T\grave{\Psi}(k)\xi(k) \\
&\quad +e(k)^T L^T Le(k)-\gamma^2\mathbf{v}(k)^T\mathbf{v}(k) \\
&\quad -(\|\bar{\mathbf{z}}(k)\|^2-\gamma^2\|\mathbf{v}(k)\|^2) \\
&= \pi_{K,\iota}(k)(\xi(k)^T\hat{\Psi}(k)\xi(k)) \\
&\quad +\sum_{\jmath\in S_{uK}^\iota(k)}\pi_{\iota\jmath}(k)\xi(k)^T\check{\Psi}(k)\xi(k) \\
&\quad -(\|\bar{\mathbf{z}}(k)\|^2-\gamma^2\|\mathbf{v}(k)\|^2)
\end{aligned}
\tag{28}
$$

where

$$
\begin{aligned}
\xi(k) &= [\mathbf{e}(k)^T \ \ \phi(\rho(k)H\mathbf{e}(k))^T \ \ \mathbf{v}(k)^T]^T \\
\hat{\Psi}(k) &= \begin{bmatrix}\hat{\Psi}_{11} & \acute{\Psi}_{12} & \acute{\Psi}_{13}\\ * & \acute{\Psi}_{22} & \acute{\Psi}_{23}\\ * & * & \hat{\Psi}_{33}\end{bmatrix} \\
\check{\Psi}(k) &= \begin{bmatrix}\check{\Psi}_{11} & \grave{\Psi}_{12} & \grave{\Psi}_{13}\\ * & \grave{\Psi}_{22} & \grave{\Psi}_{23}\\ * & * & \check{\Psi}_{33}\end{bmatrix}
\end{aligned}
\tag{29}
$$

with

$$
\begin{aligned}
\hat{\Psi}_{11} &= \acute{\Psi}_{11}+L^T L, \quad \hat{\Psi}_{33} = \acute{\Psi}_{33}-\gamma^2 I \\
\check{\Psi}_{11} &= \grave{\Psi}_{11}+L^T L, \quad \check{\Psi}_{33} = \grave{\Psi}_{33}-\gamma^2 I.
\end{aligned}
$$

Taking account into the inequality (28) in FH $[0, N]$, we can further have

$$\begin{aligned}\sum_{k=0}^{N} \Delta V(k) &= \mathbb{E}\{V(N+1) - V(0)\} \\ &\leq \sum_{k=0}^{N} \pi_{K,\iota}(k)(\xi(k)^T \hat{\Psi}(k)\xi(k)) \\ &\quad + \sum_{k=0}^{N} \sum_{\jmath \in S_{uK}^{\iota}(k)} \pi_{\iota\jmath}(k)\xi(k)^T \check{\Psi}(k)\xi(k) \\ &\quad - (\sum_{k=0}^{N} \|\bar{\mathbf{z}}(k)\|^2 - \gamma^2 \sum_{k=0}^{N} \|\mathbf{v}(k)\|^2). \quad (30)\end{aligned}$$

Utilizing the Schur complement lemma to (20), we have

$$\sum_{k=0}^{N} \pi_{K,\iota}(k)(\xi(k)^T \hat{\Psi}(k)\xi(k)) + \sum_{k=0}^{N} \sum_{\jmath \in S_{uK}^{\iota}(k)} \pi_{\iota\jmath}(k)\xi(k)^T \check{\Psi}(k)\xi(k) \leq 0. \quad (31)$$

In terms of (30) and (31), considering the FH H_∞ performance defined in Definition 1, we introduce the index $J(N)$ as follows,

$$\begin{aligned}J(N) &\triangleq \sum_{k=0}^{N} \|\bar{\mathbf{z}}(k)\|^2 - \gamma^2 \sum_{k=0}^{N} \|\mathbf{v}(k)\|^2 \\ &\quad - \gamma^2 \mathbf{e}(0)^T Q\mathbf{e}(0) \\ &< -\mathbb{E}\{V(N+1) - V(0)\} - \gamma^2 \mathbf{e}(0)^T Q\mathbf{e}(0) \\ &= -\mathbb{E}\{\mathbf{e}(N+1)^T P_{\vartheta(N+1)}(N+1)\mathbf{e}(N+1)\} \\ &\quad + \mathbf{e}(0)^T (P_{\vartheta(0)}(0) - \gamma^2 Q)\mathbf{e}(0). \quad (32)\end{aligned}$$

In term of Theorem 1, the inequalities $\mathbb{E}\{\mathbf{e}(N+1)^T P_{\vartheta(N+1)}(N+1)\mathbf{e}(N+1)\} \geq 0$ and $\mathbf{e}(0)^T (P_{\vartheta(0)} - \gamma^2 Q)\mathbf{e}(0) \leq 0$, the inequality $J(N) < 0$ can be derived. Then, based on Definition 1, the EES (17) meets the FH H_∞ performance. ■

IV. ESTIMATOR DESIGN

Based on the analysis above, we begin to design the non-fragile estimator gains in Theorem 2.

Theorem 2: Given a scalar $\gamma > 0$, the EES (17) over limited monitoring sensors and unreliable communication networks satisfies the FH H_∞ performance γ with the initial condition $P_\iota(0) < \gamma^2 Q$, if there exist matrices $P_\iota(k)_{[0,N+1]} > 0$, $\mathcal{G}_\iota(k)_{[0,N]} > 0$, $\tilde{K}_\iota(k)_{[0,N]}$ and scalars $\epsilon_1 > 0, \epsilon_2 > 0, \varepsilon_1 > 0$, such that the matrix inequalities

$$\begin{bmatrix} \hat{\Xi}_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & 0 & \Xi_{16} & 0 \\ * & \hat{\Xi}_{22} & 0 & 0 & \Xi_{25} & 0 & 0 \\ * & * & \hat{\Xi}_{33} & 0 & 0 & 0 & \Xi_{37} \\ * & * & * & -\epsilon_1 I & 0 & 0 & 0 \\ * & * & * & * & -\epsilon_1 I & 0 & 0 \\ * & * & * & * & * & -\epsilon_2 I & 0 \\ * & * & * & * & * & * & -\epsilon_2 I \end{bmatrix} < 0 \quad (33)$$

and

$$\begin{bmatrix} \check{\Xi}_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & 0 & \Xi_{16} & 0 \\ * & \check{\Xi}_{22} & 0 & 0 & \Xi_{25} & 0 & 0 \\ * & * & \check{\Xi}_{33} & 0 & 0 & 0 & \Xi_{37} \\ * & * & * & -\epsilon_1 I & 0 & 0 & 0 \\ * & * & * & * & -\epsilon_1 I & 0 & 0 \\ * & * & * & * & * & -\epsilon_2 I & 0 \\ * & * & * & * & * & * & -\epsilon_2 I \end{bmatrix} < 0, \quad \forall \jmath \in S_{uK}^{\iota} \quad (34)$$

hold for $k \in [0, N]$, where

$$\begin{aligned}
\hat{\Xi}_{11} &= \check{\Xi}_{11} = \begin{bmatrix} \hat{\Xi}_{111} & -R_1 & 0 \\ * & -2\varepsilon_1 I & 0 \\ * & * & -\gamma^2 I \end{bmatrix} \\
\Xi_{12} &= \begin{bmatrix} \Xi_{121}^T & \Xi_{122}^T & \Xi_{123}^T \end{bmatrix}^T \\
\Xi_{13} &= \begin{bmatrix} \Xi_{131}^T & 0 & \Xi_{133}^T \end{bmatrix}^T \\
\Xi_{14} &= \begin{bmatrix} \Xi_{141}^T & 0 & \Xi_{143}^T \end{bmatrix}^T \\
\Xi_{16} &= \begin{bmatrix} \Xi_{161}^T & 0 & \Xi_{163}^T \end{bmatrix}^T \\
\hat{\Xi}_{111} &= -P_\iota(k) + L^T L \\
\Xi_{121} &= A^T \mathcal{G}_\iota(k)^T - \bar{\alpha}_\iota C^T M_\iota^T \tilde{K}_\iota(k)^T \\
\Xi_{131} &= \varsigma_\iota C^T M_\iota^T \tilde{K}_\iota(k)^T \\
\Xi_{141} &= -\epsilon_1 \bar{\alpha}_\iota C^T M_\iota^T N_\iota^T \\
\Xi_{123} &= \begin{bmatrix} E^T \mathcal{G}_\iota(k)^T \\ -\bar{\alpha}_\iota D^T M_\iota^T \tilde{K}_\iota^T(k) \end{bmatrix} \\
\Xi_{161} &= \epsilon_2 \varsigma_\iota C^T M_\iota^T N_\iota^T, \quad \Xi_{122} = B^T \mathcal{G}_\iota(k)^T \\
\Xi_{133} &= \begin{bmatrix} 0 \\ \varsigma_\iota D^T M_\iota^T \tilde{K}_\iota^T(k) \end{bmatrix} \\
\Xi_{143} &= \begin{bmatrix} 0 \\ -\epsilon_1 \bar{\alpha}_\iota D^T M_\iota^T N_\iota^T \end{bmatrix} \\
\Xi_{163} &= \begin{bmatrix} 0 \\ \epsilon_2 \varsigma_\iota D^T M_\iota^T N_\iota^T \end{bmatrix} \\
\Xi_{25} &= \Xi_{37} = \mathcal{G}_\iota(k) F_\iota \\
\hat{\Xi}_{22} &= \hat{\Xi}_{33} = \hat{P}_\jmath(k+1) - \mathcal{G}_\iota(k) - \mathcal{G}_\iota(k)^T \\
\check{\Xi}_{22} &= \check{\Xi}_{33} = \check{P}_\jmath(k+1) - \mathcal{G}_\iota(k) - \mathcal{G}_\iota(k)^T, \quad (35)
\end{aligned}$$

then the estimator gains are listed as

$$K_\iota(k) = \mathcal{G}_\iota^{-1}(k)\tilde{K}_\iota(k). \quad (36)$$

Proof: Before the proof, we first define the matrices

$$\begin{aligned}
\tilde{K}_\iota(k) &= \mathcal{G}_\iota(k)K_\iota(k), \quad \bar{M}_\iota = N_\iota M_\iota \\
\Gamma_{11} &= \begin{bmatrix} -\bar{\alpha}_\iota \bar{M}_\iota C & 0 & 0 & -\bar{\alpha}_\iota \bar{M}_\iota D & 0 & 0 \end{bmatrix}^T \\
\Gamma_{21} &= \begin{bmatrix} \varsigma_\iota \bar{M}_\iota C & 0 & 0 & \varsigma_\iota \bar{M}_\iota D & 0 & 0 \end{bmatrix}^T \\
\Gamma_{12} &= \begin{bmatrix} 0 & 0 & 0 & 0 & F_\iota^T \mathcal{G}_\iota(k)^T & 0 \end{bmatrix} \\
\Gamma_{22} &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & F_\iota^T \mathcal{G}_\iota(k)^T \end{bmatrix}. \quad (37)
\end{aligned}$$

Then, according to the Lemma 2, we have

$$\Psi(k) + \epsilon_1 \Gamma_{11}\Gamma_{11}^T + \epsilon_1^{-1}\Gamma_{12}^T\Gamma_{12} + \epsilon_2 \Gamma_{21}\Gamma_{21}^T + \epsilon_2^{-1}\Gamma_{22}^T\Gamma_{22} < 0 \quad (38)$$

where

$$
\begin{aligned}
\Psi(k) &= \Psi(k)_1 + \Psi(k)_2 \\
\Psi(k)_1 &= \begin{bmatrix} \hat{\Xi}_{11} & \Xi_{12} & \Xi_{13} \\ * & \hat{\Xi}_{22} & 0 \\ * & * & \hat{\Xi}_{33} \end{bmatrix}, \quad \forall\, \jmath \in S_K^{\iota} \\
\Psi(k)_2 &= \begin{bmatrix} \check{\Xi}_{11} & \Xi_{12} & \Xi_{13} \\ * & \check{\Xi}_{22} & 0 \\ * & * & \check{\Xi}_{33} \end{bmatrix}, \quad \forall\, \jmath \in S_{uK}^{\iota}. \qquad (39)
\end{aligned}
$$

Considering the $\bar{K}_\iota(k) \triangleq K_\iota(k) + \Delta K_\iota(k)$ and $\Delta K_\iota(k) = F_\iota \Lambda(k) N_\iota$, and applying Schur lemma to matrix inequalities (38), the obtained matrices are equal to the matrix inequalities (20) pre- and post- multiplying $\mathrm{diag}_6\{I, I, I, I, \mathcal{G}_\iota(k), \mathcal{G}_\iota(k)\}$ and its transposition, because of the fact that for any matrices $P_\jmath(k+1)$ and $\mathcal{G}_\iota(k)$, the following inequality holds:

$$
\begin{aligned}
&P_\jmath(k+1) - \mathcal{G}_\iota(k) - \mathcal{G}_\iota(k)^T \\
&\qquad \geq -\mathcal{G}_\iota(k) P_\jmath(k+1)^{-1} \mathcal{G}_\iota(k)^T. \qquad (40)
\end{aligned}
$$

Thus the conditions (20) hold, which imply that the EES (17) meets the FH H_∞ performance γ. ■

V. NUMERICAL EXAMPLE

In this section, the effectiveness of the FH H_∞ non-fragile estimator design method is verified by numerical simulation, and we assume the FH is $k \in [1, 20]$. Since the monitoring sensors cannot monitor vehicle state information at all times, there are four cases occur. We consider the TVPUTP matrix as

$$
\Pi(k) = \begin{bmatrix} 0.2|\sin(k)| & \pi_{12} & \pi_{13} & 0.5|\cos(k)| \\ 0.1 & 0.1 & 0.4 & 0.4 \\ \pi_{31} & \pi_{32} & 0.2|\sin(k)| & 0.6|\cos(k)| \\ 0.1 & 0.1 & 0.3 & 0.5 \end{bmatrix} \qquad (41)
$$

where $\pi_{12}, \pi_{13}, \pi_{31}, \pi_{32}$ are unknown probabilities.

The acceleration saturation function is defined as

$$
\sigma(\bar{\boldsymbol{a}}) = \begin{cases} \boldsymbol{a}_{max}, & if\ \bar{\boldsymbol{a}} > \boldsymbol{a}_{max}; \\ \bar{\boldsymbol{a}}, & if\ -a_{max} \leq \bar{\boldsymbol{a}} \leq \boldsymbol{a}_{max}; \\ -\boldsymbol{a}_{max}, & else \end{cases}
$$

where $\bar{\boldsymbol{a}} \triangleq \rho(k) H \mathbf{x}(k)$, and we assume that the change rate of acceleration is $\rho(k) = \rho(k)_{max} \sin(k) \in [-1.5, 1.5]$, the acceleration changes within the bound $\mathcal{C} = [-0.5, 0.5]$, and saturation parameters are $h_1 = 0.6$ and $h_2 = 1$. The mathematical expectations of the cases of packet dropouts are

$$
\begin{aligned}
&\mathbb{E}\{\alpha_1(k) = 1\} = \bar{\alpha}_1 = 1, \quad \mathbb{E}\{\alpha_2(k) = 1\} = \bar{\alpha}_2 = 0.10, \\
&\mathbb{E}\{\alpha_3(k) = 1\} = \bar{\alpha}_3 = 0.10, \quad \mathbb{E}\{\alpha_4(k) = 1\} = \bar{\alpha}_4 = 0.05.
\end{aligned}
$$

Besides, we assume the sampling period $t_0 = 0.1$ and the matrices are

$$
A = \begin{bmatrix} 1 & 0.1 & 0.005 \\ 0 & 1 & 0.1 \\ 0 & 0 & 0.9\sin(k) \end{bmatrix}
$$

$$
C = \begin{bmatrix} 0.3 & 0.5 & 0.3 \\ 0.4 & 0.6 & 0.4 \end{bmatrix}, \quad D = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}.
$$

TABLE 1. The estimator gains $K_2(k)$.

1	$\begin{bmatrix} 30.3535 & 0 \\ 7.2008 & 0 \\ 21.4418 & 0 \end{bmatrix}$
2	$\begin{bmatrix} 15.0514 & 0 \\ 10.5423 & 0 \\ 7.0734 & 0 \end{bmatrix}$
3	$\begin{bmatrix} 10.5552 & 0 \\ 9.4255 & 0 \\ 4.8331 & 0 \end{bmatrix}$
4	$\begin{bmatrix} 13.3371 & 0 \\ 10.0837 & 0 \\ -4.8134 & 0 \end{bmatrix}$
5	$\begin{bmatrix} 12.7248 & 0 \\ 9.6740 & 0 \\ -4.9503 & 0 \end{bmatrix}$
$\vdots$	$\vdots$

The positive definite matrix $Q = \mathrm{diag}_3\{9.0, 9.0, 9.0\}$, the performance index $\gamma = 2.2361$, the external noises $\boldsymbol{\omega}(k) = e^{-0.1k}\sin(k)$ and $\boldsymbol{\nu}(k) = e^{-0.1k}\sin(k)$. The estimator parameter uncertainties are described by the following matrices:

$$
\begin{aligned}
F_1 &= \begin{bmatrix} 0.11 & 0.12 \\ 0.12 & 0 \\ 0 & 0.14 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 0.11 & 0.10 \\ 0 & 0.11 \\ 0.13 & 0 \end{bmatrix} \\
F_3 &= \begin{bmatrix} 0.15 & 0.10 \\ 0 & 0.12 \\ 0.10 & 0 \end{bmatrix}, \quad F_4 = \begin{bmatrix} 0.11 & 0.10 \\ 0 & 0.12 \\ 0.12 & 0 \end{bmatrix} \\
N_1 &= \begin{bmatrix} 0.15 & 0.11 \\ 0.12 & 0 \end{bmatrix}, \quad N_2 = \begin{bmatrix} 0.12 & 0.13 \\ 0.02 & 0.10 \end{bmatrix} \\
N_3 &= \begin{bmatrix} 0.10 & 0.03 \\ 0.02 & 0.14 \end{bmatrix}, \quad N_4 = \begin{bmatrix} 0.13 & 0.01 \\ 0.02 & 0.15 \end{bmatrix} \\
\Lambda(k) &= \begin{bmatrix} \sin(0.2k) & 0 \\ 0 & \sin(0.3k) \end{bmatrix}.
\end{aligned}
$$

In terms of the above parameters and the matrix inequalities (33), (34) and the estimator gains (36), we use the Matlab toolbox to compute the feasible solution and get the estimator gains $K_i(k)$ for $i \in [1, 4]$, $k \in [1, 20]$, where $K_1(k) \equiv 0$ and $K_2(k), K_3(k), K_4(k)$ are shown in TABLES $1 - 3$.

In order to illustrate the effectiveness of the non-fragile estimator by simulation, the initial states of the vehicle system and non-fragile estimator are set as

$$
\mathbf{x} = \begin{bmatrix} 1 & 0.2 & 0.5 \end{bmatrix}^T, \quad \hat{\mathbf{x}} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T.
$$

Considering the randomness of Markov chains that occurs in measurement process and random packet dropouts exist during the transmission, we repeat simulation experiment $\ell = 100$ times under the same system matrices, estimator gains, disturbance and the initial conditions. Then we get the averages of the states and their estimations as $\bar{\mathbf{x}}(k) \triangleq \sum_{\iota=0}^{\iota=\ell} \mathbf{x}_\iota(k)/\ell$, $\bar{\hat{\mathbf{x}}}(k) \triangleq \sum_{\iota=0}^{\iota=\ell} \hat{\mathbf{x}}_\iota(k)/\ell$. Based on the above, the trajectories of $\bar{\mathbf{x}}(1, k), \bar{\mathbf{x}}(2, k)$ and their estimations are shown in FIGURE 2.

TABLE 2. The estimator gains $K_3(k)$.

1	$\begin{bmatrix} 0.5064 & 39.9902 \\ -0.0026 & 7.4474 \\ 0.5338 & 26.1268 \end{bmatrix}$
2	$\begin{bmatrix} 0.5047 & 12.8371 \\ 0.4680 & 6.2744 \\ 0.5814 & 5.4427 \end{bmatrix}$
3	$\begin{bmatrix} 3.9312 & 9.5699 \\ 3.0239 & 4.9869 \\ 3.9661 & 1.2342 \end{bmatrix}$
4	$\begin{bmatrix} 4.8651 & 11.1262 \\ 3.3531 & 5.7189 \\ -3.9192 & -1.5024 \end{bmatrix}$
5	$\begin{bmatrix} 3.4639 & 8.5342 \\ 2.1714 & 4.1570 \\ -2.5654 & -2.4036 \end{bmatrix}$
$\vdots$	$\vdots$

TABLE 3. The estimator gains $K_4(k)$.

1	$\begin{bmatrix} -267.9823 & 243.5517 \\ 12.7715 & -4.8354 \\ -238.5255 & 212.8893 \end{bmatrix}$
2	$\begin{bmatrix} -412.4875 & 354.2511 \\ 47.2749 & -31.2668 \\ -314.9910 & 264.8015 \end{bmatrix}$
3	$\begin{bmatrix} -267.2182 & 228.3876 \\ 39.6603 & -29.0563 \\ -178.4375 & 147.6903 \end{bmatrix}$
4	$\begin{bmatrix} -482.7802 & 410.7690 \\ 45.8512 & -31.3176 \\ 277.3017 & -232.8872 \end{bmatrix}$
5	$\begin{bmatrix} -443.6424 & 372.9730 \\ 65.1389 & -50.2007 \\ 311.0995 & -258.1771 \end{bmatrix}$
$\vdots$	$\vdots$

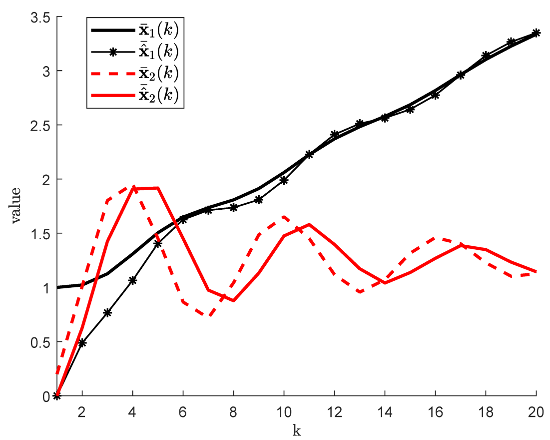

FIGURE 2. The trajectories of $\bar{x}(k)$ and their estimations.

VI. CONCLUSION

In this work, a vehicle moving with bounded acceleration on a level road has been investigated, and a saturation function $\sigma(\cdot)$ has been designed to describe the vehicle's acceleration. A Markov chain with TVPUTP has been proposed. Then, the CS dependent non-fragile estimator has been established to improve the robustness, and the EES has been derived. Two theorems have been given to ensure that the EES satisfies the FH H_∞ performance. The estimators gains have been derived by Matlab tools box. Finally, an example has been introduced to illustrate the results. In future work, this research could be carried out in the real world to help control AI vehicles.

REFERENCES

[1] S. Moon, W. Cho, and K. Yi, "Intelligent vehicle safety control strategy in various driving situations," *Vehicle Syst. Dyn.*, vol. 48, no. 1, pp. 537–554, 2010.

[2] Y. Du, C. Liu, and Y. Li, "Velocity control strategies to improve automated vehicle driving comfort," *IEEE Intell. Transp. Syst. Mag.*, vol. 10, no. 1, pp. 8–18, Jan. 2018.

[3] L. Baskar, B. De Schutter, J. Hellendoorn, and Z. Papp, "Traffic control and intelligent vehicle highway systems: A survey," *IET Intell. Transp. Syst.*, vol. 5, no. 1, pp. 38–52, Mar. 2011.

[4] M. Elhenawy, A. A. Elbery, A. A. Hassan, and H. A. Rakha, "An intersection game-theory-based traffic control algorithm in a connected vehicle environment," in *Proc. IEEE Int. Conf. Intell. Transp. Syst.*, Sep. 2015, pp. 343–347.

[5] R. Sundar, S. Hebbar, and V. Golla, "Implementing intelligent traffic control system for congestion control, ambulance clearance, and stolen vehicle detection," *IEEE Sensors J.*, vol. 15, no. 2, pp. 1109–1113, Feb. 2015.

[6] S. Li, Z. Li, Z. Yu, B. Zhang, and N. Zhang, "Dynamic trajectory planning and tracking for autonomous vehicle with obstacle avoidance based on model predictive control," *IEEE Access*, vol. 7, pp. 132074–132086, 2019.

[7] F. Bi, M. Lei, Y. Wang, and D. Huang, "Remote sensing target tracking in UAV aerial video based on saliency enhanced mdnet," *IEEE Access*, vol. 7, pp. 76731–76740, 2019.

[8] Y. Xu, R. Lu, P. Shi, H. Li, and S. Xie, "Finite-time distributed state estimation over sensor networks with round-robin protocol and fading channels," *IEEE Trans. Cybern.*, vol. 48, no. 1, pp. 336–345, Jan. 2018.

[9] L. Ma, Z. Wang, Q.-L. Han, and H.-K. Lam, "Variance-constrained distributed filtering for time-varying systems with multiplicative noises and deception attacks over sensor networks," *IEEE Sensors J.*, vol. 17, no. 7, pp. 2279–2288, Apr. 2017.

[10] N. H. Gholson and R. L. Moose, "Maneuvering target tracking using adaptive state estimation," *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-13, no. 3, pp. 310–317, 2007.

[11] S. Maresca, M. Greco, F. Gini, and L. Verrazzani, "Radar tracking of a maneuvering ground vehicle using an airborne sensor," in *Proc. Int. Radar Conf. Surveill. Safer World (RADAR)*, Oct. 2010, pp. 1–6.

[12] G.-G.-O. Ricardo, J. L.-S. Roberto, and S. Maldonado-Bascón, and A. Fernández-Caballero, "Vehicle tracking by simultaneous detection and viewpoint estimation," in *Proc. Natural Artif. Comput. Eng. Med. Appl.*, 2013, pp. 306–316.

[13] W. Jeon, A. Zemouche, and R. Rajamani, "Tracking of vehicle motion on highways and urban roads using a nonlinear observer," *IEEE/ASME Trans. Mechatronics*, vol. 24, no. 2, pp. 644–655, Jan. 2019.

[14] Y. Fang, C. Wang, W. Yao, X. Zhao, and H. Zha, "On-road vehicle tracking using part-based particle filter," *IEEE Trans. Intell. Transp. Syst.*, doi: 10.1109/TITS.2018.2888500.

[15] W. L. Yeow, C. K. Tham, and W. C. Wong, "Energy efficient multiple target tracking in wireless sensor networks," *IEEE Trans. Veh. Technol.*, vol. 56, no. 2, pp. 918–928, Mar. 2007.

[16] Y. Yu, "Distributed multimodel Bernoulli filters for maneuvering target tracking," *IEEE Sensors J.*, vol. 18, no. 14, pp. 5885–5896, Jul. 2018.

[17] L. Feng, S. Hao, M. Chen, and Q. Kong, "Non-fragile finite-time l_2-l_∞ state estimation for discrete-time Markov jump neural networks with unreliable communication links," *Appl. Math. Comput.*, vol. 271, pp. 467–481, Nov. 2015.

[18] Z.-G. Wu, P. Shi, Z. Shu, H. Su, and R. Lu, "Passivity-based asynchronous control for Markov jump systems," *IEEE Trans. Autom. Control*, vol. 62, no. 4, pp. 2020–2025, Apr. 2017.

[19] H. Shen, M. Chen, Z. Wu, J. Cao, and J. H. Park, "Reliable event-triggered asynchronous extended passive control for semi-Markov jump fuzzy systems and its application," *IEEE Trans. Fuzzy Syst.*, to be published, doi: 10.1109/TFUZZ.2019.2921264.

[20] S.-Q. Yang and L.-Y. Lin, "Dynamic output feedback finite-horizon control for Markov jump systems with actuator saturations," *IEEE Access*, vol. 7, pp. 132587–132593, 2019.

[21] Z.-X. Liu and B.-J. Huang, "The labeled multi-Bernoulli filter for jump Markov systems under glint noise," *IEEE Access*, vol. 7, pp. 92322–92328, 2019.

[22] V. M. Revathi, P. Balasubramaniam, H. P. Ju, and T. H. Lee, "H_∞ filtering for sample data systems with stochastic sampling and Markovian jumping parameters," *Nonlinear Dyn.*, vol. 78, no. 2, pp. 813–830, 2014.

[23] H. Wang, D. Zhang, and R. Lu, "Event-triggered H_∞ filter design for Markovian jump systems with quantization," *Nonlinear Anal., Hybrid Syst.*, vol. 28, pp. 23–41, May 2018.

[24] C. Zhu, B. Yang, X. Zhu, and Z. Wang, "Stabilization of linear systems over networks with limited communication capacity," *IEEE Access*, vol. 7, pp. 123625–123637, 2019.

[25] Y. Xu, J.-G. Dong, R. Lu, and L. Xie, "Stability of continuous-time positive switched linear systems: A weak common copositive Lyapunov functions approach," *Automatica*, vol. 97, pp. 278–285, Nov. 2018.

[26] H. Rao, Y. Xu, H. Peng, R. Lu, and C.-Y. Su, "Quasi-synchronization of time delay Markovian jump neural networks with impulsive-driven transmission and fading channels," *IEEE Trans. Cybern.*, to be published, doi: 10.1109/TCYB.2019.2941582.

[27] H. Rao, F. Liu, Y. Xu, H. Peng, and R. Lu, "Observer-based impulsive synchronization for neural networks with uncertain exchanging information," *IEEE Trans. Neural Netw. Learn. Syst.*, doi: 10.1109/TNNLS.2019.2946151.

[28] H. Ye and L. Wen, "Robust fault detection filter design of networked control systems," *IEEE Access*, vol. 7, pp. 141144–141152, 2019.

[29] H. Shen, M. Dai, H. Yan, and J. H. Park, "Quantized output feedback control for stochastic semi-Markov jump systems with unreliable links," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 65, no. 12, pp. 1998–2002, Dec. 2018.

[30] N. Xiao, L. Xie, and M. Fu, "Kalman filtering over unreliable communication networks with bounded Markovian packet dropouts," *Nonlinear Control*, vol. 19, no. 16, pp. 1770–1786, 2010.

[31] J. S. Harsini and F. Lahouti, "Adaptive transmission policy design for delay-sensitive and bursty packet traffic over wireless fading channels," *IEEE Trans. Wireless Commun.*, vol. 8, no. 2, pp. 776–786, Feb. 2009.

[32] R. N. Jazar, *Vehicle Dynamics: Theory Application*. New York, NY, USA: Springer, 2008.

[33] F. Yang and Y. Li, "Set-membership filtering for systems with sensor saturation," *Automatica*, vol. 45, no. 8, pp. 1896–1902, 2009.

[34] Y. Luo, Z. Wang, G. Wei, and F. E. Alsaadi, "Nonfragile l_2-l_∞ fault estimation for Markovian jump 2 D systems with specified power bounds," *IEEE Trans. Syst., Man, Cybern. Syst.*, doi: 10.1109/TSMC.2018.2794414.

[35] S. Gai and Z. Yu, "Stochastic stability for impulsive discrete-time Markovian jump systems with time-varying delay and partly unknown transition probabilities," in *Proc. IEEE Int. Conf. Cyber Technol. Autom.*, May 2013, pp. 281–286.

[36] Y. Xu, C. Liu, J.-Y. Li, C.-Y. Su, and T. Huang, "Finite-horizon H_∞ state estimation for time-varying neural networks with periodic inner coupling and measurements scheduling," *IEEE Trans. Syst., Man, Cybern. Syst.*, doi: 10.1109/TSMC.2018.2791575.

[37] L. Xie, "Output feedback H_∞ control of systems with parameter uncertainty," *Int. J. Control*, vol. 63, no. 4, pp. 741–750, 1996.

CHANG LIU was born in Xiangcheng, China, in 1992. He received the B.S. degree from Henan University, Kaifeng, China, in 2016, and the M.S. degree in control science and engineering with the Guangdong University of Technology, Guangzhou, China, in 2019, where he is currently pursuing the M.S. degree in control science and engineering.

His current research interests include networked control systems and finite time control and estimation.

FEN LIU was born in Hunan, China. She received the B.S. degree from the Changsha University of Science and Technology, Changsha, China, in 2017. She is currently pursuing the M.S. degree in control engineering with the Guangdong University of Technology, Guangzhou, China.

Her research interests include synchronization of neural networks and impulse control.

HONGXIA RAO was born in Jiangsu, China, in 1986. She received the B.S. degree from Nanchang Hangkong University, Nanchang, China, in 2007, the M.S. degree from the Nanjing University of Science and Technology, Nanjing, China, in 2009, and the Ph.D. degree in control science and engineering from the Guangdong University of Technology, Guangzhou, China, in 2019.

She is currently a Lecturer with the School of Automation, Guangdong University of Technology. Her research interests include networked control systems, Markov jump systems, and neural networks.

YONG XU was born in Zhejiang, China, in 1983. He received the B.S. degree in information engineering from Nanchang Hangkong University, Nanchang, China, in 2007, the M.S. degree in control science and engineering from Hangzhou Dianzi University, Hangzhou, China, in 2010, and the Ph.D. degree in control science and engineering from Zhejiang University, Hangzhou, in 2014.

He was a Visiting Internship Student with the Department of Electronic and Computer Engineering, The Hong Kong University of Science and Technology, Hong Kong, China, from June 2013 to November 2013, where he was a Research Fellow, from February 2018 to August 2018. He was honored the Pearl River Young Scholars Program of Guangdong Province, in 2017. He is currently an Associate Professor with the School of Automation, Guangdong University of Technology, Guangzhou, China. His research interests include PID control, networked control systems, state estimation, and positive systems.