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Enforcing the Liveness of S^4PR by Using the Approach of Allocating Resources

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ABSTRACT Liveness is very important for resource allocation systems (RASs) as it means that no deadlock can arise in the system operation. By applying the approach of allocating resources, this paper focuses on enforcing the liveness of RASs that allow for the general resource allocation and flexible routs. These considered RASs can be modeled by a subcalss of systems of sequential systems with shared resources, named as S^4PRs. Deadlocks in S^4PR can be characterized by the saturation of a kind of activity-circuits. Based on these circuits, we study the relationship between the number of initial tokens or marking of resource place and the non-saturation of some involved activity-circuits. Further, a liveness condition for S^4PR is derived which is associated with the numbers of initial tokens or markings of all resource places. An algorithm is proposed to allocate the initial number of resources so that the considered S^4PR is live. Finally the proposed method is illustrated by examples.

INDEX TERMS Resource allocation systems (RASs), Petri nets, activity-circuits, liveness-enforcing.

I. INTRODUCTION

Resource allocation systems (RASs) [1] can model well a majority of contemporary applications such as automated manufacturing systems, transport systems, and workflow management systems. All these systems involve a set of concurrent processes and a finite set of resources, where each process competes to use limited resources to complete its execution. Owing to the competition of limited resources, RAS may enter deadlock states, where a set of process instances require resources held by another instances in the same set, i.e., the resource circuit-waiting pattern is created [2]–[5]. In a deadlock state, the whole or part of RAS is permanently blocked and hence some processes cannot be performed. Thus, it is imperative to prevent deadlocks from happening in RASs.

Petri net (PN) is a powerful mathematical tool for RAS modeling and analysis. Many existing PNs modelling RASs are process-oriented [6], [7], where i) a type of resources is modelled by a (resource) place with finite tokens, and ii) a strongly connected state machine is used to model the execution plan of a process type. This paper studies a class of process-oriented PNs, namely system of simple sequential process with multiple resources (S^4PR) [8], [31]. In fact, S^4PRs can model well the disjunctive/conjunctive (D/C) RASs [9], in which each stage of a process may require multiple units of multiple types of resources to execute. The well-known PNs S^3PR [10], [30] and WS^{3}PR [11] are both proper subclasses of S^4PRs.

Liveness is an important property of S^4PR as it implies that the operation of being-modelled D/C-RAS is deadlock-free [10], [12]. To deal with the liveness issue, two structural objects siphons and perfect activity-circuits (PA-circuits) are used to characterize deadlocks in S^4PR. Generally speaking, so far there are two approaches dealing with liveness-enforcement problem in S^4PR: *supervisory control* and *initial marking configuration*. The former is to add ''external'' supervisors to the original system to ensure that the controlled system is live. Many existing works fall into this category [2], [10]–[19]. In particular, Ezpeleta *et al.* [10] show that the liveness of S^3PR, a subclass of S^4PR, is related with the absence of *empty siphons*, and they further propose a siphon-based supervisor for S^3PRs. Xing *et al.* [20] develop an optimal liveness supervisor for a class of S^3PRs. Based on PA-circuits, our previous work [3] proposes the necessary and sufficient condition associated with the liveness of S^4PR, and we also propose an algorithm to compute all saturable PA-circuits. In addition,

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other robust liveness-enforcing supervisors can be found in [17], [21], [22].

The study can be traced back to 1990s [16], [23]–[25] that designing initial marking configuration for enforcing the liveness of RASs. In particular, Zhou and DiCesare [24] consider the PN plants whose initial marking of various resources is fixed. They configure an initial marking for places that model the number of parts in the manufacturing system so as to enforce the system liveness. Compared with supervisory control, the initial marking configuration methods have the major benefit that no supervisors are added, and hence the control cost is saved. It is well-known that deadlock in RAS is caused by improper allocation of resources.

The works [26]–[28] devote to the problem of liveness enforcement in PN models by allocating resources, i.e., setting up appropriate initial markings for resource places. Specifically, Liu *et al*. [26] compute an initial resource marking to ensure the liveness of WS^3PR. The authors in [27] propose a resource configuration approach for configuring initial resources to ensure the liveness of S^3PRs. Recently, for WS^3PRs modeling RASs in which the number of parts to be processed is given, You *et al*. [28] present an algorithm for working out an initial resource marking that can enforce the liveness of WS^3PRs.

Recapitulating the above discussion, we know that there is no work in the literation can guarantee the liveness of S^4PR through resource allocation approach. Motivated by this fact and the PA-circuit-based deadlock characterization in S^4PR [3], this paper focuses on the liveness enforcement problem for S^4PR by allocating resources. We summarize the main contributions of this work as follows.

1) Based on the saturation of PA-circuits that would lead to deadlock in S^4PR [3], the relation between the initial markings of a resource place and the non-saturation of involved PA-circuits is established. This is the foundation of this work.
2) By using the proposed relation associated with the initial markings of some resources, we develop a sufficient liveness condition for S^4PR.
3) An algorithm is developed to compute an initial resource marking to guarantee the liveness of the considered S^4PR with the initial marking of idle places given only.

The remainder of this work is summarized as follows. Section II discusses the preliminaries for PNs, S^4PRs, and the PA-circuit-based deadlock characterization of S^4PRs. Section III first shows the relation between the initial resource marking and the non-saturation of some involved PA-circuits, and then develops an algorithm to compute initial resource marking for enforcing the liveness of S^4PR. Also, an example is given for illustrating the proposed method. Section IV concludes this work.

II. PRELIMINARY

A. PETRI NETS

A *generalized* PN is a 4-tuple $N = (P, T, F, W)$, where T and P are sets of *transitions* and *places*, respectively, $F \subseteq (T \times P) \cup (P \times T)$collects arcs between transitions and places. $W : F \to \mathbb{Z}$ assigns the weights to arcs such that $W(x, y) = 0$ if $(x, y) \notin F$, otherwise $W(x, y) > 0$, where $\mathbb{Z} = \{0, 1, 2, \ldots\}$. Moreover, if $\forall(x, y) \in F$, $W(x, y) = 1$, N is called an *ordinary* net.

For node $u \in T \cup P$, the post set of u is $u^\bullet = \{v \in P \cup T | (u, v) \in F\}$, the preset of u is ${}^\bullet u = \{v \in P \cup T | (v, u) \in F\}$. Furthermore, for $U \subseteq P \cup T$, we define ${}^\bullet U = \cup_{u \in U} {}^\bullet u$ and $U^\bullet = \cup_{u \in U} u^\bullet$. An ordinary PN is a *state machine* if $|{}^\bullet t| = |t^\bullet| = 1, \forall t \in T$.

A state or marking of N is a mapping $M : P \to \mathbb{Z}$. For $p \in P$, $M(p)$ is the token number in p at marking M. For $S \subseteq P$, the sum of the token numbers in S at marking M is $M(S) = \sum_{p \in S} M(p)$. The *incidence matrix* of PN N is a $|P| \times |T|$ matrix $[N]$ where $[N_{ij}] = [N_{ij}]^+ - [N_{ij}]^-$, $[N_{ij}]^- = W(p_i, t_j)$, and $[N_{ij}]^+ = W(t_j, p_i)$. PN N with initial marking (IM) M_0 is called as a marked PN, denoted by (N, M_0).

A P-vector is a mapping $I : P \to \mathbb{Z}$. If P-vector $I \geq 0$ is a P-semiflow if $I * [N] = 0$. The support of P-vector I is a place set $||I|| = \{s \in P | I(s) \neq 0\}$. A P-semiflow I is *minimal* if $||I||$ does not contain the support of any other P-semiflow. For the sake of convenience, notation $\Sigma_{s \in P} I(s)s$(resp., $\Sigma_{s \in P} M(s)s$) is used to denote P-semiflow I (resp., marking M).

For marking M, transition $t \in T$ is *enabled* at M, denoted by $M[t>$, if $\forall p \in {}^\bullet t$, $M(p) \geq W(p, t)$. If an enabled transition t at M can fire and results a new making M', denoted by $M[t > M'$, where $M'(p) = M(p) - W(p, t)$, $\forall p \in {}^\bullet t \setminus t^\bullet$; $M'(p) = M(p) + W(t, p)$, $\forall p \in t^\bullet \setminus {}^\bullet t$; $M'(p) = M(p)$, $\forall p \in P \setminus ({}^\bullet t \setminus t^\bullet) \cup (t^\bullet \setminus {}^\bullet t)$. For a positive integer k, let $\mathbb{Z}_k \equiv \{1, 2, \ldots, k\}$. A transition sequence $\delta = t_1 t_2 \ldots t_k$, is enabled at M if $M_j[t_j > M_{j+1}, j \in \mathbb{Z}_k$, where $M_1 = M$ and M_j is a marking reachable from M. Let $R(N, M_0)$ denote the set of markings reachable from M_0. Meanwhile, $M_0[\delta > M$ implies that $M = M_0 + [N] * Y$, where the i-th entry of vector Y is the occurrence number of t_i in δ.

Transition $t \in T$ is *live* if $\forall M \in R(N, M_0), \exists M' \in R(N, M)$ such that $M'[t >$., t is *dead* at M if t is not enabled at any marking in $R(N, M)$. PN (N, M_0) is live if all its transitions are live.

For a set $X \subseteq P \cup T$, the subnet generated by X is a subnet $N[X] = (P_X, T_X, F_X, W_X)$, where $P_X = X \cap P$, $T_X = X \cap T$, $F_X = F \cap (X \times X)$, and $W_X(f) = W(f)$, $\forall f \in F_X$. For $M \in R(N, M_0)$, let M_1 be a marking of $N[X]$ which is a restrictions of M to X, this is denoted by $M_1 = M|_{N[X]}$, where $M_1(p) = M(p)$, $\forall p \in X \cap P$.

A string $\varpi = x_1 x_2 \ldots x_k$ is a *path* if $x_i \in T \cup P$, and $(x_i, x_{i+1}) \in F$ for $i \in \mathbb{Z}_{k-1}$. If the nodes of a path are different from each other, it is called *elementary*. Path $\varpi = x_1 x_2 \ldots x_k$ is a *circuit* if $x_1 = x_k$.

B. S^4PRs AND ITS LIVENESS CONDITIONS

Definition 1 [4]: An S^4PR is a PN $(N, M_0) = (P_0 \cup P_A \cup P_R, T, F, W, M_0)$ such that

1) P_0, P_R and P_A, respectively, collect all *idle*, *resource*, and *activity* places, where i) $P_0 = \{p_{0i} | i \in \mathbb{Z}_m\}$; ii) $P_A = \cup_{i \in \mathbb{Z}_m} P_{Ai}$, $T = \cup_{i\in\mathbb{Z}_m} T_i$, where $\forall i \in \mathbb{Z}_m$,$P_{Ai} \neq \emptyset$, $T_i \neq \emptyset$, $\forall i, j \in \mathbb{Z}_m$, $i \neq j$, $P_{Aj} \cap P_{Ai} = \emptyset$, $T_j \cap T_i = \emptyset$; iii) $P_R = \{r_j | j \in \mathbb{Z}_n\}$; and iv) $P_R \cap (P_A \cup P_0) = \emptyset$.
2) Subnet $N_j = N[T_j \cup \{p_{j0}\} \cup P_{Aj}]$ is a strongly connected state machine and p_{j0} is contained in each circuit of N_j, where $j \in \mathbb{Z}_m$.
3) There is a unique minimal P-semiflow, denoted as I_r, for each resource place $r \in P_R$ such that $P_R \cap \|I_r\| = \{r\}$, $P_A \cap \|I_r\| \neq \emptyset$, $I_r(r) = 1$, and $P_0 \cap \|I_r\| = \emptyset$.
4) $P_A = \cup_{r \in P_R} (\|I_r\| \setminus \{r\})$.

Definition 2 [4]: Let M_0 is an initial marking of S^4PR N. M_0 is *acceptable* if $M_0(s) > 0$, $\forall s \in P_0$; (2) $M_0(p) = 0$, $\forall p \in P_A$; (3)$M_0(r) \geq \max\{I_r(p) | p \in \|I_r\|\}$, $\forall r \in \Re$.

An S^4PR with an acceptable initial marking is called *well-marked*. The state machine N_i in S^4PR (N, M_0) represents the complete machining process of i-th type parts, its initial marking is $M_0(p_{i0})$, the number of i-th type parts in the system. While $M_0(r)$ is the *capacity* of type-r resources.

According to the structure of S^4PR N, its initial marking M_0 can be denoted by $M_0 = [M_{00}, M_{0A}, M_{0R}]$, where $M_{00} = M_0|_{P0}$, $M_{0A} = M_0|_{PA}$, and $M_{0R} = M_0|_{PR}$ are the initial idle, activity, and resource markings P_0, P_A, and P_R, respectively.

The set of input and output resource places of transition $t \in T$ is denoted as ${}^{(R)}t$ and $t^{(R)}$, respectively. The set of output and input activity places of t is $t^{(A)}$ and ${}^{(A)}t$, respectively. We can apply those notations to a set, e.g., we define $K^{(R)} = \cup_{t \in K} \{t^{(R)}\}$ for $K \subseteq T$.

For $r \in P_R$, $H(r) = \|I_r\| \setminus \{r\}$ is the holder of r. We define the set of resources that $p \in P_A$ requires as $\Gamma(p) \equiv \{r \in P_R | p \in H(r)\}$.

Example 1: Consider an S^4PR (N, M_0) shown in Figure. 1, where $P_0 = \{p_1, p_6\}$, $P_R = \{r_1 - r_3\}$, $P_A = \{p_2 - p_5, p_7 - p_{11}\}$, and $T = \{t_1 - t_{11}\}$. The initial marking $M_0 = 6p_1 + 6p_6 + 6r_1 + r_2 + 3r_3 + 2r_4$ is acceptable for N, and thereby (N, M_0) is well-marked. Apparently, $M_{00} = 6p_1 + 6p_6$, $M_{0A} = 0$, and $M_{0R} = 2r_1 + r_2 + 3r_3 + 2r_4$. Note that N has two types of processes, which are modeled by the state machines generated by $\{p_1\} \cup \{p_2 - p_5\} \cup \{t_1 - t_5\}$ and $\{p_6\} \cup \{p_7 - p_{11}\} \cup \{t_6 - t_{11}\}$, respectively. The P-semiflows related with all resources in P_R are: $I_{r_1} = 4p_2 + 6p_3 + p_9 + r_1$, $I_{r_2} = p_{10} + p_{11} + r_2$, $I_{r_3} = 2p_4 + p_8 + p_9 + r_3$, and $I_{r_4} = p_2 + p_7 + p_8 + r_4$. Moreover, the set of holders of resource place r_1 is $H(r_1) = \{p_2, p_3, p_9\}$ and the set of resource places that activity place p_8 requires is $\Gamma(p_8) = \{r_3, r_4\}$.

Following our previous work [3], we know that deadlocks in S^4PR can be characterized by a kind of structure objects, namely PA-circuits. Based on this, the liveness necessary and sufficient condition for S^4PR can be derived. For the sake of completeness, we first briefly review some concepts about PA-circuits in [3].

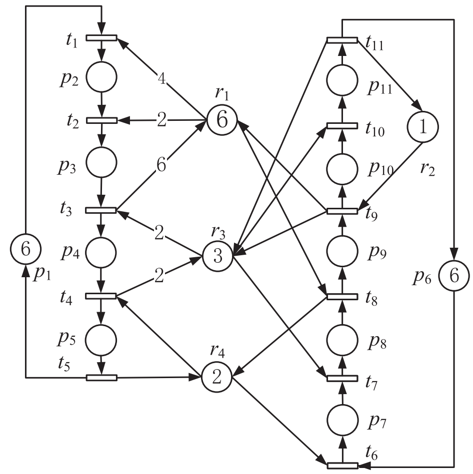

FIGURE 1. **An marked of S^4PR (N, M_0).**

Definition 3 [3]: A *single activity-path* (*SA-path*) in S^4PR is a path $\alpha = pt$ where $p \in P_A$ and $t \in T$. Let $r \in \Gamma(p)$, then we say that SA-path $\alpha = pt$ is with respect to (w.r.t.) resource r.

Let $\alpha_1 = p_1t_1$ and $\alpha_2 = p_2t_2$ be SA-paths w.r.t. r_1 and r_2, respectively. If $r_2 \in {}^{(R)}t_1$, then α_1 is *reachable* from α_2, or α_2 can *reach* α_1, this is denoted as $\alpha_1 \leftarrow \alpha_2$.

Definition 4 [3]: A sequence of SA-paths $\theta = \alpha_1\alpha_2 \ldots \alpha_n$ is called *activity-chain* if $\alpha_i = p_it_i$ is a SA-path w.r.t. resource r_i, $\forall i \in \mathbb{Z}_n$, and $\alpha_j \leftarrow \alpha_{j+1}$, i.e., α_j is reachable from α_{j+1}, $\forall j \in \mathbb{Z}_{n-1}$. Let $\Re(\theta) = \{r_i | \alpha_i \text{ is w.r.t. } r_i, i \in \mathbb{Z}_n\}$. We call θ as an *activity-chain* w.r.t. resource set $\Re(\theta)$. Then the set of transitions and activity places of θ are denoted by $\Im(\theta) = \{t_i | t_i \text{ is in } \theta\}$ and $\wp(\theta) = \{p_i \in P_A | p_i \text{ is in } \theta\}$, respectively.

Definition 5 [3]: Let $\theta = \alpha_1\alpha_2 \ldots \alpha_n$ be an activity-chain w.r.t. resource set R. If α_1 can reach α_n, i.e., $\alpha_n \leftarrow \alpha_1$, then θ is a called an *activity-circuit* w.r.t. R, or *A-circuit* for short. Any A-circuit θ is perfect if $({}^{(a)}\Im(\theta))^\bullet = \Im(\theta)$. Let Θ be the set of all perfect A-circuits (PA-circuits for short) in an S^4PR (N, M_0).

Example 2: Consider S^4PR (N, M_0) shown in Figure. 1. Since $r_1 \in \Gamma(p_2)$ and $r_1 \in {}^{(R)}t_2$, $\alpha_1 = p_2t_2$ is a SA-path w.r.t. r_1, and can reach itself, i.e.,$\alpha_1 \leftarrow \alpha_1$ and $\theta_1 = \alpha_1\alpha_1$ is an activity-chain as well as an A-circuit w.r.t. resource set $\{r_1\}$. Moreover, for SA-paths $\alpha_2 = p_3t_3$ w.r.t. r_1 and $\alpha_3 = p_8t_8$ w.r.t. r_3, since $r_3 \in {}^{(R)}t_3$ and $r_1 \in {}^{(R)}t_8$, α_2 and α_3 can reach each other, i.e., $\alpha_2 \leftarrow \alpha_3$ and $\alpha_3 \leftarrow \alpha_2$. Hence, $\theta_2 = \alpha_2\alpha_3$ is an A-circuit w.r.t. resource set $\{r_1, r_3\}$. In addition, since $({}^{(a)}\Im(\theta_1))^\bullet = \Im(\theta_1)$ and $({}^{(a)}\Im(\theta_2))^\bullet = \Im(\theta_2)$, θ_1 and θ_2 are both PA-circuits.

By the definition of P-semiflow I_r of r, $I_r(p)$ is the number of type-r resources that are used by a token (or a part) in $p \in H(r)$. Then for $S \subseteq P_A$, $\Sigma_{p \in S} I_r(p)M(p)$ is the total number of type-r resources used by parts or tokens in S at marking M, denoted as $M_{I_r}(S)$, i.e. $M_{I_r}(S) \equiv \Sigma_{p\in S} I_r(p)M(p)$.

Definition 6 [3]: A PA-circuit $\theta \in \Xi$ is *saturated* at a state M, if $M(p) \geq 1$, $\forall p \in P(\theta)$; and $\min\{W(r, t) | t \in r^\bullet \cap \Im(\theta)\} > M_0(r) - M_{I_r}(P(\theta) \cap H(r))$, $\forall r \in \Re(\theta)$.

Example 3: Consider S^4PR (N, M_0) in Figure. 1, $M = 4p_1 + p_3 + 4p_6 + 2p_8 + r_2 + r_3 + r_4$ is one of its reachable markings. For PA-circuit $\theta_2 = \alpha_2\alpha_3$ where $\alpha_2 = p_3t_3$ w.r.t. r_1 and $\alpha_3 = p_8t_8$ w.r.t. r_3, we have $\Im(\theta_2) = \{t_3, t_8\}$, $\wp(\theta_2) = \{p_3, p_8\}$, and $\Re(\theta_2) = \{r_1, r_3\}$. Since $M(p_3) = 1 > 0$, $M(p_8) = 2 > 0$, $M_0(r_1) - M_{I_{r_1}}(H(r_1) \cap \wp(\theta_2)) = 0 < \min\{W(r_1, t)|t \in r_1^\bullet \cap \Im(\theta_2)\} = 1$, and $M_0(r_3) - M_{I_{r_3}}(H(r_3) \cap \wp(\theta_2)) = 1 < \min\{W(r_3, t)|t \in r_3^\bullet \cap \Im(\theta_2)\} = 2$, we know that θ_2 is saturated at M.

The relationship between the deadlock of S^4PR and the PA-circuit saturation is as follows [3].

Theorem 1 [3]: A marked S^4PR (N, M_0) is *live* if and only if $\forall\theta \in \Theta$ and $\forall M \in R(N, M_0)$, θ is not *saturated* at M.

In order to describe and compute all PA-circuits in an easy and understandable way, our previous paper [3] defines a kind of modified S^4PRs and some of their special structural objects.

Definition 7 [3]: For a given S^4PR $N = (P_0 \cup P_R \cup P_A, T, F, W)$, its modified $S^4PR(MS^4PR)$, N^*, is defined by adding an auxiliary arc (t, r) with weight 1 for $t \in T$ and $r \in P_R$ if ${}^{(a)}t \in H(r)$ and $(t, r) \notin F$, shown by dotted arc in PN graph. Let F^* be the set of auxiliary arcs. Then the obtained MS^4PR $N^* = (P_0 \cup P_R \cup P_A, T, F \cup F^*, W)$.

Definition 8 [3]: Let $\varpi = t_1r_1 \ldots t_kr_k$ be a circuit in MS^4PR N^*. ϖ is a resource-transition circuit (RT-circuit) if $t_i \in T$ and $r_i \in P_R$ for $i \in \mathbb{Z}_k$. Let $\Im(\varpi) = \{t_i, i \in \mathbb{Z}_k\}$ and $\Re(\varpi) = \{r_i, i \in \mathbb{Z}_k\}$. An RT-circuit ϖ is perfect if $({}^{(a)}\Im(\varpi))^\bullet = \Im(\varpi)$. Let Π collect all perfect RT-circuits (PRT-circuits) in N^*.

The following theorem establishes the corresponding relationship between PA-circuits in S^4PR N and PRT-circuits in MS^4PR N^* which is w.r.t. N.

Theorem 2 [3]: Let N be an S^4PR. There exists a *one-to-one* correspondence between PA-circuits in N (i.e.,Θ) and PRT-circuits in N^*(*i.e.*, Π). Further, $\forall\theta \in \Theta$, there is unique $\varpi \in \Pi$ such that $\Im(\theta) = \Im(\varpi)$ and $\Re(\theta) = \Re(\varpi)$.

According to Theorem 2, to obtain all PA-circuit in S^4PR N, we can compute all PRT-circuits in its modified net N^* first. Since a PRT-circuit in N^* is relevant only to resource places and transitions, and its transitions belong to $P_R^\bullet \cap {}^\bullet P_R$ it must be contained in $N_R^* \equiv N^*[P_R \cup (P_R^\bullet \cap {}^\bullet P_R)]$, which is a subnet of N^*.

A PRT-circuit in N^* is a union of *elementary circuits* in N_R^*. while all elementary circuits in N_R^* can be computed by algorithm, *Find-All-Elementary-Circuits* proposed by Johnson *et al.* [32], which has the complexity $O((v + e)(c + 1))$ if N^* has v vertices e edges, and c elementary circuits. Then an algorithm for computing Θ is stated as follows.

In Algorithm CP, N_R^* is constructed first. Its vertices include all resource places of P_R and all transitions in $P_R^\bullet \cap {}^\bullet P_R$. Then all elementary circuits of N_R^* are computed by using Find-All-Elementary-Circuits. Note that $|P_R^\bullet \cap {}^\bullet P_R| \leq |T|$ and there are at most two arcs between a resource place and a transition, including auxiliary arcs. Thus the complexity of Step 2 is $O(2|T||P_R| + (|P_R| + |T|))(c + 1) \leq O(3|T||P_R|)(c + 1)$. In Step 3, all PRT-circuits in N^* are computed in an iterative manner. Let C denote the number of all different PRT-circles. The algorithm needs to compare any two PRT-circles at most, so the maximum number of comparisons is C^2, and hence, time complexity of step 3 is $O(C^2)$. Each PRT-circuit in Π is converted into a PA-circuit based on the results of Theorem 2 in Step 4. The time complexity in Step 4 is $O(C)$. Then the time complexity of Algorithm CP is $O(3|T||P_R|)(c + 1) + O(C^2) + O(C) \leq O(3|T||P_R|C^2)$. According to [3], we thus have the following conclusion.

Theorem 3 [3]: Algorithm CP can correctly output all PA-circuits of S^4PR (N, M_0) with the time complexity $O(3|T||P_R|C^2)$.

Algorithm CP (Computing all PA-Circuits in S^4PR)

Input: An S^4PR N;
Output: Θ;

Step 1. Construct N_R^* from N;
Step 2. Set $\Pi = \Theta = \emptyset$; Compute the set of elementary circuits, Ξ, by using Find-All-Elementary-Circuits(N_R^*).
Step 3. Obtain Π from Ξ in a recursive manner
(1)
1) Let $\Pi = \Xi$;
2) For each $\varpi_1 \in \Xi$ and $\varpi_2 \in \Pi$, if $\Re(\varpi_1) \cap \Re(\varpi_2) \neq \emptyset$, let $\varpi \equiv \varpi_1 \cup \varpi_2$. If ϖ is perfect and not in Π, add ϖ into Π.

Step 4. For each $\varpi \in \Pi$, let θ be its corresponding PA-circuit in N. Add θ into Θ. By Theorem 2, we know that $\Im(\theta) = \Im(\varpi)$, $\wp(\theta) = {}^{(a)}\Im(\varpi)$, and $\Re(\theta) = \Re(\varpi)$.
Step 5. Output Θ.

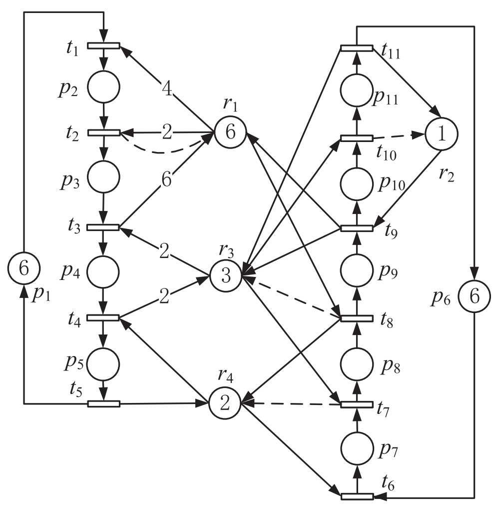

FIGURE 2. The MS^4PR (N^*, M_0) w.r.t. S^4PR (N, M_0) in FIGURE 1.

Example 4: Consider S^4PR (N, M_0) shown in Figure. 1. The corresponding MS^4PR N^* is illustrated in Figure. 2, where (t_2, r_1), (t_{10}, r_2), (t_8, r_3), and (t_7, r_4) are auxiliary arcs. Correspondingly, N_R^* shown in Figure. 3, contains four elementary circuits $\varpi_1 - \varpi_4$ as described in Figure. 4.

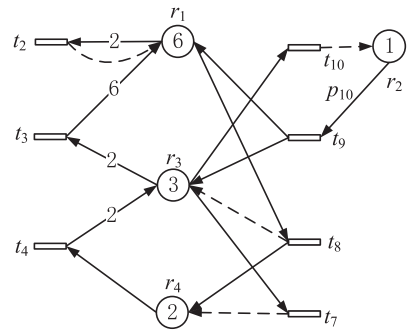

FIGURE 3. $N_R^* = N^*[P_R \cup (P_R^\bullet \cap^\bullet P_R)]$, the subnet of N^* in FIGURE 2.

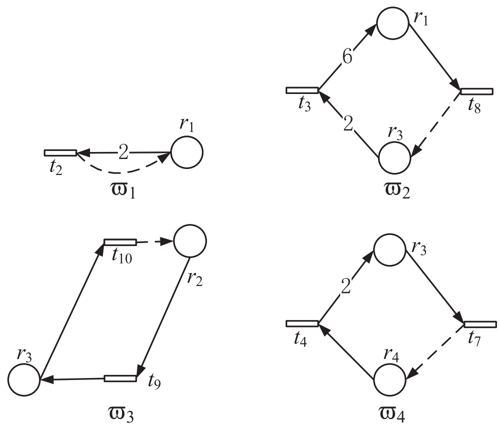

FIGURE 4. Four elementary circuits, $\varpi_1 - \varpi_4$, of N_R^* in FIGURE 3.

According to Algorithm CP, we have $\Pi = \{\varpi_1 - \varpi_{10}\}$,where $\varpi_5 = \varpi_1 \cup \varpi_2, \varpi_6 = \varpi_2 \cup \varpi_3, \varpi_7 = \varpi_3 \cup \varpi_4, \varpi_8 = \varpi_1 \cup \varpi_2 \cup \varpi_3, \varpi_9 = \varpi_2 \cup \varpi_3 \cup \varpi_4, \varpi_{10} = \varpi_1 \cup \varpi_2 \cup \varpi_3 \cup \varpi_4$. According to the one-to-one correspondence relation between Θ and Π in Theorem 2, we can obtain all PA-circuits in N, i.e., $\Theta = \{\theta_i | \theta_i$ corresponds to ϖ_i, $i \in \mathbb{Z}_{10}\}$. Note that θ_i and ϖ_i have the same set of transitions and resources, respectively. Table 1 shows the details of all PRT-circuits and PA-circuits.

III. DESIGNING INITIAL MARKING FOR AN S^4PR TO ENSURE ITS LIVENESS

Given an S^4PR N with its initial idle marking M_{00}, we intend to establish an initial resource marking, M_{0R}, such that the resulting net (N, M_0) is well-marked and live, where $M_0 = [M_{00}, M_{0A}, M_{0R}]$.

Note that all places of P_A must be emptied at the initial marking according to Definition 2, that is, $M_{A0} = 0$. Hence, we just need to consider the initial marking for resource places, M_{0R}. By Theorem 1, the liveness of marked S^4PR is related to the non-saturation of PA-circuits in Θ. In this paper, we use this structural property to design the initial marking, M_{0R}, for the given initial markings M_{00} and M_{0A}, so that all PA-circuits cannot be saturated during the system evolution, and hence, will not lead to deadlock.

First, we study the relation between the non-saturation of PA-circuit θ and the initial marking of resource place

TABLE 1. PA-circuits in S^4PR in Figure. 1.

ϖ/θ	$\Re(\varpi)$ or $\Re(\theta)$	$\Im(\varpi)$ or $\Im(\theta)$
ϖ_1/θ_1	r_1	t_2
ϖ_2/θ_2	$r_1\, r_3$	$t_3\, t_8$
ϖ_3/θ_3	$r_2\, r_3$	$t_9\, t_{10}$
ϖ_4/θ_4	$r_3\, r_4$	$t_4\, t_7$
ϖ_5/θ_5	$r_1\, r_3$	$t_2\, t_3\, t_8$
ϖ_6/θ_6	$r_1\, r_2\, r_3$	$t_3\, t_8\, t_9\, t_{10}$
ϖ_7/θ_7	$r_2\, r_3\, r_4$	$t_4\, t_7\, t_9\, t_{10}$
ϖ_8/θ_8	$r_1\, r_2\, r_3$	$t_2\, t_3\, t_8\, t_9\, t_{10}$
ϖ_9/θ_9	$r_1\, r_2\, r_3\, r_4$	$t_3\, t_4\, t_7\, t_8\, t_9\, t_{10}$
ϖ_{10}/θ_{10}	$r_1\, r_2\, r_3\, r_4$	$t_2\, t_3\, t_4\, t_7\, t_8\, t_9\, t_{10}$

$r \in \Re(\theta)$. Further, an algorithm is proposed to design an initial resource marking, M_{0R}, for S^4PR where only M_{00} and M_{0A} are given.

A. RELATIONS BETWEEN THE NON-SATURATION OF PA-CIRCUITS AND THE INITIAL MARKING OF AN INVOLVED RESOURCE PLACE

For a positive number y, let $\lfloor y \rfloor$ represent the maximal integer less than y, for instance, $\lfloor 2.3 \rfloor = 2$.

Definition 9: For well-marked S^4PR (N, M_0), let $\theta \in \Theta$ and $r \in \Re(\Theta)$. We define

$$\lambda(\theta, r) = \Sigma_{p \in H(r) \cap \wp(\theta)} I_r(p), \quad \text{and}$$

$\omega(\theta, r) = \{k \in \mathbb{Z}_{\lambda(\theta,r)-1} | \min_{x_p \in \{0,1,\ldots,\lfloor k/I_r(p) \rfloor\}} \{k - \Sigma_{p \in H(r) \cap \wp(\theta)} I_r(p) x_p \in \mathbb{Z}\} \geq \min\{W(r, t) | t \in \Im(\theta) \cap r^\bullet\}\}$.

In the above definition, $\lambda(\theta, r)$ represents the sum of $I_r(p)$ for $p \in H(r) \cap \wp(\theta)$; while $\omega(\theta, r)$ collects the positive integer $k < \lambda(\theta, r)$ which satisfies $\min_{x_p}\{k - \Sigma_{p \in H(r) \cap \wp(\theta)} I_r(p) x_p\} \geq \min\{W(r, t) | t \in \Im(\theta) \cap r^\bullet\}$ where $x_p \in \{0, 1, \ldots, \lfloor k/I_r(p) \rfloor\}$ for $p \in H(r) \cap \wp(\theta)$. Since the net is finite, $\mu \equiv \max\{\lambda(\theta, r) | \theta \in \Theta, r \in \Re(\theta)\}$ is a finite constant.

By the following Algorithm EE, all elements of $\omega(\theta, r)$ can be enumerated.

Algorithm EE(Enumerating all Elements in $\omega(\theta, r)$)

Input: $\theta \in \Theta$ and $r \in \Re(\theta)$;
Output: $\omega(\theta, r)$;

1: Compute $\lambda(\theta, r)$; set $\omega(\theta, r) = \emptyset$
2: If($\lambda(\theta, r) = 1$){ output $\omega(\theta, r)$; exits;} .
3: for ($i \in \mathbb{Z}_{\lambda(\theta,r)-1}$){
4: solve the following ILP, obtain G^{ILP}
ILP$_2$: $G^{ILP}(i) = \min\{i - \Sigma_{p \in H(r) \cap \wp(\theta)} x_p I_r(p)\}$
s.t. $x_p \in \{0, 1, \ldots, \lfloor i/I_r(p) \rfloor\}, \forall p \in H(r) \cap \wp(\theta)$;
$i - \Sigma_{p \in H(r) \cap \wp(\theta)} x_p I_r(p) \geq 0$;
5: if ($G^{ILP} \geq \min\{W(r, t) | t \in \Im(\theta) \cap r^\bullet\}$);
6: $\omega(\theta, r) = \omega(\theta, r) \cup \{i\}$;
7: }
8:Output $\omega(\theta, r)$.

Note that in ILP$_2$ of Algorithm EE, for the given θ and r, $\lambda(\theta, r) \leq \mu$, and x_p and p have $\lfloor i/I_r(p)\rfloor(\leq i)$ and $|\wp(\theta) \cap H(r)|(\leq |P_A|)$ different values, respectively. Solving ILP$_2$ for a given i is equivalent to calculate the minimum of up to $i * |P_A|$ different values. Hence, the complexity of ILP2 for i will not exceed $O(i * |P_A|)$, and the computational complexity of Algorithm EE is $O(\mu^2 * |P_A|)$.

Example 5: Reconsider S^{4}PR in Figure. 1 as well as PA-circuit $\theta_4 \in \Theta$. From Table 1 we know that $\Re(\theta_4) = \{r_3, r_4\}$, $\Im(\theta_4) = \{t_4, t_7\}$, and $\wp(\theta_4) = \{p_4, p_7\}$. Then for $r_3 \in \Re(\theta_4)$, we have $\wp(\theta_4) \cap H(r_3) = \{p_4\}$, $\lambda(\theta_4, r_3) = \Sigma_{p\in H(r_3)\cap\wp(\theta_4)} I_{r_3}(p) = I_{r_3}(p_4) = 2$, and $\mathbb{Z}_{\lambda(\theta_4,r_3)-1} = \{1\}$. Then the ILP$_2$ is as follows.

$$G^{ILP}(1) = \min\ \{1 - x_{p_4} I_r(p_4)\}$$
$$s.t.\ x_{p_4} \in \{0\};$$
$$1 - x_{p_4} I_r(p_4) \geq 0$$

The above ILP is simple and has the unique solution $x_{p_4} = 0$, and $G^{ILP}(1) = 1$. Thus, $\omega(\theta_4, r_3) = \{1\}$.

Lemma 1: Let (N, M_0) be a well-marked S^{4}PR, $\theta \in \Theta$, and $r \in \Re(\theta)$. If $M_0(r) < \lambda(\theta, r)$, then at any $M \in R(N, M_0)$, θ cannot be saturated.

Proof: We assume that PA-circuit θ is saturated at M. Then $M(p) > 0$ for $p \in \wp(\theta) \cap H(r)$. Hence, $M_{I_r}(\wp(\theta) \cap H(r)) \geq \Sigma_{p\in\wp(\theta)\cap H(r)} I_r(p) = \lambda(\theta, r)$. On the other hand, as all places in $\wp(\theta) \cap H(r)$ use resource r, we have $M_{I_r}(\wp(\theta) \cap H(r)) \leq M_0(r)$, and hence $M_0(r) \geq \lambda(\theta, r)$. This contradicts with the condition in this lemma. Therefore, θ cannot be saturated at M. ♣

Lemma 2: Let (N, M_0) be a well-marked S^{4}PR, $\theta \in \Theta$, $r \in \Re(\theta)$, and $\omega(\theta, r) \neq \emptyset$. If $M_0(r) = \lambda(\theta, r) + k$, where $k \in \omega(\theta, r)$, then θ cannot be saturated at any $M \in R(N, M_0)$.

Proof: Assume that θ is saturated at M. Then we have $M_0(r) - M_{I_r}(H(r) \cap \wp(\theta)) < \min\{W(r, t)|t \in r^\bullet \cap \Im(\theta)\}$ according to Definition 6. That is,

$$M_0(r) < \min\{W(r, t)|t \in \Im(\theta) \cap r^\bullet\} + M_{I_r}(H(r) \cap \wp(\theta)) \quad (1)$$

Note that $M_0(r) = \lambda(\theta, r) + k$, where $k \in \omega(\theta, r)$. By combing this equation and (1), we have

$$\lambda(\theta, r) + k < \min\{W(r, t)|t \in \Im(\theta) \cap r^\bullet\} + M_{I_r}(H(r) \cap \wp(\theta)) \quad (2)$$

However, recall that $\lambda(\theta, r) = \Sigma_{p\in H(r)\cap\wp(\theta)} I_r(p)$, and according to Definition 6, for place $p \in \wp(\theta) \cap H(r)$, we have $M(p) \geq 1$. Hence, we have $M_{I_r}(H(r) \cap \wp(\theta)) = \Sigma_{p\in H(r)\cap\wp(\theta)} I_r(p)(M(p) - 1) + \Sigma_{p\in\wp(\theta)\cap H(r)} I_r(p) = \Sigma_{p\in H(r)\cap\wp(\theta)} I_r(p)(M(p)-1) + \lambda(\theta, r)$. By combing this equation into (2), we have

$$k - \Sigma_{p\in\wp(\theta)\cap H(r)} I_r(p)(M(p) - 1) < \min\{W(r, t)|t \in \Im(\theta) \cap r^\bullet\} \quad (3)$$

For each $p \in \wp(\theta) \cap H(r)$, we know that $M(p) \geq 1$, that means, at least $\Sigma_{p\in\wp(\theta)\cap H(r)} I_r(p) = \lambda(\theta, r)$ units of resource r have been occupied in $\wp(\theta) \cap H(r)$. Thus, $M(p) \leq 1 + \lfloor(M_0(r) - \lambda(\theta, r))/I_r(p)\rfloor = 1 + \lfloor k/I_r(p)\rfloor$, and further we obtain $M(p) - 1 \leq \lfloor k/I_r(p)\rfloor$. That is, $M(p) - 1 \in \mathbb{Z}_{\lfloor k/I_r(p)\rfloor}$. However, since $k \in \omega(\theta, r)$, (3) is impossible by Definition 9. Thus, θ cannot be saturated at M. ♣

Next, we denote the set of PA-circuits associated with $r \in P_R$ by $\Theta(r) = \{\theta \in \Theta | r \in \Re(\theta)\}$ and further define $\Delta(r) = \cup_{\theta\in\Theta(r)}(\wp(\theta) \cap H(r))$. For example, consider S^{4}PR in Figure. 1, from Table 1 we know that $\Theta(r_1) = \{\theta_1, \theta_2, \theta_5, \theta_6, \theta_8, \theta_9, \theta_{10}\}$, and $\Delta(r_1) = \cup_{\theta\in\Theta(r_1)}(\wp(\theta) \cap H(r_1)) = \{p_2, p_3, p_9\}$.

Definition 10: Let (N, M_0) be a well-marked S^{4}PR, $r \in P_R$, and $N_i = (P_{Ai} \cup \{p_{i0}\}, T_i, F_i)$, $i \in \mathbb{Z}_m$, denote the i-th state machine in N. We define

$$B(N, r) = M_0(r) - \Sigma_{i\in\mathbb{Z}_m} \max\{I_r(p)|p \in \Delta(r) \cap p_{Ai}\} M_0(p_{i0})$$

Example 6: Consider S^{4}PR (N, M_0) in Figure. 1. For $r_1 \in P_R$, we have $\Delta(r_1) = \{p_2, p_3, p_9\}$. Note that there are two state machines in N where p_1 and p_6, represent the corresponding idle places, respectively. We have $M_0(p_1) = M_0(p_6) = 6$, $P_{A0} = \{p_2 - p_5\}$, and $P_{A1} = \{p_7 - p_{11}\}$. we have $B[N, r_1] = 6 - \Sigma_{i\in\mathbb{Z}_m} \max\{I_r(p)|p \in \Delta(r_1) \cap p_{Ai}\} M_0(p_{i0}) = 6 - \max\{I_{r_1}(p)|p \in \{p_2, p_3\}\} M_0(p_1) - \{I_{r_1}(p)|p \in \{p_9\}) M_0(p_6) = 6 - 6 \times 6 - 6 \times 1 = -36$.

Lemma 3: Given a well-marked S^{4}PR (N, M_0) and $r \in P_R$. If $B[N, r] \geq \min\{W(r, t)|t \in \Im(\theta) \cap r^\bullet, \theta \in \Theta(r)\}$, then each PA-circuits in $\Theta(r)$ is not saturated at any marking in $R(N, M_0)$.

Proof: Assume that PA-circuit $\theta \in \Theta(r)$ is saturated at $M \in R(N, M_0)$. Since sets P_{Ai}, $i \in \mathbb{Z}_m$, are disjoint and $P_A = \cup_{i\in\mathbb{Z}_m} P_{Ai}$, we can get

$$M_{I_r}(H(r) \cap \wp(\theta)) = \Sigma_{i\in\mathbb{Z}_m} M_{I_r}(H(r) \cap \wp(\theta) \cap P_{Ai}) \quad (4)$$

For $p \in \wp(\theta) \cap H(r) \cap P_{Ai}$, we have $I_r(p) \leq \max\{I_r(p)|p \in \Delta(r) \cap p_{Ai}\}$ since $\wp(\theta) \cap H(r) \cap P_{Ai} \subseteq \Delta(r) \cap p_{Ai}$. Thus, we further obtain

$$M_{I_r}(H(r) \cap \wp(\theta) \cap P_{Ai}) \leq \Sigma_{p\in\wp(\theta)\cap H(r)\cap P_{Ai}} M(p) \max\{I_r(p)|p \in \Delta(r) \cap p_{Ai}\} \quad (5)$$

Also, we have $\wp(\theta) \cap H(r) \cap P_{Ai} \subseteq P_{Ai}$. Hence (5) can be rewritten as

$$M_{I_r}(H(r) \cap \wp(\theta) \cap P_{Ai}) \leq \max\{I_r(p)|p \in \Delta(r) \cap p_{Ai}\} \Sigma_{p\in P_{Ai}} M(p) \quad (6)$$

On the other hand, since N_i is strongly connected and all places in $P_{Ai} \cup \{p_{i0}\}$ constitute a P-semiflow support, we have $\Sigma_{p\in P_{Ai}} M(p) + M(p_{i0}) = M_0(p_{i0})$. That means, $\Sigma_{p\in P_{Ai}} M(p) \leq M_0(p_{i0})$. By combing this and (6), we know that

$$M_{I_r}(H(r) \cap \wp(\theta) \cap P_{Ai}) \leq \max\{I_r(p)|p \in \Delta(r) \cap p_{Ai}\} M_0(p_{i0}) \quad (7)$$

After combing (7) and (4), we have

$$M_{I_r}(H(r) \cap \wp(\theta)) \leq \Sigma_{i\in\mathbb{Z}_m} \max\{I_r(p)|p \in \Delta(r) \cap p_{Ai}\} M_0(p_{i0}) \quad (8)$$

It is assumed that θ is saturated at M. Hence, $M_0(r) - M_{I_r}(H(r) \cap \wp(\theta)) < \min\{W(r,t)|t \in \Im(\theta) \cap r^\bullet\}$. Note that $\min\{W(r,t)|t \in \Im(\theta) \cap r^\bullet\} \le \min\{W(r,t)|t \in r^\bullet \cap \Im(\theta_1), \theta_1 \in \Theta(r)\}$. Thus, we have

$$M_0(r) - M_{I_r}(H(r) \cap \wp(\theta)) < \min\{W(r,t)|t \in \Im(\theta_1) \cap r^\bullet, \theta_1 \in \Theta(r)\} \quad (9)$$

In addition, from (8) we know that

$$B[N,r] = M_0(r) - \Sigma_{i \in \mathbb{Z}_m} \max\{I_r(p)|p \in \Delta(r) \cap p_{Ai}\} M_0(p_{i0}) \le M_0(r) - M_{I_r}(H(r) \cap \wp(\theta)) \quad (10)$$

By combing (10) and (9), we have

$$B[N,r] < \min\{W(r,t)|t \in \Im(\theta_1) \cap r^\bullet, \theta_1 \in \Theta(r)\} \quad (11)$$

This is impossible and hence Lemma 3 is proved. ♣

By combining Lemmas 1–3 and Theorem 1, we have a new conclusion.

Theorem 4: Given a well-marked S^{4}PR (N, M_0). Then (N, M_0) is *live* if $\theta \in \Theta$, $\exists r \in \Re(\theta)$, such that one of the following is met

(1) $\lambda(\theta, r) > M_0(r)$;
(2) $M_0(r) = \lambda(\theta, r) + k$, where $k \in \omega(\theta, r)$; and
(3) $B[N,r] \ge \min\{W(r,t)|t \in \Im(\theta_1) \cap r^\bullet, \theta_1 \in \Theta(r)\}$

B. COMPUTING INITIAL MARKING FOR S^{4}PR

Recall that $\Theta(r) = \{\theta \in \Theta | r \in \Re(\theta)\}$ for resource r. Then for a given subset $\Psi \subseteq \Theta(r)$, the following algorithm is developed to compute an initial marking for r such that all PA-circuits in Ψ will never be saturated.

Algorithm CAR (Computing an Initial Marking for A Resource Place)

Input: $r \in P_R$ and $\Psi \subseteq \Theta(r)$;
Output: $M_{0R}(r)$, an initial marking of r;

```
1: By using Algorithm EE; compute ω(θ, r), θ ∈ Ψ;
2: K := max{λ(θ, r) + n|θ ∈ Ψ, n ∈ ω(θ, r) ≠ ∅};
3: M_0R(r) = 0
4: for (i = 1,...,K){
5:   Flag = 0;
6:   if (i ≥max{I_r(p)|p ∈ H(r)}){
7:    for (θ ∈ Ψ){
8:     if(λ(θ, r))≤i and ∄k∈ω(θ, r) s.t. i = k + λ(θ, r)){
9:      Flag = 1;
10:      break;}
11:    }
12:    if(Flag = 0){
13:     M_0R(r) = i;
14:     break;}
15: }}
16: output M_0R(r);
```

Essentially, Algorithm CAR tries to compute an initial marking for resource r based on the conclusions of Lemmas 1 and 2. In particular, Lemma 2 implies that a PA-circuit $\theta \in \Psi$ will never be saturated if $M_{0R}(r) = \lambda(\theta, r) + n$, $n \in \omega(\theta, r)$. Thus, in Algorithm CAR we only check that weather an integer in $\mathbb{Z}_K$ could be the sought initial marking of r. At the beginning, we compute $\omega(\theta, r)$, $\theta \in \Psi$ and set $M_{0R}(r) = 0$. Due to the requirement of M_{0R} to be an acceptable initial marking, we are only considering integer $i \in \mathbb{Z}_K$ and $i \ge \max\{I_r(p)|p \in H(r)\}$. There are two steps in the loop process for i:

Step 1 (Line 6–11): If $\exists \theta \in \Psi$ s.t. $\lambda(\theta, r) \le i$ and $\nexists k \in \omega(\theta, r)$ s.t. $i = \lambda(\theta, r) + k$, then apparently, the initial marking of r cannot be i because the conditions in Lemmas 1 and 2 are not met. That is, θ may be saturated if $M_{0R}(r)$ is set to be i. Thus, i is not the sought initial marking of r. We need to continue looping $i + 1$.

Step 2 (Line 12–16): If the value of Flag is not changed and still 0, then for each PA-circuit $\theta \in \Psi$, there are two sub-cases: i) $\lambda(\theta, r) > i$, or ii) $i = \lambda(\theta, r) + k$ where $k \in \omega(\theta, r)$. Apparently, the sub-case i) [resp. case ii)] equates to the condition in Lemma 1 [resp. Lemma 2]. Hence, we set $M_{0R}(r) = i$ and then θ will never be saturated according to the conclusion in Lemmas 1 and 2.

Note that to compute $\omega(\theta, r)$ for $\theta \in \Psi$, Algorithm EE is called $|\Psi|$ times, and the computational complexity is $O(\mu^2|P_A|\Psi|)$; while in Algorithm CAR, the loop for $i \in \mathbb{Z}_K$ and $\theta \in \Psi$ is repeated up to $K|\Psi|$ times, and $K < 2\lambda(\theta, r) \le 2\mu$ since the maximum element in $\omega(\theta, r)$ is $\lambda(\theta, r) - 1 (\le \mu - 1)$. So the complexity of Algorithm CAR is $O(\mu^2|P_A|\Psi|) + O(2\mu|\Psi|) = O(\mu^2|P_A|\Psi|)$.

From the above analysis, we can draw the following result.

Lemma 4: Let N be an S^{4}PR, $r \in P_R$, and $\Psi \subseteq \Theta(r)$. If $M_{0R}(r) \ge 1$ output by using Algorithm CAR for inputs r and Ψ, then all PA-circuits in Ψ will never be saturated.

Given an S^{4}PR N with initial idle and activity markings M_{00} and M_{0A}, Algorithm CIR is proposed to compute M_{0R} so that all PA-circuits in Θ will never be saturated.

Algorithm CIR (Computing an Initial Resource marking)

Input: S^{4}PR $N = (P_0 \cup P_R \cup P_A, T, F, W)$ with $M_{00}, M_{0A} = 0$;
Output: M_{0R}, an initial resource marking of N;

```
By using Algorithm CP, compute Θ;
set Ω = Θ;
for (r ∈ P_R){
  if (Θ(r) ∩ Ω = ∅)
   let M_0R(r) =max{I_r(p)|p ∈ H(r)}
  else{
   M_0R(r) = Algorithm CAR(r, Θ(r) ∩ Ω);
   if (M_0R(r) = 0);
    M_0R(r)=Σ_{i∈Z_m} max{I_r(p)|Δ(r)∩p_Ai}M_0(p_i0)
              +min{W(r, t)|t∈ℑ(θ)∩r•, θ∈Θ(r)};
    let Ω = Ω \ Θ(r);
   }
}
output M_0R;
```

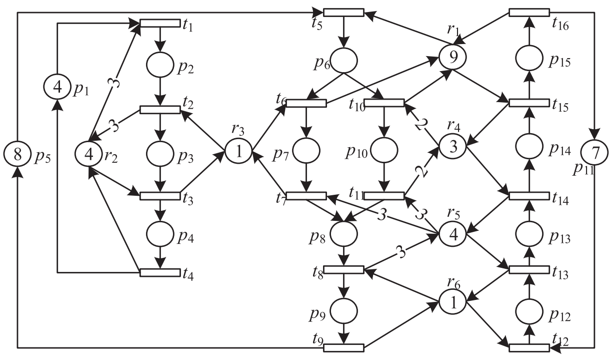

FIGURE 5. An example of S^4PRs.

In Algorithm CIR, let Ω represent the set of PA-circuits that might be saturated. Initially, we set $\Omega = \Theta$. For $r \in P_R$, if $\Theta(r) \cap \Omega = \emptyset$, then the initial marking of r will not affect the saturation of any PA-circuit in Ω. Hence we directly set $M_{0R}(r) = \max\{I_r(p)|p \in H(r)\}$ so as to save resources. On the contrary, if $\Theta(r) \neq \emptyset$, we let $M_{0R}(r)$ be the output of Algorithm CAR with r and $\Theta(r) \cap \Omega$ being its input, i.e., $M_{0R}(r)$ = Algorithm CAR(r, $\Theta(r) \cap \Omega$). If $M_{0R}(r) > 0$, we know that all PA-circuits in $\Theta(r)$ will never be saturated according to Lemma 4; otherwise, we set $M_{0R}(r) = \min\{W(r,t)|t \in \Im(\theta) \cap r^\bullet, \theta \in \Theta(r)\} + \Sigma_{i\in\mathbb{Z}_m}\max\{I_r(p)|\Delta(r) \cap p_{Ai}\}M_0(p_{i0})$, and the corresponding $B[N,r] = \min\{W(r,t)|t \in r^\bullet \cap \Im(\theta), \theta \in \Theta(r)\}$ by Definition 10, and further all PA-circuits in $\Theta(r)$ will never be saturated according to Lemma 3. Therefore, M_{0R}, the output of Algorithm CIR can ensures that all PA-circuits in Ω will never be saturated.

To compute Θ, Algorithm CP is called. For each r in P_R, the algorithm loops once, in which Algorithm CAR is called. So the complexity of the loop process is $O(\mu^2|P_A||\Theta|) + O(2\mu|\Theta|) = O(\mu^2|P_A|C) + O(2\mu C) \leq O(\mu^2|P_A|C)$ where C is the number of all different PRT-circles in Θ. Then the complexity of Algorithm CIR is $O(3|T||P_R|C^2) + O(\mu^2|P_A|C)$.

Thus, we obtain the following conclusion.

Theorem 5: Let N be an S^4PR with M_{00} and $M_{0A} = 0$. M_{0R} is an initial resource marking computed by Algorithm CIR. Then (N, M_0) is well-marked and live, where $M_0 = [M_{00}, M_{0A}, M_{0R}]$.

Example 7: Consider S^4PR N in Figure. 1 with $M_{00} = 6p_1 + 6p_6$ and $M_{0A} = 0$. Firstly, by using Algorithm CP, we obtain $\Theta = \{\theta_1 - \theta_{10}\}$ as shown in Table 1. Then set $\Omega = \Theta$. We will execute the loop process for each resource place in $P_R = \{r_1 - r_4\}$.

For r_2, we have $\Theta(r_2) \cap \Omega = \{\theta_3, \theta_6 - \theta_{10}\}$. Note that $\lambda(\theta_i, r_2) = 1$, $\omega(\theta_i, r_2) = \emptyset$, $\forall i \in \{3, 6, \ldots, 9\}$. By Algorithm CIR, we can set $M_{0R}(r_2) = \min\{W(r,t)|t \in \Im(\theta) \cap r^\bullet, \theta \in \Theta(r)\} + \Sigma_{i\in\mathbb{Z}_m}\max\{I_r(p)|\Delta(r) \cap p_{Ai}\}M_0(p_{i0}) = 1 + 6 = 7$. Then, set $\Omega := \Omega \setminus \Theta(r_2) = \{\theta_1, \theta_2, \theta_4, \theta_5\}$.

For r_1, we have $\Theta(r_1) \cap \Omega = \{\theta_1, \theta_2, \theta_5\}$. Then $\lambda(\theta_1, r_1) = 4$, $\omega(\theta_1, r_1) = \{2, 3\}$; $\lambda(\theta_2, r_1) = 6$, $\omega(\theta_2, r_1) = \{1, 2, 3, 4, 5\}$; $\lambda(\theta_5, r_1) = 10$, $\omega(\theta_5, r_1) = \{1, 3, 5, 7, 9\}$. According to Algorithm CAR, we have $M_{0R}(r_1) = 7$. In fact, since $7 - \lambda(\theta_1, r_1) = 3 \in \omega(\theta_1, r_1)$, $7 - \lambda(\theta_2, r_1) = 1 \in \omega(\theta_2, r_1)$, θ_1 and θ_2 cannot be saturated by Lemma 2. Meanwhile, since $M_{0R}(r_1) = 7 < \lambda(\theta_5, r_1) = 10$, θ_5 cannot be saturated either by Lemma 1. Set $\Omega := \Omega \setminus \Theta(r_1) = \{\theta_4\}$.

For r_3, we have $\Theta(r_3) \cap \Omega = \{\theta_4\}$, $\lambda(\theta_4, r_3) = 2$ and $\omega(\theta_3, r_2) = \{1\}$. By using Algorithm CAR, we set $M_{0R}(r_2) = \lambda(\theta_4, r_3) + 1 = 3$. Then, Set $\Omega := \Omega \setminus \Theta(r_3) = \emptyset$.

For r_4, we have $\Theta(r_4) \cap \Omega = \emptyset$, then directly let $M_{0R}(r_4) = \max\{I_{r_4}(p)|p \in H(r_4)\} = 1$.

Therefore, Algorithm CIR outputs $M_{0R} = 7r_1 + 7r_2 + 3r_3 + r_4$. By combing M_{0R}, we establish an initial marking $M_0 = [M_{00}, M_{0A}, M_{0R}]$ for S^4PR N in Figure. 1, where $M_{00} = 6p_1 + 6p_6$, $M_{A0} = 0$. We can check that (N, M_0) is well-marked S^4PR and live.

Example 8: Let us consider S^4PR N shown in Figure. 5 whose initial marking for idle places and operation places are $M_{00} = 4p_1 + 8p_5 + 7p_{11}$ and $M_{0A} = 0$, respectively. By using CIR, we will design an initial marking for resource places to enforce the liveness of this S^4PR. First, according to Algorithm CP, we obtain $\Theta = \{\theta_1 - \theta_{12}\}$ shown in Table 2. Set $\Omega = \Theta$.

TABLE 2. All PA-circuits of S^4PR in Figure. 5.

θ	$\Re(\theta)$	$\Im(\theta)$
θ_1	$r_2\,r_3$	$t_2\,t_3$
θ_2	$r_4\,r_5$	$t_{11}\,t_{14}$
θ_3	$r_5\,r_6$	$t_8\,t_{13}$
θ_4	$r_1\,r_3\,r_4\,r_5$	$t_6\,t_7\,t_{10}\,t_{14}\,t_{15}$
θ_5	$r_1\,r_3\,r_4\,r_5$	$t_6\,t_7\,t_{10}\,t_{11}\,t_{14}\,t_{15}$
θ_6	$r_1\,r_2\,r_3\,r_4\,r_5$	$t_2\,t_3\,t_6\,t_7\,t_{10}\,t_{14}\,t_{15}$
θ_7	$r_1\,r_2\,r_3\,r_4\,r_5$	$t_2\,t_3\,t_6\,t_7\,t_{10}\,t_{11}\,t_{14}\,t_{15}$
θ_8	$r_4\,r_5\,r_6$	$t_8\,t_{11}\,t_{13}\,t_{14}$
θ_9	$r_1\,r_3\,r_4\,r_5\,r_6$	$t_6\,t_7\,t_8\,t_{10}\,t_{13}\,t_{14}\,t_{15}$
θ_{10}	$r_1\,r_3\,r_4\,r_5\,r_6$	$t_6\,t_7\,t_8\,t_{10}\,t_{11}\,t_{13}\,t_{14}\,t_{15}$
θ_{11}	$r_1\,r_2\,r_3\,r_4\,r_5\,r_6$	$t_2\,t_3\,t_6\,t_7\,t_8\,t_{10}\,t_{13}\,t_{14}\,t_{15}$
θ_{12}	$r_1\,r_2\,r_3\,r_4\,r_5\,r_6$	$t_2\,t_3\,t_6\,t_7\,t_8\,t_{10}\,t_{11}\,t_{13}\,t_{14}\,t_{15}$

For r_1, we have $\Theta(r_1) \cap \Omega = \{\theta_4 - \theta_7, \theta_9 - \theta_{12}\}$. Then $\lambda(\theta_i, r_1) = 1$, $\omega(\theta_i, r_1) = \emptyset$, $\forall i \in \{4, 5, 6, 7, 9, 10, 11, 12\}$. By Algorithm CIR, we can set $M_{0R}(r_1) = \min\{W(r,t)|t \in \Im(\theta) \cap r^\bullet, \theta \in \Theta(r)\} + \Sigma_{i\in\mathbb{Z}_m}\max\{I_r(p)|\Delta(r) \cap p_{Ai}\}M_0(p_{i0}) = 1 + 8 \times 1 = 9$. Set $\Omega := \Omega \setminus \Theta(r_1) = \{\theta_1, \theta_2, \theta_3, \theta_8\}$.

For r_4, we have $\Theta(r_4) \cap \Omega = \{\theta_2, \theta_8\}$. Then $\lambda(\theta_i, r_4) = 2$, $\omega(\theta_i, r_4) = \{1\}$, $\forall i \in \{2, 8\}$. According to Algorithm CAR, we have $M_{0R}(r_4) = 3$. Set $\Omega := \Omega \setminus \Theta(r_4) = \{\theta_1, \theta_2\}$.

For r_5, we have $\Theta(r_5) \cap \Omega = \{\theta_2\}$, and $\lambda(\theta_2, r_5) = 3$, and $\omega(\theta_2, r_5) = \{1, 2\}$. By using Algorithm CAR, we set $M_{0R}(r_5) = \lambda(\theta_2, r_5) + 1 = 4$. Set $\Omega := \Omega \setminus \Theta(r_5) = \{\theta_1\}$.

For r_2, we have $\Theta(r_2) \cap \Omega = \{\theta_1\}$, $\lambda(\theta_1, r_2) = 3$, and $\omega(\theta_1, r_2) = \{1, 2\}$. By using Algorithm CAR, we set $M_{0R}(r_2) = \lambda(\theta_1, r_2) + 1 = 4$. Set $\Omega := \Omega \setminus \Theta(r_2) = \emptyset$.

For r_3 and r_6, we have $\Theta(r_3) \cap \Omega = \emptyset$, $\Theta(r_6) \cap \Omega = \emptyset$, then directly let $M_{0R}(r_3) = \max\{I_{r_3}(p)|p \in H(r_3)\} = 1$ and $M_{0R}(r_6) = \max\{I_{r_6}(p)|p \in H(r_6)\} = 1$.

Therefore, $M_{0R} = 9r_1 + 4r_2 + r_3 + 3r_4 + 4r_5 + r_6$ is the output of Algorithm CIR. Further, we can establish an initial marking $M_0 = [M_{00}, M_{0A}, M_{0R}]$ for S^{4}PR N in Figure. 5, where $M_{00} = 4p_1 + 8p_5 + 7p_{11}$, $M_{A0} = 0$. It is checked that(N, M_0) is a well-marked S^{4}PR and live.

IV. CONCLUSION

This paper addresses the liveness enforcement problem for a class of PN, S^{4}PRs, which are able to model complex RASs with the most general resource acquisition and flexible routings. Our previous paper [3] points out that deadlocks in S^{4}PR are caused by *saturated* PA-circuit. Based on that structural property, first we investigate the relation between the initial resource marking and the non-saturation of involved PA-circuits. In other words, we try to establish some condition for the initial marking of a resource place; if it is hold some PA-circuits containing this resource will never be saturated. Second, for a given S^{4}PR, we develop its liveness condition associated with the initial markings of all resource places. Finally, for an S^{4}PR where only the initial idle and activity markings are given, an algorithm is proposed to compute an initial resource marking so that S^{4}PR with the obtained initial marking is live.

So far, only the works [7], [22], [28] propose liveness-enforcing approaches for PN models by configuring the initial resource marking. But their PN models are S^{3}PR and WS^{3}PR, which are proper subclass of S^{4}PR. Hence, their method cannot be applied to S^{4}PR but, on the contrary, ours can enforce the liveness of S^{3}PR and WS^{3}PR.

In future, we intend to study the problem of liveness-enforcing for S^{4}PR by designing initial resource marking with multiple objectives such as save resources and improve permissiveness.

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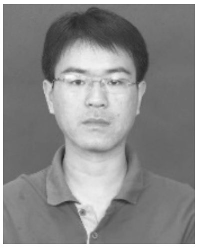

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